Softening the Gap between Wöhler and Paris

– New Approaches for Fatigue Analysis –

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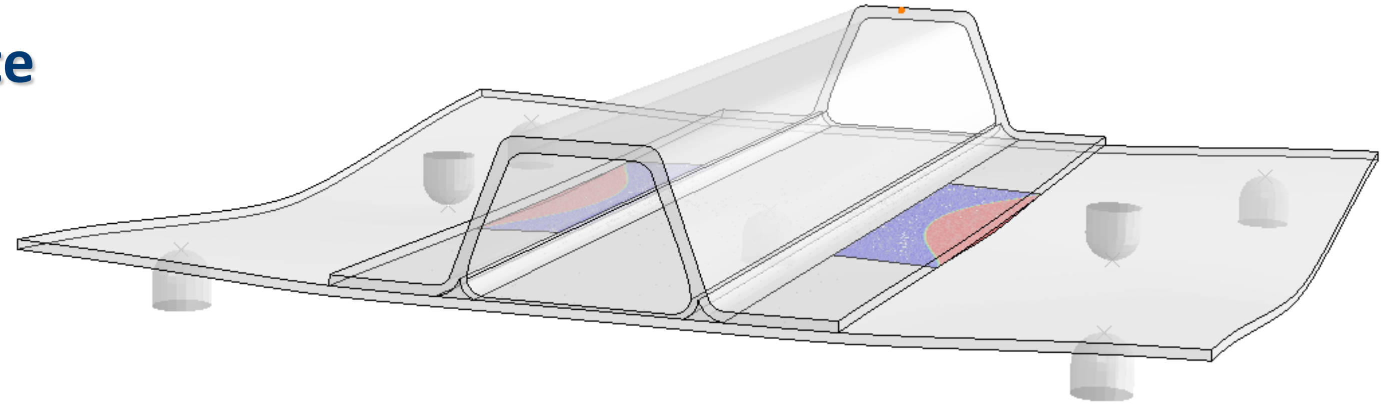
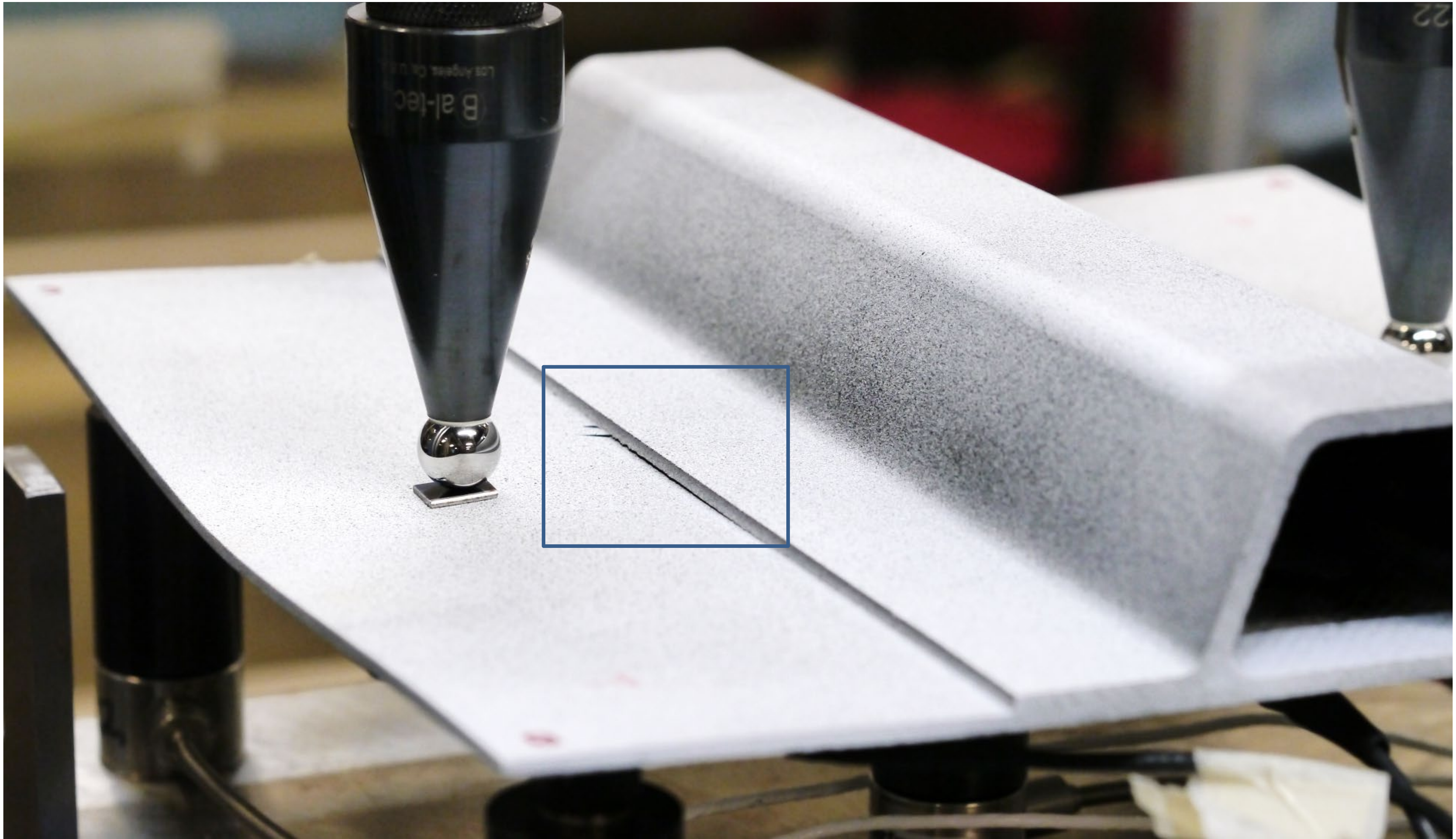
(NASA – retired)

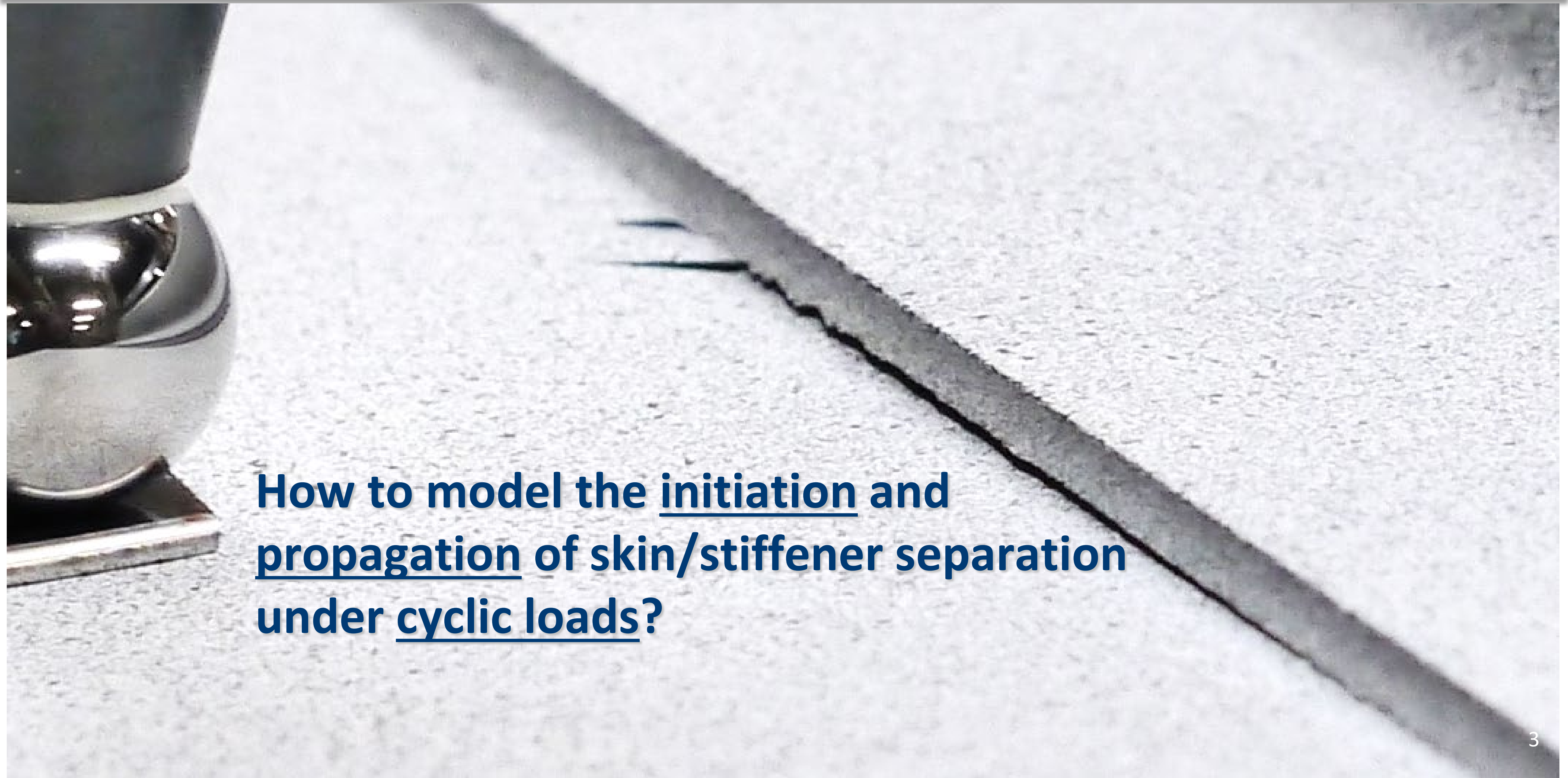


5th International Conference on Mechanics of Advanced Materials and Structures
1-3 July 2026, Institut Clément Ader, Toulouse, France

Motivation: Skin/Stiffener Separation



Motivation: Skin/Stiffener Separation



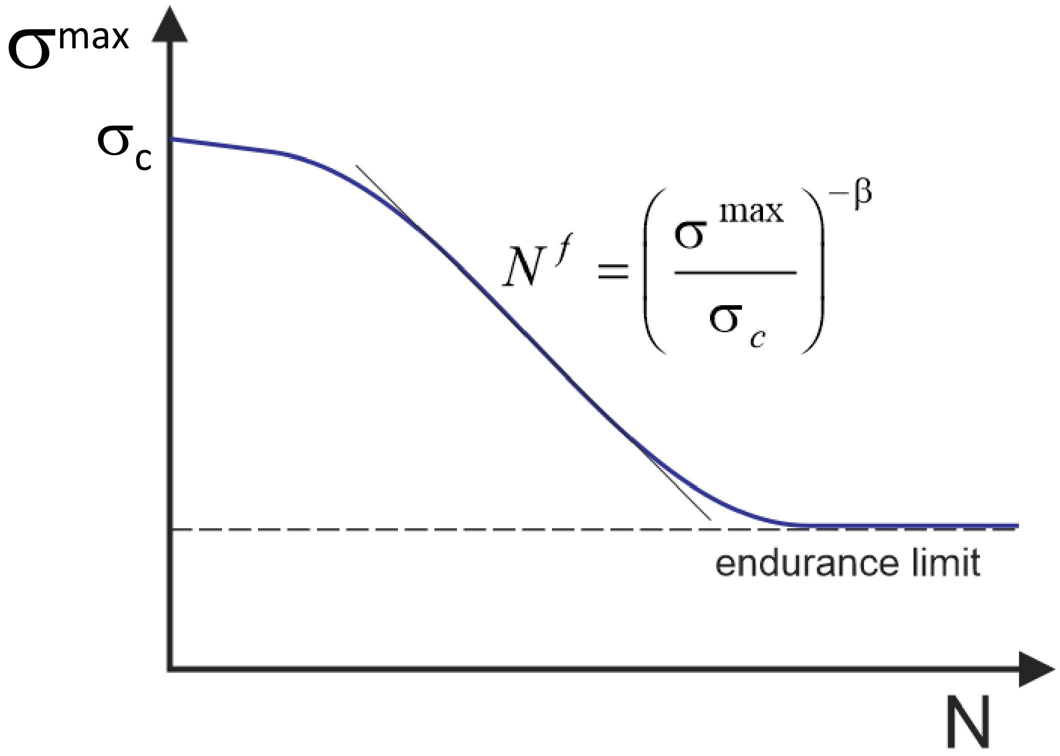
How to model the initiation and propagation of skin/stiffener separation under cyclic loads?

Fatigue Analysis

What fatigue analysis tools do you use?

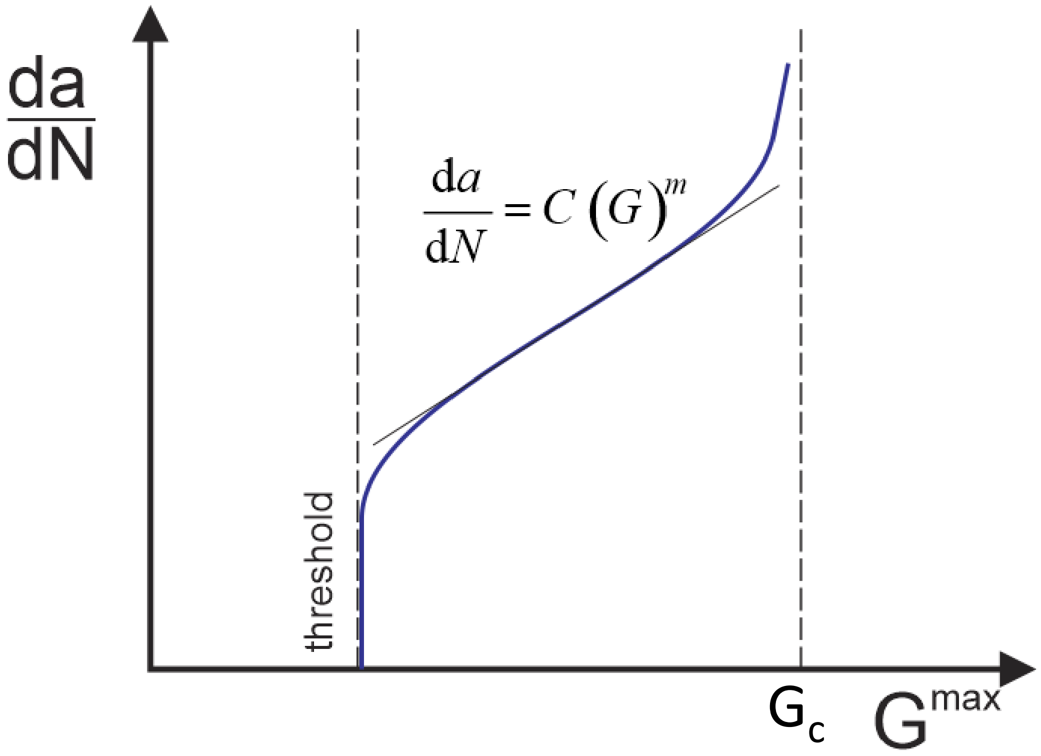
Fatigue: Strength or Fracture Mechanics?

Strength: Wöhler (S-N Curve)



- Safe-life design

Fracture: Paris Law

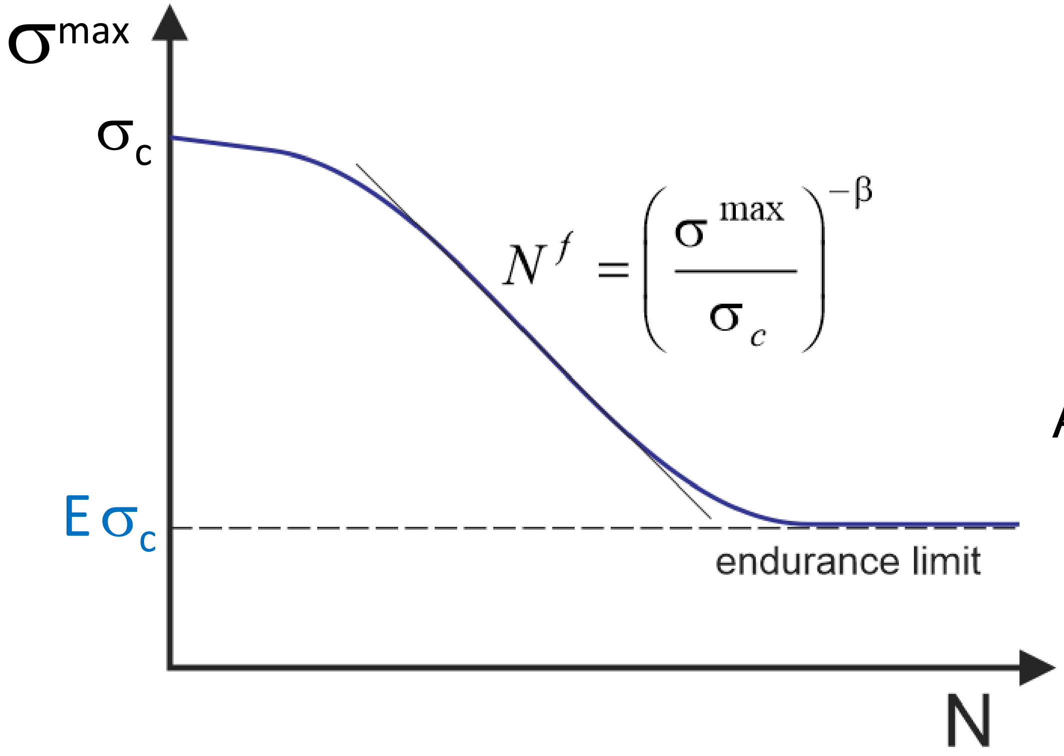


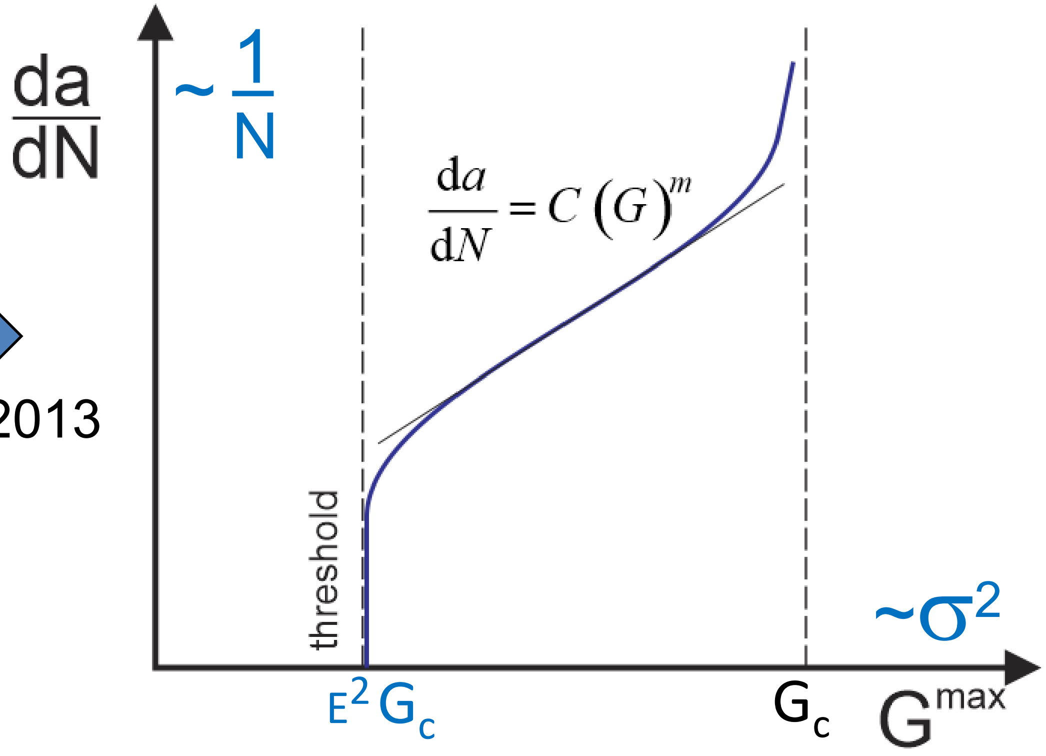
- Damage tolerant design:
  - Slow-growth
  - Fail-safe
  - No-growth

Fatigue: Strength or Fracture Mechanics?

Strength: Wöhler (S-N Curve)

Fracture: Paris Law





Allegri, 2013

S-N Curve and Paris law are related: $\beta = 2m$

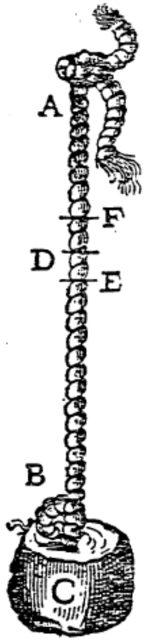
The Issue of Strength Scaling
(when analysis tools were linear)

Scaling of Structural Strength

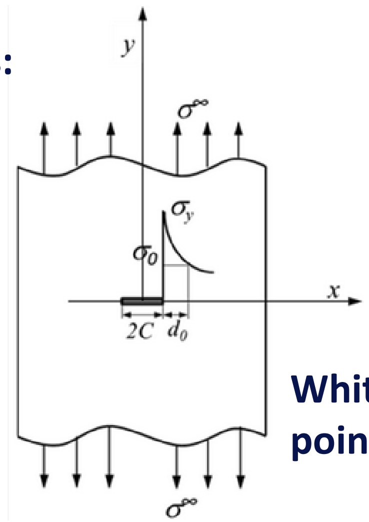
Strength of Materials

"resistance increases with cross-section"

da Vinci, 1505
Galileo, 1638



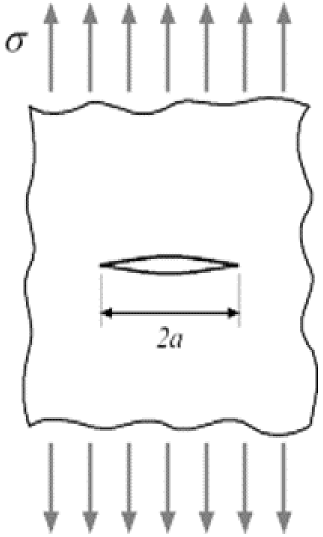
Notches:



Whitney-Nuismer point stress criterion

Fracture Mechanics (Griffith, 1921)

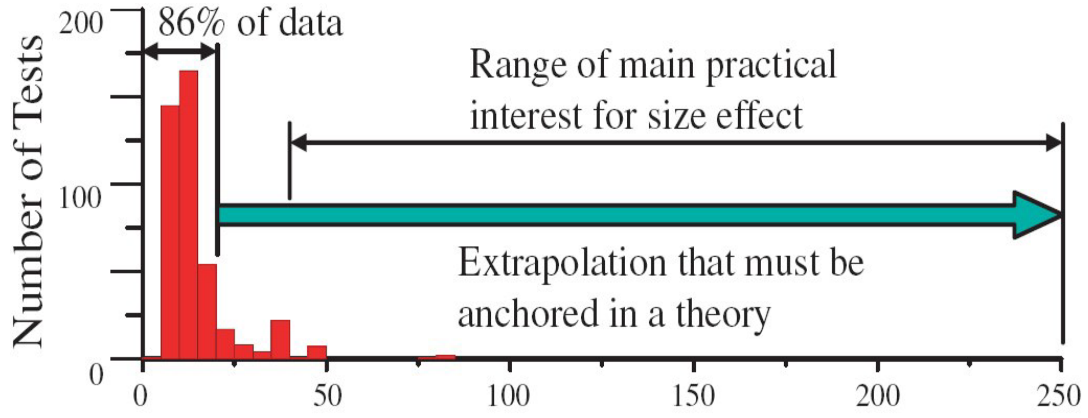
"weakness is due to the presence of flaws"

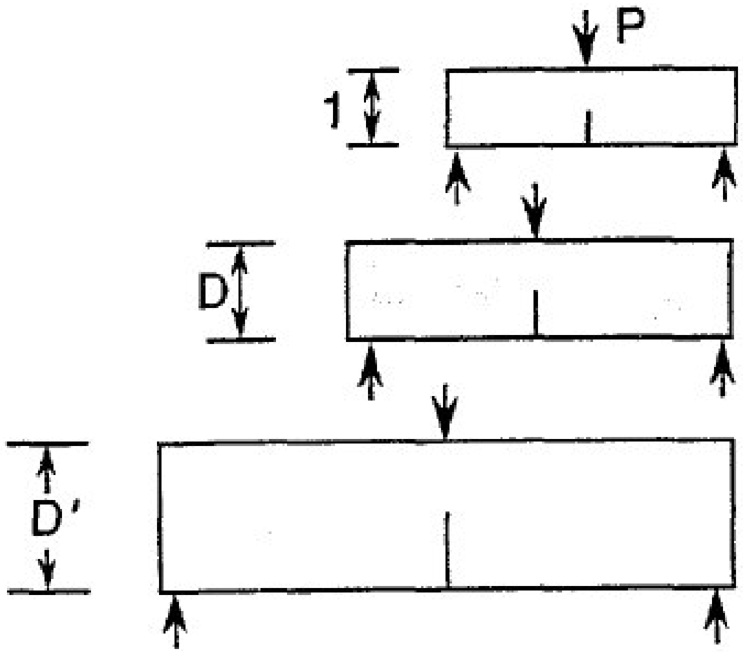


$$\sigma_u = \sqrt{\frac{E\,G_c}{\pi\,a}}$$

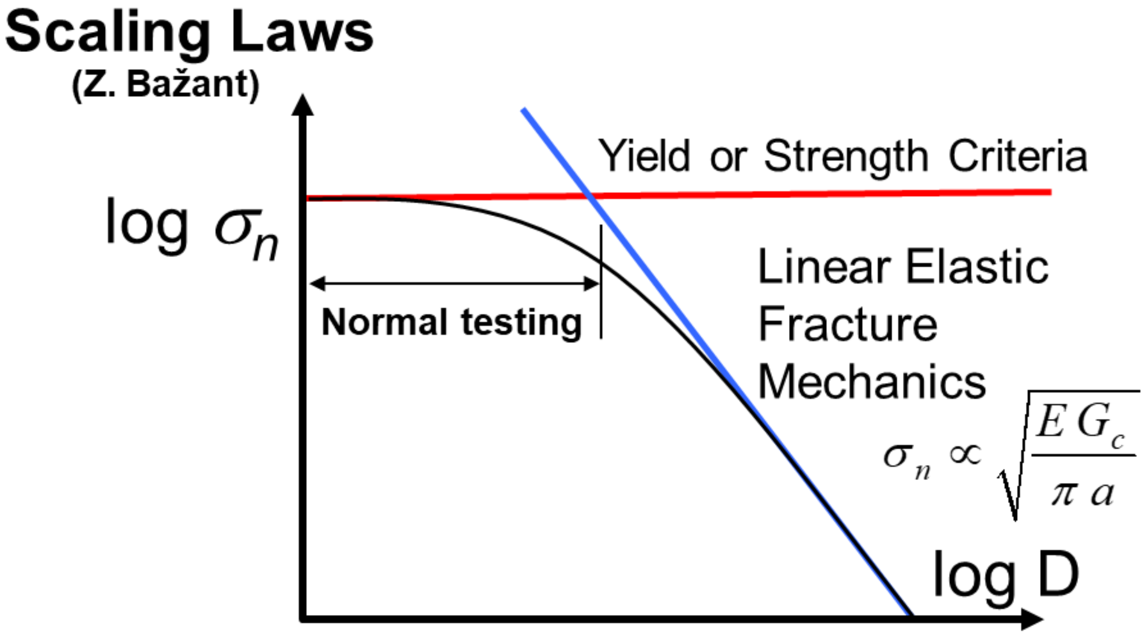
While the aerospace engineers were arguing, the civil engineers…

Scaling: the Effect of Structure Size on Strength





Z. Bažant, Scaling of Structural Failure, 1997



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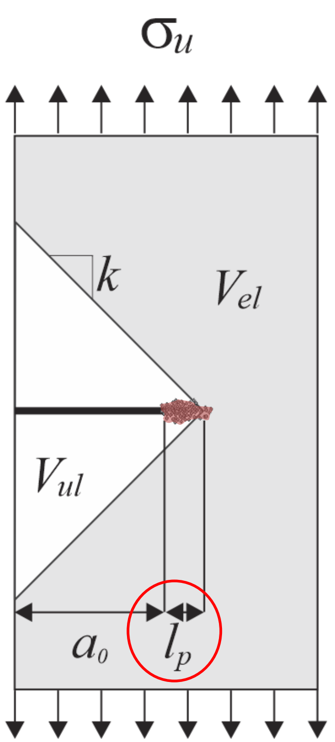
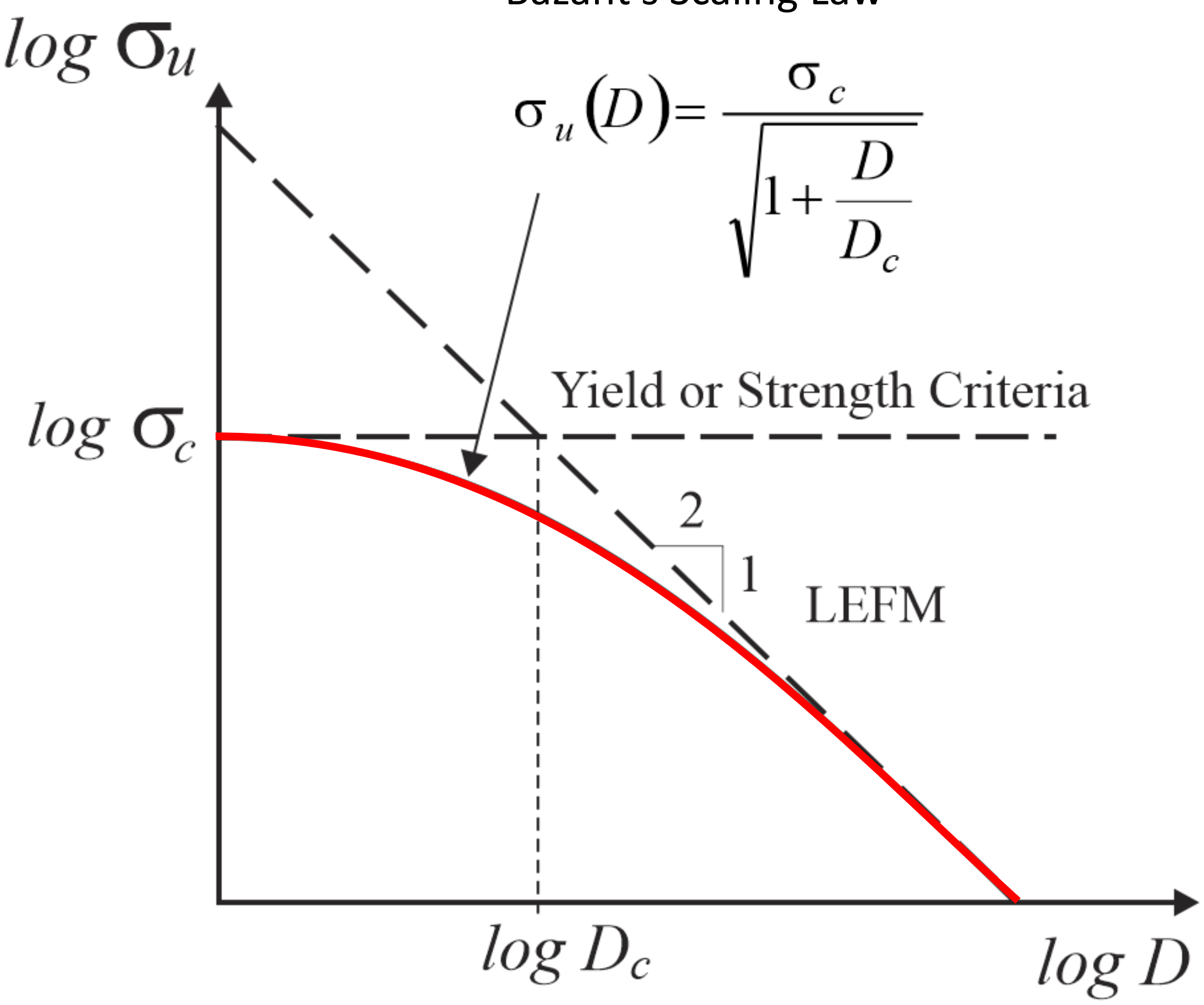


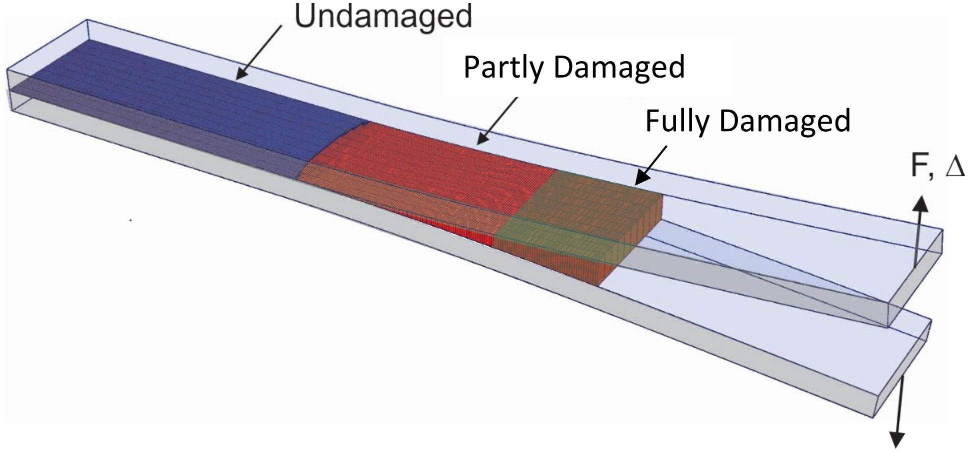
$$\sigma_u = \sqrt{\frac{EG_c}{k(a_0 + l_p)}}$$

Softening is the bridge between strength scaling theories

Bazant's Scaling Law



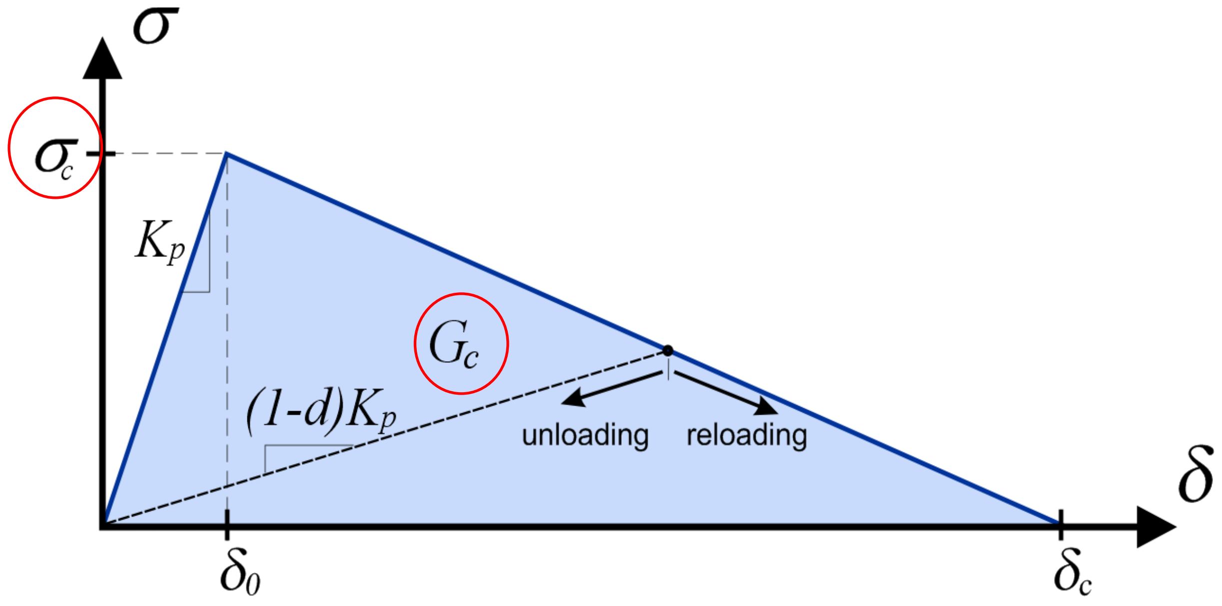
Cohesive Laws: Two Criteria in One Softening Law

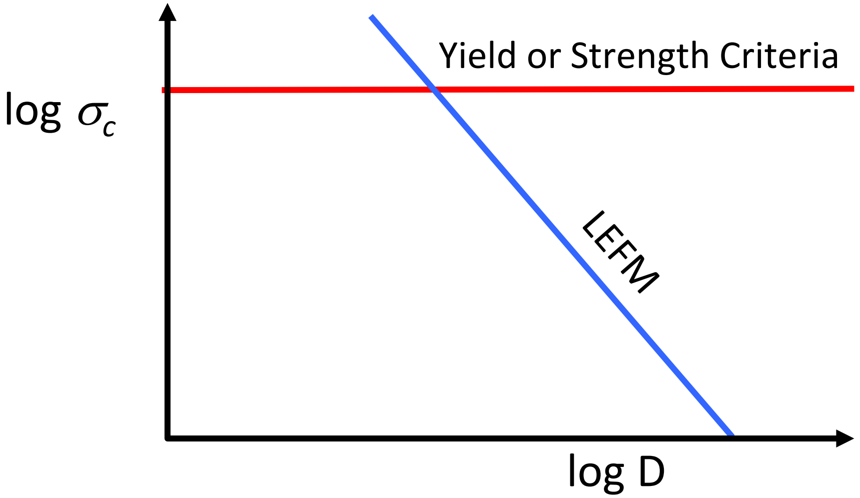


Two material properties:

- G_c Fracture toughness
- σ_c Strength

Bilinear Traction-Displacement Law

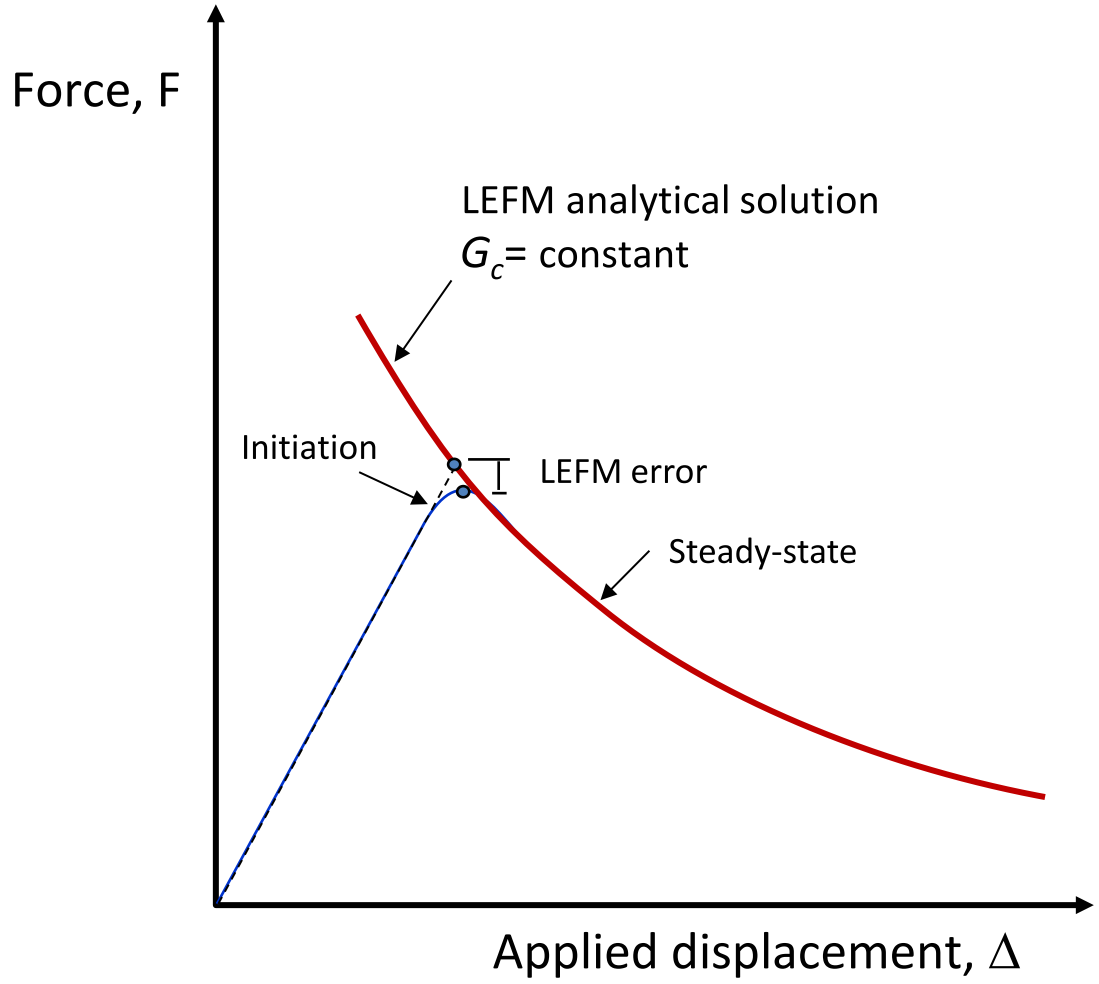


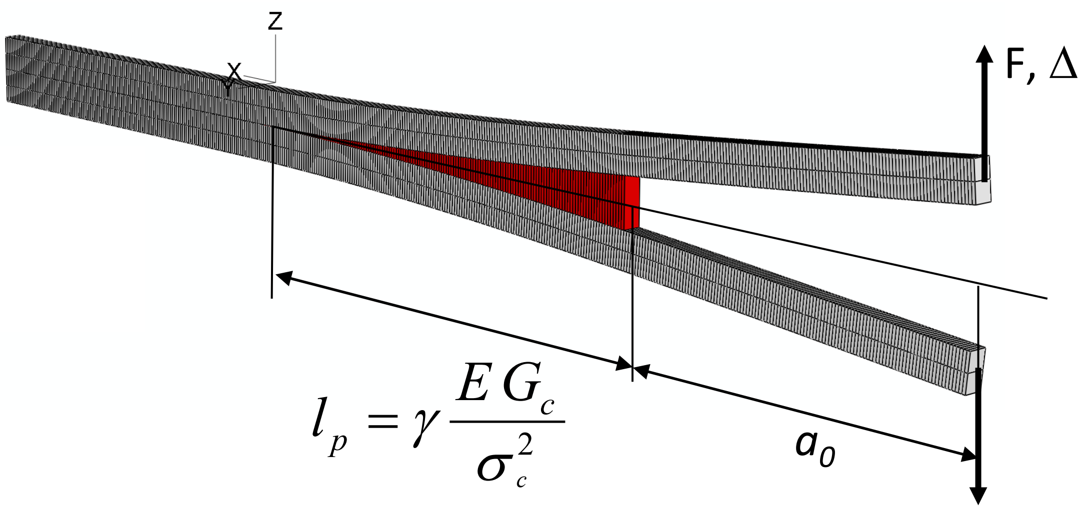


Characteristic Length:

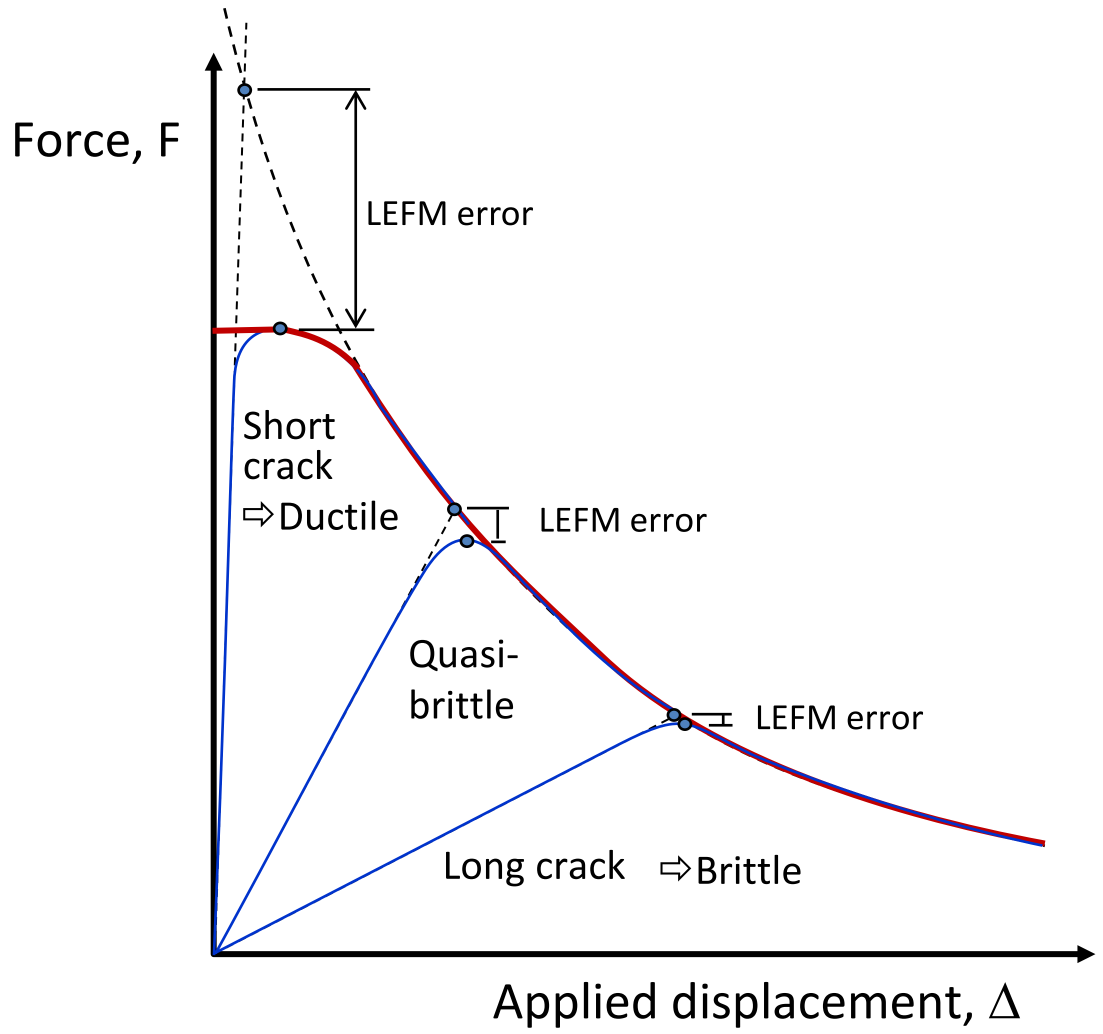
$$l_c = \gamma \frac{E\,G_c}{\sigma_c^2}$$

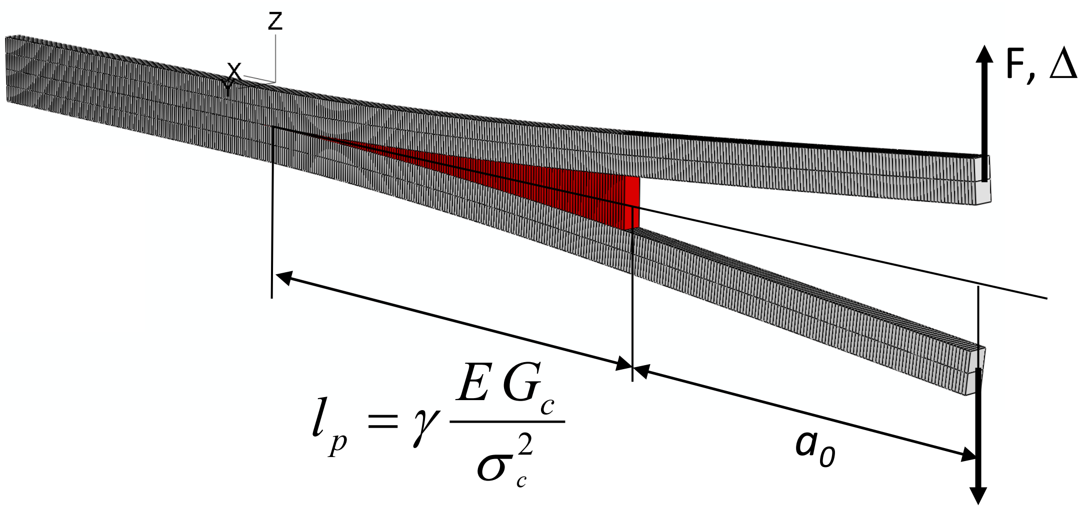
Crack Length and Process Zone





Crack Length and Process Zone





$$l_p = \gamma \frac{E\,G_c}{\sigma_c^2}$$

Brittle:

$$a_0 > 100\ l_p$$

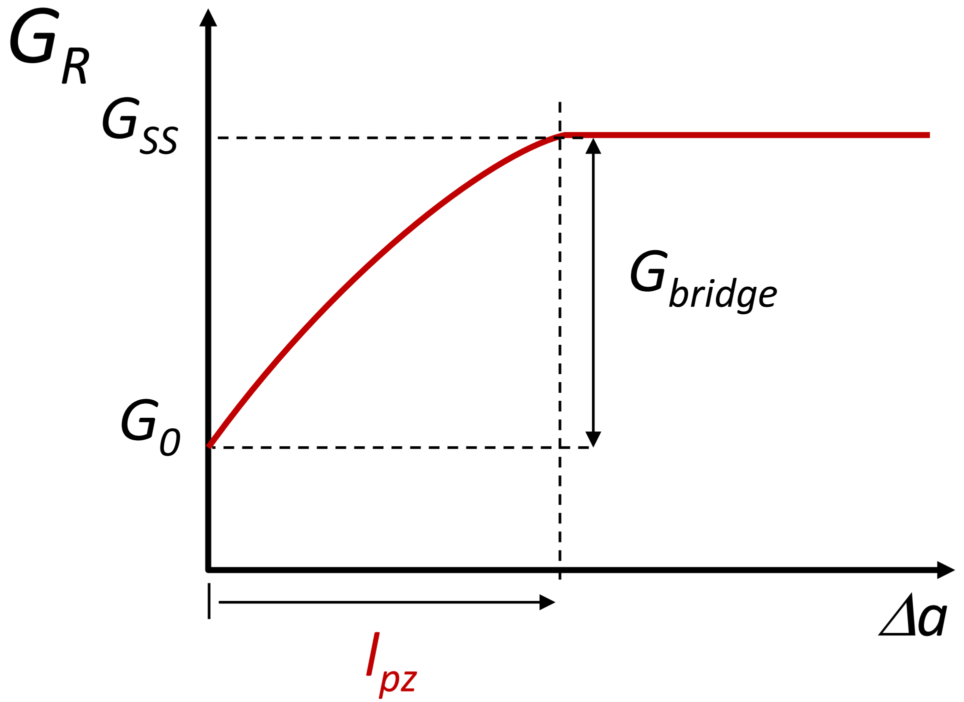
Quasi-brittle:

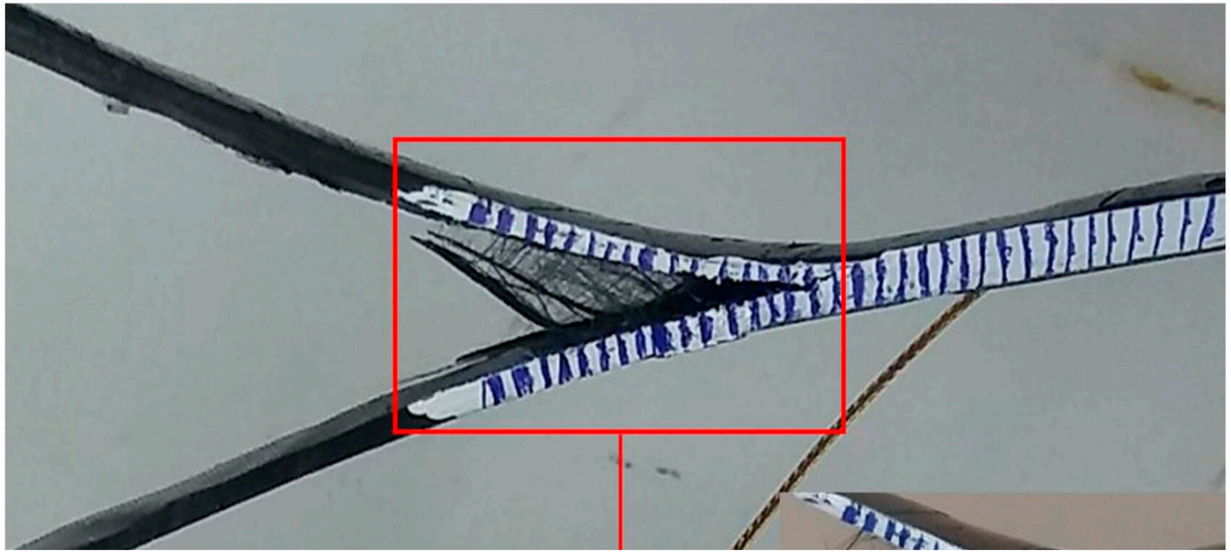
$$100\ l_p \geq a_0 \geq 5 l_p$$

Ductile:

$$5\ l_p > a_0$$

R-Curve Effects in Composites

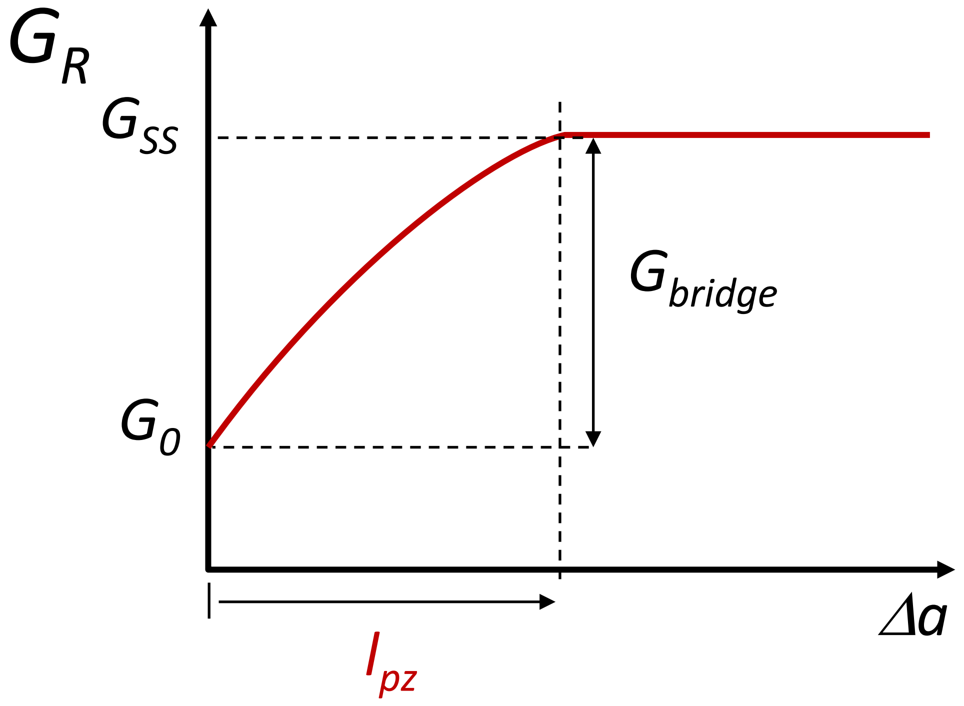


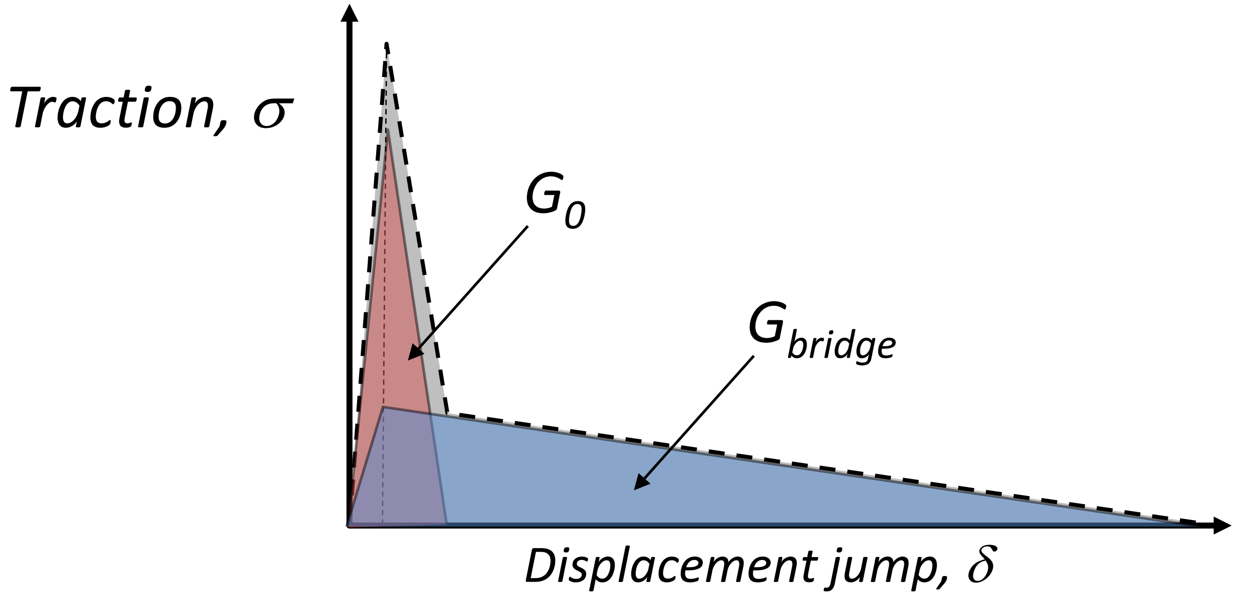


M. Farajpour, JCM, 2023

Mechanisms such as fiber bridging can induce a gradual increase in the critical energy release rate, G_R. (R-curve effect)

Cohesive Model of R-Curve Effects





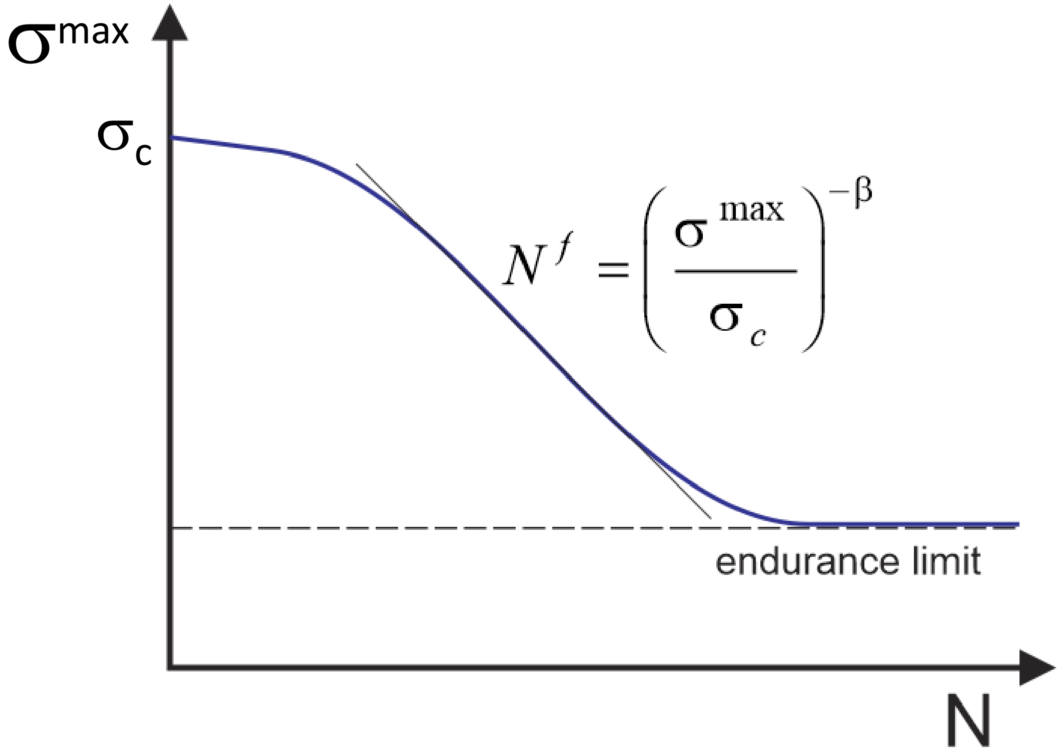
- The shape of the traction-separation (TS) induces different R-curves.
- TS with longer "tails" result in longer l_{pz}.

OK, material softening bridges the gap between strength and fracture theories…

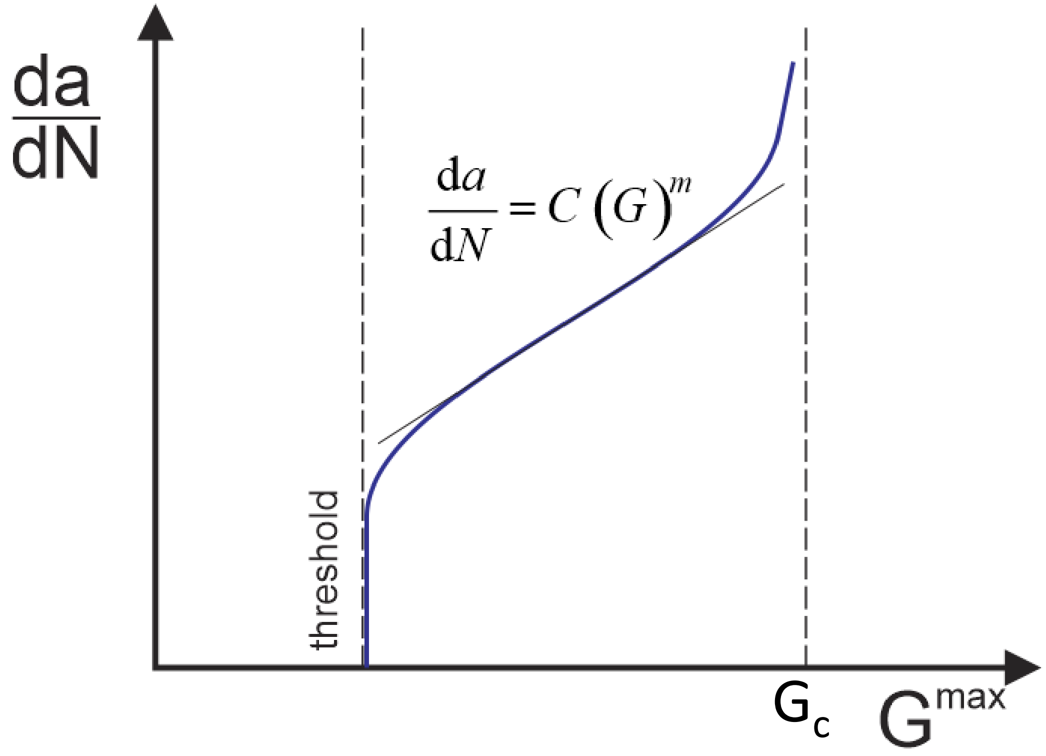
But what about fatigue?

Fatigue: Strength or Toughness?

Strength: Wöhler (S-N Curve)



Fracture: Paris Law

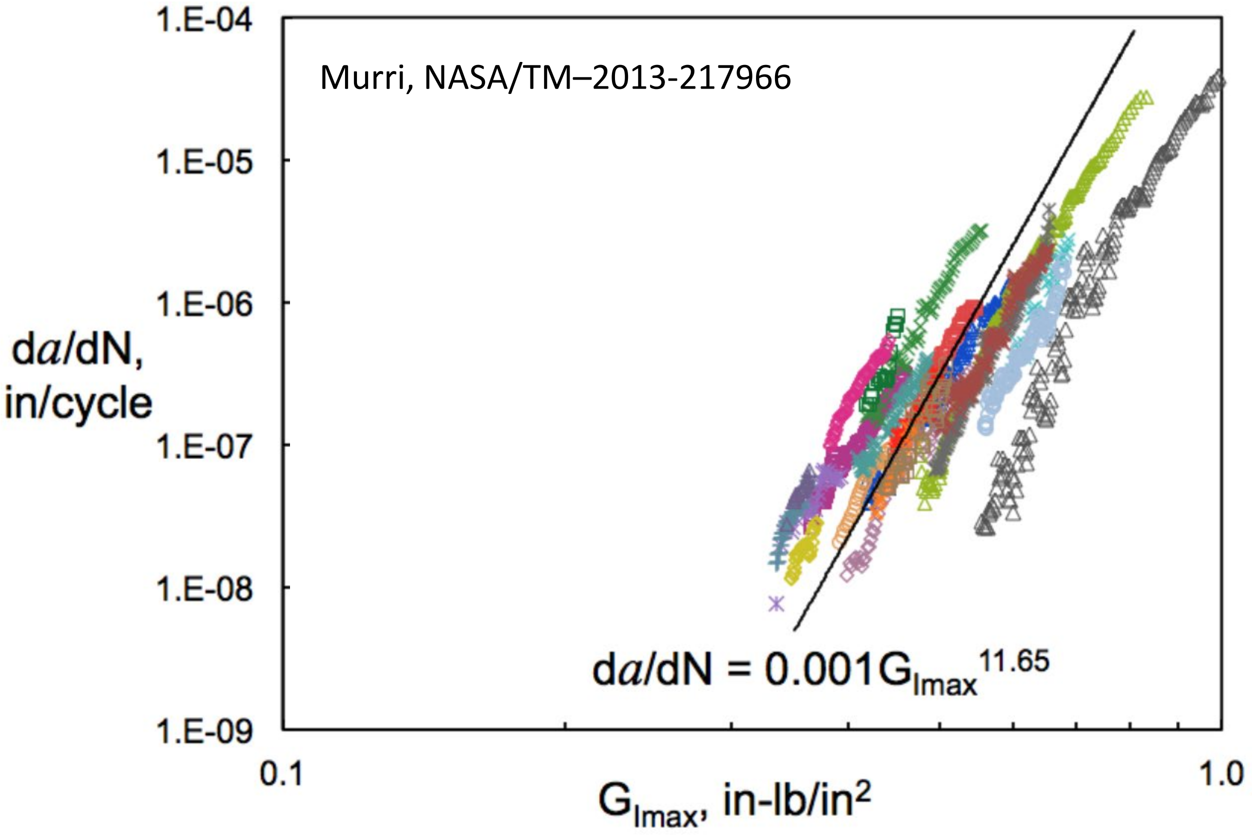


Propagation:

- Most fatigue cohesive constitutive models are based on Paris law

Drawbacks of Models Based on the Paris Law

Paris Law: $$\frac{da}{dN} = C\left(G_{max}\right)^m$$

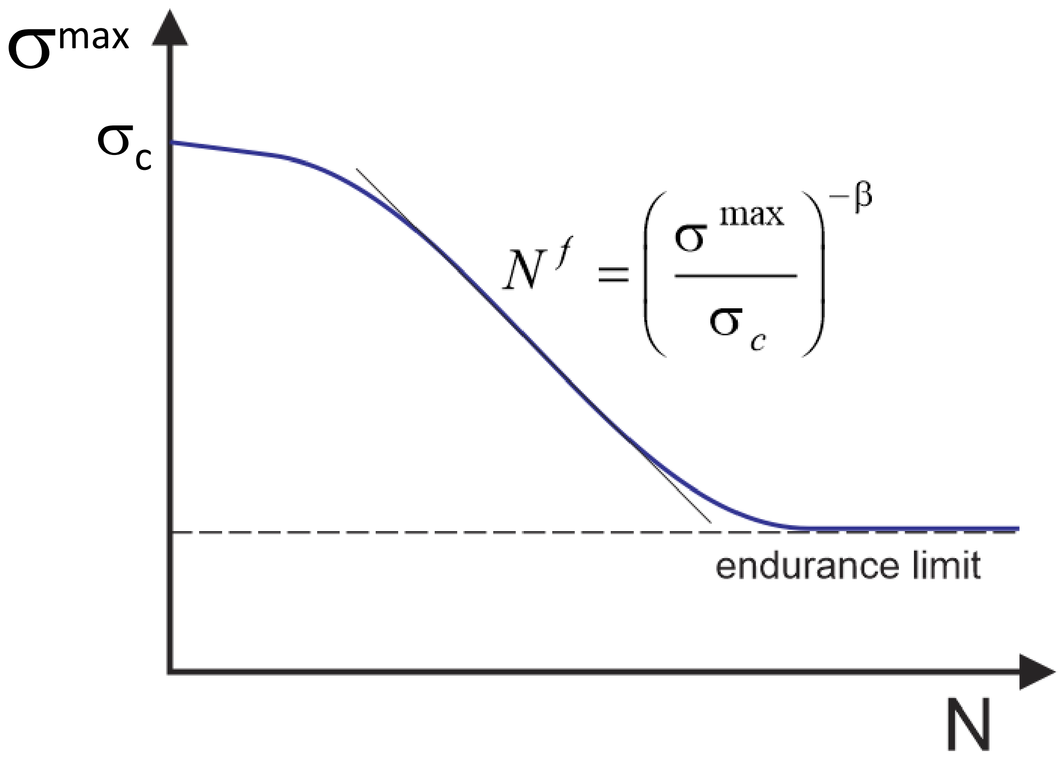


Model Limitations and Drawbacks:

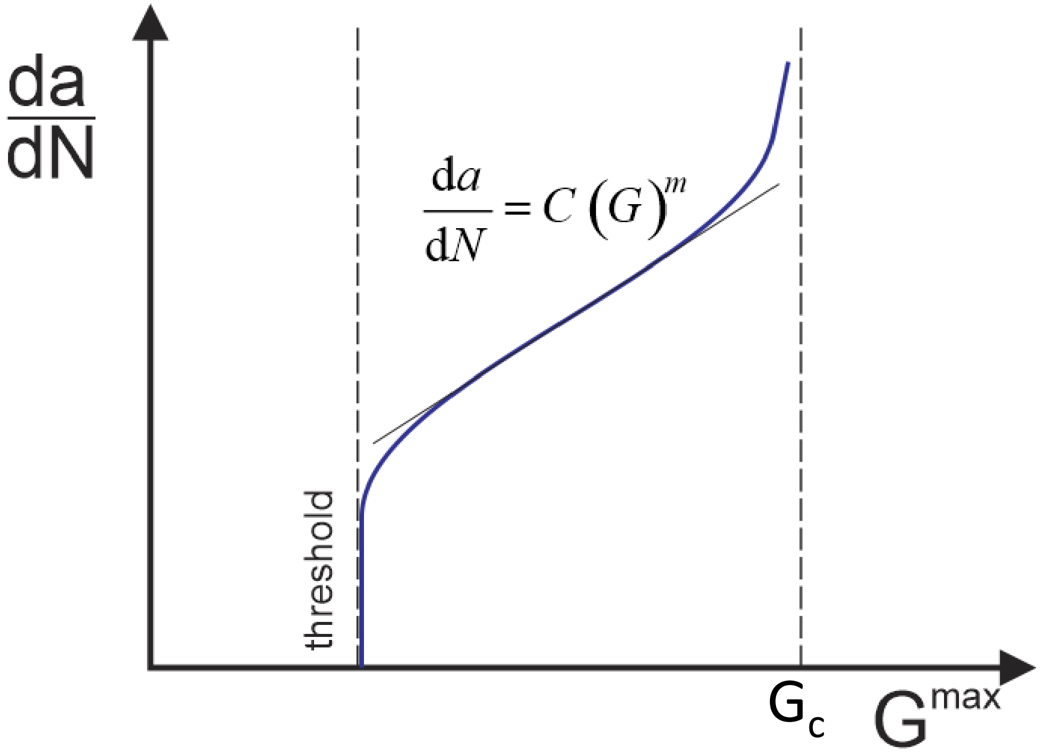
- Paris data (LEFM) does not apply to crack initiation.
- Characterization tests are expensive.
- Tests must be conducted at different stress ratios *R* and mode mixities *B*.
- R-curves induce multiple Paris laws.
- Implementation of Paris law into a cohesive law is difficult.

Fatigue: Strength or Toughness?

Strength: Wöhler (S-N Curve)



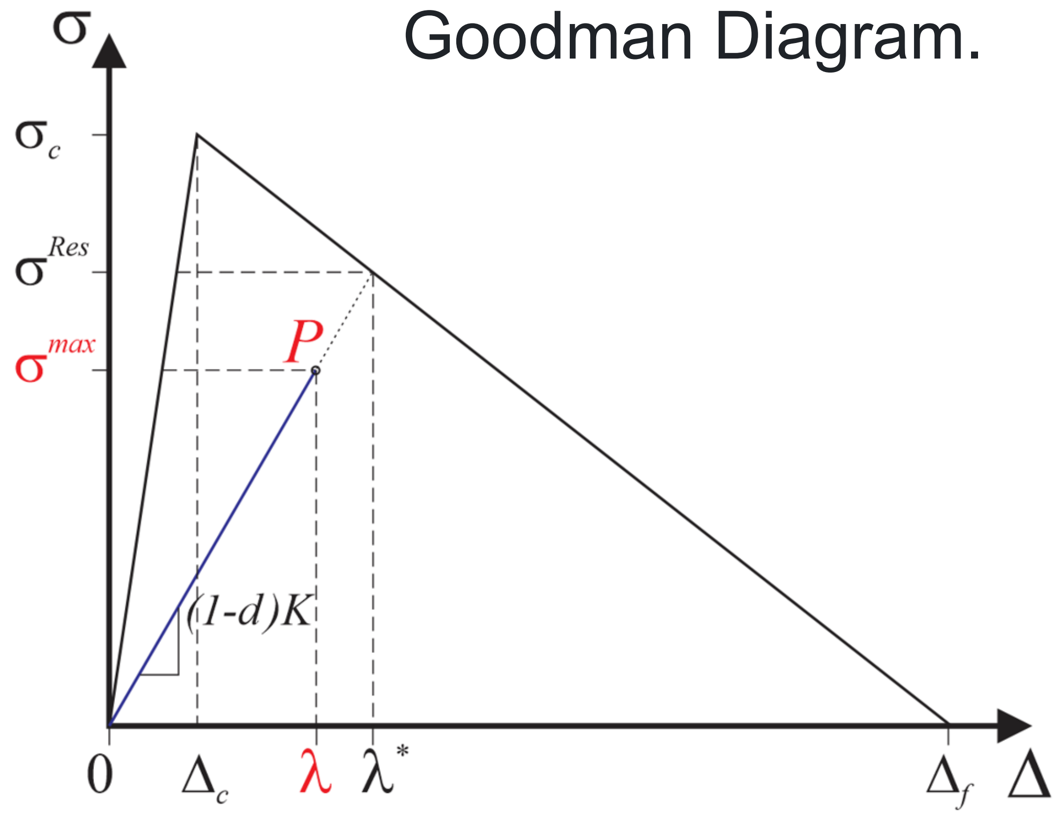
Fracture: Paris Law



Can we develop a cohesive model based on S-N that can also predict crack propagation?

CF22 Cohesive Fatigue Model

- CF22 is a reduced-input cohesive fatigue model that can predict the initiation and propagation of fatigue cracks.
- Relies on intrinsic relationships between S-N and da/dN.
- Effects of stress ratio and mode mixity are predicted using the Goodman Diagram.



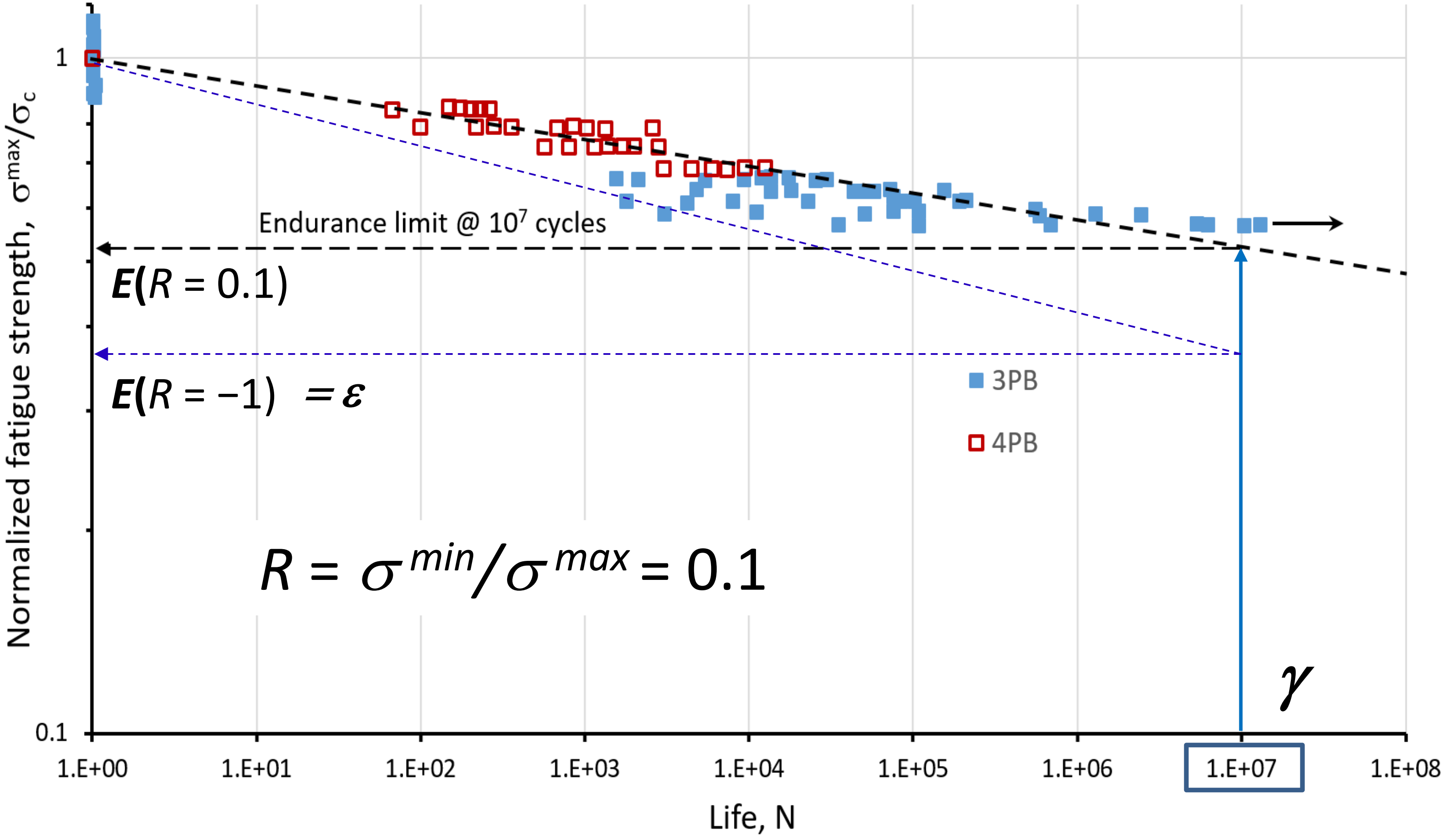
$$\frac{dD}{dN} = \frac{1}{\gamma} \frac{(1-D)^{-q}}{E^{\beta}(\beta + q + 1)} \left(\frac{\lambda}{\lambda^*}\right)^{\beta}$$

Endurance: $E = \dfrac{2\varepsilon}{\varepsilon + 1 + (\varepsilon - 1)R}$

S-N exponent: $\beta = \dfrac{-\log \gamma}{\log E}$

S-N Diagram for Delamination

- A single line fit the S-N curve from N = 1 to 10^7

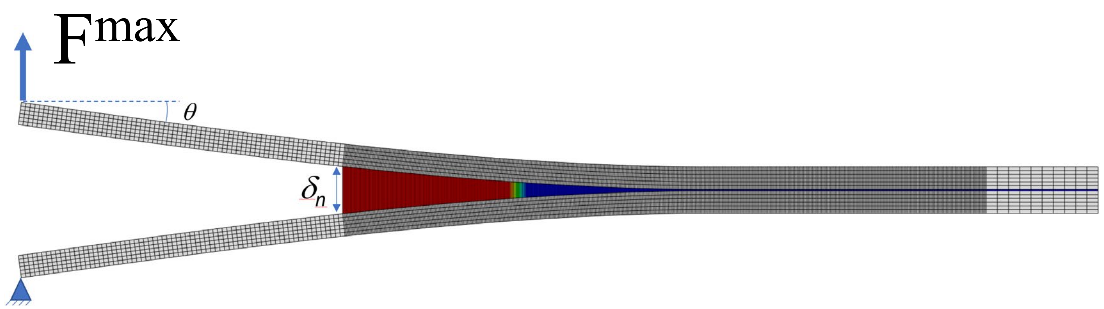


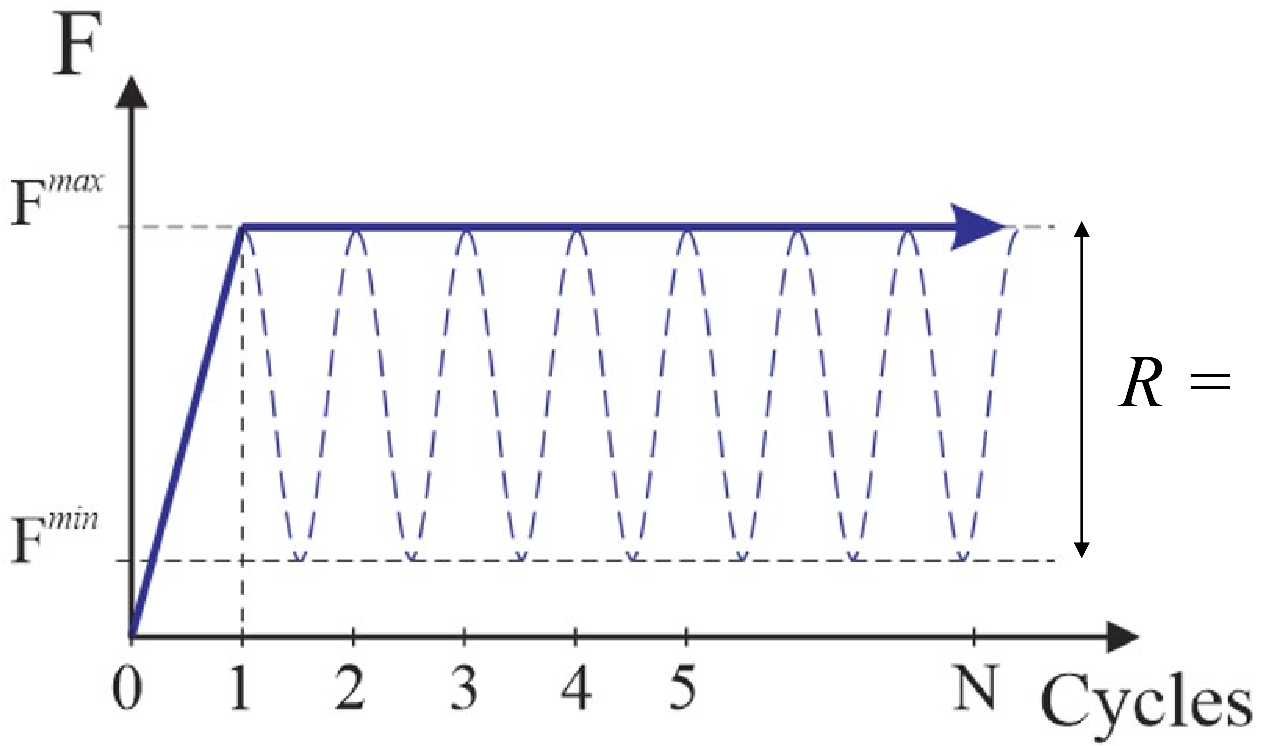
The S-N curve is anchored at point (γ, E).

E(R) can be calculated using Goodman Diagram.

Fatigue Analysis - Simplified Cyclic Loading (SCL)

- F^{max} is held constant
- Constitutive model is informed of cyclic loading through value of *R*

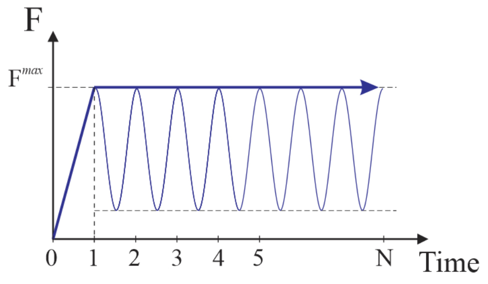


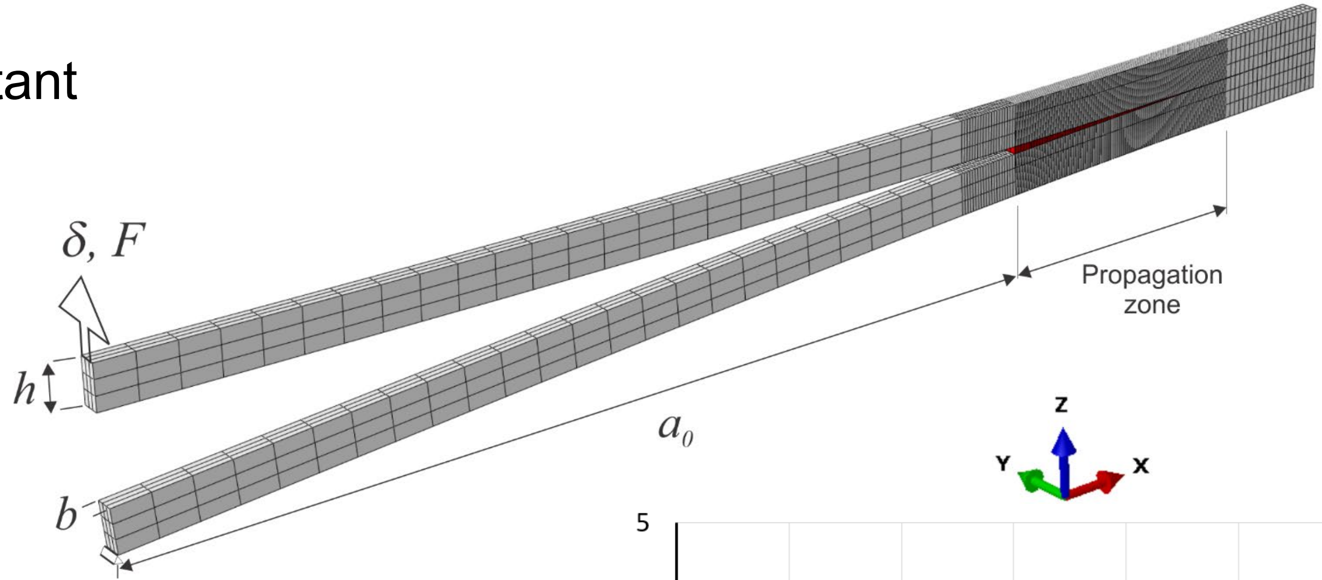


$$R = \frac{F^{min}}{F^{max}}$$

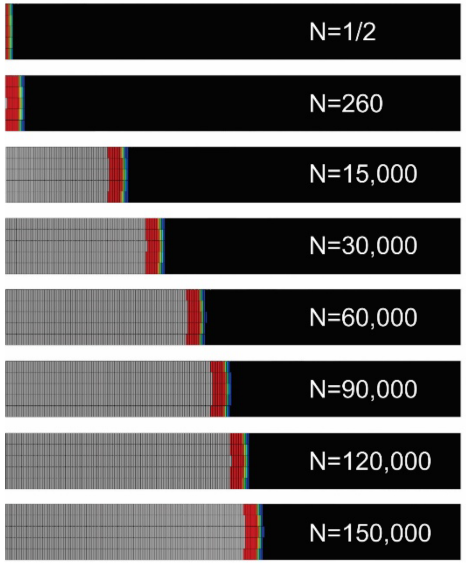
Analysis of Double Cantilever Beam Specimen (DCB)

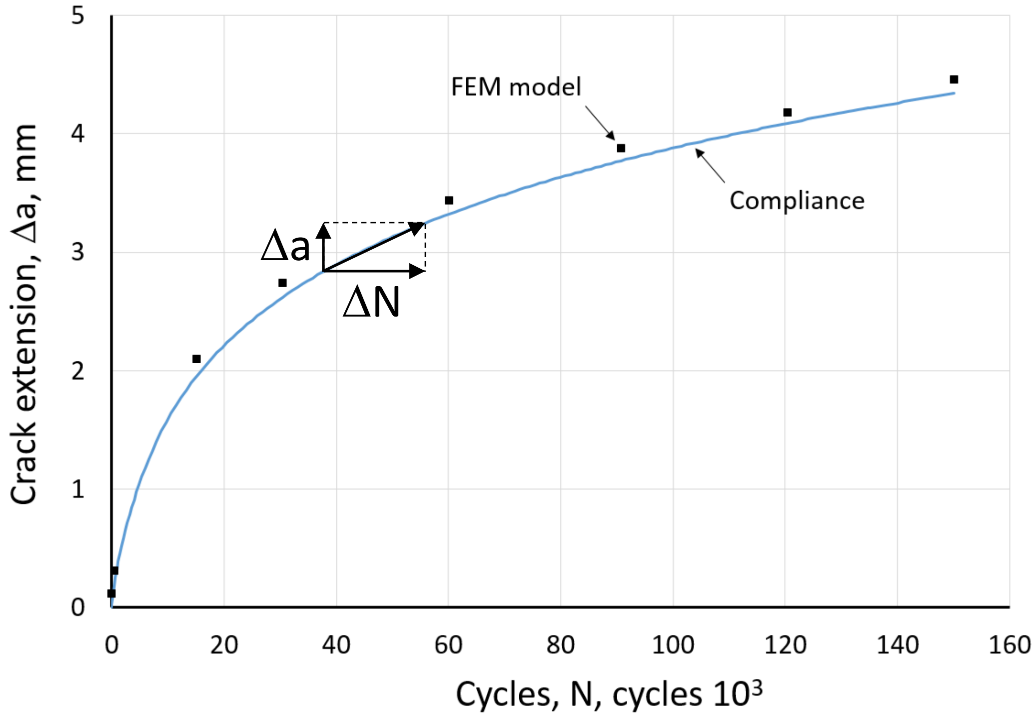
Applied load remains constant



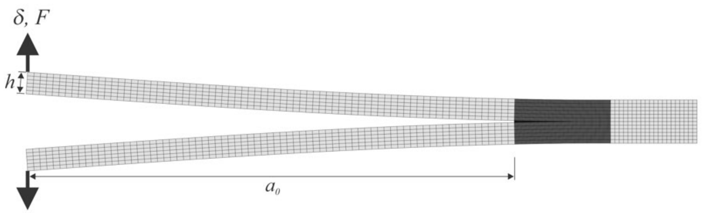


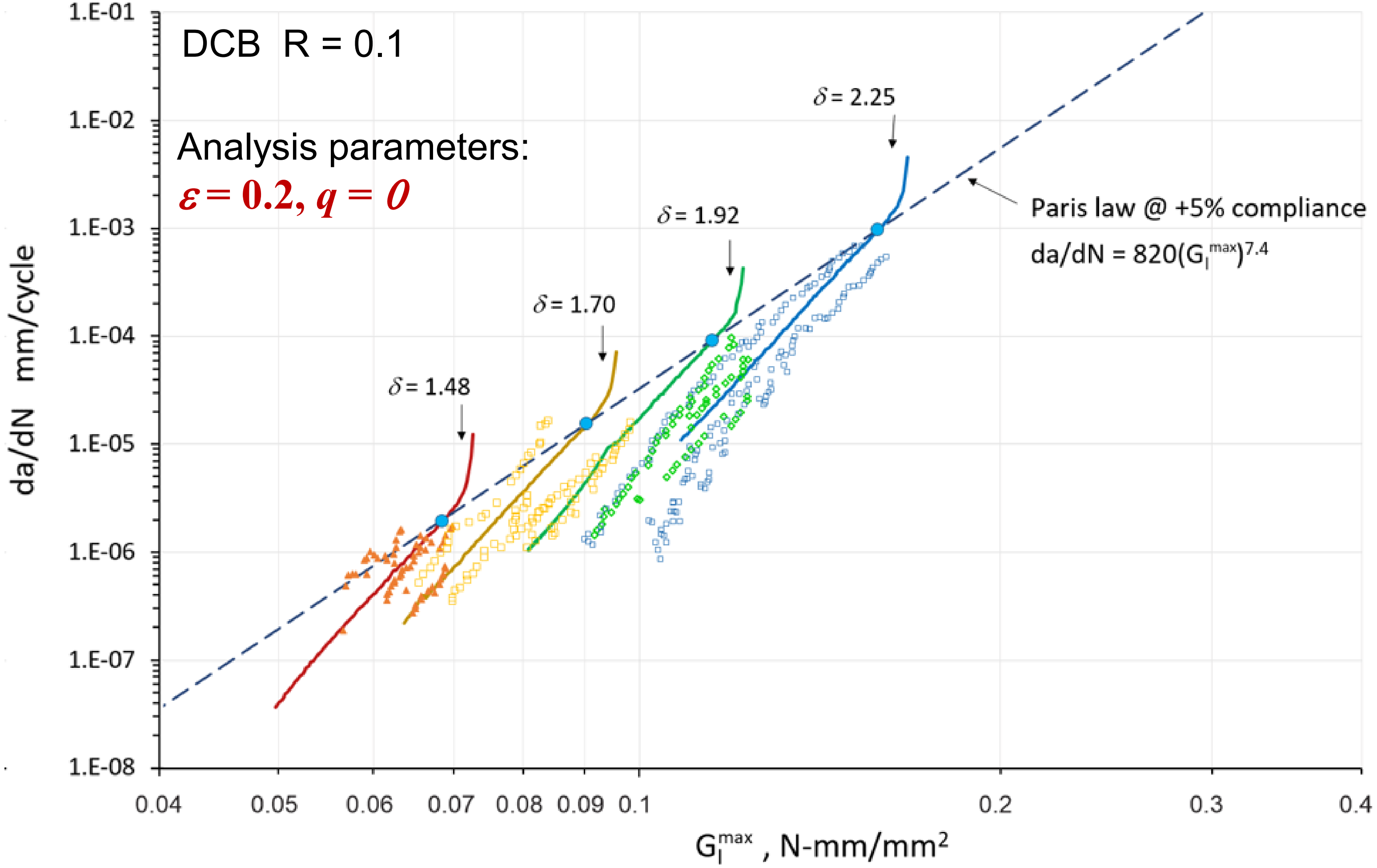
Detail of propagation zone



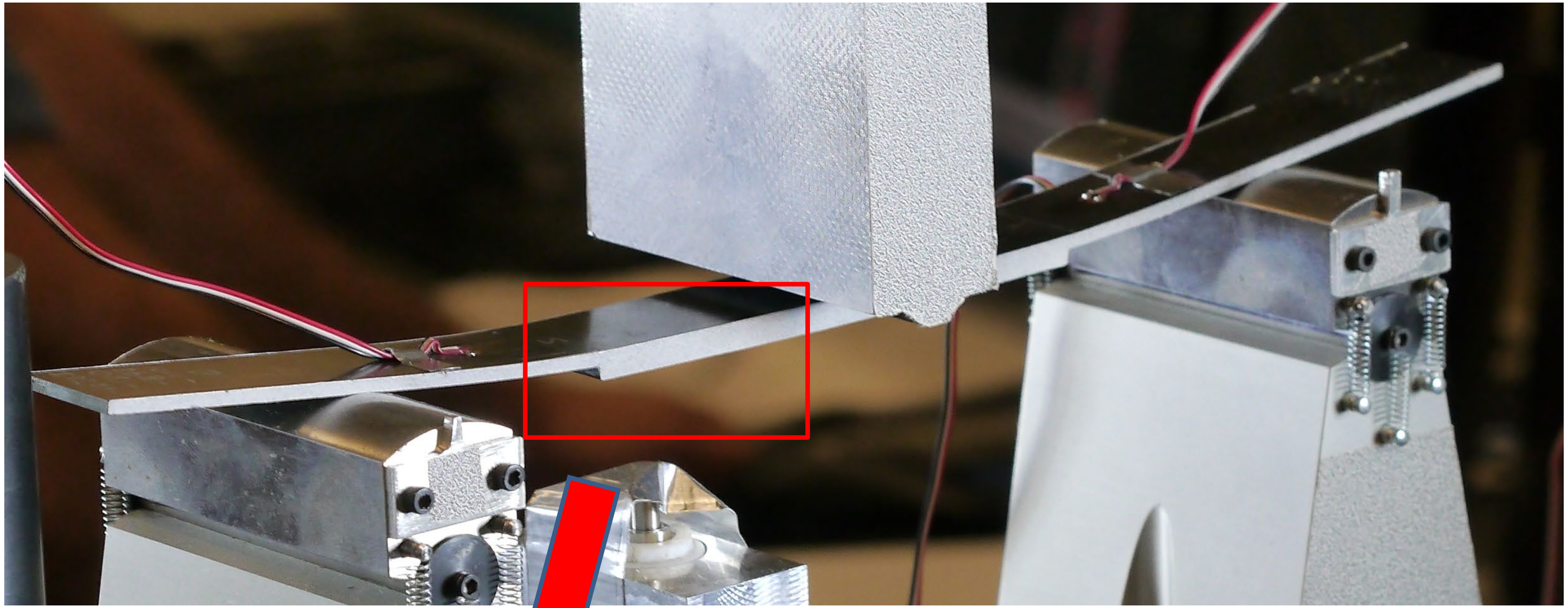


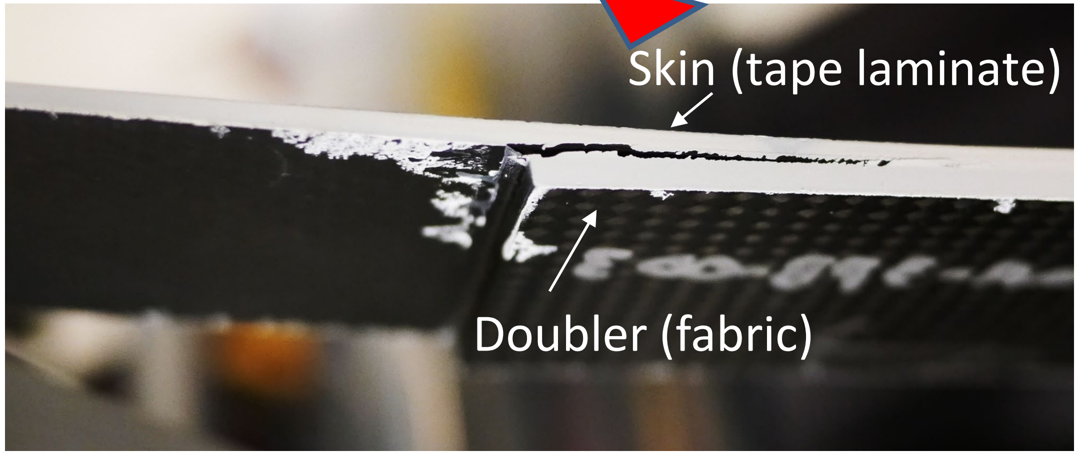
Fatigue Analysis of DCB Test with R-Curve Effect

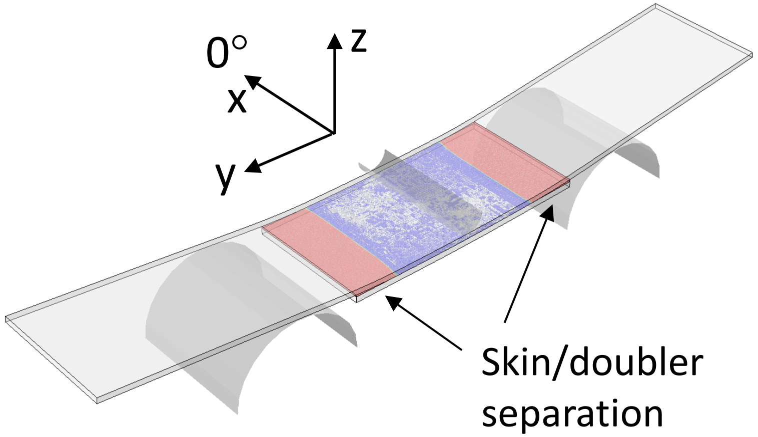




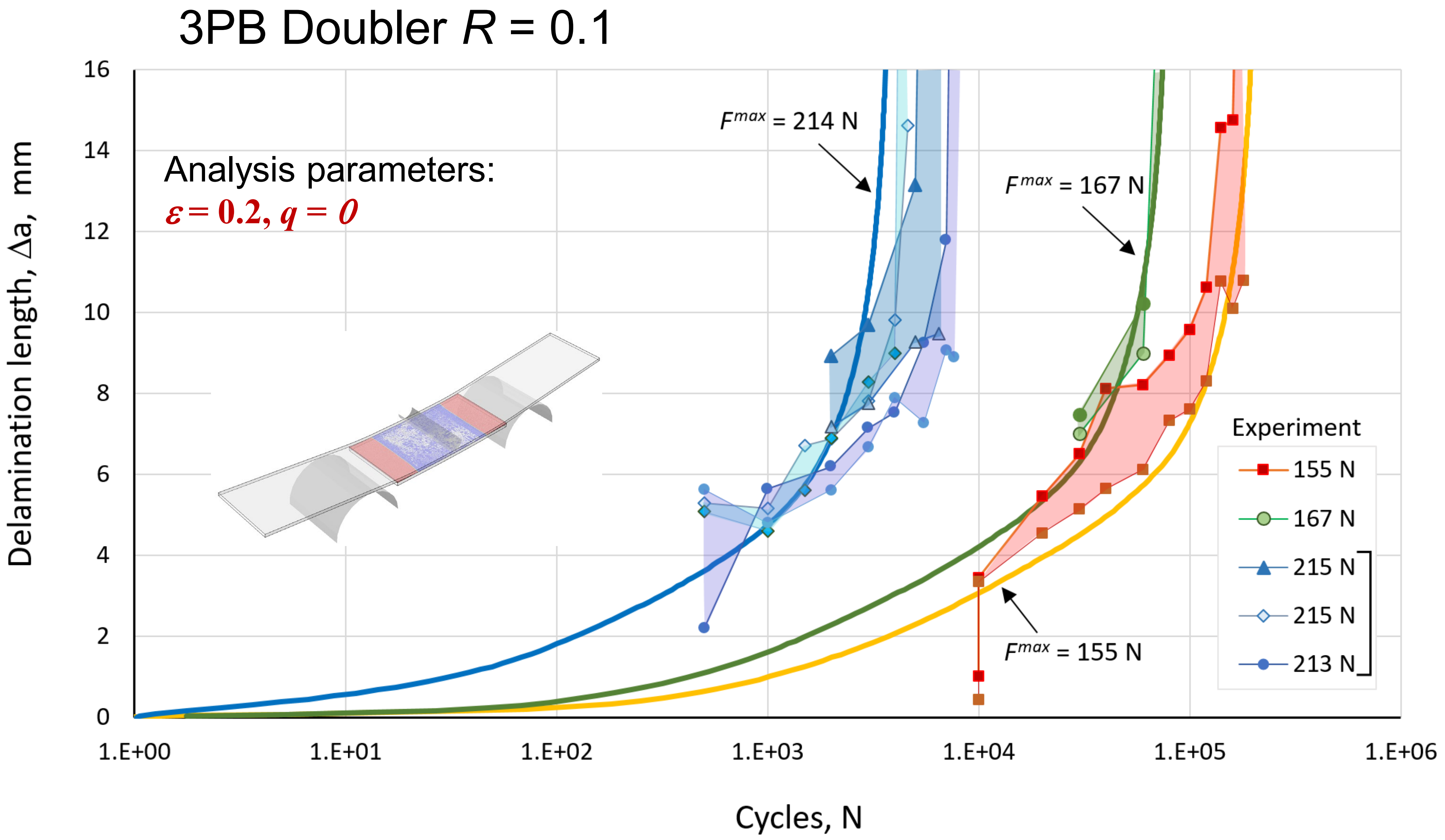
Three Point Bend (3PB) Doubler Specimen







Delamination length in 3PB Doubler



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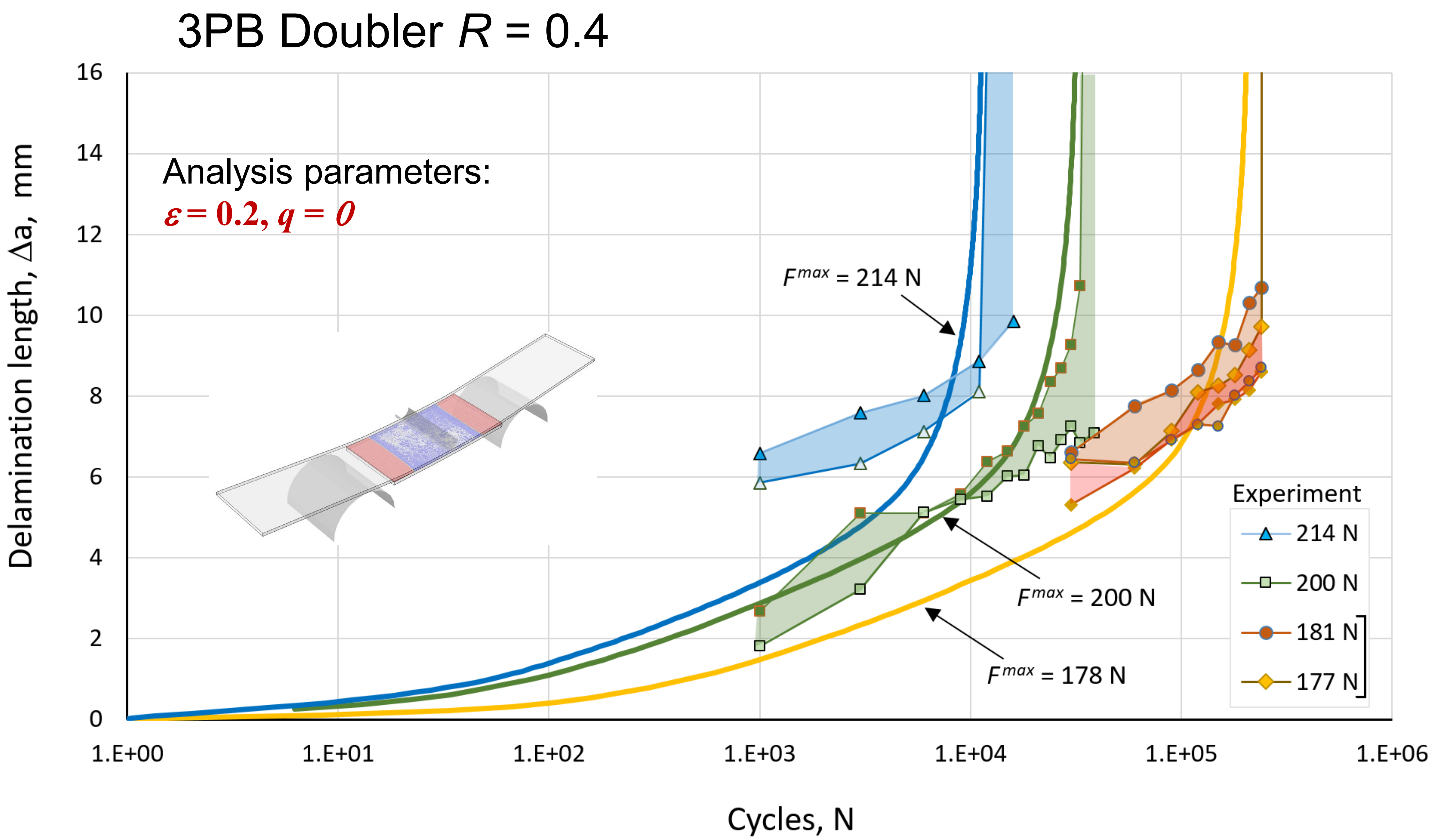

Softening the Gap between Wöhler and Paris

- **Material softening blends strength and fracture theories.**
- **A common cohesive thread ties <u>fatigue threshold</u>, <u>S-N</u>, and the <u>Paris rate.</u>**
- **Transient rates of propagation can only be predicted with softening models.**