

Reduced-Order Modeling Approach to Blunt-Body Aerodynamic Modeling

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Blunt-body entry vehicles induce complex flow phenomena that result in dynamic instabilities in the low supersonic to transonic flight regime. Dynamic stability coefficients are typically calculated through parameter identification and trajectory regression techniques using both physical test data and computational fluid dynamics (CFD) simulations. The objective of this work is to advance the quantification of the dynamic stability behavior of blunt-body entry vehicles by leveraging high-fidelity CFD data through reduced-order modeling (ROM). ROMs are capable of leveraging high-fidelity aerodynamic data in a cost-effective manner by finding a low-dimensional representation of the full-order model. ROMs based on proper orthogonal decomposition have shown success in recreating CFD analyses of parametric ROM applications and time-varying ROM applications. Results of this research demonstrated success in constructing two ROMs of a blunt-body entry vehicle to recreate heatshield and backshell pressure distributions from forced oscillation trajectories. The ROM was more successful at reconstructing the heatshield pressure distribution, with challenges arising in predicting the chaotic response of backshell latent coordinates.

Nomenclature

A	=	amplitude of the forcing function
A_i	=	batch set of the snapshot matrix
C_A	=	axial force coefficient
C_l	=	rolling moment coefficient
C_m	=	pitching moment coefficient
C_N	=	normal force coefficient
C_n	=	yaw moment coefficient
C_P	=	coefficient of pressure
C_Y	=	side force coefficient
d	=	reduced-order modeling dimension
f	=	generic function operator
f^*	=	dimensional frequency of forcing function
ff	=	forget factor of streaming singular value decomposition
g	=	regression function operator
i, j	=	generic indices
k	=	preserved left singular vectors
M_∞	=	Mach number
m	=	number of snapshots
n	=	degrees of freedom of samples
p	=	vector of design variables
r	=	covariance matrix rank
S	=	data covariance matrix
t^*	=	dimensional time
v	=	number of validation snapshots
X	=	matrix of snapshots
x	=	single snapshot
Z	=	matrix of latent coordinates
z	=	single latent coordinate

α	=	angle of attack
$\dot{\alpha}$	=	rate of change of angle of attack
δ	=	desired information content
μ	=	average of the input snapshot matrix
σ	=	singular value
Φ	=	proper orthogonal decomposition mode matrix
ϕ	=	proper orthogonal decomposition mode

I. Introduction

FOR decades, the solution to planetary entry, descent, and landing (EDL) has been the blunt-body entry vehicle, which is capable of generating the large amounts of drag and dissipating the heat of entry over a large area. Unfortunately, atmospheric entry vehicles can be dynamically unstable in the supersonic to transonic flight regimes, becoming more unstable as the vehicle decelerates [1]. Understanding the physical phenomena that drive blunt-body dynamic instability is a major research effort [2–5]. Identifying dynamic stability coefficients from physical testing methods [1,6–9] and computational fluid dynamics (CFD) simulations [3,6,10–16] is a complex task. To calculate these coefficients, linear and quasi-static aerodynamic assumptions are made, and an assumed functional form is utilized via trajectory-fitting techniques [1]. These assumptions may be flawed, as the literature has noted the nonlinear dynamic nature of these vehicles [6]. Therefore, there is a need to develop new methodologies to quantify the dynamic behavior of entry capsules.

Recent research efforts in dynamic stability quantification have been focused on finding innovative ways to quantify blunt-body dynamic stability, moving away from traditional, linearized functional forms and dynamic stability coefficients. McKown et al. used 1-degree-of-freedom (1-DOF) free-flight CFD (FF-CFD) to generate vehicle trajectories of the Genesis entry capsule to investigate the use of nonlinear pitch damping coefficient curves [17]. In an effort to eliminate the need for dynamic stability coefficients, Ernst et al. used feed-forward and time-delay neural networks to predict forces and moments of a supersonic inflatable aerodynamic decelerator (SIAD) using data generated by a CFD–rigid body dynamics (RBD) modeling framework [12]. The neural networks outperformed the traditional limit cycle analysis outlined in [1] when used to predict aerodynamic forces and moments [12], indicating the benefit of a lack of functional form and benefit in omitting dynamic stability coefficients. These findings demonstrate that it is possible that the dynamic response of entry capsules cannot be isolated to coefficients; thus, developing advanced functional forms could be insufficient. Additionally, utilizing analytical functional forms to predict vehicle aerodynamic responses can limit model accuracy and expressiveness if the incorrect functional form is utilized [1,18]. This indicates a need

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to deviate from prescribed functional forms to capture vehicle dynamic responses [12].

The existing data reduction techniques utilized in current research efforts omit the rich data that are obtained from CFD simulations. Currently, CFD simulations are used to conduct simulated ballistic range shots, providing more samples of the vehicle state during flight than what is achieved in physical test campaigns [12,17]. This omitted aerodynamic information could be critical to discovering new methods for dynamic stability quantification. Instead of focusing on the coefficient and vehicle state outputs of a CFD simulation, the resulting flowfield could be explored to find new methods of predicting the dynamic response of the vehicle. Fundamentally, the vehicle dynamics are driven by aerodynamic forces and moments, oftentimes simplified to a single force and moment acting on the vehicle. There are only two basic sources of aerodynamic forces and moments on a vehicle: the *pressure distribution* over the body surface and the *shear distribution* over the body surface [19]. As stated by Anderson, pressure and shear distributions on the body surface are the only mechanisms that create a force on a body moving through a fluid [19]. Utilizing CFD simulations enables the sampling of pressure and shear distributions across the vehicle as a result of the flowfield. If a methodology could be developed that predicted the dynamic changes of aerodynamic surface distributions (both pressure and shear), then the resulting forces and moments from those distributions could capture more physically consistent vehicle dynamics than the vehicle dynamics achieved by linear and quasi-static dynamic stability coefficients.

Reduced-order modeling (ROM) is a form of surrogate modeling with the ability to leverage high-fidelity aerodynamic data in a computationally efficient manner [20–26]. In a single field model, a ROM can produce a field of responses, mimicking the results produced by CFD simulations [26]. ROMs handle the size and complexity of aerodynamic data by finding a low-dimensional representation of the data called the *latent space* that captures fundamental features of the original high-dimensional data [20,27]. This latent space can be leveraged to make predictions of the high-dimensional field at unseen parameter points [20]. ROMs have been applied to numerous applications similar to the blunt-body entry vehicle problem, including vortex shedding behind a rotating cylinder [28], turbulent flows [29], axisymmetric turbulent wake behind a disk [30], parametric applications in compressible flow [21], and unsteady aerodynamics [31]. Additionally, Peters et al. utilized ROMs to model store separation, studying several different formulations to reconstruct surface pressures and freestream flow for a pitch-up store [32]. If the pressure and shear distributions on an entry vehicle's surface can be generated by a ROM, then the time-accurate vehicle dynamics can be predicted in a computationally efficient manner, enabling a new methodology to predict the dynamic response of entry capsules.

The objective of the research described in this paper is to advance the ability to quantify the dynamic stability of blunt-body entry vehicles by leveraging high-fidelity CFD data to create an aerodynamic ROM of an entry capsule.

II. Reduced-Order Modeling Methodology

In general, ROMs are classified in one of two ways: intrusive and nonintrusive [20,33]. Intrusive ROMs are provided by the governing equations used to generate the full-order model (FOM), often requiring modifications to source code to enable their use [34–37]. Nonintrusive ROMs are a data-driven method that only requires the input and output data for training, a favorable characteristic in aerospace applications that often leverage “black box” analysis codes [20,38]. Additionally, the ROM literature primarily focuses on modeling two types of fluid flows: parametrically varying fluid flows and time-varying fluid flows. Parametrically varying ROMs focus on the parametric prediction of steady flowfields, most commonly varying Mach and angle of attack (α) [33]. Examples of parametric ROMs include airfoils at different Mach and α configurations [20,21,23,39–42], hypersonic aerodynamics [24], and rotorcraft aerodynamics [43]. Time-varying ROMs focus on decomposing unsteady fluid flows, focusing on modal decomposition and identifying dominant structures in the flow. Examples of time-varying ROMs include

cylinder wakes [28,44–46], aeroelastic analysis [47], and wind turbine analysis [48,49]. In entry capsule aerodynamic modeling, it is critical to capture both the parametric and time-varying impacts of the flow on the dynamic response of the capsule. Constructing a parametric and time-varying ROM of an entry capsule will enable the dynamic prediction of aerodynamic fields, providing time-accurate predictions of the vehicle response in a computationally efficient manner. However, the ROM literature combining parametric and time-varying applications is an emerging research area. Therefore, an additional objective of the research described in this paper is to advance parametrically time-varying applications of ROMs.

A ROM can be divided into four main parts: training data generation, dimensionality reduction (DR), interpolation or regression, and backmapping. Each part of the ROM is summarized below, outlining any necessary mathematical formulations for implementation.

A. Training Data Generation

The first step is to identify the input state space to be represented by the ROM. Initially, this research focused on developing a ROM capable of predicting surface pressure distributions for an entry capsule. The synthesized methodology will be equally applicable for reconstructing surface shear distributions in future work. Parameters of the vehicle input state space need to be identified, establishing the upper and lower bounds that need to be sampled. This research focused on an entry capsule input space with a constant Mach number, varying in angle of attack, α , and rate of change of angle of attack, $\dot{\alpha}$. This input space was generated by sampling forced oscillation CFD simulations. Forced oscillation is a dynamic testing technique where the vehicle is “forced” through prescribed oscillatory motions, most commonly by simulated sinusoidal rotational motion [1,50]. The forced oscillation motion was simulated using Fully Unstructured Navier–Stokes 3D (FUN3D), a CFD solver developed by NASA Langley Research Center [51]. In FUN3D, the vehicle motion in forced oscillation is controlled by

$$\alpha(t) = A \sin(2\pi f^* t^*) \quad (1)$$

the resulting $\dot{\alpha}$ is given by differentiating Eq. (1):

$$\dot{\alpha}(t) = 2\pi f^* A \cos(2\pi f^* t^*) \quad (2)$$

where A is the amplitude of oscillation, f^* is the dimensional frequency of oscillation, and t^* is the dimensional time. Through combinations of A and f^* , a series of trajectories can be simulated spanning an $[\alpha, \dot{\alpha}]$ state space of the entry vehicle. Each sampled time step in a CFD simulation provides a single snapshot for the input matrix of the ROM. Therefore, a single forced oscillation trajectory can provide thousands of parametrically time-varying snapshots for the ROM to train with.

B. Dimensionality Reduction

After generating the training set of data, a DR technique must be selected to construct the low-dimensional space of the ROM. DR is a form of unsupervised machine learning where high-dimensional data is projected into the latent space that preserves some quality of the original data [20]. Applications of parametric ROMs and time-varying ROMs have leveraged principal component analysis (PCA), also called proper orthogonal decomposition (POD). POD is a linear DR technique that uses eigenvalue decomposition to determine the dominant features present in the data [52]. POD is capable of deciphering spatial structures present in the FOM when leveraged in parametric ROMs and is capable of deciphering temporal structures of a FOM when applied in time-varying ROMs [46]. POD was first introduced as a mathematical technique to extract structures present in turbulent fluid flows [52]. An objective algorithm in POD decomposes the FOM into a minimal number of *modes*, or basis functions, that capture the highest amount of energy of the FOM [52]. This research implemented a nonintrusive, or data-driven, variant of POD called *snapshot POD*. The original derivation of POD by Berkooz et al. can be found in [29].

From the sampled forced oscillation simulations, an input snapshot matrix $\mathbf{X}$ is generated with m snapshots and n degrees of freedom of the response variable of interest. In this entry vehicle ROM, m represents the number of samples of the CFD simulation, and n represents the number of degrees of freedom of the pressure data. For this application, the pressure data was calculated at discretized points associated with the CFD grid. Depending on the sampled CFD solver, the CFD grid can produce pressure data associated with grid cells or grid nodes. The n dimension of the input snapshot matrix quickly becomes large in entry vehicle applications due to CFD grid sizes. FUN3D is a node-based CFD solver [51]; therefore, in this application, n represents the number of nodes on the vehicle surface, with each node producing a C_p value to be predicted by the ROM. The result of the CFD simulations is $\mathbf{X} = [\mathbf{x}^1, \dots, \mathbf{x}^m] \in \mathbb{R}^{n \times m}$. Before applying POD, $\mathbf{X}$ is mean centered, resulting in [20]

$$\bar{\mathbf{x}} = \frac{1}{m} \sum_j \mathbf{x}_j = 0 \quad (3)$$

POD projects the high-dimensional snapshot matrix $\mathbf{X}$ into the low-dimensional latent space spanned by orthonormal basis vectors called *modes* [20]. The columns of the mode matrix Φ consist of a set of orthonormal modes, $\Phi = [\phi^1, \dots, \phi^d] \in \mathbb{R}^{n \times d}$, where d defines the dimension of the latent space and the number of preserved modes [20]. By optimizing the Rayleigh quotient [53],

$$\max_{\phi_j} \frac{\phi_j^T \mathbf{S} \phi_j}{\phi_j^T \phi_j} \quad (4)$$

the number of preserved POD modes can be determined, where $\mathbf{S} = (1/m)\mathbf{X}\mathbf{X}^T$ is a sample data covariance matrix [20]. The mode matrix Φ is calculated by performing singular value decomposition (SVD) on the input snapshot matrix $\mathbf{X}$ [20]:

$$\mathbf{X} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^T \quad (5)$$

where $\mathbf{U} \in \mathbb{R}^{n \times n}$, $\mathbf{V} \in \mathbb{R}^{m \times m}$, and $\mathbf{\Sigma} = \text{diag}(\sigma_1, \dots, \sigma_r) \in \mathbb{R}^{n \times m}$ such that $\sigma_1 \geq \sigma_2 \geq \dots \geq \sigma_r \geq 0$ and $r = \min(n, m)$ [20,37]. The eigenvectors of $(1/m)\mathbf{X}^T\mathbf{X}$ are the columns of $\mathbf{V}$, and the eigenvectors of $(1/m)\mathbf{X}\mathbf{X}^T$ are the columns $\mathbf{U}$, with corresponding eigenvalues defined by $\sigma_1^2, \dots, \sigma_r^2$ [20].

The main challenge in POD is that SVD can be computationally expensive with large matrices. Traditional SVD methods result in computation scaling with $O(nm^2)$, requiring $O(nm)$ of memory [54]. Recall in this entry capsule problem that n represents the number of nodes on the surface of the vehicle and m represents the number of snapshots or CFD time step samples. To improve the computational efficiency of the SVD component in POD, a streaming SVD algorithm developed by Maulik and Mengaldo [54] was implemented. Instead of performing SVD on the full input matrix $\mathbf{X}$, a QR decomposition is applied to a batch set of data, A_i , and then SVD is applied to the resulting upper triangle matrix R [54]. Additionally, the user specifies a value k to define the amount of preserved left singular vectors or largest coherent structures to preserve, and a forget factor, $ff \in [0, 1]$, to control the effect of older batches on the final result of the streaming SVD [54]. In this application, $ff = 1.0$ so that the SVD utilizes all the batches equally [54]. Due to the significant number of CFD time steps being sampled to construct this ROM, a value k is selected where $k \ll m$ and $k > d$ to preserve as much variance as possible in each batch before optimizing the number of preserved POD modes. The streaming SVD method reduces the computation cost of SVD to $O(nmk)$ operations and $O(nk)$ of memory, enabling the rapid eigenvalue decomposition of the entry capsule input snapshot matrix [54].

The columns of $\mathbf{U}$ represent the POD modes, ϕ_j [20]. A low-rank matrix $\mathbf{U}' \in \mathbb{R}^{n \times d}$ can be obtained by determining a rank d , where $d < r$ [53]. This is achieved by arranging the $\mathbf{\Sigma}$ and the corresponding columns of $\mathbf{U}$ in descending order and then preserving the first d columns of $\mathbf{U}$ that correspond to the largest singular values in $\mathbf{\Sigma}$ [20].

The relative information content (RIC) of including d basis vectors in a reduced rank model is defined as [20,53]:

$$\text{RIC}(d) = \frac{\sum_{i=1}^d \sigma_i^2}{\sum_{j=1}^r \sigma_j^2} \quad (6)$$

The number of basis vectors d to be preserved in a POD approximation must be specified by the user [20]. The desired information content $\delta \in [0, 1]$ is selected, resulting in the preservation of d basis vectors that achieve $\text{RIC}(d) \geq \delta$ [20,37]. In the literature, values of $\delta \geq 0.99$ are commonly seen [20,42]. After selecting the number of preserved modes, the latent space coordinates can be calculated by $\mathbf{Z} = \Phi^T \mathbf{X}$, where the matrix $\mathbf{Z} = [\mathbf{z}^1, \dots, \mathbf{z}^m] \in \mathbb{R}^{d \times m}$ [20]. The latent space matrix $\mathbf{Z}$ can become increasingly large as thousands of m snapshots are provided by forced oscillation CFD simulations, regardless of the amount of POD modes identified by DR.

C. Interpolation or Regression

After the latent space matrix $\mathbf{Z}$ is generated, a parametric mapping is generated that can be used to predict unsampled latent space coordinates [20]. Interpolation or regression functions g_i are trained using input state space variables $\mathbf{p}$ to predict coordinates in the latent space $\mathbf{z}_i$, resulting in $g_i: \mathbf{p} \mapsto \mathbf{z}_i$ [20]. With this research, the input state space $\mathbf{p}$ is composed of α and $\dot{\alpha}$ pairings. These pairings are mapped to the generated latent space coordinates, allowing for the parametric prediction of unsampled latent space coordinates. A variety of interpolation and regression models have been used in the ROM literature, including cubic splines [21], radial basis functions (RBFs) [24,42], Kriging [20], and artificial neural networks (ANNs) [55]. In this research, a Kriging regressor with a Matérn 3/2 kernel optimized with a maximum log-likelihood criterion was leveraged to predict unseen latent coordinates [56].

D. Backmapping

Leveraging the regression to predict an unsampled design point $\mathbf{p}^*$, the predicted latent space coordinates can be transformed into the FOM through *backmapping* techniques. By implementing POD as the DR technique, backmapping is achieved by the linear projection $\mathbf{x}(\mathbf{p}^*) = \Phi \mathbf{z}(\mathbf{p}^*)$ [20]. This relationship makes POD a favorable DR method, as there is a direct linear relationship between the latent coordinates and the full-order space [20]. The resulting ROM from the established mathematical formulations is of the form shown in Eq. (7). This equation summarizes the linear relationship between POD modes and the overall response of the ROM.

$$\hat{\mathbf{f}}(\mathbf{x}; \mathbf{p}) = \mu(\mathbf{x}) + \sum_{i=1}^d \mathbf{z}_i(\mathbf{p}) \phi_i(\mathbf{x}) = \mu + \Phi \mathbf{z} \quad (7)$$

E. Evaluating the ROM Performance

To evaluate the accuracy of the ROM, a validation forced oscillation trajectory is generated, resulting in v validation points. The predictive accuracy of the ROM is evaluated across the validation trajectory using a series of quantitative and qualitative metrics. The total mean absolute error (MAE) is used to assess the accuracy of the ROM at predicting the entire field and is calculated using

$$\text{MAE}_{\text{total}} = \frac{\sum_{j=1}^v |\mathbf{x}_j(\mathbf{p}^*) - \mathbf{x}_j^*|}{v} \quad (8)$$

where $\mathbf{x}_j(\mathbf{p}^*)$ is the predicted field at the unsampled vehicle state, and $\mathbf{x}_j^*$ is the actual field at the unsampled point. The main sources of error in the ROM can be decomposed into two main sources: compression error and regression error. The compression error measures the error that is incurred when an FOM solution is projected into the ROM subspace, which can be computed using:

$$\text{MAE}_{\text{comp}} = \frac{\sum_{j=1}^v |\Phi \Phi^T x_j^* - x_j|}{v} \quad (9)$$

where $\Phi \Phi^T x_j^*$ is the reconstructed field using the POD modes, and x_j is the original field from the FOM. The regression error measures the accuracy with which the regression is able to predict the latent space coordinates at unseen points and is generated using

$$\text{MAE}_{\text{reg}} = \frac{\sum_{j=1}^v |z_j(p^*) - z_j|}{v} \quad (10)$$

where $z_j(p^*)$ is the predicted latent coordinates of an unsampled vehicle state, and z_j is the actual latent coordinates of the unsampled vehicle state. By splitting up the error between compression and regression, the ROM can be evaluated in a modular manner to assess the accuracy of each process.

Additionally, aerodynamic coefficients C_A , C_Y , C_N , C_m , C_n , and C_l can be calculated by integrating the predicted ROM pressure field and comparing the results to the aerodynamic coefficients generated by FUN3D. Figure 1 shows the coordinate system for the vehicle, noting α , β , and relevant aerodynamic coefficients. The training data for the ROM is in the FUN3D reference frame; therefore, the coordinate system displayed in Fig. 1 is in the FUN3D reference frame. This is slightly different from the common trajectory simulation reference frames, where the z -axis is pointed in the opposite direction. Due to the forced pitching motion of the entry capsule, only C_A , C_N , and C_m will result in coherent trends. The other coefficients will only be a function of the chaotic nature of the wake; therefore, they will not be reported. Additionally, the predicted flowfield and actual flowfields can be compared visually and analytically to demonstrate the effectiveness of the ROM to generate the unsampled flowfield.

III. Application: Genesis Sample Return Capsule

To achieve the objective of this research of advancing the ability to quantify dynamic stability of blunt-body entry vehicles, an application or test case needs to be identified. This research used the Genesis Sample Return Capsule (SRC), specifically the ballistic range model of the capsule, to construct a ROM using the methodology outlined in the previous section. Figure 2 shows a section view of the ballistic range model of the Genesis SRC from Desai and Cheatwood [57]. Ballistic range testing identified dynamic instabilities, including a tumbling risk of the vehicle during longer flights [58]. Using the Genesis SRC as a test case for the ROM allows the results to be compared to ballistic range results and compared to other ongoing studies on the Genesis SRC.

The CFD data for this ROM was generated by Willier et al. using an outer mold line provided by the NASA Langley Research Center [59]. Figure 3 displays a quarter view of the CFD grid utilized to

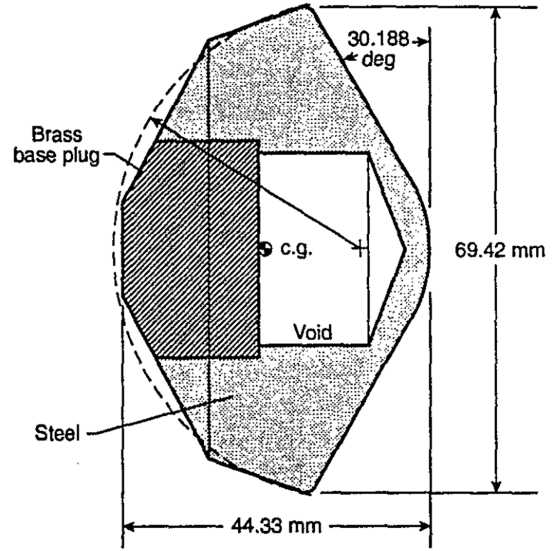


Fig. 2 Genesis ballistic range model [57].

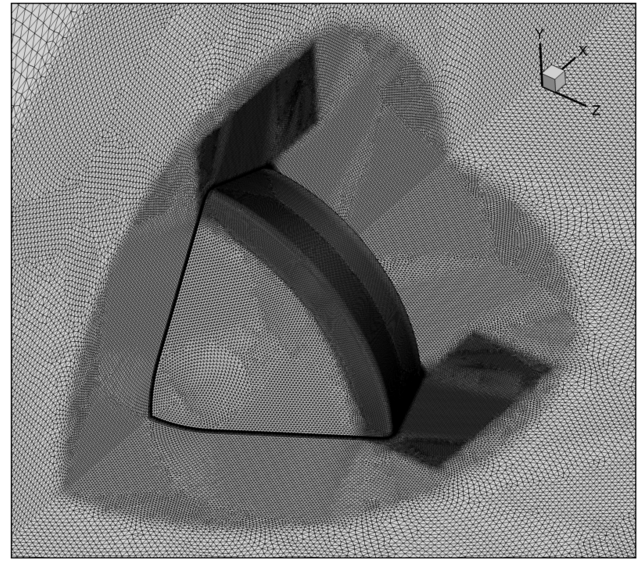


Fig. 3 Genesis quarter grid [59].

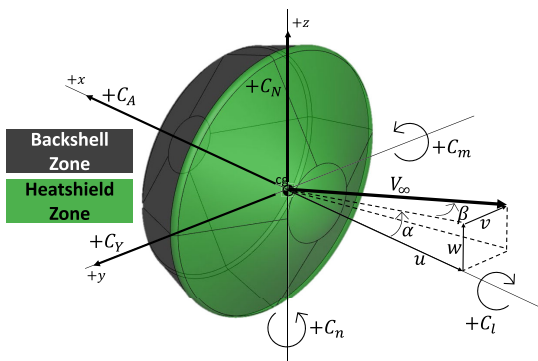


Fig. 1 Vehicle coordinate system. (Graphic represents the FUN3D reference frame, which is slightly different from common trajectory simulation reference frames, where the z -axis is pointed in the opposite direction. Caution should be used when comparing the results of this research to other analyses.)

generate the training data for the ROM. The CFD grid totaled 53.8 million tetrahedral nodes, with 6 vehicle lengths in the front of the vehicle and 15 vehicle lengths behind the vehicle [59]. The primary drivers for grid refinement were the resolution of pressure and shear distributions on the surface of the vehicle, the metrics used as training data for the ROM [59]. Details about grid refinement and structure, CFD settings, and flow initialization used to generate the ROM training data can be found in [59]. The results of Willier et al. indicate that aerodynamic coefficients produced from these CFD simulations align with data produced from the ballistic range shots conducted by Cheatwood et al. [58]. The CFD grid results in 103,856 nodes on the vehicle surface used to track pressure data.

Table 1 summarizes the amplitudes and frequencies used to generate the input state space sampled in this research. Each forced oscillation CFD simulation was simulated at $M_\infty = 1.5$ and run for 20,000 time steps with $\Delta t^* = 582.9$ ns between each snapshot, resulting in 4 periods of oscillation for $f^* = 343.11$ Hz and 8 periods of oscillation for $f^* = 686.22$ Hz. The first 5000 time steps, 1 period of oscillation for $f^* = 343.11$ Hz and 2 periods of oscillation for $f^* = 686.22$ Hz, of each sampled trajectory were removed from the data set to eliminate transients present in the CFD solution. A grand total of 240,000 snapshots of surface pressure data were generated from the forced oscillation CFD simulations.

Table 1 Summary of forced oscillation CFD simulation settings

Frequency, f^* (Hz)	Amplitude, A (deg)	Classification
343.11	5	Training
—	6	Training
—	7	Training
—	8	Training
—	9	Training
—	10	Validation
—	11	Training
—	12	Training
—	13	Training
—	14	Training
—	15	Training
686.22	5	Training
—	7.5	Training
—	10	Training
—	12.5	Training
—	15	Training

Table 2 ROM snapshot matrix dimensions

Component	Grid nodes, n	Snapshots, m
Heatshield	39,988	22,500
Backshell	63,868	22,500

Two ROMs were generated: a heatshield ROM and a backshell ROM. The data zones for each ROM are identified in Fig. 1. Splitting the data aids in reducing the dimensionality of each ROM and isolates the contribution of the heatshield and backshell pressure fields on the overall vehicle aerodynamic response. The same ROM methodology was applied to the heatshield and the backshell, the only difference being the RIC used for each ROM. Each input snapshot matrix was composed of CFD data from every 10th CFD time step. The data provided by the $A = 10^\circ$, $f^* = 343.11$ Hz forced oscillation trajectory was used as a validation data set and was not included in the input snapshot matrices. The resulting snapshot matrix dimensions are summarized in Table 2.

IV. Results

A. Evaluating Dimensionality Reduction

Figure 4 shows the RIC plot for the constructed POD ROMs of the Genesis SRC heatshield and backshell. This plot indicates the variance stored in each mode and the necessary number of modes to achieve the specified RIC. Figure 4a shows that 10 modes are required to achieve an RIC ≥ 0.999 . Figure 4b shows that 46 modes are required to achieve an RIC ≥ 0.99 . A lower RIC was selected for the backshell ROM to minimize the computational requirements of training the regression on the backshell latent space. A significant amount of compression can be achieved using POD with streaming

Table 3 MAE summary

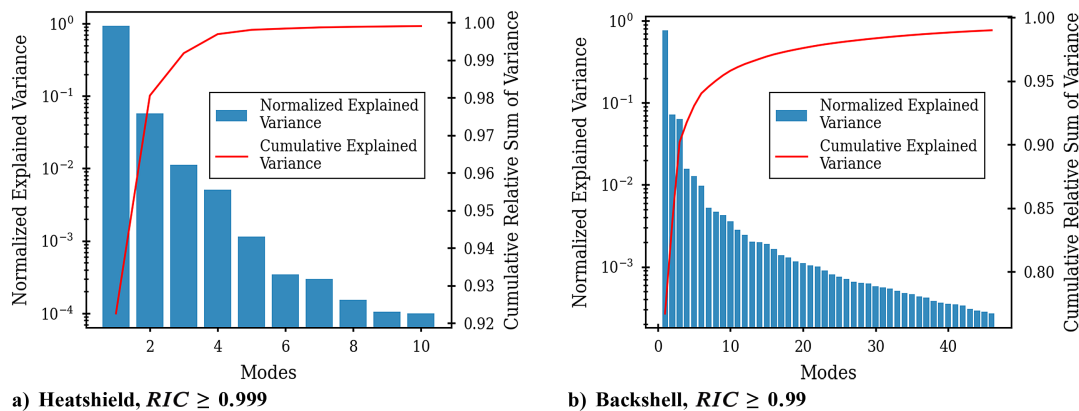
Component	Compression error	Interpolation error	Total error
Heatshield	0.7535 ± 0.0018	0.0 ± 0.2031	0.7539 ± 0.0034
Backshell	-0.3064 ± 0.0012	0.0 ± 0.1913	-0.2952 ± 0.0103

SVD, reducing the number of degrees of freedom by about three orders of magnitude on both the heatshield and backshell. The MAE for the compression step was calculated using Eq. (9). The true mean of the pressure coefficient C_p across all the heatshield nodes is 0.7535, with an MAE at each node equal to ± 0.0018 . The value can be equated to changes in C_A and C_N by adding and subtracting the MAE to the pressure field and integrating the field to produce aerodynamic coefficients. Therefore, the MAE of C_p on the heatshield results in a confidence interval in C_A equal to ± 0.00028 and the confidence interval in C_N equal to ± 0.00028 . The true mean of C_p across all the backshell nodes is -0.3064 , with an MAE at each node equal to ± 0.0012 . This MAE in C_p on the backshell results in a confidence interval in C_A of ± 0.00019 and a confidence interval in C_N of ± 0.00019 . These metrics are summarized in Table 3.

Figure 5 shows a visualization of the three most significant POD modes on the heatshield of the vehicle, and Fig. 6 shows the three most significant POD modes on the backshell of the vehicle. In total, there are 10 modes being utilized to reconstruct the pressure distribution of the heatshield, and 46 modes are used to reconstruct the pressure distribution of the backshell. While literature notes that POD modes can represent nonphysical features [46], these modes could provide insight into the root cause of the dynamic response of entry capsules. POD modes represent the dominant structures in the data that were recovered through DR, and visualizing the POD modes provides an understanding of what behaviors are driving the observed responses.

Figure 7 shows the values of the three most significant latent spaces for the heatshield ROM throughout the forced oscillation trajectories, with each color in the plot representing a different amplitude and frequency. The latent space is spanned by α and $\dot{\alpha}$, with an associated latent coordinate value for each pairing. The three most significant latent spaces of the heatshield display noticeable functional behaviors in the α and $\dot{\alpha}$ space, a strong indicator a regression can be fit successfully to predict unseen latent coordinates.

Figure 8 shows the three most significant latent spaces for the backshell ROM. The backshell latent spaces display a more chaotic response resulting from changes in α and $\dot{\alpha}$, compared to the functional response seen in the three most significant heatshield latent spaces. This is a direct result of the nature of vortex shedding and subsonic recirculation occurring on the backshell. The chaotic behavior of the backshell latent coordinates will impact the regression stage of the ROM and the ability to predict unseen latent coordinate values. The characteristics displayed in the latent space of the heatshield and backshell align with literature about the chaotic aerodynamic interactions on entry capsules. These plots also indicate that there is uncertainty associated with latent

**Fig. 4** RIC plot for pressure ROM.

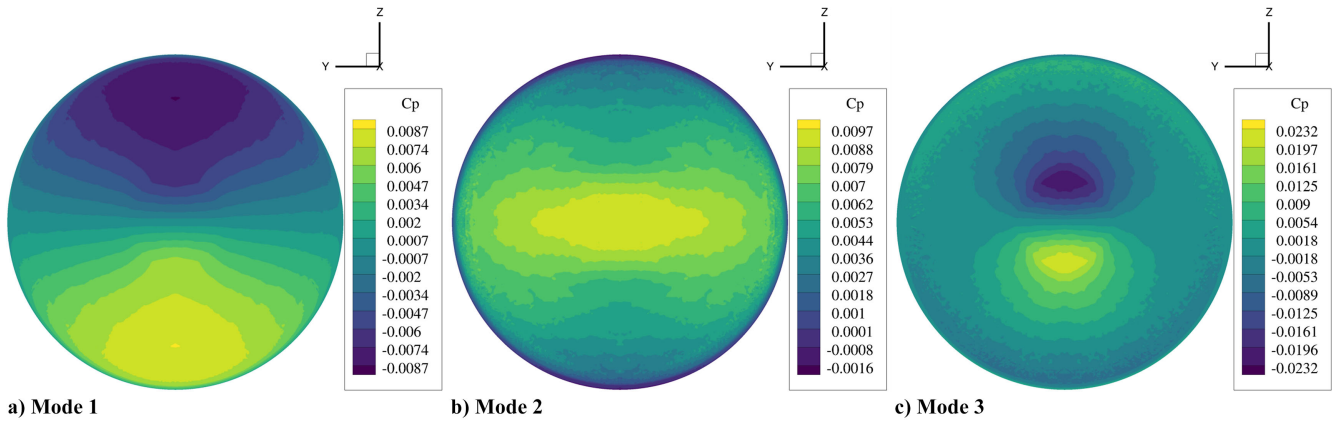


Fig. 5 Three most significant POD modes for heatshield ROM.

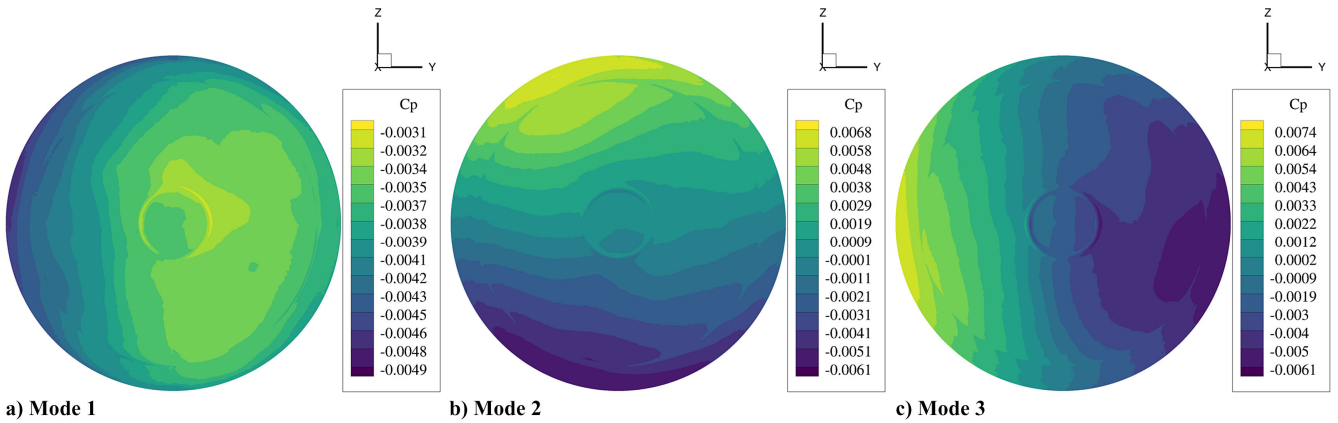


Fig. 6 Three most significant POD modes for backshell ROM.

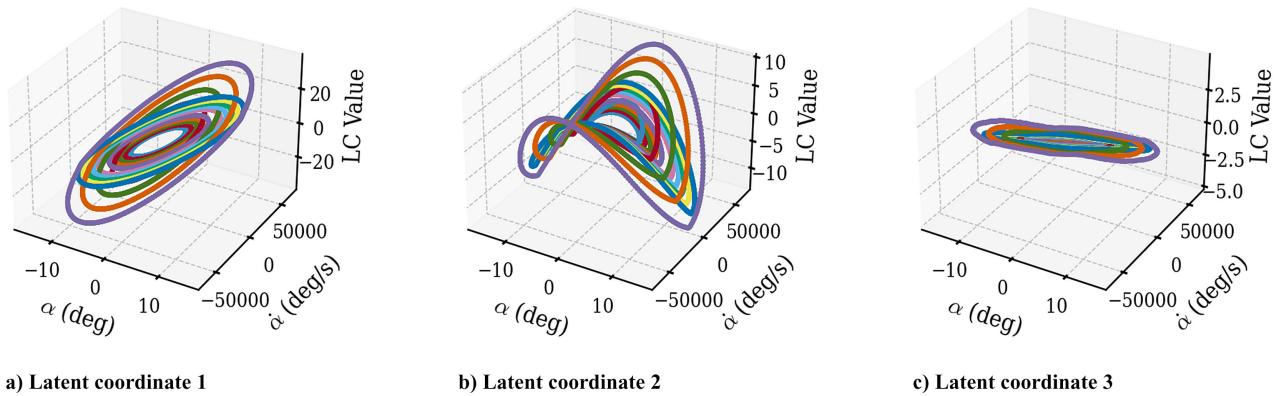


Fig. 7 Three most significant latent spaces for heatshield ROM.

coordinates for the backshell ROM. For identical vehicle states (i.e., the same $[\alpha, \dot{\alpha}]$ values), a spread of latent coordinates can be seen from the same forced oscillation trajectory. This trend is clear in the purple data shown in Fig. 8a, representing the latent space spanned by the forced oscillation trajectory from $A = 15^\circ$, $f^* = 686.11$ Hz. The multiple periods of forced oscillation data being represented result in a spread of latent coordinates. Latent coordinates are the low-dimensional representation of the full-order space, in this case the full surface pressure distribution. Therefore, this uncertainty in the latent space represents uncertainty associated with pressure distributions of the backshell. This spread will impact the ROM's ability to approximate unsampled pressure distributions.

Time-series statistics, such as autocorrelation, can be used to determine underlying structures in the data throughout a trajectory [60]. In this example, autocorrelation can be used to decipher signal correlations between latent space coordinates to determine if the underlying physical phenomena observed in the FOM are captured in the latent space and if there are other underlying signals that are being recovered by DR. Figure 9 shows the autocorrelation plots for the three most significant latent spaces for the heatshield ROM. The x-axis of these plots represents the number of lags or previous time steps of latent space information, and the y-axis represents the autocorrelation value. In these plots, there is a strong, sinusoidal-like function being resolved in the correlations. This trend is to be expected since the vehicle is

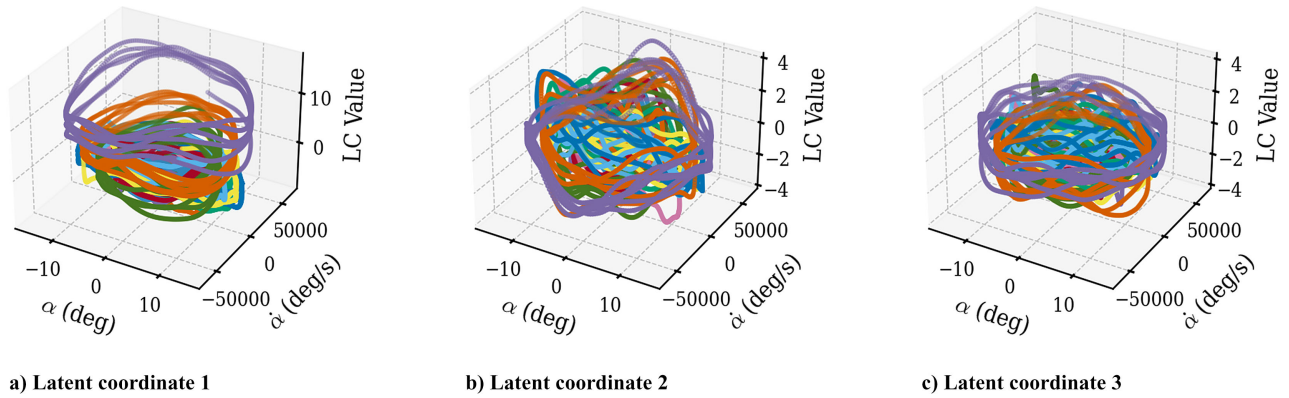


Fig. 8 Three most significant latent spaces for backshell ROM.

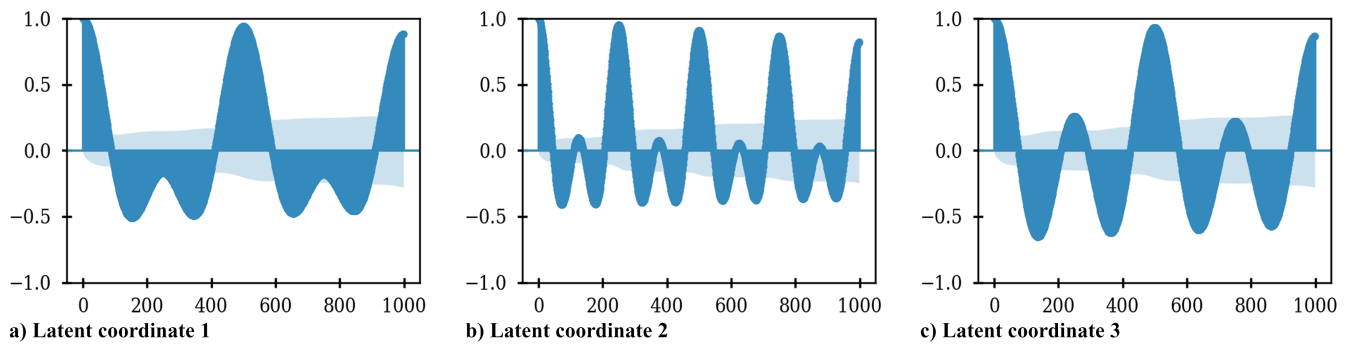


Fig. 9 Autocorrelation of three most significant latent spaces for heatshield ROM.

being forced into a prescribed sinusoidal behavior. Furthermore, a Fourier analysis on the autocorrelation histories of each latent coordinate reveals dominant frequencies of 343.1, 686.2, and 1372 Hz. These dominant frequencies are all harmonics of the forcing frequencies considered in this problem, which is expected. Furthermore, the resulting pattern of peaks and troughs in Fig. 9 is consistent with the constructive and destructive interference of these frequencies. Between the latent space plots shown in Fig. 7 and the autocorrelation plots shown in Fig. 9, it appears that the heatshield data is dominated by the quasi-static pressures resulting from the oncoming flow. The changes in the latent coordinates can be linked directly to changes in the vehicle's α . Additionally, the training data include three periods of oscillation per forced oscillation CFD simulation. In the low-frequency simulation ($f^* = 343.11$ Hz), this results in an oscillation period of 500 nondimensional time steps. In the high-frequency simulation ($f^* = 686.22$ Hz), this occurs every 250 nondimensional time steps. These points align directly with the peaks in the autocorrelation plots shown in Fig. 9.

Figure 10 shows the autocorrelation plots for the three most significant latent spaces for the backshell ROM. The autocorrelation history of these coordinates also exhibits an oscillatory behavior but likely is driven by a wider spectrum of underlying frequencies due to the turbulent wake. To examine this further, a fast Fourier transform (FFT) is used to identify the underlying frequencies and their power spectral density (PSD) for these latent coordinate histories. The underlying frequencies and their PSD values are shown in Fig. 11. In this plot, it is observed that a wide range of frequencies can be observed in the latent coordinate time series, with the PSD decaying for higher frequencies. Furthermore, it is observed that the rate of decay in PSD versus frequency exhibits an $f^{-5/3}$ relationship (shown by the dashed line), which is consistent with the energy cascade from turbulent theory.

Leveraging the latent space generated by POD, the full-order space for all the training points was reconstructed using backmapping. Recall that, with POD, this is achieved by $\mathbf{x}(p^*) = \Phi \mathbf{z}(p^*)$. This creates a low-dimensional form of the input snapshot matrix $\mathbf{X}$. Each

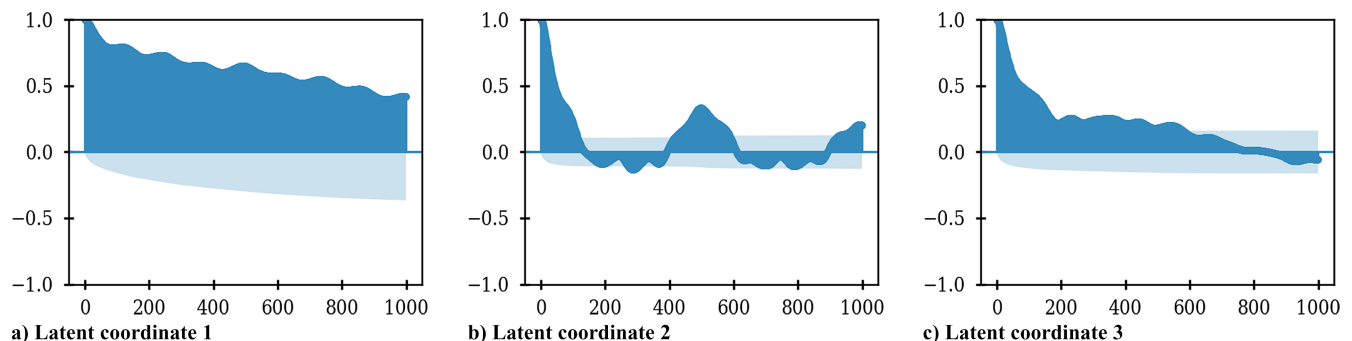


Fig. 10 Autocorrelation of three most significant latent spaces for backshell ROM.

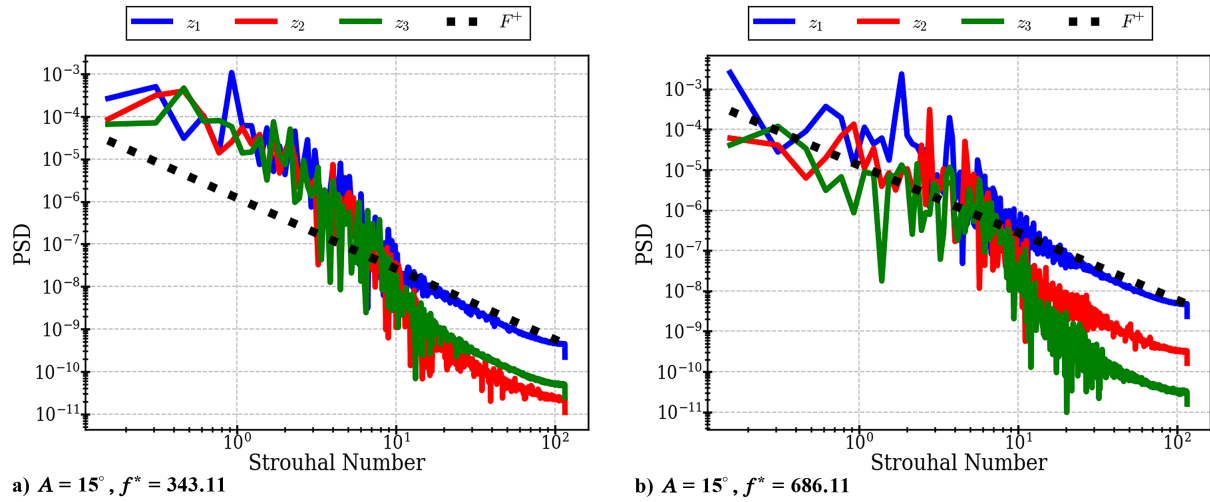


Fig. 11 Frequency domain analysis of three most significant latent spaces for backshell ROM.

snapshot was then integrated to produce forces and moments and then nondimensionalized to produce aerodynamic coefficients. Figure 12 shows the C_A actual versus reconstructed aerodynamic coefficient for both the heatshield and backshell for the $A = 13^\circ$, $f^* = 343.11$ Hz forced oscillation trajectory. In these plots, a single time step represents an α and $\dot{\alpha}$ pairing resulting from the forced trajectory. The red line indicates the C_A value resulting from the CFD simulation data. The green line indicates the C_A value resulting from reconstructing the flowfield from the preserved POD modes. Due to the similarities between the actual values and reconstruction, residual plots for the

aerodynamic coefficients are also generated. For both the heatshield and backshell, the ROM can precisely reconstruct the sampled training points. If larger discrepancies were present in these two curves, then it would indicate that POD cannot properly project the FOM into a linear subspace. However, Fig. 12 indicates that the data can be reduced to a linear subspace.

Figure 13 shows the C_N actual versus reconstructed aerodynamic coefficient for both the heatshield and backshell for the $A = 13^\circ$, $f^* = 343.11$ Hz forced oscillation trajectory. As was seen in Fig. 13, the POD ROM demonstrates that it can reduce the data

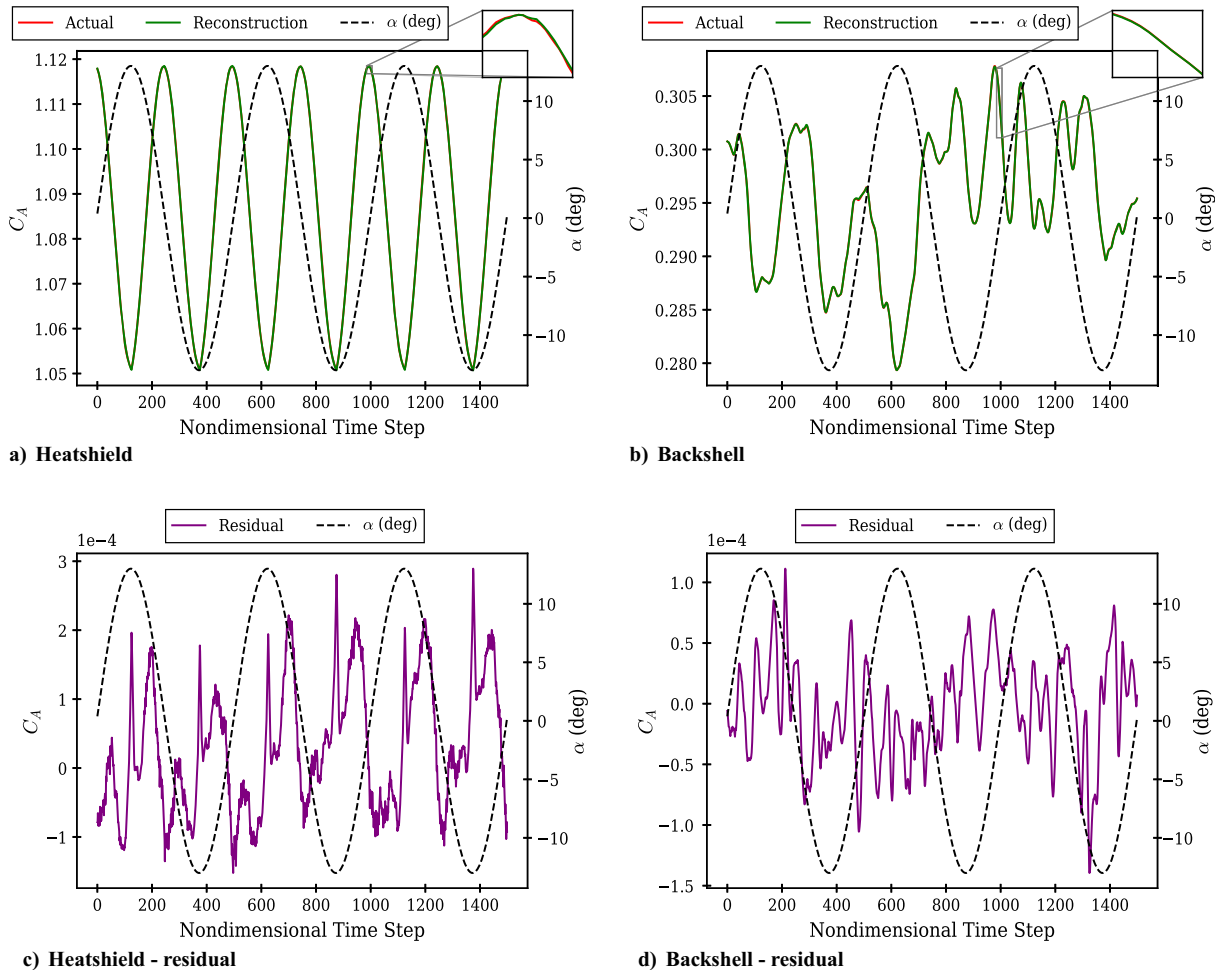


Fig. 12 Actual vs reconstructed C_A across $A = 13^\circ$, $f^* = 343.11$ forced oscillation trajectory.

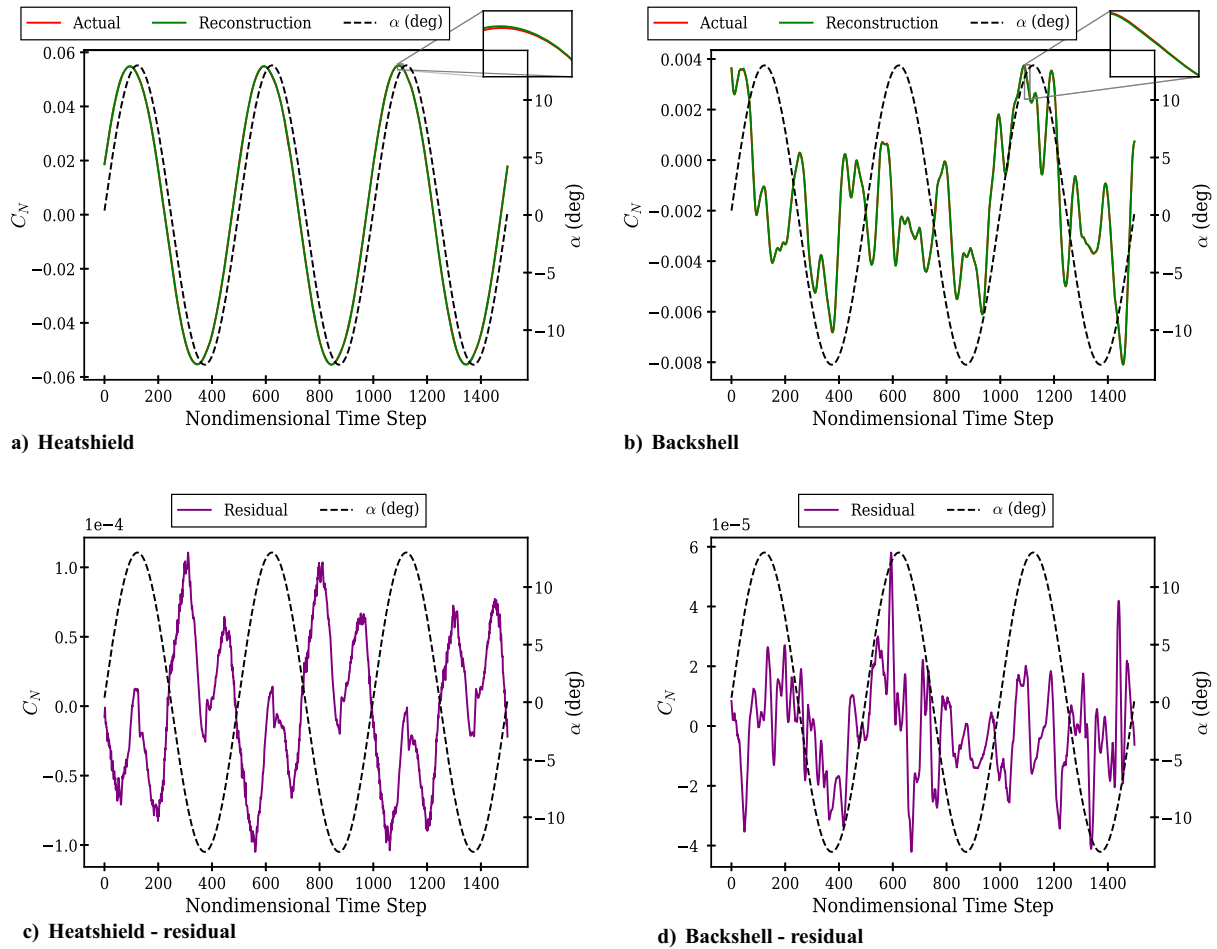


Fig. 13 Actual vs reconstructed C_N across $A = 13^\circ$, $f^* = 343.11$ forced oscillation trajectory.

into a linear subspace and reconstruct pressure fields that preserve both C_A and C_N behaviors identical to the CFD data the ROM was trained on.

Figure 14 shows the C_m actual versus reconstructed aerodynamic coefficient for both the heatshield and backshell for the $A = 13^\circ$, $f^* = 343.11$ Hz forced oscillation trajectory. This plot is the first time the CFD-based coefficient line (red) can be isolated from the ROM-reconstructed coefficient line (green). Though, this difference is subtle, indicating again that the POD ROM can reconstruct pressure fields that preserve trends in C_m across the training points. The key element now is the performance of the ROM at predicting unsampled training points.

B. Predicting Unsampled Trajectories

A Kriging regression with a Matérn 3/2 kernel was applied to the ROM to predict the unseen motion of a forced oscillation trajectory where $A = 10^\circ$, $f^* = 343.11$ Hz, resulting in 1500 $[\alpha, \dot{\alpha}]$ pairings to predict the full trajectory. The previously calculated POD modes were applied to the CFD-generated pressure field data from FUN3D, generating the actual latent space of the unseen motion. The Kriging regression was used to predict the unseen latent coordinates at each $[\alpha, \dot{\alpha}]$ pair. Figure 15 shows the predicted versus actual plots of the three most significant latent coordinates of the heatshield ROM. Additionally, residual plots have been calculated due to similarities between actual latent space values and ROM-predicted latent space values. As was hypothesized, the Kriging regression could more easily recover the functional form of the three most significant latent coordinates of the heatshield ROM. The MAE of the regression is calculated using Eq. (10). For the heatshield, the MAE of the latent coordinate value is ± 0.2031 . Given the scale of the latent coordinates shown in Fig. 15, this value is well within acceptable bounds. This MAE is summarized in Table 3.

Figure 16 shows the predicted versus actual plots of the first three latent coordinates of the backshell ROM. Residual plots were not included for the backshell predictions, as it is clear where the prediction error is occurring. The Kriging regression has a much harder time predicting the chaotic response of the backshell latent coordinates. This will impact the ability to approximate an accurate pressure field using the backshell ROM. The MAE of the regression is calculated using Eq. (10). For the backshell, the MAE of the latent coordinate value is ± 0.1913 . Given the magnitude of the backshell latent coordinates shown in Fig. 16, this MAE is more significant, indicating a poor prediction by the Kriging regression on the backshell. This MAE is summarized in Table 3.

Using linear transformations, the pressure distribution can be approximated from the predicted latent coordinates. For each $[\alpha, \dot{\alpha}]$ pair in the 1500 steps that make up the full trajectory, a pressure distribution is reconstructed. A percent difference between the predicted pressure field and actual pressure field at each step is calculated, and then every step is averaged together to condense the ROM accuracy into a single plot. Figure 17 shows the resulting percent difference of the heatshield and backshell ROMs across all 1500 snapshots in the predicted trajectory. Figure 17a shows that the percent difference between the actual and predicted heatshield pressure fields is around 0–10%. The largest errors appear at the shoulder of the vehicle. These discrepancies could be the result of complex aerodynamic interactions like expansion waves that the ROM has challenges accurately predicting. The difference in the shoulder region could be a result of the loss of fidelity as a result of DR, causing the low-frequency modes contributing to the results of this region to be discarded. It is also possible that the CFD grid leveraged in this research needs further refinement in the shoulder region to improve the predictive capabilities of the ROM at the shoulder. The MAE of the total field was calculated using Eq. (8), with the mean of

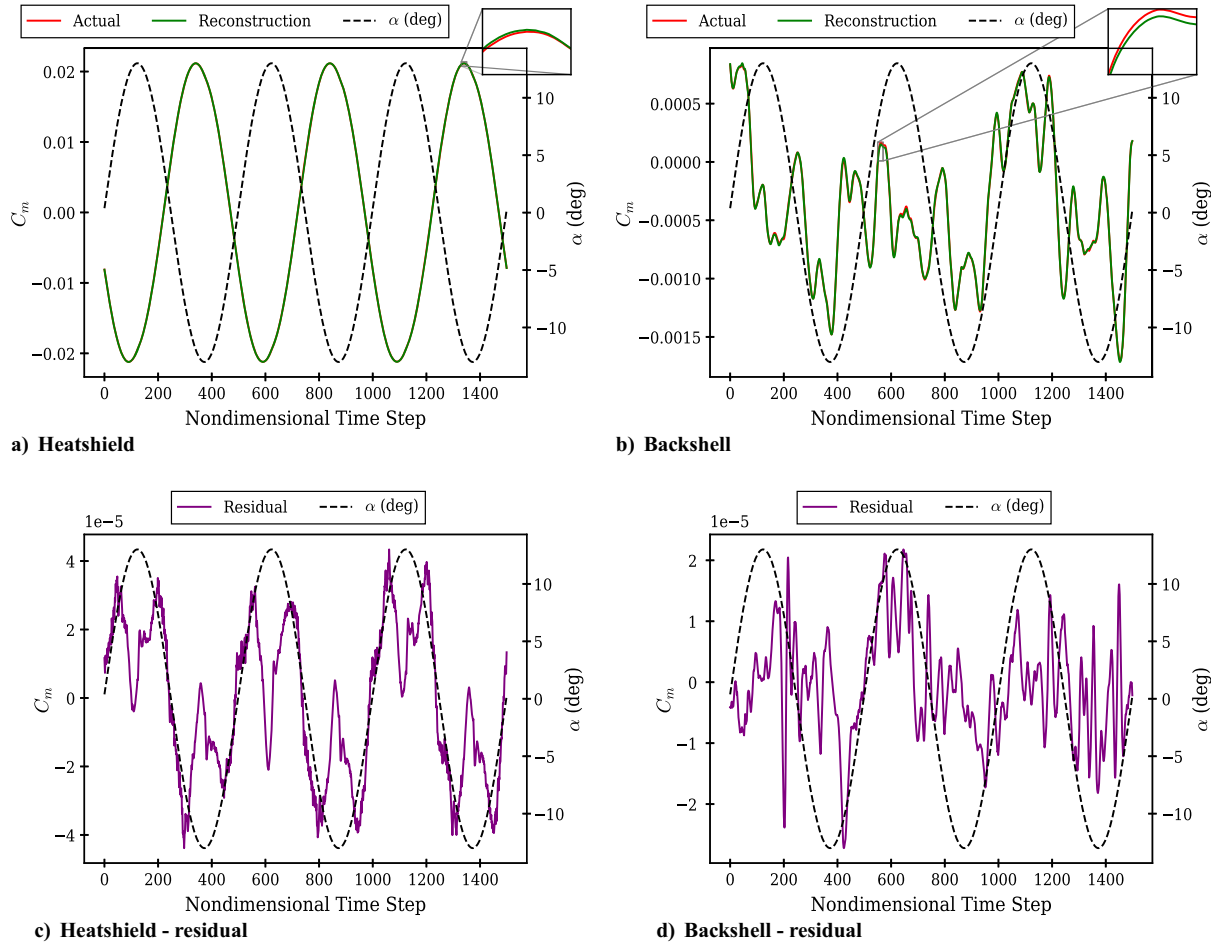


Fig. 14 Actual vs predicted C_m across $A = 13^\circ$, $f^* = 343.11$ forced oscillation trajectory.

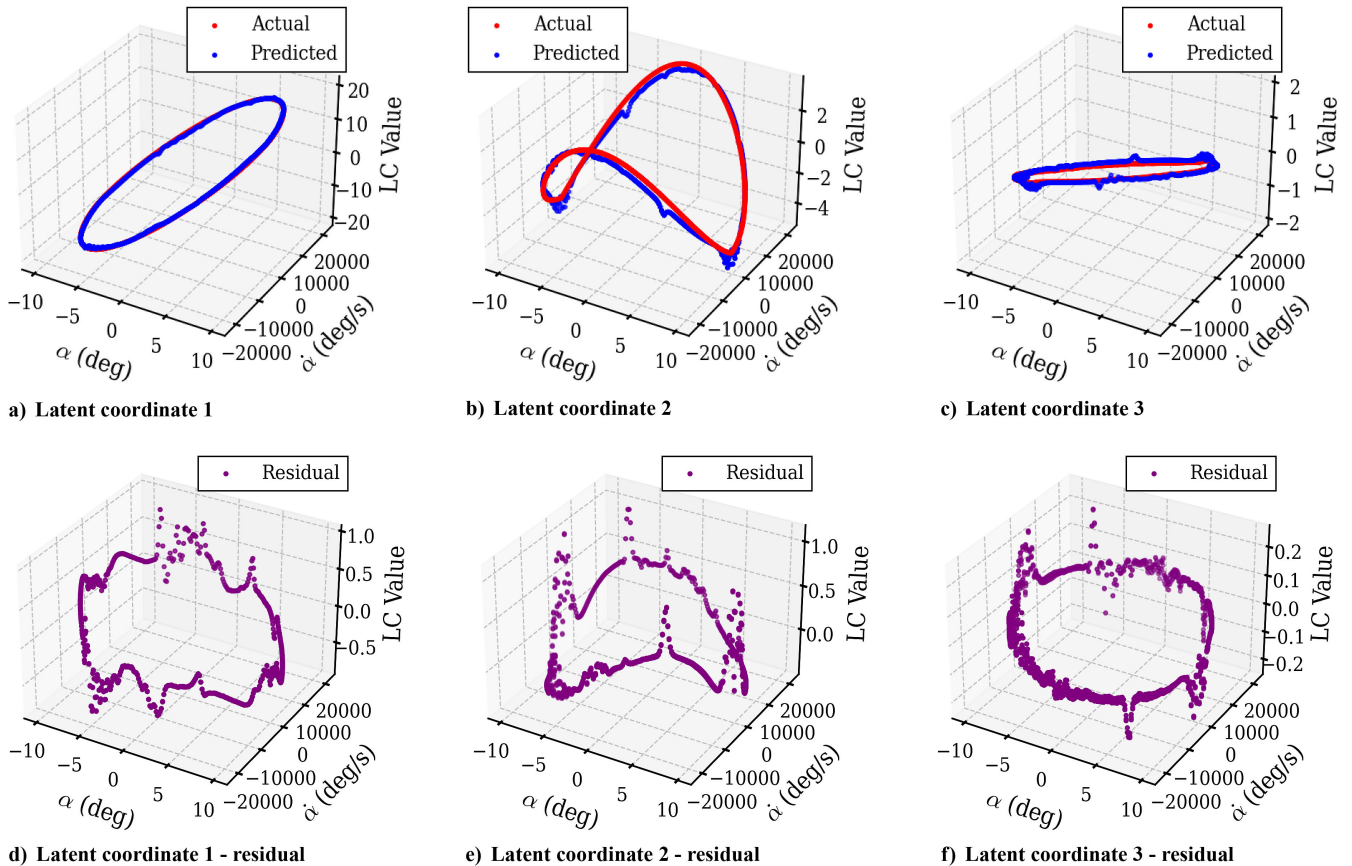


Fig. 15 Predicted vs actual of first three latent coordinates for heatshield ROM.

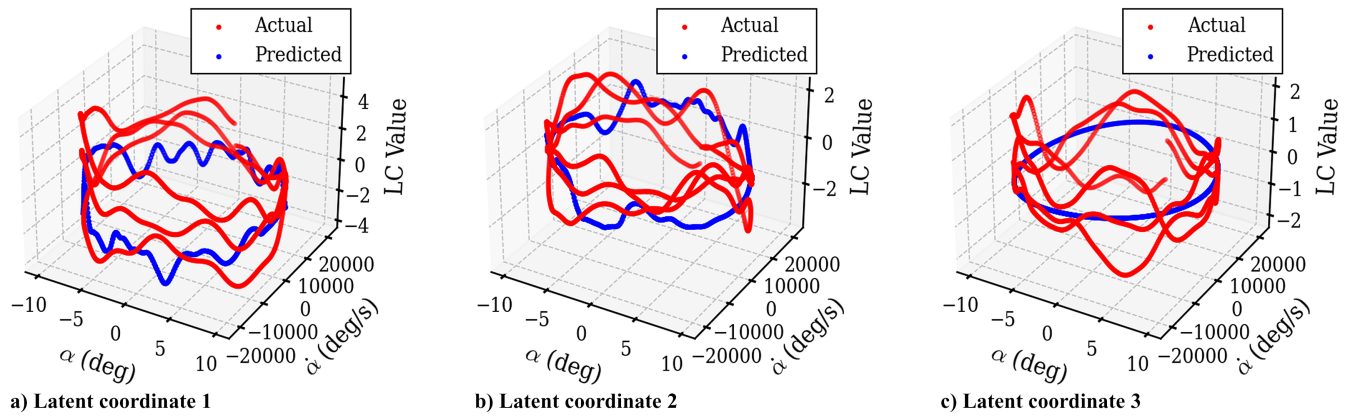


Fig. 16 Predicted vs actual of first three latent coordinates for backshell ROM.

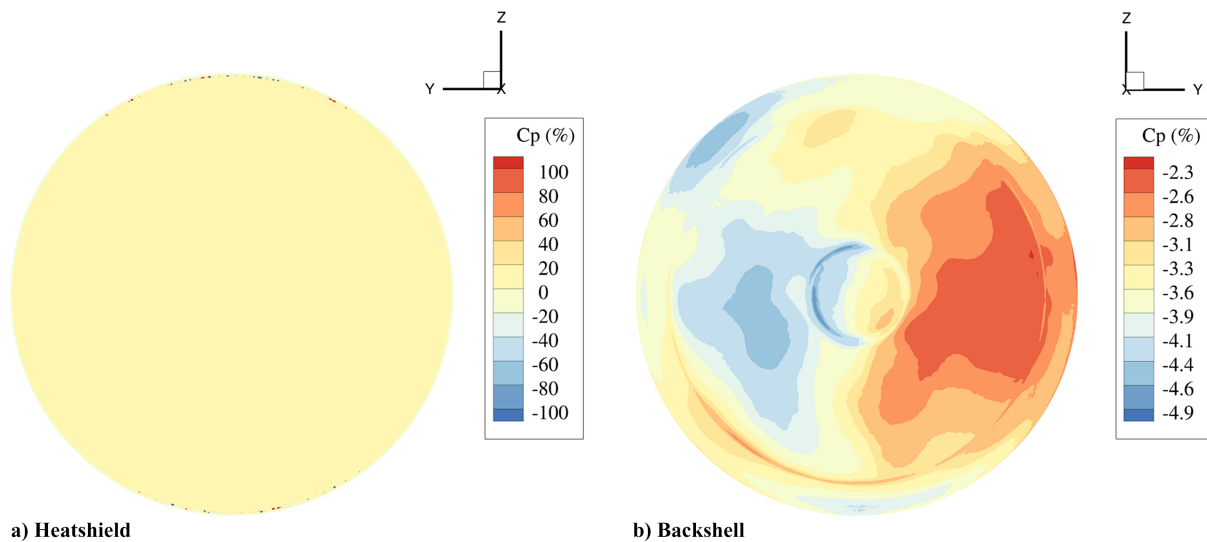


Fig. 17 Average percent difference across 1500 snapshots.

the heatshield field equal to 0.7539 and the MAE at each node equal to ± 0.0034 . This is summarized in Table 3.

Figure 17b shows much larger discrepancies in the percent difference between the predicted versus actual pressure distributions on the backshell. The percent difference range is lower than on the heatshield; however, there are more significant regions of inaccuracies on the backshell. The first three latent coordinate predictions for the backshell (shown in Fig. 16) showed that the predicted latent coordinates are much different from the actual latent coordinates. Therefore, the resulting predicted pressure fields will be different from the actual

pressure fields. The mean of the total backshell field is equal to -0.2952 , with the MAE at each node equal to ± 0.0103 . This is summarized in Table 3.

The predicted pressure field from the ROM can be integrated into the resulting aerodynamic coefficients to explore the accuracy of the predictive capabilities of the ROM. Additionally, it is critical that the ROM is capable of parametric and time-varying prediction of aerodynamic coefficients to explore the dynamic response of the vehicle. Figure 18 shows the axial aerodynamic response across the sampled trajectory. Figure 18a indicates that this aerodynamic

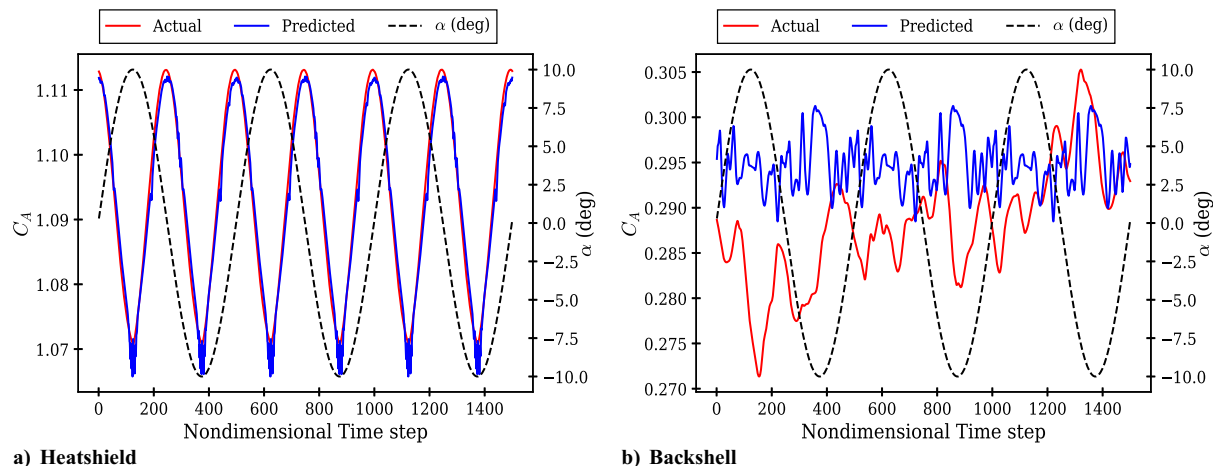


Fig. 18 Actual vs predicted C_A across the validation trajectory.

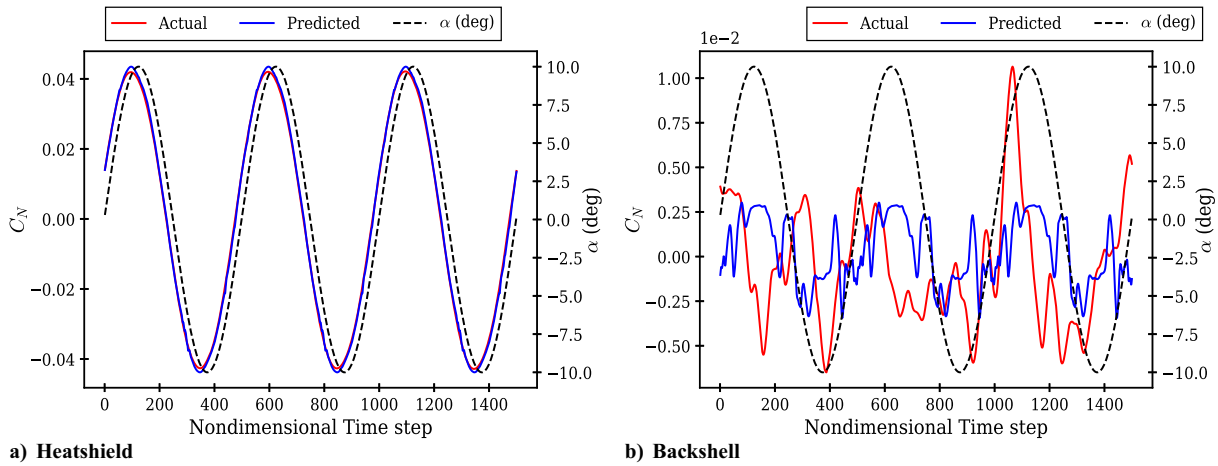


Fig. 19 Actual vs predicted C_N across the validation trajectory.

coefficient can be accurately predicted for the heatshield across a validation trajectory, with subtle discrepancies occurring at the peaks of the oscillation cycle. This is likely a result of discrepancies occurring in the predicted latent space, specifically in the second latent coordinate where the predicted field around $\dot{\alpha} = 0$ is noticeably different than the actual latent coordinates (see Fig. 15b). As expected, challenges in predicting the latent space of the backshell propagate through the predicted pressure field, resulting in discrepancies in the predicted C_A values across the sampled trajectory for the backshell (shown in Fig. 18b). However, the resulting aerodynamic coefficients on the backshell are of a similar magnitude, indicating that the ROM is doing a decent job in its predictions.

Figure 19 shows the normal aerodynamic coefficient across the sampled trajectory. As was seen with C_A , the heatshield coefficient (Fig. 19a) was accurately predicted across the validation trajectory, with slight differences occurring at peak oscillation amplitudes. The backshell coefficient (Fig. 19b) was harder to accurately predict; however, the magnitude of the response is still captured by the ROM.

Figure 20 shows the actual versus predicted aerodynamic moment coefficient for the heatshield and backshell across the sampled trajectory. Similar trends can be seen in C_m , where the heatshield coefficients line up almost exactly, and the backshell coefficients are of the same magnitude but do not align as precisely as the heatshield. Only C_A , C_N , and C_m are explored in this research because the data set was produced from purely pitching motion. The other resulting aerodynamic coefficients display noisy behavior at significantly small magnitudes.

V. Conclusions

The results of this research showed promise in using a POD ROM to quantify the dynamic aerodynamics of blunt-body entry vehicles. POD

leveraging streaming SVD increased computational efficiency by reducing the large input snapshot matrices of the heatshield and backshell into usable data. POD was able to reduce the FOM into a linear subspace and precisely approximate the time histories of aerodynamic coefficients that matched results from CFD simulations. A kriging regression was used to predict unseen latent coordinates, accurately predicting heatshield surface pressure distributions but having more challenges predicting backshell surface pressure distributions. The ROM was capable of accurately predicting pressure distributions on the heatshield for unsampled trajectories. The predicted pressure distributions on the backshell were less accurate, but the magnitude of the aerodynamic response was captured. While the focus of this work was to predict the dynamic response of an entry capsule, this methodology can also provide insight into the root cause of dynamic instabilities of entry capsules. The latent space identified in DR isolated chaotic trends in the backshell data set, emphasizing the unsteadiness that contributes to the dynamic response of the vehicle. Additionally, while it is important to note that POD modes are data driven and can sometimes represent nonphysical phenomena, the POD modes could indicate surface pressure field characteristics that are driving dynamic instabilities of blunt-body entry vehicles. The extracted POD modes isolate high-energy dimensions in the original data field, which might isolate dominant features leading to dynamic instabilities, particularly on the backshell of the entry vehicle. Experimental data on vehicle surface distributions could provide validation to unsteady CFD surface distributions, allowing for more definitive conclusions to be made about trends displayed by the POD modes. For example, advances in pressure-sensitive paint (PSP) testing and other surface distribution mapping techniques could provide the necessary validation of unsteady CFD surface distributions and potentially provide data that could be included in future ROM developments.

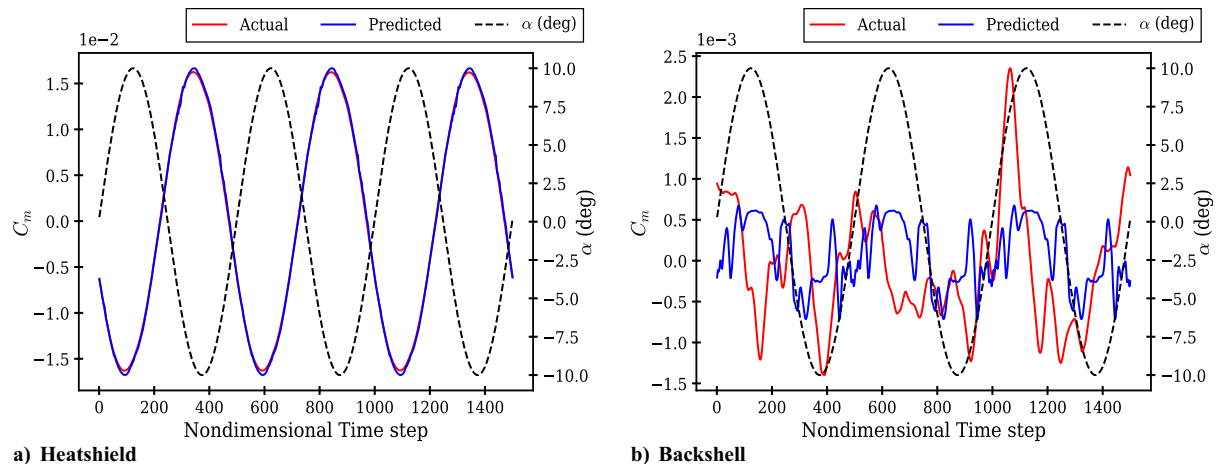


Fig. 20 Actual vs predicted C_m across the validation trajectory.

This research has many avenues of future work to improve and advance these capabilities. The first step is improving the regression stage of the ROM. A Kriging regression has known computational limits, with the computational expense scaling with $O(m^3)$ [56]. While a Kriging regression was implemented on this large data set, the amount of training points is only expected to increase, increasing the burden on the regression. There is literature available that attempts to approximate and scale Gaussian process regressions [61–63], which can be implemented and leveraged in this problem. Due to the chaotic nature of the backshell latent space, it is possible that additional information (e.g., flowfield information, input space parameters, sampled trajectories) would help improve the predictions. All these avenues are being explored in future research. Future work should explore applying dynamic mode decomposition (DMD) to the data and make comparisons between the performance of POD and DMD. The goal of the ROM is to improve upon databases of scalar dynamic stability coefficients by predicting pressure fields at each time step of a 6-DoF trajectory simulation. With these predictive models in place, it is possible in future work to leverage these models to extract scalar coefficients of interest and compare existing methods and experimental results to further validate the proposed approach. Additionally, this work was constructed on forced oscillation data, which can produce an unrealistic vehicle behavior because the vehicle is being “forced” to move in a sinusoidal behavior. Future work in this research will apply this methodology to free-flight CFD data, progressing this methodology to represent more physically consistent entry capsule behavior.

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