

Turbulence-driven losses in sonic booms using multiple scattering theory

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Abstract

Accurate prediction of sonic-boom propagation through the turbulent atmospheric boundary layer is essential for the certification and public acceptance of next-generation low-boom aircraft such as NASA’s X-59. Flight test data show that turbulence produces mean loudness reduction that grow with both turbulence strength and propagation distance, effects not fully captured by current sonic boom prediction tools such as PCBoom. Models based on a nonlinear parabolic equation combine the Burgers equation for nonlinear absorption with the linear parabolic equation for turbulence effects. The one-way parabolic formulation conserves mean energy density as it cannot represent turbulence-induced backscattering losses. To overcome this limitation, the multiple scattering theory (MST) has been used that employs the three-dimensional wave equation and explicitly accounts for scattering from turbulent refractive-index fluctuations. The MST framework reveals two effective propagation modes whose interference produces spectral features observed in flight test data. Extension of the analysis to the Bethe–Salpeter equation yields an expression for the mean energy density that includes both coherent modes and turbulence-induced modal coupling, enabling the prediction of energy losses absent from parabolic models. The expression establishes a physics-based approach for sonic boom prediction and supports the development of next-generation propagation tools for low-boom aircraft.

1 Introduction

The realization of overland commercial supersonic transport critically depends on predicting and minimizing the ground footprint of shockwaves generated by next-generation, low-boom experimental aircraft, such as NASA’s X-59. Idealized, standard ray-tracing propagation models typically represent acoustic shockwaves, called sonic booms, as sharp N-waves or shaped-boom signatures traveling through an otherwise unperturbed atmosphere. A prominent example is NASA’s PCBoom propagation suite [1, 2], which has been extensively validated for N-waves measured above the atmospheric boundary layer (ABL). However, empirical measurements of sonic booms that have propagated through the ABL reveal discrepancies between numerical simulations and field data, due to the effects of atmospheric turbulence present in the ABL. Specifically, PCBoom consistently overpredicts the ensemble mean of acoustic loudness metrics [3].

To investigate the effects of turbulence on sonic boom propagation under strictly controlled conditions, NASA conducted the Sonic Booms Atmospheric Turbulence (SonicBAT) flight test campaign. The campaign, which aimed to isolate the mechanisms responsible for attenuation anomalies in the mean field, took place at Edwards Air Force Base [4]. The test featured a NASA F-18 research aircraft executing supersonic steady level flights from an East-North-East (ENE) trajectory shown in Fig. 1a. A TG-14 research motorglider, shown in Fig. 1b, was deployed to fly directly beneath the F-18’s supersonic path at an altitude

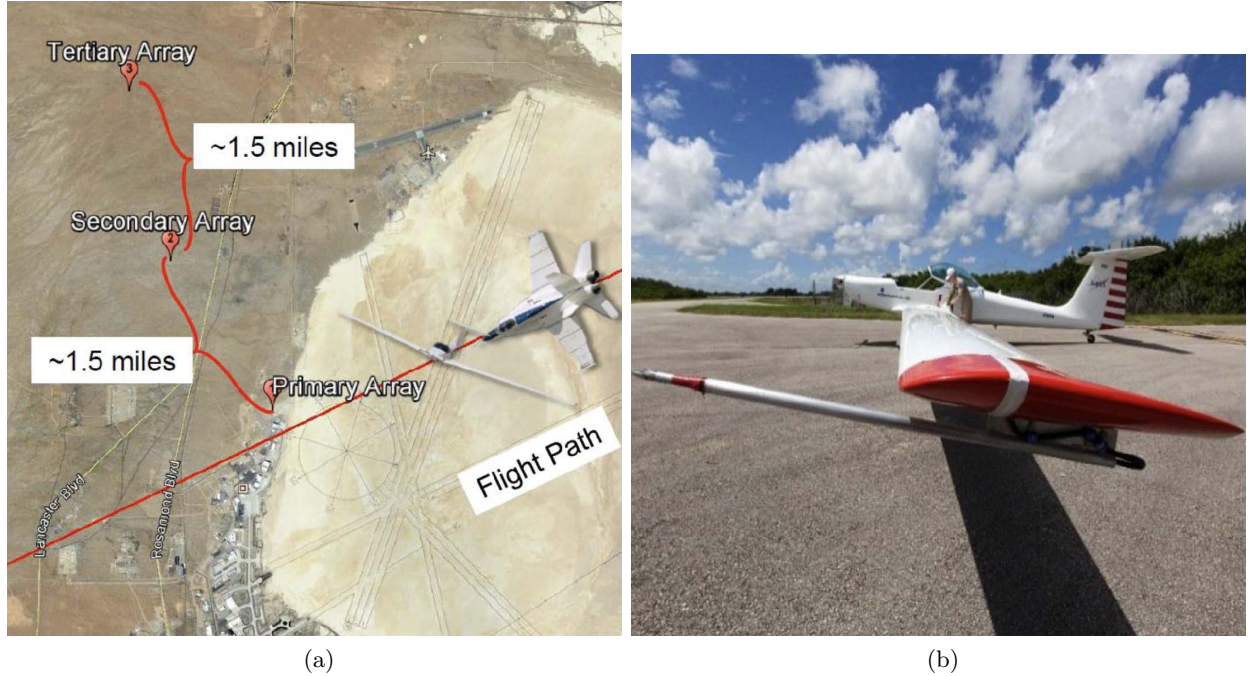


Figure 1: SonicBAT supersonic flight track (a) and the NASA TG-14 motorglider (b). (Adapted from a NASA Report [4]).

ranging between 4,000 and 12,000 ft across all supersonic flights. The motorglider operated at an altitude above the turbulent ABL.

Outfitted with a specialized microphone mounted at its wingtip, the TG-14 was capable of capturing and measuring the clean or “unturbulized” acoustic shockwave uncorrupted by turbulence before it encountered any atmospheric boundary layer interactions. To characterize how this same waveform evolved after propagating through turbulence, a network of ground-based microphones was deployed across the desert floor. Shown in Fig. 1a adapted from Ref. [4], this network consisted of three microphone arrays positioned at increasing lateral distances from the aircraft ground track. The Primary Array was located nearly directly undertrack, while the Secondary and Tertiary Arrays were positioned 1.5 and 3 miles away, respectively, from the undertrack line. By recording sonic booms at progressively greater propagation distances through the ABL, this multi-array configuration enabled a detailed mapping of the spatial evolution of turbulence-induced acoustic variability. The sensors in the Primary Array were spaced about 30 m apart and arranged linearly parallel to the flight track, providing high-resolution sampling of the boom structure immediately after its passage through the boundary layer.

2 Model Validation and Boundary Layer Discrepancies

The acoustic measurements obtained from the TG-14 have been used to validate PCBoom for unturbulized N-waves [5, 6]. An example of the near-perfect agreement between predicted and measured waveforms is shown in Fig. 2a, where the two waveforms closely match. The corresponding distribution of ΔPL , defined as the difference between predicted and measured Perceived Levels (PL), was computed across nearly all supersonic passes. The

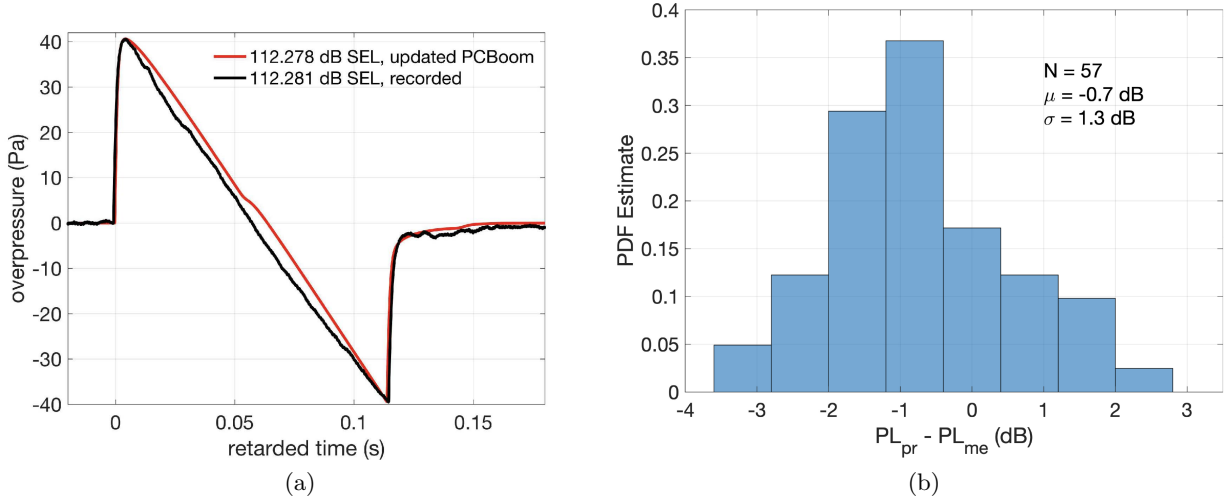


Figure 2: Comparison of prediction and measurement obtained above the ABL. (a) A representative supersonic flight shows excellent agreement between the predicted and measured N-waves, with loudness level expressed as the unweighted sound exposure level (SEL). (b) The probability density function (PDF) of the discrepancy between the predicted (PL_{pr}) and measured (PL_{me}) Perceived Levels is shown for nearly all supersonic flights, illustrating the distribution of prediction error across the dataset.

resulting histogram displayed in Fig. 2b exhibits a mean μ close to zero and a small standard deviation σ , demonstrating excellent overall agreement in turbulence-free conditions.

In contrast, PCBoom in its current form consistently overpredicts the ensemble mean of both the peak overpressure and loudness levels when turbulence is present [7, 8]. Sonic booms measured by the microphones in the Primary Array from one supersonic pass reveal substantial variability as shown in Fig. 3a. Some waveforms exhibit noticeably larger peak overpressures, and the overall N-wave shape is visibly distorted relative to the unturbulized waveform recorded above the ABL (see Fig. 2a). The mean waveform derived from these ground-based measurements is substantially quieter than the PCBoom prediction, with the perceived level reduced by roughly 10 dB PL, reflecting turbulence-induced attenuation that is not captured in the model.

To quantify this discrepancy, the loudness reduction due to turbulence was examined as a function of propagation distance. Here, the mean loudness reduction is defined as the mean of the PLs of the measured waveforms with turbulence minus the predicted PL without turbulence. The resulting probability distributions for the Primary, Secondary, and Tertiary Arrays are shown in Fig. 4a-4c. As summarized in Fig. 4d, the mean loudness reduction increases systematically with lateral distance from the undertrack, demonstrating that loudness reduction grows with the propagation distance through the turbulent ABL [3, 9].

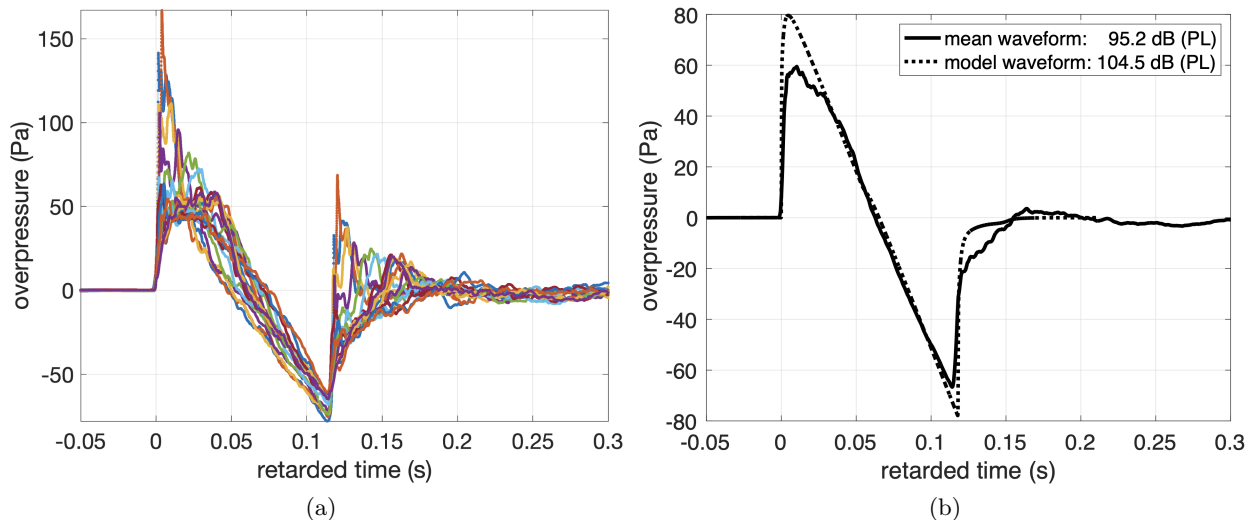


Figure 3: Comparison of measured and predicted sonic booms. (a) Waveforms measured by the Primary Array during a representative supersonic pass show substantial variability in peak overpressure and waveform shape. (b) The ensemble-averaged (mean) waveform from these measurements is compared with the predicted waveform in the absence of turbulence. The measured mean exhibits a loudness level approximately 10 dB PL lower than the prediction, highlighting the turbulence-induced attenuation not captured by the model.

3 Turbulence Effects and Limitations of the Parabolic Equation Approximation

A key implication of the preceding results is that any predictive model for sonic boom propagation must explicitly account for turbulence-induced losses. One approach currently pursued at NASA is the use of KZK-based turbulence filters [10, 11], derived from the Khokhlov–Zabolotskaya–Kuznetsov (KZK) equation, a nonlinear parabolic equation for moving inhomogeneous media [12, 13, 9]). The KZK equation may be written [14] as

$$\frac{\partial p}{\partial x} = (\mathcal{D} + \mathcal{T} + \mathcal{N} + \mathcal{A})p, \quad (1)$$

where the parabolic-equation (PE) component consists of the diffraction operator $\mathcal{D}$ and the turbulence operator $\mathcal{T}$, while the Burgers-equation component contains the nonlinearity $\mathcal{N}$ and absorption $\mathcal{A}$. The coordinate system uses x to denote the axis of the primary propagation and $\mathbf{r}_\perp = (y, z)$ for the transverse direction.

The PE is the standard one-way approximation and, therefore, cannot represent backscattering (e.g., see Refs. [15, 16, 17, 18]), the mechanism responsible for turbulence-driven energy loss. Because full KZK solvers are computationally expensive and slow, NASA developed KZK turbulence filters as signal-processing surrogate models. These filters rely on a linear implementation, which requires that nonlinear effects remain sufficiently small so that the commutators of the nonlinear operator $\mathcal{N}$ with the other operators in Eq. (1) can be neglected.

In most numerical implementations, the KZK equation is solved using a split-step method (e.g., see Refs. [12, 13, 4]), with $\mathcal{N}$ and $\mathcal{A}$ included in the Burgers equation, while $\mathcal{D}$ and $\mathcal{T}$

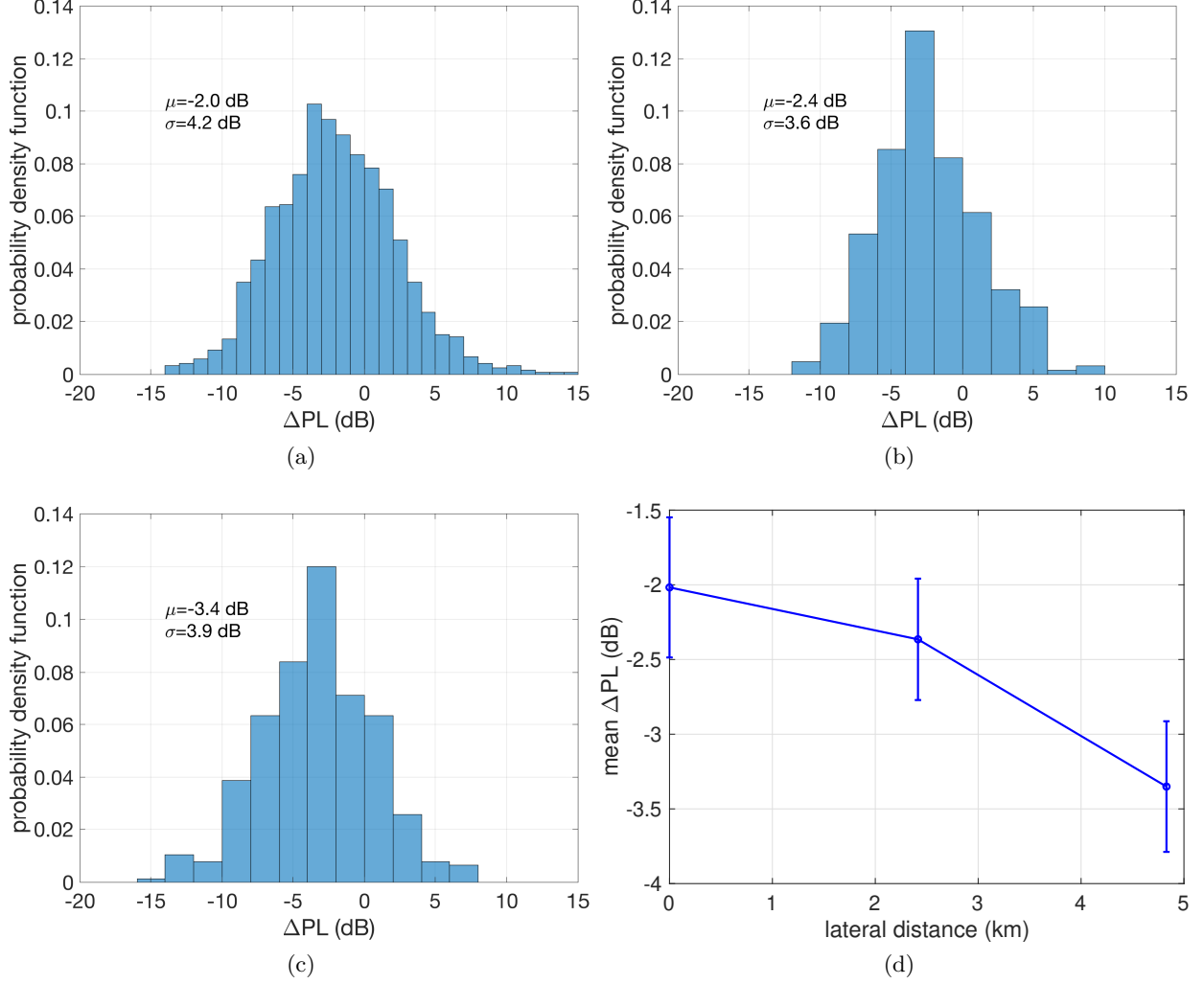


Figure 4: Probability density functions of the mean loudness reduction for sonic booms propagating through turbulence, measured at the (a) Primary Array, (b) Secondary Array, and (c) Tertiary Array. The mean loudness reduction μ is defined as the mean perceived level (PL) of the individual turbulized waveforms minus the PL of the predicted waveform without turbulence. (d) The mean loudness reduction, shown with error bars corresponding to one standard deviation (1σ), is plotted as a function of lateral distance. The trend demonstrates that loudness reduction increases systematically with increasing lateral propagation distance through the turbulent ABL.

included in the PE. In the frequency domain, the PE takes the form [14]

$$\frac{\partial P}{\partial x} = \left(\frac{i}{2\kappa_o} \nabla_{\perp}^2 - \mathbf{m}'_{\perp} \cdot \nabla_{\perp} - i\kappa_o n'_e \right) P, \quad (2)$$

where $\nabla_{\perp} = \partial/\partial \mathbf{r}_{\perp}$ and $n'_e(\mathbf{r}) = (c'(\mathbf{r}) + v'_x(\mathbf{r}))/\bar{c}$ is the fluctuation of the nondimensionalized effective refractive index along the axis of the primary propagation x , and c' and v'_x are the fluctuations of the sound speed and velocity component along x , respectively. Additionally, $\bar{c}$ is the mean sound speed, $\kappa_o = \omega/\bar{c}$ is the free-space wavenumber, and ω is the angular frequency. Note that $\mathbf{m}'_{\perp}$, the transverse velocity component, vanishes when the medium is quiescent or is set to zero when the contribution of the transverse velocity fluctuation is neglected [19, p. 41]. In this situation, an analytical expression for the mean energy density $\langle |P(\mathbf{r}, \omega)|^2 \rangle$ is available. As shown in the derivation of Rytov, Kravtsov, and Tatarskii (RKT) [16, pp. 45-47],

$$\langle |P(\mathbf{r}, \omega)|^2 \rangle = |A(\omega)|^2, \quad (3)$$

where $A(\omega)$ is the complex-valued amplitude of the incident plane wave at $x = x_o$ and is independent of both $\mathbf{r}_{\perp}$ and x . Thus, the PE approximation to the full wave equation conserves the mean energy density along the axis of the primary propagation. Since this is an exact solution to Eq. (2), any numerical implementation based on the PE must satisfy this constraint.

In the presence of velocity fluctuations, the RKT derivation of the mean energy density conservation can be extended. Multiplying Eq. (2) on the left by P^* and adding the resulting equation to its complex conjugate yield

$$\frac{\partial}{\partial x} |P|^2 = \frac{i}{2\kappa_o} (P^* \nabla_{\perp}^2 P - P \nabla_{\perp}^2 P^*) - \mathbf{m}'_{\perp} \cdot \nabla_{\perp} |P|^2. \quad (4)$$

We now use the identity $P^* \nabla_{\perp}^2 P - P \nabla_{\perp}^2 P^* = \nabla_{\perp} \cdot (P^* \nabla_{\perp} P - P \nabla_{\perp} P^*)$ and let $P(x, \mathbf{r}_{\perp}, \omega) = Q(x, \mathbf{r}_{\perp}, \omega) e^{iS(x, \mathbf{r}_{\perp}, \omega)}$, where Q and S denote the wave amplitude and phase, respectively. We also invoke the approximation $\nabla_{\perp} \cdot \mathbf{m}'_{\perp} = 0$ used in the derivation of the original KZK equation for moving media [12]. Taking the ensemble average $\langle \cdot \rangle$, Eq. (4) then reduces to

$$\frac{\partial}{\partial x} \langle |P|^2 \rangle = -\nabla_{\perp} \cdot \mathbf{J}(x, \mathbf{r}_{\perp}, \omega), \quad (5)$$

where $\mathbf{J}(x, \mathbf{r}_{\perp}, \omega) = \langle Q^2 \nabla_{\perp} S \rangle / \kappa_o + \langle \mathbf{m}'_{\perp} Q^2 \rangle$ is the transverse energy flux.

We now assume that the incident field is a plane wave with initial amplitude $A(\omega)$ at $x = x_o$, and that the fluctuation is statistically homogeneous. Following the RKT transverse-symmetry argument [16, p. 47], the quantities $\langle Q^2 \nabla_{\perp} S \rangle$ and $\langle \mathbf{m}'_{\perp} Q^2 \rangle$ are each independent of $\mathbf{r}_{\perp}$. Because $\nabla_{\perp}$ operates on these quantities, the right-hand side of Eq. (5) vanishes, yielding Eq. (3). This result expresses conservation of the mean energy density along the primary propagation axis, now including the effects of velocity fluctuations.

The spatial stationarity in the transverse plane of all statistical moments can also be used to show that the right-hand side of Eq. (5) vanishes. In a statistically-homogeneous, random medium with an incident plane wave, all multi-point correlation functions and single-point statistical moments are stationary with respect to transverse translations [20, 16]. Stationarity requires that any spatial shift by an arbitrary transverse displacement vector

$\mathbf{d}_\perp$ leave the ensemble expectation invariant, giving $\langle \mathbf{J}_\perp(x, \mathbf{r}_\perp + \mathbf{d}_\perp, \omega) \rangle = \langle \mathbf{J}_\perp(x, \mathbf{r}_\perp, \omega) \rangle$. Because this identity holds for every $\mathbf{d}_\perp$ in the two-dimensional real coordinate space $\mathbb{R}^2$, the expected transverse flux must be spatially constant across the transverse plane [21] giving $\langle \mathbf{J}_\perp(x, \mathbf{r}_\perp, \omega) \rangle = \mathbf{K}(x, \omega)$, where $\mathbf{K}$ depends only on the longitudinal propagation coordinate x and the frequency ω . Taking the transverse divergence of a spatially constant field yields $\nabla_\perp \cdot \langle \mathbf{J}_\perp \rangle = 0$. Substituting this result into Eq. (5) shows directly that the right-hand side vanishes in the ensemble average, thereby establishing conservation of the mean energy density as expressed in Eq. (3).

The foregoing analysis shows that energy conservation within the PE approximation continues to hold even in a moving medium. However, measured sonic booms do not satisfy this constraint. Field data consistently demonstrate a decrease in mean energy density with propagation distance through turbulence. This discrepancy highlights the need for models that incorporate the full physics of multiple scattering and backscattering, beyond the capabilities of the PE framework.

4 Multiple Scattering Theory and the Bimodal Mean Field

The Multiple Scattering Theory (MST) provides a physically rigorous framework capable of capturing the reduction in mean energy density that occurs as a sonic boom propagates through atmospheric turbulence. The formulation begins with the full three-dimensional Helmholtz equation, which may be recast into the integral representation as

$$P(\mathbf{r}) = P_o(\mathbf{r}) + 2\kappa_o^2 \int d^3\mathbf{r}' G_o(\mathbf{r}, \mathbf{r}') n'_e(\mathbf{r}') P(\mathbf{r}'), \quad (6)$$

where $P(\mathbf{r}) = P(\mathbf{r}, \omega)$ is the acoustic pressure field, $P_o(\mathbf{r}) = P_o(\mathbf{r}, \omega)$ is the incident acoustic pressure, $n'_e(\mathbf{r})$ is the fluctuation of the refractive index defined in Sec. 3, and $G_o(\mathbf{r}, \mathbf{r}') = e^{i\kappa_o|\mathbf{r}-\mathbf{r}'|}/4\pi|\mathbf{r}-\mathbf{r}'|$ is the free-space Green's function in 3D space, where $\mathbf{r}'$ refers to the vector coordinate of the source. In a turbulent medium, the quantity of interest is the ensemble-averaged field, which also forms the basis for computing the mean energy density. A recently published article [22, Eq. (18)] shows that the mean field may be written as

$$\langle P(\mathbf{r}, \omega) \rangle = \int_{-\infty}^{\infty} \frac{dk_x}{2\pi} \frac{S(\omega) e^{ik_x x}}{D(k_x)}, \quad (7)$$

where $S(\omega)$ is the source strength and $D(k_x)$ is the dispersion relation given by $D(k_x) = k_x^2 - \kappa_o^2 - \tilde{\Sigma}(k_x)$. The dispersion relation contains the self-energy $\tilde{\Sigma}$, which represents the interaction energy between the acoustic field and the fluctuations of the effective refractive index $\tilde{n}_e$. For a turbulent field with an exponential correlation model, the self-energy takes the closed form [22, Eq. (17)]

$$\tilde{\Sigma}(k) = \sum_{\nu=1}^3 \frac{4\sigma_\nu^2 \kappa_o^4 \ell_\nu^2}{(k^2 - \kappa_o^2) \ell_\nu^2 - 2i\kappa_o \ell_\nu + 1},$$

where ℓ_ν is the correlation length, and σ_ν^2 is the variance of the turbulent fluctuation. The subscript ν denotes the sound speed, shear-produced velocity, and buoyancy-produced velocity fluctuations. For other correlation models, such as the von Kármán spectrum, both

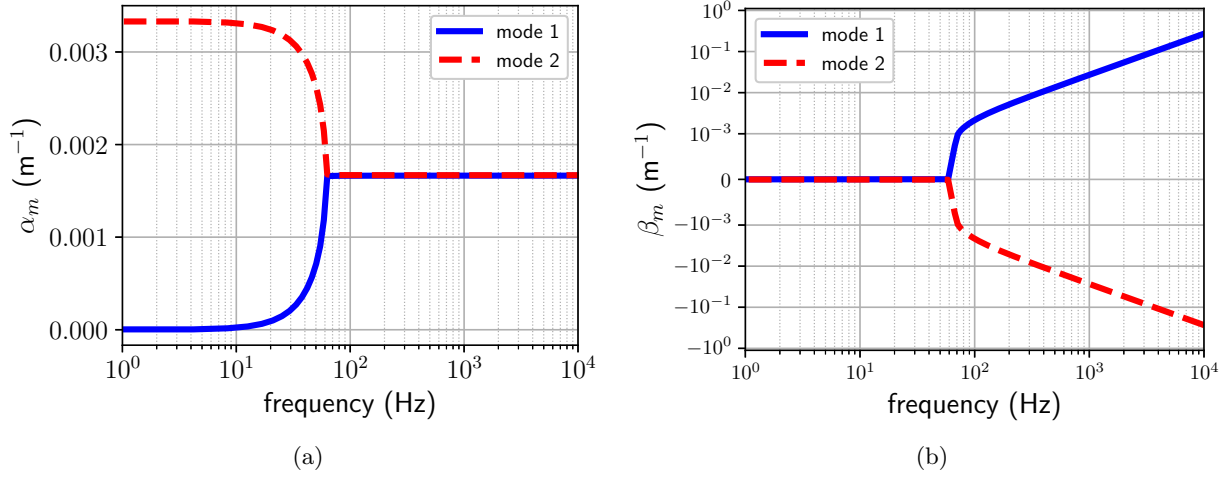


Figure 5: Scattering wavenumbers associated with the two modes in Eq. (9). (a) Extinction coefficient and (b) phase shift for the primary mode (mode 1, shown in blue) and the secondary mode (mode 2, shown in red).

the self-energy and the roots of the dispersion relation can be computed numerically, as discussed in the prior work [22]. In that earlier study, only the dominant mode was retained. The effective wavenumber k_e is obtained by solving $D(k_e) = 0$.

A more complete representation of the mean field requires inclusion of the second mode inherent in the dispersion relation. The effective wavenumber has two solutions,

$$k_{e,\mp} = \kappa_o \sqrt{1 - \frac{1}{2\kappa_o^2 \ell_\nu^2} \left(a_\nu \mp \sqrt{a_\nu^2 + 16\kappa_o^4 \ell_\nu^2 \sigma_\nu^2} \right)}, \quad (8)$$

where $a_\nu = 1 - 2i\kappa_o \ell_\nu$. The minus sign corresponds to the dominant or primary mode, while the plus sign corresponds to the secondary mode that is ignored in Ref. [22, Eq. (20)]. The effective wavenumber gives rise to the scattering wavenumber k_s , defined as $k_{s,\mp} = k_{e,\mp} - \kappa_o$ whose real part defines the phase shift β and whose imaginary part defines the extinction coefficient α .

As shown in Fig. 5a, the extinction coefficient of the primary mode is much smaller than that of the secondary mode below a transition frequency of approximately 60 Hz, whereas the two become identical above this frequency. In contrast, the phase shifts are identical below the transition frequency and become distinct — indeed exact negatives of one another — above it.

Using contour integration in the upper half of the complex k_x -plane, the resulting mean field given by Eq. (7) becomes

$$\langle P(\mathbf{r}, \omega) \rangle = \sum_{m=1}^2 \frac{2\kappa_o A(\omega)}{D'(k_{e,m})} e^{ik_{e,m}x}, \quad (9)$$

where $D'(k_{e,m}) = \partial D / \partial k_x$ evaluated at $k_x = k_{e,m}$, and $A(\omega) = iS(\omega)/2\kappa_o$ is the complex amplitude of the incident plane wave at $x = x_o$. Note that $m = 1$ corresponds to the mode associated with the “−” branch in Eq. (8), and $m = 2$ corresponds to the “+” branch. In Ref.

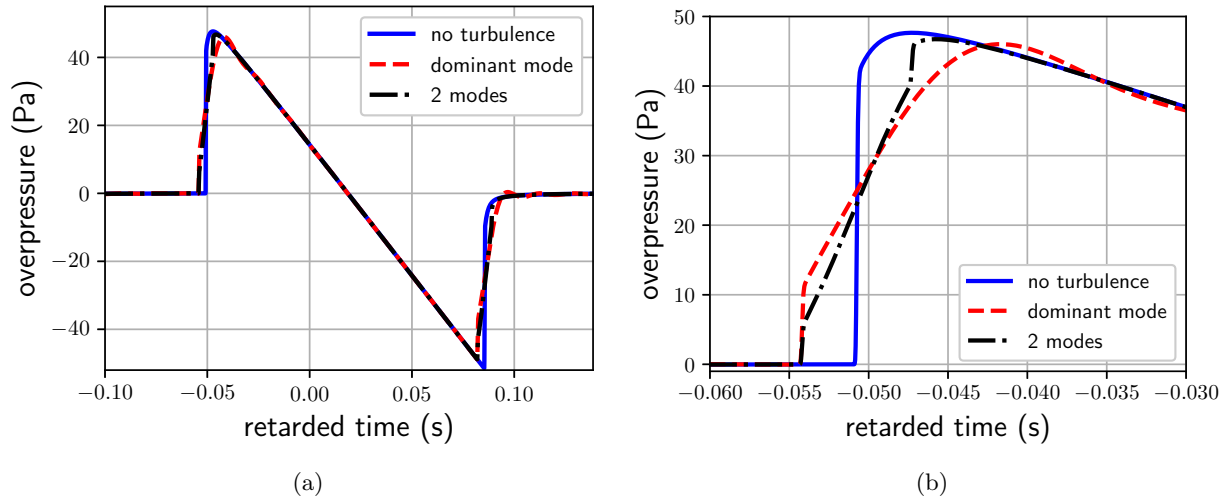


Figure 6: Effect of turbulence-induced modal attenuation on a sonic boom waveform. (a) The reference waveform without turbulence (solid blue) is compared with predictions with turbulence using only the primary mode (dashed red) and using both primary and secondary modes (dash-dot black). (b) Magnified view of the leading shock region shown in (a).

[22, Eq. (21)], $D'(k_{e,m})$ is approximated by $2\kappa_o$; however, this approximation is not adopted in the present work because the effects of the secondary mode are explicitly retained.

Time-domain simulations illustrate the consequences of including both modes. As shown in Fig. 6, the model without turbulence produces a sharp N-wave, whereas the inclusion of turbulence, whether through the dominant mode alone or through both modes, yields a rounded and thickened shock front [22], with a rise-time increase of approximately 10 ms. Such behavior is routinely observed in flight tests first reported in Ref. [23] as well as in laboratory experiments [24]. Although the secondary mode has little influence on the overall waveform shape, its presence is clearly manifested in the energy spectral density shown in Fig. 7a. The closely spaced spectral dips (comb structure) arise from the impulsive nature of the signal, while the more widely spaced dips result from interference between the primary and secondary modes. These interference patterns are also evident in the the mean spectrum (Fig. 7b) of the ground-measured sonic booms shown in Fig. 3. As displayed in Fig. 7b, the energy density of the mean field exhibits a similar modal spacing, suggesting that such features may be exploited to infer turbulence parameters.

5 Mean Energy Density and the Bethe-Salpeter Solution

The mean energy density is another quantity of central importance in characterizing sonic boom propagation through turbulence. Because the parabolic equation (PE) approximation conserves mean energy density, the Multiple Scattering Theory (MST) is being developed to capture its reduction due to backscattering. The expression for the mean energy density follows from the integral formulation, Eq. (6), of the full three-dimensional wave equation. From this formulation, one obtains the Bethe-Salpeter equation (e.g., see Ref. [16, pp. 125-130]), from which the mean energy density can be written as [25]

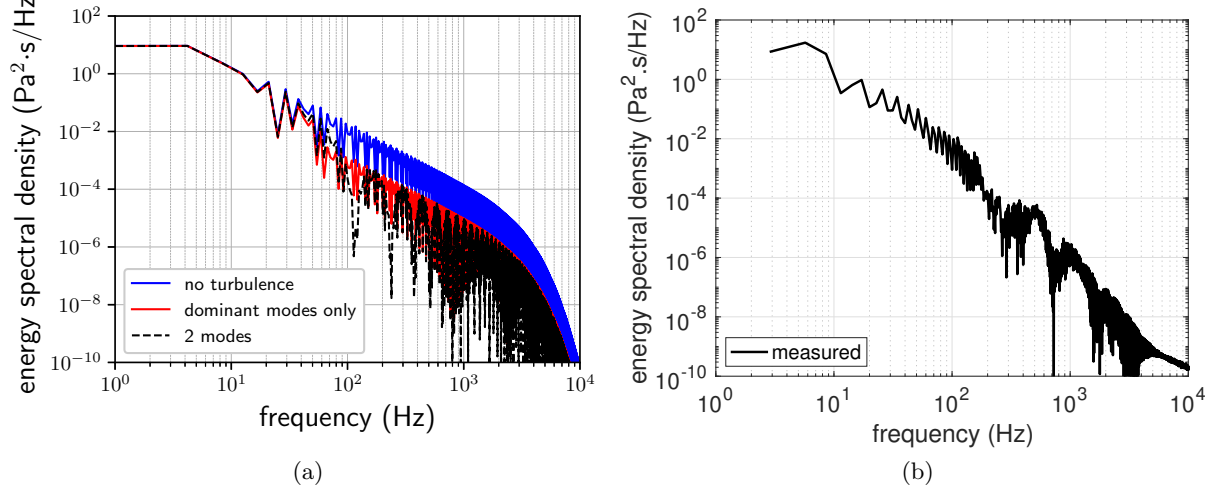


Figure 7: Interference patterns in the energy spectral density. (a) The spectra of the waveforms in Fig. 6 show an interference pattern when both the primary and secondary modes (black) are included, a feature not present when only the primary mode (red) is used. (b) A similar interference pattern appears in the spectrum of the mean field computed from the ground-measured sonic booms shown in Fig. 3.

$$\langle |P(\mathbf{r})|^2 \rangle = |\langle P(\mathbf{r}) \rangle|^2 + 4\kappa_o^4 \int d^3\mathbf{r}' d^3\mathbf{r}'' \langle G(\mathbf{r}, \mathbf{r}') \rangle \langle G^*(\mathbf{r}, \mathbf{r}'') \rangle \varphi(\mathbf{r}' - \mathbf{r}'') \langle P(\mathbf{r}') P^*(\mathbf{r}'') \rangle$$

in the primary axis of propagation. Here, $|\langle P(\mathbf{r}) \rangle|^2$ is the energy density of the mean acoustic field, $\langle G \rangle$ is the mean Green's function, and φ is the two-point correlation function of the turbulent refractive-index fluctuations. This expression shows that the mean energy density consists of two contributions: the energy density of the mean field and the mean energy density associated with the scattered field.

To obtain a tractable solution, the mean energy density is solved in the wavenumber domain using the Fourier transform. The resulting expression takes the form

$$\langle \tilde{P}(\mathbf{k}') \tilde{P}^*(\mathbf{k}' - \mathbf{k}) \rangle = \langle \tilde{P}(\mathbf{k}') \rangle \langle \tilde{P}^*(\mathbf{k}' - \mathbf{k}) \rangle + \tilde{J}(\mathbf{k}, \mathbf{k}') \langle \tilde{P}(\mathbf{k}') \tilde{P}^*(\mathbf{k}' - \mathbf{k}) \rangle, \quad (10)$$

where

$$\tilde{J}(\mathbf{k}, \mathbf{k}') = 4\kappa_o^4 \int \frac{d^3\mathbf{k}''}{(2\pi)^3} \langle \tilde{G}(\mathbf{k}'') \rangle \langle \tilde{G}^*(\mathbf{k} - \mathbf{k}'') \rangle \Phi(\mathbf{k}'' - \mathbf{k}')$$

acts as a weighting factor and is an energy-transfer kernel that represents the coupling of two propagators (Green's functions) mediated by the turbulence spectrum Φ . Equation (10) can be expanded as a Dyson series (see Ref. [22] and the references therein) and, after rearranging, yields

$$\langle \tilde{P}(\mathbf{k}') \tilde{P}^*(\mathbf{k}' - \mathbf{k}) \rangle = \frac{\langle \tilde{P}(\mathbf{k}') \rangle \langle \tilde{P}^*(\mathbf{k}' - \mathbf{k}) \rangle}{1 - \tilde{J}(\mathbf{k}, \mathbf{k}')}.$$

The mean energy density in the physical space is then obtained by taking the inverse Fourier transform, yielding

$$\langle |P(\mathbf{r})|^2 \rangle = \int \frac{d^3k d^3k'}{(2\pi)^6} e^{i\mathbf{k}\cdot\mathbf{r}} \frac{\langle \tilde{P}(\mathbf{k}') \rangle \langle \tilde{P}^*(\mathbf{k}' - \mathbf{k}) \rangle}{1 - \tilde{J}(\mathbf{k}, \mathbf{k}')}. \quad (11)$$

This equation enables the prediction of mean energy losses that lie outside the scope of the parabolic equation approximation. The numerator of the integrand represents the mean-field energy density, now modulated by the weighting introduced through the denominator. The denominator itself encodes turbulence-induced modal coupling, which governs how energy is redistributed among modes. A complete analysis and interpretation of Eq. (11) will be presented in a forthcoming publication.

6 Conclusion

Flight test data clearly demonstrate that turbulence plays a central role in predicting the propagation of sonic booms, with measured loudness reductions increasing both with turbulence strength and with the distance traveled through the turbulent atmospheric boundary layer. Existing prediction tools based on the KZK equation provide a useful framework for modeling nonlinear propagation, atmospheric absorption, and turbulence through a combination of the Burgers equation and the parabolic equation. However, because the parabolic equation is inherently a one-way propagation approximation and therefore neglects backscattering, it is known to conserve mean energy density and cannot reproduce the turbulence-induced energy losses observed in field measurements. Moreover, full KZK solvers remain computationally expensive, limiting their practicality for large-scale parametric studies.

To address these limitations, a Multiple Scattering Theory (MST) has been developed to explicitly account for energy losses due to backscattering effects. The MST framework reveals that the mean acoustic field comprises two distinct propagation modes, whose interference produces spectral features that are consistent with those observed in flight test datasets. These modal interactions provide a physics-based explanation for energy losses and interference effects in measured sonic booms through turbulent conditions. Ongoing work is focused on extending the MST framework to higher-order statistical moments with the goal of developing a comprehensive turbulence-aware prediction model for next-generation low-boom aircraft.

AUTHOR DECLARATIONS

Conflict of Interest

The author declares that there are no conflicts of interest to disclose.

Data

The data and plots in this article were presented at the 190th Meeting of the Acoustical Society of America. The presentation is available for download at <https://ntrs.nasa.gov/citations/20260003623>.

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