

A vast valley network beneath the Greenland Ice Sheet

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Introduction

This Supporting Information contains additional methodological detail and 11 figures that provide additional methodological and graphical detail on our methods and results.

Text S1. Ice Flow Perturbation Analysis

S1.1. IFPA implementation overview

We estimate subglacial topography for the GrIS interior following a modification of the Ice Flow Perturbation Analysis (IFPA) method described by Ockenden et al. (2022, 2023, 2026). IFPA solves for basal conditions using a weighted least-squares inversion (Ockenden et al., 2022; Thorsteinsson et al., 2003). We do not restate the numerical implementation of IFPA here except for clarity or to note where improvements were made.

IFPA treats ice in a planar slab (implemented as a square “patch”) aligned in the direction of ice flow and describes the steady-state spatial variability of the ice-surface elevation s , surface-velocity components u and v , subglacial topography b , and basal slipperiness c as functions of projected spatial coordinates x and y . These fields are non-dimensionalized (denoted by capital letters S , U , V , B , C) and Fourier transformed (denoted by circumflexes) so that their spatial variability can be described as a function of wavenumbers k and l aligned parallel (x) and perpendicular to flow (y), respectively. In the forward model, the non-dimensionalized Fourier transforms of perturbations in ice-surface elevation ($\hat{S}$) and velocity ($\hat{U}$, $\hat{V}$) are calculated from the non-dimensionalized Fourier transforms of perturbations in subglacial topography ($\hat{B}$) and basal slipperiness ($\hat{C}$):

$$\hat{S}(k, l) = T_{sb}(k, l) \hat{B}(k, l) + T_{sc}(k, l) \hat{C}(k, l), (1)$$

$$\hat{U}(k, l) = T_{ub}(k, l) \hat{B}(k, l) + T_{uc}(k, l) \hat{C}(k, l), (2)$$

$$\hat{V}(k, l) = T_{vb}(k, l) \hat{B}(k, l) + T_{vc}(k, l) \hat{C}(k, l), (3)$$

where T_{sb} , T_{ub} , T_{vb} , T_{sc} , T_{uc} , T_{vc} , are the wavenumber-specific, non-dimensional Full-Stokes (FS) transfer functions that describe the amplitude ratio of perturbations in bed properties (second subscript) relative to the ice surface properties (first subscript) and vary with surface slope a and mean non-dimensional slipperiness $\bar{C}$. We noted two typographic errors in the FS transfer functions T_{ub} and T_{vc} in the Supplement of Ockenden et al. (2023); however, the transfer functions were implemented correctly in the code repository accompanying their study.

IFPA implements a weighted least squares inversion modulated with a filter to solve for $\hat{B}(k, l)$ and $\hat{C}(k, l)$, as described in Appendix C of Ockenden et al. (2022). Weighting factors Σ_s , Σ_U , and Σ_V balance the relative sizes of perturbations in surface elevation and velocity in the inversion, respectively. The filter suppresses artificial amplification of short wavelengths (p_{filt} and wc_B) and removes features directly aligned with ice flow (θ_B), which fall within the nullspace of the inversion (Thorsteinsson et al., 2003).

A core assumption of IFPA is that spatial variability in the input surface fields can be interpreted in terms of local perturbations from regional mean values of either subglacial topography or basal slipperiness, i.e., the ratio of the basal sliding and deformation speeds. However, abrupt perturbations in slipperiness are also readily identifiable by abrupt changes in surface velocity, which are most apparent at shear margins. Following Ockenden et al. (2026), we only focus on interpreting IFPA in terms of subglacial topography, but we directly mitigate the potential for misinterpretation of changes in slipperiness.

S1.2. Modifications to original IFPA algorithm

Our first improvement is our use of ice thickness as a patch size parameter. The theoretical foundation of IFPA treats the ice sheet as an infinitely long, planar slab aligned with ice flow. We divide the GrIS into overlapping square “patches” that are each rotated to align with ice flow. We allow the patch size to vary across the GrIS by defining overlapping 150-km square “tiles” spaced 75 km apart, in which the uniform patch size is ten times the mean ice thickness of the tile (5.4–30.6 km), minimizing the patch size and hence the region over which the ice thickness and flow direction are assumed to be uniform. IFPA is applied to each tile independently and sinusoidal weighting is used to merge them.

Our second improvement concerns the mean non-dimensional slipperiness ($\bar{C}$) across the GrIS. This single, depth-averaged parameter collectively represents several poorly known subsurface conditions, including the rheologies of both the ice column and the bed, both of which are expected to vary significantly across the GrIS. The smaller the assumed value of $\bar{C}$, the greater the amplitude of IFPA-inferred perturbation to subglacial topography based on surface observations. Parts of the GrIS are likely frozen to the bed (MacGregor et al., 2022) where $\bar{C} = 0$ by definition, and other parts are likely thawed and sliding rapidly, implying $\bar{C} \gg 1$. After testing a larger parameter space for $\bar{C}$ than Ockenden et al. (2023), following Ockenden et al. (2026) we

ultimately opt to use $\bar{C} = 50$ across the GrIS interior – including where the bed is likely frozen – with two significant exceptions ([Figure S2](#)). The Northeast Greenland Ice Stream (*NEGIS*) is an exceptional feature of the GrIS interior and it is also unusual in the context of mapping subglacial topography because the extent of its fast flow reaches far inland from where MC is generally applied. Whatever the “correct” value of $\bar{C}$ may be for the NEGIS, it should clearly be higher than that used elsewhere in the generally slower-flowing GrIS. For each IFPA tile that overlaps with the NEGIS, we prescribe a larger, spatially uniform $\bar{C}$ value (75–250) based on modeling of basal shear stress in that region by Sergienko et al. (2014). We elected not to use results from all four regions modeled by Sergienko et al. (2014) to prescribe $\bar{C}$ because the other regions (Petermann Glacier and Sermeq Kujalleq) do not have as pronounced a surface perturbation in either the flow-aware hillshade or strain rates. We also prescribe $\bar{C} = 1000$ for tiles near the edge of the ice sheet that contain <50% ice-sheet pixels to reduce extreme inferred relief near the irregular ice-sheet margin and to improve blending of IFPA with BMG subglacial topography, described below. Because the tiles overlap, the “effective” $\bar{C}$ value for pixels near the ice sheet edge is between 50 and 1000.

Our third improvement is the blending of IFPA-inferred subglacial topography with BMG. We first run IFPA across the entire GrIS. We identify four different types of regions where the GrIS-wide IFPA output is disfavored and where BMG is preferred and weight each map appropriately to generate a blended topography. First, where MC is used by BMG, typically along faster-flowing margins and outlet glaciers, we assume that MC generally produces a more realistic subglacial topography, but that IFPA may be able to better capture short-wavelength (≤ 5 km) perturbations in subglacial topography where ice flow is relatively slow. Thus, we transition from IFPA where surface speeds are ≤ 25 m yr⁻¹ to BMG where they are ≥ 300 m yr⁻¹. Second, across 77 manually mapped regions in northern GrIS (100–16,000 km²), oblong surface perturbations in the GrIMP flow-aware hillshade suggest a subglacial perturbation as their origin. However, based on our radargram inspection and comparison with previous mappings, they appear to be instead due to an englacial perturbation (disrupted basal ice), ([Figure S5](#)). Third, where absolute lateral shear strain rates are high ($\geq 10^{-4}$ yr⁻¹), which only occurs in the GrIS interior around NEGIS, we expect that any surface perturbation is due to a short-wavelength change in slipperiness rather than topography. Fourth, across parts of some sharper ice divides within the GrIS, IFPA malfunctions due to the abrupt change in surface-velocity azimuth. Those problematic shear margins and divides are mapped manually. In all cases, a 2-km wide buffer is used to transition between regions where IFPA is used exclusively and where it is not ([Figure S3](#)).

S1.3. IFPA parameter selection

Below, we summarize the parameters used and our modifications, where an asterisk (*) indicates values chosen to be consistent with Ockenden et al. (2022, 2023, 2026).

- **Ice-sheet domain:** Greenland ice sheet
We apply IFPA to the entire ice sheet, including the margins where BedMachine Greenland v6 (*BMG*) does not apply a mass conservation (*MC*) technique. Because of its reliance on Fourier transforms, IFPA requires a gapless dataset within the entire patch to function correctly, i.e., it does not output meaningful results near irregular ice-sheet

margins without further modification. To reach the GrIS margin, we create synthetic fields for ice thickness h , u , and v , such that h increases logarithmically from 0–1000 m, and the magnitude of surface velocity increases logarithmically from 0–500 m yr⁻¹, over a 33 km buffer around the ice sheet, which is the minimum distance between the ice-sheet edge and edge of the BMG domain. We find the azimuth of the observed velocity, rotate it by 180°, and approximate it past the ice-sheet edge to generate synthetic u and v fields. The surface elevation beyond the ice sheet is modified to be the sum of the BMG surface and this synthetic buffer thickness, and the surface-velocity fields are similarly adjusted. While this produces some small pockets of unrealistic values for b at the very edges of the ice sheet due to small thicknesses and velocities, we filter these out, and this method fills in large regions where MC was not applied ($\sim 2 \times 10^5$ km²).

- **Region of interest:** adj = variable

Due to its use of overlapping patches, IFPA can theoretically be run for an infinitely large region of interest (*ROI*), but computational resources limit the practical size of a region of interest. As a result of our code improvements, we can run IFPA for a 150-km square region efficiently, enabling us to define patch sizes independently for each ROI. Thus, the number of patches – and the number of sub-regions over which sufficient patches overlap (adj) – are determined dynamically by the desired dimensions of the entire rectangular ROI. To run IFPA for all the Greenland Ice Sheet (*GrIS*), we define 150-km square tiles that overlap by 75 km. Each tile is run independently so that the patch size may vary, generating perturbations that more closely resemble radar observations. A sinusoidal weight is applied to each tile when combining the results for the whole ice sheet.

- **Number of overlapping patches*:** $n = 3$

See Figure 4a in Ockenden et al. (2022) and Figure S2 in Ockenden et al. (2026).

- **Patch dimensions:** square patch size = $10\bar{H}_{tile}$

For each tile, the patch size is 10 times the mean ice thickness of the tile, although we impose a minimum patch size of 5.4 km, and the tile is divided into the appropriate number of patches to achieve an overlap of nine patches at any given point in the domain. The IFPA inversion is run for each patch independently, and a sinusoidal weight is applied to each patch to combine all patches and achieve a smooth result for the tile.

- **Tapering parameter*:** $tapering = 0.1$

A simple tapering function avoids edge effects in the inversion.

- **Transfer functions*:** Full-Stokes (*FS*) flow transfer functions, following Gudmundsson (2003) and Ockenden et al. (2026). As originally designed, IFPA could employ either the FS linearized transfer functions derived by Gudmundsson (2003), or the simplified transfer functions that employ the Shallow-ice-Stream Approximation (*SSA*) derived by Gudmundsson (2008) and further developed by Sergienko (2012). Because we refactored the IFPA code and substantially accelerated it, we only use the FS transfer functions, as the SSA underestimates subglacial topography for wavelengths shorter than about ten ice thicknesses (Gudmundsson, 2008; Ockenden et al., 2023), and virtually all subglacial valleys that we seek to resolve are smaller than that threshold.

- **Weighting factor:** $\Sigma_U = 1^*$, $\Sigma_V = 1^*$, $\Sigma_S = \frac{\max(S)}{100 * \max(\sqrt{U^2 + V^2})}$

Ockenden et al. (2022, 2023, 2026) applied a weighting factor $\Sigma_s = 10^{-3}$ to balance the relative magnitudes of the non-dimensionalized ice-surface elevation and velocities; however, we define Σ_s from the data to allow more flexibility when applying IFPA to many different regions in Greenland. The factor of 100 in the denominator is used to maintain consistency in order of magnitude with the default value in the aforementioned studies.

- **Directional filtering parameters***: $\theta_B = 10^\circ$

- **Filtering parameters***: $p_{filt} = -2$, $w_{cb} = 1$

$p_{filt} = -2$ generally preserves wavenumbers on the order of 10^3 for any value of $\bar{C}$, and we maintain that this value is a good choice when $\bar{C}$ is unknown.

- **Ice thickness for redimensionalization***: $\bar{h}$

The inverted, filtered, and non-dimensionalized IFPA perturbation field is re-dimensionalized using the mean ice thickness from BMG for each patch.

- **Non-dimensional slipperiness**: $\bar{C} = 50$ (with exceptions)

The non-dimensional slipperiness represents several poorly known basal conditions such as basal friction and rheology, ice rheology, and hydrology, which are expected to vary significantly across the island. We test a variety of methods to estimate either spatially uniform mean non-dimensional slipperiness $\bar{C}$ or spatially variable non-dimensional slipperiness C for Ryder Glacier ([Figure S2](#)):

1. Using uniform $\bar{C}$ values between 5–1000. We ultimately decide to use $\bar{C} = 50$, which produces perturbations consistent with radar observations and yields acceptable RMSEs for the test region.
2. Using a spatially variable C computed from the driving stress and deformation speed, which requires several assumptions about ice rheology. This method produces realistic dimensional c values, consistent with those used in other studies (e.g., Sergienko, 2012), but smaller C values that result in predicted subglacial topography with large-amplitude features not observed in the radar data. However, we use this method to compute variable C values for the NEGIS and Nioghalvfjærdsbræ regions defined by Sergienko (2014). We ultimately compute a uniform $\bar{C}$ value to be used in the inversion for each of the tiles overlapping these regions by taking the mean of C at each tile's vertices.
3. Optimizing $\bar{C}$ for each patch by performing the inversion with uniform $\bar{C}$ values ranging from 1–1000 and choosing the values that minimizes the RMSE between IFPA and the radar observations. Typically, the maximum RMSE occurs at $\bar{C} \sim 5$, with local minima both lower and higher than this value, such that the optimized $\bar{C}$ value for each patch is typically either <5 or $>>5$. This may be the result of a numerical instability that was not apparent in the parameter space (25–200) considered by Ockenden et al. (2023). The complex spatial distribution of optimized $\bar{C}$ values between patches is thus difficult to interpret, and the number of iterations needed to test each $\bar{C}$ value adds significant computation time.
4. Running IFPA to estimate spatially variable C for each patch or for an entire tile, using the same parameters as Ockenden et al. (2026), and then running the inversion with uniform $\bar{C}$ (computed as the mean or median of C) for b . This

approach requires an initial guess for $\bar{C}$ but does not produce substantially different results than if the inversion is run for b with a uniform $\bar{C}$ (method 1).

S1.4. IFPA mapping of anomalous feature

Here we briefly highlight a remarkable feature in northwestern Greenland that is resolved by IFPA for the first time ([Figure S11](#)). This conspicuously circular surface feature was also identified by others independently (J.L. Bamber & M.A. Cooper, pers. comm., 2017; A. Grinsted, pers. comm., 2018). It has been surveyed glancingly only once, revealing a modest yet broad (≥ 100 m) depression. IFPA resolves a deeper, ~ 9 -km-wide and circular depression centered at $\sim 74.25^\circ\text{N}$, 47.64°W that is reminiscent of other possible subglacial impact craters in Greenland (Kjær et al., 2018; MacGregor et al., 2019), although significantly smaller than those structures. Curiously, this depression lies at or close to the origin of one of this region's most prominent subglacial valleys, which continues linearly to the northwest > 300 km to the terminus of Kjer Glacier. Regardless of its origin, we highlight this feature as one worthy of future investigation given its potential significance to Earth's impact history.

Text S2. Feature mapping

We apply manual tracing and Hough transform analysis to map features in both BMG and the blended subglacial topography. The centerlines of subglacial valleys are traced in QGIS at a scale of 1:750,000, focusing on those that are ≥ 10 km long and ≥ 1 km wide, i.e., the same criteria as MacGregor et al. (2024). Superposition of elevation and unidirectional hillshades at multiple azimuths are used to identify valley centerlines. We record whether valleys are visible in both BMG and the blended topography, whether valleys visible in the blended topography are extensions of valleys visible in BMG, and whether valleys visible in either are associated with major outlet glaciers.

We use Hough transforms to semi-automatically evaluate the dominant azimuths of major features in the blended subglacial topography, whether they are subglacial valleys or not (Wang & Howarth, 1990). We divide the GrIS into 75-km squares (the intersecting area of four overlapping tiles). The squares are half the size of the 150-km square IFPA tiles used for modeling because we found that mean azimuths could be confidently resolved at a finer scale. For each square, we generate 12 distinct unidirectional hillshades at 15° illumination-azimuth intervals. For each hillshade, we use Canny edge detection (Canny, 1986) to generate a binary image which identifies dominant features and record the 100 most prominent peaks in azimuth θ (the angle of the perpendicular projection from the origin to a line that passes through the edge feature) and offset ρ (the perpendicular distance from the origin to the edge feature). Normalization is used to reduce bias toward central features (Fitton & Cox, 1998). This process is illustrated in **Figure S4**; **Figure S4D** shows θ and ρ for five of the most prominent normalized Hough peaks in the example region.

Text S3. Valley network morphology

Automated mapping of subglacial valleys using TopoToolbox provides an independent estimate of valley orientations in Greenland, which can then be interpreted in terms of the tectonic and geologic structure of the island, and to measure branching angles as a proxy for the prevailing climate in which these valley networks may have formed (e.g. Seybold et al., 2017). For processing in TopoToolbox, the rebounded, blended IFPA DEM was filled using the default settings in TopoToolbox v3, requiring a minimum upstream drainage area of 100 km² for identifying individual “stream” segments. However, we excluded segments less than 2 km in length and areas in which disrupted basal units affect the surface topography, as IFPA cannot yet accurately estimate subglacial topography in these areas. These omissions induce discontinuities in stream networks in northern Greenland (**Figure 3D**) and stream orders are thus unlikely to be reliable directly adjacent to those areas. We also modified the TopoToolbox function *networksegment.m* to obtain stream order and to calculate branching angles. To measure branching angles, we used the Strahler stream order to separate segments and included angles between all stream segments of either the same order or different orders.

Supplementary Figures

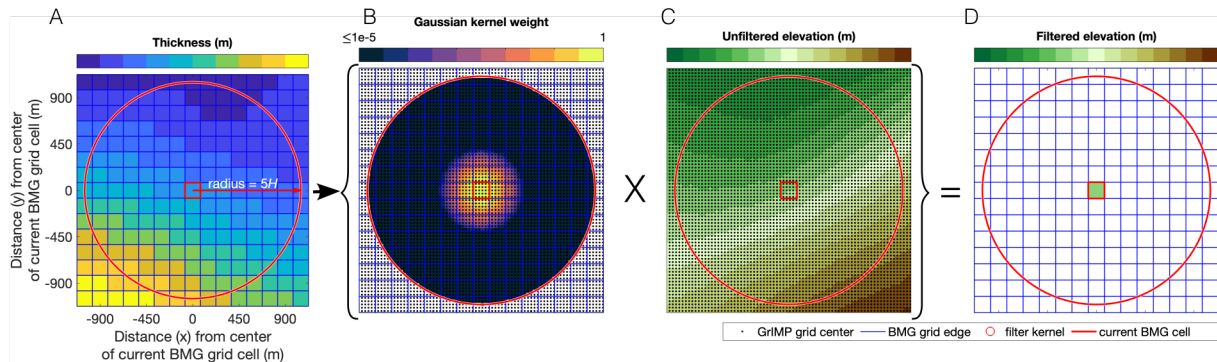


Figure S1.

Schematic of Gaussian filtering method. All panels feature either the 150 m BMG grid (blue gridlines) and the 30-m GrIMP grid (black dots), or both. The diameter of the Gaussian kernel (outlined by red circle) is ~5 times the ice thickness (A) for the BMG grid cell under consideration (red square). The filtered elevation (D) for the current grid cell is the product of the normalized Gaussian kernel (B) and the unfiltered GrIMP elevations (C).

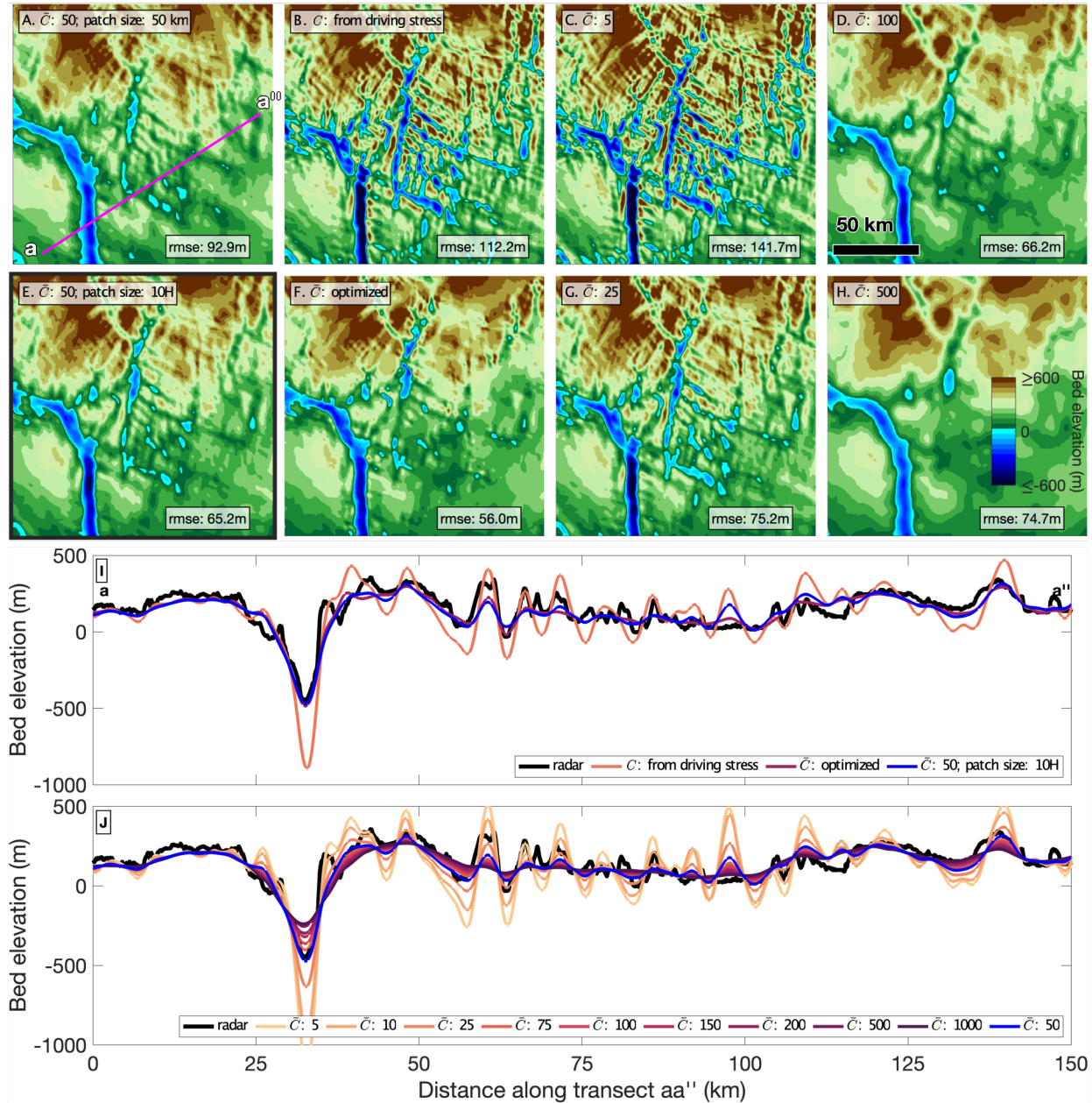


Figure S2.

Comparison of different treatments of slipperiness ($\bar{C}$) in IFPA at the Ryder Glacier onset region. (A–H) show the subglacial topography from selected experiments. (A) and (E) show the effect of changing the patch size with a uniform $\bar{C}$; experiments shown in all other panels used a patch size of $10\bar{H}_{tile}$ and show the effect of using spatially variable (B, F) or uniform (C–D, G–F) slipperiness. (I) and (J) show the radar bed elevation along survey line a–a'' (black line) and the bed elevation resulting from different treatments of slipperiness; both include the results from a uniform $\bar{C} = 50$ (corresponding to (E)). (I) shows results using a variable C value calculated from driving stress ("var"; (B)) and using a $\bar{C}$ value optimized for each patch ("opt"; (F)). (J) shows results from using a uniform $\bar{C}$ value for the tile.

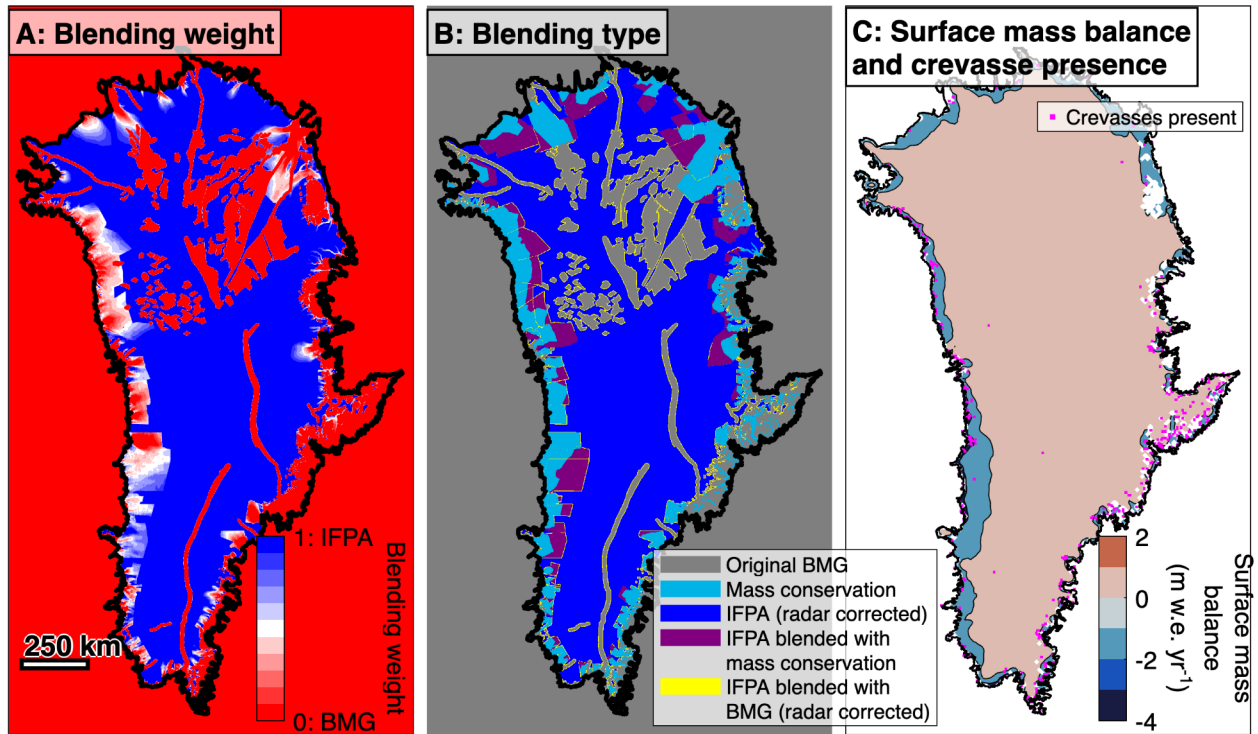


Figure S3.

Weights used to blend BMG (0) and IFPA (1) bed elevations before radar correction (A) and final blending type after radar correction (B) across the entire BMG domain (both on- and off-ice). (C) Multi-year (1979-2019) average surface mass balance from MAR v3.11 (GrIS-wide average 0.1 m yr⁻¹; Fettweis, 2022) and the presence of crevasses in 5-km square grid cells derived from 2-m crevasse presence/absence maps from Chudley et al. (2021). The 5-km grid cells containing crevasses have been dilated to 10 km for better visibility.

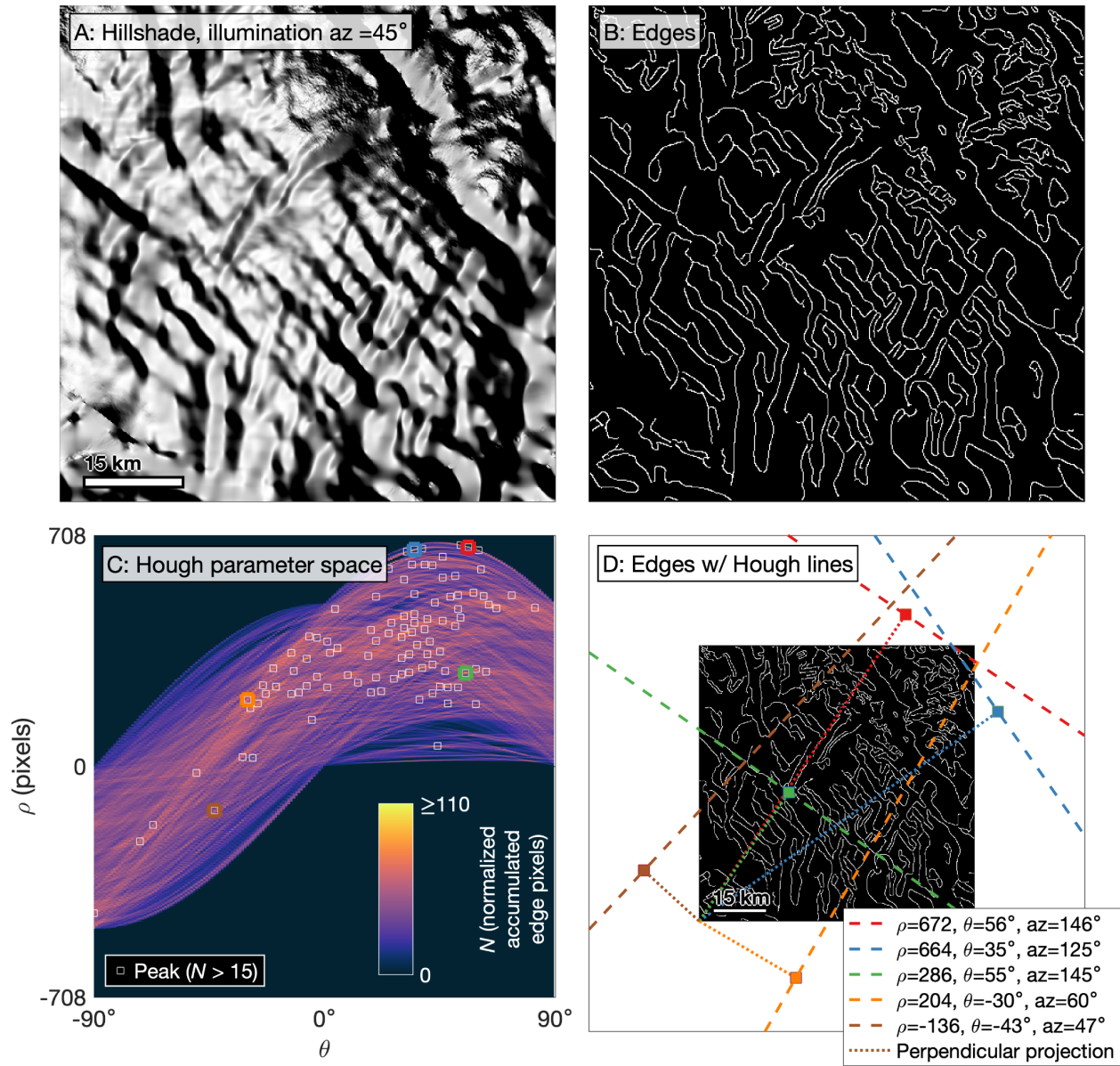


Figure S4.

Example workflow for Hough transform. (A) IFPA hillshade for a 75-km square region illuminated from an azimuth of 45°. (B) Edges of the hillshade extracted using the Canny method. (C) Hough parameter space, normalized to reduce bias toward identifying peaks in the center of the image. White squares highlight the 100 most prominent perpendicular projections, and colored squares highlight five of the most prominent ones, which are plotted on top of the edge map in (D) along with their respective lines that may follow potential linear features.

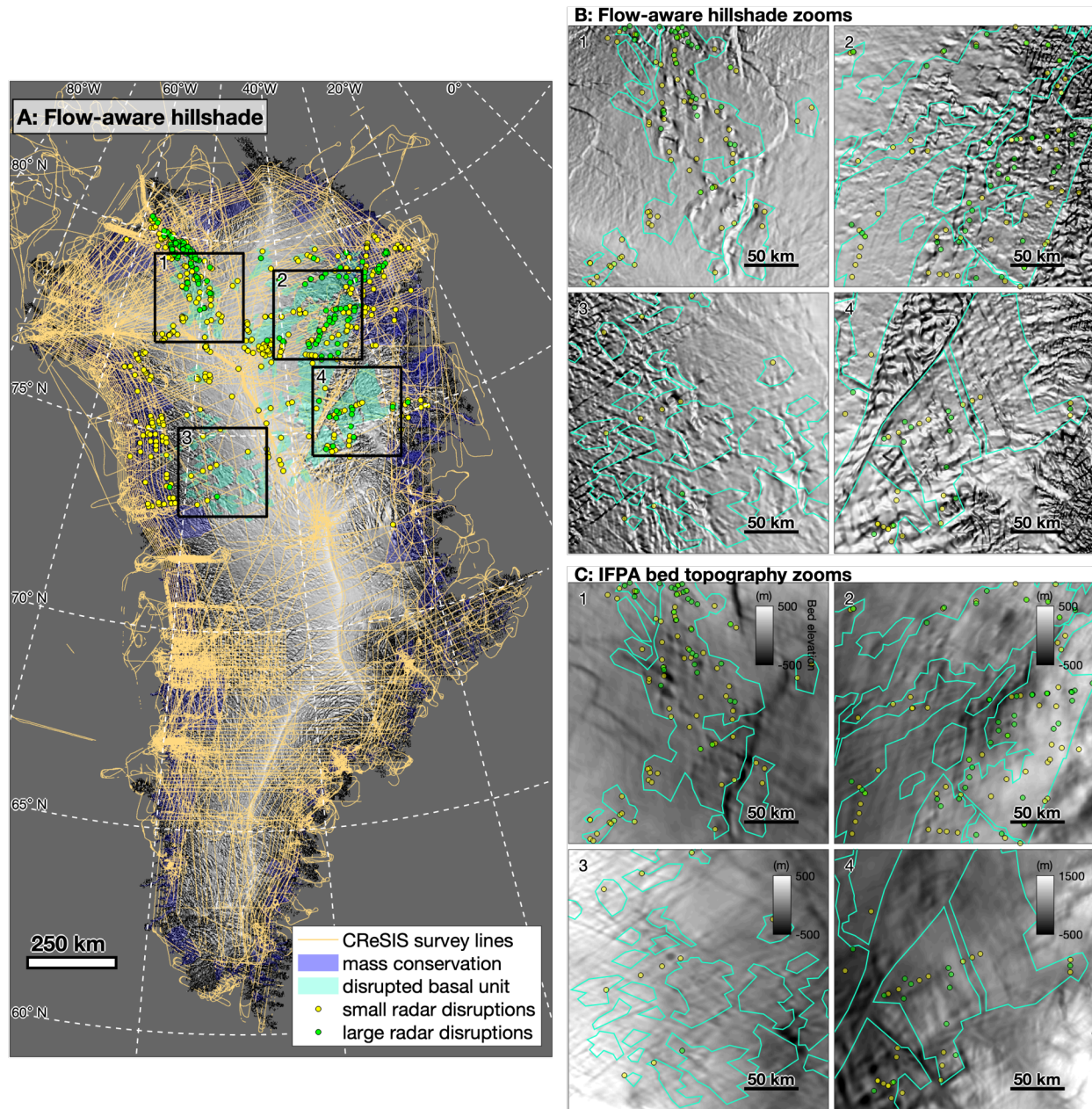


Figure S5.

Disrupted basal units in Northern Greenland. (A) Subaerial perspective from the GrIMP flow-aware hillshade showing CReSIS surveys, mass conservation regions, disrupted basal units mapped in this study, and small and large basal unit disruptions mapped by Leysinger Vieli et al. (2018). (B) Same as (A), zoomed-in on square regions. (C) Zoom-ins of raw IFPA bed topography showing erroneous subglacial topography induced by disrupted basal units.

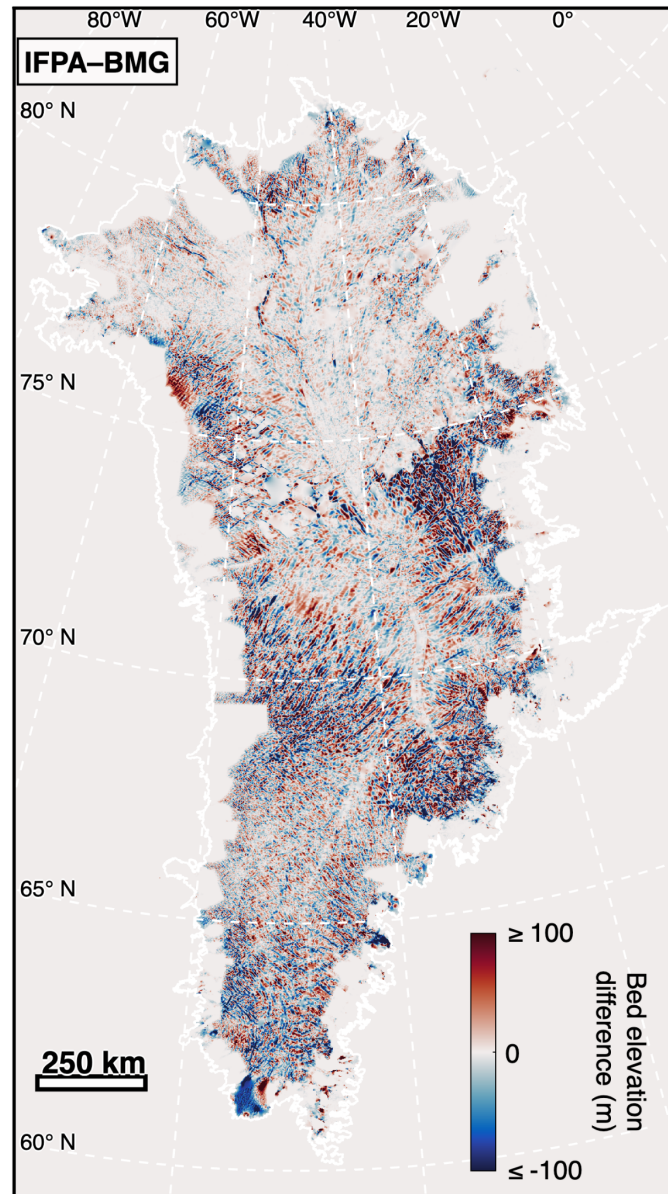


Figure S6.
Difference between the blended IFPA and BMG bed elevation.

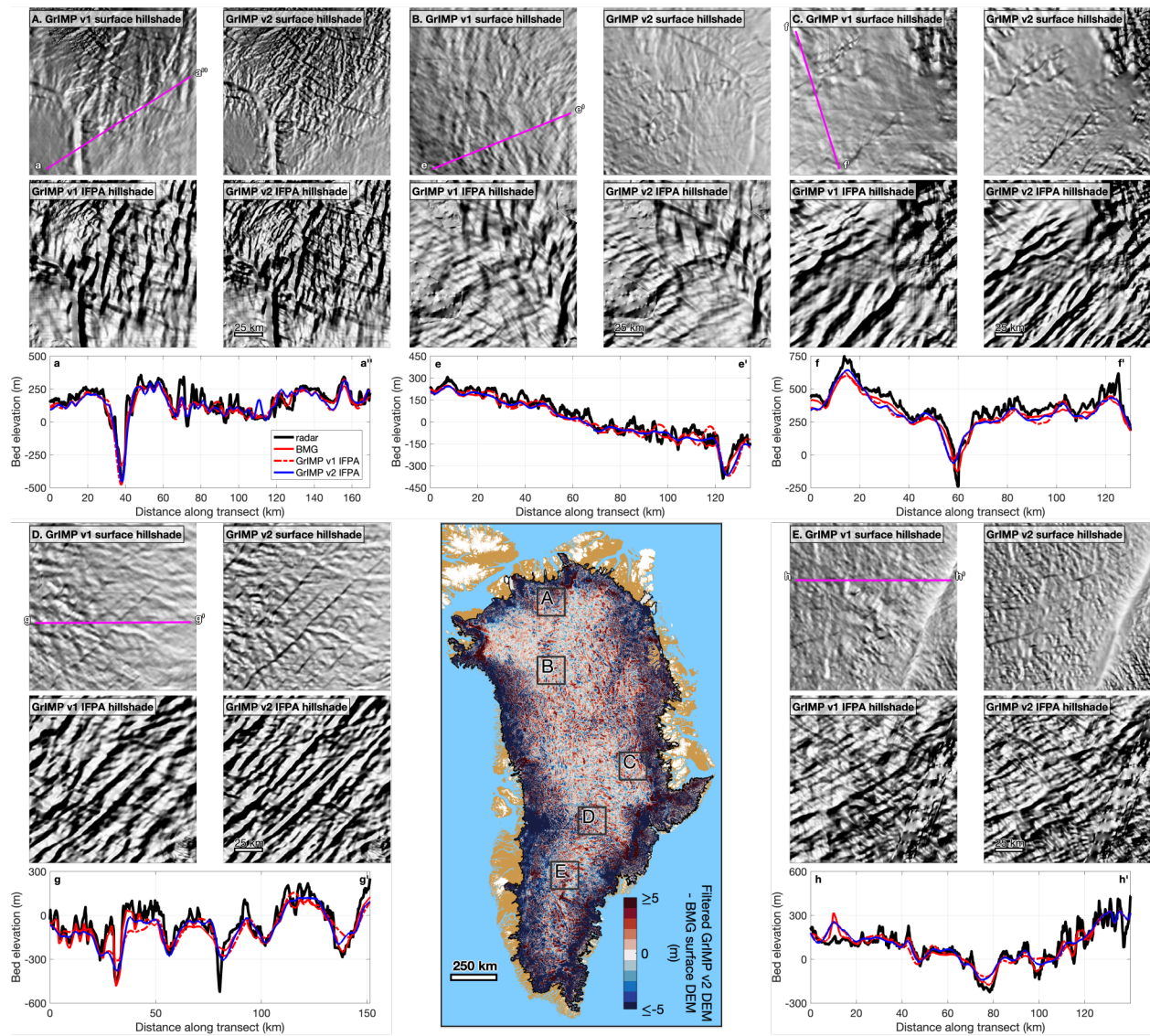


Figure S7.

Comparison of GrIMP v1 (left panels) and v2 (right panels) flow-aware hillshades (top panels), IFPA bed hillshades (middle panels), and bed elevations along CReSIS surveys (profiles/lower panels) for five regions. Contiguous surface features are more prominent in GrIMP v2 than v1.

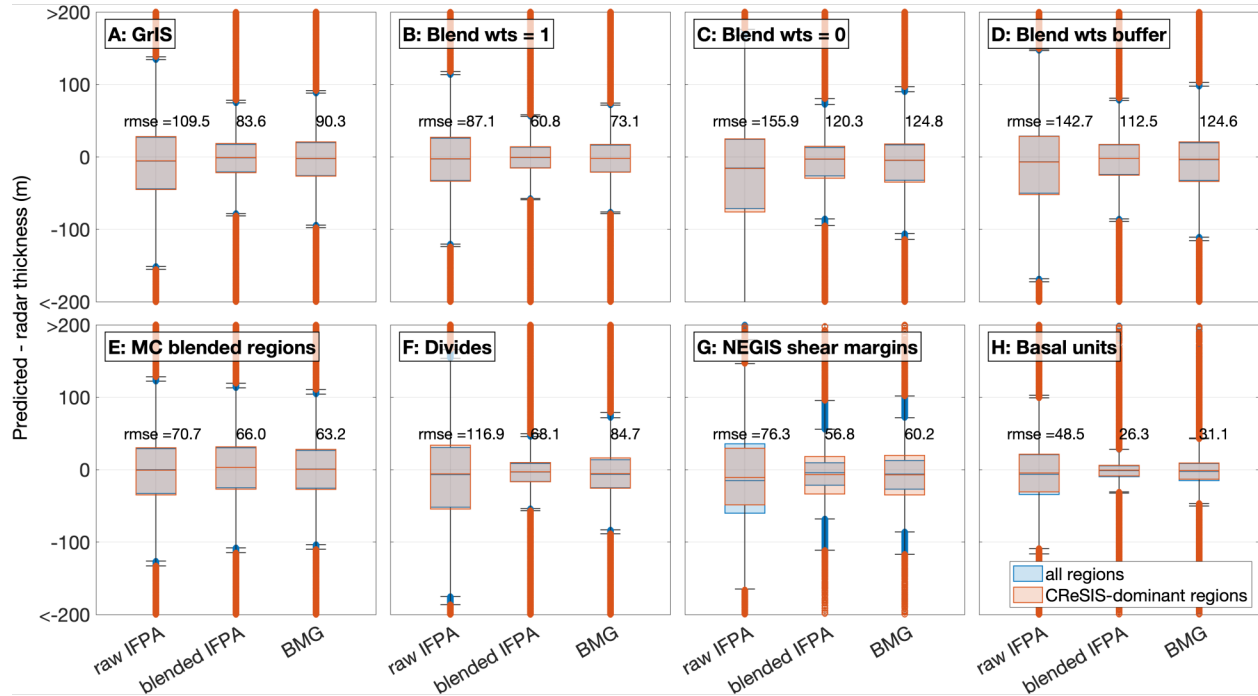


Figure S8.

Box plots comparing thickness residuals between CReSIS observations and IFPA (both raw and blended) and BMG across various ROIs. The line inside each box is the sample median, the upper and lower edges of each box are the upper and lower quartiles, respectively, the whiskers extend to the nonoutlier maximum and minimum values, and outliers are shown as circles (which overlap here). “Blend wts” refers to regions where the blended weights for IFPA vs. BMG (Figure S3) are either unity (IFPA only), zero (BMG only), or within in between those values (“buffer”). In (F–H), the box plots for blended IFPA and BMG should be nearly identical; discrepancies arise from using the GrIMP DEM to obtain thickness from the blended IFPA bed whereas BMG uses the GrIMP v1 DEM.

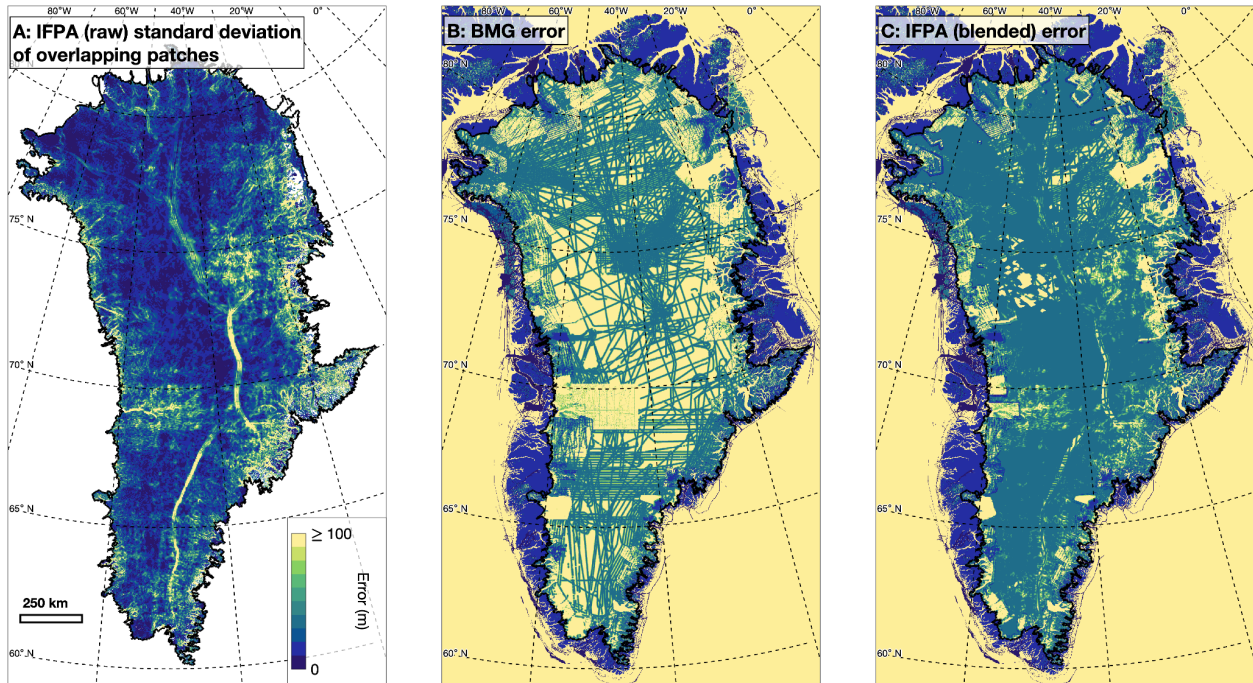


Figure S9.

Uncertainty estimates for each subglacial topography map. (A) A proxy for uncertainty in the IFPA method, which is the standard deviation of bed elevation of overlapping patches. Although not a true error measurement for the IFPA method, the standard deviation of overlapping patches indicates regions where the assumptions behind the method break down, particularly the assumption of planar flow in a slightly inclined slab, e.g., in mountainous terrain with steep slopes or in where surface-velocity azimuths change abruptly, such as at ice divides, where the standard deviation is high (yellow). (B) Reported error in subglacial topography for BMG. (C) Uncertainty estimate of the blended subglacial topography, computed as the root sum of squares of (A) and (B) blended using the same weights for absolute subglacial topography ([Figure S3](#)).

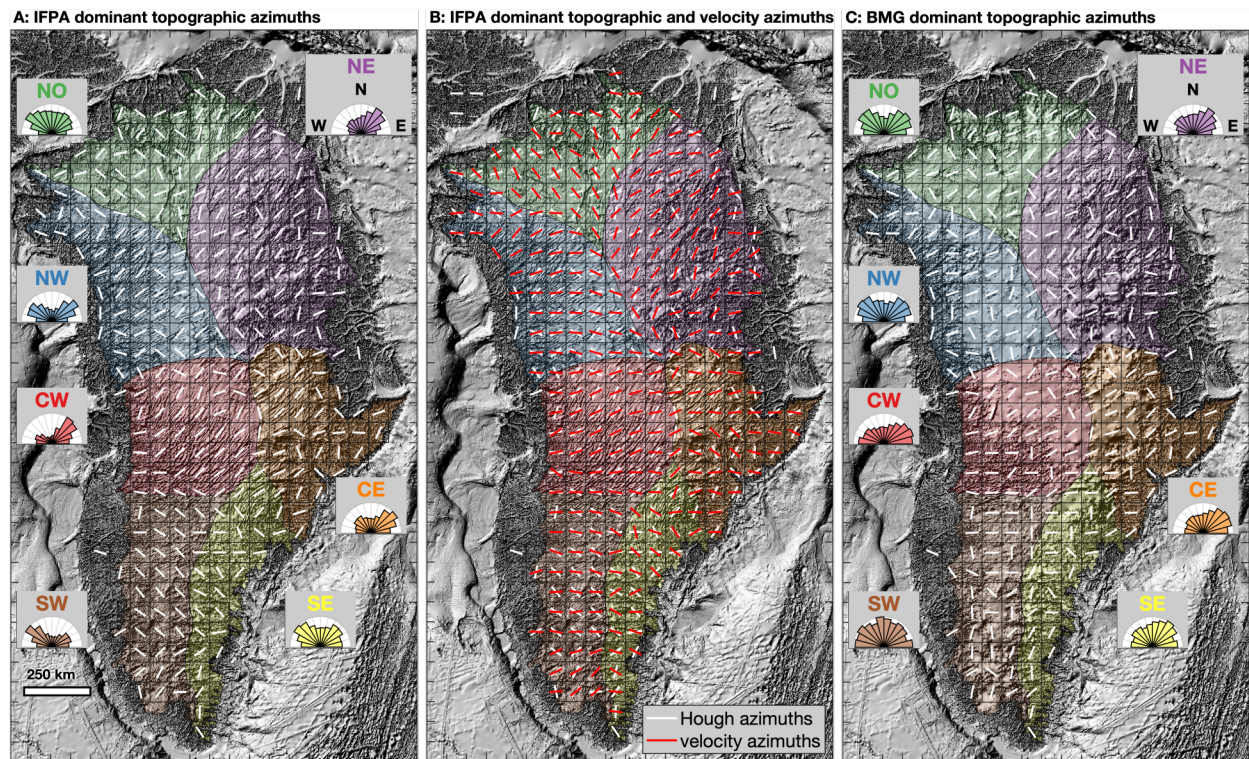


Figure S10.

(A) Same as [Figure 3B](#) (Hough azimuths for IFA). (B) Same as [Figure 3B](#), but including mean surface-velocity azimuths for each region. (C) Same as [Figure 3B](#), but using BMG. Histograms are normalized by the azimuthal bin with the maximum value.

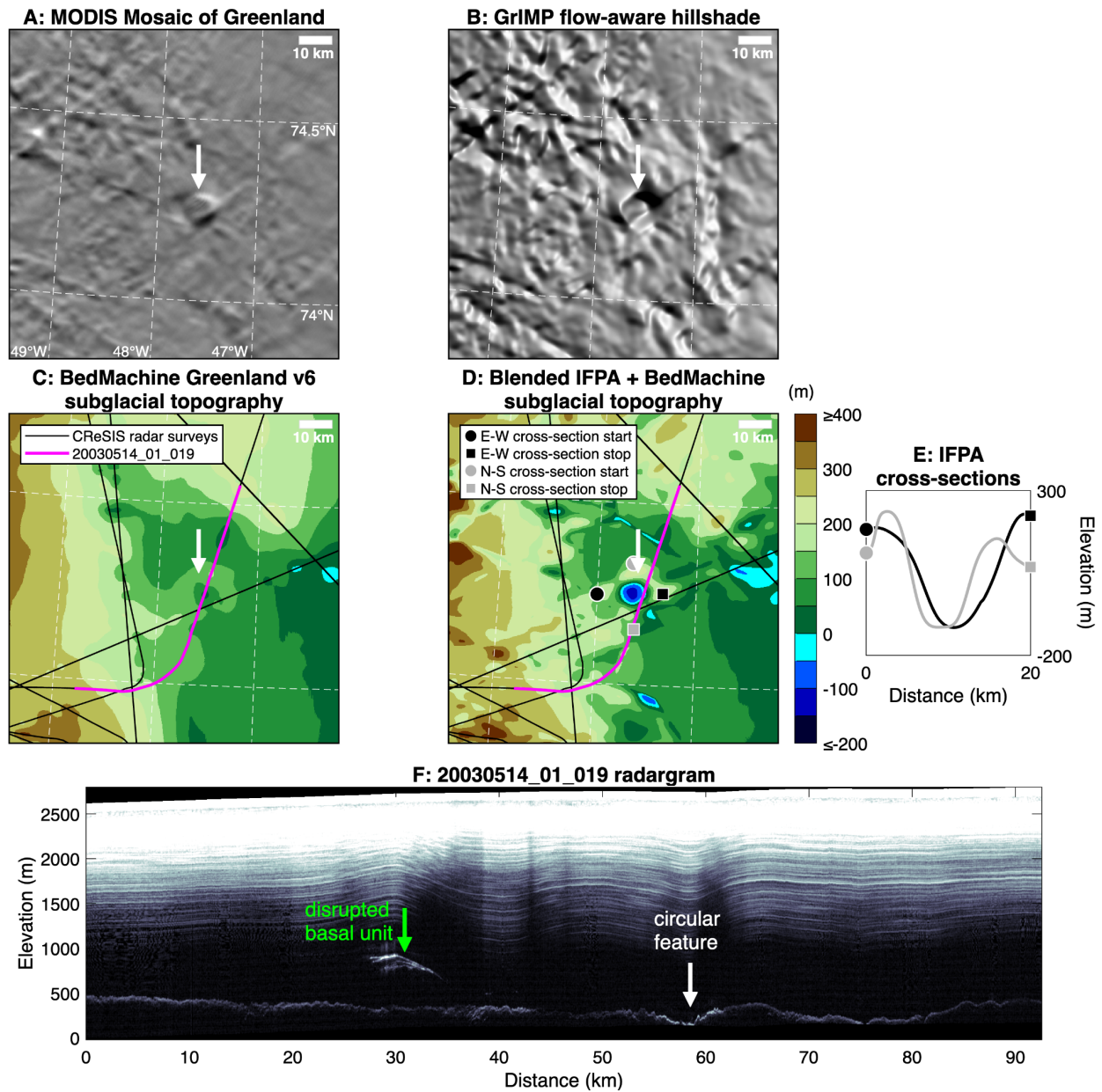


Figure S11.

Overview of an anomalous circular depression (hereafter “feature”) in northwestern Greenland. (A, B) Subaerial perspective from the MODIS Mosaic of Greenland (Haran et al., 2013) and the GrIMP flow-aware hillshade, respectively. (C, D) BMG and IFPA subglacial topography, respectively. White arrow in (A–D, F) highlights the location of the feature. (E) Cross-sections through the feature from blended IFPA subglacial topography. (F) The sole radargram from 2003 that has overflowed the feature, grazing the eastern flank of the depression (CReSIS, 2025). A disrupted basal unit is also highlighted.