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ANALOG STUDY OF DESCENTS FROM LUNAR ORBIT

By Joseph N. Sivo and Carl E. Campbell

Lewis Research Center
Cleveland, Ohio

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
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Analog study of open-loop soft lunar landings from circular orbit was made to determine the effects of orbit altitude, retrothrust magnitude and direction, and techniques of retrothrust staging on propellant consumption, range, and velocity components near touchdown. Descents to the surface were investigated over a range of orbit altitudes to 300,000 feet and a range of thrust-Earth-weight ratios to 1.5 for various thrust-vector directions relative to the velocity vector. Both continuous thrust and two-stage thrusting modes were used.

For the continuous-thrust techniques discussed herein, an initial thrust-Earth-weight ratio of about 0.5 resulted in near optimum propellant consumption for orbit altitudes up to 150,000 feet. With two-stage or interrupted thrust, propellant consumption decreased with increasing thrust-weight ratio, but the reductions were small at thrust-Earth-weight ratios greater than about 1.0. Interrupted-thrust methods were more efficient than continuous-thrust methods for orbit altitudes above about 50,000 feet. Any significant range control (range extension only) during constant-continuous-thrust descents was costly in terms of propellant consumption. Efficient range control was available during interrupted-thrust descents with control of first-retrothrust cutoff velocity and thrust-vector angle.

INTRODUCTION

The safe accomplishment of a lunar soft landing by a manned vehicle demands that the most advantageous mode of descent to the lunar surface be intensively studied. Lunar descent from a close lunar orbit has several major advantages over a radial lunar approach. The close lunar orbit permits careful surveillance of the intended landing site, provides time for orbit adjustments prior to landing commitment, and allows time to check out the landing vehicle system prior to commitment. Descent from lunar orbit is a logical step in the manned lunar program, which is first to include circumlunar and lunar orbiting phases.

The descent-from-orbit technique also offers the advantage of a safer abort capability than the radial descent method. In any landing abort maneuver, it is of primary importance to operate on the vertical velocity component to assure a safe abort once the descent to the surface is initiated. For a radial descent trajectory, the vertical velocity is the total velocity vector, whereas in the
Descent from orbit trajectory, the vertical velocity is only a small component of the total velocity vector. Thus, the descent from orbit allows more time and requires less thrust and energy to abort a landing than the radial descent. Investigations dealing with the problem of descent from orbit are described in references 1 to 4. A typical study relating to radial descent is reported in reference 5.

It is the intent of this analysis to investigate the propulsion and control requirements of descents from lunar orbit to the lunar surface. An analog study was made of open-loop soft lunar landings from lunar orbit to determine effects on fuel requirements, range, and velocity components near touchdown of such variables as orbit altitude, retrothrust magnitude and direction, and techniques of retrothrust staging. Terminal approach techniques that resulted in both vertical and horizontal vehicle attitude at thrust termination were considered. The descents to the surface were analyzed over a range of orbit altitudes to 300,000 feet and a range of thrust - Earth-weight ratios to 1.5 for a range of thrust-vector direction schedules relative to the total velocity vector. Both continuous thrust and two-stage thrusting modes were used. Comparisons of several basic descent maneuvers are made in terms of fuel requirements and range capability.

ANALYSIS OF BASIC MANEUVERS

The vehicle is assumed to be a point mass concentration at its center of gravity moving in an inverse-square gravity field. The moon is assumed to be spherical and nonrotating. Lunar descents may well originate from the pericynthion of an elliptical orbit subsequent to a Hohman type (elliptical) transfer from a higher initial orbit altitude. For this analysis, however, the descents were assumed to originate from circular orbits to simplify the choice of initial velocity at each orbit altitude. A specific impulse of 424 seconds (hydrogen and oxygen) was used during all thrusting periods throughout the investigation. A few descent maneuvers were also run with a specific impulse of 300 seconds to illustrate the performance differences resulting from use of a typical storable propellant.

The general equations used in this analysis are as follows:

\[ \dot{V} = \frac{-F}{m} \cos \beta - g \sin \theta \quad (1) \]

\[ \dot{\theta} = \frac{1}{V} \left( \frac{-F}{m} \sin \beta - g \cos \theta \right) + \frac{V \cos \theta}{r_m + h} \quad (2) \]

\[ \dot{h} = V \sin \theta \quad (3) \]

\[ \dot{R} = \frac{r_m}{r_m + h} V \cos \theta \quad (4) \]
\[
\frac{W_p}{W_c} = \frac{F}{m_0} \cdot t_b \quad \text{(5)}
\]

\[
\Delta V = g_c I_{sp} \ln \left(1 - \frac{W_p}{W_c}\right) \quad \text{(6)}
\]

\[
m = m_0 - \frac{F t_b}{g_c I_{sp}} \quad \text{(7)}
\]

(All symbols are defined in the appendix.)

A force diagram indicating vector directions is shown in figure 1.

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**Continuous-Thrusting Mode**

The independent variables considered in this descent maneuver were thrust magnitude (maintained constant during the maneuver), orbit altitude (circular orbit velocity at each altitude), and thrust direction. The thrust vector was maintained either along the velocity vector or at some fixed angle relative to the velocity vector during the maneuver. In the latter case, two types of thrust-vector direction control were used. The first used only positive values of \( \beta \) (thrust-vector angle relative to velocity vector) during the entire maneuver. This resulted in an initial downward thrust component. The second provided a \( +\beta \) direction (force down) followed by a \( -\beta \) direction (force up) of
equal magnitude. The purpose of this maneuver was to provide horizontal vehicle attitude at touchdown. The altitude at which \( \beta \) was changed from plus to minus was determined by iteration. Whenever the velocity was not zero at zero path angle, the thrust-vector angle was adjusted to maintain horizontal flight until the velocity reached zero.

Interrupted-Thrusting Mode

This descent was characterized by two thrusting periods separated by a free-fall period. The thrust magnitude was equal during both thrusting periods; however, the thrust-vector direction was not necessarily the same. The first thrusting period was terminated when the absolute velocity reached a selected value. Thrust was reinitiated at the altitude that resulted in zero velocity at zero altitude. This altitude was found by iteration. Two general techniques regarding the thrust-vector angle were used. The first technique used zero or positive values of \( \beta \) during the first thrust period and \( \beta = 0 \) during the second thrust period. The second technique utilized zero or positive values of \( \beta \) during the first thrust period and negative values during the second thrust period with horizontal flight angle as the objective when zero altitude was reached. A modification of the interrupted-thrust maneuver utilized a low level of thrust instead of a free-fall period between the two main thrusting periods.

The general procedure in obtaining data was to hold all but one of the independent variables constant while iterating with the others until the objective of zero velocity or zero path angle at near zero altitude was reached. All thrust-vector angle changes and thrust terminations were assumed to occur instantaneously.

DISCUSSION

It is the intent of this analysis to evaluate generally the effects of some primary variables on fuel consumption, surface range, component velocities, and maneuver times during descent from lunar orbit to the lunar surface. For reference purposes, the characteristic velocity of the maneuver (related to the propellant consumption by eq. (6)) is presented with the data, but it is not discussed in the text. Although direct descents will not actually be attempted to the lunar surface but to some reasonable hover altitude, it is felt that the performance values presented herein are good approximations of realistic orbit-to-hover descent maneuvers, and that the performance comparisons of the various types of maneuvers are quite valid. Propulsion aspects of the translation maneuver that may be required between hover and touchdown are discussed in reference 6.

Continuous Thrust

Thrust along velocity vector. - This thrusting mode (sketch (a) of fig. 2) requires the matching of orbit altitude with thrust magnitude for zero velocity
at touchdown. Trajectory 1 of figure 2 illustrates the result of too high a
thrust level for a given orbit altitude: the velocity reaches zero before zero

altitude is reached and this necessitates a free-fall period and an engine re-
start. Reducing the thrust to the proper value for the orbit altitude results
in trajectory 2. This not only avoids an engine restart but reduces the pro-
pellant consumption.

The performance of properly matched descents with continuous constant thrust
along the velocity vector is shown in figure 3. The variation of orbit altitude
with thrust required is shown to be approximately hyperbolic (fig. 3(a)). Any
change in thrust level by a factor of 2 results in an inverse change in orbit
altitude by a factor of about 4.4. This sensitivity of orbit altitude to thrust
level could result in sizable errors in final altitude for nominal thrust-level
errors in an open-loop descent maneuver. If a reasonably safe orbit altitude
is considered to be 50,000 feet or higher (considering lunar oblateness and
mountain height), a thrust - Earth-weight ratio below 0.5 would be required with
this thrusting mode.

Propellant consumption was fairly insensitive to thrust level at thrust -
Earth-weight ratios above about 0.6 (fig. 3(b)) but increased rapidly as thrust
level was reduced appreciably below this value (as orbit altitude increased).
Surface range and maneuver time increased at about the same rate with decreased thrust or increased orbit altitude (figs. 3(c) and (d)). The curve of horizon surface range in figure 3(c) indicates that the landing site is not visible from the start of this type maneuver unless the orbit altitude is above about 280,000 feet. The final vehicle attitude was always vertical with this thrusting mode and most of the flight-path-angle change (0° to 90°) occurred toward the end of the maneuver. Analog measurements of these rapid angle changes were not very accurate, but vehicle pitching rates of the order of 4° in the final second were the highest recorded with this type maneuver.
Force-down maneuver. - In the force-down maneuver, the thrust-vector direction was maintained at a fixed angle to the velocity vector as shown in sketch (b) of figure 2 such that a downward component of thrust existed during the initial portion of the maneuver. Force-down maneuvers were investigated as a means of reducing range (improving landing site visibility) and of possibly reducing propellant consumption by the reduction of gravity losses. The effect of specific impulse on descent performance was investigated for several force-down maneuvers and is discussed at the end of this section. A typical force-down trajectory is shown by trajectory 3 of figure 2 and the performance of all the force-down maneuvers investigated is presented in figure 4.

The combination of thrust level and thrust-vector direction ($+\theta$) necessary for zero velocity at touchdown from orbit altitudes of 50,000, 100,000, and 150,000 feet is shown in figure 4(a). At a given altitude, larger $+\theta$'s were required for higher thrust levels, and for a given thrust level, higher $+\theta$'s were also required for higher orbit altitudes. Minimum propellant consumption was obtained with thrust - Earth-weight ratios of 0.7, 0.5, and 0.4 for orbit altitudes of 50,000, 100,000, and 150,000 feet, respectively, but a thrust - Earth-weight ratio of about 0.5 would give near optimum propellant consumption at
all three altitudes. Thrust levels less than that required for thrusting along the velocity vector from a given altitude require a force-up or -β maneuver; however, propellant consumption increased much more rapidly with -β than it did with a corresponding +β or force-down maneuver. The minimum propellant consumption obtainable at a given orbit altitude increased as the orbit altitude was raised. The minimum propellant consumption ratios for continuous-thrust descents from orbit altitudes of 50,000, 100,000, and 150,000 feet were 0.341, 0.351, and 0.361, respectively.

As an adjunct to this analog study, constant-continuous-thrust descents were also investigated on a digital computer by a calculus-of-variations technique adapted from the method of reference 7. This study yielded the minimum propellant consumption for a given altitude-thrust combination with constant continuous thrusting by allowing an optimal variation of β. The minimum propellant consumption ratios determined by this method were less than 1 percentage point lower than the values obtained in this investigation with a constant β throughout the descent maneuver.

Figure 4. - Concluded. Performance for continuous-thrust force-down maneuvers.
Surface range and maneuver time both decreased with increasing thrust level (and increasing values of +β) (figs. 4(b) and (c)) but were only slightly affected by orbit altitude because of the higher degree of force down imposed on the trajectory from higher altitudes; however, the apparently small effect of orbit altitude on maneuver time results in the significant effect on propellant consumption shown in figure 4(d). The vehicle attitude at touchdown was nearly vertical for all force-down maneuvers, but because of the offset thrust vector, the vertical position actually occurs slightly before the velocity reaches zero. Force-down maneuvers from 50,000 feet resulted in pitching rates as high as 15° per second at the end of the maneuver. If a force-down maneuver were used, the thrust vector would be aligned with the velocity vector at whatever rate the attitude control system could tolerate before the vertical position was reached. The performance values presented here would not, however, be significantly different.

The performance of continuous-thrust force-down maneuvers with a lower energy propellant combination (Isp = 300 instead of 424 sec) is shown by the dashed curves of figure 4. The major effect of the lower Isp was an increase in propellant consumption ratio of about 0.10 for a given thrust - Earth-weight ratio (fig. 4(d)), which meant a reduction in landed vehicle weight of at least 16 percent. The higher propellant consumption (lower average vehicle weight) at the same thrust level meant a higher average deceleration during the descent maneuver with the low-energy propellant. This resulted in a lower maneuver time (fig. 4(c)), a higher force-down angle (fig. 4(a)), and a shorter range (fig. 4(b)) compared with the high-energy-propellant descent maneuvers.

A performance map of the continuous-thrust maneuvers discussed so far is presented in figure 5. It can be seen that a slight force-down maneuver
(β ≈ 5°) gives slightly lower propellant consumption than thrusting along the velocity vector from a given orbit altitude. There are two reasons for this effect: (1) The thrust level required for this amount of force down is near the optimum thrust for descent from a given altitude (as determined by the calculus-of-variations technique), and (2) this amount of force-down angle resulted in a flight path that more closely approached the optimum (variable β) flight path. A slight amount of force down (and the increased thrust level that accompanied it) also reduced the range compared with thrusting along the velocity vector so that the landing site was visible throughout the descent maneuver. A fair amount of range reduction was possible with large increases in thrust level and force-down angle. At orbit altitudes near 50,000 feet a 50-percent range reduction increased the propellant consumption only about 1 percent, but at higher orbit altitudes, the large range reductions began to be costly in propellant consumption. These variations in range resulted from the selection of different fixed thrust levels and should not be confused with the problem of making major range changes after initiation of the descent. This would require variable-thrust engines to maintain high efficiency.

Force-down and flare maneuver. - In order to reduce vehicle pitching rates and sink rates (the vertical velocity component) near touchdown, the force-down maneuver was modified to consist of a force-down (±β) maneuver followed by a force-up (-β) or flare maneuver, as described in the section ANALYSIS OF BASIC MANEUVERS. The vehicle attitude is horizontal when zero velocity is reached instead of vertical as with the previous maneuvers. A typical force-down and flare maneuver is shown by trajectory 4 of figure 2. The performance of several flare maneuvers from an orbit altitude of 100,000 feet is shown in figure 6 for a thrust - Earth-weight ratio of 0.63.

Figure 6. - Continuous-thrust force-down and flare maneuvers from 100,000-foot orbit. Thrust - Earth-weight ratio, 0.6; thrust-vector angle during final horizontal run, -10°.
The force-down and flare maneuver that resulted in zero velocity at pullout occurred with $\beta = \pm 15^\circ$. The propellant consumption (fig. 6(a)) was about equal to that for the simple force-down maneuver from the same altitude, but the range (fig. 6(b)) was slightly longer. Increasing the value of $\beta$ resulted in an increasing horizontal velocity component at pullout as shown in figure 6(c). This necessitated constant-altitude horizontal runs (thrust-vector angle of approximately $-10^\circ$) to allow time for the horizontal velocity to reach zero. Increasing the thrust angle $\beta$ also decreases the deceleration term $\frac{F}{m} \cos \theta$ and results in an increased maneuver time. An increased propellant consumption and increased range are associated with the increased time.

**Velocity components.** Vertical and horizontal velocity components near touchdown are shown as a function of altitude in figure 7 for several continuous-thrust descent maneuvers. To accomplish a soft landing, it will be necessary to cope with the indicated velocities if efficient descents are to be made. This will require accurate knowledge of the velocity components and hence accurate sensor devices.

The general effects of the independent variables on velocity components were as follows: (1) raising the orbit altitude at a given thrust level increased vertical velocity and decreased horizontal velocity; (2) raising the thrust level at a given orbit altitude increased the vertical velocity but had little effect on the horizontal velocity; and (3) raising the orbit altitude and decreasing the thrust as required for thrusting along the velocity vector ($\beta = 0$) increased the vertical velocity, except for the final few thousand feet of altitude, and decreased the horizontal velocity. The vertical velocity trends are directly related to maneuver time, vertical velocity being the average value determined from the required altitude change and the total maneuver time. The dashed curve of figure 7(a) shows that a force-up thrusting mode during the final part of the maneuver instead of force down all the way reduces the vertical velocity near touchdown, but the reduction is at the expense of increased horizontal velocity.

**Interrupted Thrust**

The rapid increase in propellant consumption with increasing orbit altitude for continuous-thrust descent maneuvers (see fig. 5) led to the investigation of interrupted thrusting modes for orbit altitudes from 100,000 to 300,000 feet. Interrupted-thrust descents were characterized by two thrusting periods separated by a free-fall period. The thrust magnitude was the same during both thrusting periods, but the thrust-vector direction varied. The thrust vector was always along the velocity vector during the final thrusting period except for force-up ($-\beta$) maneuvers during the final thrusting period when horizontal vehicle attitude was desired at touchdown.

The independent variables considered with continuous thrusting, orbit altitude, thrust magnitude, and thrust direction, were also considered with interrupted thrusting along with an additional variable, the velocity increment.
(a) Thrust - Earth-weight ratio $F/W_E$, 0.62.

(b) Initial altitude $h_0$, 100,000 feet.

(c) Thrust-vector angle $\beta$, 0.

Figure 7. - Vertical and horizontal velocity components as function of altitude for continuous thrust.
Figure 8. - Variation of propellant consumption with orbit altitude for interrupted thrust.
Figure 9. - Variation of range with orbit altitude for interrupted thrust.
removed during the first retrothrust \( \Delta V_1 \). Removal of all the velocity (approx. 5500 ft/sec) in the first retrothrust period resulted in the shortest range but is the least efficient descent maneuver (see trajectory 1 of fig. 2). The other extreme would be an elliptical (Hohman type) transfer from orbit to the surface with practically all the velocity removed in the final retrothrust period. This is the most efficient landing maneuver but would be impractical from the standpoint of safety and range (180°).

A reasonable compromise between the two extremes should exist at some intermediate division of velocity removal between the initial and final retrothrust periods. The values of \( \Delta V_1 \) considered were 1500, 2500, and 3500 feet per second. Some typical interrupted-thrust trajectories are shown in curves 5, 6, and 6a of figure 2.

The propellant consumption increased with increasing orbit altitude as shown in figure 8, but the rate of increase was lower than with continuous-thrust maneuvers (see fig. 5). All interrupted-thrust descents from an orbit altitude of 100,000 feet had lower propellant consumption ratios than the minimum of 0.351 obtained with continuous thrust from the same altitude. Interrupted-thrust maneuvers with a first retrothrust \( \Delta V \) of 1500 feet per second had propellant ratios of 0.351 or less for orbit altitudes as high as 300,000 feet. The crossing of \( \beta_1 = 0 \) and \( \beta_1 = 10^\circ \) curves at high altitudes indicates that a small amount of force down gives optimum propellant consumption just as in the continuous-thrust case. Range increased with increasing orbit altitude, as shown in figure 9, but the landing site was visible throughout the descent for all but the 1500-foot-per-second \( \Delta V_1 \) maneuvers with force-down thrust vectors less than about 10°.

Typical effects of the various independent variables on propellant consumption and range are shown in figure 10 for an orbit altitude of 200,000 feet. A fair amount of range reduction was possible with force-down thrust vectors with little or no increase in propellant consumption for force-down angles up to about 15°. Large range reductions were possible by selection of a large first retrothrust \( \Delta V \) but resulted in significant increases in propellant consumption. Some range control is possible with interrupted thrusting with control of the first-retrothrust cutoff velocity and thrust-vector angle. Increasing the thrust decreased both range and propellant consumption, but the reductions were small for the relatively large increases in thrust.

Velocity components near touchdown for several interrupted-thrust maneuvers are shown in figure 11. The general effects of the independent variables on velocity components were as follows: (1) Raising the orbit altitude increased vertical velocity and decreased horizontal velocity; (2) increasing the thrust level increased both vertical and horizontal velocities; (3) increasing the first retrothrust \( \Delta V \) increased vertical velocity and decreased horizontal velocity; (4) a force-down (+\( \beta \)) maneuver during the first retrothrust had no significant effect on either velocity component near touchdown; and (5) a force-up (-\( \beta \)) maneuver during the final retrothrust appreciably lowered the vertical velocity and increased the horizontal velocity considerably. Maximum vehicle pitching
rates were quite high for most interrupted-thrust descents as the velocity vector went from a fairly shallow angle to 30° in the final few seconds. The analog data, although only crudely accurate for high angular velocities, indicated pitching rates as high as 25° in the final second. A modified maneuver that would recognize actual vehicle-attitude-system limitations would not appreciably alter the performance values presented herein inasmuch as only the final few seconds of the maneuvers would be affected by the modification.

**Steped Thrust**

A modification of the interrupted-thrust maneuver was investigated that utilized a low level of thrust instead of a free-fall period in order to avoid a main engine shutdown and restart. The low thrust level could be provided by either extreme throttling of the main engine or by the use of a pilot thrust system that would serve as an igniter for the main engine. Thrust levels
(a) First-retrothrust velocity change $\Delta V_1$, 3500 feet per second; first- and second-retrothrust thrust-vector angles $\beta_1$ and $\beta_2$, 0.

(b) Initial altitude $h_0$, 300,000 feet; thrust - Earth-weight ratio $F/N_E$, 0.62; first- and second-retrothrust thrust-vector angles $\beta_1$ and $\beta_2$, 0.

(c) Initial altitude $h_0$, 300,000 feet; thrust - Earth-weight ratio $F/N_E$, 0.62; first-retrothrust velocity change $\Delta V_1$, 1500 feet per second.

Figure 11. - Vertical and horizontal velocity components as function of altitude for interrupted thrust.
of one-twentieth to one-fifth of the main engine thrust were considered in this investigation. A typical stepped-thrust trajectory is shown by curve 7 of figure 2 and the stepped-thrust performance (with all thrusting along the velocity vector) is presented in figures 12 and 13.

(a) Thrust - Earth-weight ratio, 0.62.
(b) Thrust - Earth-weight ratio, 1.24.
Figure 12. - Effect of ratio of stepped thrust to main thrust on propellant consumption.

(a) Thrust - Earth-weight ratio, 0.62.
(b) Thrust - Earth-weight ratio, 1.24.
Figure 13. - Effect of ratio of stepped to main thrust on surface range.
For stepped-thrust descents from orbit altitudes around 100,000 feet propellant consumption (fig. 12) was not appreciably higher than for the free-fall case for stepped-thrust levels that were as high as one-fifth of the main engine thrust. Stepped thrust could, however, become costly in propellant consumption at high orbit altitudes with high values of $\Delta V_1$ if large stepped-thrust levels were required for reliability (or with limited variable-thrust range). The range always decreased with increasing stepped-thrust level (fig. 13), as could be expected, but the reductions were small except for high orbit altitudes with low values of $\Delta V_1$. In general, it appears that the stepped-thrust principle could be utilized for most descent conditions and would provide improved reliability and some range reduction with little cost in propellant consumption.

Comparison of Landing Techniques

Performance comparisons of continuous- and interrupted-thrust descents are shown in figures 14 to 17 for ranges of the primary variables of orbit altitude and thrust - Earth-weight ratio. The effect of orbit altitude on propellant consumption for both maneuver techniques is shown in figure 14. Propellant consumption increased rapidly with increasing orbit altitude for continuous thrusting along the velocity vector. Force-down maneuvers (optimum combinations of $\beta$ and $F/W_e$) gave lower propellant consumption than the $\beta = 0$ thrusting mode at a given altitude, but the increase in propellant consumption with increasing orbit altitude was just as rapid. Lower propellant consumption was possible with interrupted thrusting than with continuous thrusting for orbit altitudes above about 50,000 feet, as shown by the dashed curves of figure 14, which represent the near optimum ($\beta = 0^\circ$ and $\Delta V = 1500$ ft/sec) interrupted-thrust descents at two thrust levels. At an orbit altitude of 300,000 feet, interrupted thrusting permitted a 6 or 7 percent higher landed vehicle weight than continuous thrusting. The increase in propellant consumption with increasing orbit altitude for interrupted thrust was quite gradual for orbit altitudes up to 300,000 feet and closely paralleled the minimum-energy curve. This curve represents the theoretically most efficient descent maneuver from any orbit altitude, that is, an impulsive thrust applied $180^\circ$ from the landing site such that the pericynthion of the resultant elliptical orbit occurs at zero altitude with the final $\Delta V$ also applied impulsively (Hohman transfer).

The effect of thrust level on propellant consumption for both continuous- and interrupted-thrust descents is shown in figure 15 for an orbit altitude of 100,000 feet. The minimum energy value discussed previously ($W_p/W_o = 0.334$) is also shown as a reference. With continuous thrust the minimum propellant consumption ratio of 0.351 occurred at a thrust - Earth-weight ratio of about 0.5 and increased rapidly as the thrust was appreciably changed from the optimum value. With interrupted thrust, propellant consumption decreased continually with increasing thrust level, but the reductions were small for thrust - Earth-weight ratios greater than about 1.0 ($W_p/W_o = 0.340$).
The effect of orbit altitude on the surface range covered during continuous- and interrupted-thrust descents is summarized in figure 16. The range increased appreciably with increasing orbit altitude with continuous thrusting along the velocity vector \((\beta = 0)\), but it was not noticeably affected by the orbit altitude for continuous-thrust force-down maneuvers. The range increased with increasing orbit altitude for all interrupted-thrust descents at about the same rate as with continuous thrusting along the velocity vector. The landing site was not visible
Figure 16. - Effect of orbit altitude on surface range for both continuous- and interrupted-thrust maneuvers.

Figure 17. - Effect of thrust - Earth-weight ratio on surface range for both continuous- and interrupted-thrust maneuvers. Orbit altitude, 100,000 feet.
from the start of the maneuver with continuous thrusting along the velocity vector except at very high orbit altitudes. Landing site visibility throughout the maneuver was possible for continuous-thrust force down and interrupted-thrust descents from any orbit altitude with the proper selection of independent variables.

Typical effects of thrust level on the surface range covered during continuous- and interrupted-thrust descents from a given orbit altitude are shown in figure 17. The range decreased markedly with increasing thrust level (and increasing force-down angle) for continuous-thrust descents. The thrust level had little effect on range for interrupted-thrust descents, especially with low values of $\Delta V_1$ where the first retrothrust was applied for a relatively short time when compared with the total maneuver time.

**SUMMARY OF RESULTS**

An analog study of open-loop lunar landings from circular orbit was made to determine the effects of orbit altitude and thrust management on descent trajectories and propellant consumption. The following summarizes the results of this study:

1. For the continuous-thrust techniques discussed herein, an optimum thrust-weight ratio $F/W_E$ occurs at approximately 0.5 for orbit altitudes up to 150,000 feet.

2. For interrupted thrust, propellant efficiency increases with increasing $F/W_E$, but gains at $F/W_E$s higher than 1.0 are insignificant. Near optimum efficiency is reached with an $F/W_E$ of 1.0.

3. For initial altitudes of about 50,000 feet, both thrust methods require the same propellant consumption. At higher altitudes, interrupted-thrust methods are more efficient than continuous-thrust methods. At 300,000 feet a 6- or 7-percent higher landed vehicle weight is available with interrupted thrust.

4. Any significant range control (range extension only) during constant-continuous-thrust descents was costly in terms of propellant consumption.

5. Efficient range control was available with interrupted-thrust methods by control of first-retrothrust cutoff velocity and thrust-vector angle.

Lewis Research Center
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APPENDIX - SYMBOLS

F  main thrust, lb

F_s  stepped thrust, lb

g  local gravity, ft/sec^2

g_c  Earth gravity, 32.17 ft/sec^2

g_m  lunar surface gravity, 5.32 ft/sec^2

h  altitude, ft

Isp  specific impulse, sec

m  mass, lb-sec^2/ft

R  lunar surface range, ft

r_m  radius of moon, 5.702×10^6 ft

\( t_b \)  burning time, sec

\( u \)  vehicle attitude angle, deg

V  velocity, ft/sec

W  weight, lb

\( \beta \)  thrust-vector angle, deg

\( \theta \)  flight-path angle, radians

Subscripts:

E  Earth

c  initial (circular) orbit

p  propellant

1  first retrothrust period

2  second retrothrust period

Superscript:

'  first derivative with respect to time
REFERENCES


