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Understanding the Main-Sequence Stars

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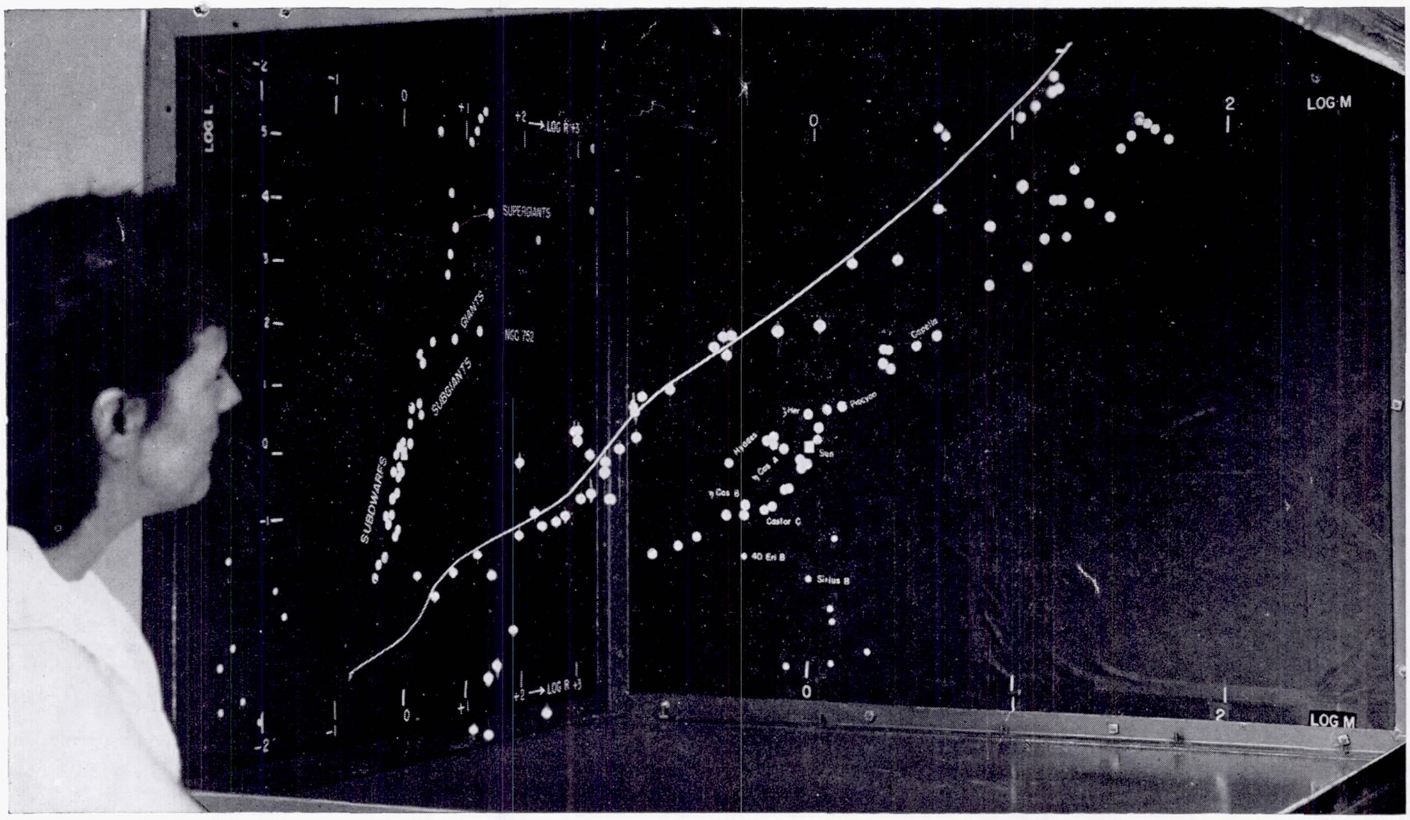

Many years ago at Yerkes Observatory, G. P. Kuiper constructed this three-dimensional model of stellar characteristics, with stars represented by white beads on thin wires. Luminosities increase upward, masses rightward, and stellar radii to the back. Thus, projecting the ball pattern on the rear panel gives the mass-luminosity relation, on the left panel the radius-luminosity relation. The base would represent radius against mass. Several white dwarf stars, including Sirius B, are shown at lower left by beads well below the white thread that marks the main sequence.

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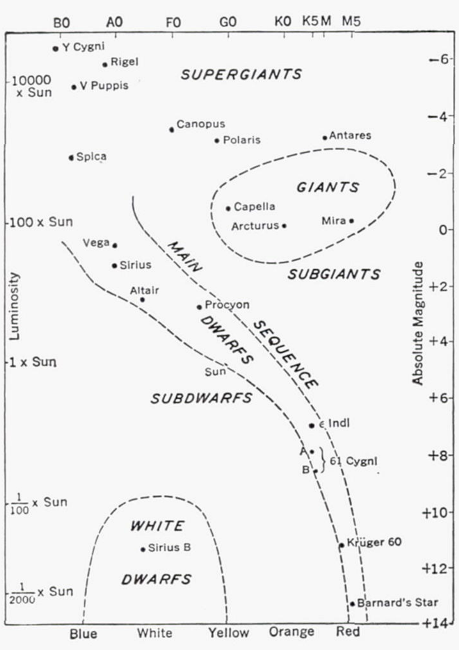


Well-known examples of important varieties of stars are shown in this Hertzsprung-Russell diagram. Adapted from "Astronomy," by W. T. Skilling and R. S. Richardson, Henry Holt and Co., 1947.

MOST of the stars we observe in the sky lie on the main sequence of the Hertzsprung-Russell diagram, in which their luminosities are plotted against their spectral types. At upper left in the diagram we find blue-white stars with effective temperatures of 25,000° or more and luminosities 10,000 times that of the sun. Both temperature and luminosity decrease down the main sequence to the red dwarfs, at lower right, where effective temperatures are only a few thousand degrees and luminosities as low as 0.0001 the sun's.

From an observational point of view, this correlation between luminosity and temperature over such a wide range is a basic characteristic of the main sequence. Furthermore, these differences are accompanied by corresponding ones in the masses of the stars.

For example, one component of the *O*-type spectroscopic binary star Delta Orionis has a mass of some 30 suns. On the other hand, the two far cooler *M* stars forming Castor C have masses only 0.6 times the sun's.

Extensive observations have led to two basic relations. First, a star's luminosity (L) is proportional to its mass (M) raised to some power. This is known as the *mass-luminosity* relation. In the case of stars on the main sequence above the sun's position, $L \propto M^4$.

Second, we also know that the energy radiated from a star's surface — its luminosity — is determined by its effective temperature and radius. Since each point on the H-R diagram represents a luminosity and a temperature, it must also represent a particular radius. Thus we can construct a series of curves, each corresponding to a constant radius, as in the accompanying chart. A main-sequence star of a given mass must have a definite radius. Hence, in addition to the mass-luminosity relation we have a *mass-radius* relation for main-sequence stars. How can we explain these two theoretically?

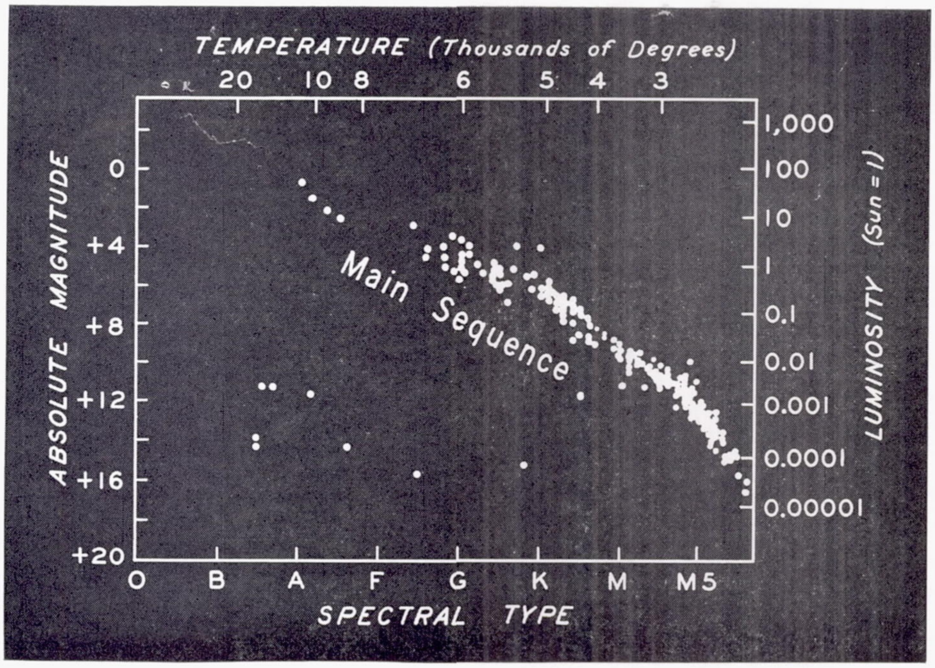
Since A. S. Eddington nearly 50 years ago pioneered the modern study of stellar structure, theoretical astrophysicists have tried to answer this question. It now appears that they have succeeded. A full explanation would require our plunging into a web of differential equations. Instead, let us avoid difficult mathematics and try to apply intuitive reasoning to explain the mass-luminosity and mass-radius relations in main-sequence stars.

We know from observation that stars are composed of gas. Molecules in gases on earth move rapidly in random directions, and will not stay together unless they are sealed in a container. Most of us have had the distressing experience of hearing the air molecules rushing out of a leaking tire.

No material container, however, confines the gases of a star. Its atoms, ions, and electrons are held together by their self-gravitation. According to Newton's law of universal gravitation, any two particles attract each other with a force proportional to the product of their masses and inversely proportional to the square of the distance between them. Thus the mutual gravitation between the parts of a star holds it together.

Should the gaseous particles be stationary, this mutual attraction would set all of them rushing toward the center and the star would collapse. Actually, their rapid motions prevent the collapse. The gases seek a compromise between the mutual attraction that tends to squeeze them together and the tendency to fly apart because of their own velocities. There is a balance when the total kinetic energy, which results from the motions of the particles, is the same order of magnitude as the total gravitational potential energy. Indeed, such a state of equilibrium exists in all main-sequence stars.

The kinetic energy of a gas depends on the total count of particles in it and on its temperature. The number of particles in a star increases with its mass, M, while the temperature is simply a measure of the average speed of the particles. Thus, the star's kinetic energy is



In this H-R diagram are plotted only stars within 10 parsecs (32.6 light-years) of the sun, most of them falling on the main sequence. White dwarfs (at lower left) are hard to detect. Adapted from "Elementary Astronomy," by O. Struve, B. Lynds, and H. Pillans, Oxford University Press, 1959.

proportional to M times T. On the other hand, potential energy is proportional to M^2/R. Since these two opposing conditions balance, we find that $MT \propto M^2/R$, and that the average temperature in a star depends on M/R.

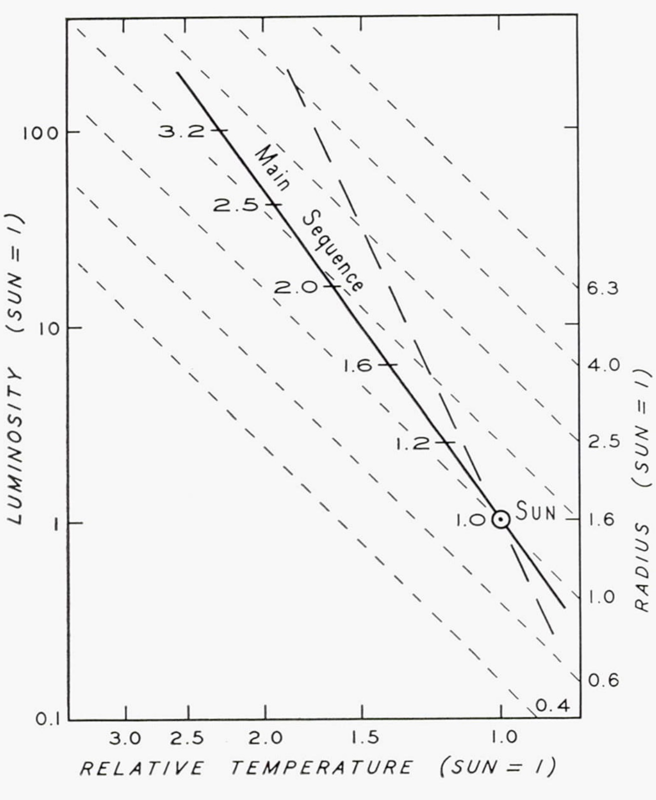
From this it follows that, for a given mass, small stars must be at higher average temperatures than large ones. And for a given radius, the more massive stars have higher temperatures in their interiors.

Both these statements follow readily from our physical intuition. A small radius means relatively short distances between constituent parts, which consequently attract each other more strongly, and particles with higher velocities are held together. Since average particle speed increases with temperature, a higher temperature is expected with a smaller radius. For a given radius, a larger mass also provides a larger holding force, therefore allowing a higher temperature.

We can now investigate how a star shines. The tremendous energy radiated from the surface of a main-sequence star comes from thermonuclear reactions that convert hydrogen into helium. Because the rate of energy production falls rapidly with decreasing temperature, these reactions can be assumed to take place only in a small, very hot region in the center of the star. Energy generated in this core leaks outward through the stellar envelope in the form of radiation and escapes at the surface. The luminosity of the star can be estimated from the amount of energy flowing through the envelope.

This situation is similar to that of water flowing through a pipe. The amount of water coming out per unit time increases with the pressure gradient (the difference in pressure at the pipe's two ends divided by its length). At the same time, the water's viscosity acts as a brake; the flow is inversely proportional to it.

Consider a column of stellar matter extending from the center to the surface of a star. The amount of energy flowing in this column is proportional to the gradient of radiation pressure. The latter pressure varies as the central temperature raised to the fourth power and is inversely proportional to the radius. As-



If the temperature and luminosity of any star relative to the sun are known, the star can be plotted in this H-R diagram. Follow the slanting dashed lines to the right margin in order to read off the star's radius in solar units. The solid line is the theoretical main sequence, on the assumption that opacity is proportional to density squared. Numbers along the solid-line main sequence indicate mass in solar units. An alternative main sequence (dashed) embodies the assumption that opacity is linearly proportional to density.

suming for our purposes that the central temperature differs from the average only by a constant factor, we can say that the pressure gradient is proportional to T^4/R.

Stellar matter is not completely transparent. Its opacity arising from photoionization (detachment of electrons from atoms by the intense radiation) is proportional to $\rho^2 T^{-3.5}$, where ρ is the material's density (varies as M/R^3). The energy flow through our hypothetical column is proportional to the pressure gradient divided by the opacity, in the same way as the water flow through the pipe is proportional to the pressure gradient divided by the viscosity.

The star's total luminosity is the flow from the column multiplied by the star's surface area. From this and the relation obtained earlier, $T \propto M/R$, we can show that

$$L \propto M^{5.5}/R^{0.5}.$$

Because this formula involves the radius, it is not the desired mass-luminosity relation. We must find another relation of luminosity, mass, and radius. Then by combining the two we can eliminate the radius.

Let us consider the energy produced by the thermonuclear reactions. The total output is calculated by nuclear physicists to be proportional to the product of the star's mass, density, and the 18th power of the central temperature, that is, to $M\rho T^{18}$. This output must equal the luminosity, because the star is supposed to be in equilibrium. Remembering that the temperature is proportional to M/R, we find

$$L \propto M^{20}/R^{21}.$$

Now we can combine the two proportionalities, and the luminosity turns out to depend on about the fifth power of the mass:

$$L \propto M^{5.15}.$$

This is the theoretical mass-luminosity relation. We also find that $R \propto M^{0.7}$. Thus we have arrived at two theoretical relations necessary to explain the existence of the main sequence, though the powers of M differ somewhat from the observed ones.

Stefan's radiation law tells us that the energy radiated by a unit area on a star's surface is proportional to the effective temperature raised to the fourth power. A simple algebraic calculation, using this law, leads to the result that the luminosity of the star is proportional to $T^{5.6}$. This is the solid line plotted in the schematic H-R chart on page 137, and is the theoretical locus of the main sequence.

Both this line and the theoretical mass-luminosity relation depend strongly upon the obstructing power of stellar material — the opacity. In our calculation, we assumed that it is proportional to the square of the density. Had we used the density itself, the luminosity would have turned out to vary as $T^{8.7}$ instead of $T^{5.6}$. At the same time, the luminosity would change with M^3, instead of about M^5. The main sequence corresponding to these values is plotted as a dashed line on the same diagram. Here we see how important the opacity is to internal structure, and consequently to the external characteristics of a star.

The foregoing discussion applies only to stars on the main sequence above the sun. For those below it, the rate of energy generation is proportional to T^4 rather than T^{18}. It is less temperature-sensitive because of differences in the reactions that ultimately convert hydrogen into helium. As a result, we can no longer assume that energy generation is confined to a small central region of such a star. Our previous arguments have to be modified, but the general principles are the same. Gravitational attraction is balanced by the motion of the particles, and the total energy leaking through the star's surface is equal to that generated inside.

Of great interest is the star's ability to adjust itself so that the amount of energy generated and that radiated are exactly the same. To see how this is done, suppose a star produces more energy than it is radiating. The excess will cause it to expand, thereby cooling it off. The rate of energy generation, being temperature-sensitive, will drop rapidly until the excess is erased and the balance restored. If, on the other hand, too little energy were being produced, the star would contract. This would raise the temperature and increase the energy production.

Thus, we can now define the main sequence as a stage of stellar evolution in which the energy radiated away is completely compensated by the conversion of hydrogen into helium in the star's core, and this state of equilibrium persists as long as there is hydrogen in the core. Since a star is originally composed mainly of hydrogen, this phase can last a long time — longer than all the other stages of a star's life (except perhaps the white dwarf stage, which is simply a cooling off). Because each star spends such a large part of its life on the main sequence, we observe the majority of them there.

During the other phases of a star's lifetime, when it is contracting prior to reaching the main sequence, or expanding after leaving it, the star is not in equilibrium. During contraction, the central temperature steadily rises. Upon leaving the main sequence, a star with larger mass than the sun's burns hydrogen in a shell surrounding its central core. According to modern investigators, the hydrogen-exhausted core contracts and the envelope expands as the burning shell moves outward. Thus the star's thermal state changes continuously.

We may say that stars seek to evolve and change because of their internal gravitation and because of the radiation of energy from their surfaces. But the release of nuclear energy arrests the evolution as long as the supply of hydrogen in the central region lasts. Thus only the main-sequence stage shows a constancy in physical characteristics.

In one sense, the history of a star parallels that of a living organism. The life span can be divided into three parts: growth, a long stage of maturity, and a final decline leading to the end of activity. The main sequence represents the long stage of maturity of the stars.