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# ACCELERATION OF CHARGED PARTICLES BY HYDROMAGNETIC SHOCK WAVES

J. W. Kern

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# ACCELERATION OF CHARGED PARTICLES BY HYDROMAGNETIC SHOCK WAVES

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## PREFACE

This Memorandum is part of a continuing theoretical study of fields and particles, sponsored by the National Aeronautics and Space Administration under Contract NASr-21(05).

## ABSTRACT

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Two mechanisms are discussed for the acceleration of particles trapped in the magnetosphere. Both are associated with the propagation of strong hydromagnetic (HM) shock waves through the magnetosphere. One accelerates thermal electrons throughout the magnetosphere. The other affects energetic particles trapped in the magnetosphere and can lead to changes in their energy and pitch angle distributions. A single HM shock wave is shown to be capable of producing, by either (or both) mechanisms, significant ring currents.

## ACKNOWLEDGMENTS

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## I. INTRODUCTION

Local accelerations of charged particles by hydromagnetic (HM) shock waves have been discussed in connection with the ring current of a geomagnetic storm (Dessler, Hanson, and Parker, 1961; Parker, 1962; Kern, 1962). Such accelerations could be expected to modify the distributions of particle pitch angles and energies. It is therefore of interest to clarify the physical bases for such accelerations so that observations can be made to test the HM shock wave hypotheses.

At least two mechanisms can be invoked by which HM shock waves can accelerate trapped particles. One mechanism accelerates a low energy (thermal) plasma, while the other accelerates energetic particles. Both mechanisms are outlined briefly in the present paper.

In an earlier paper (Kern, 1962), the author employed a model for the acceleration of particles that was not applicable for HM shock waves, since it neglected the gas motion behind the front. This gas motion is taken into account here in discussing particle accelerations produced by rather realistic models of HM shock waves. It is shown that relatively few HM shock waves will suffice to impart sufficient energy to particles already trapped in the magnetosphere to produce the main-phase of a geomagnetic storm. We will assume that an HM shock wave propagates through the magnetosphere, created by the impact of a solar stream on the sunward side of the magnetosphere. We identify the arrival of the magnetic field change at the earth with the sudden commencement of a geomagnetic storm. The structure of this shock wave is assumed to be a step-wise increase in the local magnetic field coupled with a gas flow behind the shock front. This model is consistent with solutions of the problem of a conducting piston compressing the magnetosphere (Colgate, 1959; Cole, 1959). More sophisticated models and treatment of the problem seem inappropriate at present, since the structure of HM shock waves in an essentially collisionless plasma is still in doubt (Gardner, et al., 1958; Fishman, Kantrowitz and Petschek, 1960; Morawetz, 1961).

An elaborate model seems unwarranted because the physical constitution of the distant magnetosphere, through which the shock wave propagates, is also in doubt. Preliminary calculations are therefore made only for the rather simple model described by Colgate and Cole.

Recent observations (Davis and Williamson, 1962) indicate that a substantial ring current of energetic protons is present during magnetically quiet periods. It will be shown that such a ring current may be enhanced by a single HM shock wave associated with a sudden commencement. Further accelerations of low-energy particles may also occur that will contribute to the main-phase ring current of a geomagnetic storm. Actual calculations are made here of: (1) a distribution of kinetic energy density resulting from the acceleration of thermal electrons, and (2) the enhancement of an existing ring current of energetic protons by a single HM shock wave. Both calculations indicate significant increases in energy density. Hence both mechanisms may be important for a complete theory of geomagnetic storms.

The acceleration of particles by HM shock waves is only one aspect of the local acceleration of particles trapped in the magnetosphere. Auroral particles and particles associated with ionospheric current systems can be accounted for if sufficient particle energy is available in the magnetosphere (Kern, 1962). The acceleration of particles by HM shock waves may therefore account for the energy necessary for aurora, ionospheric currents, and other dumping phenomena.

## II. MODEL OF STRONG HYDROMAGNETIC SHOCK WAVE

The model employed here for the structure of a strong HM shock wave is the same as that used previously by Kern (1962), and earlier by Cole (1959) and others. Our attention will be newly focused here on the gas motion behind the shock front and on the electric fields associated with this gas motion.

Consider a strong HM shock wave propagating transverse to magnetic field lines. This wave is characterized by a sudden increase in the local magnetic field. Take a right-handed xyz coordinate system with z parallel to the direction of the magnetic field B, x parallel to the direction of the wave's motion, and y in the plane of the shock front, as shown in Fig. 1. The shock wave can be regarded as driven by gas moving behind the front with a velocity u in the x direction. The velocity of the shock front is denoted by U. For the two-dimensional geometry considered here, the velocity U of the shock front and the gas velocity u are related by continuity of magnetic flux through the shock front. The rate at which flux enters the shock front (per cm of length perpendicular to B) is $UB_o$, where $B_o$ is the field ahead of the shock front. This must be equal to the rate at which flux leaves the back of the shock front, $(U - u)B_1$, where $B_1$ is the field in back of the shock front. It follows that $u = U(1 - B_o/B_1)$.

An electric field is associated with the gas motion behind the shock front. This field is given by $\underset{\sim}{E} = - \underset{\sim}{u} \times \underset{\sim}{B}_1 = E_y$ and is equivalent to the electric field required to produce the force-free drift of particles with the velocity u in the x direction. Figure 1 shows the relation of $E_y$ to the magnetic field B, the gas velocity u and the shock-front velocity U. This electric field will be referred to later in connection with the acceleration of energetic particles. As mentioned above, the shock front itself is assumed to be thin compared to the cyclotron radius of even very low-energy protons. The following analysis will neglect motion of particles along magnetic field lines. The model is therefore two-dimensional and the particle motions considered will be those transverse to magnetic field lines.

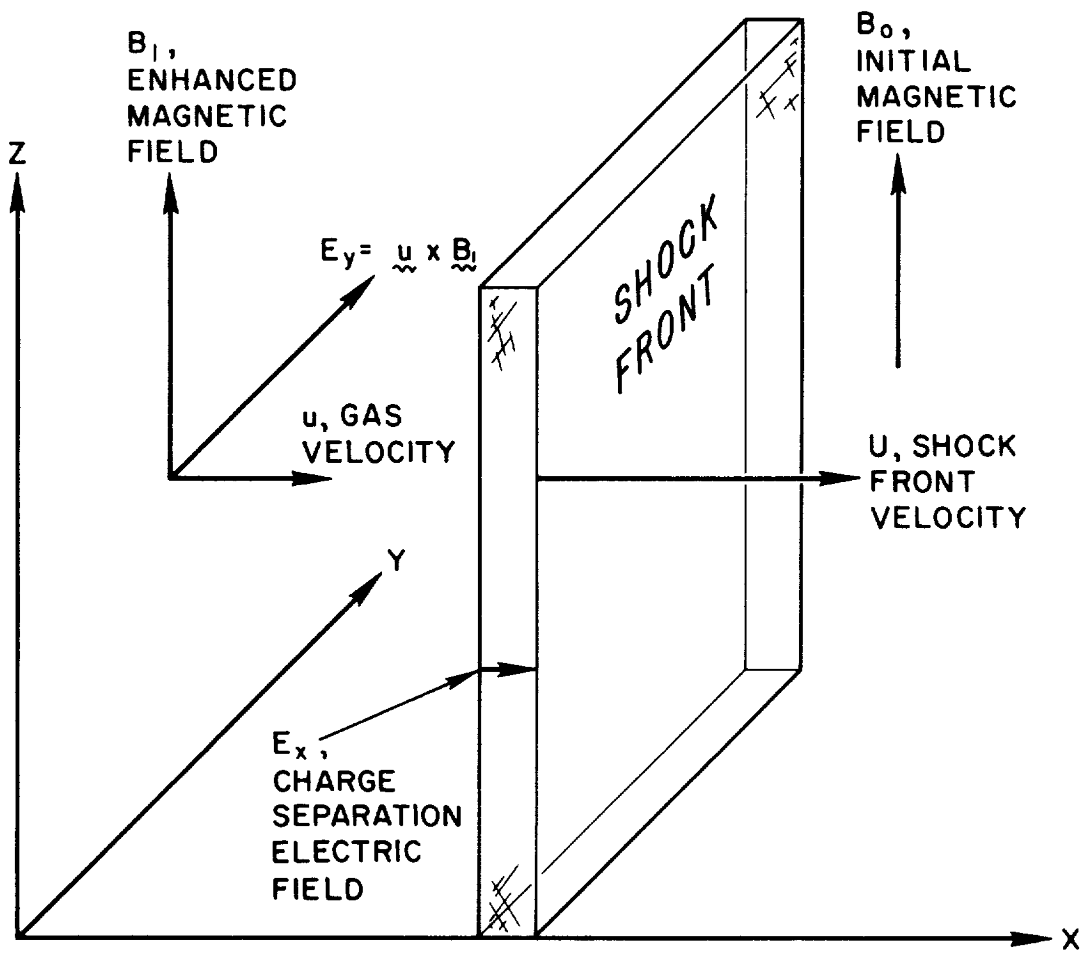


Fig. 1 — Model of hydromagnetic shock wave in a collisionless plasma

## III. ACCELERATION OF THERMAL PARTICLES

Colgate (1959) points out that charge separation can occur in a hydromagnetic shock front, resulting in an electric field in the direction of the front's advance. This field can then accelerate both electrons and protons that pass through the shock front. Such accelerations can constitute an important dissipation mechanism for an HM shock wave.

The origin of this electric field can be seen from the following considerations. The gas velocity u behind the front is the mass velocity of ions after the shock front arrives, the net motion being initially taken as zero. The mass velocity u applies also to the guiding centers of energetic particles. Drifting also with this same velocity, u, are the electrons behind the front. Now since the mass of the moving gas behind the front is concentrated in the ions, there is a problem of accelerating those ions initially at rest to the drift velocity u as they pass through the shock front. If we consider as constant an ion's total energy (kinetic plus electrostatic), the only possible acceleration mechanism without collisions seems to be an electric field in the x-direction, the direction of shock-front advance. This field must be such that the gain in kinetic energy of the gas is matched by a decrease in electrostatic potential energy. The kinetic energy of a given ion changes from about zero to $Mu^2/2$, and hence by $Mu^2/2$, where M is the ion mass and u is the gas velocity behind the front, for a frame of reference at rest with respect to the shock front. The electrostatic potential $\varphi$ through the shock front that is required to accelerate the ions to the drift velocity is given by

$$e\varphi = M(U - u)^2/2 - MU^2/2$$

in a frame of reference moving with the shock front, where e is the ion charge and M is the ion mass. The first term is the kinetic energy relative to the shock front of an ion that has passed through the front, while the second term is the kinetic energy of an ion with respect to the shock front before the front passes it. This is equivalent to

$$e\varphi = -(MU^2/2)\,[1 - (B_o/B_1)^2]$$

where $B_o$ is the initial magnetic field, and $B_1$ is the enhanced magnetic field behind the shock front.

This potential difference will also accelerate the electrons passed by the shock front. The energy of drift motion is negligible for the electrons, so the acceleration must affect mainly the cyclotron motion of the electrons transverse to B. The electrons acquire only half of this potential energy in the form of kinetic energy of cyclotron motion about B, as can be seen with the help of Fig. 2. The figure shows an electron orbit after the electron is accelerated through a very thin shock front (Colgate, 1959). The electron, it turns out, is reflected backwards a number of times from the back side of the receding shock front, and drifts parallel to the electric field in the y-direction. Colgate (1959) points out that the receding of the shock front during the multiple reflections of the electron effectively doubles the volume of phase space occupied by the electron (from one-half orbit to one full orbit). This is equivalent to an adiabatic expansion of an electron gas and reduces the orbital kinetic energy by 1/2.

Thus, relative to the shock front, an electron gains 1/2 the kinetic energy that a thermal ion gains in passing the shock front. The change in kinetic energy is thus given by $e\varphi/2 = (MU^2/4)[1 - (B_o/B_1)^2]$. We can express $B_1$ as $B_o + b$, where b is an increment in the initial magnetic field associated with the passage of the shock front. Then the energy of an accelerated thermal electron is

$$W_\perp = (MU^2/4)\,\frac{2b/B_o + (b/B_o)^2}{(1 + b/B_o)^2} .$$

This last equation can be used to calculate a possible ring current generated by a single HM shock wave in the magnetosphere. The model for

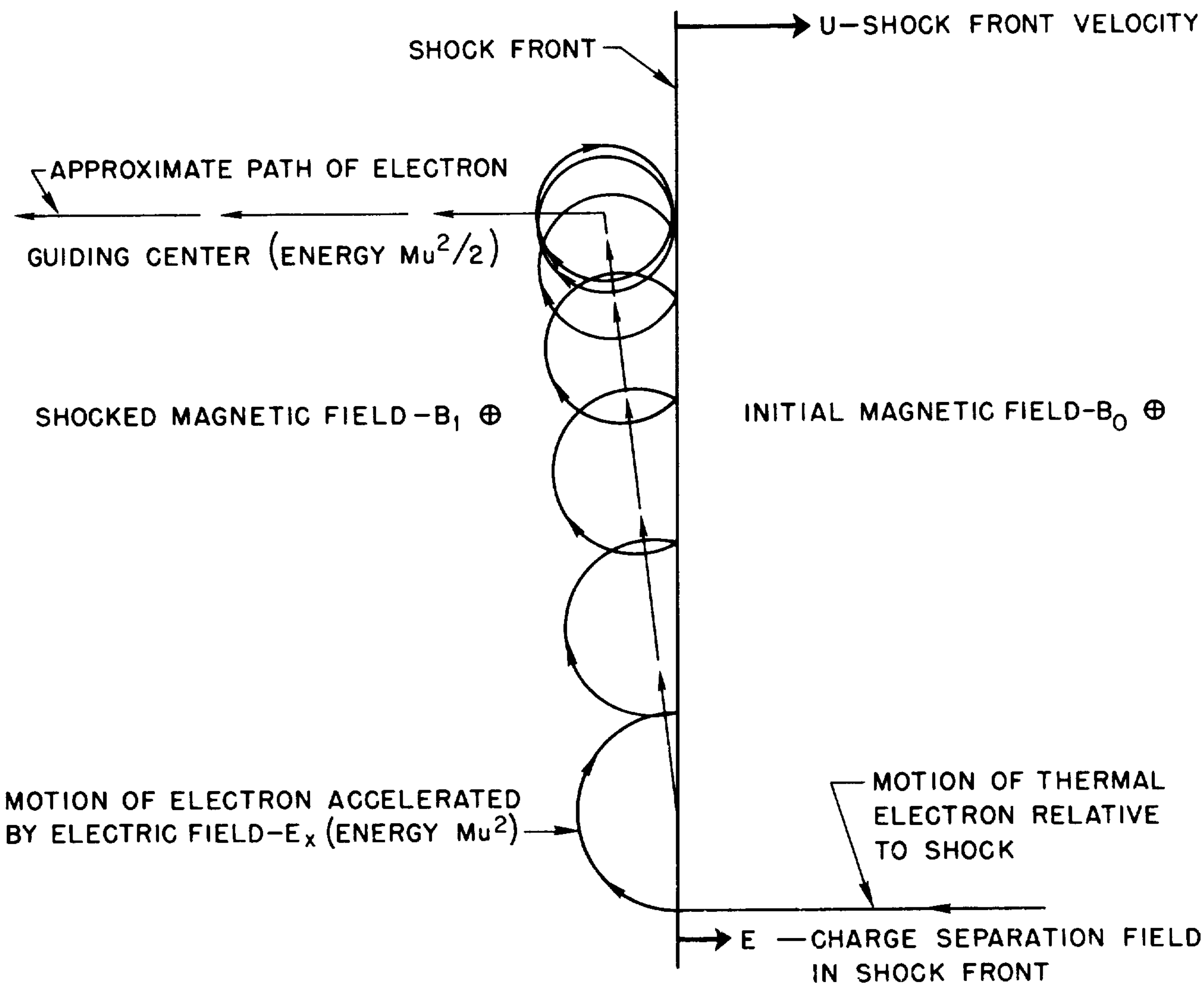


Fig. 2—Acceleration of a thermal electron by charge-separation electric field in hydromagnetic shock wave-front

the shock is as follows: A solar-stream impinges on the sunward side of the magnetosphere and compresses the geomagnetic field containing trapped protons and electrons. A strong HM shock wave is generated in the magnetosphere by this compression. This shock wave travels from the sunward side of the magnetosphere toward the night side, accelerating thermal electrons through the charge-separation field across the shock front discussed above. Acceleration of electrons by an electric field due to charge separation damps the HM shock wave. In the magnetosphere, the electric field will accelerate thermal electrons. The electric field will not appreciably affect the energetic particles present, if these particles have large Larmor radii, so that they make many cyclotron orbits through the shock front moving past them. This is because the energetic particles traverse the electric field due to charge separation in opposite directions each time they penetrate the shock front. The effect of the interaction of energetic particles with the wave will be discussed in more detail in Section IV.

We can show the effect of the acceleration of thermal particles by a charge separation field in an HM shock front. Let the number density of thermal electron-proton pairs vary with geocentric distance as $r^{-\alpha}$, where r is given in earth radii ($r_e$) and $\alpha$ is a constant. The velocity u of the gas behind the shock front can be taken simply as $u = U(1 - B_o/B_1)$, where U is the velocity of the shock front, $B_o$ is the local field in front of the shock, and $B_1$ is the enhanced field behind the shock. We take U to be a constant, and $B_1 = B_o + b$. Then the energy density $E_1$ of accelerated electrons in the compressed field $B_1$ (assuming the original energy negligible) will be

$$E_1 = (nMU^2/4)\,\frac{2b/B_o + (b/B_o)^2}{(1 + b/B_o)^2}$$

where n is the local number density of electron-proton pairs. Following relaxation of the compression the energy density becomes

$$E(r) = E_1(B_o/B_1) = (nMU^2/4)\,\frac{2b/B_o + (b/B_o)^2}{(1 + b/B_o)^3}$$

Let the number density be $n_o$ at $4r_e$, then beyond this distance $n = n_o(4/r)^\alpha$. The energy density E then is given by

$$E = (n_o MU^2/4)(4/r)^\alpha \frac{2b/B_o + (b/B_o)^2}{(1 + b/B_o)^3}$$

If $b/B_o = 1$ at, say, $8r_e$, $n_o = 100/cm^3$ and $U = 10^8$ cm/sec, we have $E(8r_e) = 125\ kev/cm^3 \cdot (1/2)^\alpha$. Let $b/B_o$ vary as $c(r/4)^\gamma$ beyond $4r_e$ where $c = b/B_o$ at $4r_e$, and $\gamma$ is a constant that will be determined. We then have

$$E(r) = (n_o MU^2/4)(4/r)^\alpha \frac{c(r/4)^\gamma}{[1+c(r/4)^\gamma]^3} [2 + c(r/4)^\gamma]$$

$\gamma$ will in general be greater than 3, since the amplitude of the shock wave is damped as it moves inward. A sudden commencement of 20$\gamma$ at the earth's surface would correspond to $b/B_o = c = 4 \times 10^{-2}$ at $4r_e$, if damping is neglected between the surface and $4r_e$. To obtain $b/B_o = 1$ at $8r_e$, $\gamma$ must be taken as about 4.6. By adopting this value, in effect we fix the damping of the HM shock wave by the thermal electrons present. Beyond $8r_e$, the compression presumably increases to an upper limit of $(B_o+b)/B_o = 3$ (Colgate, 1959). The variation of E(r) with r is shown in Fig. 3 for $\alpha = 2$ and $\alpha = 4$. Of course, E(r) must vanish if the density of thermal particles goes to zero. The outer boundary of the ring current is determined primarily by the distribution of the thermal particles in the magnetosphere. We note that $\alpha = 2$ is consistent with the HM wave velocities calculated for the magnetosphere by Dessler, Francis, and Parker (1960) and MacDonald (1961). The distribution of particle energy for $\alpha = 2$ constitutes a ring current of significance to geomagnetic storm theory.

The change in magnetic field that a ring current produces at the earth's surface can be related to the total energy of the particles composing the ring current (Parker, 1962) A total energy of $3 \times 10^{15}$ joules will produce a main-phase decrease of 100$\gamma$. We can estimate the energy in the calculated distributions of accelerated thermal electrons.

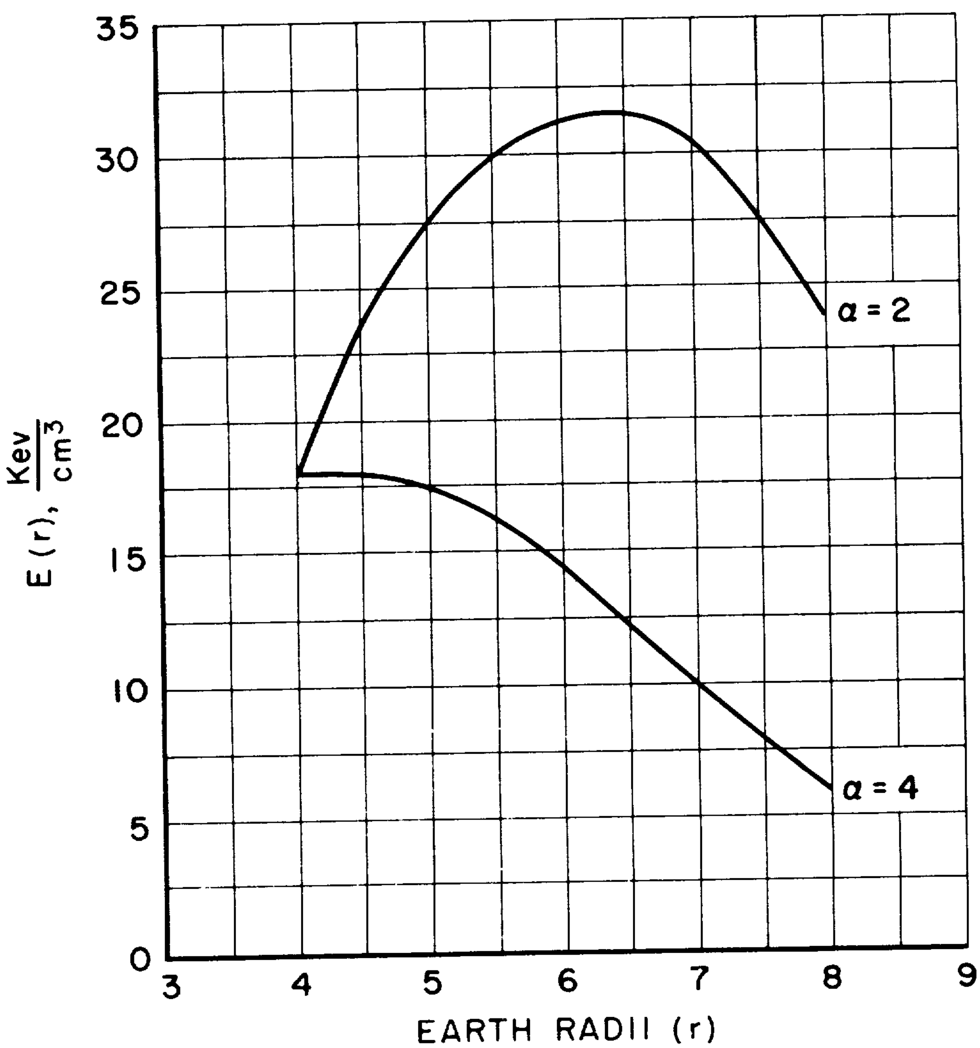


Fig. 3—Energy density of thermal electrons accelerated by shock wave in magnetosphere for two models of initial number density

Take the energy densities given in Fig. 3 to be distributed above and below the equatorial plane in a cylinder of height $4r_e$. An approximate integration of the indicated energy densities over the volume from $4r_e$ to $8r_e$ gives the total energies $\delta U = 0.7 \times 10^{15}$ joules for $\alpha = 2$, and $\delta U = 0.2 \times 10^{15}$ joules for $\alpha = 4$. These total energies will be sufficient to produce respective main-phase decreases of about $25\gamma$ and $6\gamma$. The effects of particles beyond $8r_e$ are neglected here. From Fig. 3, such particles will not contribute significantly to a decrease in the surface field.

We also note that the maximum in this model of a ring current is at about $6r_e$, well inside the distribution of thermal particles. We can obtain larger energy densities by varying parameters in the above relations. For example, if the shock wave velocity U is $2 \times 10^8$ cm/sec, the total energy of the accelerated electrons is larger by a factor of 4, and main-phase decreases of about $100\gamma$ and $25\gamma$ would be expected for $\alpha = 2$ and $\alpha = 4$. This calculation only illustrates the possible effect of a single HM shock wave on thermal particles in the magnetosphere.

## IV. ACCELERATION OF ENERGETIC PARTICLES

Dorman and Freidman (1959) suggest that HM shock waves can be responsible for the acceleration of solar cosmic rays during solar flare activity. Their examination of the mechanics of this kind of acceleration mechanism indicates that it can indeed be a first-order effect, and that it is characterized by a greater speed of energy acquisition by particles than that of the Fermi mechanism or other statistical mechanisms. They suggest a source for the small increase in cosmic-ray intensity on earth before the start of a magnetic storm: that the additional cosmic-ray particles may be reflected from the shock wave that is formed by the leading edge of the corpuscular stream ejected from the sun. Dorman and Freidman also note that intense shock waves can propagate in the solar corona, in the interplanetary and interstellar medium, and in the shells of novae and supernovae. Acceleration of particles by HM shock waves may therefore play an important role in the origin of cosmic-ray primaries, as well as solar cosmic rays. The present paper carries this speculation one step further by examining the acceleration of energetic particles trapped in the magnetosphere by a single HM shock wave that is formed when a solar stream impinges on the magnetosphere.

Following Dorman and Freidman (1959), let us consider an HM shock wave in a medium with a frozen-in magnetic field parallel to the plane of the front as indicated in Fig. 1. The velocity of the front is U. In the undisturbed medium the field is $B_o$; in the shocked medium the field is $B_1$. The mass velocity of the shocked medium is u in a rest-frame. In a coordinate system that is at rest, the shock front will overtake and move across the orbit of a spiralling particle. In the undisturbed medium, a particle's guiding center is stationary, since $\nabla B_o = 0$, except at the shock front. After the front has passed, the particle's guiding center drifts with the velocity u under the influence of the electric field $\mathbf{E}_y = - \mathbf{u} \times \mathbf{B}_1$ shown in Fig. 1. Near the front, the proton orbit is partly in the undisturbed medium with no electric field and partly in the shocked medium with the field $\mathbf{E}_y$.

The integral of the electric field over a complete orbit is no longer zero, as is the case for a proton spiralling in a uniform electric field. The orbital velocity of the proton is therefore accelerated, since the proton travels parallel to the electric field $E_y$ over a part of each orbit. At the same time, the proton guiding center drifts parallel to the shock front in just such a manner as to lose the same amount of electrostatic potential energy as the proton gains in kinetic energy. This drift motion parallel to the shock front is indicated in Fig. 4 in a frame of reference moving with the shock front. This figure also shows schematically the motion of the proton's guiding center and the envelope for the proton's orbital motion across the shock front. Note that in this frame of reference, the electric field is the same on both sides of the shock front (since $\nabla \times \underline{E} = - \frac{\partial \underline{B}}{\partial t} = 0$). The electrostatic energy lost in the total drift parallel to the shock front in this electric field $\underline{E} = - \underline{U} \times \underline{B}_o$ corresponds to the gain in kinetic energy of the particle. It is evident that this acceleration mechanism will work if (1) the cyclotron radius of the proton is large compared to the thickness of the shock front, and (2) the cyclotron period of the proton is a small fraction of the time required for the shock front to cross the orbit of a proton. The latter requirement simply implies that the orbital velocity of a particle must be very much greater than the velocity of the shock front.

A particle with sufficient velocity will penetrate the moving shock front a number of times. This acceleration can also be physically interpreted as a reflection of an energetic particle from the magnetized plasma moving at the gas velocity u behind the shock front. This interpretation is reminiscent of the Fermi acceleration of cosmic rays by moving interstellar clouds containing magnetic fields (Fermi, 1954). The multiple reflections that occur in the present case greatly multiply the energy gained by an energetic particle from a single shock wave.

Particle trajectories of this kind are needed for propagation of a "weak" shock wave in a collisionless medium with no charge

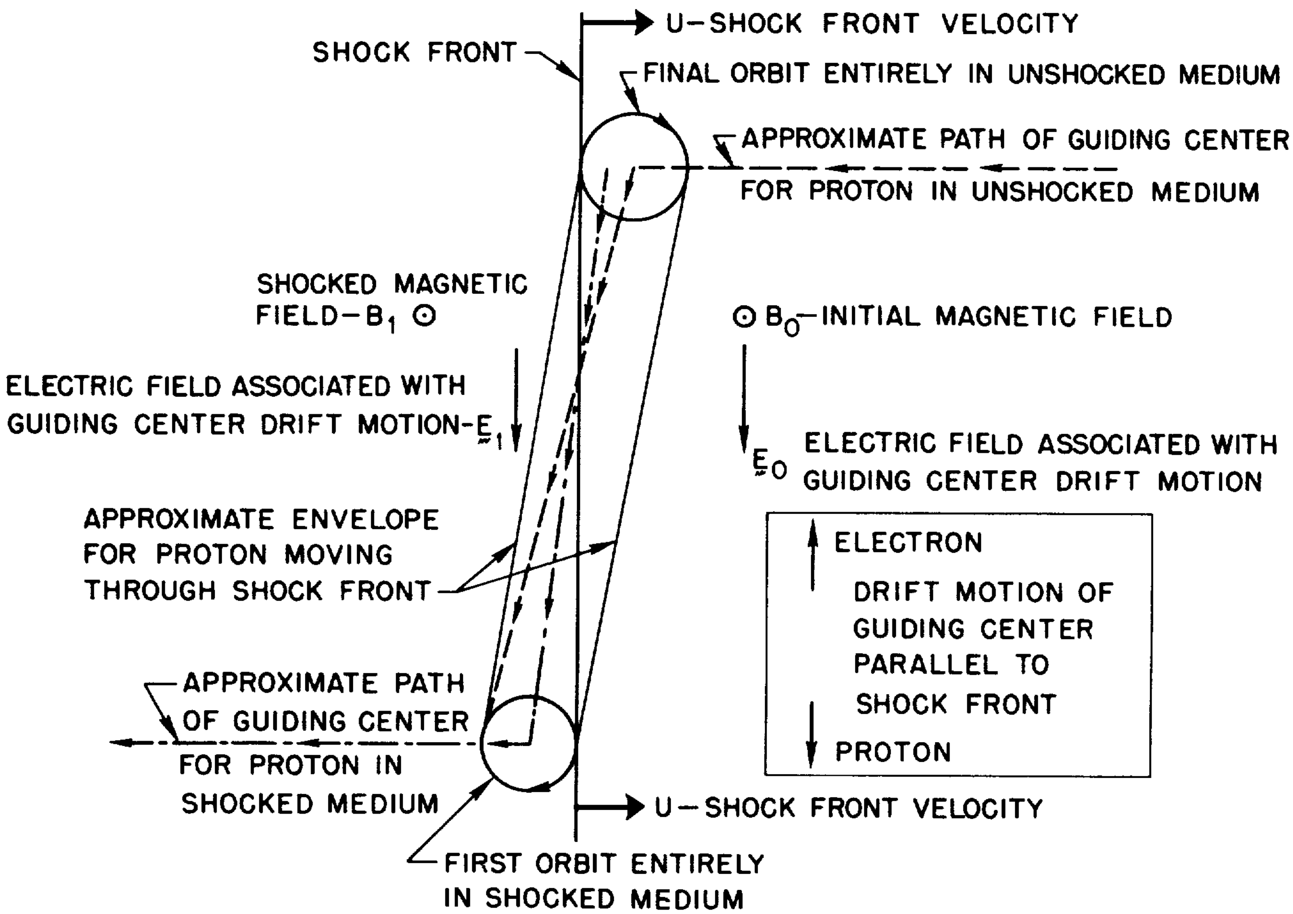


Fig. 4 — Acceleration of energetic proton by hydromagnetic shock wave

separation in the shock front (Morawetz, 1961). Apparently, the "reflections" of energetic particles from the shock front damp the wave, just as collisions do in a gas-dynamic shock wave. This reflection process is the basis of the solution that Shabanskii (1962) obtains for the acceleration of energetic particles by an HM shock wave. He selects a particle velocity $v_o$ much greater than the shock wave velocity U, then approximates the shock structure by a stepwise increase in the magnetic field combined with a finite gas velocity u behind the shock front. This approximation may be appropriate for strong HM shock waves (Colgate, 1959; Cole, 1959). Shabanskii obtains the momentum after the particle has passed through the shock front as

$$p = p_o \exp I(\psi_o)$$

where p is the momentum of the particle after the shock front has passed, $p_o$ is the initial momentum, and $I(\psi_o)$ is an integral function of the initial angle of incidence of the particle on the shock front $\psi_o$. $I(\psi_o)$ is given by

$$I(\psi_o) = \int_{\psi_o}^{\pi/2} \frac{4 \cos^2\psi \, d\psi}{s\pi - 2\psi - 4 \sin \psi \cos \psi}$$

where $s = (1 + \omega_o/\omega_1)/(1 - \omega_o/\omega_1)$ and $\psi$ is the angle of incident of the particle on the shock front. This expression can be evaluated for different values of $\psi_o$ and s. However, for a particle velocity $v_o \gg U$, the initial angle $\psi_o$ is always very nearly $- \pi/2$.

We chose here to compute a change in momentum of energetic trapped particles as a function of the field compression $B_1/B_o$. This is done by evaluating $I(- \pi/2)$ for values of $B_1/B_o$ between 1 and 3. The change in magnetic moment of a particle can be computed from the modified momentum, as indicated below. Shabanskii evaluates the above integral in the limits of weak and strong shock waves. He uses gas-dynamic relations to determine the compression of the magnetic field. The collisionless shock may not be governed by these relations.

For example, Colgate (1959) shows that the maximum compression of the magnetic field in a strong HM shock without collisions is $B_1/B_o = 3$. This corresponds to the maximum compression in a gas dynamic shock for a gas with 2 degrees of freedom. The compression due to a solar stream impinging on the magnetosphere would be expected to decrease as an HM shock wave moves into the geomagnetic field, since the local field increases toward the earth.

Consider a model in which a single strong HM shock wave passes through a collisionless plasma, with a subsequent slow relaxation of the compression driving the shock. As the shock front passes, the magnetic field increases from $B_o$ to $B_1$. Energetic particles are accelerated; their momenta transverse to the magnetic field are increased such that $p_1/p_o = \exp I(-\pi/2)$. Following the shock, the energy of nonrelativistic particles changes as $p_1^2/p_o^2$. Now the magnetic moments of such particles are given by $\mu = E_\perp /B$, where $E_\perp$ is the energy of motion transverse to magnetic field lines, and B is the magnetic field. Figure 5 shows the ratio $E/E_o$ (the post-shock energy of nonrelativistic particles over their pre-shock energy) as a function of the field compression $B_1/B_o$. We assume that following the shock the field B is slowly decreased to its original value $B_o$. The formula of Shabanskii (1962) has been used, hence the indicated changes apply rigorously only to particles for which the initial velocity $v_o$ is much greater than the shock-front velocity U. The preceding ideas can be applied to ultrarelativistic particles for which the energy changes as $p_1/p_o$. For such particles (e.g., Mev range electrons), $E/E_o = (p_1/p_o)(B_o/B_1)$, and the modification of energies is much less significant, as shown in Fig. 5.

For nonrelativistic particles, we note that for $B_1/B_o = 1 + \delta$, where $\delta < 0.5$, $E/E_o = 1 + \delta$ to within less than 1%. Repeated HM shock events can be treated as enhancing the energy of particles by a factor of $(1 + \delta)$ for each event. If the $\delta$'s are about equal and very much less than 1, n events would give

$$E_n/E_o = (1 + \delta)^n \cong 1 + n\delta$$

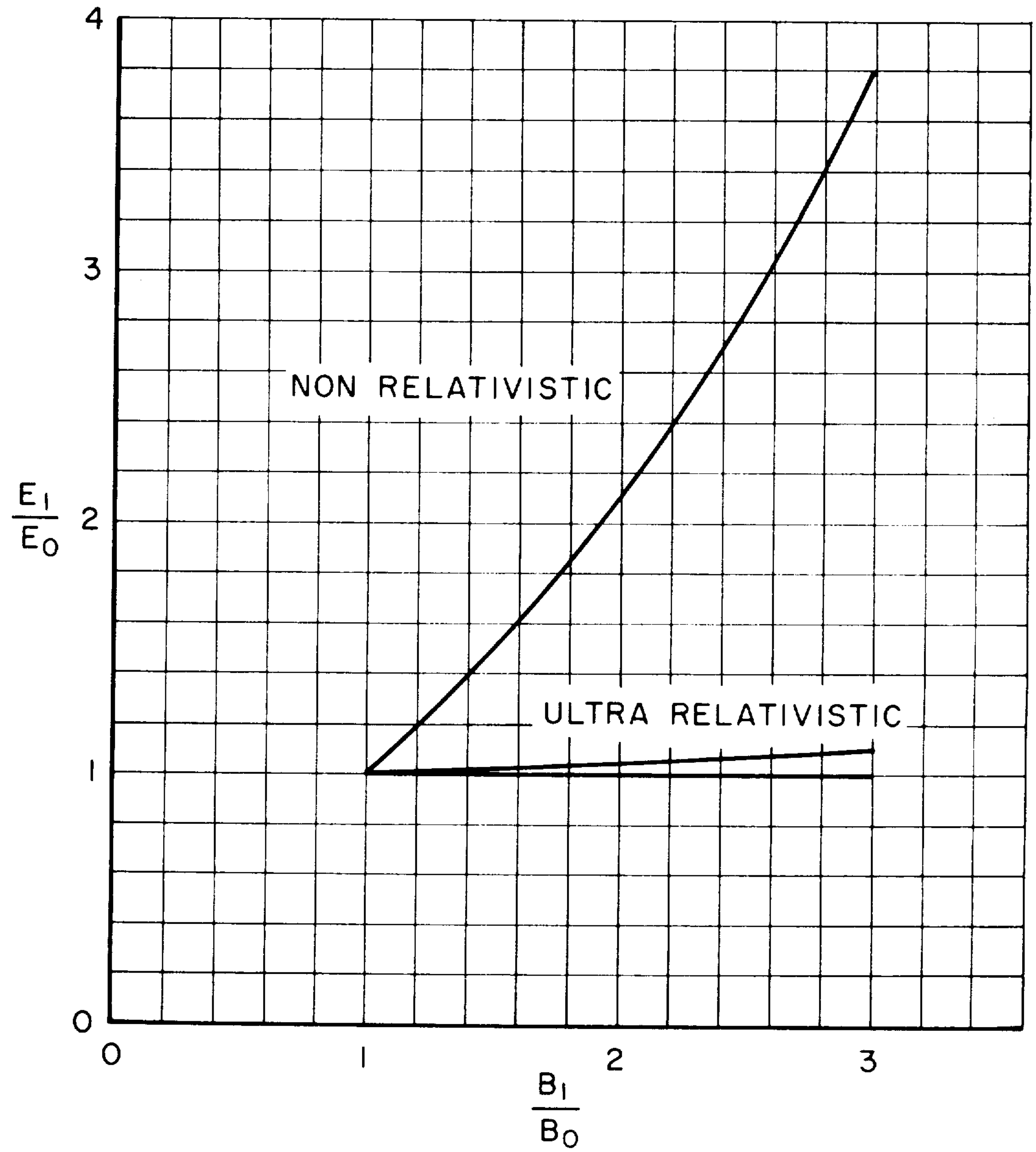


Fig. 5—Ratio of final energy to initial energy ($E_1/E_0$) for energetic (both non-relativistic and ultra-relativistic) particles accelerated by HM shock wave as function of ratio of final magnetic field to initial magnetic field ($B_1/B_0$)

Thus 10 shocks for which $\delta = 0.01$ would give about the same energy change as a single shock for which $\delta = 0.1$.

The observations of Davis and Williamson (1962) show an existing ring current of energetic particles that might be accelerated by HM shock waves in the magnetosphere. We can test this hypothesis by calculating the enhancement of a ring current for which $\beta$ is constant between the geocentric distances $R_1$ and $R_o$ and zero outside this interval. $\beta$ is given by $E_\perp / (B^2/8\pi)$, where $E_\perp$ is the energy density of particles resulting from motion transverse to the magnetic field B, and $B^2/8\pi$ is the energy density in the magnetic field.

An HM shock wave will be damped by energetic particles due to the acceleration mechanisms discussed above. We can calculate this damping for a simple model of an HM shock wave propagating through a magnetosphere containing trapped particles of high energy and constant $\beta$. We will assume the magnetic energy in the shock wave is not renewed. This is the case if the solar-stream interface driving the shock wave has been stopped by the increased magnetic field and by plasma pressure inside the cavity. Let the change in magnetic field at the geocentric distance $R_o$ be $b_o$. The initial energy in the magnetic field change is $b_o^2/8\pi$. Particles are accelerated at the expense of this magnetic energy. In fact, we can set

$$\frac{d}{dr}\left(\frac{b^2}{8\pi}\right) = - K\delta E(r)$$

where $\delta E(r)$ is the change in the local energy density of the particles as a function of geocentric distance r, and K is a constant to be evaluated. Now for the range of field changes considered, we can approximate $\delta E$ by $(b/B)E_i$, where B is the local magnetic field, and $E_i$ is the initial energy density of the energetic particles. This is equivalent to considering only the irreversible changes in particle energy. We can see from Fig. 5 that the energy added to accelerated particles is nearly proportional to the change in B. It follows that $(b/4\pi)db/dr = - K\, E_i(b/B)$ or $(1/B)db/dr = - (K/2)8\pi\, E_i/B^2$. Now $\beta_1 = 8\pi E_i/B^2$, hence $db/dr = - (K/2)\beta_i B$ where $\beta_i$ is the initial ratio of the energy density of particles to the local energy density of the magnetic field, and B is the local magnetic field. We evaluate $b(r)$ for the case $\beta_i$ = constant between $r = R_o$ and $r = R_1$, where $b(R_o) = b_o$ and $R_o > R_1$. Also, we let $b(R_1) = b_1$ and $B = B_{eq}/r^3$. It follows that $b_o - b = + (K/4)\beta_i B_{eq}(1/R_o^2 - 1/r^2)$ or

$$b = b_o + (K/4)\beta_i B_{eq}(1/r^2 - 1/R_o^2)$$

while

$$b_o - b_1 = (K/4)\beta_i B_{eq}(1/R_o^2 - 1/R_1^2)$$

Thus, evaluating K,

$$b = b_o + (b_o - b_1)\,\frac{(R_o^2 - r^2)}{r^2 R_o^2}\;\frac{R_1^2 R_o^2}{(R_1^2 - R_o^2)}$$

or finally

$$b = b_o + (b_o - b_1)\,\frac{R_1^2}{R_o^2 - R_1^2} - (b_o - b_1)\,\frac{R_o^2 R_1^2}{R_o^2 - R_1^2}\;\frac{1}{r^2}$$

It follows that

$$\frac{b}{B} = \frac{1}{B_{eq}}\left[b_o - (b_o - b_1)\,\frac{R_1^2}{R_o^2 - R_1^2}\right] r^3 - \frac{1}{B_{eq}}\,(b_o - b_1)\,\frac{R_o^2 R_1^2}{R_o^2 - R_1^2}\, r$$

or $b/B = K_1 r^3 - K_2 r$, where $K_1$ and $K_2$ are the coefficients in the above expression. Also note that due to the linearity of $\delta E$ and $\delta B$, the modification of $\beta$ is simply $\delta\beta = \beta_i\, b/B$, where $\beta_i$ is the initial value $\beta$. We take $\beta_i$ = constant between $R_1$ = 2.5 earth radii and $R_o$ = 8.0 earth radii, as suggested by the data of Davis and Williamson (1962). The above equations have been used to calculate $b$ and $\delta\beta/\beta_i$ as functions of $r$ between these distances. Also assumed are $b_o/B(R_o) = 1$ and $b_1 = 20\gamma$, corresponding to a typical sudden commencement of a geomagnetic storm. Figure 6 shows the results.

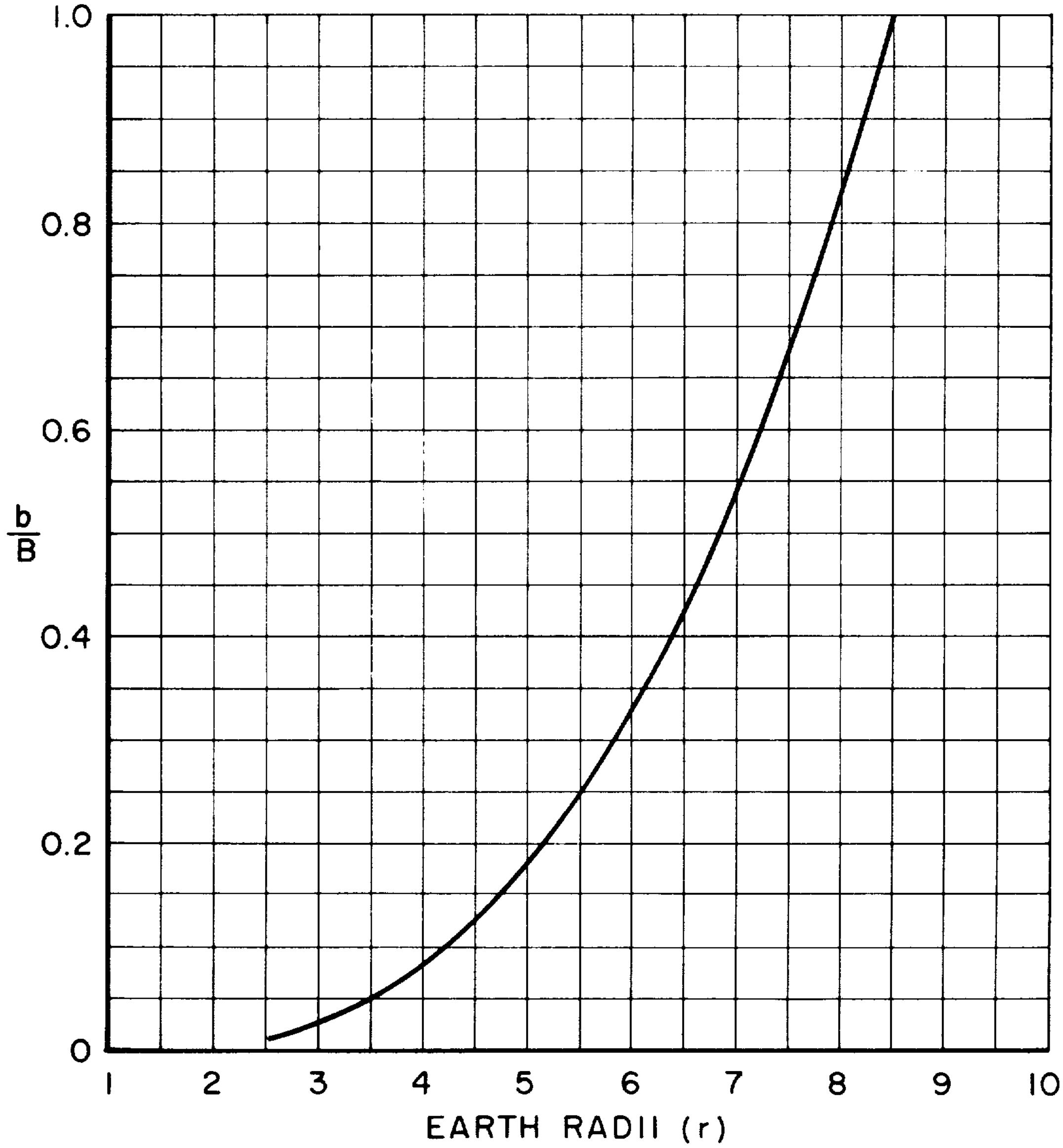


Fig. 6—Ratio of change in magnetic field b to initial field B as a function of geocentric distance for model of HM shock wave in magnetosphere

Note that the maximum enhancement of $\beta$ is at the outer edge of the ring current. Any HM shock waves impinging on the magnetosphere after such an enhancement of a ring current would be more strongly damped near the outer boundary.

In our model with constant $\beta_i$, the energy density of the particles varies as $r^{-6}$ initially. This energy density is modified by the HM shock wave such that the energy density of the particles $E = E_i \beta/\beta_i$. Figure 7 shows how E changes for an HM shock producing the changes in $\beta$ indicated in Fig. 6. The local increase in the energy density of the particles is $\delta E = E_i\ \delta\beta/\beta_i = E_i\ b/B$. The curve $\delta E/E_{2.5}$ in Fig. 7 can be regarded as an additional ring current resulting from the acceleration of energetic particles following the sudden commencement of a geomagnetic storm.

The above discussion indicates that HM shock waves can be one source of energy for the local acceleration of energetic trapped particles. An HM shock wave may therefore sufficiently enhance the energy densities of a quiet-time ring-current so that they contribute to the main phase of a geomagnetic storm.

We calculate the approximate total energy gained by the trapped particles in a quiet-time ring-current in the following manner. Let the volume occupied by the ring current be a cylinder of height h. The enhancement in the energy density $\delta E$ for the case considered here is given as a function of geocentric distance by $\beta_i (B^2/8\pi) b/B$. The change in the total energy $\delta U$ of the particles is given by the integral

$$\delta U = 2\pi a^2 h \int r\ \beta_i (B^2/8\pi) b/B\ dr$$

between $R_1$ and $R_0$. Substituting $B = B_{eq}/r^3$ and $b/B = K_1 r^3 - K_2 r$, we have

$$\delta U = (1/4) a^2 h \beta_i B_{eq}^2 \int_{R_1}^{R_o} (K_1 r^{-2} - K_2 r^{-4}) dr$$

$$= \left[ (1/4)\ a^2 h \beta_i B_{eq}^2\ - K_1 r^{-1} + (K_2/3) r^{-3} \right]_{R_1}^{R_o}$$

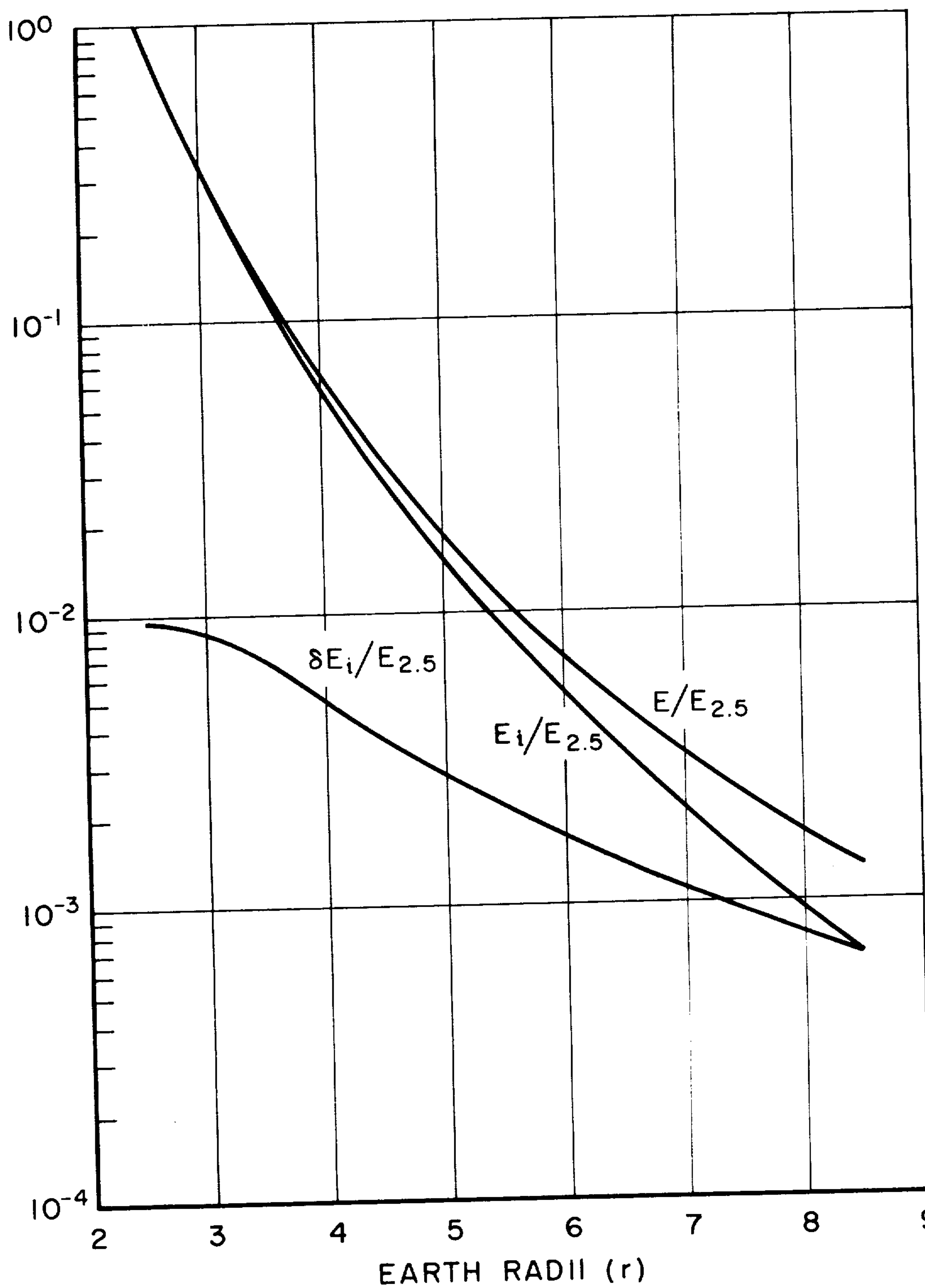


Fig. 7—Ratios of (1) initial energy density ($E_i$), (2) enhanced energy density E following HM shock wave, and (3) difference ($\delta E = E - E_i$) to initial energy density at 2.5 earth radii ($E_{2.5}$) as a function of geocentric distance r (in earth radii)

$$\delta U = (1/4)a^2 h\beta_i B_{eq}^2 \; K_1\left(\frac{1}{R_1} - \frac{1}{R_o}\right) - (K_2/3)\left(\frac{1}{R_1^3} - \frac{1}{R_o^3}\right)$$

For $R_o = 8.5\ r_e$ and $R_1 = 2.5\ r_e$, we have $K_1 = 1.73 \times 10^{-3}$, $K_2 = 7.05 \times 10^{-3}$; also $a = 6.3 \times 10^8$ cm, $B_{eq} = 1/3$ gauss. We take $h = 4a$ and obtain $\delta U \sim 10^{15}\ \beta_i$ joules. For greater compressions of the outer magnetosphere, $\delta U$ increases. For example, if $b/B = 1$ at $7.5\ r_e$, $\delta U \sim 1.3 \times 10^{15}\ \beta_i$ joules; and if $b/B = 1$ at $6.5\ r_e$, $\delta U \sim 2 \times 10^{15}\ \beta_i$ joules. A $\delta U$ of about $3 \times 10^{15}$ joules is required to produce a main phase decrease in $B_{eq}$ of $100\gamma$ at the equator (Parker, 1962). Davis and Williamson (1962) show that $\beta_i \sim 0.1$ between $4r_e$ and $8r_e$ for a magnetically quiet period. For this $\beta_i$, the $\delta U$'s calculated above would give $3\gamma$, $4\gamma$, and $7\gamma$ main phase decreases in $B_{eq}$.

To obtain larger decreases in $B_{eq}$ we can postulate shock events resulting from variations in the solar stream's structure. Each such shock event would add energy to the trapped particles. Thus the total energy gained by a ring current from HM shock waves in a geomagnetic storm might be much greater than that calculated above for the single shock wave accompanying a sudden commencement. Successive shock events would produce successive changes in a ring current resembling those calculated in the author's earlier paper (Kern, 1962).

The acceleration of energetic particles is accomplished at the expense of the energy of gas motion behind the shock front. Energy extracted from the shock wave is ultimately supplied by the solar stream. Thus the acceleration of energetic particles by the mechanism described above must contribute to the deceleration of the interface between the magnetosphere and the solar stream. The presence of energetic particles must therefore be taken into account in any theory describing the dynamics of a solar stream's impact on the magnetosphere.

## V. CONCLUSIONS

The acceleration of trapped particles by HM shock waves may contribute to the energy available in the magnetosphere for (1) the ring current of the geomagnetic storm, (2) auroras and airglow due to dumping of charged particles, and (3) ionospheric current systems associated with particle dumping. Two acceleration mechanisms appear likely. First, thermal particles can be accelerated by charge separation fields in the front of HM shock waves. Second, energetic particles can be accelerated by multiple reflections from the front of an HM shock wave. Calculations presented here indicate that both of these mechanisms can produce changes in the energy density of trapped particles which are of major significance. The above calculations are based on oversimplified models and are intended to show merely the orders of magnitude of the changes produced.

The acceleration of energetic particles by an HM shock wave would, because of their large acceleration transverse to B, produce gross changes in pitch-angle distributions for both energetic protons and electrons. Large increases in the proportion of particles with pitch angles near $90^{\circ}$ have been observed in the trapped radiation following the sudden commencement of a geomagnetic storm (Hoffman, Arnoldy, and Winckler, 1962). Such increases are consistent with the acceleration of energetic particles by an HM shock wave that is associated with a sudden commencement. The acceleration of thermal electrons by HM shock waves could supply large numbers of Kev range electrons in the regions conjugate to the auroral zones. The appearance of relatively large fluxes of Kev range electrons in the outer magnetosphere following a sudden commencement would be strong evidence in support of the acceleration of thermal particles by a single HM shock wave.

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