Technical Report No. 32-397

Fusion Propulsion System Requirements for an Interstellar Probe

Dwain F. Spencer

Rob Roy McDonald, Chief
Engineering Research Section

JET PROPULSION LABORATORY
California Institute of Technology
Pasadena, California
May 15, 1963
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ABSTRACT

An examination of the engine constraints for a fusion-propelled vehicle indicates that minimum flight times for a probe to a 5 light-year star will be approximately 50 years. The principal restraint on the vehicle is the radiator weight and size necessary to dissipate the heat which enters the chamber walls from the fusion plasma. However, it is interesting, at least theoretically, that the confining magnetic field strength is of reasonable magnitude, $2 \times 10^5$ gauss, and the confinement time is approximately 0.1 sec.

I. INTRODUCTION

The prospect of interstellar exploration has aroused considerable speculation during the past few years. Previous authors have stated that exploration beyond the solar system is impossible without the photon (annihilation) rocket. As pointed out in Ref. 1, this is not necessary if the full potential of the fission or fusion nuclear reactions can be realized in a multistage vehicle. The purpose of this analysis is to examine in more detail the requirements on a fusion propulsion system to drive an interstellar spacecraft on a probe mission.
II. LIMITATIONS ON TRANSIT TIME FOR A FUSION-PROPELLED VEHICLE

In general, the fraction of fuel which is utilized in a nuclear reactor is less than the theoretical limit. This is the so-called burnup fraction, \( b \), which is a number less than or equal to unity. The equation for the exhaust velocity, \( w \), of a particular stage can be generalized to:

\[ w = c \left[ eb (2 - eb) \right]^{1/2} \quad (1) \]

In order to determine the effect of burnup on system performance, we recall that for optimum staging (Ref. 2), the burnout velocity of the \( n \)th stage is:

\[ \left( \frac{u_n}{c} \right) = \left( \frac{8^{n-1} w_{o/e} - 1}{8^{n-1} w_{o/e} + 1} \right) \quad (2) \]

Figure 1 shows the performance of a fusion vehicle with an acceleration of 1 \( g \)/stage, and a stage-mass ratio of 10. It should be noted that unless burnups of greater than 1% can be achieved, there is little chance of the fusion vehicle performing interstellar missions to 5 light years with flight times of less than 50 years.

Figure 2 exemplifies the penalty in transit time when the average vehicle acceleration is less than 1 \( g \). It is obvious that accelerations greater than \( 10^{-3} \) \( g \) are required if the vehicle is to have a reasonable transit time to a 5 light-year star. The achievable acceleration with a fusion vehicle will be given later.

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\(^{1}\)See Nomenclature for definition of symbols.
Fig. 2. Effect of acceleration on transit time to a 5 light-year distance
III. GENERAL CHARACTERISTICS OF A FUSION ENGINE

Figure 3 presents a schematic of a typical continuous-feed fusion engine. The basic components of the system are the plasma injector, the fusion plasma, the superconducting coils, the structural vessel (including insulation), a refrigeration cycle and low temperature radiator to dissipate the heat developed in the coils (principally neutron heating), and a primary coolant system and radiator to reject the heat developed in the pressure vessel and shield structure (cyclotron radiation, bremsstrahlung, and neutron heating). For purposes of discussion, the heat load to the coils was neglected, and all energy escaping the plasma was assumed to be absorbed in the structure.

Now the thrust of the engine is simply

\[ F = \dot{m}_w w \]  

and the required fusion exhaust power is

\[ P_{ex} = 10^{-13} F w/2 \]

The total power output required from the fusion reactor is

\[ P_t = P_{ex} [1 - (\gamma + \alpha)] \]

where \( \gamma \) is the fractional power carried by the neutrons and \( \alpha \) is the fractional power lost from the fuel due to bremsstrahlung and cyclotron radiation. The power which is absorbed in the engine walls is then

\[ P_{abs} = (\gamma + \alpha) / [1 - (\gamma + \alpha)] P_{ex} \]  

As pointed out in Ref. 3, a D – He\(^3\) fuel is of particular interest for rocket propulsion since the products are all charged particles which can be trapped by the external magnetic field. Now consider the competing reactions in such an engine (Ref. 4), and their energy yields:

\[
\begin{align*}
D + He^3 & \rightarrow He^4 + He^3 & 3.6 \text{ MeV} & + & 14.7 \text{ MeV} \\
D + D & \rightarrow He^3 + n & 0.52 \text{ MeV} & + & 2.45 \text{ MeV} \\
D + D & \rightarrow T + H & 1.01 \text{ MeV} & + & 3.02 \text{ MeV} \\
D + T & \rightarrow He^4 + n & 3.5 \text{ MeV} & + & 14.1 \text{ MeV}
\end{align*}
\]

If we neglect the last reaction (since the amount of tritium present is small), the fractional energy release which is imparted to the neutrons can be estimated. Let \( y \) represent the fuel fraction of He\(^3\) and then \((1 - y)\) is the fuel fraction of D. Define the fraction of power carried by the neutrons as \( \gamma = (P_{ne})/(P_t) \).

Then,

\[ \gamma = \frac{0.5 (1-y)^2 (\overline{\sigma v})_{1,1} E_{ne}}{y (1-y) (\overline{\sigma v})_{1,2} (E_{1,2}) + 0.5 (1-y)^2 (\overline{\sigma v})_{1,1} (E_{1,1})} \]

where \( \overline{\sigma v} \) determines the reaction rate for a Maxwellian velocity distribution, and \( E \) represents the reaction energy.

The fractional energy lost by bremsstrahlung and cyclotron radiation, \( \alpha \), is defined as \( \alpha = a_{br} + a_{cr} \).

The equation for \( a_{br} \) (Ref. 4) is

\[ a_{br} = \frac{5.35 \times 10^{-21} N_t (N_z Z_{t,2}^2 + N_z Z_{t,3}^2) (T_e')^{1/4}}{2.93 \times 10^{-12} N_t N_t (\overline{\sigma v})_{1,2}} \]

Rearranging and using the definitions of the He\(^3\) and D fractions given above

\[ a_{br} = \frac{1.8 \times 10^{-18} (T_e')^{1/4} (3y + 1) (y + 1)}{y (1-y) (\overline{\sigma v})_{1,2}} \]
The fractional power going into cyclotron radiation (Ref. 5) is approximately

\[ \theta = \frac{8.5 \times 10^{-21} \left[ (y + 1) T_i^2 + (y + 1)^2 (T_e^2)^{y} \right]}{y \left(1 - y\right)(\sigma \bar{v})_{1.2}} \left[ 1 + \frac{T_e}{204} \right] \]

(10)

Due to self-absorption of the cyclotron radiation in the plasma and reflection from the chamber walls (if properly designed), the fractional power lost through this mode may be reduced. In the region of interest for these studies, a rough estimate of this fractional energy loss is approximately 1% of \( \theta \); thus

\[ \alpha_{c,r} = 1 \times 10^{-1} \theta \]

(11)

Figure 4 shows the fractional power entering the wall vs. the \( \text{He}^3 \) fuel fraction for various ion temperatures. In all cases an ion-to-electron temperature ratio of 2 is assumed, as this appears to be a reasonable value for injection mechanisms of interest. From Fig. 4, there is an optimum operating temperature of 100 to 200 kev in the region from 0.5 to 0.7 \( \text{He}^3 \) fuel fraction. It should be noted, however, that the minimum fractional energy escaping the fuel is approximately 20%. This simply means that 20% of the generated energy must be dumped by a thermal radiator. A similar problem has been well

The remaining equations which are necessary to determine the performance of the system will now be considered. The rest mass of fuel exhausted is generalized to

\[ \dot{m}_{ex} = \dot{m}_i \left(1 - b \epsilon\right) \]

(12)

and the rest mass of fuel burned is

\[ \left(\dot{m}^*_f\right)_b = b \dot{m}_i \]

(13)

But this is governed by the reaction rate in the chamber. Then, neglecting the DD and DT contributions,

\[ \left(\dot{m}_i\right)_b = \left(\frac{M_1 + M_2}{N_{R_0}}\right) N_1 N_2 (\sigma \bar{v})_{1.2} V_f \]

(14)

where \( V_f \) is the volume of the fuel.

The thrust is given by

\[ F = \left(\frac{M_1 + M_2}{N_{R_0}}\right) N_1 N_2 (\sigma \bar{v})_{1.2} V_f \frac{c}{\epsilon} \left(1 - b \epsilon\right) \]

\[ \times \left(\frac{e \left(2 - b \epsilon\right)}{b}\right)^{1/2} \]

(15)

If the engine thrust and size are specified, (along with the reaction temperature), the required fuel concentration may then be determined from Eq. 15. This, in turn, sets the required magnetic field for confinement. Under optimum conditions, the confining magnetic field strength is simply

\[ B = \left(8\pi N_i k T\right)^{1/2} \]

(16)

Another quantity of interest is the confinement time of an average fuel ion necessary to obtain a certain burn-up fraction. The fuel flow rate from the confined volume is

\[ \dot{m}_i \left(1 - b \epsilon\right) = \frac{V_f}{t_c N_{R_0}} \left(N_1 M_1 + N_2 M_2\right) \]

(17)
Combining Eq. 13, 14, and 17 and solving for $t_c$,

$$t_c = \frac{(N_1 \mathcal{M}_1 + N_2 \mathcal{M}_2) b}{(\mathcal{M}_1 + \mathcal{M}_2) N_1 N_2 (\sigma_v)_{1,2} (1 - b \varepsilon)}$$

(18)

A very important result can be seen by examining Eq. 1, 15, and 18. By simply decreasing the burnup, not only is the required confinement time decreased, but also the powerplant thrust is increased. The penalty for this is, of course, a decrease in the specific impulse of the engine. This factor, however, will be shown to be of importance if the total burning time becomes excessive.

### IV. SYSTEM WEIGHTS AND PERFORMANCE

The two most significant weights of a particular stage are the pressure vessel and the primary waste-heat radiator. The pressure vessel weight is, of course, determined by the internal magnetic pressure which it must withstand. For a cylinder, the usual equation for hoop stress is simply

$$s = \frac{p r}{z}$$

(19)

where $r$ is the internal radius of the cylinder and $z$ the thickness, but,

$$p = B^2/8\pi$$

(20)

Then the required thickness is

$$z = B^2 r/8\pi s$$

(21)

The weight of the pressure shell is

$$W_s = 2\pi rz \rho$$

(22)

For a cylinder with an $l/d = 2$, and utilizing Eq. 21, the weight of the shell in pounds is

$$W_s = 2.24 \times 10^{-6} (\rho / s) B^2 r^2$$

(23)

Since the amount of heat to be rejected by the primary radiator is quite large, a conventional radiator design does not appear interesting. Rather, the concept proposed in Ref. 7 will be considered where the authors present an analysis for a so-called "belt-type radiator." For an optimum system the belt weight is given by

$$W_B = \left[2.0 P_{abs}^{5/2} (AR)^{1/2} / C (U / 3000) (1.8 T_B)^6 / 1000 \right]$$

(24)

where $(AR)$ is the aspect ratio of the belt, $C$ the specific heat, $U$ the belt speed, and $T_B$ the belt maximum temperature.

Dr. L. Jaffe has suggested the use of pyrographite for the belt material, since we desire a very high radiating temperature. Since the coolant first passes through the shell, it too would be pyrographite. To maximize the strength of the structure, a radiating temperature of 3200°K is assumed. The tensile strength of pyrographite at this temperature is approximately 60,000 psi and it has a heat capacity of 0.5 cal/g°C. With this material, an assumed belt speed of 3,000 cm/sec, and an additional allowance for the heat-transfer mechanism to the belt and enclosure, the total radiator weight is given by

$$W_{rad} = 2.1 \times 10^{-5} (P_{abs})^{5/2} (AR)^{1/2} + 320 (P_{abs})^{1/2}$$

(25)

*Private communication with L. D. Jaffe at JPL.
The total stage dead weight is then the sum of Eq. 22 and 25. It should be noted that with the high operating temperature of the shell, substantial insulation of the superconducting coils may be required. However, this is considered to be a negligible weight compared to that of the shell and radiator.

The performance is calculated by considering that the size of each stage (including the fuel volume) varies linearly with thrust level; thus, each stage has the same initial acceleration. A fuel diameter of 10 m was selected for a thrust of $10^4$ lb.

The burning time of the $j$th stage is

$$t_{b,j} = [(1 - \epsilon b) I_j]/[(1 + x_j)a_{o,j}]$$

(26)

where $I_j$ is the specific impulse of the $j$th stage, $x_j$ is the $j$th stage fraction, and $a_{o,j}$ the initial acceleration of the $j$th stage. From the preceding arguments, the burning of all stages is the same and the propulsion time is simply

$$P.T. = 3.18 \times 10^{-8} n t_{b,j}$$

(27)

The total distance traveled during propulsion is given by

$$X_i = \sum_{j=1}^{n} X_j$$

(28)

where

$$X_j \equiv u_{j-1} t_{b,j} + \frac{c^2}{\ddot{a}_j \gamma} \left[ \left( 1 + \frac{\ddot{a}_j^2 g^2 t_{b,j}^2}{c^2} \right)^{\frac{1}{2}} - 1 \right]$$

(29)

and

$$\ddot{a}_j = (a_{o,j}/2) \left[ (2x_j + 1)/x_j \right]$$

(30)

The coast time to a 5 light-year star is

$$C.T. = 3.18 \times 10^{-8} \left[ (5)(9.5 \times 10^{17}) - X_i \right]/u_{n}$$

(31)

and the total transit time is

$$T.T.T. = P.T. + C.T.$$

(32)

In order to determine the required engine characteristics, an interstellar probe mission is considered. The required gross-payload weight to perform this mission is estimated to be 10,000 lb. The principal portion of this weight is necessary to provide telecommunications capability. Using X-band communication to a 200-fet terrestrial dish, an information rate of 1 bit/min requires a 1-Mwe power transmitter at a distance of 5 to 10 light years. The auxiliary powerplant necessary to provide this power will probably weigh on the order of 2000 to 5000 lb. This weight is consistent with the payload weight of 10,000 lb that has been assumed.

Figure 5 presents the required initial acceleration of each stage vs. the fuel-burnup fraction for radiator-aspect ratios of 1 and 10. The higher initial acceleration permissible for a given burnup fraction at an aspect ratio of 1.0 is a result of the lower radiator weight at the aspect ratio of 1.0. It should be noted that initial accelerations are approximately $2 \times 10^{-3}$ g in the region of interest, so the fusion vehicle would have to be boosted into earth orbit and would have an initial weight of $10^7$ lb in this design.

*Fig. 5. Initial acceleration of each stage vs. fuel burnup fraction*

The total transit time and propulsion time are shown as functions of the fuel burnup fraction in Fig. 6. Note that the minimum flight time to a 5 light-year star is approximately 50 years and occurs with continuous propulsion. The total propulsion time, however, can be halved with an increase in transit time of only 10% near the minimum flight time. The burning time of each stage is, of course, $1/5$ of the propulsion time due to the assumption made in the analysis. The decrease in radiator dead

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*Private communication with S. Golomb.*
weight at a lower aspect ratio results in a decrease in flight time by approximately 8 years due to the higher acceleration of each stage; however, as can be seen in Fig. 7, this requires a larger radiator area, as the power to be rejected is greater. Thus, in order to cut the radiated power and radiator size for at least the first stage, it may be more efficient to utilize an aspect ratio of 10. Even with this, the radiated power is approximately $10^8$ Mw from the first stage. This is $10^6$ times that for any other system now being considered; however, developments over the next 50 years may show that this is not inconceivable.

Figures 8 and 9 present the requirements on fuel concentration, magnetic field strength, and plasma confinement time vs. burnup fraction. Due to the method used in scaling the vehicle, these values are the same for each stage. Fuel concentrations on the order of $10^{15}$ to $10^{16}$ particles/cm$^3$ and magnetic field strengths of 200,000 to 300,000 gauss are required. These do not seem inconceivable; however, there are certainly problems which must be solved before these values are achieved. The confinement time for an average fuel ion of approximately 0.1 sec is also reasonable.
Fig. 8. Required fuel concentration and magnetic field strength of the fusion engines vs. burnup fraction

Fig. 9. Plasma confinement time vs. burnup fraction
V. CONCLUSIONS

This analysis points out the difficulty in approaching the theoretical performance for an interstellar spacecraft given in Ref. 1; however, it indicates that flight times of less than 50 years to a 5 light-year star may be approached with a fusion-propelled vehicle, if certain engineering problems can be solved.

 NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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<tbody>
<tr>
<td>AR</td>
<td>aspect ratio of belt</td>
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<tr>
<td>a</td>
<td>acceleration, earth g</td>
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<tr>
<td>B</td>
<td>magnetic field strength, gauss</td>
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<tr>
<td>b</td>
<td>fuel burnup fraction</td>
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<tr>
<td>C</td>
<td>specific heat of belt material, cal/g°C</td>
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<tr>
<td>C.T.</td>
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<td>c</td>
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<td>burnout velocity, cm/sec</td>
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<tr>
<td>V</td>
<td>volume, cm³</td>
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$v$ relative velocity of particles, cm/sec
$W$ weight, lb
$w$ engine exhaust velocity, cm/sec
$X$ distance traveled during propulsion, cm
$y$ He$^3$ fraction of fuel
$Z$ atomic number
$z$ thickness of structure, cm
$a$ fractional power lost from fuel due to bremsstrahlung and cyclotron radiation
$\beta$ stage dead-weight fraction
$\gamma$ fraction of power carried by neutrons
$\delta$ stage-mass ratio
$\theta$ fractional power going into cyclotron radiation
$\epsilon$ fraction of fuel mass converted to energy
$\rho$ density of structural material (pyrographite), g/cm$^3$
$\sigma$ microscopic reaction cross section, cm$^2$
$\chi$ stage burnout-weight fraction

Subscripts
$abs$ absorbed
$B$ belt
$b$ burned
$br$ bremsstrahlung
$c$ confinement
$c.r.$ cyclotron radiation
$e$ electron
$ex$ exhaust
$f$ fuel
$i$ ion
$j$ $j$th stage ($j = 1$ to $n$)
$n$ final
$ne$ neutron
$pay$ payload
$rad$ radiator
$s$ shell
$t$ total
$0$ initial
$1$ species 1 (D)
$2$ species 2 (He$^3$)

Superscripts
$-$ average value
$'$ temperature in kev
REFERENCES


