Flutter of Two Parallel Flat Plates
Connected by an Elastic Medium

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Nomenclature

\( A_0 \) = \( \frac{R_{ss}}{2(n/b)^4} \)
\( a, b \) = plate length and width, see Fig. 1
\( B_0 \) = \( \left( \frac{\Omega a^2}{\pi^4} + (a/b)R_{ss} - (a/b)^4 \right) \)
\( D_0 \) = plate flexural stiffness
\( h_0 \) = thickness of plate
\( k \) = elastic spring constant
\( t \) = lateral aerodynamic load
\( M \) = Mach number
\( N_{y0}, N_{y1} \) = midplane force intensities, positive in compression
\( q \) = dynamic pressure, \( \rho U^2/2 \)
\( R_{ss} \) = \( a^4N_{y0}/\pi^4D_0 \)
\( S_0 \) = spring stiffness parameter, \( ka^4/\pi^4D_0 \)
\( t \) = time
\( U \) = freestream velocity
\( w_{x0} \) = lateral deflection of plate
\( x, y \) = Cartesian coordinates, see Fig. 1
\( \beta \) = \( (M^2 - 1)^{1/2} \)
\( \gamma \) = mass density of plate
\( \lambda \) = \( 2a^4/\pi^2 D_0 \)
\( \rho \) = mass density of air
\( \omega \) = circular frequency
\( \Omega_{x0} \) = \( \omega^0 + \gamma h_0/2D_0 \)
\( +, - \) = subscripts refer to upper and lower plate, respectively

Introduction

The flutter behavior of a structural configuration consisting of two rectangular, simply supported, parallel plates laterally connected by many closely spaced linear springs is investigated. The configuration analyzed is shown in Fig. 1. The upper plate has air flowing at supersonic speed over the
upper surface, and both plates are subjected to midplane loadings. This configuration is an idealization of a micrometeoroid bumper that is attached to a primary structure by a light, soft filler material. The aeroelastic behavior of such a configuration may be important in the design of structural components of a manned space station which are exposed to an airstream during launch.

**Analysis**

The equilibrium equations and appropriate boundary conditions are

\[
\frac{D_x}{D_x} \nabla w_+ + N_{xx} + N_{xy} + \frac{\partial^2 w_+}{\partial y^2} + \frac{\partial w_+}{\partial y} + \frac{\partial w_+}{\partial x} + \frac{\partial w_+}{\partial z} + k(w_+ - w_-) = f(x,y,t) \quad (1)
\]

\[
\frac{D_x}{D_x} \nabla w_- + N_{xx} + N_{xy} + \frac{\partial^2 w_-}{\partial y^2} + \frac{\partial w_-}{\partial y} + \frac{\partial w_-}{\partial x} + \frac{\partial w_-}{\partial z} + k(w_- - w_+) = 0 \quad (2)
\]

\[
\frac{\partial^2 w_+}{\partial x^2}(x,0,t) = \frac{\partial^2 w_+}{\partial y^2}(x,b,t) = \frac{\partial^2 w_-}{\partial x^2}(0,y,t) = \frac{\partial^2 w_-}{\partial y^2}(a,y,t) = 0 \quad (3)
\]

where \(f(x,y,t)\) is the lateral load per unit area due to aerodynamic pressure. For static strip theory the lateral load is given by the simple Ackeret value \(I(x,y,t) = -(2q/\beta) × (\partial w_+ / \partial x)\).

A two-term Galerkin solution is pursued. Solutions that satisfy the boundary conditions for simply supported edges are assumed as follows:

\[
w_{\pm}(x,y,t) = \left[ C_{1\pm} \sin \left( \frac{\pi x}{a} \right) \sin \left( \frac{\pi y}{b} \right) + C_{2\pm} \sin \left( \frac{2\pi x}{a} \right) \sin \left( \frac{\pi y}{b} \right) \right] e^{int} \quad (4)
\]

where \(l(x,y,t)\) is the lateral load per unit area due to aerodynamic pressure. For static strip theory the lateral load is given by the simple Ackeret value \(I(x,y,t) = -(2q/\beta) × (\partial w_+ / \partial x)\).

Flutter occurs with the coalescence of two natural frequencies as the dynamic pressure parameter \(\lambda\) increases (see Ref. 1). The procedure is to solve Eq. (5) for \(\lambda\) and maximize the resulting expression with respect to the frequency to obtain a critical value of the dynamic pressure parameter \(\lambda_c\). In many cases, more than one critical value exists corresponding to the coalescence of different pairs of modes, and it is necessary to seek the lowest critical value to define a flutter boundary.

In order to illustrate the general flutter characteristics exhibited by this configuration, calculations were made for the simplified case for which \(h_+ = h_-, D_+ = D_-, R_{zv} = 0, a/b\)
Flutter boundary for $R_{z+} = 0$.

Figures 2-4 present flutter boundaries ($\lambda_\omega$ vs $R_\omega$) for square plates having various combinations of midplane loads and a spring parameter of $S = 20$. At the present time realistic values of $S$ are not clearly defined; the value chosen ($S = 20$) might be typical of a configuration with a very soft filler material. The flutter boundary for a single flat plate with the same physical properties as either the upper or the lower plate considered in the present analysis is also shown in each of the figures (see Ref. 1).

Figure 2 is a plot of the boundary for the configuration when there is no load in the lower plate. This boundary becomes asymptotic to the single plate boundary for large negative values of $R_\omega$, i.e., large midplane tension, as do all of the boundaries considered herein.

Figure 3 is a plot of the boundary when the midplane loads are the same in each plate. Here the tuning effect can be very significant since it is possible to have a zero flutter speed with a tensile load, and a peak exists which is much higher than the corresponding value for the single plate.

Figure 4 is a plot of the boundary when there is no load in the upper plate. This case is perhaps the most realistic combination of loading if the configuration is considered to be a micrometeoroid protection device. This boundary has characteristics very similar to the boundary in Fig. 2 except that here the tuning effect is more prevalent. In particular, for streamwise tension, a condition that can be expected from bending loads on a space vehicle during launch, the elastically supported plate is more prone to flutter than the single plate alone.

The results of this analysis indicate that if a configuration similar to this one is used for applications where supersonic airflows are encountered, a very careful flutter analysis is in order to insure that undesirable flutter characteristics are not present.

Reference