Technical Memorandum No. 33-167

JPL Ephemeris Tapes E9510, E9511, and E9512

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JET PROPULSION LABORATORY
California Institute of Technology
Pasadena, California

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ABSTRACT

The first issue of JPL Ephemeris Tapes is described. These tapes carry the positions and velocities of the planets and of the Moon, plus nutations and nutation rates in longitude and obliquity, together with second and fourth modified differences, for the interval December 30, 1949, to January 5, 2000.

1. INTRODUCTION

The JPL Ephemeris Tapes E9510, E9511, and E9512, which constitute the first release from the JPL Ephemeris Tape System, are described in this Memorandum. The time intervals spanned by the tapes are as follows:

<table>
<thead>
<tr>
<th>Tape</th>
<th>Julian date (Calendar date)</th>
<th>Julian date (Calendar date)</th>
</tr>
</thead>
<tbody>
<tr>
<td>E9510</td>
<td>243 3280.5 (Dec. 30, 1949)</td>
<td>244 0584.5 (Dec. 29, 1969)</td>
</tr>
<tr>
<td>E9511</td>
<td>243 9500.5 (Jan. 10, 1967)</td>
<td>244 6796.5 (Jan. 1, 1987)</td>
</tr>
<tr>
<td>E9512</td>
<td>244 5708.5 (Jan. 9, 1984)</td>
<td>245 1548.5 (Jan. 5, 2000)</td>
</tr>
</tbody>
</table>

The data contained on these tapes are the positions and velocities of the Moon and of the planets Mercury, Venus, the Earth–Moon barycenter, Mars, Jupiter, Saturn, Uranus, Neptune, and Pluto, plus the nutations and nutation rates in longitude and obliquity, together with second and fourth central differences modified by throwing back sixth and eighth differences according to the formulas

\[ \Delta_m^2 = \delta^2 + (-0.01312) \delta^8 + (0.0043) \delta^8 \]

and

\[ \Delta_m^4 = \delta^4 + (-0.27827) \delta^8 + (0.0685) \delta^8 \]
Positions and velocities are referred to the rectangular equatorial coordinate system of the mean equator and equinox of “1950.0” = Julian ephemeris date 243 3282.423357. Planetary data are heliocentric and are expressed in AU and AU/day, and lunar data are geocentric and are expressed in units called “Earth radii” and “Earth radii”/day. The value of the factor for converting lunar positions to kilometers must be chosen carefully in order to maintain consistency among the physical constants. The value

\[ 1 \text{"Earth radius"} = 6378.327 \text{ km} \]

is recommended. The value 6378.3255 km is given in Ref. 1.

Positions of the Moon and of the inner planets were obtained either from evaluations of certain general perturbation theories (source positions) or from numerical integrations in which epoch values were chosen so as to obtain the least-squares fit to the source positions; positions so obtained by numerical integration will be referred to in this Memorandum as “fitted” positions. Velocities were obtained either by numerical differentiation of source positions, using a central difference formula in which eighth differences are retained, or by numerical integration. Positions and velocities of the outer planets have been similarly fitted; here, however, the source positions were based upon an earlier numerical integration.

The following Parts contain details concerning the data carried on these JPL Ephemeris Tapes, including special checks made. In addition to these checks, all data were tested for consistency by plotting sixth differences and by playing back the final data recorded on the JPL Ephemeris Tapes against the tapes that carried the data originally, as described in Ref. 2.

The format of the JPL Ephemeris Tapes is described in Appendix A. Other features relevant to the development of these tapes are given in Ref. 2.

II. PLANETARY EPHEMERIDES

A. Fitted Ephemerides

Fitted position and velocity data are recorded for all planets except Mercury and Neptune. Long arc fits of Mercury positions will be deferred until the completion of a modified numerical integration program that permits inclusion of relativistic corrections. Difficulty with fitting Neptune positions has been experienced, and inclusion of fitted Neptune position and velocity data is postponed to a later issue of JPL Ephemeris Tapes.

The sequence in which numerical integration fits for the different planets were generated is as follows: Jupiter, Saturn, Uranus, Pluto, Mars, the Earth-Moon barycenter, and Venus. In each case an interim set of JPL Ephemeris Tapes was used to provide the positions of the perturbing planets from which the perturbing attractions on the planet being fitted were calculated. For Jupiter this set consisted of source positions only. A fitted Jupiter ephemeris was used in fitting Saturn; Jupiter was refitted with this fitted Saturn ephemeris, with no significant change observed; and Uranus and Pluto were fitted using the previous fits to Jupiter and Saturn. The remaining fits were generated using fitted Jupiter, Saturn, Uranus, and Pluto ephemerides.
The statistical summary and epoch values of the Venus integration are listed in Table 1. Plots of the residuals between the source data and the fitted data are given in Fig. 6 to 20. It is seen that the uncertainty introduced by the integration is still less than the uncertainty in the source positions.

The source position tape is E9002; the fitted position-velocity tape is E9102.

D. The Ephemeris of the Earth–Moon Barycenter

The coordinates and velocity components of the Earth–Moon barycenter were obtained as a numerical integration fit to source positions. The source positions were generated by an IBM 7094 program developed by N. Block (Ref. 3), which evaluated the Newcomb theory of the motion of the Earth (Ref. 9) after application of corrections to the mean elements deduced by R. Duncombe (Ref. 8). The values of the mean elements used in the evaluation are stated in Ref. 3.

The accuracy of the source positions is felt to be of the order of 0.1 sec of arc in latitude and longitude after the application of the Duncombe corrections. This estimate is confirmed by published accounts of the reduction of radar observations of Venus. The uncertainty in the radius vector, and therefore the uncertainty in the rectangular coordinates, is essentially the uncertainty in the astronomical unit.

The machine evaluation was checked against hand computations at selected epochs. In addition, the evaluation program was checked by comparing the evaluation of the Newcomb theory without the Duncombe corrections, but with a correction of the form 4.78 T to the mean longitude, with a similar evaluation, by P. Herget, that was recorded onto tape by the Livermore Radiation Laboratory. Plots of this comparison are shown in Fig. 21 to 25.

The statistical summary and epoch values of the Earth–Moon integration are listed in Table 2. Plots of the residuals between the source data and the fitted data are given in Fig. 26 to 40.

The source position tape is E9003; the fitted position-velocity tape is E9103.

E. The Ephemeris of Mars

The coordinates and velocity components of Mars were obtained as a numerical integration fit to source positions. The source positions were obtained from the USNO in the form of punched cards; they constitute an evaluation of the third-order Hansen theory developed by G. M. Clemence (Ref. 10, Ref. 11), with provisional values of the mean elements.

It is expected that the source positions will prove to be accurate to a few hundredths of a second of arc in latitude and longitude.

The statistical summary and epoch values of the Mars integration are listed in Table 3. Plots of the residuals between the source data and the fitted data are given in Fig. 41 to 55.

The source position tape is E9004; the fitted position-velocity tape is E9104.

F. The Ephemeris of Jupiter

The coordinates and velocity components of Jupiter were obtained as a numerical integration fit to source positions. The source positions were obtained by the simultaneous integration of the motion of the five outer planets performed on the Selective Sequence Electronic Calculator by W. J. Eckert, D. Brouwer, and G. M. Clemence (Ref. 12); the source positions were recorded on tape at the Livermore Radiation Laboratory. These source positions were converted to true heliocentric coordinates by applying the corrections for the perturbations of the inner planets developed by Clemence (Ref. 13). The corrections were evaluated and applied by an IBM 7094 program developed by N. Block (Ref. 14).

The machine evaluation of the corrections was checked against hand computations at selected epochs. Plots of the corrections are given in Fig. 56 and 57.

The statistical summary and the epoch values of the Jupiter integration are listed in Table 4. Plots of the residuals between the corrected source data and the fitted data are given in Fig. 58 to 63.

The corrected source position tape is E9005; the fitted position-velocity tape is E9105.
In each case the numerical integration used a second-sum method with eighth differences retained. A 2-day integration step size was used for Venus, for the Earth-Moon barycenter, and for Mars; a 4-day step was used for the outer planets. Values of the Sun/planet mass ratios that were used are listed as follows:

<table>
<thead>
<tr>
<th>Planet</th>
<th>Inverse mass ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mercury</td>
<td>6,000,000.0</td>
</tr>
<tr>
<td>Venus</td>
<td>408,000.0</td>
</tr>
<tr>
<td>Earth-Moon barycenter</td>
<td>339,390.0</td>
</tr>
<tr>
<td>Mars</td>
<td>3,093,500.0</td>
</tr>
<tr>
<td>Jupiter</td>
<td>1,047.355</td>
</tr>
<tr>
<td>Saturn</td>
<td>3,501.6</td>
</tr>
<tr>
<td>Uranus</td>
<td>22,869.0</td>
</tr>
<tr>
<td>Neptune</td>
<td>19,314.0</td>
</tr>
<tr>
<td>Pluto</td>
<td>360,000.0</td>
</tr>
</tbody>
</table>

Equations of condition for ecliptic latitude and ecliptic longitude factored by the cosine of the latitude were formed at 4-day intervals for the inner planets and at 40-day intervals for the outer planets. These equations were accumulated into normal equations at the completion of the integration, which were then solved for differential corrections to the ecliptic Keplerian elements \( \alpha, e, I, \Omega, \omega, \text{and } M_e \), which osculate at the epoch Julian ephemeris date 243 3280.5. The process was repeated until no further significant reduction in the sum of squares of residuals could be made.

The integration was performed in the equatorial system, mean of 1950.0. The value assigned to the obliquity in order to transform between the equatorial and ecliptic systems was \( \varepsilon = 0.40920619 \) rad.

The mean residuals, mean square residuals, and standard deviations, as well as epoch values of equatorial position and velocity, osculating ecliptic elements, and the ecliptic components of the \( P, Q, \) and \( R \) vectors, are listed in the tables for each of the fitted ephemerides.

B. The Ephemeris of Mercury

The coordinates of Mercury were generated by an IBM 7094 program developed by N. Block (Ref. 3), which evaluated the Newcomb theory of Mercury (Ref. 4) after application of the corrections to the mean elements deduced by G. M. Clemence (Ref. 5). The values of the mean elements used in the evaluation are given in Ref. 3.

The accuracy of the Mercury ephemeris is felt to be of the order of 0.1 sec of arc in latitude and longitude (Ref. 6). No estimate of the accuracy in the radius vector or in the rectangular components is available.

Rectangular velocity components were generated by numerical differentiation of position components.

The machine evaluation was checked by hand at selected epochs. In addition, the evaluation program was checked by a comparison of the evaluation of the Newcomb theory without the Clemence corrections with the data obtained on punched cards from the U.S. Naval Observatory (USNO) for the interval 1960–1981.

The source position tape is E9001.

C. The Ephemeris of Venus

The coordinates and velocity components of Venus were obtained as a numerical integration fit to source positions. The source positions were generated by an IBM 7094 program developed by N. Block (Ref. 3), which evaluated the Newcomb theory of Venus (Ref. 7) after application of corrections to the mean elements deduced by R. Duncombe (Ref. 8). The values of the mean elements used in the evaluation are stated in Ref. 3.

The accuracy of the source positions is felt to be of the order of 0.1 sec of arc in latitude and longitude after application of the corrections (Ref. 6). This estimate has been confirmed by published accounts of the reduction of radar observations of Venus. The uncertainty in the radius vector, and therefore the uncertainty in the rectangular coordinates, is essentially the uncertainty in the value of the astronomical unit.

The machine evaluation was checked against hand computations at selected epochs. In addition, the evaluation program was checked by comparing the evaluation of the Newcomb theory with the Duncombe corrections with an earlier evaluation performed by P. Herget and recorded on tape at the Livermore Radiation Laboratory. Plots of this comparison are given in Fig. 1 to 5.
G. The Ephemeris of Saturn

The coordinates and velocity components of Saturn were obtained as a numerical integration fit to source positions. The basis of the source positions, including the corrections applied to the source positions, is the same as for Jupiter, as described in Section II-F.

Plots of the corrections are given in Fig. 64 and 65. The statistical summary and the epoch values of the fitted integration are given in Table 5. Plots of the residuals between the corrected source data and the fitted data are given in Fig. 66 to 71.

The corrected source position tape is E9006, and the fitted position-velocity tape is E9106.

H. The Ephemeris of Uranus

The coordinates and velocity components of Uranus were obtained as a numerical integration fit to source positions. The basis of the source positions, including the corrections applied to them, is the same as for Jupiter, as described in Section II-F.

Plots of the corrections are given in Fig. 72 and 73. The statistical summary and the epoch values of the fitted integration are given in Table 6. Plots of the residuals between the corrected source data and the fitted data are given in Figures 74 to 79.

The corrected source position tape is E9007; the fitted position-velocity tape is E9107.

I. The Ephemeris of Neptune

The coordinates of Neptune were taken directly from the basic source positions without application of the corrections for the perturbations of the inner planets described in Section II-F. Velocities were obtained by numerical differentiation of the source positions.

Difficulty was encountered in fitting the Neptune ephemeris. A fitted ephemeris will be distributed in a subsequent set of JPL Ephemeris Tapes.

Plots of the corrections that can be applied are given in Fig. 80 and 81. The uncorrected source position tape is E9018.

J. The Ephemeris of Pluto

The coordinates and velocity components of Pluto were obtained as a numerical integration fit to source positions. The source positions and the corrections applied to them are the same as the ones used for Jupiter (see Section II-F).

Plots of the corrections are given in Fig. 82 and 83. The statistical summary and the epoch values of the fitted integration are listed in Table 7. Plots of the residuals between the corrected source data and the fitted data are given in Fig. 84 to 89.

The corrected source position tape is E9009; the fitted position-velocity tape is E9109.

III. THE LUNAR EPHemeris AND nUTrations

The rectangular coordinates of the Moon were generated by a set of IBM 7094 programs developed by N. Block (Ref. 15). The major computation was the evaluation of the Brown Improved Lunar Theory (Ref. 16), which yielded ecliptic latitude and longitude, semi-diameter, parallax, and the nutations in longitude and obliquity. These quantities are referred to the mean ecliptic of date and are corrected for aberration.

A second program was used to convert the data to rectangular coordinates, to rotate to the mean equator of 1950.0, and to remove the aberration correction.

The evaluation of the Brown Improved Lunar Theory was checked by detailed hand computations at selected epochs, and by comparison with the previous evaluation of the theory as described in Ref. 16 over the interval...
1960–1974. Plots of the comparison are given in Fig. 90 to 98.

While the theory has been evaluated at half-day steps over the interval 1850–2000, only the last 50 years are recorded on the JPL Ephemeris Tapes.

Uncertainty in the latitude of the Moon is of the order of a few hundredths of a second of arc. The same is true of the longitude; however, systematic errors in the longitude of the Moon would be assimilated into $\Delta t_{\text{U}}$, which is the difference between Ephemeris Time and Universal Time. Radar measurements of the Moon indicate that the uncertainty in the radius vector is of the order of one kilometer, if one uses the value for the “Earth radius” suggested in Part I.

The lunar velocity components and nutation rates were obtained by numerical differentiation. The source rectangular position tapes are E9000 and E9010.

**NOMENCLATURE**

- $a$  
  semimajor axis, osculating at epoch, astronomical units
- $B$  
  ecliptic latitude, radians
- $dB$  
  residual in ecliptic latitude, radians
- $dL$  
  residual in ecliptic longitude, radians
- $dx, dy, dz$  
  residuals in rectangular equatorial coordinates, astronomical units
- $d\rho$  
  residual in radius vector, astronomical units
- $e$  
  eccentricity, osculating at epoch
- $I$  
  inclination to the ecliptic, osculating at epoch, radians
- $M_0$  
  mean anomaly at epoch, radians
- $n$  
  mean motion, osculating at epoch, radians/day
- $P$  
  unit vector directed toward perihelion, osculating at epoch
- $Q = R \times P$  
- $R$  
  unit vector normal to the orbital plane, osculating at epoch
- $x, y, z$  
  rectangular equatorial coordinates, astronomical units
- $\dot{x}, \dot{y}, \dot{z}$  
  rectangular equatorial velocity components, astronomical units/day
- $\Omega$  
  ecliptic longitude of the ascending node, osculating at epoch, radians
- $\varpi$  
  ecliptic longitude of perihelion, osculating at epoch, radians
REFERENCES


REFERENCES (Cont’d)


The same format is used for all JPL Ephemeris Tapes: two information records at the beginning of each tape followed by the data records. The record format may be described as follows:

A. The first record of each tape contains 24 BCD words written in binary. These 24 words serve to describe the general nature of the information on the tape.

B. The second record of each tape contains the following information in the order listed:
   1. Number of bodies on tapes = 10.
   2. A floating point 50., which denotes a buffered tape.
   3. Initial Julian date for which data are provided.
   4. Final Julian date for which data are provided.
   5. Step size of the logical data record = 8.0 days.
   6-25. Ten pairs of numbers. The first number of the pair denotes the body in increasing order out from the Sun, with a zero used for lunar data. The second number of each pair is the step size of data provided for that body.

C. The JPL Ephemeris Tapes contain data in buffered and overlapped 8-day logical records. The end points of the 8-day span are repeated as the first points of the succeeding 8-day record. This format allows ease of handling by the interpolation program.

The format for the JPL Ephemeris Tape records is listed in Table A-1. All data, with the exception of those for nutations, are double precision, so that the total record size is 1863 words. The step size for lunar data and nutations is one-half day. Mercury data are given in 2-day steps, and all other data, in 4-day steps.

The Julian date is the epoch (Ephemeris Time, ET) of the start of the data record. Lunar positions and velocities are referred to the geocentric equatorial rectangular reference frame of the mean equator and equinox of 1950.0 = JD 243 3282.423. They are expressed in units called “Earth radii” and “Earth radii”/mean solar day. Planetary positions and velocities are referred to the heliocentric equatorial rectangular frames of 1950.0, and are expressed in units of AU and AU/mean solar day.

The conversion of position and velocity tabulations to laboratory units, such as kilometers and kilometers per second, requires scaling by the conversion factors kilometers/AU and kilometers/“Earth radius.” Conversion of
planetary data from a heliocentric to a geocentric frame of reference requires specification of the Earth–Moon mass ratio \( \mu \), in order to locate the Earth–Moon barycenter in the geocentric frame. Finally, if data are required for a particular epoch in Universal Time (UT), the time correction \( \Delta t_0 = E_T - U_T \) must be specified.

In particular, the Brown Improved Lunar Theory is based upon values of ratios of solar to lunar parallax and of the Earth–Moon mass that are no longer considered the best estimates of these quantities. Thus, use of a best estimate of the actual value of the “Earth radius” in kilometers will not yield best estimates of the position of the Moon in kilometers, and an artificial value of the “Earth radius” is preferred for the scaling referred to above. Derivation of the value of the “Earth radius” to be used is given in Ref. 15 and Ref. 17. A consistent set of constants, and the set used at JPL, is given in Ref. 1. The ones pertinent to usage of JPL Ephemeris Tapes are

\[
1 \text{ AU} = 149599000 \text{ km}
\]
\[
1 \text{ “Earth radius”} = 6378.3255 \text{ km}
\]

and

\[
\mu = \text{Earth/Moon mass ratio} = 81.3015
\]

The listed Earth/Moon mass ratio corresponds to

\[
G_M = 398603.2 \text{ km}^3/\text{sec}^2
\]

and

\[
G_m = 4902.7779 \text{ km}^3/\text{sec}^2
\]
Table 1. The fitted Venus ephemeris

a. Statistical summary

<table>
<thead>
<tr>
<th>Residuals in position</th>
<th>Mean residual</th>
<th>Mean square residual</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_B$, rad</td>
<td>0.263 × 10^{-8}</td>
<td>0.169 × 10^{-13}</td>
<td>0.130 × 10^{-8}</td>
</tr>
<tr>
<td>$\cos B , d_l$, rad</td>
<td>0.228 × 10^{-10}</td>
<td>0.139 × 10^{-13}</td>
<td>0.373 × 10^{-4}</td>
</tr>
<tr>
<td>$d_P$, AU</td>
<td>0.125 × 10^{-4}</td>
<td>0.901 × 10^{-11}</td>
<td>0.949 × 10^{-7}</td>
</tr>
<tr>
<td>$d_x$, AU</td>
<td>0.134 × 10^{-7}</td>
<td>0.447 × 10^{-13}</td>
<td>0.211 × 10^{-8}</td>
</tr>
<tr>
<td>$d_y$, AU</td>
<td>0.117 × 10^{-7}</td>
<td>0.335 × 10^{-13}</td>
<td>0.183 × 10^{-8}</td>
</tr>
<tr>
<td>$d_z$, AU</td>
<td>0.464 × 10^{-8}</td>
<td>0.123 × 10^{-11}</td>
<td>0.111 × 10^{-8}</td>
</tr>
</tbody>
</table>

b. Epoch values at JD 243 3280.5

$x = 0.14297 \ 05927 \ 52417 \ 36 \ \text{AU}$

$y = 0.64700 \ 40914 \ 66046 \ 70 \ \text{AU}$

$z = 0.28248 \ 15304 \ 99832 \ 21 \ \text{AU}$

$\dot{x} = -1.98937 \ 92042 \ 18857 \ 9 \times 10^{-3} \ \text{AU/day}$

$\dot{y} = 0.31132 \ 35168 \ 83586 \ 04 \times 10^{-5} \ \text{AU/day}$

$\dot{z} = 0.26594 \ 96678 \ 99819 \ 59 \times 10^{-7} \ \text{AU/day}$

$P = -0.65498 \ 69613 \ 30167 \ 09$

$Q = -0.75344 \ 90074 \ 96176 \ 83$

$R = 0.05750 \ 36833 \ 43745 \ 698$

$\alpha = 0.72333 \ 51942 \ 17628 \ 65 \ \text{AU}$

$\epsilon = 0.00681 \ 09381 \ 55057 \ 9789$

$\Omega = 1.33051 \ 93188 \ 69337 \ 6 \ \text{rad}$

$M_e = 5.37820 \ 46362 \ 98368 \ 9 \ \text{rad}$

$n = 0.02796 \ 22799 \ 28307 \ 715 \ \text{rad/day}$

$l = 0.05923 \ 91465 \ 87757 \ 471 \ \text{rad}$

$\tilde{\omega} = 0.95628 \ 60526 \ 23776 \ 47 \ \text{rad}$

$0.75409 \ 02436 \ 89187 \ 48$

$-0.65661 \ 96803 \ 10495 \ 50$

$-0.01408 \ 89958 \ 29702 \ 161$

$0.75409 \ 02436 \ 89187 \ 48$

$-0.65661 \ 96803 \ 10495 \ 50$

$-0.01408 \ 89958 \ 29702 \ 161$

$0.04837 \ 33900 \ 98475 \ 910$

$0.03413 \ 48580 \ 18875 \ 405$

$0.99824 \ 58748 \ 26325 \ 49$
Table 2. The fitted Earth–Moon barycenter ephemeris

a. Statistical summary

<table>
<thead>
<tr>
<th>Residuals in position</th>
<th>Mean residual</th>
<th>Mean square residual</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>dB, rad</td>
<td>0.989 \times 10^{-4}</td>
<td>0.739 \times 10^{-14}</td>
<td>0.854 \times 10^{-4}</td>
</tr>
<tr>
<td>cos8 dL, rad</td>
<td>0.149 \times 10^{-4}</td>
<td>0.149 \times 10^{-14}</td>
<td>0.386 \times 10^{-4}</td>
</tr>
<tr>
<td>dp, AU</td>
<td>-0.807 \times 10^{-7}</td>
<td>0.516 \times 10^{-13}</td>
<td>0.212 \times 10^{-4}</td>
</tr>
<tr>
<td>dx, AU</td>
<td>0.187 \times 10^{-7}</td>
<td>0.125 \times 10^{-13}</td>
<td>0.333 \times 10^{-4}</td>
</tr>
<tr>
<td>dy, AU</td>
<td>-0.615 \times 10^{-7}</td>
<td>0.644 \times 10^{-13}</td>
<td>0.246 \times 10^{-4}</td>
</tr>
<tr>
<td>dz, AU</td>
<td>-0.159 \times 10^{-7}</td>
<td>0.186 \times 10^{-13}</td>
<td>0.136 \times 10^{-4}</td>
</tr>
</tbody>
</table>

b. Epoch values at JD 243 3280.5

\[
\begin{align*}
x &= -0.13636 \ 09306 \ 41671 \ 92 \ AU \\
y &= 0.89339 \ 77915 \ 04971 \ 95 \ AU \\
z &= 0.38745 \ 85503 \ 98339 \ 49 \ AU \\
\dot{x} &= -0.01732 \ 00148 \ 83331 \ 809 \ AU/day \\
\dot{y} &= -0.00224 \ 42645 \ 65737 \ 4915 \ AU/day \\
\dot{z} &= -0.00097 \ 33401 \ 91888 \ 5169 \ AU/day \\
\sigma &= 1.00000 \ 77903 \ 17566 \ 6 \ AU \\
\epsilon &= 0.01675 \ 00264 \ 14521 \ 240 \\
\Omega &= 5.88805 \ 39233 \ 92999 \ 1 \ rad \\
M_0 &= 6.21359 \ 22390 \ 38032 \ 3 \ rad \\
\dot{\omega} &= 0.01720 \ 19240 \ 48966 \ 068 \ rad/day \\
\dot{l} &= 3.09381 \ 72278 \ 60804 \ 1 \times 10^{-8} \ rad \\
\ddot{\omega} &= 2.17702 \ 51921 \ 56052 \ 2 \ rad \\
P &= -0.20953 \ 31439 \ 30791 \ 21 \\
Q &= -0.97780 \ 15450 \ 96155 \ 02 \\
R &= -0.00000 \ 11909 \ 01373 \ 78624 \ 63 \\
0.97780 \ 15450 \ 83785 \ 70 \\
-0.20953 \ 31439 \ 28268 \ 81 \\
-0.00000 \ 28554 \ 26230 \ 40813 \ 13 \\
0.00000 \ 25425 \ 06871 \ 03940 \ 72 \\
-0.00000 \ 17627 \ 71638 \ 65812 \ 27 \\
0.99999 \ 99999 \ 95104 \ 42 \\
\end{align*}
\]
### Table 3. The fitted Mars ephemeris

#### a. Statistical summary

<table>
<thead>
<tr>
<th>Residuals in position</th>
<th>Mean residual</th>
<th>Mean square residual</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( dB, \text{ rad} )</td>
<td>(-0.157 \times 10^{-4} )</td>
<td>(0.881 \times 10^{-14})</td>
<td>(0.293 \times 10^{-7})</td>
</tr>
<tr>
<td>( \cos B \ dL, \text{ rad} )</td>
<td>(0.212 \times 10^{-16})</td>
<td>(0.189 \times 10^{-13})</td>
<td>(0.137 \times 10^{-4})</td>
</tr>
<tr>
<td>( d_n, \text{ AU} )</td>
<td>(-0.147 \times 10^{-7})</td>
<td>(0.124 \times 10^{-13})</td>
<td>(0.110 \times 10^{-4})</td>
</tr>
<tr>
<td>( dx, \text{ AU} )</td>
<td>(0.106 \times 10^{-7})</td>
<td>(0.137 \times 10^{-13})</td>
<td>(0.116 \times 10^{-4})</td>
</tr>
<tr>
<td>( dy, \text{ AU} )</td>
<td>(0.161 \times 10^{-7})</td>
<td>(0.353 \times 10^{-13})</td>
<td>(0.187 \times 10^{-4})</td>
</tr>
<tr>
<td>( dz, \text{ AU} )</td>
<td>(0.448 \times 10^{-8})</td>
<td>(0.881 \times 10^{-14})</td>
<td>(0.937 \times 10^{-7})</td>
</tr>
</tbody>
</table>

#### b. Epoch values at JD 243 3280.5

\[
x = -1.36983 \ 15114 \ 19633 \ 8 \text{ AU} \\
y = 0.84313 \ 67641 \ 15894 \ 78 \text{ AU} \\
z = 0.42383 \ 59192 \ 28453 \ 10 \text{ AU} \\
x' = -0.00738 \ 45877 \ 85441 \ 0887 \text{ AU/day} \\
y' = -0.00947 \ 73458 \ 76822 \ 0827 \text{ AU/day} \\
z' = -0.00415 \ 16544 \ 70878 \ 2360 \text{ AU/day} \\
\sigma = 1.52374 \ 94504 \ 78361 \ 1 \text{ AU} \\
\epsilon = 0.09326 \ 08172 \ 85580 \ 042 \\
\Omega = 0.85818 \ 45355 \ 14872 \ 56 \\
\omega = 2.93885 \ 03070 \ 84425 \ 4 \text{ rad} \\
\dot{n} = 0.00914 \ 55782 \ 23467 \ 6807 \text{ rad/day} \\
\dot{\epsilon} = 0.03228 \ 82225 \ 77917 \ 681 \text{ rad} \\
\ddot{\omega} = 4.99151 \ 08908 \ 68736 \ 9 \text{ rad} \\
P = 0.90712 \ 62577 \ 59614 \ 19 \\
Q = 0.42014 \ 90958 \ 64392 \ 70 \\
R = 0.02442 \ 68595 \ 85680 \ 632 \\
-0.41971 \ 28478 \ 57047 \ 13 \\
0.90741 \ 14997 \ 13459 \ 86 \\
-0.21106 \ 76690 \ 68972 \ 23 \\
-0.03103 \ 32023 \ 22564 \ 610 \\
0.00889 \ 42358 \ 92012 \ 5475 \\
0.99947 \ 87806 \ 07032 \ 71
Table 4. The fitted Jupiter ephemeris

a. Statistical summary

<table>
<thead>
<tr>
<th>Residuals in position</th>
<th>Mean residual</th>
<th>Mean square residual</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>dB, rad</td>
<td>-0.225 \times 10^{-9}</td>
<td>0.170 \times 10^{-18}</td>
<td>0.346 \times 10^{-9}</td>
</tr>
<tr>
<td>cos8 dl, rad</td>
<td>-0.151 \times 10^{-9}</td>
<td>0.272 \times 10^{-18}</td>
<td>0.521 \times 10^{-8}</td>
</tr>
<tr>
<td>dp, AU</td>
<td>-0.124 \times 10^{-1}</td>
<td>0.881 \times 10^{-18}</td>
<td>0.269 \times 10^{-7}</td>
</tr>
<tr>
<td>dx, AU</td>
<td>-0.122 \times 10^{-7}</td>
<td>0.626 \times 10^{-18}</td>
<td>0.218 \times 10^{-7}</td>
</tr>
<tr>
<td>dy, AU</td>
<td>-0.182 \times 10^{-8}</td>
<td>0.849 \times 10^{-18}</td>
<td>0.291 \times 10^{-7}</td>
</tr>
<tr>
<td>dz, AU</td>
<td>-0.175 \times 10^{-8}</td>
<td>0.149 \times 10^{-18}</td>
<td>0.121 \times 10^{-7}</td>
</tr>
</tbody>
</table>

b. Epoch values at JD 2433280.5

\[
x = 3.34935 \, 66566 \, 82054 \, 3 \, \text{AU}
\]
\[
y = -3.47376 \, 60083 \, 43569 \, 3 \, \text{AU}
\]
\[
z = -1.57216 \, 12094 \, 85446 \, 9 \, \text{AU}
\]
\[
\dot{x} = 0.00558 \, 56549 \, 54254 \, 2162 \, \text{AU/day}
\]
\[
\dot{y} = 0.00496 \, 22486 \, 67480 \, 5555 \, \text{AU/day}
\]
\[
\dot{z} = 0.00199 \, 22660 \, 81200 \, 1201 \, \text{AU/day}
\]
\[
\begin{align*}
\alpha &= 5.20265 \, 54230 \, 23991 \, 9 \, \text{AU} \\
e &= 0.04891 \, 07594 \, 65016 \, 249 \\
\Omega &= 1.74467 \, 38009 \, 37960 \, 7 \, \text{rad} \\
M_0 &= 5.27964 \, 68743 \, 47353 \, 2 \, \text{rad} \\
n &= 0.00145 \, 02773 \, 92580 \, 9626 \, \text{rad/day} \\
f &= 0.02281 \, 00753 \, 14931 \, 982 \, \text{rad} \\
\omega &= 4.77738 \, 51017 \, 57879 \, 2 \, \text{rad}
\end{align*}
\]
\[
P = 0.97154 \, 94330 \, 36208 \, 41 \quad 0.23656 \, 34462 \, 91335 \, 31 \\
Q = -0.23659 \, 17146 \, 06593 \, 56 \quad 0.97160 \, 80310 \, 75493 \, 65 \\
R = 0.02246 \, 41822 \, 49845 \, 630 \quad 0.00394 \, 58612 \, 18236 \, 5349 \\
\]
Table 5. The fitted Saturn ephemeris

a. Statistical summary

<table>
<thead>
<tr>
<th>Residuals in position</th>
<th>Mean residual</th>
<th>Mean square residual</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( d_B ), rad</td>
<td>-0.172 \times 10^{-9}</td>
<td>0.757 \times 10^{-19}</td>
<td>0.214 \times 10^{-8}</td>
</tr>
<tr>
<td>( \cos B ) ( d_L ), rad</td>
<td>-0.823 \times 10^{-10}</td>
<td>0.330 \times 10^{-18}</td>
<td>0.574 \times 10^{-8}</td>
</tr>
<tr>
<td>( d_P ), AU</td>
<td>-0.143 \times 10^{-4}</td>
<td>0.136 \times 10^{-14}</td>
<td>0.368 \times 10^{-7}</td>
</tr>
<tr>
<td>( d_x ), AU</td>
<td>-0.127 \times 10^{-1}</td>
<td>0.195 \times 10^{-14}</td>
<td>0.423 \times 10^{-7}</td>
</tr>
<tr>
<td>( d_y ), AU</td>
<td>-0.717 \times 10^{-4}</td>
<td>0.201 \times 10^{-14}</td>
<td>0.442 \times 10^{-7}</td>
</tr>
<tr>
<td>( d_z ), AU</td>
<td>-0.423 \times 10^{-4}</td>
<td>0.370 \times 10^{-14}</td>
<td>0.188 \times 10^{-7}</td>
</tr>
</tbody>
</table>

b. Epoch values at JD 243 3280.5

\[
\begin{align*}
\alpha &= 9.52268 \ 86471 \ 63906 \ 1 \text{ AU} \\
e &= 0.05349 \ 26945 \ 47401 \ 222 \\
\Omega &= 1.97652 \ 99822 \ 74839 \ 72 \text{ rad} \\
M_0 &= 1.18009 \ 55514 \ 40889 \ 1 \text{ rad} \\
\dot{x} &= -0.00185 \ 82735 \ 03439 \ 4825 \text{ AU/day} \\
\dot{y} &= -0.00498 \ 38513 \ 60975 \ 9502 \text{ AU/day} \\
\dot{z} &= -0.00198 \ 02491 \ 18772 \ 8693 \text{ AU/day} \\
\pi &= 5.88309 \ 62072 \ 67269 \ 2 \text{ rad} \\
P &= -0.00598 \ 21733 \ 60788 \ 5730 \quad 0.99983 \ 89593 \ 39908 \ 74 \\
Q &= -0.99918 \ 52803 \ 57757 \ 30 \quad -0.00530 \ 12316 \ 94687 \ 0475 \\
R &= 0.03991 \ 22677 \ 66651 \ 736 \quad 0.01714 \ 50369 \ 41066 \ 200 \\
\end{align*}
\]
Table 6. The fitted Uranus ephemeris

a. Statistical summary

<table>
<thead>
<tr>
<th>Residuals in position</th>
<th>Mean residual</th>
<th>Mean square residual</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d B$, rad</td>
<td>$-0.66 \times 10^{-2}$</td>
<td>$0.118 \times 10^{-18}$</td>
<td>$0.665 \times 10^{-19}$</td>
</tr>
<tr>
<td>$\cos \delta dL$, rad</td>
<td>$0.326 \times 10^{-8}$</td>
<td>$0.903 \times 10^{-18}$</td>
<td>$0.893 \times 10^{-19}$</td>
</tr>
<tr>
<td>$d p$, AU</td>
<td>$-0.469 \times 10^{-6}$</td>
<td>$0.324 \times 10^{-19}$</td>
<td>$0.180 \times 10^{-19}$</td>
</tr>
<tr>
<td>$d x$, AU</td>
<td>$0.623 \times 10^{-8}$</td>
<td>$0.333 \times 10^{-19}$</td>
<td>$0.159 \times 10^{-19}$</td>
</tr>
<tr>
<td>$d y$, AU</td>
<td>$-0.313 \times 10^{-4}$</td>
<td>$0.262 \times 10^{-18}$</td>
<td>$0.689 \times 10^{-19}$</td>
</tr>
<tr>
<td>$d z$, AU</td>
<td>$-0.280 \times 10^{-4}$</td>
<td>$0.553 \times 10^{-18}$</td>
<td>$0.689 \times 10^{-19}$</td>
</tr>
</tbody>
</table>

b. Epoch values at JD 243 3280.5

\[
\begin{align*}
x &= -1.00291 \ 32748 \ 89426 \ 7 \ AU \\
y &= 17.32349 \ 19228 \ 51997 \ AU \\
z &= 7.60483 \ 60538 \ 20472 \ 2 \ AU \\
\dot{x} &= -0.00395 \ 52571 \ 36522 \ 0751 \ AU/day \\
\dot{y} &= -0.00037 \ 59007 \ 25259 \ 65507 \ AU/day \\
\dot{z} &= -0.00010 \ 88436 \ 97028 \ 67699 \ AU/day \\
a &= 19.16371 \ 87081 \ 38772 \ AU \\
e &= 0.04620 \ 99622 \ 58820 \ 451 \\
\Omega &= 1.28736 \ 33344 \ 44398 \ 4 \ rad \\
M_0 &= 5.00583 \ 15710 \ 84931 \ 3 \ rad \\
\dot{a} &= 0.00020 \ 50554 \ 21083 \ 97609 \ rad/day \\
\dot{e} &= 0.01347 \ 38497 \ 21806 \ 080 \ rad \\
\dot{\Omega} &= 1.70362 \ 64784 \ 99619 \ 3 \ rad \\
P &= -0.98859 \ 44291 \ 41681 \ 44 \\
Q &= -0.15004 \ 57195 \ 82359 \ 18 \\
R &= 0.01293 \ 56658 \ 52154 \ 074 \\
0.15000 \ 90167 \ 60570 \ 34 & 0.15335 \ 47350 \ 14638 \ 363 \\
-0.98867 \ 74488 \ 55836 \ 86 & -0.00178 \ 44211 \ 74098 \ 6327 \\
-0.00376 \ 78926 \ 57973 \ 9758 & 0.99999 \ 92290 \ 20571 \ 83 \\
\end{align*}
\]
Table 7. The fitted Pluto ephemeris

a. Statistical summary

<table>
<thead>
<tr>
<th>Residuals in position</th>
<th>Mean residual</th>
<th>Mean square residual</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d\beta$, rad</td>
<td>$0.121 \times 10^{-6}$</td>
<td>$0.480 \times 10^{-13}$</td>
<td>$0.692 \times 10^{-4}$</td>
</tr>
<tr>
<td>$\cos \beta , d\lambda$, rad</td>
<td>$-0.344 \times 10^{-6}$</td>
<td>$0.480 \times 10^{-13}$</td>
<td>$0.429 \times 10^{-4}$</td>
</tr>
<tr>
<td>$d\rho$, AU</td>
<td>$-0.119 \times 10^{-8}$</td>
<td>$0.157 \times 10^{-12}$</td>
<td>$0.390 \times 10^{-4}$</td>
</tr>
<tr>
<td>$dx$, AU</td>
<td>$0.890 \times 10^{-8}$</td>
<td>$0.881 \times 10^{-12}$</td>
<td>$0.297 \times 10^{-4}$</td>
</tr>
<tr>
<td>$dy$, AU</td>
<td>$0.390 \times 10^{-8}$</td>
<td>$0.638 \times 10^{-12}$</td>
<td>$0.697 \times 10^{-4}$</td>
</tr>
<tr>
<td>$dz$, AU</td>
<td>$-0.134 \times 10^{-8}$</td>
<td>$0.763 \times 10^{-12}$</td>
<td>$0.280 \times 10^{-4}$</td>
</tr>
</tbody>
</table>

b. Epoch values at JD 243 3280.5

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$ = $-26.23175$ &amp; 78230 &amp; 46232 &amp; AU</td>
<td>$a$ = $39.37364$ &amp; 76522 &amp; 73551 &amp; AU</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$y$ = $20.56153$ &amp; 05924 &amp; 86850 &amp; AU</td>
<td>$e$ = $0.24880$ &amp; 34057 &amp; 56600 &amp; 34</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$z$ = $14.44369$ &amp; 10491 &amp; 54213 &amp; AU</td>
<td>$\Omega$ = $1.91433$ &amp; 71561 &amp; 99444 &amp; 1 rad</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\dot{x}$ = $-0.00131$ &amp; 57216 &amp; 36860 &amp; 2943 &amp; AU/day</td>
<td>$M_0$ = $5.26530$ &amp; 24509 &amp; 38841 &amp; 5 rad</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\dot{y}$ = $-0.00261$ &amp; 98367 &amp; 63761 &amp; 8888 &amp; AU/day</td>
<td>$n$ = $0.00006$ &amp; 96263 &amp; 40474 &amp; 02263 &amp; 5 rad/day</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\dot{z}$ = $-0.00042$ &amp; 70513 &amp; 54029 &amp; 74011 &amp; AU/day</td>
<td>$I$ = $0.29967$ &amp; 06990 &amp; 98890 &amp; 99 rad</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$P$ = $0.68084$ &amp; 18086 &amp; 73955 &amp; 33</td>
<td>$\dot{\omega}$ = $1.99558$ &amp; 15747 &amp; 74785 &amp; 5 rad</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q$ = $0.67763$ &amp; 91709 &amp; 56164 &amp; 49</td>
<td>$-0.68125$ &amp; 59301 &amp; 78684 &amp; 30</td>
<td>$0.26896$ &amp; 98667 &amp; 64513 &amp; 34</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R$ = $0.27795$ &amp; 60857 &amp; 09170 &amp; 80</td>
<td>$-0.72526$ &amp; 10553 &amp; 63070 &amp; 01</td>
<td>$-0.12166$ &amp; 16437 &amp; 48094 &amp; 04</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$-0.99943$ &amp; 21839 &amp; 50796 &amp; 088</td>
<td>$0.95543$ &amp; 37524 &amp; 29428 &amp; 41</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
RESID S PXYZ VS. TIME IN DAYS PAST 2433282.5 GIL OROZCO
THEMIS-BLOCK SOURCE FOR THE PLANET VENUS X=1E7, Y=1E7, Z=1E7

Fig. 1. Venus rectangular position residuals (Newcomb minus Block) for 1950–1960
RESIDS PXYZ VS. TIME IN DAYS PAST 2433282.5 GIL DROZCO THEMIS-BLOCK SOURCE FOR THE PLANET VENUS X\times10^7, Y\times10^7, Z\times10^7

Fig. 2. Venus rectangular position residuals (Newcomb minus Block) for 1960–1971
Fig. 3. Venus rectangular position residuals (Newcomb minus Block) for 1971–1982
RESIDS PXYZ VS. TIME IN DAYS PAST 2433282.5 GIL OROZCO
THEMIS-BLOCK SOURCE FOR THE PLANET VENUS X*1E7, Y*1E7, Z*1E7

Fig. 4. Venus rectangular position residuals (Newcomb minus Block) for 1982–1993
Fig. 5. Venus rectangular position residuals (Newcomb minus Block) for 1993–2000
Fig. 6. Venus rectangular position residuals (Block with Duncombe corrections minus fitted) for 1950–1960
Fig. 7. Venus rectangular position residuals (Block with Duncombe corrections minus fitted) for 1960–1970
Fig. 8. Venus rectangular position residuals (Block with Duncombe corrections minus fitted) for 1970–1980
Fig. 9. Venus rectangular position residuals (Block with Duncombe corrections minus fitted) for 1980–1990
RESIDUALS (SOURCE - PLOD2)
VENUS (10 YEARS)

Fig. 10. Venus rectangular position residuals (Block with Duncombe corrections minus fitted) for 1990–2000
Fig. 11. Venus polar position residuals (Block with Duncombe corrections minus fitted) for 1950–1960
Fig. 12. Venus polar position residuals (Block with Duncombe corrections minus fitted) for 1960–1970
Fig. 13. Venus polar position residuals (Block with Duncombe corrections minus fitted) for 1970–1980

RESIDUALS (SOURCE - PL002)
VENUS (10 YEARS)
Fig. 14. Venus polar position residuals (Block with Duncombe corrections minus fitted) for 1980–1990
Fig. 15. Venus polar position residuals (Block with Duncombe corrections minus fitted) for 1990–2000
Fig. 16. Venus rectangular velocity residuals (Block with Duncombe corrections minus fitted) for 1950–1960.
RESID VS TIME IN DAYS PAST 2433282.5 GIL OROZCO
SOURCE-PLOD RESIDUALS FOR THE PLANET VENUS

Fig. 17. Venus rectangular velocity residuals (Block with Duncombe corrections minus fitted) for 1960–1971
RESIDS VXYZ VS. TIME IN DAYS PAST 2433282.5 GIL OROZCO
SOURCE - PLOD RESIDUALS FOR THE PLANET VENUS X*1E7, Y*1E7, Z*1E8

Fig. 18. Venus rectangular velocity residuals (Block with Duncombe corrections minus fitted) for 1971–1982
RESIDS VXYZ VS. TIME IN DAYS PAST 2433282.5 GIL DROZCO
SOURCE - PLOD RESIDUALS FOR THE PLANET VENUS X*1E7, Y*1E7, Z*1E8

Fig. 19. Venus rectangular velocity residuals (Block with Duncombe corrections minus fitted) for 1982–1993
Fig. 20. Venus rectangular velocity residuals (Block with Duncombe corrections minus fitted) for 1993–2000
RESIDS PXYZ VS. TIME IN DAYS PAST 2433282.5 GIL OROZCO
THEMIS-BLOCK SOURCE FOR THE EARTH-MOON BARYCENTER X*1E6, Y*1E6, Z*1E6

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RESIDS PXYZ VS. TIME IN DAYS PAST 2433282.5 G1L DROZCO
THEMIS-BLOCK SOURCE FOR THE EARTH-MOON BARYCENTER X×1E6, Y×1E6, Z×1E6

Fig. 22. Earth–Moon barycenter rectangular position residuals (Newcomb minus Block with Herget corrections) for 1960–1971
Fig. 23. Earth–Moon barycenter rectangular position residuals (Newcomb minus Block with Herget corrections) for 1971–1982
RESIDS PXYZ VS. TIME IN DAYS PAST 2433282.5 GIL OROZCO

THEMIS-BLOCK SOURCE FOR THE EARTH-MOON BARYCENTER X*1E6, Y*1E6, Z*1E6

Fig. 24. Earth–Moon barycenter rectangular position residuals (Newcomb minus Block with Herget corrections) for 1982–1993
RESIDS PXYZ VS. TIME IN DAYS PAST 2433282.5 GIL OROZCO
THEMIS-BLOCK SOURCE FOR THE EARTH-MOON BARYCENTER X*1E6, Y*1E6, Z*1E6

Fig. 25. Earth-Moon barycenter rectangular position residuals (Newcomb minus Block with Herget corrections) for 1993–2000
RESIDUALS (SOURCE - PL002)
ERTHMN (10 YEARS)

Fig. 26. Earth–Moon barycenter rectangular position residuals (Block with Duncombe corrections minus fitted) for 1950–1960
Fig. 27. Earth–Moon barycenter rectangular position residuals (Block with Duncombe corrections minus fitted) for 1960–1970
RESIDUALS (SOURCE - PLOD2)
ERTHMN (10 YEARS)

Fig. 28. Earth–Moon barycenter rectangular position residuals (Block with Duncombe corrections minus fitted) for 1970–1980
Fig. 29. Earth–Moon barycenter rectangular position residuals (Block with Duncombe corrections minus fitted) for 1980–1990
RESIDUALS (SOURCE - PL0D2)
ERTHMN (10 YEARS)

Fig. 30. Earth–Moon barycenter rectangular position residuals (Block with Duncombe corrections minus fitted) for 1990–2000
RESIDUALS \(\text{SOURCE} - \text{PL0D2}\)
\(\text{ERTHMIN} (10\ \text{YEARS})\)

Fig. 31. Earth–Moon barycenter polar position residuals (Block with Duncombe corrections minus fitted) for 1950–1960
Fig. 32. Earth–Moon barycenter polar position residuals (Block with Duncombe corrections minus fitted) for 1960–1970
Fig. 33. Earth–Moon barycenter polar position residuals (Block with Duncombe corrections minus fitted) for 1970–1980
Fig. 34. Earth–Moon barycenter polar position residuals (Block with Duncombe corrections minus fitted) for 1980–1990
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RESIDS VXYZ VS. TIME IN DAYS PAST 2433282.5 GIL DROZCO
SOURCE-PLOD RESIDUALS FOR THE EARTH-MOON BARYCENTER X*1E7, Y*1E7, Z*1E8

Fig. 36. Earth-Moon barycenter rectangular velocity residuals (Block with Duncombe corrections minus fitted) for 1950-1960
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RESIDS VXYZ VS. TIME IN DAYS PAST 2433282.5 GIL OROZCO
SOURCE-PLOD RESIDUALS FOR THE EARTH-MOON BARYCENTER X×1E7, Y×1E7, Z×1E8

Fig. 38. Earth–Moon barycenter rectangular velocity residuals (Block with Duncombe corrections minus fitted) for 1971–1982
RESIDS VXYZ VS. TIME IN DAYS PAST 2433282.5 GIL OROZCO
SOURCE-PLOD RESIDUALS FOR THE EARTH-MOON BARYCENTER X=1E7, Y=1E7, Z=1E8

Fig. 39. Earth–Moon barycenter rectangular velocity residuals (Block with Duncambe corrections minus fitted) for 1982–1993
RESIDS VXYZ VS. TIME IN DAYS PAST 2433282.5 GIL OROZCO
SOURCE - PLOD RESIDUALS FOR THE EARTH-MOON BARYCENTER X=1E7, Y=1E7, Z=1E8

Fig. 40. Earth—Moon barycenter rectangular velocity residuals (Block with Duncombe corrections minus fitted) for 1993–2000
Fig. 41. Mars rectangular position residuals (USNO minus fitted) for 1950–1960

RESIDUALS (SOURCE - PLOD2)
MARS (10 YEARS)

Fig. 41. Mars rectangular position residuals (USNO minus fitted) for 1950–1960
RESIDUALS (SOURCE - PLOD2)
MARS (10 YEARS)

Fig. 42. Mars rectangular position residuals (USNO minus fitted) for 1960–1970
RESIDUALS (SOURCE - PLOD2)
MARS (10 YEARS)

Fig. 43. Mars rectangular position residuals (USNO minus fitted) for 1970–1980
Fig. 44. Mars rectangular position residuals (USNO minus fitted) for 1980–1990
RESIDUALS (SOURCE - PL0D2)
MARS (10 YEARS)

Fig. 45. Mars rectangular position residuals (USNO minus fitted) for 1990–2000
Fig. 46. Mars polar position residuals (USNO minus fitted) for 1950–1960
Fig. 47. Mars polar position residuals (USNO minus fitted) for 1960–1970
Fig. 48. Mars polar position residuals (USNO minus fitted) for 1970–1980
RESIDUALS (SOURCE - PL0D2) MARS (10 YEARS)

Fig. 49. Mars polar position residuals (USNO minus fitted) for 1980–1990
RESIDUALS (SOURCE = PL0D2)
MARS (10 YEARS)

Fig. 50. Mars polar position residuals (USNO minus fitted) for 1990–2000
Fig. 51. Mars rectangular velocity residuals (USNO minus fitted) for 1950–1960
Fig. 52. Mars rectangular velocity residuals (USNO minus fitted) for 1960-1971
Fig. 53. Mars rectangular velocity residuals (USNO minus fitted) for 1971–1982
Fig. 54. Mars rectangular velocity residuals (USNO minus fitted) for 1982–1993
**RESIDS VXYZ VS. TIME IN DAYS PAST 2433282.5 GIL OROZCO**

**SOURCE-PLOD RESIDUALS FOR THE PLANET MARS X*1E7, Y*1E7, Z*1E7**

*Fig. 55. Mars rectangular velocity residuals (USNO minus fitted) for 1993–2000*
Fig. 56. Jupiter rectangular position corrections for the perturbations caused by the inner planets for 1950–1977.
Fig. 57. Jupiter rectangular position corrections for the perturbations caused by the inner planets for 1977–2000
Fig. 58. Jupiter rectangular position residuals (corrected SSEC minus fitted) for 1950–1975
Fig. 59. Jupiter rectangular position residuals (corrected SSEC minus fitted) for 1975–2000
Fig. 60. Jupiter polar position residuals (corrected SSEC minus fitted) for 1950–1975
Fig. 61. Jupiter polar position residuals (corrected SSEC minus fitted) for 1975–2000
Fig. 62. Jupiter rectangular velocity residuals (corrected SSEC minus fitted) for 1950–1977
Fig. 63. Jupiter rectangular velocity residuals (corrected SSEC minus fitted) for 1977–2000
Fig. 64. Saturn rectangular position corrections for the perturbations caused by the inner planets for 1950–1977
RESIDS PXYZ VS. TIME IN DAYS PAST 2433282.5 GIL OROZCO
SUN SOURCE-THEMIS FOR THE PLANET SATURN X*1E5, Y*1E5, Z*1E5

Fig. 65. Saturn rectangular position corrections for the perturbations caused by the inner planets for 1977–2000
RESIDUALS (SOURCE - PL0D2) SATURN (25 YEARS)

Fig. 66. Saturn rectangular position residuals (corrected SSEC minus fitted) for 1950–1975
RESIDUALS (SOURCE - PL0D2)
SATURN (25 YEARS)

Fig. 67. Saturn rectangular position residuals (corrected SSEC minus fitted) for 1975–2000
Fig. 68. Saturn polar position residuals (corrected SSEC minus fitted) for 1950–1975
Fig. 69. Saturn polar position residuals (corrected SSEC minus fitted) for 1975–2000
Fig. 70. Saturn rectangular velocity residuals (corrected SSEC minus fitted) for 1950–1977
RESIDS VXYZ VS. TIME IN DAYS PAST 2433282.5 GIL DROZCO
SOURCE-PLOD RESIDUALS FOR THE PLANET SATURN X×1E9, Y×1E9, Z×1E9

Fig. 71. Saturn rectangular velocity residuals (corrected SSEC minus fitted) for 1977–2000
RESIDS PXYZ VS. TIME IN DAYS PAST 2433282.5 GIL OROZCO
SUN SOURCE-THEMIS FOR THE PLANET URANUS X*1E5; Y*1E5; Z*1E5
Fig. 72. Uranus rectangular position corrections for the perturbations caused by the inner planets for 1950–1977
RESIDS PXYZ VS. TIME IN DAYS PAST 2433282.5 GIL OROZCO
SUN SOURCE-THEMIS FOR THE PLANET URANUS X×1E5, Y×1E5, Z×1E5

Fig. 73. Uranus rectangular position corrections for the perturbations caused by the inner planets for 1977–2000
RESIDUALS (SOURCE - PLOD2)
URANUS (25 YEARS)

Fig. 74. Uranus rectangular position residuals (corrected SSEC minus fitted) for 1950–1975
RESIDUALS (SOURCE - PL002)
URANUS (25 YEARS)

Fig. 75. Uranus rectangular position residuals (corrected SSEC minus fitted) for 1975–2000
Fig. 76. Uranus polar position residuals (corrected SSEC minus fitted) for 1950–1975
Fig. 77. Uranus polar position residuals (corrected SSEC minus fitted) for 1975–2000
RESIDS VXYZ VS. TIME IN DAYS PAST 2433282.5 GIL DROZCO
SOURCE - PLOD RESIDUALS FOR THE PLANET URANUS X*1E9, Y*1E9, Z*1E9

Fig. 78. Uranus rectangular velocity residuals (corrected SSEC minus fitted) for 1950–1975
RESIDS VXYZ VS. TIME IN DAYS PAST 2433282.5 GIL OROZCO
SOURCE - PLOD RESIDUALS FOR THE PLANET URANUS X*1E9, Y*1E9, Z*1E9

Fig. 79. Uranus rectangular velocity residuals (corrected SSEC minus fitted) for 1975–2000
Fig. 80. Neptune rectangular position corrections for the perturbations caused by the inner planets for 1950–1977
RESIDS PXYZ VS. TIME IN DAYS PAST 2433282.5 GIL OROZCO
SUN SOURCE THEMIS FOR THE PLANET NEPTUNE \times 10^5; Y \times 10^5; Z \times 10^5

Fig. 81. Neptune rectangular position corrections for the perturbations caused by the inner planets for 1977–2000.
Fig. 82. Pluto rectangular position corrections for the perturbations caused by the inner planets for 1950–1977
Fig. 83. Pluto rectangular position corrections for the perturbations caused by the inner planets for 1977–2000
RESIDUALS (SOURCE - PL0D2)  
PLUTO (25 YEARS) 

Fig. 84. Pluto rectangular position residuals (corrected SSEC minus fitted) for 1950–1975
Fig. 85. Pluto rectangular position residuals (corrected SSEC minus fitted) for 1975-2000.
Fig. 86. Pluto polar position residuals (corrected SSEC minus fitted) for 1950–1975
Fig. 87. Pluto polar position residuals (corrected SSEC minus fitted) for 1975–2000
RESID XSYZ VS. TIME IN DAYS PAST 2433282.5 GIL ORÓZCO
SOURCE-PLOD RESIDUALS FOR THE PLANET PLUTO X*1E9, Y*1E9, Z*1E9

Fig. 88. Pluto rectangular velocity residuals (corrected SSEC minus fitted) for 1950–1975
RESIDS VXYZ VS. TIME IN DAYS PAST 2433282.5 GIL OROZCO
SOURCE - PLOD RESIDUALS FOR THE PLANET PLUTO X*1E9, Y*1E9, Z*1E9

Fig. 89. Pluto rectangular velocity residuals (corrected SSEC minus fitted) for 1975–2000
Fig. 90. Lunar rectangular position residuals (JPL–STL minus Block) for 1950–1952
RESIDS PXYZ VS. TIME IN DAYS PAST 2433282.5 GIL OROZCO
(JPL-STL)-BLOCK EVALUATION LUNAR POSITION DATA X×1E5, Y×1E5, Z×1E5

Fig. 91. Lunar rectangular position residuals (JPL–STL minus Block) for 1952–1955
RESIDS X=, Y=, Z= VS. TIME IN DAYS PAST 2433282.5

Fig. 92. Lunar rectangular position residuals (JPL—STL minus Block) for 1955–1958
Figure 93. Lunar rectangular position residuals (JPL–STL minus Block) for 1958–1960.
Fig. 94. Lunar rectangular position residuals (JPL–STL minus Block) for 1960–1963
Fig. 95. Lunar rectangular position residuals (JPL-STL minus Block) for 1963–1966
Fig. 96. Lunar rectangular position residuals (JPL–STL minus Block) for 1966–1969
Fig. 97. Lunar rectangular position residuals (JPL–STL minus Block) for 1969–1971
RESIDS PXYZ VS. TIME IN DAYS PAST 2433282.5 GIL DROZCO
(JPL, STL) - BLOCK EVALUATION LUNAR POSITION DATA X*1E5, Y*1E5, Z*1E5

Fig. 98. Lunar rectangular position residuals (JPL-STL minus Block) for 1971-1974
Table 1. The fitted Venus ephemeris

a. Statistical summary

<table>
<thead>
<tr>
<th>Residuals in position</th>
<th>Mean residual</th>
<th>Mean square residual</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>dB, rad</td>
<td>$0.263 \times 10^{-8}$</td>
<td>$0.169 \times 10^{-33}$</td>
<td>$0.130 \times 10^{-4}$</td>
</tr>
<tr>
<td>cos8 dl, rad</td>
<td>$0.228 \times 10^{-18}$</td>
<td>$0.139 \times 10^{-32}$</td>
<td>$0.373 \times 10^{-8}$</td>
</tr>
<tr>
<td>dp, AU</td>
<td>$0.125 \times 10^{-5}$</td>
<td>$0.901 \times 10^{-54}$</td>
<td>$0.949 \times 10^{-7}$</td>
</tr>
<tr>
<td>dx, AU</td>
<td>$0.134 \times 10^{-7}$</td>
<td>$0.447 \times 10^{-33}$</td>
<td>$0.211 \times 10^{-6}$</td>
</tr>
<tr>
<td>dy, AU</td>
<td>$0.117 \times 10^{-7}$</td>
<td>$0.335 \times 10^{-32}$</td>
<td>$0.183 \times 10^{-6}$</td>
</tr>
<tr>
<td>dz, AU</td>
<td>$0.464 \times 10^{-5}$</td>
<td>$0.123 \times 10^{-33}$</td>
<td>$0.111 \times 10^{-6}$</td>
</tr>
</tbody>
</table>

b. Epoch values at JD 243 3280.5

\[
\begin{align*}
\hat{x} &= 0.14297 \ 05927 \ 52417 \ 36 \ AU \\
\hat{y} &= 0.64700 \ 40914 \ 66046 \ 70 \ AU \\
\hat{z} &= 0.28248 \ 15304 \ 99832 \ 21 \ AU \\
\dot{\hat{x}} &= -0.01989 \ 37920 \ 42168 \ 579 \ AU/day \\
\dot{\hat{y}} &= 0.00311 \ 32351 \ 68835 \ 8604 \ AU/day \\
\dot{\hat{z}} &= 0.00265 \ 94966 \ 78998 \ 1959 \ AU/day \\
\end{align*}
\]

\[
\begin{align*}
a &= 0.72333 \ 51942 \ 17628 \ 65 \ AU \\
e &= 0.00681 \ 09381 \ 55057 \ 9789 \\
\Omega &= 1.33051 \ 93188 \ 69337 \ 6 \ rad \\
M_0 &= 5.37820 \ 46362 \ 98368 \ 9 \ rad \\
\dot{\hat{n}} &= 0.02796 \ 22799 \ 28307 \ 715 \ rad/day \\
\dot{l} &= 0.05923 \ 91465 \ 87757 \ 471 \ rad \\
\dot{\omega} &= 0.95628 \ 60526 \ 23776 \ 47 \ rad \\
\end{align*}
\]

\[
\begin{align*}
P &= -0.65498 \ 69606 \ 71256 \ 06 \\
Q &= -0.75344 \ 90080 \ 63958 \ 81 \\
R &= 0.05750 \ 36833 \ 43476 \ 514 \\
\end{align*}
\]
### Table 2. The fitted Earth–Moon barycenter ephemeris

#### a. Statistical summary

<table>
<thead>
<tr>
<th>Residuals in position</th>
<th>Mean residual</th>
<th>Mean square residual</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>dB, rad</td>
<td>0.989 × 10⁻⁴</td>
<td>0.739 × 10⁻¹⁴</td>
<td>0.854 × 10⁻⁷</td>
</tr>
<tr>
<td>cos8 dl, rad</td>
<td>0.149 × 10⁻⁹</td>
<td>0.149 × 10⁻¹²</td>
<td>0.386 × 10⁻⁶</td>
</tr>
<tr>
<td>dp, AU</td>
<td>−0.807 × 10⁻¹</td>
<td>0.516 × 10⁻¹²</td>
<td>0.212 × 10⁻⁴</td>
</tr>
<tr>
<td>dx, AU</td>
<td>0.187 × 10⁻⁷</td>
<td>0.125 × 10⁻¹²</td>
<td>0.353 × 10⁻⁶</td>
</tr>
<tr>
<td>dy, AU</td>
<td>−0.615 × 10⁻¹</td>
<td>0.644 × 10⁻¹³</td>
<td>0.246 × 10⁻⁶</td>
</tr>
<tr>
<td>dz, AU</td>
<td>−0.139 × 10⁻⁷</td>
<td>0.186 × 10⁻¹³</td>
<td>0.136 × 10⁻⁶</td>
</tr>
</tbody>
</table>

#### b. Epoch values at JD 243 3280.5

\[
\begin{align*}
x &= -0.13636 \, 09306 \, 41671 \, 92 \text{ AU} \\
y &= 0.89339 \, 77915 \, 04971 \, 95 \text{ AU} \\
z &= 0.38745 \, 85503 \, 98339 \, 49 \text{ AU} \\
\dot{x} &= -0.01732 \, 00148 \, 83331 \, 809 \text{ AU/day} \\
\dot{y} &= -0.00224 \, 42645 \, 63737 \, 4915 \text{ AU/day} \\
\dot{z} &= -0.00097 \, 33401 \, 91888 \, 5169 \text{ AU/day} \\
\alpha &= 1.00000 \, 77903 \, 17566 \, 6 \text{ AU} \\
\epsilon &= 0.01675 \, 00264 \, 14521 \, 240 \\
\Omega &= 5.88805 \, 39223 \, 92999 \, 1 \text{ rad} \\
\lambda &= 6.21359 \, 22390 \, 38032 \, 3 \text{ rad} \\
\dot{\alpha} &= 0.01720 \, 19240 \, 48966 \, 068 \text{ rad/day} \\
\dot{\epsilon} &= 3.09381 \, 72278 \, 60804 \, 1 \times 10⁻⁶ \text{ rad} \\
\dot{\Omega} &= 2.17702 \, 51921 \, 56052 \, 2 \text{ rad} \\
\end{align*}
\]

\[
\begin{align*}
P &= -0.20953 \, 31440 \, 21208 \, 32 \\
Q &= -0.97730 \, 15450 \, 77102 \, 54 \\
R &= -0.00000 \, 11909 \, 01373 \, 79842 \, 77
\end{align*}
\]

\[
\begin{align*}
0.97780 \, 15450 \, 74522 \, 22 & \quad 0.00000 \, 25425 \, 06870 \, 87354 \, 76 \\
-0.20953 \, 31440 \, 17177 \, 54 & \quad -0.00000 \, 17627 \, 71638 \, 90804 \, 89 \\
-0.00000 \, 28354 \, 26230 \, 40112 \, 61 & \quad 0.99999 \, 99999 \, 95214 \, 16
\end{align*}
\]
Table 3. The fitted Mars ephemeris

a. Statistical summary

<table>
<thead>
<tr>
<th>Residuals in position</th>
<th>Mean residual</th>
<th>Mean square residual</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>dB, rad</td>
<td>$-0.157 \times 10^{-6}$</td>
<td>$0.881 \times 10^{-14}$</td>
<td>$0.293 \times 10^{-7}$</td>
</tr>
<tr>
<td>cosB dB, rad</td>
<td>$0.212 \times 10^{-9}$</td>
<td>$0.189 \times 10^{-13}$</td>
<td>$0.137 \times 10^{-9}$</td>
</tr>
<tr>
<td>dQ, AU</td>
<td>$-0.147 \times 10^{-1}$</td>
<td>$0.124 \times 10^{-13}$</td>
<td>$0.110 \times 10^{-8}$</td>
</tr>
<tr>
<td>dX, AU</td>
<td>$0.106 \times 10^{-7}$</td>
<td>$0.137 \times 10^{-14}$</td>
<td>$0.116 \times 10^{-8}$</td>
</tr>
<tr>
<td>dY, AU</td>
<td>$0.161 \times 10^{-7}$</td>
<td>$0.353 \times 10^{-13}$</td>
<td>$0.187 \times 10^{-8}$</td>
</tr>
<tr>
<td>dZ, AU</td>
<td>$0.448 \times 10^{-9}$</td>
<td>$0.881 \times 10^{-14}$</td>
<td>$0.937 \times 10^{-7}$</td>
</tr>
</tbody>
</table>

b. Epoch values at JD 243 3280.5

\[
\begin{align*}
    x &= -1.36983 15114 19633 8 \text{ AU} \\
    y &= 0.84313 67641 15894 78 \text{ AU} \\
    z &= 0.42383 59192 28453 10 \text{ AU} \\
    \dot{x} &= -0.00738 45877 85441 0887 \text{ AU/day} \\
    \dot{y} &= -0.00947 73458 76822 0827 \text{ AU/day} \\
    \dot{z} &= -0.00415 16544 70878 2360 \text{ AU/day} \\
    a &= 1.52374 94504 78361 1 \text{ AU} \\
    e &= 0.09326 08172 83380 042 \\
    \Omega &= 0.85918 45355 14972 56 \text{ rad} \\
    \omega &= 2.93885 03070 84425 4 \text{ rad} \\
    \Omega &= 0.00914 55782 23467 6807 \text{ rad/day} \\
    I &= 0.03228 82225 77917 681 \text{ rad} \\
    \omega &= 4.99151 08908 68736 9 \text{ rad/day} \\
    P &= 0.90712 62673 16662 79 \\
    Q &= 0.42014 90969 58384 59 \\
    R &= 0.02442 68595 85142 808 \\
    \dot{P} &= -0.41971 28488 58721 62 \\
    \dot{Q} &= 0.90741 14992 07287 31 \\
    \dot{R} &= -0.02110 67669 07296 699 \\
    \omega &= -0.03103 32023 32892 158 \\
    \Omega &= 0.00889 42358 54586 6587 \\
    \omega &= 0.99947 87806 25946 47
\end{align*}
\]
Table 4. The fitted Jupiter ephemeris

**a. Statistical summary**

<table>
<thead>
<tr>
<th>Residuals in position</th>
<th>Mean residual</th>
<th>Mean square residual</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>dB, rad</td>
<td>(-0.225 \times 10^{-6})</td>
<td>(0.170 \times 10^{-15})</td>
<td>(0.346 \times 10^{-9})</td>
</tr>
<tr>
<td>(\cos \theta \Delta L), rad</td>
<td>(-0.151 \times 10^{-6})</td>
<td>(0.272 \times 10^{-16})</td>
<td>(0.321 \times 10^{-8})</td>
</tr>
<tr>
<td>(d\rho), AU</td>
<td>(-0.124 \times 10^{-7})</td>
<td>(0.881 \times 10^{-16})</td>
<td>(0.269 \times 10^{-7})</td>
</tr>
<tr>
<td>(dx), AU</td>
<td>(-0.122 \times 10^{-7})</td>
<td>(0.626 \times 10^{-15})</td>
<td>(0.218 \times 10^{-7})</td>
</tr>
<tr>
<td>(dy), AU</td>
<td>(-0.182 \times 10^{-8})</td>
<td>(0.849 \times 10^{-15})</td>
<td>(0.291 \times 10^{-7})</td>
</tr>
<tr>
<td>(dz), AU</td>
<td>(-0.175 \times 10^{-8})</td>
<td>(0.149 \times 10^{-13})</td>
<td>(0.121 \times 10^{-7})</td>
</tr>
</tbody>
</table>

**b. Epoch values at JD 243 3280.5**

\[
\begin{align*}
\text{x} &= 3.34935 \ 66566 \ 82054 \ 3 \ \text{AU} & \sigma &= 5.20265 \ 54230 \ 23991 \ 9 \ \text{AU} \\
\text{y} &= -3.47376 \ 60083 \ 43569 \ 3 \ \text{AU} & \epsilon &= 0.04891 \ 07594 \ 65016 \ 249 \\
\text{z} &= -1.57216 \ 12094 \ 85446 \ 9 \ \text{AU} & \Omega &= 1.74467 \ 38009 \ 37960 \ 7 \ \text{rad} \\
\dot{x} &= 0.00558 \ 56549 \ 54254 \ 2162 \ \text{AU/day} & \nu &= 0.00145 \ 02773 \ 92580 \ 9626 \ \text{rad/day} \\
\dot{y} &= 0.00496 \ 22486 \ 67480 \ 5555 \ \text{AU/day} & \iota &= 0.02281 \ 00753 \ 14931 \ 982 \ \text{rad} \\
\dot{z} &= 0.00199 \ 22660 \ 81200 \ 1201 \ \text{AU/day} & \omega &= 4.77738 \ 51017 \ 57879 \ 2 \ \text{rad} \\
\end{align*}
\]

\[
\begin{align*}
\text{P} &= 0.97134 \ 94329 \ 36575 \ 89 & 0.23656 \ 34667 \ 91785 \ 63 & -0.02275 \ 99379 \ 56469 \ 106 \\
\text{Q} &= -0.23659 \ 17150 \ 90821 \ 03 & 0.97160 \ 80309 \ 57364 \ 59 & 0.00148 \ 13942 \ 25633 \ 8869 \\
\text{R} &= 0.02246 \ 41822 \ 50331 \ 029 & 0.00394 \ 58612 \ 18321 \ 7649 & 0.99973 \ 98615 \ 11518 \ 28
\end{align*}
\]
Table 5. The fitted Saturn ephemeris

a. Statistical summary

<table>
<thead>
<tr>
<th>Residuals in position</th>
<th>Mean residual</th>
<th>Mean square residual</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( dB, \text{rad} )</td>
<td>(-0.172 \times 10^{-9} )</td>
<td>(0.757 \times 10^{-10})</td>
<td>(0.214 \times 10^{-8})</td>
</tr>
<tr>
<td>( \cos B , dl, \text{rad} )</td>
<td>(-0.823 \times 10^{-9} )</td>
<td>(0.330 \times 10^{-10})</td>
<td>(0.374 \times 10^{-8})</td>
</tr>
<tr>
<td>( dp, \text{AU} )</td>
<td>(-0.143 \times 10^{-4} )</td>
<td>(0.136 \times 10^{-11})</td>
<td>(0.368 \times 10^{-7})</td>
</tr>
<tr>
<td>( dr, \text{AU} )</td>
<td>(-0.127 \times 10^{-7} )</td>
<td>(0.195 \times 10^{-14})</td>
<td>(0.432 \times 10^{-7})</td>
</tr>
<tr>
<td>( dy, \text{AU} )</td>
<td>(-0.717 \times 10^{-4} )</td>
<td>(0.201 \times 10^{-14})</td>
<td>(0.442 \times 10^{-7})</td>
</tr>
<tr>
<td>( dz, \text{AU} )</td>
<td>(-0.423 \times 10^{-4} )</td>
<td>(0.370 \times 10^{-14})</td>
<td>(0.188 \times 10^{-7})</td>
</tr>
</tbody>
</table>

b. Epoch values at JD 243 3280.5

\[
\begin{align*}
  x &= -8.97249 \, 44447 \, 14922 \, 9 \, \text{AU} \\
  y &= 2.27971 \, 23065 \, 89970 \, 9 \, \text{AU} \\
  z &= 1.33036 \, 83255 \, 31707 \, 7 \, \text{AU} \\
  \dot{x} &= -0.00185 \, 82755 \, 03439 \, 4825 \, \text{AU/day} \\
  \dot{y} &= -0.00498 \, 38513 \, 60975 \, 9502 \, \text{AU/day} \\
  \dot{z} &= -0.00198 \, 02491 \, 18772 \, 8693 \, \text{AU/day} \\
  a &= 9.52268 \, 86447 \, 14922 \, 9 \, \text{AU} \\
  e &= 0.05349 \, 26945 \, 47401 \, 222 \\
  \Omega &= 1.97652 \, 98227 \, 48397 \, 2 \, \text{rad} \\
  M_0 &= 1.18009 \, 55514 \, 40889 \, 1 \, \text{rad} \\
  n &= 0.00058 \, 54692 \, 21659 \, 67757 \, \text{rad/day} \\
  \dot{I} &= 0.04345 \, 26116 \, 04031 \, 310 \, \text{rad} \\
  \omega &= 5.88309 \, 62072 \, 67269 \, 2 \, \text{rad} \\
  P &= -0.00598 \, 21729 \, 96448 \, 7647 \\
  Q &= -0.99918 \, 52803 \, 59906 \, 34 \\
  R &= 0.03991 \, 22677 \, 67386 \, 926 \\
  0.99983 \, 89593 \, 60259 \, 18 \\
  -0.00530 \, 12313 \, 35481 \, 8541 \\
  0.01714 \, 50369 \, 41382 \, 138 \\
  -0.01691 \, 94843 \, 78497 \, 251 \\
  0.04000 \, 84048 \, 47266 \, 022 \\
  0.99905 \, 60838 \, 06080 \, 96
\end{align*}
\]
Table 6. The fitted Uranus ephemeris

a. Statistical summary

<table>
<thead>
<tr>
<th>Residuals in position</th>
<th>Mean residual</th>
<th>Mean square residual</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>dB, rad</td>
<td>-0.660 x 10^-16</td>
<td>0.118 x 10^-18</td>
<td>0.865 x 10^-9</td>
</tr>
<tr>
<td>cos8 dL, rad</td>
<td>0.326 x 10^-9</td>
<td>0.903 x 10^-18</td>
<td>0.893 x 10^-8</td>
</tr>
<tr>
<td>dp, AU</td>
<td>-0.469 x 10^-3</td>
<td>0.324 x 10^-17</td>
<td>0.180 x 10^-7</td>
</tr>
<tr>
<td>dx, AU</td>
<td>0.823 x 10^-3</td>
<td>0.333 x 10^-17</td>
<td>0.163 x 10^-7</td>
</tr>
<tr>
<td>dy, AU</td>
<td>-0.313 x 10^-3</td>
<td>0.262 x 10^-18</td>
<td>0.159 x 10^-7</td>
</tr>
<tr>
<td>dz, AU</td>
<td>-0.280 x 10^-3</td>
<td>0.553 x 10^-18</td>
<td>0.689 x 10^-9</td>
</tr>
</tbody>
</table>

b. Epoch values at JD 243 3280.5

\[
\begin{align*}
  x &= -1.00291 \ 32748 \ 89426 \ 7 \ AU \\
  y &= 17.32349 \ 19328 \ 51997 \ AU \\
  z &= 7.60483 \ 60538 \ 20472 \ 2 \ AU \\
  \dot{x} &= -0.00395 \ 52571 \ 3622 \ 0751 \ AU/day \\
  \dot{y} &= -0.00037 \ 59007 \ 25259 \ 65007 \ AU/day \\
  \dot{z} &= -0.00010 \ 88436 \ 97028 \ 67699 \ AU/day \\
  a &= 19.16371 \ 87081 \ 38772 \ AU \\
  e &= 0.04620 \ 99622 \ 58820 \ 451 \\
  \Omega &= 1.28736 \ 33344 \ 44398 \ 4 \ rad \\
  M_o &= 5.00583 \ 15710 \ 84931 \ 5 \ rad \\
  n &= 0.00020 \ 50554 \ 21083 \ 97609 \ rad/day \\
  l &= 0.01347 \ 38497 \ 21906 \ 080 \ rad \\
  \omega &= 1.70362 \ 64784 \ 99619 \ 3 \ rad \\
  P &= -0.98859 \ 44293 \ 13043 \ 70 \\
  Q &= -0.15004 \ 57187 \ 18859 \ 02 \\
  R &= 0.01293 \ 58658 \ 52665 \ 412 \\
  0.15000 \ 90158 \ 94858 \ 85 \\
  -0.98867 \ 74489 \ 86863 \ 77 \\
  -0.00376 \ 78926 \ 58122 \ 8593 \\
  0.01335 \ 47550 \ 13593 \ 079 \\
  -0.15017 \ 44211 \ 85763 \ 5285 \\
  0.99990 \ 92290 \ 60097 \ 60
\end{align*}
\]
Table 7. The fitted Pluto ephemeris

a. Statistical summary

<table>
<thead>
<tr>
<th>Residuals in position</th>
<th>Mean residual</th>
<th>Mean square residual</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_B$, rad</td>
<td>$0.121 \times 10^{-10}$</td>
<td>$0.480 \times 10^{-18}$</td>
<td>$0.692 \times 10^{-8}$</td>
</tr>
<tr>
<td>$\cos B , dl$, rad</td>
<td>$-0.544 \times 10^{-8}$</td>
<td>$0.480 \times 10^{-18}$</td>
<td>$0.429 \times 10^{-8}$</td>
</tr>
<tr>
<td>$d_p$, AU</td>
<td>$-0.119 \times 10^{-5}$</td>
<td>$0.157 \times 10^{-11}$</td>
<td>$0.390 \times 10^{-8}$</td>
</tr>
<tr>
<td>$dx$, AU</td>
<td>$0.890 \times 10^{-6}$</td>
<td>$0.881 \times 10^{-12}$</td>
<td>$0.297 \times 10^{-8}$</td>
</tr>
<tr>
<td>$dy$, AU</td>
<td>$0.390 \times 10^{-6}$</td>
<td>$0.658 \times 10^{-12}$</td>
<td>$0.697 \times 10^{-8}$</td>
</tr>
<tr>
<td>$dz$, AU</td>
<td>$-0.134 \times 10^{-6}$</td>
<td>$0.963 \times 10^{-13}$</td>
<td>$0.280 \times 10^{-8}$</td>
</tr>
</tbody>
</table>

b. Epoch values at JD 243 3280.5

$x = -26.23175 \ 78330 \ 46232 \ AU$
$y = 20.56153 \ 05924 \ 86850 \ AU$
$z = 14.44369 \ 10491 \ 54213 \ AU$

$\dot{x} = -0.00131 \ 57216 \ 36860 \ 2943 \ \text{AU/day}$
$\dot{y} = -0.00261 \ 98367 \ 63761 \ 8888 \ \text{AU/day}$
$\dot{z} = -0.00042 \ 70513 \ 54029 \ 74011 \ \text{AU/day}$

$a = 39.37364 \ 76522 \ 73551 \ AU$
$e = 0.24880 \ 34057 \ 56600 \ 34$
$\Omega = 1.91433 \ 75561 \ 99444 \ 1 \ \text{rad}$
$M_p = 5.26530 \ 24509 \ 38841 \ 5 \ \text{rad}$
$n = 0.00006 \ 96263 \ 40374 \ 02263 \ 5 \ \text{rad/day}$
$l = 0.29967 \ 06990 \ 98890 \ 99 \ \text{rad}$
$\omega = 1.99558 \ 15747 \ 74785 \ 5 \ \text{rad}$

$P = -0.68084 \ 18087 \ 46617 \ 09$
$Q = 0.67763 \ 91708 \ 61034 \ 31$
$R = 0.27795 \ 60856 \ 99330 \ 29$

$-0.68125 \ 59300 \ 52014 \ 01$
$-0.72526 \ 10554 \ 58257 \ 92$
$0.09943 \ 21839 \ 47347 \ 375$

$0.26896 \ 98667 \ 72469 \ 62$
$-0.12166 \ 16437 \ 10512 \ 50$
$0.95543 \ 37523 \ 96290 \ 58$
SUBJECT: Errata for TM 33-167

Gentlemen:

It is requested that the following alterations be made in your copy of Jet Propulsion Laboratory Technical Memorandum No. 33-167, entitled "JPL Ephemeris Tapes E9510, E9511, and E9512," by P. R. Peabody, J. F. Scott, and E. G. Orozco, dated March 2, 1964:

1. On page 3, change the last line of the third paragraph to read:

"systems was \( \epsilon = 0.40920 \ 61941 \ 42990 \ 53 \text{ rad.} \)"

2. In the Nomenclature, page 6, the \( P, Q, R \) vectors are expressed in ecliptic coordinates. To the Nomenclature, add:

"\( \omega \) ecliptic argument of perihelion, osculating at epoch, radians."

3. The attached pages 11-17 contain corrections of clerical and typographical errors in Tables 1b through 7b as follows:

a. All \( P, Q, R \) matrices in TM 33-167 were copied from the wrong pages of the computer output.

b. Typographical errors occurred in \( \phi \) in Table 5b and \( \phi \) and \( n \) in Table 7b.

c. The argument of perihelion was mislabeled \( \tilde{\omega} \) instead of \( \omega \).

Very truly yours,

{signature}

N. F. White, Assistant Manager
Technical Information Section

NFW:oju