Interpretive Report on Cumulative Fatigue Damage in the Low Cycle Range

Methods to overcome limitations of the Palmgren-Miner method of Linear Cumulative Damage are outlined, and potential research programs are suggested

BY S. S. MANSON

Introduction

Cumulative damage in fatigue has received a great deal of attention in recent years, and considerable literature which has been developed proposes various methods of predicting fatigue life in spectrum loading and the correlation of available data by these methods. Most published analyses relate to the high cyclic life range, but the methods lend themselves to application in the range of low cyclic life as well. Also, because support for the research associated with this problem has come largely from the aerospace industry and governmental organizations associated with this industry, most experimental work in this field has been performed on materials of special interest to aerospace applications, so that materials of interest to other applications must as yet be studied.

No attempt will be made in this brief treatment to summarize all the methods that have been used to predict cumulative fatigue damage, especially since a recent report by Chrichlow, *et al.*,[1] provides a very thorough analysis of these methods. However, three new methods have appeared within the past three years. These do not appear in Crichlow[1] and will be outlined; their potential application will be discussed in relation to research programs that should prove of value to the pressure vessel industry.

Limitations of the Palmgren-Miner Method

The earliest and perhaps the simplest method of accounting for damage accumulation in fatigue is popularly referred to as the Palmgren-Miner[2,3] method of Linear Cumulative Damage. The basis for the method is extremely simple: If n_1 cycles are applied at a stress or strain level at which failure occurs in N_1 cycles, then it is hypothesized that the fraction of life used up is the "cycle ratio" n_1/N_1, so that the remaining fraction is $(1 - n_1/N_1)$. If this service is followed by cycle ratios of n_2/N_2, $n_3/N_3, \ldots n_p/N_p$, then the remaining life fraction is $[1 - n_1/N_1 - n_2/N_2 \ldots n_p/N_p] = 1 - \sum_{q=1}^{p} n_q/N_q$. At failure there is no remaining life, so that:

$$\sum_{p=1}^{q} n_p/N_p \pm n_1/N_1 + n_2/N_2 + n_3/N_3 \ldots n_q/N_q = 1$$

Two very important limitations become evident upon comparing the implications of eq (1) with the experimental data that have been generated since the method was first proposed:

1. The life is independent of the order of application of the stresses. Thus, if n_1/N_1 is interchanged with n_2/N_2 in eq (1) no difference results in the life relationship because of the linear manner with which the terms enter into the equation. Experiments have indicated, however, that the order of application of the stresses can be of considerable importance, depending on the nature of the loading spectrum. If, for example, a smooth specimen is subjected first to a relatively high stress, and the test run to completion at a lower stress, the sum of the cycle ratios is less than unity, whereas if the order of the stresses is reversed the sum will be greater than unity. Thus, high stresses applied early in the history are more damaging than if applied later. The importance of order of load application assumes less significance as the number of blocks in which repetitive loading occurs increases. In Fig. 1, for example, the loading consists of a series of blocks *ABCDEFGH*, *A'B'C'D'E'F'G'H'*... in which four stress levels are applied in sequence, a stress level of intermediate magnitude being applied first. The sequence can, however, be regarded as that shown in Block *1'*, Block *2'*... in which the highest stress level is applied first. The difference between the two ways of breaking up the total operation into the two types of blocks consists largely of the prior application of history *ABCDE*. Now if the total number of blocks is large, it makes relatively little difference whether *ABCDE* is applied or not. If however, the total number of blocks is small, the order of loading may be extremely important. Under circumstances in which the order of load application is unimportant, the linear damage rule is more likely to be valid.

The order the loads are applied within a block may still be important even if it is unimportant whether the block loading starts with the highest stress in the cycle or the lowest. This fact is illus-

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This report was prepared for the Subcommittee on Plastic Fatigue Strength of the Fabrication Division, Pressure Vessel Research Committee, as a brief review of cumulative damage in low cycle fatigue and as a recommendation for a possible research program in this area that could be sponsored by PVRC. The effect of cumulative damage in low cycle fatigue is one of the Research Topics submitted to PVRC by the ASME Special Committee to Review Code Stress Basis in 1959.

During the 1963–64 fiscal year of PVRC, the PVRC Subcommittee on Research on High Strength Steels included in its research program at Lehigh University the determination of the validity of Manson's general power law relation at elevated temperatures and also whether or not the material constants in Manson's equation can be obtained from hot tensile tests.

A is not fully equivalent to Block *B* even though both start with the highest stress in the cycle, and the stresses within the blocks are the same (although of different order).

From the standpoint of order-dependence it can be argued that the Palmgren-Miner Linear Damage Rule is most likely to be valid where a large number of repetitions of blocks of loading are present, and where the loading within a given block is highly random. In any case, however, it is the failure of the ability of the Linear Damage Rule to allow for order dependence of the stresses that constitutes one of its serious limitations.

2. Stresses below the endurance limit (for infinite life) of the virgin material do not have any effect on the life no matter how many cycles are applied, and regardless of where in the history of loading they are applied. The reason for the implication is obvious, since for any stress below the endurance limit the value of the denominator N is infinite, hence any finite value of n in the numerator does not contribute a cycle ratio of finite magnitude which reduces life at other stress levels.

It is this limitation that is the cause of the most serious discrepancies that have been associated with the method. While the virgin material may demonstrate an infinite life endurance limit, prior operation at a stress level higher than this value may reduce the endurance limit, perhaps by introducing imperfections or even cracks in the material. Subsequent operation at a stress level lower than the initial endurance limit may be able to propagate the imperfection because of the stress concentration associated with it, although it was unable to do any damage before the imperfection was introduced by the higher stress level. Thus, failure to include the damaging effect of stresses below the initial endurance limit may result in analyses that are unconservative.

One of the first investigations to take into account the reduction in endurance limit caused by prior stress history is that of Henry.[4] Other methods outlined in Crichlow[1] also account in one way or another for the reduction in endurance limit, as do two of the three recent methods discussed later in this survey. If the linear damage rule were extended to take into account prior history—for example, by hypothesizing a rational basis for extending the basic S–N curve to lower levels than the original endurance limit—then it is possible that the predictions yielded by the method would be greatly improved. However, all the methods that have sought to overcome this objection to the linear damage rule have also involved additional assumptions which becloud the relative contributions achieved by overcoming each of the objections.

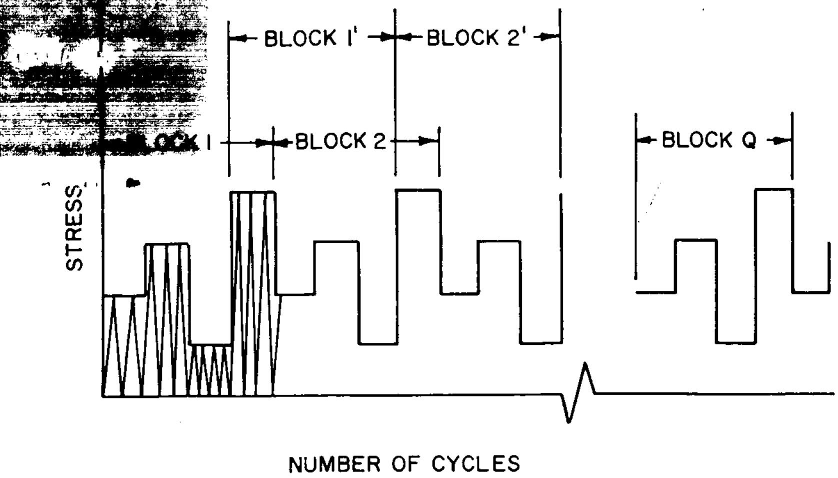

Fig. 1—Spectrum loading of blocks of repetitive load sequences. The eight horizontal lines from left to right (Blocks 1 and 2) are AB,CD,EF,GH,A'B', C'D',E'F',G'H'

Methods That Attempt to Overcome Some of the Limitations

A large number of methods have been proposed which in one way or other attempt to overcome some deficiencies of the linear damage rule. As already indicated, no attempt is made here to outline or discuss all of the methods. Three methods which have appeared only within approximately the past three years will, however, be outlined because they have not been summarized in any single report to date. It will be seen that these methods differ considerably in the technique used to overcome the Palmgren-Miner deficiencies; all have some advantages accompanied by new deficiencies the seriousness of which requires further investigation.

The Method of Manson

Figure 3 shows the method pro-

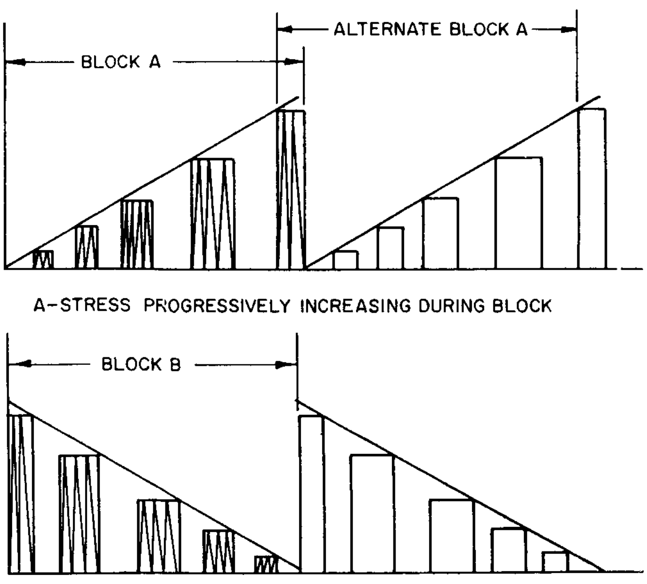

Fig. 2—Spectrum loadings consisting of progressively increasing or decreasing stresses within a given block

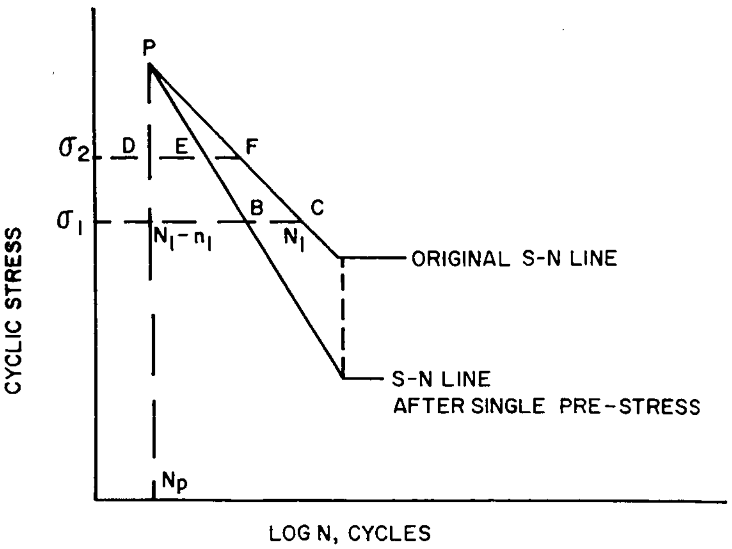

Fig. 3—Rotation of S-N curve as a result of prior stress cycling. "N", is "N_f" in LOG N cycles; N_1-n_1 extends from y-axis to S-N line after single pre-stress

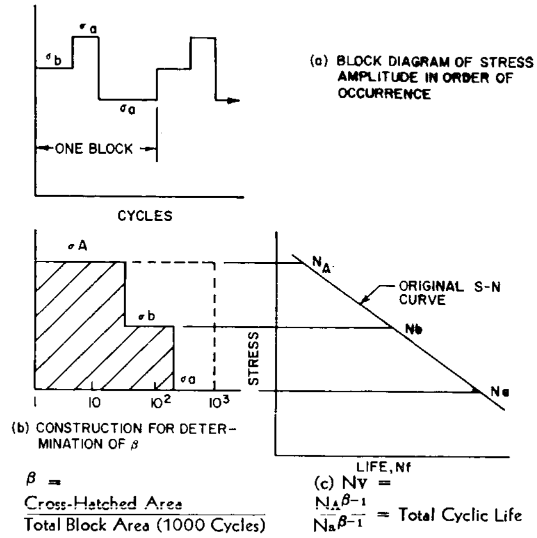

Fig 4—Fuller's method of determining total cyclic life when stress history consists of a large number of repeated stress blocks. N_V actually= $N_A{}^{\beta}/N_a{}^{\beta-1}$. Also correct terms are $\sigma_A, \sigma_b, \sigma_a, N_b N_a$, and N_f in "LIFE,Nf"

posed by Manson,[5] and experimentally verified by Manson, Nachtigall and Freche[6] to overcome both of the foregoing limitations of the linear damage rule. The method is based on the hypothesis that prior fatigue history produces the effect of rotating the σ-N_f (stress vs. life) curve rather than displacing it parallel to itself as required by the Palmgren-Miner hypothesis. Thus, in Fig. 3, if PFC is the initial σ-N_f curve of the material, application of n_1 cycles at stress σ_1 for which the life is N_1 rotates the σ-N_f line into *PEB*. The position of this line is determined from:

1. The point *P* on the initial σ-N_f line of the material. Equivalently, this point is identified by the value of N_P, which is regarded as a material constant.

2. The point *B* which is at the prestress σ_1. Thus, since life N_1 is known at the prestress, and n_1 is used up, the remaining life at σ_1 is $N_1 - n_1$; hence the line *P–B* can be constructed.

The effect of a complex prior history can be compounded by constructing the successive rotations associated with each increment of prior history. It can readily be shown that this procedure produces the proper effect of order dependency. Thus, the early application of a high stress level has a more detrimental effect on life at a subsequent lower stress level than if the order of applying the stresses is reversed.

The method also provides an allowance for reducing the endurance limit due to prior fatigue history. Manson, Nachtigall and Freche[6] assumed that the altered endurance limit could be estimated by extending the revised σ-N_f line to the life corresponding to the knee of the original σ-N_f curve. As shown experimentally in Fig. 4 of Manson, *et al.*,[6] this procedure is conservative, since the endurance limit is rarely reduced by as large a factor as the method indicates. One important program is, therefore, to determine a more accurate rule for estimating the effect of prior history on endurance limit.

The Method of Fuller

Fuller's method[7] is based on the empirical analyses of available cumulative fatigue data rather than on a foundation of rational analysis. Because it does make use of experimental data, it is likely to be consistent with actual material behavior, at least in some ranges. On the other hand, lacking a rational basis, its extension to situations not specifically covered in the empirical rules may sometimes be difficult.

The method is applicable only to cases in which a relatively large number of blocks of stress pattern are applied. Within a given block the variation of stress may be fairly random, but there must be a large number of repetitions of the block for the method to be meaningful. The procedure associated with the proposed method can best be illustrated with the use of Fig. 4.

In the upper left section of Fig. 4 is shown the block diagram of the stress-time history assumed here for purposes of discussion. The method concentrates on the stress pattern within one of the blocks which is assumed to be repeated a large number of times. The stresses within the block are first arranged in order of decreasing magnitude, regardless of order of occurrence in service, and an auxilliary block diagram is drawn as shown in Fig. 4 (b). In this auxiliary block diagram the number of applications of each stress level is normalized to a total of 1000 cycles. Thus, if in Fig. 4 (a) the number of applications of stress σ_A within block 1 is 100, whereas the *total* number of stress cycles in Block 1 is 2000, then in constructing the block diagram of Fig. 4 (b) the width of the bar representing σ_A is 50 cycles. The horizontal scale (cycles per thousand) is logarithmic; the vertical scale (stress) is linear. It will be seen that, because of the logarithmic scale and because the stresses are arranged in order of decreasing magnitude, greater emphasis is placed on the higher stress levels.

Having constructed a diagram of stress vs. cycles per 1000 as shown in Fig. 4 (b), the method requires the calculation of a quantity β as shown in Fig. 4 (c). It is the ratio of the area under the block diagram to the area of the rectangle bounded between the highest and lowest stress over the 1000 cycle range.

The value of β so determined is then used to calculate N_v, the total number of cycles that can be withstood, as also shown in Fig. 4 (c). It will be noticed that N_v depends only on β and on the fatigue lives (as determined from the basic σ-N_f curve of the material) at only the highest and lowest stress levels within the cycle.

As can be seen from the procedure

outlined, the method cannot take account of order of application of stress amplitude. In the first place it is applicable only to cases in which there is a large number of repetitions of the same pattern of loading, and hence its applicability is inherently limited to situations in which order of load or stress application is secondary. In the second place, whatever effect might occur due to order of stress application *within* a given block of loading is obviously ignored by constructing the block diagram according to decreasing order of magnitude regardless of actual order of application within the loading cycle.

Provision is, however, made for including the effect of applying stress cycles of subendurance level within the basic block loading. In such instances, Fuller has suggested that the fatigue curve (assumed to be linear when constructed on a semilog plot of σ vs. N_f) be linearly extended below the endurance limit, so that a definite life can be associated with each stress level below the endurance limit. Stresses below the endurance limit are then included in the analysis just as are stresses above the endurance limit. Whether this procedure produces accurate predictions requires considerable analysis of various types of stress sequences typically encountered in practice.

The Method of Valluri

Of all the methods that have thus far been proposed for the analysis of cumulative fatigue, that very recently proposed by Valluri[8] has the most elaborate rational basis. Within its range of validity it would be the most powerful of available methods, because its analytical form is such as to bring many otherwise intractable problems within scope of analysis. Unfortunately, the method does not yield predictions that are in all cases in agreement with the experimental evidence (although producing for some problems remarkably accurate predictions); thus refinements may be necessary before it can be applied with confidence. It is of special significance in the present discussion because it purports to be valid in the low-cycle range; in fact, disclaims validity in the high-life range.

As already indicated, the method has a rational basis. However, a careful analysis of the derivation reveals that many of the quantitative relations are expressed on the basis of analogy and on intuition rather than on well-established natural laws. The starting point of the analysis is a relation drawn from dislocation theory; during the course of the analysis use is being made of the Griffith concept that ultimate fracture is due to the unstable propagation of a critically-sized crack. In its final form the theory expresses the manner in which a crack grows as a function of the number of applied cycles of a given maximum stress and stress range. It follows the crack from its "initial" size, l_0, associated inherently with the static ultimate tensile strength via the Griffith equation, to its "final" size at which the applied maximum stress is sufficient to propagate the crack cataclysmically.

The method can best be described graphically with the aid of Fig. 5. By application of the final equation the curves *OAB* and *OCD* can be constructed representing the progress of the crack length, l, for two fatigue tests of constant stress amplitude σ_1 and σ_2. The stress σ_2 is greater than σ_1, so that for a given number of cycles the crack length along *OCD* is greater than the crack length along *OAB*. However, the crack length which can be propagated cataclysmically to fracture at the high stress σ_2 is lower than that corresponding to the lower stress σ_1. Thus, the end-point *D* of curve *OCD* is at a value of l_2 which is less than l_1, the end-point at *B*. In fact, the ratio of l_1 to l_2 is proportional to the square of the ratio of σ_2 to σ_1.

The ability to construct the curves *OAB*, *OCD*, or in fact, the curve for any stress amplitude, depends on the knowledge of four material constants. Two of these, the ultimate tensile strength and the endurance limit, are conventionally reported properties. The other two —an effective initial crack length associated with unstable crack propagation at a stress equal to the ultimate tensile strength according to the Griffith Equation, and an effective grain size parameter—are best determined indirectly. Valluri chooses to determine these constants so that the end points of N_f vs. σ agrees with the conventional fatigue curve of the material at two strategically chosen points at intermediate fatigue lives.

Once the material constants are determined it becomes possible to construct curves such as *OAB*, *OCD*, etc., for each stress level. Each curve shows the progress of the crack length during a fatigue test in which the stress amplitude and range is maintained constant. To compound the effect of varying stresses, it is merely necessary to keep track of the progress of the crack size, using this size as a measure of the fatigue damage. Thus, if n_1 cycles are applied at σ_1, we can proceed to point *A* in Fig. 5 along the curve associated with σ_1. If the stress is then changed to σ_2, we can proceed from *A* to *C* along a line of constant crack length, and then the remaining number of cycles to failure at σ_2 is from *C* to *D*. If several stress levels are involved the method can be extended, establishing the starting point at any one stress level from the crack length developed by application of all prior stress levels.

The method of Valluri thus has built into it the capability of accounting for the order of stress application, since the order of proceeding along the various curves affects the final answer. Unfortunately, the method implies that if the high stresses are applied first a greater number of cycles can be withstood than if the low stresses are applied first (for the case of tests involving only two stress levels). As corroboration of this implication, Valluri points to some NASA experiments in which these results were indeed observed. But these tests involved notched specimens, in which the application of a high stress level first caused plastic flow at the root of the notch, thereby producing residual compressive stresses which were beneficial upon the subsequent application of the low stress. But since the theory, as applied, refers to the smooth specimens, not involving the residual stress, the implications for this case appear to be in error. Further analysis is necessary to determine the validity of the implications of the method regarding order of stress application.

As for the effect of subendurance stress levels, the method cannot take these into account because it is based on crack propagation curves at each stress level, and no such curve can be drawn for any stress below the initial endurance limit.

Relation to Pressure Vessels

It will be observed that the methods discussed thus far, as well as those discussed by Crichlow, *et al.*,[1] refer primarily to stress or load cycling. In a pressure vessel only part of the loading is due to the pressure for which the stresses can be computed either elastically or including the effect of plasticity. That part of the loading which is due to thermal gradients is, however, of the type in which the strain per cycle is more likely to be known, rather than the stress. Also, when stress concentrations are present, the strains are better known than the stresses. Thus, although most of the cumulative damage theories have been developed for cases involving stress cycling, the application here is to strain cycling, or worse yet, to a combination of stress and strain cycling. In particular, the fact that the mean stress is not zero causes a requirement for adaptation of any applicable cumulative damage theory.

It may also be noted that almost all experimental data available for checking of cumulative fatigue theories have been obtained at room temperature. Most of the pressure vessels of interest operate at higher temperatures, which introduces several unknowns:

1. Determination of the basic relations between strain and life at the higher temperatures.
2. The application of the basic relations to a cumulative damage theory.
3. The importance of strain rate, which may alter the percentage of a given total strain that develops in the form of creep instead of plastic flow, thereby appreciably affecting the life even though the total strain range is constant.

The effect of fatigue damage on residual static strength of the vessel is of some importance since the development of a crack may appreciably weaken the vessel against overloads. The Valluri theory is capable of taking this effect into account, but preliminary calculations by the author indicate that the predictions are overconservative—that is, they imply greater loss of static strength than experiment corroborates.

Since the most universally acceptable criterion for the amount of cumulative damage sustained by a structure is the state of crack development, it may be advisable to support research that will contribute to the art of such detection and interpretation of observations.

Potential Research Programs

In view of the many unknowns involved in this program, it is to be expected that any one research project can only provide a limited amount of contributory information, and that many research projects will be needed if substantial progress is to be made. In the following discussion a number of potential research projects are outlined. It is hoped that some of these can find support from the Pressure Vessel Research Committee, and that others will find support by individual members, educational institutions and other interested organizations.

Projects Relating to the Basic Relation

Almost any cumulative damage theory involves the application of data obtained under simple laboratory conditions. For this reason, it is important to establish these relations for the materials of interest and conditions of interest to the pressure vessel industry. For this application it is generally agreed that strain cycling data will be of the greatest interest, later to be modified for the effect of superimposed load cycling. Accordingly, several projects that might be undertaken are as follows:

The Basic Relation Between Strain Range and Life at Elevated Temperature. The general power law relation proposed by Manson, $\epsilon_p = MN_f^z$, (where ϵ_p is plastic strain, N_f cyclic life, and M and z material constants), and the simplified relation suggested by Coffin $\epsilon_p = D/2\ N_f^{-1/2}$ (when D is the ductility in the tensile test) have been studied for many materials at room temperature. Although the relation $\epsilon_p = D/2\ N_f^{-1/2}$ has been found to be a reasonable first approximation to the behavior of several materials, the more general relation $\epsilon_p = MN_f^z$ has been found superior, and more desirable for precise design. More recently Manson[4] has suggested a method for determining the constants M and z from tensile test data so that fatigue tests may be avoided, if necessary, even in the use of the more general relation. However, the amount of data that has been collected on materials at high temperature is very limited. An important part of the program, therefore, is to determine the validity of the power law relations for pressure vessel materials at the temperatures of interest and, in particular, to determine whether the approximations for the constants in these equations can be determined from tensile tests at elevated temperatures.

Although the power law relationships between life and plastic strain are adequate for determining life in the range up to 10,000 cycles, it becomes inadequate above this life. When cyclic lives in the vicinity of 100,000 cycles are involved, it is almost imperative to use total strain range as the criterion for life instead of plastic strain. The method recently developed by Manson[9] appears to be remarkably accurate for at least 20 materials thus far evaluated, but no attempt has been made as yet to evaluate the method at elevated temperatures. A program to predict the strain-cycling fatigue properties of pressure vessel materials in the temperature range of interest from elevated temperature static properties would be of great value in establishing the basic strain-life

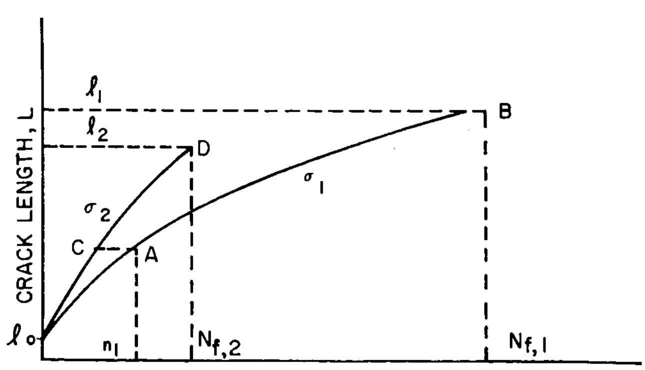

Fig. 5—Crack growth during stress cycling. Curves CD and AB intersect at O

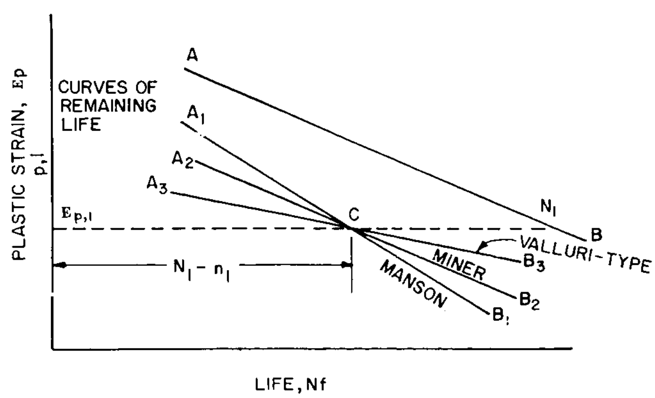

Fig. 6—Curves of remaining life after prestrain at one plastic strain level according to several theories

relationships later to be applied in cumulative fatigue considerations. If the method proves valid, it should serve a valuable purpose in screening materials for given pressure vessel applications.

Attention should also be directed to the method[9] outlined for optimizing the heat treatment in order to obtain maximum life for specified magnitude of strain range. Thus, on the basis of the analyses presented, the optimum combination of ductility and tensile strength can be determined if the operating strain range is given. High strain range favors the specification of high ductility, and relatively low strain range favors the specification of high tensile strength. Evaluation of the graphical method[9] for elevated temperature application, while not directly concerned with the problem of cumulative fatigue, may prove of great value in pressure vessel application.

The Importance of Alternate Straining at High and Low Temperature. The basic data to be used in cumulative damage computations are those obtained at constant temperature. Presumably, if the temperature changes during the course of the test, the curve for the proper temperature would be used in determining the increment of damage for that portion of the loading. However, when cyclic thermal strains occur in a pressure vessel an additional phenomenon takes place and cannot readily be incorporated in the calculations in this way. During each single cycle the compressive strain occurs (generally) while the local temperature is high, while the tensile strain takes place at a lower temperature. If the material is sensitive to this phenomenon, the results may be entirely different from those that occur at constant temperature, either the low, the high or some intermediate value.

Both Coffin[10] and Majors[11] have observed large reductions in life when the strain was thermally induced, as compared to isothermal mechanical tests, and only Swindeman and Douglas[12] have found no difference between the two types of loading. Their work was, however, on Inconel, which appeared to be insensitive to temperature of straining. Hence, if thermal strain cycling is involved in the cumulative fatigue loading, it is important that the basic fatigue data reflect the sensitivity of the material to this type of loading.

Of course, the best type of data to obtain is thermal strain cycling behavior. Some materials, notably Type 347 stainless steel, have already been studied. However, it must be recognized that further materials of interest to the pressure vessel industry must be studied if a general pattern of behavior is to be established. How such data would be incorporated into a cumulative damage theory is not clear at this time (unless it be in the simple Miner Theory), but it must at least be pursued if the significance of this important variable is to be determined.

The Relative Importance of Creep and Plastic Flow. When the cyclic plasticity is imposed rapidly, the flow is of the slip type, and it occurs over a large number of slip planes. If, however, the deformation occurs slowly, as a result of forces acting over a long period of time, the "plastic flow" is more like creep than slip plasticity, and some of it takes place in the grain boundaries. It has been found that for the same amount of cyclic plastic flow, the number of cycles to failure is considerably lower for the creep type of deformation. The best recent description of a program involving this type of research is by Morrow,[13] which was presented at the International Conference on Creep in August 1963. Professor Morrow is anxious to extend this type of work, and it is possible that a program could be arranged with him.

Although it is, of course, desirable to work with pressure vessel materials at temperatures simulating operation, it is possible that the early work in this field could be done with materials of low melting temperature, so that creep can be studied even at room temperature. The program would involve the determination of a family of $\epsilon_p - N_f$ curves for different strain rates, so that ϵ_p can be made to be nearly all creep or all slip plasticity, and comparing these curves. If there is a large difference among the curves, it is evident that the curve in which the strain rate most closely simulates the condition in a given pressure vessel application must be used in the cumulative damage analysis.

Importance of Mean Stress, Stress Gradient and Stress State. The cumulative damage relations, as a general rule, make use of the basic behavior of the material in the most simple type of laboratory test—usually uniaxial, completely reversed stress or strain cycling at constant temperature. But pressure vessels present more complex situations than the simple compounding of the basic behavior. For example, the presence of biaxial stresses may affect the life in a way that is not directly predictable from uniaxial stress data. The presence of a mean stress, upon which alternating stress and strain is superimposed, may also be of considerable importance and not readily predictable from completely reversed loading. Finally, high stress gradients, resulting from geometric discontinuities and design features, and from sharp temperature gradients, may also have their peculiar effects, not easily predictable from uniform stress state data.

Before, therefore, or at least concurrently with, a program on cumulative damage under varying stress can be interpreted for the complex conditions involved in a pressure vessel, it is desirable to determine the behavior of pressure vessel materials under more simple conditions which include the important variables. Even if, in the final analysis, it is not possible to use "simple" laboratory tests for the compounding of cumulative damage predictions, it may still be possible to use the "simply-determined" laboratory tests for such compounding. Thus, for example, suppose we are interested in the cumulative fatigue behavior of a material with a notch in it under conditions of variable loading. The normal objective in a cumulative damage study might be to try to predict the results from uniaxial fatigue data on smooth specimens. But it will be recognized that a notch introduces many complications. Inherently there is a high stress and strain gradient. Plastic flow in the early cycles may introduce a residual stress which acts as a mean stress about which the alternating loading fluctuates. And constraint may introduce a multiaxial stress state that can affect the behavior. Whether it is possible to include all these effects in a cumulative damage analysis may be questionable. But it is possible that a series of laboratory tests on notched specimens under known conditions of constant loading amplitude may be helpful in predicting the behavior under variable load amplitude.

Perhaps one of the simplest programs is the study of a series of notched specimens under strain cycling conditions. Just what strain amplitude to maintain constant may require further study, but it would appear that it should be the nominal strain across a gage length which includes the notch, rather than becoming involved with the localized strain within the notch.

In any case, the effect of high strain range on the life of notched materials would be a valuable adjunct to the subsequent analysis of cumulative fatigue of notched geometries.

Studies involving a mean stress of known and controllable amplitude can readily be conducted with currently available machines. Mean stress may be maintained by means of a load cell, while superimposed strain can be achieved by means of microswitches.

Multiaxial stress effects can be studied by means of tubular specimens under internal pressure with superimposed axial or torsional load. Professor Joseph Marin of Pennsylvania State University has signified an interest in such studies in the low cycle range at room and elevated temperatures. Such studies would be especially useful in the interpretation of tests involving varying stress or strain amplitudes.

Projects Relating to the Cumulative Damage Law

The second area requiring research is how to utilize the basic life relations obtained under simple laboratory conditions, such as at constant temperature and stress, to predict the behavior under conditions of varying loading. A large number of tests have been conducted over the past two decades to evaluate existing cumulative damage theories, and to develop new ones, for fatigue in the high cyclic life range.

In evolving suitable test programs for the low cycle range it is important to benefit from the experience gained in the previous studies. For example, one type of test that has been conducted is to subject specimens to block type loadings of varying types, and to determine the sum of the cycle ratios (for evaluation of the Miner Hypothesis or Linear Damage Rule). Finding that the sum of the cycle ratios lies between 0.8 and 1.3 does not necessarily mean that the assumed value of 1.0 is good because it gives no insight as to other conditions under which the error may be very much greater. Nor is the oft-quoted fact that the sum of the cycle ratios is far from unity meaningful in evaluating the Linear Damage Rule, because the special conditions under which the large discrepancy is attained may be so far removed from the practical condition of interest as to be meaningless.

To have the greatest value, the test should be designed as close as possible to a "critical" test—one which tests the "heart" of the theory and which gives insight into the reality of the basic assumptions involved in a theory. The ideal experiments have not yet been evolved. In fact, the ideal experiments depend to a great extent on the particular aspect of the theory that is of the greatest interest for a given application. However, some progress has been made in the evolution of significant cumulative damage tests, and this progress should be carried over into the low cycle range. Several possible approaches will now be outlined.

The Two-Strain Block Test. One of the simplest cases of cumulative fatigue damage is that in which the first part of the test is conducted at one stress level, and the remainder of the life is used up at a second stress level. For the case of low-cycle fatigue it is probably best to modify the test by substituting "strain range" for "stress." Since the different cumulative damage theories predict different behaviors in such a test, the basic assumptions involved in the theories can best be tested by comparing the experimental behavior to the predictions of each theory.

Consider, for example, Fig. 6, in which the line AB represents the basic life characteristic of a material. Let it be assumed that a series of specimens are subjected to plastic strain $\epsilon_{p,1}$ for n_1 cycles, while failure at this plastic strain level occurs at N_1 cycles. The "remaining" $\epsilon_p - N_f$ curve of the precycled specimen is then determined by subjecting these specimens to other plastic strain levels and cycling until failure. Obviously the remaining life curve passes through point C at a strain level $\epsilon_{p,1}$ and life level $(N_1 - n_1)$, regardless of which cumulative damage theory applies.

The rest of the curve can, however, shed light on which cumulative damage theory is most accurate. For example, if the remaining life curve is A_2B_2, parallel to AB then the Linear Damage Rule of Miner applies; if it tends to converge toward AB at a low life, such as does line A_1B_1, the theory of Manson is more accurate; if it tends to converge toward AB at high life, as does A_3B_3, then the approach of Valluri* (and Henry, Gatts and others as well) better characterize the behavior. Such a series of experiments, therefore, serves to make a major distinction among the theories available, and sheds light on their relative merits.

It should be recognized, however, that experiments of this type are more difficult to perform and more costly than appears at first. For the line of remaining life (A_1B_1 or A_2B_2 or A_3B_3) to be sufficiently distinct from AB, and for its inclination to be clearly definitive of the applicable cumulative damage theory, the value of n_1 must be fairly close to N_1, say between 60 and 80% of N_1. This fact increases appreciably the amount of scatter that results in the determination of the curve A_1B_1 (or A_2B_2 or A_3B_3), and places the requirements of running many duplicate tests in order to achieve statistically significant results.

Assume, for example, that N_1 is 1000 ± 200—that is, the life at the prestrain condition lies between 800 and 1200 cycles. Let it also be assumed that n_1 is 700 cycles. Therefore, if 10 specimens are all prestrained for 700 cycles, their remaining lives at the prestrain level is between 100 and 500 cycles. This "scatter" will probably be carried over to lives at other strain levels in the determination of the remaining $\epsilon_p - N_f$ curve. Obviously, the percentage scatter is greatly increased in the determination of the latter curve, more so than in the determination of the original curve.

To obtain results that can be interpreted with confidence, a large number of duplicate tests should be conducted, and the data treated statistically. In Manson, *et al.*,[6] approximately 11 duplicate tests were conducted at each stress level, and the median was used to determine the lines for verifying the cumulative damage theory proposed therein.

It should also be pointed out that, although experiments of this type tend to bring out large differences in the predictions of the various theories, such large differences do not necessarily exist under other test conditions. Hence, the fact that one theory turns out to be far superior to the others in such tests may not reflect the degree of superiority—if it is superior at all—in more practical cases. But such tests constitute a first step in the evaluation of cumulative damage theories.

Repetitive Block Tests. Block tests of the types shown in Fig. 1

* In Fig. 6 the line A_3B_3 is designated as Valluri-type, because it contains the general feature of what might be expected from a theory which contains the assumptions involved in the Valluri theory. The latter gives no direct predictions of what a curve of ϵ_p vs. N_f appears like because it is formulated on the basis of stress rather than plastic strain. But it, like several other theories that have been proposed, contains the general assumptions that result in curves that bear the relation to AB that A_3B_3 does in Fig. 6.

can also be used to check the various theories. These blocks should be typical of strain levels encountered in service for various pressure vessel applications. It is important that the block pattern for successive tests be chosen in a systematic manner—for example, by keeping the amplitude in the highest block constant, while varying the amplitude in one of the other blocks constant—and comparing the results of tests with predictions by several cumulative damage theories. When the tests are systematic, not only does it become possible to compare the predictions of the theories in a more meaningful way, but it is also possible to generalize test results for their practical value regardless of whether or not they agree with any particular theory.

Blocks of the type shown in Fig. 2 can also be used to determine order-dependence of experimental results; they can also be used for comparison with the various theories.

Block tests in which the strain amplitude is completely random are also of great interest. It may, for example, be found that the linear damage rule has greater validity the more random the loading; this observation has, in fact, been made in limited stress cycling tests reported in the literature. A careful study of typical strain patterns normally to be encountered in pressure vessel service will serve to indicate what type of strain block pattern is best suited to study.

Projects Relating to the Basic Mechanism of Cumulative Fatigue Damage and Its Detection

From a practical point of view, the most directly applicable type of information relates to the method for incorporating cumulative fatigue damage theories into the design equations. However, an understanding of what fatigue damage is and how it can be detected is of the utmost importance as a long-range objective. Out of such understanding may come principles of material design that will minimize damage, operational principles as yet unknown but which will later be recognized as good practice, and detection methods that will make possible maximum service life and optimum retirement times prior to costly or catastrophic failure.

It should be apparent at the outset that it will not be easy to assign a simple number to represent the state of "accumulated damage." The meaning of this statement can best be illustrated by considering a simple test in which 40% of the life at a given strain level is used up. According to the linear damage rule, it might be said that the accumulated damage is 40%, implying that 60% is still available for later use. However, it is apparent that the state of damage depends on how the residual properties of the specimen are to be determined.

If, for example, we are interested in the remaining life at a small alternating stress level (well below the initial endurance limit of the material), there may be no damage at all, for the remaining life may still be infinite. If we are interested in the remaining life at a strain level somewhat lower than the prestrain level, we may find that the damage is more than 40%, because the remaining life is less than 60% of what it was before the prestrain. At a strain level somewhat above the prestrain value the remaining life is more than 60% of what it was; hence, the damage as measured by such a test will be less than 40%.

If the damage is to be measured by determining the remaining static tensile strength, it may be found to be 95% of what it was before the prestraining. Thus, it might be said that the accumulated damage was 5% on this basis. But if there is a requirement that the specimen be capable of sustaining the static strength that it had prior to prestraining, then the damage is 100%, since the prestraining had completely destroyed the ability of the specimen to withstand this load. Similarly, impact tests, or other methods of evaluation will rate the initial damage associated with the 40% of the cyclic life in different ways.

An alternate approach is to identify the damage by some physical representation, such as the progress of cracks that have been caused by the cycling. This is a better approach, but is more difficult to define, and even when defined does not constitute the complete answer. Thus, for example, Sessler and Weiss[14] have pointed out that the early stages of cycling causes both the formation of cracks and loss of available ductility due to hardening, but the multiplicity of small cracks formed during the major part of the cyclic life do not produce as detrimental an effect as single notches because of the "multiple notch effect" (i.e., the reinforcement of the stress fields due to the several notches, producing a greater strength than if only one notch were present).

Only in the very latter stages of life is the size of the dominant crack governing in relation to further crack growth. Also it is well known that some cracks grow to a critical size, and then stop propagating, particularly if they are associated with a residual compressive stress pattern unfavorable to such growth. Nevertheless, the determination of damage as defined by the dominant crack, and crack distribution in its vicinity could be a useful tool in determining the cumulative damage sustained by a material. Several possible approaches will now be outlined.

Crack Detection by Use of Cryogenic Temperatures. Many materials become extremely brittle when tested at very low temperatures. Thus, if cracks are developed during cumulative fatigue tests at normal temperatures, subsequent testing at a very low temperature where no plastic flow takes place, may serve to preserve the state of crack development, and thereby provide a measure of the damage that had accumulated. Sessler and Weiss[14] have used this technique, testing at liquid nitrogen temperature of −320° F. This temperature was not, however, low enough to completely embrittle the pressure vessel material studied, and it is possible that the use of liquid hydrogen temperature (−423° F), or even liquid helium (−452° F) would better serve the purpose. Tests could be conducted first at a constant strain amplitude, and the damage measured by such cryogenic tests; then complex block patterns could be imposed, and cryogenic tests used to determine state of cumulative damage by comparison with the constant amplitude tests.

It is also possible to use impact tests, perhaps in conjunction with cryogenic temperatures, to augment the embrittling effect by adding high strain rate to low temperature.

Interpretation of the results of such tests in terms of cumulative damage will require some experience. Such interpretation may proceed along the lines of Sessler and Weiss[14], by determining from the test results, together with Neuber's equations for stresses in the vicinity of cracks, the effective crack length developed by the fatigue process. Or, other means may be used to study the fracture surface. Optical as well as electron microscopes may be valuable in such a study.

Crack Detection by Acoustical Means. When a crack develops, sound waves are emitted. Acoustical pickups on the test specimen can be very useful in detecting the onset as well as the propagation of fatigue cracks. Of course the results are only qualitative, since crack sizes are hard to measure directly in this way. However, the method may prove invaluable, particularly if conducted as an adjunct to other methods.

A program of this type may prove costly because the acoustics of the loading system must not interfere with the low level of sound developed by fatigue cracking. But since the frequencies involved are fairly characteristic, it is possible that acoustic filtering may be able to preserve only the frequencies of interest while attenuating undesired sounds. Some progress has already been made in the study of fatigue by this method but it has not been used to study cumulative fatigue damage.

Crack Detection by Damping Measurements. If a low amplitude vibration is imposed on a specimen containing cracks, rubbing of the crack surfaces produces hysteresis damping which may be a measure of the state of crack development. It may, in fact, be possible to develop a technique whereby localized vibrations may be imposed, and thereby to isolate regions of crack development. Correlation with experience may provide a criterion whereby removal of a part from service may be effected before danger develops of catastrophic failure.

Since means of inducing vibration are relatively simple, this method may prove very promising. The amount of energy input to produce a given amplitude of vibration may serve as a measure of hysteresis.

Crack Detection by Magnetic and Ultrasonic Means. If the material is magnetic, it is obviously possible to use the evidence of disturbance of the magnetic field in the vicinity of a crack as a means of detecting the crack. For surface cracks the magnaflux method has already been developed to a high degree for this purpose, but other techniques can be developed for detection of internal cracks. Ultrasonics may also prove useful for the measurement of internal cracks. A high frequency vibration is imposed on the material, and the presence of cracks is observed by the reflection of these waves from the crack surfaces. Ultrasonics as a means of detecting internal flaws is not new, and its application to the study of fatigue damage should not prove too difficult.

Concluding Remarks

In the foregoing discussion an attempt has been made to outline a large number of programs, any of which should ultimately prove useful in pressure vessel application. It is left to the Pressure Vessel Research Committee to combine these remarks with their experienced knowledge of current problems relating to pressure vessels to select a suitable research program for their support. A few subjective remarks may, however, be in order as a guide to the final selections.

Regarding the choice of a suitable cumulative damage theory, it is recognized that a large number of methods have been proposed, and much effort has been exerted to prove or to disprove these theories, or to test their regions of validity. Such research is very useful, but it can generally be expected that it will be performed regardless of support by the Pressure Vessel Research Committee.

Proponents of each of the methods were probably supported by their parent or other sponsoring organizations, and probably will continue to receive support in further pursuance of their work. If they need further support, they will probably solicit it themselves. Furthermore, the areas of divergence of the various theories are not so great that they require clarification of first magnitude. Since funds are limited, therefore, it is felt that this type of research may best be left to others, or to later study. As a first approximation the Linear Damage Rule may be adopted without fear of gross error for most practical cases.

Other areas also seem to be getting some support. At the Fourth Pacific Area National Meeting of the ASTM, held Sept. 30–Oct. 5, 1962 at Los Angeles, a symposium was presented in which it was clear that mean stress[15] and stress state[16] are currently receiving attention.

One area that would appear to be of great importance, but which does not appear to be getting a great deal of support at the present time, is the effect of thermal stress, either under constant amplitude or under cumulative fatigue conditions, and the relation of such thermal stress fatigue to fatigue by mechanical cycling. It has already been indicated that large discrepancies have been observed. If these discrepancies are indeed real, then the designer of a pressure vessel must give the proper attention to the thermal stress fatigue aspect in predicting life; if they are due to experimental technique, this should be clarified. In any case, it would appear that this problem is of sufficient importance to merit special consideration by the Pressure Vessel Research Committee, especially since it is not receiving much attention elsewhere.* Using pressure vessel materials, and temperature ranges and thermal strains representative of pressure vessel service, comparisons should be made of experimental cyclic lives and predictions based on behavior in mechanical strain cycling. Such comparisons should prove of great value in future design of pressure vessels in which thermal stresses play an important role in life determination.

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* Note, however, that Taira[17] has presented interesting information on this subject.