RENDEZVOUS CAPABILITY OF
HORIZONTAL-TAKE-OFF LAUNCH VEHICLE
WITH AIR-BREATHING PROPULSION

by Charlie M. Jackson, Jr.

Langley Research Center
Langley Station, Hampton, Va.
RENDEZVOUS CAPABILITY OF
HORIZONTAL-TAKE-OFF LAUNCH VEHICLE WITH
AIR-BREATHING PROPULSION

By Charlie M. Jackson, Jr.

Langley Research Center
Langley Station, Hampton, Va.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

For sale by the Office of Technical Services, Department of Commerce,
Washington, D.C. 20230 -- Price $0.75
RENDEZVOUS CAPABILITY OF
HORIZONTAL-TAKE-OFF LAUNCH VEHICLE WITH
AIR-BREATHING PROPULSION

By Charlie M. Jackson, Jr.
Langley Research Center

SUMMARY

An investigation was made to determine the rendezvous capability of a horizontal-take-off launch vehicle with an air-breathing propulsion system. A simplified closed-form analysis was made in order to calculate the offset launch capability and allowable launch-time error. Although a more detailed investigation would undoubtedly indicate additional improvements in the rendezvous capability, it is believed that the present analysis gives an indication of this capability which is consistent with the accuracy of current component performance estimates for such a vehicle.

The results of the investigation indicate large increases in offset launch capability for a vehicle with aerodynamic lift and air-breathing propulsion in comparison with that of a vehicle utilizing an orbital-plane-change maneuver. The maneuver associated with the improved offset capability of the air-breathing launch vehicle is, however, not without problems. Aerodynamic heating is, as indicated by this investigation, a serious constraint inasmuch as the most efficient cruise velocities are consistent with the most severe heating conditions.

Influence coefficients were determined for the significant parameters assumed for this analysis, and none were found to be so important that reasonable variations would affect the consideration of offset capability as an advantage for the horizontal-take-off launch vehicle with air-breathing propulsion.

Investigation of the allowable launch-time error with respect to reduction of the maximum time in the parking orbit indicated that for a target orbit at an altitude of 300 nautical miles, a reduction of only a few hours was possible with an orbital phasing maneuver or a combination aerodynamic-orbital phasing maneuver. A comparison of the two types of maneuvers indicated no distinct advantage for either.

INTRODUCTION

The subject of near-earth rendezvous has received considerable attention in the past few years both analytically (using the techniques of orbital
mechanics) and experimentally (using simulators to determine the usefulness of a pilot in the control loop). The analytical investigations have indicated several methods of increasing the rendezvous capability of ballistic launch vehicles; the use of rendezvous-compatible satellite orbits (ref. 1) is one such method. Another approach to rendezvous is the phasing or chasing method (ref. 2) with which launch-time errors can be corrected by remaining in a parking orbit until transfer to the target's orbit and position can be made. These techniques increase rendezvous capability for the ballistic launch vehicle without incurring significant fuel expenditures. However, to effect a significant increase in the launch-time window (time period in one earth day during which the interceptor may be launched into an orbit co-planar with the target orbit) the launch vehicle must be capable of placing the payload in an orbit which is offset from the launch point. One method of obtaining this offset at near minimal fuel cost (described in ref. 2) is by launching into an orbital plane with minimum inclination to the target orbital plane and by applying an impulse at the intersection of the planes (90° from launch).

The concept of a recoverable winged launch vehicle with some aerodynamic lift capability offers the possibility of obtaining offset range (perpendicular distance from launch site to orbit plane) with an aerodynamic assisted turn. The low specific impulse associated with an all-rocket propulsion system precludes its use for an offset maneuver. However, an air-breathing horizontal-take-off launch vehicle (ref. 3) offers relatively good specific impulse, and if the fuel is available it can efficiently perform the cruise-turn maneuver required for offset launch. Another possible advantage of the air-breathing launch vehicle is the correction (by loiter maneuver) of launch-time error when, due to operating problems, the interceptor is launched before the time for direct rendezvous (subsequently referred to in this report as "lead time"). The loiter capability of the air-breathing vehicle eliminates the need to correct small injection time errors by the chasing orbit technique and therefore has the potential of reducing the maximum time from launch to rendezvous. These considerations indicate the possibility of an increased rendezvous capability for the air-breathing launch vehicle compared with that of a ballistic launch vehicle or a rocket-propelled winged launch vehicle.

The present report is a preliminary analysis of several possible combinations of first-stage cruise and cruise-turn maneuvers (including subsonic loiter and/or cruise, high-speed cruise, and high-speed turn segments) required to place the second stage in an offset orbital plane. An effort is made to point out some of the more important design problems and compromises associated with an offset launch mission composed of the particular flight segments considered.

SYMBOLS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>range during acceleration from take-off velocity to offset-maneuver velocity (BC in fig. 1), ft</td>
</tr>
<tr>
<td>b</td>
<td>subsonic cruise range (OB in fig. 1), ft</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>$C_{D,i}$</td>
<td>induced-drag coefficient, $\frac{\text{Induced drag}}{Q_S}$</td>
</tr>
<tr>
<td>$C_{D,\text{min}}$</td>
<td>minimum-drag coefficient, $\frac{\text{Minimum drag}}{Q_S}$</td>
</tr>
<tr>
<td>$C_I$</td>
<td>nondimensional influence coefficient, percent change of offset distance during high-speed cruise and turn divided by percent change of parameter</td>
</tr>
<tr>
<td>$C_L$</td>
<td>lift coefficient, $\frac{\text{Lift}}{Q_S}$</td>
</tr>
<tr>
<td>$C_{L,\alpha}$</td>
<td>lift-curve slope, $\frac{1}{\text{rad}}$</td>
</tr>
<tr>
<td>$D$</td>
<td>total drag, lb</td>
</tr>
<tr>
<td>$D_i$</td>
<td>induced drag, lb</td>
</tr>
<tr>
<td>$g$</td>
<td>acceleration due to earth's gravitational field, 32.174 ft/sec$^2$</td>
</tr>
<tr>
<td>$h$</td>
<td>altitude above earth's surface, ft</td>
</tr>
<tr>
<td>$I_{sp}$</td>
<td>installed specific impulse, sec</td>
</tr>
<tr>
<td>$L$</td>
<td>lift, lb</td>
</tr>
<tr>
<td>$M$</td>
<td>Mach number</td>
</tr>
<tr>
<td>$m$</td>
<td>vehicle mass, slugs</td>
</tr>
<tr>
<td>$Q_{\text{offset}}$</td>
<td>total heat input at stagnation point for offset maneuver, Btu/ft$^2$</td>
</tr>
<tr>
<td>$Q_{\text{accel}}$</td>
<td>total heat input at stagnation point for acceleration from take-off to stage separation, Btu/ft$^2$</td>
</tr>
<tr>
<td>$\dot{Q}$</td>
<td>convective heat-transfer rate, Btu/ft$^2$-sec</td>
</tr>
<tr>
<td>$q$</td>
<td>dynamic pressure, lb/ft$^2$</td>
</tr>
<tr>
<td>$R$</td>
<td>earth radius, $20.89 \times 10^6$ ft</td>
</tr>
<tr>
<td>$r$</td>
<td>turn radius, ft</td>
</tr>
<tr>
<td>$S$</td>
<td>reference area, ft$^2$</td>
</tr>
<tr>
<td>$T$</td>
<td>thrust, lb</td>
</tr>
</tbody>
</table>
time, sec

\( t_\text{l} \)  
\( t_\text{l} \) time lapse from end of offset maneuver to 100-nautical-mile orbit, sec

\( t_\eta \)  
\( t_\eta \) time for target satellite to traverse \( \eta \), sec (see fig. 4)

\( V \)  
\( V \) velocity, ft/sec

\( W \)  
\( W \) vehicle weight, lb

\( \dot{W} \)  
\( \dot{W} \) fuel flow rate, lb/sec

\( x \)  
\( x \) distance from launch point to rendezvous measured parallel to target orbit, ft (see fig. 4)

\( y \)  
\( y \) perpendicular distance from launch point to target orbit, ft (see fig. 1)

\( \alpha \)  
\( \alpha \) angle of attack, deg or rad

\( \beta \)  
\( \beta \) atmospheric density decay parameter, \( 4.2553 \times 10^{-5} \frac{1}{\text{ft}} \)

\( \gamma \)  
\( \gamma \) flight-path angle measured positive up from horizontal, deg

\( \eta \)  
\( \eta \) distance along orbital path from point nearest the launch site to target satellite at time of launch, ft

\( \rho \)  
\( \rho \) atmospheric density, \( \rho_0 \exp(-\beta h) \), slugs/ft\(^3\)

\( \phi \)  
\( \phi \) bank angle, rad

\( \psi \)  
\( \psi \) heading angle or turn angle, rad (see fig. 1)

Subscripts:

\( 0, A, B, C, D, E, F \)  
\( 0, A, B, C, D, E, F \) points along first-stage trajectory as described in figure 1

bank  
banked

cruise  
high-speed cruise

max  
maximum

min  
minimum

\( \text{o} \)  
\( \text{o} \) sea level or take-off

\( \text{opt} \)  
\( \text{opt} \) optimum

\( s \)  
\( s \) target satellite
In order to delineate some of the problems associated with an offset cruise and turn maneuver, a boost trajectory with the simplified offset maneuver described by figure 1 was analyzed for a winged air-breathing first-stage vehicle with an internally stored ballistic rocket second stage. In figure 1 the vehicle is launched at point O. A subsonic loiter (path OAO) involving a 360° turn, the turn radius being defined by the desired loiter time, was considered in addition to a subsonic cruise phase (path OB). The acceleration to high-speed cruise velocity is represented by the segment BC, high-speed cruise by CD, turn into the orbital plane at cruise altitude and velocity by DE, and finally acceleration from cruise velocity to stage-separation velocity by EF. For the present investigation the velocity at stage separation is assumed to be 8000 ft/sec. The performance for all cruise and turn portions of the flight path is calculated from steady-state analysis which assumes average values of weight, aerodynamics, and propulsion characteristics. Acceleration phases are analyzed by stepwise integration of the two-dimensional equations of motion. The best offset distance (maximum with respect to the fuel consumed) is obtained for a specified amount of fuel available for the necessary cruise and turn maneuver. The return maneuver is not considered in this investigation. However, it can be of considerable importance if the offset distance is larger than the glide-range capability of the vehicle and if no alternate landing point is available.

Aerodynamic Maneuvers

**Cruise flight segments.** The fuel consumption associated with a cruise maneuver was obtained by assuming a constant fuel flow rate:
For the subsonic cruise (OB), a velocity of 900 ft/sec and a dynamic pressure of 900 lb/ft² were used. For the high-speed cruise (CD), constant velocities which varied over the range from 2000 to 8000 ft/sec and a dynamic pressure of 1300 lb/ft² were used.

**Turn flight segments.**- For turn maneuvers at constant velocity, altitude, and turn radius the approximate total fuel required consists of the fuel used for a cruise with range equal to arc length and the fuel used to maintain the banked condition of the vehicle. (The increased thrust required to overcome the increase in drag due to lift, which may be as much as 50 percent, is assumed available.) In order to calculate the additional fuel due to the banked condition, consider the following sketch in which it has been assumed that the angles of attack are small enough such that the vertical thrust component is negligible.

The assumptions of small angles of attack and constant velocity allow the following expression:

\[ T + \Delta T = D + \Delta D = D_{\text{min}} + D_1 + \Delta D_1 \]  

(2)

where \( \Delta T \) and \( \Delta D \) are the increments in thrust and drag due to the banked condition. By using a parabolic drag polar, equation (2) can be written as follows:

\[ T + \Delta T = D_{\text{min}} + \frac{C_D, i}{C_L^2} \frac{(L + \Delta L)^2}{\bar{q}S} = D_{\text{min}} + \frac{C_D, i}{C_L^2} \frac{L^2 + 2L \Delta L + \Delta L^2}{\bar{q}S} \]  

(3)

Since the cruise thrust \( T \) and cruise drag \( D \) are equal, the thrust increment can be expressed:
\[ \Delta T = \frac{D_1}{L^2} \Delta L (2L + \Delta L) \]  \hfill (4a)

Substituting \( L (\sec \phi - 1) \) for \( \Delta L \) in equation (4a) gives

\[ \Delta T = \frac{D_1}{L} \tan^2 \phi \]  \hfill (4b)

For a constant bank angle and constant velocity (constant turn radius), equation (4b) can be used to approximate the additional fuel consumed due to the banked condition in terms of the specific impulse, turn time, and angle through which the flight path has turned. The resulting expression is

\[ \Delta W_{\text{bank}} = \frac{C_{D,1}}{C_L^2} \frac{\bar{W}^2\psi^2}{g^2 I_{sp} \bar{g}} t_{\text{turn}} \]  \hfill (5)

where \( \bar{W} \) is the average weight during the maneuver.

Combining the equations for cruise and bank fuel results in the following equation:

\[ \Delta W_{\text{turn}} = \frac{C_{D,1}}{C_L^2} \frac{\bar{W}^2\psi^2}{g^2 I_{sp} \bar{g}} t_{\text{turn}} \bar{W} + \frac{t_{\text{turn}}}{(L/D) I_{sp}} \]  \hfill (6)

Optimization of high-speed cruise and turn maneuvers. - The fuel used for the high-speed cruise and turn maneuvers (CDE), obtained from equations (1) and (6) and the geometry of figure 1, is

\[ \Delta W = \left( \frac{\bar{W}}{(L/D) I_{sp}} \left[ \frac{y}{V \sin \psi} - \frac{a + b}{V} - \frac{r(1 - \cos \psi)}{V \sin \psi} \right] \right) + \left( \frac{\bar{W} \psi r}{(L/D) V I_{sp}} \right) + \left( \frac{C_{D,1}}{C_L^2} \frac{\bar{W}^2\psi^2}{g^2 I_{sp} \bar{g}} \frac{\psi V}{r} \right) \]  \hfill (7)

where the first braced term represents the fuel weight for the high-speed cruise (CD), the second braced term represents the fuel expended to maintain a normal cruise for the arc length of the turn DE, and the last braced term represents that additional fuel required to maintain the banked condition during the turn.

Since equation (7) indicates that the fuel required for the high-speed cruise and turn maneuvers is a strong function of the turn radius and initial heading angle \( \psi \), it would be interesting to investigate the values of these parameters with respect to obtaining a mission with minimum fuel expenditure for a fixed offset distance \( y \). The expression for values of \( r \) which yield minimum fuel expenditure can be obtained mathematically by setting \( \frac{\partial \Delta W}{\partial r} \).
equal to zero. The resulting equation is:

\[ r_{opt} = \frac{\sqrt{\frac{c}{\rho}} V}{g} \sqrt{\frac{C_{D,i} L \bar{W}}{C_{L}^2 D S}} \left( \frac{\psi}{\sin \psi} \right) \quad (8) \]

The best offset distance was calculated by replacing \( r \) in equation (7) by \( r_{opt} \) as given in equation (8) and solving for \( y \) where \( 0 < \psi < \psi_{\text{limit}} \) and the limiting value of \( \psi \) was determined by the available fuel \( \Delta W \). The high-speed cruise and turn maneuvers were examined for the range of \( \psi \) to determine the best value of \( y \) consistent with the present analysis.

**Acceleration segments.** Calculation of the trajectory parameters for the acceleration segments of the boost mission (BC and EF in fig. 1) was accomplished by stepwise integration of the two-dimensional equations of motion. The basic assumptions associated with these equations are: spherical nonrotating earth, exponential atmosphere, and inverse square variation of the gravitational field. The equations of motion with attendant auxiliary equations are as follows:

\[ \frac{d\gamma}{dt} = \frac{T \sin \alpha}{mV} + \frac{C_{L} q}{mV} \left[ g \cos \gamma + g \left( 1 - \frac{V^2}{g(R + h)} \right) \right] \quad (9) \]

\[ \frac{dV}{dt} = \frac{T \cos \alpha - qS \left( C_{D,\text{min}} + \frac{C_{D,i}}{C_{L}^2} C_{L}^2 \right)}{m} - g \sin \gamma \quad (10) \]

\[ q = \frac{\rho_o \exp(-\beta h) V^2}{2} \quad (11) \]

\[ \frac{dh}{dt} = V \sin \gamma \quad (12) \]

\[ \frac{dm}{dt} = - \frac{T}{gI_{\text{sp}}} \quad (13) \]

\[ \alpha = (\alpha)_{C_{L}=0} + \frac{C_{L}}{C_{L\alpha}} \quad (14) \]

\[ g = g_o \left( \frac{R}{R + h} \right)^2 \quad (15) \]
For the acceleration segments of the mission, a typical trajectory for a horizontal-take-off launch vehicle with an air-breathing propulsion system was simulated by stepwise integration of equations (9) to (15) with the estimated aerodynamic and propulsion characteristics presented in figure 2. The flight path consisted of maintaining a constant dynamic pressure of 1500 lb/ft². The results of these calculations, presented in figure 3, provide the necessary input parameters (weight and range) for the steady-state cruise and turn calculations.

Figure 2.- Assumed aerodynamic and propulsion characteristics of typical launch vehicle.

Figure 3.- Trajectory for typical launch vehicle.

Aerodynamic heating considerations.- Since consideration is given to high-speed cruise maneuvers, the question of aerodynamic heat input must be evaluated. In the present investigation only the convective heat input is considered for the stagnation region on a sphere of radius equal to 1 foot. These conditions were imposed in the interest of generality and simplicity. A more detailed calculation of heating characteristics involves a knowledge of the vehicle shape and heat-transfer characteristics as well as a stepwise integration of the heat input (to determine surface temperature rise). Basing the analysis on only the convective heat transfer is considered justified because reference 4 indicates that for constant surface temperature the radiation component of heat transfer is small compared with the convective heat transfer for the small-diameter leading edges of high L/D configurations such as the horizontal-take-off launch vehicle. A simple calculation indicated that for a leading edge with
active cooling to a temperature of \(1000^\circ\) R the radiation-heat-transfer rate would amount to a small percentage of the convective heating rate. For example, at a cruise velocity of \(4000\) ft/sec and a dynamic pressure of \(1500\) lb/ft\(^2\) the radiation heating rate was 2 percent of the convective heating rate.

The semi-empirical equation used in the present analysis to compute the stagnation convective heat-transfer rate for laminar flow over a sphere of radius equal to 1 foot is:

\[
\dot{Q} = 120 \times 10^{-12} \left( \frac{\rho}{\rho_0} \right)^{1/2} v^{3.22}
\]  

(16)

and is discussed in reference 5 for heating rates at near satellite velocities. A comparison of the heating rate given by equation (16) was made with the more rigorous methods of reference 6. For the flight conditions of interest in the present analysis the results indicated that, although the heating rate was in error by as much as an order of magnitude, the variation with flight velocity was in general agreement. Therefore, the heating rates presented in this analysis should be used only for the purpose of indicating trends in the coolant requirements for variations in offset distance and velocity.

Space Maneuvers and Comparisons

Space maneuvers.- A cursory evaluation of the rendezvous capability of a boost system with orbital maneuver capability can be obtained by allowing offset distances to be corrected by orbital-plane changes.

If the orbital-plane change desired is assumed to occur \(90^\circ\) down range from the launch point (a near optimum condition), the fuel cost can be expressed:

\[
\frac{\Delta W}{W} = 1 - \exp \left[ - \frac{V_S (\text{Range})}{3435 g I_{sp}} \right]
\]

(17)

where the offset range is expressed in nautical miles and \(\Delta W/W\) is the fuel fraction required at the plane change.

Launch-time errors can be corrected by the parking-orbit technique. For example, if the launch is late then the interceptor goes into an orbit at an altitude lower than the target orbit (orbital period less than that of the target), makes up the time misalignment with an appropriate number of orbits (periods), and then transfers to the target orbit for rendezvous. In the present analysis the lower altitude limit for a parking orbit was assumed to be 100 nautical miles in order to avoid the problems of orbit decay due to atmospheric drag. It is of interest to note that with the impulsive change in energy technique used in the present analysis, utilizing a parking orbit at altitude lower than the target orbit requires the same energy or fuel as does direct launch to the target orbit. However, for parking orbits at higher altitudes than that of the target orbit (conditions necessary to correct launch lead time)
the energy requirements are more than those for a direct launch since the inter-
ceptor must achieve a higher energy parking orbit and then expend additional
energy to reduce the parking orbit to the energy level of the target orbit. For
the present analysis the nominal mission is assumed to be a Hohmann transfer
from circular orbital conditions at an altitude of 100 nautical miles to the
target orbital altitude of 300 nautical miles. The fuel requirements for
orbital maneuvers, including transfer to parking orbits above and below the tar-
get orbits, have been calculated with the use of equation (17) and an assumed
specific impulse of 450 seconds.

Method of comparison of space and aerodynamic maneuvers.- One way to com-
pare the offset efficiency of space maneuvers with that of aerodynamic maneuvers
is to examine the reduction of orbital weight for a boost system with constant
gross take-off weight. Equation (17) directly gives the fuel fraction required
for the offset by a space maneuver as a function of specific impulse. However,
the effect of the fuel consumed early in the boost phase on the orbital weight
is dependent on the boost-system performance and weights. In order to evaluate
the change in orbital weight for the aerodynamic maneuvers, consider the booster
weight at the points on the ascent path defined by the following sketch:

![Diagram showing the ascent path with points O, C, E, F, G, and H.
- First-stage maneuvers
- Upper-stage maneuvers
- Decreasing weight
- Velocity]

where points O, C, E, and F correspond to figure 1, point G refers to conditions
after stage separation, and point H refers to conditions at orbital injection.
By assuming constant weight ratios during the acceleration segments, \( \frac{W_C}{W_O} \),
\( \frac{W_F}{W_E} \), and \( \frac{W_R}{W_G} \), and a constant inert weight ratio for the first stage,
\( \frac{W_F - W_G}{W_O} \), the weight at orbital conditions can be expressed in terms of these
constant ratios and the fuel ratio available for the aerodynamic offset maneuver
\( \left( \frac{W_C - W_E}{W_O} \equiv \frac{\Delta W}{W_O} \right) \) as

\[
\frac{W_H}{W_O} = \frac{W_H}{W_G} \left[ \frac{W_F}{W_E} \left( \frac{W_C}{W_O} - \frac{\Delta W}{W_O} \right) - \frac{W_F - W_G}{W_O} \right] 
\]

Equation (18) gives the weight at orbital conditions for the nominal ascent
path when \( \frac{\Delta W}{W_O} \) becomes zero; thus, the expression for the relative orbital
weight due to a fuel expenditure, \( \Delta W \), in the first stage can be written:
Relative orbital weight \( \frac{W_H}{W_O} = 1 - \frac{W_F}{W_E} \) (19)

For the present analysis the ratios \( \frac{W_H}{W_G} \) and \( \frac{W_F}{W_E} \) were obtained from figure 3, and \( \frac{W_H}{W_O} \) was assumed to be 0.132.

**Launch-Lead-Time Correction Maneuver**

The launch-lead-time error can have a large effect on the time required to rendezvous if the low-parking-orbit technique is used to compensate for the resulting interceptor-satellite phase misalignment. The low-parking-orbit technique results in maximum time to rendezvous for small lead-time errors. (Interceptor is just ahead of optimum rendezvous position in a lower orbit and must chase the target satellite to make up nearly 360° phase misalignment.) In the present analysis the maximum time to rendezvous is defined as the time required to maneuver the interceptor from a circular orbit at an altitude of 100 nautical miles and co-planar with the satellite orbit (altitude of 300 nautical miles) to a position coincident with the satellite and in the same orbit regardless of the satellite position in the orbit at the time of interceptor launch. Small lead-time errors can be corrected with small parking-orbit time at considerable fuel expense by parking in an orbit with a longer period (higher orbit) than that of the target orbit. From the standpoint of system design it is important to know the maximum time to rendezvous and the fuel cost of reducing this time. The philosophy of the present analysis considers that a specific amount of fuel is available in order to enhance the rendezvous potential of the horizontal-take-off launch vehicle with an air-breathing propulsion system. A maneuver has been investigated which uses the available fuel to increase the offset capability of the vehicle. It is also in order to investigate the use of some or all of this fuel to reduce the maximum time to rendezvous, especially for those cases of offset which do not require all the available fuel.

The lateral maneuver considered for correction of launch-lead-time error is represented in figure 4. As previously mentioned, for a given fuel available and offset distance a range of possible heading angles exists for the offset maneuver. Two such maneuvers are represented by paths 1 and 2 in figure 4. Path 1 represents the on-time launch condition and is assumed to be the path for minimum fuel consumption; hence, there is no subsonic cruise (0 and B1 coincide) and no subsonic loiter (vehicle accelerates directly to cruise velocity at C1). Path 2 on the other hand incorporates the subsonic loiter (OA20) and/or subsonic cruise (OB2) and therefore requires more fuel than path 1. The time interval between the direct rendezvous for path 1 (on-time launch) and the direct rendezvous for path 2 (late launch) is the launch-time correction capability available for the difference in fuel expenditure between the two paths. Since only the correction capability is of interest here (not a specific problem), an early launch can be corrected by simply designating path 2 as the
on-time path (retaining the subsonic cruise and loiter) and using the direct path (path 1) for correction.

Figure 4.- Geometry of launch-lead-time correction maneuver.

In order to determine the time differences for two such paths and the respective fuel expenditures, consider the time interval required for the target satellite to cover the distance $\eta$ indicated in figure 4.

$$t_\eta = \left[ t_{0AO} + t_{OB} + t_{BC} + t_{CE} + t_L \right] - \frac{x}{V_s} \tag{20}$$

The bracketed term on the right of equation (20) is the summation of the times for the respective segments of the boost maneuver where $t_L$ includes the segments from the end of the offset maneuver to a 100-nautical-mile orbit. The second term represents the time required for the target satellite to move the distance $x$. For a specific offset distance and quantity of fuel there exists a path for maximum time and one for minimum time required for the target satellite to cover the distance $\eta$. The allowable launch lead (or lag) time is the difference between these extremes. The use of equation (20) to determine these extremes involves the assumption of a nonrotating earth (launch point has not moved during the time $\Delta t_\eta$). The reason for this assumption is that of generality, since the actual velocity of the satellite relative to the launch point is dependent on orbital inclination and launch point coordinates.

The additional fuel expenditures for subsonic loiter and cruise were calculated by the techniques previously outlined. An average wing loading of 65 lb/ft$^2$ was assumed for the maneuver. The aerodynamic and propulsion characteristics presented in figure 2 were assumed to be average operating conditions at a flight velocity of 900 ft/sec and a dynamic pressure of 900 lb/ft$^2$. The fuel cost of the subsonic cruise was calculated by assuming a constant fuel
Figure 5. Method of determining maximum acceptable launch-lead-time error.

$V_{\text{cruise}} = 4000$ ft/sec; offset range = 400 n. mi.
flow rate (eq. (1)), and the loiter fuel was calculated from equation (6) with the assumption that \( \psi = 360^\circ \).

An example of the method used to determine the combination of subsonic loiter, subsonic cruise, and high-speed cruise-turn segments for the largest allowable lead-time error is represented graphically in figure 5 for an offset maneuver velocity of 4000 ft/sec and an offset distance of 400 nautical miles. Similar figures are necessary to complete the investigation of the range of offset distances considered in this analysis. The fuel cost of the offset maneuver is presented in figure 5(a) as a function of heading angle for various values of subsonic range. If the total fuel fraction available for the maneuver is 0.06 at take-off then for a large range of heading angles the offset (even with subsonic cruise) does not require the total fuel fraction. The time, \( t_\eta - t_\zeta \), is increased considerably by using the fuel \( \Delta W_{OAO} \) (fig. 5(a)) for a loiter maneuver (increasing \( t_\eta - t_\zeta \) by \( t_{OAO} \), fig. 5(b)). Figure 5(b) shows that the offset maneuver can be accomplished for a range of heading angles and from a minimum time, \( t_\eta - t_\zeta \), of about 950 seconds (\( \psi = 80^\circ \), no subsonic cruise or loiter) to a maximum of about 2850 seconds (\( \psi = 55^\circ \), subsonic cruise for 340 nautical miles and no loiter), which results in a lead-time-error correction capability, \( \Delta t_\eta \), of about 1900 seconds for this situation.

RESULTS AND DISCUSSION

Offset Capability

Aerodynamic maneuver capability.- The techniques outlined in the section entitled "Analysis" were used to analyze the fuel cost of the offset maneuver as defined in the present paper. Of primary interest is the effect of maneuver velocity on fuel consumption since the cruise efficiency and the aerodynamic heating rate are strong functions of velocity. The fuel cost for best offset distances obtained at velocities from 2000 to 8000 ft/sec is presented in figure 6 for a typical launch vehicle with air-breathing propulsion and aerodynamic lift capability. Consideration of the effects of maneuver velocity on the offset capability of the system with aerodynamic cruise indicates that for large values of offset distance (\( y >> a \)) the maneuver efficiency increases with velocity. This increase is due to: first, a general increase of the cruise efficiency or range factor \( V(L/D)I_{sp} \) for the aerodynamic and propulsion characteristics used (fig. 2), and second, the increased offset distance due to the change in the acceleration range (indicated by the lower terminal points of curves) associated with
increased cruise velocity (for $y \gg a$ the heading angle $\psi$ was near 90$^\circ$ in all cases). As the available fuel (hence best offset distance obtainable) decreases ($y$ approaches $a$) the maneuver efficiency at the lower cruise velocities increases because the increase in acceleration range with cruise velocity, now a disadvantage, requires the heading angle $\psi$ to depart from 90$^\circ$. This effect of maneuver velocity is a result of the framework of the present analysis and probably would not exist with a more practical integrated maneuver (turn during acceleration).

Since the high-speed cruise and turn maneuvers considered thus far have been "best" with respect to maximum offset distance for minimum fuel fraction, it is in order to examine some of the dependent parameters which may offer constraints to these "best" results. Figure 7 represents the variation of some operating parameters with velocity during the high-speed cruise and turn maneuvers. The band represents the variation due to all turn radii considered in the present investigation. The optimum turn radius varied from 80 nautical miles at a velocity of 2000 ft/sec to 200 nautical miles at 8000 ft/sec. Examination of the ranges of values of the operating parameters of figure 7 indicates that these parameters impose no serious constraints with the possible exceptions of transverse acceleration, which for maneuver velocities less than 4000 ft/sec exceeds 2g, and the thrust requirements, which for high velocities amount to about a 50-percent thrust margin.

Intuitively, one might expect aerodynamic heating to be a serious operating constraint since the vehicle is required to cruise at near peak heating conditions as $V_{\text{cruise}}$ approaches 8000 ft/sec. The simplified heating analysis described in the section entitled "Analysis" was applied to the offset maneuvers considered thus far and the resulting ratio of total heat input during offset maneuver to total heat input without offset maneuver (a measure of the additional cooling capacity required) is presented in figure 8 as a function of offset distance for the cruise velocities considered and three fuel fractions required for offset range (0.015, 0.030, and 0.060). According to the results of the present analysis (see fig. 8), the coolant requirements for hypersonic cruise
(V_{cruise} > 4000 \text{ ft/sec}) are significantly increased with little increase in offset range. These effects are due primarily to increased heating rates at the higher velocities and to the decreased air-breathing propulsion efficiency. As a result of this cursory heating analysis an offset maneuver velocity of 4000 \text{ ft/sec} was chosen for further analysis.

Since the aerodynamic and propulsion characteristics (fig. 2) used in the investigation thus far have been typical of the state-of-the-art estimates, it is in order to investigate, at least to a limited extent, the effects of deviation from these estimates. Nondimensional influence coefficients are presented in figure 9 for the offset-range effects of the trajectory, aerodynamic, and propulsion parameters associated with the best offset maneuver at a velocity of 4000 \text{ ft/sec}, with an average wing loading of 59 \text{ lb/ft}^2 and for a fuel cost of 3 percent of the gross take-off weight. The variation of these parameters was assumed to take place only during the high-speed cruise and turn maneuvers and is compared with the offset distance obtained by these segments. Figure 9 indicates that the dynamic pressure or maneuver altitude has considerable influence on the portion of the offset range due to high-speed cruise and turn. For example, an increase of 1.0 percent in dynamic pressure results in a decrease of cruise-turn offset distance of about 1.22 percent. This effect is due to the reduction in operating L/D. Although all of the parameters considered have a significant effect on the offset distance, for the case considered none is so important that reasonable variations would affect the consideration of the improved offset capability as an advantage for the horizontal-take-off launch vehicle with air-breathing propulsion.

A comparison of the simplified method of calculating high-speed cruise-turn performance used in the present analysis was made with the more complete results of a stepwise integration of the differential equations of motion. By using the differential equations of motion, the additional thrust and fuel consumed due to the turn were evaluated with consideration of the effects of angle of attack on the thrust vector.
The lateral displacement was computed by stepwise integration of the lateral acceleration. The following table presents a comparison of some of the performance parameters for a high-speed cruise-turn maneuver (typical of those presented in the present report) computed by the two techniques under discussion:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Stepwise integration of differential equations</th>
<th>Closed-form equations</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{CE}$, ft/sec</td>
<td>4000</td>
<td>4000</td>
</tr>
<tr>
<td>$W_D/W_0$</td>
<td>0.8580</td>
<td>0.8580</td>
</tr>
<tr>
<td>$W_E/W_0$</td>
<td>0.8498</td>
<td>0.8486</td>
</tr>
<tr>
<td>$\psi$, deg</td>
<td>90</td>
<td>90</td>
</tr>
<tr>
<td>$\phi$, deg</td>
<td>60.9</td>
<td>60.9</td>
</tr>
<tr>
<td>Time, sec</td>
<td>111.0</td>
<td>108.7</td>
</tr>
<tr>
<td>Lateral range, n. mi.</td>
<td>465.8</td>
<td>472.0</td>
</tr>
<tr>
<td>Longitudinal range, n. mi.</td>
<td>455.2</td>
<td>455.2</td>
</tr>
</tbody>
</table>

This comparison indicates that the closed-form equations overestimate the amount of fuel used during the cruise-turn maneuver by 14.0 percent, probably a result of the neglect of centrifugal effects and the thrust component normal to the flight path. Although the typical example chosen showed some discrepancy between the simplified closed-form analysis and the more complete stepwise integration of the equations of motion, the closed-form method appears to be adequate for the scope of this investigation.

Space maneuvers and comparisons.- The results of the offset efficiency analysis presented in figure 6 for the air-breathing launch vehicle are compared in figure 10 with an orbital-plane-change maneuver. The basis of comparison is relative orbital weight for a vehicle with constant take-off weight. The results of figure 10 indicate a significant increase in offset capability with the use of an aerodynamic maneuver (obtainable with horizontal-take-off air-breathing launch vehicle) compared with an orbital-plane-change maneuver (obtainable with any vehicle).

Launch-Lead-Time Correction Capability

The launch-lead-time correction capability is presented in figure 11 as a function of attainable offset distance for different fuel fractions. Figure 11 indicates, for example, with an available fuel fraction of 0.03 and an offset requirement of 300 nautical miles, the vehicle can be launched as much as 750 seconds.
before design launch time and complete the rendezvous maneuver without the use of a parking orbit. The shape of the curves of figure 11 (characterized by a knee) reflects the advantage of using a subsonic cruise to obtain the offset distance. The subsonic cruise, although not the most efficient range maneuver, results in little loss of available lead time as indicated in figure 11 by the portion of the curve to the left of the knee. However, as y increases beyond the range available at subsonic cruise conditions for the specified fuel expenditure (to the right of the knee), it is necessary to use a high-speed cruise in order to meet the range requirement although the available lead time decreases rapidly.

Lead-time correction capability has obvious operational advantages by providing a margin of error for the design launch time of a specific launch. In addition, it potentially reduces the time spent in parking orbit for the launch conditions consistent with maximum parking-orbit time. Since the maximum possible interceptor-target misalignment is slightly less than one target orbital period and the lead-time correction directly reduces this misalignment, it follows that the lead-time correction will have a profound effect on the parking-orbit time necessary to correct the misalignment.

Figure 12 compares the effectiveness of correcting launch-time errors by using the parking-orbit technique in two different ways when an aerodynamic offset maneuver is required as well. In each case the excess maneuver fuel, above that required for the offset maneuver, has been employed to reduce the parking-orbit time from the calculated maximum.

For the first case (solid lines in fig. 12) the excess fuel is employed for an aerodynamic loiter and then a direct ascent is made to a 100-nautical-mile parking orbit. The reduction in maximum time to rendezvous shown is the decrease in the time spent in the 100-nautical-mile parking orbit prior to ascent to the target orbit of 300 nautical miles. For example, if a 300-nautical-mile offset is required
and a fuel fraction of 0.03 is available for offset and the parking-orbit-time reduction, then figure 11 indicates a lead-time capability of 750 seconds to be available for reduction of the maximum possible interceptor-target misalignment (note that this is about 14 percent of the parking-orbit period, 5250 seconds at 100 nautical miles), which must be corrected by a parking orbit. From another viewpoint, figure 12 indicates that the maximum time to rendezvous (consistent with a misalignment of slightly less than one orbital period) is 17.8 hours and that the reduced parking-orbit time, which is due to the available lead time of 750 seconds, is 15.3 hours, again resulting in a reduction of about 14 percent. For the second case considered (dashed lines in fig. 12) a direct ascent to a high parking orbit is made after the offset maneuver, with no aerodynamic correction available for interceptor-satellite misalignment. The excess fuel, above that required for the offset, is used to obtain a parking orbit at an altitude above the target orbit and then to descend to the target orbit for rendezvous. As an example, consider again an offset distance of 300 nautical miles and an available fuel fraction of 0.03 for cruise velocities of 4000 ft/sec. Figure 6 indicates that a fuel fraction of 0.018 is required to get into the orbital plane (correct the offset), which leaves an excess of 0.012 for orbital maneuver. This fuel excess will permit ascent to a parking orbit of 326 nautical miles and return to the target orbit. From figure 12 it can be seen that this technique will give a reduction in maximum time to rendezvous of 2.0 hours (17.8 hours minus 15.8 hours).

The results of the investigation of the reduction in maximum rendezvous time by an orbital phasing maneuver and a combination aerodynamic-orbital phasing maneuver indicated that a time reduction of a few hours is possible with small offset required. No distinct advantage of either system was evident.

**CONCLUDING REMARKS**

An investigation was made to determine the rendezvous capability of a horizontal-take-off launch vehicle with air-breathing propulsion. A simplified closed-form analysis was made in order to calculate the offset launch capability and allowable launch-lead-time error. Although a more detailed investigation would undoubtedly indicate additional improvements in the rendezvous capability, it is believed that the present analysis gives an indication of this capability which is consistent with the accuracy of current component performance estimates for such a vehicle.

The results of the investigation indicate large increases in offset launch capability for a vehicle with aerodynamic lift and air-breathing propulsion in comparison with that of a vehicle utilizing an orbital-plane-change maneuver. The maneuver associated with the improved offset capability of the air-breathing launch vehicle is, however, not without problems. Aerodynamic heating is, as indicated by this investigation, a serious constraint inasmuch as the most efficient cruise velocities are attended by the most severe heating conditions.

Influence coefficients were determined for the significant parameters assumed for this analysis, and none were found to be so important that reasonable
variations would affect the consideration of offset capability as an advantage for the air-breathing launch vehicle.

Investigation of the allowable launch-time error with respect to reduction of the maximum time in parking orbit indicated that for a target orbit at an altitude of 300 nautical miles, a reduction of only a few hours was possible with use of an orbital phasing maneuver or a combination aerodynamic-orbital phasing maneuver. A comparison of the two types of maneuvers indicated no distinct advantage for either.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Station, Hampton, Va., August 26, 1964.

REFERENCES


"The aeronautical and space activities of the United States shall be conducted so as to contribute . . . to the expansion of human knowledge of phenomena in the atmosphere and space. The Administration shall provide for the widest practicable and appropriate dissemination of information concerning its activities and the results thereof."

—National Aeronautics and Space Act of 1958

NASA SCIENTIFIC AND TECHNICAL PUBLICATIONS

TECHNICAL REPORTS: Scientific and technical information considered important, complete, and a lasting contribution to existing knowledge.

TECHNICAL NOTES: Information less broad in scope but nevertheless of importance as a contribution to existing knowledge.

TECHNICAL MEMORANDUMS: Information receiving limited distribution because of preliminary data, security classification, or other reasons.

CONTRACTOR REPORTS: Technical information generated in connection with a NASA contract or grant and released under NASA auspices.

TECHNICAL TRANSLATIONS: Information published in a foreign language considered to merit NASA distribution in English.

TECHNICAL REPRINTS: Information derived from NASA activities and initially published in the form of journal articles.

SPECIAL PUBLICATIONS: Information derived from or of value to NASA activities but not necessarily reporting the results of individual NASA-programmed scientific efforts. Publications include conference proceedings, monographs, data compilations, handbooks, sourcebooks, and special bibliographies.

Details on the availability of these publications may be obtained from:

SCIENTIFIC AND TECHNICAL INFORMATION DIVISION

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

Washington, D.C. 20546