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RADIANT HEAT TRANSFER TO ABSORBING GASES WITH FLOW AND CONDUCTION

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INTRODUCTION

During the past five years there has been a significant increase of interest in the problem of radiant heat transfer to absorbing gases. Recent activity in this area has been motivated by the advent of heat transfer problems arising in space vehicle reentry, MHD energy conversion, and energy transport in a gaseous nuclear reactor.

Hottel was one of the earliest workers in this field, having investigated radiant heat transfer from furnace gases as early as 1927. Although other workers have recently become active in the field by obtaining analytical solutions for heat transfer and temperature distributions (1), (2), (3), Hottel's analysis (4) of the heat transfer problem remains probably the most realistic since it is applicable to real (nongray) gases and may be applied to geometries of almost arbitrary shape.

There has been very little done so far in obtaining generalized results for radiation to a flowing gas or for combined radiation and conduction. Adrianov, (3), investigated radiation heat transfer to a flowing gas, but some of the assumptions in his analysis render his results valid only for the case of weak absorbtion in the gas. Viskanta, (2), has presented a rather complete analytical treatment of combined radiation and conduction between two infinite parallel plates separated by an absorbing gas, but only a few specific results were presented.

It is the purpose of this investigation to expand on the work done by both Adrianov and Viskanta which is cited above. The method used in the present analysis is a modification of Hottel's zoning technique (4). Results are obtained for a gray gas in a rectangular channel formed by two black, parallel flat plates of finite length, and are presented in generalized form for the case of radiation to flowing gases and also for combined radiation and conduction.

ANALYSIS

A two-dimensional analysis of radiation heat transfer is made for a gray gas enclosed in a rectangular channel formed by two black parallel flat plates of finite length and infinite width. A sketch of this configuration is shown in figure 1. A rigorous treatment of this problem requires the solution of the following two dimension integro-differential equation which represents the heat balance on an infinitesimal volume, dV, at any point in the channel.

$$-4k\sigma T^4(\vec{r}_o) + Gc_p \left.\frac{\partial T(\vec{r})}{\partial x}\right|_{\vec{r}=\vec{r}_o} + \lambda \left.\frac{\partial^2 T(\vec{r})}{\partial y^2}\right|_{\vec{r}=\vec{r}_o}$$

$$= k\iiint \sigma T^4(\vec{r}) f(\vec{r} - \vec{r}_o) d\tau + k\iint \sigma T_s^4(\vec{r}) g(\vec{r} - \vec{r}_o) da \qquad (1)$$

An explanation of the various terms in equation (1) is given below

k	absorbtion coefficient of the gas
$-4k\sigma T^4(\vec{r}_o)$	energy emitted per unit volume at $\vec{r} = \vec{r}_o$
$Gc_p \left.\frac{\partial T}{\partial x}\right\|_{r=r_o}$	rate of enthalpy increase of the flowing gas at $\vec{r} = \vec{r}_o$
$\lambda \frac{\partial^2 T(\vec{r})}{\partial y^2}$	net conduction heat transfer per unit volume
integrals on the right hand side	radiation absorbed from emission due to surrounding gas volume and radiation absorbed from emission of flat plate surfaces, respectively

If the conduction and convection terms are removed in equation (1), its solution is greatly simplified since the equation will then be linear in the emissive power σT^4. If in addition the length of the channel also becomes infinite, the integrals can be partially evaluated in closed form in terms of exponential-integral functions, as was done by Usiskin (1).

In the absence of these simplifying restrictions, the only feasible way to solve this problem appears to be application of a method similar to Hottel's where the two-dimensional integral-differential equation (1) is approximated by a system of algebraic equations. This is done by first dividing the region between the plates up into a 10×10 array of rectangular zones, 100 zones in all. The gas temperature variation within each of these zones may be approximated by a two-dimensional linear function. Heat balance equations in the form of equation (1) are now written for infinitesimal volume elements located at the center of each of 100 zones. The integral on the right of equation (1) is approximated by a finite summation over all the zones and surfaces. The derivatives are replaced by numerical approximations obtained from fitting a paraboloid through the temperatures of the zone in question and those of its adjacent neighbors in both directions. Each of the 100 zones is labeled by a dual subscript (i,j), i is the position along the length of the channel and j represents the position in the transverse direction. The resulting system of nonlinear algebraic equations, which is given below, are then solved for the 100 values of T_{ij}, the gas temperatures at the center of the zones, for a given set of boundary conditions (plate and end temperatures).

The conditions of the two ends of the channel are represented by assuming that the ends are porous black plugs having the same temperature as the effective gas temperature at the ends. The equations to be solved are thus:

$$-4k\sigma T_{ij}^4 + G_{cp}\frac{(T_{i+1,j} - T_{i-1,j})}{2\,\Delta x} + \lambda\frac{(T_{i,j+1} + T_{i,j-1} - 2T_{ij})}{\Delta y^2}$$

$$= k\sum_{m,n=1}^{10} \sigma T_{m,n}^4 f(\vec{r}(m,n) - \vec{r}(i,j))$$

$$+ k\sum_{m,n=1}^{10} \sigma T_s^4(m,n) g(\vec{r}(m,n) - \vec{r}(i,j)) \qquad i=1,10;\ j=1,10 \qquad (2)$$

The principal difference between the present approach and that of Hottel's is that here the heat balance is taken on an infinitesimal volume of the center of the zone, rather than on the entire zone. This approach results in a large reduction in the effort required to compute the "exchange factors" $f(\vec{r})$ and $g(\vec{r})$. Since the system of 100 equations (2) is generally nonlinear due to the presence of convection and conduction terms, it was solved using the Newton-Raphson iterative method for nonlinear algebraic equations. This method rapidly converges and the correct solution is obtained after only a few iterations.

RESULTS AND DISCUSSION

The system of equations described by equation (2) was solved on an IBM 7090 computer for a gray gas of constant absorptivity enclosed in a black walled rectangular channel of aspect ratio, L/D, of ten and for a range of optical thickness τ_o, $(0.1 \le \tau_o \le 6)$, $\tau_o = kD$, where k is the gas absorptivity and D is the plate spacing. Cases were run for either combined convection and radiation or combined conduction and radiation, for a wide range of gas flow rates and thermal conductivities. The walls of the channel were considered to be at constant temperatures. To establish a reference several cases were run for radiation only, without flow or conduction.

Figure 2 illustrates the heat transfer between the two plates for the case of a stagnant nonconducting gas. Since in this case equation (1) is linear in the emissive power, σT^4, the heat transfer results may be normalized by dividing by the difference in the emissive powers of the two plates. The results given in figure 2 compare favorably with those obtained by Usiskin, (1), for infinite parallel flat plates. The agreement is not perfect because of end effects due to the finite length of the channel.

In order to discuss the effect of conduction on radiation in a stagnant gas it is desirable to present the results in some type of nondimensional parametric form to obtain complete generality. Equation (1) may be nondimensionalized by dividing through by $k\sigma T_*^4$ where T_* is the temperature of the hotter plate. It can also be shown that the integrals on the right, for a given temperature field, are a function only of τ_o, and the channel aspect ratio L/D. On the basis of the above observations it can be shown that the solution of equation (1) is a unique function of the boundary conditions (ratio of cold plate and end temperatures to T_*), and of the following set of dimensionless parameters $\left(\frac{\lambda/D}{\sigma T_*^3}, \tau_o, \frac{L}{D}\right)$. For the sake of brevity the parameter $\frac{\lambda/D}{\sigma T_*^3}$ will be designated N_{CR}. Figure 3 shows the effect of combined conduction and radiation on the transverse temperature profile for the cases of both weak gas radiation ($\tau_o = 0.2$), and a relatively opaque gas ($\tau_o = 3.0$). In both cases, the temperature gradient and therefore the heat conduction at the cold wall were substantially higher than those for pure conduction (linear temperature profile). At the hot wall the temperature gradient may be either greater or less than that for pure conduction, depending upon the value of N_{CR} and the opacity of the gas.

Another effect of combined radiation and conduction is that the total heat transferred is greater than the sum of the heat transfer for radiation and conduction, taken separately. This effect is shown in figure 4 where the ratio of combined heat transfer to the sum of both components taken separately is plotted versus τ_o for various values of N_{CR}. This augmentation is due to interaction between radiation and conduction resulting from the nonlinear nature of the combined process. There is no augmentation for the case $\tau_o = 0$ since the gas is then not involved in the radiation transfer process, and the energy transport by conduction and that by radiation are independent. For large values of τ_o, the radiation transport becomes similar to a diffusion process (Rosseland approximation) and again the interaction with conduction diminishes.

For the discussion of the effect of gas flow on radiation without conduction it is again desirable to present the results in the form of dimensionless parameters. In the discussion to follow both plates are at the same, constant temperature T^* and the gas enters the channel from one end through the black porous plug at a temperature T_i. The gas flow is in the direction of length L, as shown in figure 1. The results to be given are for slug flow, G is constant. In a manner similar to that described earlier, the solution to equation (1) is a function of the dimensionless inlet temperature (T_i/T_*) and the following set of dimensionless parameters $\left(\frac{Gc_p}{\sigma T_*^3}, \tau_o, \frac{L}{D}\right)$. The parameter $\frac{Gc_p}{\sigma T_*^3}$ is known as the Boltzmann number N_{Bo} and was also used by Adrianov (3) in presenting his results. In figure 5, the parameter $\Lambda = \frac{T_o - T_i}{T_* - T_i}$ represents the ratio of the actual heat transferred to the gas to the maximum theoretically possible. The striking feature of figure 4 is that for a given value of N_{Bo} the heat transfer to the gas goes through a maximum with increasing τ_o. The reason for this behavior is that as τ_o increases the layers of gas next to the wall effectively absorb most of the radiation from the wall and thereby shield the bulk of the gas stream from any direct radiation. Because of this the bulk of the gas absorbs less radiation with increasing τ_o, resulting in reduced overall heat transfer.

The effect of this self shielding can also be seen by comparing the transverse temperature profiles at the channel exit for a weak and a strongly absorbing gas as shown in figure 6. At $\tau_o = 1.0$ there is very little self shielding and the temperature profile across the channel is fairly flat. For a higher degree of gas opacity at the same value of Λ, $\tau_o = 4.0$, the temperature profile shows the variation expected, the temperatures next to the wall being higher but the temperatures in the remainder of the stream being reduced due to the heavy self shielding. The preceding remarks have indicated some of the effects of conduction and gas flow on radiant heat transfer to absorbing gases. The interaction between conduction and radiation results in an increase in net heat transfer in comparison to the sum of the conduction and radiation heat transfer taken separately. For the case of radiation to a flowing gas from a constant temperature surface, the net heat transferred to the gas goes through a maximum as the gas absorbtivity is increased.

REFERENCES

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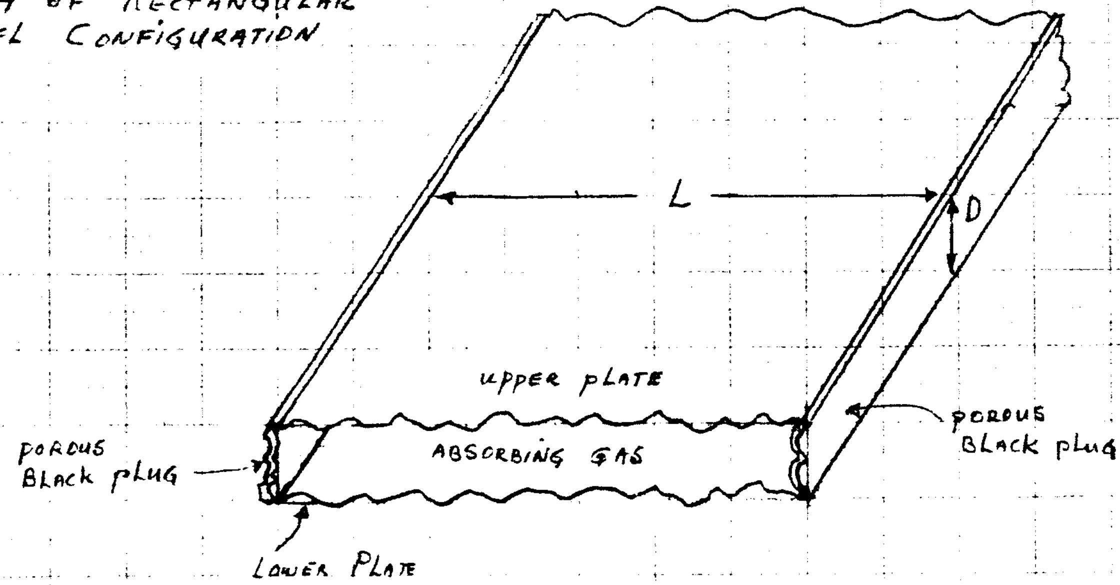
Fig 1.
Sketch of Rectangular Channel Configuration



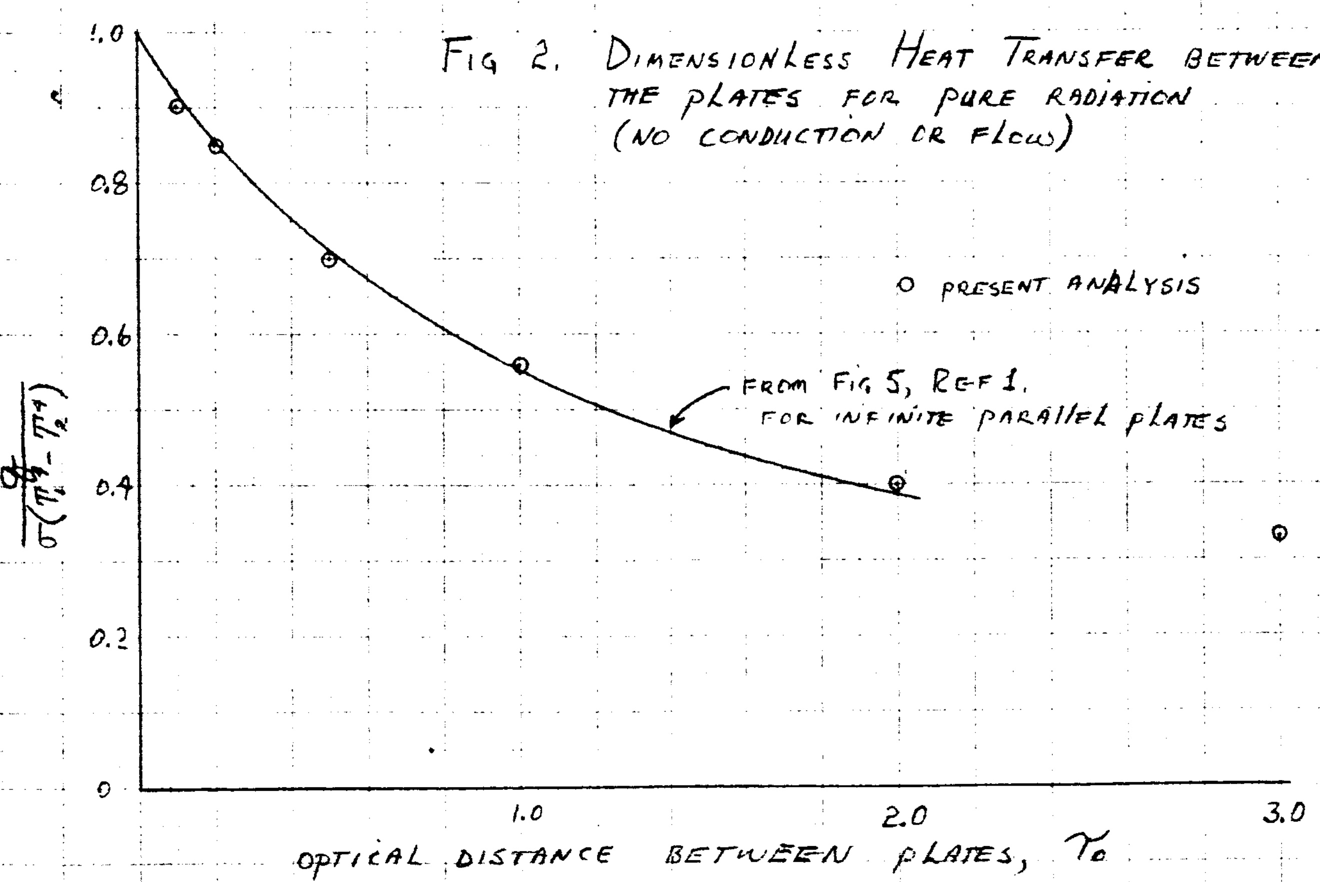


Fig 2. Dimensionless Heat Transfer between the plates for pure radiation (no conduction or flow)

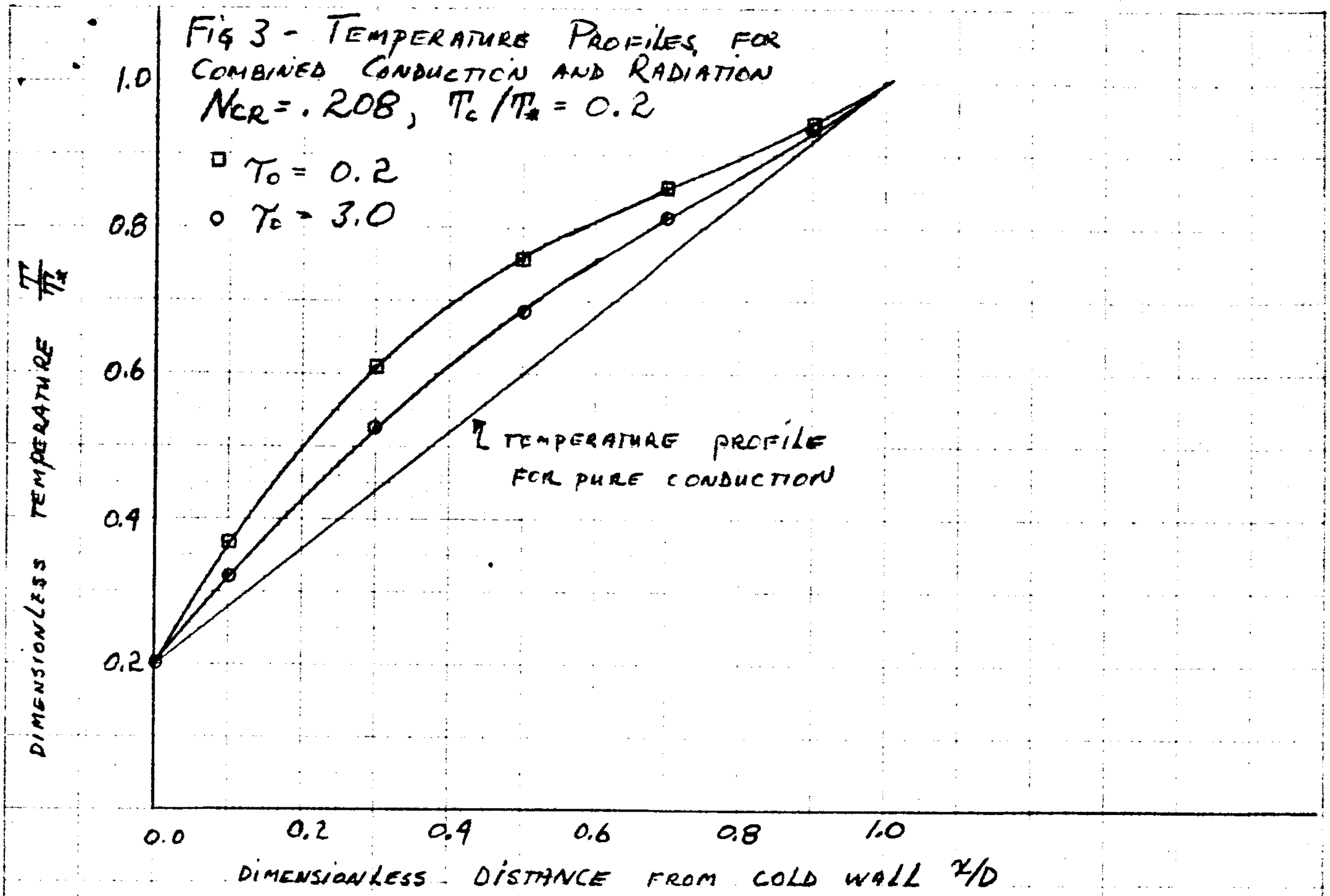


Fig 3 - Temperature Profiles for Combined Conduction and Radiation
$N_{CR} = .208$, $T_c/T_* = 0.2$
□ $\tau_0 = 0.2$
○ $\tau_0 = 3.0$
Temperature profile for pure conduction
Dimensionless temperature T/T_*
1.0
0.8
0.6
0.4
0.2
0.0
0.2
0.4
0.6
0.8
1.0
Dimensionless distance from cold wall x/D

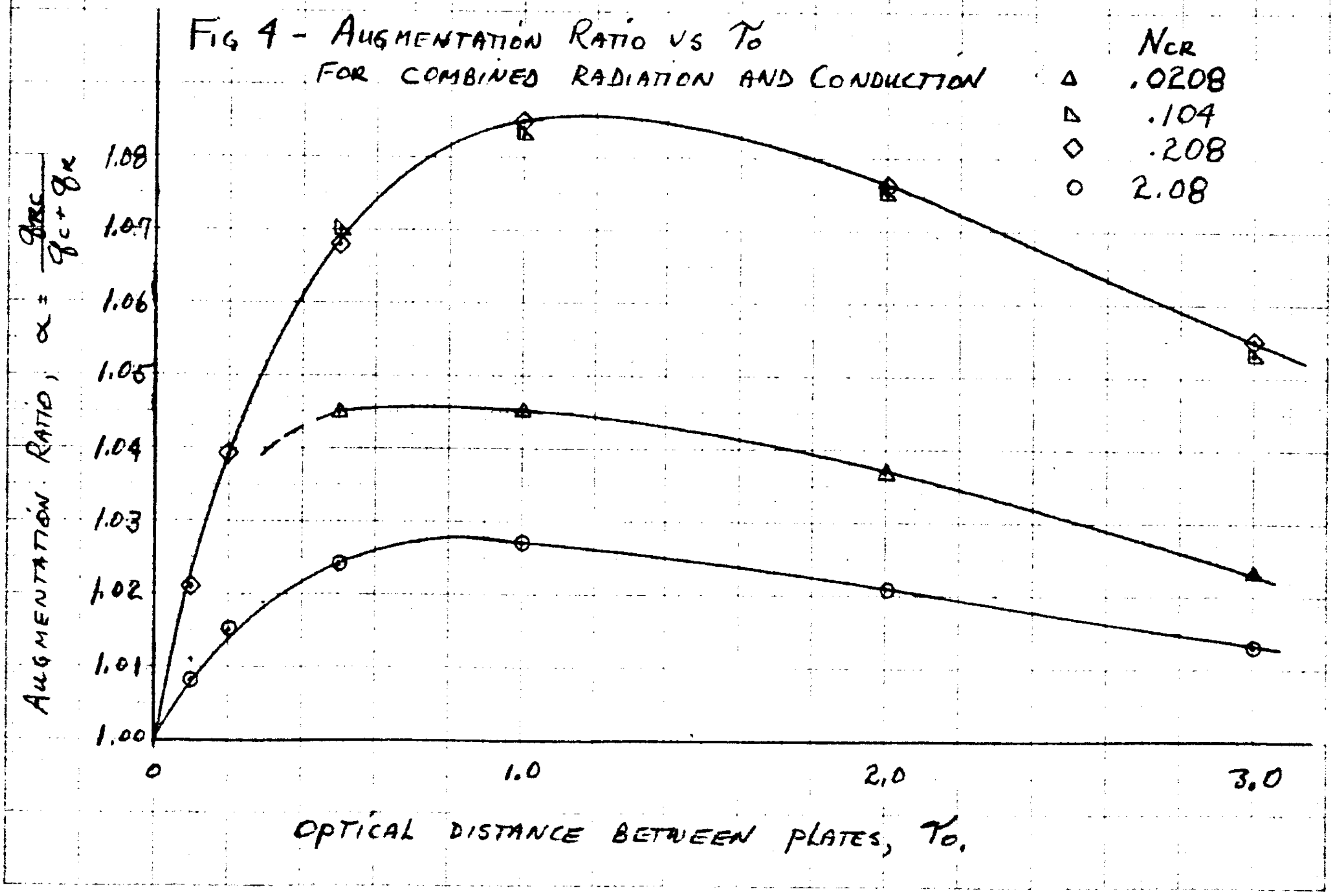


Fig 4 - Augmentation Ratio vs τ_0 for Combined Radiation and Conduction
N_{CR}
△ .0208
◭ .104
◇ .208
○ 2.08
Augmentation Ratio, $\alpha = \frac{q_{RC}}{q_c + q_R}$
1.08
1.07
1.06
1.05
1.04
1.03
1.02
1.01
1.00
0
1.0
2.0
3.0
Optical distance between plates, τ_0.

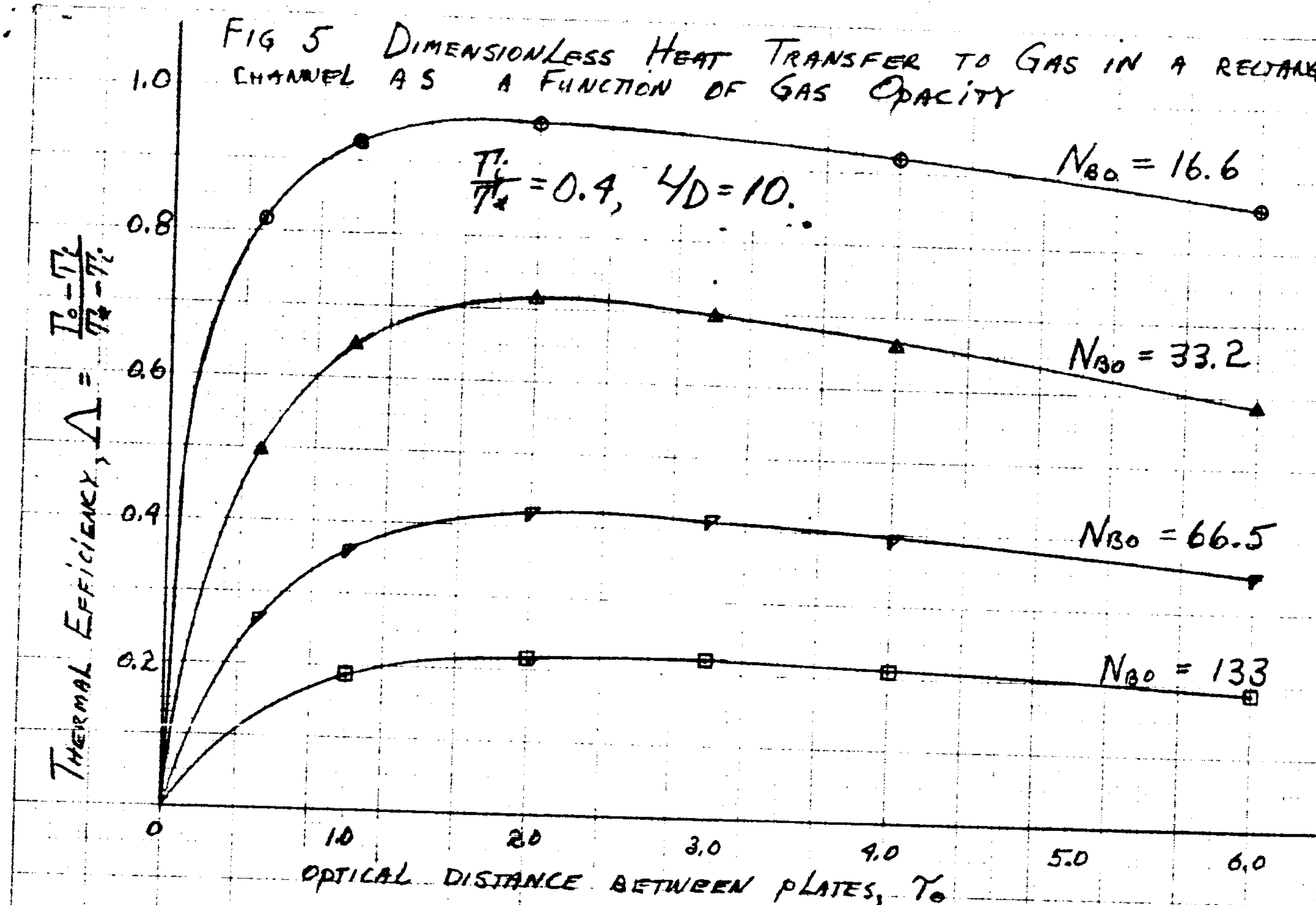


FIG 5 DIMENSIONLESS HEAT TRANSFER TO GAS IN A RECTANGULAR CHANNEL AS A FUNCTION OF GAS OPACITY

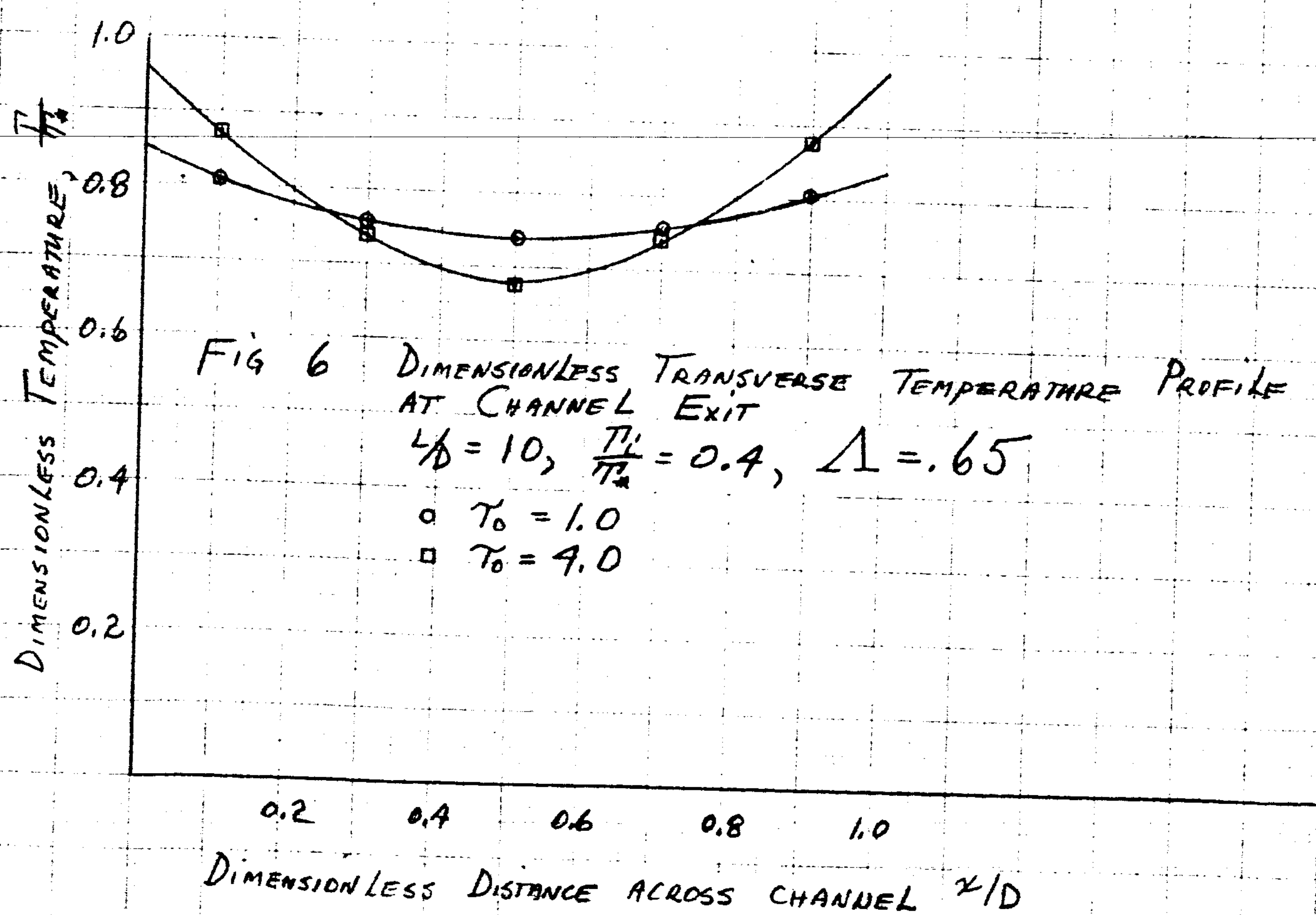


FIG 6 DIMENSIONLESS TRANSVERSE TEMPERATURE PROFILE AT CHANNEL EXIT

$L/D = 10$, $\frac{T_i}{T_*} = 0.4$, $\Lambda = .65$

○ $\tau_o = 1.0$

□ $\tau_o = 4.0$