NASA TECHNICAL NOTE

NASA TN D-2791

NASA TN D-2791

FACILITY FORM 602

N65 23165
(ACCESSION NUMBER)

(THRU)

14
(PAGES)

1
(CODE)

(NASA CR OR TMX OR AD NUMBER)

25
(CATEGORY)

GPO PRICE $

CFSTI
~~OTS~~ PRICE(S) $ 1.00

Hard copy (HC)

Microfiche (MF) .50

HYDROMAGNETIC STABILITY

by S. P. Talwar

Goddard Space Flight Center
Greenbelt, Md.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • MAY 1965

NASA TN D-2791

HYDROMAGNETIC STABILITY

By S. P. Talwar

Goddard Space Flight Center
Greenbelt, Md.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

For sale by the Clearinghouse for Federal Scientific and Technical Information
Springfield, Virginia 22151 – Price $1.00

HYDROMAGNETIC STABILITY

by
S. P. Talwar
Goddard Space Flight Center

SUMMARY

23165

In this article, the problem of hydromagnetic instability is reviewed. The emphasis throughout is on the physical understanding rather than the mathematical rigor. The problem of gravitational instability is discussed in detail, and the effects of finite Larmor radius, finite Larmor frequency, and resistivity are reviewed. Only macroscopic instabilities are dealt with, and no attempt is made to cover all aspects of plasma stability.

Author

CONTENTS

HYDROMAGNETIC STABILITY*

by
S. P. Talwar†
Goddard Space Flight Center

INTRODUCTION

Investigation of the stability of a physical system is of paramount importance, since every system in nature is subject to many perturbations, small or large. The stability of magnetized plasmas finds application in various astrophysical situations, such as magnetic stars, solar phenomena, interstellar matter, and plasma confinement in the laboratory for realization of controlled thermonuclear fusion.

In investigating stability, we inquire as to how a configuration of equilibrium (not necessarily static) responds to a perturbation. Specifically we ask, "If the system is disturbed, will the disturbance die down, or will it grow in amplitude with time?" We say that the system is, in the former case, stable with respect to the particular disturbance and, in the latter case, unstable. If therefore, the perturbation is time-dependent of the form e^{nt}, the problem reduces to a characteristic value problem for n. It can, in general, be real or complex. If n is real, then $n > 0$ implies that the system is unstable and $n < 0$ implies stability ($n = 0$ defining the states of marginal stability). If n is complex, the system will be stable if $\mathrm{Re}(n) < 0$ (periodically damped). If $\mathrm{Re}(n) > 0$, the system is said to be "overstable" in that the system becomes subject to restoring forces which, during an oscillation, push the material back toward the undisturbed state with a velocity greater than its original outward velocity; and thus the system is rendered unstable through growing waves.

In discussing the question of stability, we should remember that in the most general sense the system is stable only if it is shown to be stable for all conceivable perturbations to which it could be subjected. Even if the system is unstable to a very special, and rare, type of perturbation, it must be considered unstable.

We also should remember that the perturbation analysis used in the stability investigation emphasizes the smallness of the perturbation; and so it is possible that a system in stable equilibrium for small perturbations ceases to be so for finite amplitude disturbances. Similarly it could as well happen that an initially unstable perturbation of small amplitude ultimately may be

*Text of a lecture given by the author to the Theoretical Division of Goddard Space Flight Center on November 2, 1964.

†Senior Research Associate of National Academy of Sciences (on leave from the Department of Physics, University of Delhi, Delhi, India).

replaced by an oscillation of a large, but limited, amplitude. Under that situation, the linearized analysis of the perturbation theory ceases to be valid and the subsequent behavior of the system will be governed by nonlinear equations whose analysis is, in general, difficult to carry out.

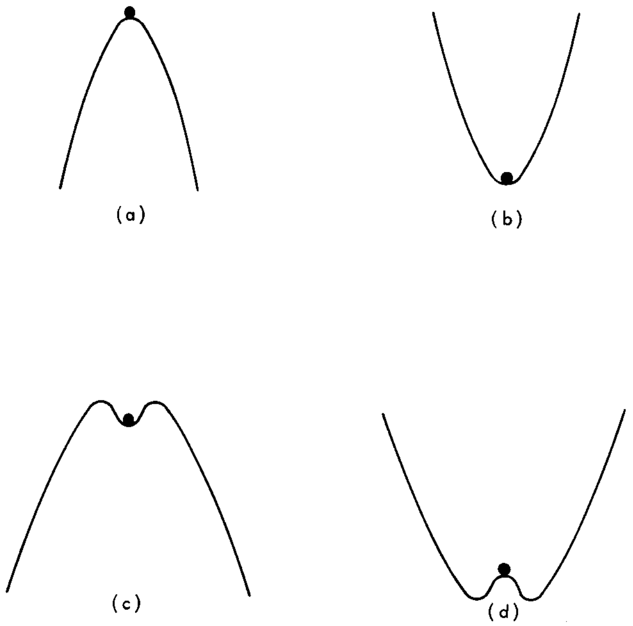


Figure 1—A mechanical example illustrating stable and unstable states of equilibrium. The ball is in unstable and stable states in (a) and (b), respectively; (c) depicts stability for only small displacements; and (d) shows instability of restricted amplitude.

The above remarks about stability are illustrated by the examples in Figure 1. In Figure 1(a), the ball is resting in equilibrium at the top of a hill, and any small perturbation causes a large displacement and represents instability. In Figure 1(b), the ball rests at the bottom of a valley where it oscillates about the position of equilibrium, representing stability. In Figure 1(c), it rests in a small depression on the top of a hill; and so the ball, although stable for small oscillations, will go over the ridge and become unstable if the perturbation is large enough. Finally, in Figure 1(d), the hill is only a small hump in the bottom of a valley; and the instability is limited in amplitude by the walls of the valley.

Again, it should be mentioned that in an unstable situation we are primarily interested in the rate of growth of instability as compared with the characteristic time of observation on the system. Various astrophysical configurations may be inherently unstable, but the instability may develop extremely slowly so that the system appears to be permanent and stable. In plasma confinement problems, the upper limit is several seconds for a fusion reactor of reasonable size and magnetic field, in the temperature range of 10 to 100 kev. This is adequate for the thermonuclear problem, and we need to confine a plasma in stabilized form for times of this order.

The question of the stability of a system is investigated, basically, in two major ways: by the *normal mode analysis*, and by the *energy method.*

In applying the *normal mode analysis,* we set up the linearized perturbation equations, derive a single perturbation equation in a single perturbation parameter, and solve this equation under a set of appropriate boundary conditions for the system. This then gives a dispersion relation for n, the parameter determining the stability of the system. This method is quite general and has extensive applications. It yields complete information about stability, including the growth rates of any unstable perturbations. However, in many problems the normal mode equations are extremely complicated in solution; and, in such cases, it is profitable to obtain information, although limited in scope, about stability by employing the energy method.

The *energy method* consists in examining the sign of the change in potential energy of the system under all displacements. If the potential energy characterizing the system decreases under some displacement, the kinetic energy is available for motion away from the equilibrium state; and the system is unstable. This method is particularly useful in handling complicated configurations, but is applicable only to a *static* system characterized by a constant potential energy. Naturally, therefore, it is not applicable for *dissipative* systems and systems endowed with initial kinetic energy of fluid motions. In such systems the energy of steady motion may well be converted into the kinetic energy of instability without causing any decrease in potential energy; in fact, the potential energy may even increase while instability occurs. One merit of the energy method is that it makes the energetics of the problem clear; that is, it tells what particular form of energy is enhanced, and which is depleted, during instability.

A plasma carrying a magnetic field may be subject to a "jungle" of instabilities, depending on its character. The various instabilities may be classified roughly into two main categories: *macroscopic*, and *microscopic*. The first group (macroscopic, or low-frequency instabilities), which we discuss mostly in this review, comprises those arising in dense plasmas which are described by collision-dominated single fluid equations. The microinstabilities, on the other hand, are associated with deviations of the velocity distribution of particles from Maxwellian character [e.g., a peak in Maxwellian tail, difference in longitudinal and transverse (to magnetic field) temperatures, or mutually streaming beams]. Such instabilities arise in dilute plasmas where collisions are not frequent enough to insure isotropic distribution, and may result in local fluctuations of density and electromagnetic fields in plasma.

ENERGY PRINCIPLE FOR A GRAVITATING CONFIGURATION

An energy principle for a static hydromagnetic system with zero dissipation has been developed by Bernstein et al. (Reference 1), among others. We shall develop in outline the energy principle for self-gravitating, ideally conducting plasmas. It is worthwhile to mention that, in fact, three energy principles—based on the hydromagnetic equations, the double-adiabatic fluid equations (Reference 2), and the Vlasov equation—have been given to date. In the hydromagnetic energy principle, we deal with a collision-dominated plasma in which collisions are so strong that the gas pressure is isotropic but still so weak as to neglect resistance. The double-adiabatic energy principle corresponds to less dense plasmas in which the collisions are not strong enough to keep the pressure isotropic but are yet sufficiently strong to render heat flow negligible. The energy principle based on the Vlasov equation refers to no collisions and in the limit that gyration radius for each type of particle and Debye length are infinitely small compared with macroscopic quantities. We shall, later, compare the results regarding stability as obtained from these three energy principles.

The general equations governing a hydromagnetic fluid are written as follows.

$$\rho \frac{\partial \mathbf{v}}{\partial t} = -\nabla p - \rho \nabla U + \frac{\mathbf{J} \times \mathbf{B}}{c} \quad \Big| \quad -\rho(\mathbf{v}\cdot\nabla)\mathbf{v} + \frac{\rho\nu}{3}\nabla\nabla\cdot\mathbf{v} + \rho\nu\nabla^2\mathbf{v} \quad , \tag{1}$$

$$\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho\mathbf{v}) = 0 \quad , \tag{2}$$

$$\nabla\cdot\mathbf{B} = 0 \quad , \tag{3}$$

$$\nabla\cdot\mathbf{E} = 0 \quad , \tag{4}$$

$$\nabla \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} \quad , \tag{5}$$

$$\nabla \times \mathbf{B} = \frac{4\pi}{c}\mathbf{J} \quad , \tag{6}$$

$$\frac{1}{p}\frac{dp}{dt} = \frac{\gamma}{\rho}\frac{d\rho}{dt} \quad \Big| \quad + \epsilon \quad , \tag{7}$$

$$\nabla^2 U = 4\pi G\rho \quad ,$$

$$\mathbf{E} = -\frac{1}{c}(\mathbf{v} \times \mathbf{B}) \quad \Big| \quad + \frac{m}{Ne^2}\frac{\partial \mathbf{J}}{\partial t} + \frac{1}{cNe}(\mathbf{J} \times \mathbf{B}) - \frac{1}{Ne}\nabla p_e - \frac{\mathbf{J}}{\sigma} \quad , \tag{8}$$

and

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) \quad \Big| \quad + \frac{c^2}{4\pi\sigma}\nabla^2\mathbf{B} + \frac{c^2}{\omega_{p_e}^2}\frac{\partial}{\partial t}\nabla^2\mathbf{B} - \frac{c}{4\pi Ne}\nabla \times \left[(\nabla \times \mathbf{B}) \times \mathbf{B}\right] \quad . \tag{9}$$

Here p, ρ, U denote the pressure, density, and gravitational potential; and $\mathbf{v}$, $\mathbf{B}$, $\mathbf{J}$, $\mathbf{E}$ stand for the velocity and electromagnetic vectors at a point. The last two terms in Equation 1 represent viscous forces, and ϵ in Equation 7 includes energy dissipation due to finite electrical conductivity, viscosity, and thermal conductivity. The terms on the left-hand side of the vertical line are the ones defining an ideal nondissipative static plasma. Thus the terms on the right side of the vertical line are discarded in the hydromagnetic energy principle. These terms (like fluid motions, viscosity, finite resistivity) may have a profound influence on the stability, but their inclusion can be done only by normal mode analysis. We shall see later how each term influences the character of stability. It may be mentioned that, in the above equations, displacement current and space charge have been neglected in the spirit of hydromagnetic approximation. An order-of-magnitude analysis shows that the electron inertia term, Hall current term, and electron pressure term contribute significantly only when the plasma is dilute and is characterized by high temperature and high magnetic fields.

Using Equations 1 to 9, we may write finally the perturbation equation for displacement ξ in a nondissipative, initially static plasma:

$$\rho_0 \frac{\partial^2 \xi}{\partial t^2} = F(\xi) \quad , \tag{10}$$

where

$$F(\xi) = \nabla\left[\gamma p_0 \nabla\cdot\xi + (\xi\cdot\nabla)p_0\right] + \frac{1}{4\pi}\left[(\nabla \times B_0) \times b + (\nabla \times b) \times B_0\right]$$

$$+ 4\pi G \rho_0^2 \xi - \rho_0 \nabla \times X + \nabla\cdot(\rho_0 \xi) \nabla U_0 \quad , \tag{11}$$

where quantities with the suffix 0 denote the unperturbed values and b stands for the change in magnetic field at a point; X is an arbitrary function of space coordinates satisfying the equation

$$\nabla \times \nabla \times X = 4\pi G \nabla \times (\rho_0 \xi) \quad . \tag{12}$$

The operator F, containing equilibrium quantities and their space derivatives alone, can be shown to be a self-adjoint operator. This leads to an important conclusion about the stability of a static, nondissipative plasma: that the eigenvalues are either real (positive or negative) or pure imaginary. Thus, complex values of n in the expression e^{nt} for time-dependence of perturbation are ruled out, meaning that a static, nondissipative plasma cannot become "overstable." The expression for the change in potential energy is written (to the second order in ξ) as

$$\delta W = -\frac{1}{2}\int \xi \cdot F(\xi)\, d\tau_0 \quad , \tag{13}$$

where the integration is over the unperturbed volume of the configuration.

It may be noted that δW should be a point function rather than a path function of displacement; otherwise, δW will not be unique. This is, however, guaranteed by F's being a self-adjoint operator. For a confined plasma we may write

$$\delta W = \delta W_v + \delta W_s + \delta W_F \quad , \tag{14}$$

where δW_v, δW_s, δW_F respectively stand for the contribution to change in potential energy from the vacuum region, deformed surface, and the plasma fluid region. Each contribution is separately written as

$$\left.\begin{aligned}
\delta W_v &= \frac{1}{8\pi}\int d\hat{\tau}\, |\hat{b}|^2 > 0 \ , \\
\delta W_s &= \frac{1}{2}\int dS_0 (n_0\cdot\xi)^2\, n_0 \left[\!\left[\nabla\left(p_0 + \frac{B_0^2}{8\pi}\right)\right]\!\right] , \\
\text{and} \quad \delta W_F &= \frac{1}{2}\int d\tau_0 \left[\frac{b^2}{4\pi} - J_0 \cdot b \times \xi + \gamma p_0 (\nabla\cdot\xi)^2 + (\xi\cdot\nabla p_0)\nabla\cdot\xi \right. \\
&\qquad \left. - (\xi\cdot\nabla U_0)\nabla\cdot(\rho_0\xi) - 4\pi G \rho_0^2 \xi^2 + \rho_0 \xi\cdot\nabla \times X\right] \ ;
\end{aligned}\right\} \tag{15}$$

here quantities with caret on top refer to the vacuum region, and the double brackets stand for the difference (along the direction of normal n_0) in the quantity on the two sides of the interface.

Among various important conclusions regarding plasma stability as derived from the application of the energy principle, we may particularly mention "Teller's Criterion," which says that the configuration of a *field-free, homogeneous, nongravitating* plasma separated from a vacuum region having a magnetic field is stable only if the lines of force on the interface are everywhere convex toward the plasma. This follows quite easily from Equation 15. We see that $\delta W_v > 0$, $\delta W_F > 0$ for a homogeneous, field-free, nongravitating plasma and that δW_s is written as

$$\begin{aligned}\delta W_s &= -\frac{1}{2}\int dS_0 \, (\mathbf{n}_0\cdot\boldsymbol{\xi})^2 \, \mathbf{n}_0\cdot\nabla\left(\frac{\hat{B}_0^2}{8\pi}\right) \\ &= -\frac{1}{8\pi}\int dS_0 \, (\mathbf{n}_0\cdot\boldsymbol{\xi})^2 \, \mathbf{n}_0\cdot\mathbf{R}\left(\frac{|\hat{B}_0|^2}{R^2}\right)\end{aligned} \tag{16}$$

Clearly, δW_s also is positive if $\mathbf{n}_0\cdot\mathbf{R}$ is negative everywhere on the surface (that is, the surface is convex toward the plasma). Teller's Criterion shows therefore that the static pinch carrying surface current and the mirror geometry (Figure 2) should be grossly unstable, whereas the configurations produced by one, two, or four wires carrying currents (as shown in Figure 3) should be stable. An equivalent requirement for $\delta W_s > 0$ is that the vacuum magnetic field should increase outward from the plasma surface, and this forms the essential basis of plasma confinement in a "mirror-cum-cusp" kind of geometry given by Ioffe recently.

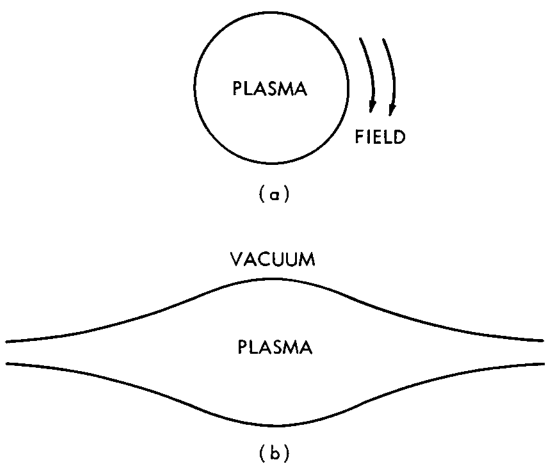


Figure 2—Two thermonuclear fusion geometries are shown: (a) the "conventional pinch," and (b) the "mirror geometry."

It may be emphasized that Teller's conclusions about guessing the stability from the curvature of the lines of force does not apply if there are two plasmas or if the plasma itself carries a magnetic field. Only detailed calculation of δW from each region can decide the question.

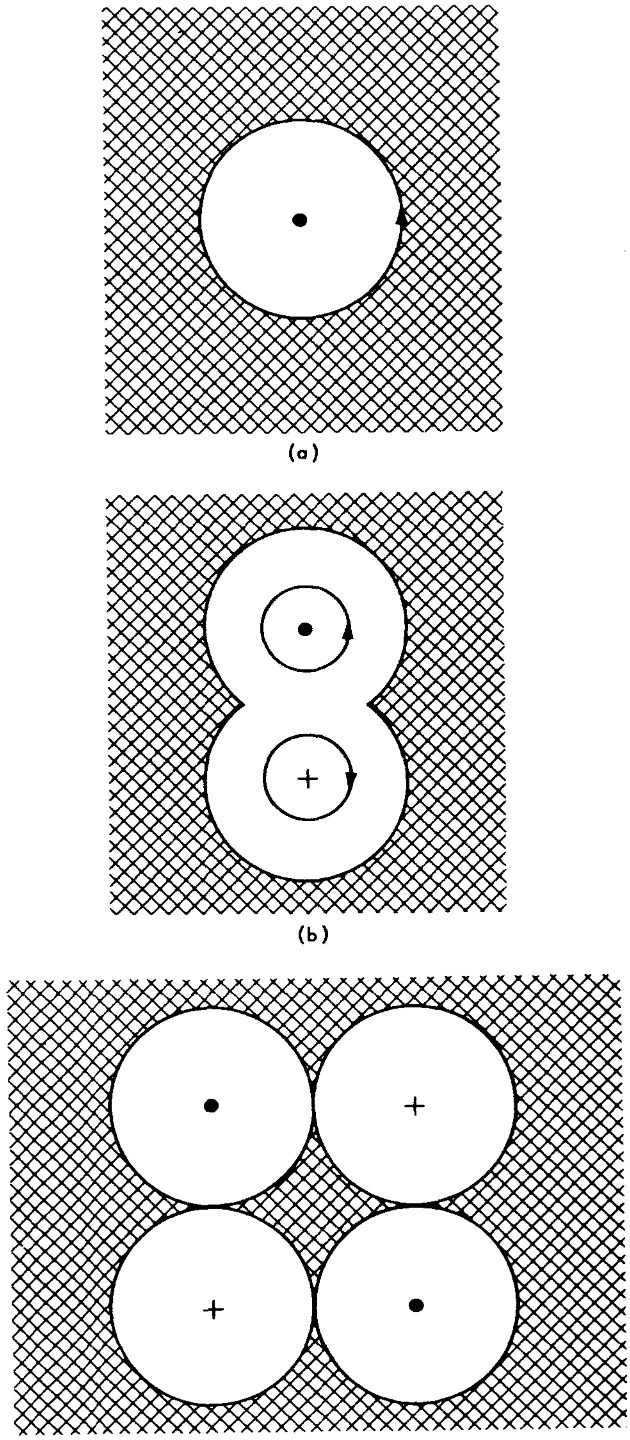
RAYLEIGH-TAYLOR INSTABILITY IN PLASMAS

The classical Rayleigh-Taylor instability arises, as is well known, whenever density in a liquid increases upward anywhere in a downward gravitational field. Its analogue in hydromagnetics was first worked out by Kruskal and Schwarzschild (Reference 3), who showed that the configuration of a semi-infinite, uniform, incompressible ideal plasma separated from a vacuum region is unstable gravitationally when both plasma and vacuum carry uniform (but unequal) parallel magnetic fields along the planar interface. In particular, they found that for flute disturbances (which do not bend the lines of force) the growth rate of instability is $\sqrt{gk}$, where $k(=2\pi/\lambda)$ is the wavenumber of perturbation. This result is identical, as expected, to that given by the classical Rayleigh-Taylor formula when one fluid is replaced by vacuum. The calculations may be generalized to include the effects of two ideal plasmas superposed on each other and partaking in a uniform rotation, the axis being parallel to the direction of gravity. The dispersion formula obtained for nondissipative incompressible plasmas is written as (Reference 4)

$$
\begin{aligned}
gk(\rho_2 - \rho_1) &= \rho_1 \left[n^2 + (\mathbf{k}\cdot\mathbf{V}_1)^2\right] \\
&\quad \cdot \left\{1 + \frac{4\Omega^2 n^2}{\left[n^2 + (\mathbf{k}\cdot\mathbf{V}_1)^2\right]^2}\right\}^{1/2} \\
&\quad + \rho_2 \left[n^2 + (\mathbf{k}\cdot\mathbf{V}_2)^2\right] \\
&\quad \cdot \left\{1 + \frac{4\Omega^2 n^2}{\left[n^2 + (\mathbf{k}\cdot\mathbf{V}_2)^2\right]^2}\right\}^{1/2} . \qquad (17)
\end{aligned}
$$

A general result which emerges from this investigation is that both the rotation and the magnetic field have a stabilizing influence, the former for large wavelengths and the latter for short wavelengths. It may be noted that, whereas the stabilizing effect of the field vanishes for flute disturbances for which $\mathbf{k}\cdot\mathbf{B} = 0$, that of rotation does not.

The gravitational instability, as worked out by Kruskal and Schwarzschild on hydromagnetics, was later reconciled on orbit theory by Rosenbluth and Longmire (Reference 5). They showed that a disturbance at the interface leads to a charge separation, since both ions and



(a)

(b)

(c)

Figure 3—Three plasma configurations, stable in Teller's Criterion, are given: (a) a single wire carrying current, (b) two wires carrying opposite currents, and (c) four wires carrying current as shown. (The plasma region is depicted by crosshatching.)

electrons drift in opposite directions along the interface under the action of gravity. This results in an electric field which produces an increase in the amplitude of perturbation (instability) due to the charge-independent $\mathbf{E} \times \mathbf{B}$ force (Figure 4). Later experiments with mirror geometry revealed that the mirror geometry, although unstable, is not as violently unstable as demanded by Rosenbluth and Longmire's theory. Also, we know that the Van Allen radiation belts, unstable on ideal hydromagnetics, do show (particularly the inner belt) a reasonable amount of stability. The question therefore arises as to how the violence of the gravitational instability could be reduced. Some of the possibilities suggested are as follows.

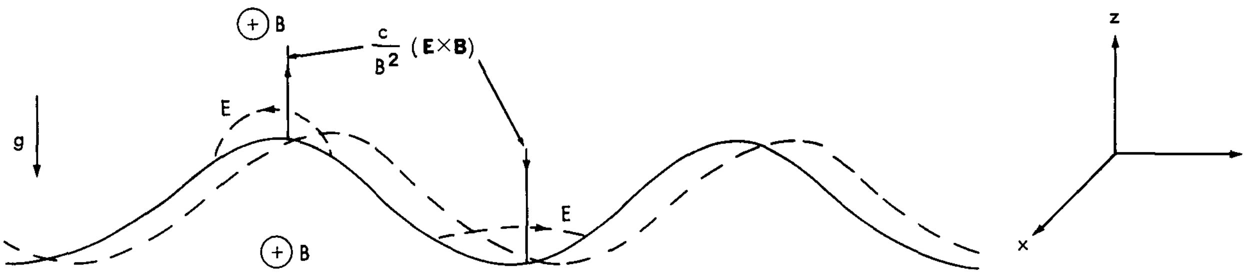


Figure 4—The mechanism for gravitational instability.

Shear Field

If the plasma and vacuum regions carry nonparallel uniform magnetic fields along the interface, clearly a perturbation for which $\mathbf{k} \cdot \mathbf{B}_0 = 0$ everywhere in the configuration cannot be invoked. Thus the perturbation must result in bending the lines of force, thereby producing a restraining influence on the growth of perturbation. In particular, if

$$
\begin{aligned}
B_y(z) &= \alpha_P B_0, & B_x(z) &= 0, & &\text{(Plasma region, } z > 0) \\
&= \alpha_V B_0, & &= B_0, & &\text{(Vacuum region, } z < 0)
\end{aligned}
\tag{18}
$$

then the stability requires

$$
\frac{B_0^2}{4\pi\rho_0} \left[\left(k_x + \alpha_V k_y\right)^2 + \left(\alpha_P k_y\right)^2 \right] > gk \quad , \tag{19}
$$

where $k \left(= \sqrt{k_x^2 + k_y^2}\right)$ denotes the wavenumber of perturbation.

For $k_x = 0$ and $\alpha_P = \alpha_V = \alpha$, the above condition for stability reduces to

$$
\alpha^2 B_0^2 > g\rho_0 \lambda \quad . \tag{20}
$$

Thus the effect of shear in a magnetic field is to suppress the short wavelength instability. We therefore may surmise that, in natural situations like the Van Allen belts, instability may be suppressed because of a shear in the prevailing magnetic field or because of finite β (the ratio of kinetic to magnetic pressure), as even then any perturbation should result in a change in the magnetic field and hence to restoring forces.

Conducting End Plates

As the cause of gravitational instability is basically the charge separation due to gravity drift, it can be suppressed if we can somehow short circuit the charge separation. This can be achieved by two conducting plates perpendicular to the prevailing magnetic field. In the Van Allen belts such short circuiting has to be provided by the upper parts of the ionosphere.

Finite Ion Larmor Frequency

An implicit assumption of the hydromagnetic description of gravitational instability is that the Larmor frequency of charged particles is effectively infinite. Lehnert, who recently (Reference 6) investigated the effect of a finite ion Larmor frequency, found that small wavelength instability can be effectively suppressed because of the finiteness of ion Larmor frequency ω_i . The argument is somewhat as follows: The charge-dependent gravitational drift plays a double role. In the initial stages it drives the instability but, if the gravitational drift is fast enough so that the time required to drift a half wavelength is smaller than the growth time of instability, then the initially unstable charge distribution may be reversed in phase, causing a stable oscillation. Thus, only the first role has been included in Rosenbluth and Longmire's theory.

In the framework of hydromagnetics, the finite ion Larmor frequency can be taken care of by using a generalized Ohm's law (i. e., including the first three terms on the right side of the vertical line with the terms on the left side in Equation 8.

Finite Ion Larmor Radius

In Rosenbluth and Longmire's description of gravitational instability, the Larmor radius of the charged particles was assumed neglibibly small. Rosenbluth, Krall, and Rostoker (Reference 7) showed that a reasonable amount of stabilization for short wavelengths can be effected if the effect of finiteness of the ion Larmor radius (which is much larger than the electron Larmor radius) is included. Their reasoning is somewhat as follows: Because of the different Larmor radii of electrons and ions, the mean electric field as seen by a gyrating electron is different from the mean electric field as seen by an ion. Thus, the characteristic velocity $c/B^2\ (\mathbf{E} \times \mathbf{B})$ with which the particle moves in the direction of gravity to give rise to instability is different for the ions and the electrons. The ions drift along the direction of gravity more slowly than the electrons, with the result that a current along the direction of gravity is generated, trying to build up a charge distribution out of phase with the original charge separation, because of gravity drift, responsible for producing flute instability. They showed that, when the inequality $(a_i k)^2 > n_0/\omega_i$ is satisfied, the monotonic flute instability is replaced by a stable oscillation. Here a_i is the Larmor radius for the ion, k is the wave number of perturbation, and n_0 and ω_i respectively denote the growth rate given by the hydromagnetic theory and the ion Larmor frequency.

Roberts and Taylor (Reference 8) pointed out that the finite Larmor radius effect can be incorporated into the hydromagnetic equation by including certain transport-like terms. These represent a type of viscosity, independent of any collisions, where the mean free path is replaced by the ion Larmor radius. They recovered the results obtained by Rosenbluth et al. by using hydromagnetic equations which include the modifications to both the ion pressure term and Ohm's law.

NONIDEALIZED HYDROMAGNETICS AND RESISTIVE EFFECTS

In the last section we discussed situations (mirror geometry and the Van Allen belts) which are found observationally to be less violently unstable than predicted by ideal (infinite electric conductivity, infinite Larmor frequency, and zero Larmor radius) hydromagnetic theory. We tried to invoke some operative mechanisms which could suppress instability. We also may come across cases which in actual observation reveal gross instability, although shown to be completely stable in ideal hydromagnetics. One such typical example is the "hard core pinch." In such cases, again, some mechanism must be operating which is responsible for instability and which is not taken care of in the ideal theory. We have seen that finite Larmor radius and frequency produce a stabilizing influence on the system. Looking at the various terms on the right-hand side of the vertical line in Equations 1 to 9, we find that the only operative mechanism not discussed so far (except for fluid motions) is a finite dissipation in the medium (e.g., finite resistivity). Resistivity (and other dissipative forces) may play a vital role in deciding the question of a system's stability. It is clear that the eigenvalues need not be real or pure imaginary, for the equations are not self-adjoint as against the case for static nondissipative plasmas.

Exhaustive work on resistive instability has been done by Furth, Killeen, and Rosenbluth (Reference 9) who, treating a sheared field in a slab geometry, found three resistive modes labeled rippling, tearing, and gravitational modes. Recently Coppi (Reference 10) studied the joint effect of resistivity and finite Larmor radius and found that the growth rates of resistive modes are slowed down by the finite Larmor radius, although not completely stabilized. The gravitational mode is most strongly affected through a viscosity-like term in the equation of motion, whereas the tearing mode is slowed down by a Hall term in the generalized Ohm's law.

Physically speaking, resistivity plays a dual role in the stability determination. We may think that finite transport processes should be stabilizing since they (finite conductivity or viscosity) lead to dissipation of energy by converting ordered motion into random motion (e.g., Joule heat); and, since an instability is a particular type of ordered motion, should they not cause instabilities to be damped? If resistance (and viscosity) were to occur only in the energy equation (Equation 7), this probably would be so; but, in addition, viscosity appears in the equation of motion (Equation 1), and the electrical resistivity in the electromagnetic induction equation (Equation 9). In the equation of motion the viscous stresses are forces tending to restrict the relative motion of neighboring elements of the fluid, and this is a stabilizing effect. The presence of finite resistivity allows matter to slip across the field, and thus the restraining influence of the field in infinite conductivity (glued field) is lost—which means that finite conductivity has a destabilizing influence.

If the Joule term in the energy equation is neglected (i. e., currents are kept constant, as in Furth, Killeen, and Rosenbluth's work), the finite resistivity effect is destabilizing. It may, however, be noted that with the change in conductivity we may envisage a corresponding change in the electric field and/or the current density; and the effect of resistivity with constant current or constant electric field may be quite different regarding the stability of the configuration.

COMPARISON OF THE THREE ENERGY PRINCIPLES

As noted previously (page 3), three energy principles dealing with the different limiting situations of a static, nondissipative plasma have been worked out. The method of establishment is basically the same. We can, therefore, state a comparison theorem for the potential energy variations derived from the ideal hydromagnetics, the Chew-Goldberger-Low (CGL) double-adiabatic approximation, and the Vlasov equation in $M/e \to 0$ limit. The theorem applies for a system which initially is in static equilibrium with *isotropic* plasma pressure. When the collision time is much larger than the oscillation or growth time of instability, the pressure will not remain isotropic during the course of perturbation. The ideal hydromagnetic equations could not be used consistently, since they assume an isotropic pressure to prevail at every instant of time.

The results obtained for the change in potential energy for plasma with an initially isotropic pressure distribution are

$$\delta W_{MHD} < \delta W_{kinetic} < \delta W_{CGL} \quad .$$

This shows that the CGL approximation gives the most optimistic prediction of stability. Conversely, if we obtain stability in the MHD approximation, the stability is certainly guaranteed in the more exact and tedious kinetic theory in the $M/e \to 0$ limit as well as in the CGL approximation. Thus we need only to insure that a system having isotropic equilibrium pressure is stable for ideal hydromagnetics. It must be remembered however that, if the unperturbed system is characterized by anisotropic pressure, new instabilities can arise which are not given by the ideal MHD theory—a typical example being the "fire-hose" and "mirror" instability in a homogenous dilute plasma subject to a uniform magnetic field.

CONCLUSION

In addition to the macroscopic instabilities, the plasma may suffer from a bewildering variety of microscopic instabilities, and their number seems to be ever increasing. The best solution, perhaps, is to eliminate the intolerable instabilities and learn to live with the ones which cannot be eliminated.

(Manuscript received November 17, 1964.)

REFERENCES

1. Bernstein, I. B., Frieman, E. A., Kruskal, M. D., and Kulsrud, R. M., "An Energy Principle for Hydromagnetic Stability Problems," *Proc. Roy. Soc. A* 244:17-40, February 25, 1958.
2. Chew, G. F., Goldberger, M. L., and Low, F. E., "The Boltzmann Equation and the One-Fluid Hydromagnetic Equations in the Absence of Particle Collisions," *Proc. Roy. Soc. A* 236:112-118, July 10, 1956.
3. Kruskal, M., and Schwarzschild, M., "Some Instabilities of a Completely Ionized Plasma," *Proc. Roy. Soc. A* 223:348-360, May 6, 1954.
4. Talwar, S. P., "Stability of a Conducting Rotating Fluid of Variable Density," *J. Fluid Mech.* 9(4):581-592, December 1960.
5. Rosenbluth, M. N., and Longmire, C. L., "Stability of Plasmas Confined by Magnetic Fields," *Ann. Phys.* 1(2):120-140, May 1957.
6. Lehnert, B., "Stability of a Plasma Boundary in a Magnetic Field," *Phys. Fluids* 4(7):847-854, July 1961; and "Stability of a Plasma with a Continuous Density Distribution," *Phys. Fluids* 4(8):1053-1054, August 1961.
7. Rosenbluth, M. N., Krall, N. A., and Rostoker, N., "Finite Larmor Radius Stabilization of 'Weakly' Unstable Confined Plasmas," *Nuclear Fusion* Suppl. Pt. 1:143-150, 1962.
8. Roberts, K. V., and Taylor, J. B., "Magnetohydrodynamic Equations for Finite Larmor Radius," *Phys. Rev. Letters* 8(5):197-198, March 1, 1962.
9. Furth, H. P., Killeen, J., and Rosenbluth, M. N., "Finite-Resistivity Instabilities of a Sheet Pinch," *Phys. Fluids* 6(4):459-484, April 1963.
10. Coppi, B., "Influence of Gyration Radius and Collisions on Hydromagnetic Stability," *Phys. Fluids* 7(9):1501-1516, September 1964.

NASA-Langley, 1965