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Abstract

This paper is a review of the basic theoretical dynamical properties of an atmosphere with an extended temperature strongly bound by gravity. The review begins with the historical developments leading up to the realization that the only dynamical equilibrium of an atmosphere with extended temperature is supersonic expansion. Through the course of the theoretical developments of the basic principles it is shown that sufficient conditions for supersonic expansion are $T(r)$ declining asymptotically less rapidly than $1/r$, or the density at the base of the corona being less than N_B given by (40) if no energy is available except through thermal conductivity, or the temperature falling within the limits given by (18) if $T \propto N^{\alpha-1}$ throughout the corona. Less extended temperatures lead to equilibria which are subsonic or static. The hypothetical case of a corona with no energy supply other than thermal conduction from its base is considered at some length because the equations may be solved by analytical methods and illustrate the transition from subsonic to supersonic equilibrium as the temperature becomes more extended. Comparison with the actual corona shows that the solar corona is actively heated for some distance into space by wave dissipation, etc.

The dynamical stability of the expanding atmosphere is demonstrated, and

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in a later section the radial propagation of acoustic and Alfven waves through the atmosphere and wind is worked out. The calculations show that the magnetometer will probably detect waves more easily than the plasma instrument, but that both are needed to determine the mode and direction of the wave. An observer in the wind at the orbit of Earth can "listen" to disturbances generated in the corona near the sun and in turbulent regions in interplanetary space.

The possibility that the solar corona is composed of small-scale filaments near the sun is considered. It is shown that such filamentary structure would not be seen at the orbit of Earth. It is pointed out that the expansion of a non-filamentary corona seems to lead to too high a calculated wind density at the orbit of Earth to agree with the present observations, unless $T(r)$ is constant or increases with r. A filamentary corona, on the other hand, would give the observed wind density for declining $T(r)$.

It is shown that viscosity plays no important role in the expansion of an atmosphere either with or without a weak magnetic field. The termination of the solar wind, presumably between 10 - 10^3 a.u., is discussed briefly. The interesting development here is the interplanetary L_α recently observed, which may come from the interstellar neutral hydrogen drifting into the outer regions of the solar wind.

Theory is at the present time concerned with the general dynamical principles which pertain to the expansion equilibrium of an atmosphere. It is to be expected that the rapid progress of direct observations of the corona and wind will soon permit more detailed studies to be carried out. It is important that the distinction between detailed empirical models and models intended to illustrate general principles be kept clearly in mind at all times.

Dynamical Theory of the Solar Wind

I. Introduction

The solar wind was first known as the phenomenological "solar corpuscular radiation" responsible for auroral and geomagnetic activity. General recognition of the existence of solar corpuscular radiation began with Störmer's proposal, at the turn of the century, that the aurora is caused by monoenergetic beams of fast particles from the sun (See Störmer, 1955). It is now known that Störmer's picture is incorrect, but it served at the time to focus attention on the existence and importance of corpuscular radiation from the sun.

Somewhat later Chapman and Ferraro (1931) pointed out that geomagmetic activity is of solar corpuscular origin and began work on a general model which has developed into the present ideas of the geomagnetic storm.

More recently the cosmic ray variations, first discovered by Forbush (1938, 1954), have been shown from observations to be of interplanetary magnetic origin (Meyer and Simpson, 1955, 1957; Neher and Anderson, 1962; Masley, et al, 1962). The variations result from the expulsion of cosmic rays from the inner solar system by the magnetic fields transported in the solar corpuscular radiation (Morrison, 1954; Parker, 1958c, 1961a, 1963, 1965).

The intermittent nature of auroral and geomagnetic activity at most observing stations led to the view that solar corpuscular radiation is emitted in isolated clouds and/or narrow beams from special active regions on the surface of the sun. The mechanism for producing the beams of energetic particles was not known, but was generally believed to be electromagnetic in nature. Unfortunately this phenomenological

picture is not correct. The solar corpuscular radiation is not restricted to narrow beams and its origin is not restricted to special active regions on the sun. It was Biermann's (1951, 1957) analysis of comet tails which first showed the general nature of solar corpuscular radiation. Biermann showed that gaseous comet tails point away from the sun as a consequence of the pressure of solar corpuscular radiation. Then, since comets with gaseous tails are observed never to fail to point away from the sun, he noted that solar corpuscular radiation fills interplanetary space at all times. The nearly radial direction of the comet tail indicated corpuscular velocities of at least several hundred km/sec.

The next important step was made by Chapman (1957, 1959) who pointed out that energy transport by thermal conduction in the outer solar atmosphere exceeds the radiative energy losses, with the result that the 10^{6} °K observed near the sun must extend beyond the orbit of Earth with but little decline. The usual assumption of hydrostatic equilibrium led to a calculated density of 10^{2} atoms/cm^{3} at the orbit of Earth, so that Chapman was able to state that the solar corona extended throughout the inner solar system.

Now an interplanetary magnetic field of as little as 10^{-8} gauss is sufficient to prevent the free passage of Biermann's general solar corpuscular radiation through Chapman's ambient coronal gases. Yet neither Biermann nor Chapman could be denied. Resolution of the difficulty lay in recognizing that the solar corpuscular radiation and the extended solar corona must be the same thing, from which it followed that the dynamical origin of the solar corpuscular radiation must be sought in the dynamical properties of the extended solar atmosphere (Parker, 1958a).

The basic facts concerning the extended solar corona are that the atmosphere is tightly bound to the sun by the solar gravitational field and the atmosphere has an extended temperature ($T \propto 1/r^{2/7}$ according to Chapman's calculations). The thermal energy of each atom in the lower corona is of the order of one tenth the gravitational energy.* A tightly bound atmosphere traditionally has been considered to be in hydrostatic equilibrium. Escape from the atmosphere is limited to evaporation, or thermionic emission, of the small portion of atoms in the outer layers whose thermal velocity exceeds the escape velocity, as discussed by Jeans (see for instance, Pickelner, 1950; van de Hulst, 1953). But evaporation from the solar corona gives low particle velocities in space and cannot explain the solar corpuscular emission, of 300-2000 km/sec. It was evident therefore, that a re-examination of the theory of the dynamical equilibrium of a stationary atmosphere was required. The novel feature of the solar corona was its extended temperature**. It was a straight forward matter to show from the dynamical equations that the equilibrium of an atmosphere with a sufficiently extended temperature is supersonic expansion (Parker, 1958a), in constrast to the static equilibrium of an ordinary atmosphere.

The present paper summarizes the dynamical equilibrium theory of the tightly bound, extended atmosphere as it has been worked out to understand the

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* The terrestrial atmospheric hydrogen is bound to Earth to about the same degree.

**The basic energy source for the corona is believed to be the dissipation of hydrodynamic and/or hydromagnetic waves from the photosphere (Alfven, 1947; Schwarzschild, 1948; Biermann, 1948; Schatzman, 1949). The observed outward expansion of the temperature (see summary in van de Hulst, 1953) is explained at least by the thermal conductivity (Chapman, 1957, 1959) if not be extended dissipation of waves (see section IV C).

origin of the solar corpuscular radiation (Parker, 1958a, 1960, 1964a-c, 1965b). The major portion of the paper is devoted to theoretical considerations, but it is important to note that rapid progress is now being made in direct measurements of the solar corpuscular radiation in space (Shklovskii, et al, 1960; Gringauz, et al, 1960; Neugebauer and Snyder, 1963, 1965; Bridge, et al, 1964; Coleman, et al, 1960; Ness, et al, 1964) and in observations of coronal conditions near the sun (Ney, et al, 1961; Billings, 1963). The observations and measurements have now established the main qualitative features of the theory, and provide hourly and daily numbers for the velocity and density of the expanding corona at the orbit of Earth. In addition, the direct measurements are providing information on the details of temperature, inhomogeneity, turbulence, time variation, etc. which neither theory nor indirect inference from geophysical and cometary phenomena could anticipate. Some idea of the extent of active coronal heating near the sun can already be inferred from the recent observations. But, on the other hand, for all the recent rapid progress, detailed quantitative observations of the corona near the sun and its expansion throughout interplanetary space are not yet sufficiently comprehensive to permit construction of a detailed quantitative model of the corona and wind. Theoretical study is still aimed at understanding the basic dynamical principles.

II. The Equilibrium of a Steady Atmosphere

To get at the basic dynamical principles of atmospheric equilibrium consider an atmosphere with spherical symmetry about a nonrotating spherical parent body of radius a, mass M_*, surrounded by a cold void at infinity.*

*The typical interstellar pressures of 10^{-14}-10^{-12} dynes/cm^2 are negligible for the present purposes.

If the atmosphere is sufficiently dense that the mean free path is small compared to the scale height and/or the radial distance r from the parent body, then the ordinary hydrodynamic equations are appropriate. This is the situation in the solar corona. If the atmosphere is so tenuous that the mean free path is long, the universal presence of weak magnetic fields and the attendant instabilities maintain approximate statistical isotropy in the thermal motions, and it turns out that again the large-scale, low frequency variations of the gas are describable, in at least an approximate way, by the conventional hydrodynamic equations with isotropic pressure. This appears to be the situation in the expanding corona at the orbit of Earth.

The calculations are carried out for ionized hydrogen, with M denoting the mass of the hydrogen atom, $N(r)$ the number of atoms per cm^3, and $T(r)$ the local effective temperature, so that the pressure is $2N(r)\,k\,T(r)$. Generalization to mixtures of gas is straightforward and is left to the reader. Hydrostatic equilibrium requires that

$$\frac{d}{dr}\,2NkT + \frac{GM_* NM}{r^2} = 0\,, \tag{1}$$

so that if the density and temperature are N_0 and T_0 at the base of the atmosphere at $r = a$, integration of (1) yields

$$N(r)\,T(r) = N_0 T_0 \exp\left[-\frac{GM_* M}{2k}\int_a^r \frac{dr}{r^2\,T(r)}\right] \tag{2}$$

at some radial distance r. It is evident from the integral in the exponential that if $T(r)$ declines more rapidly than $1/r$, the integral

increases without bound and $\lim N(r)\, k\, T(r) = 0$, $r \to \infty$. This is the classical static atmosphere. The starting point of the theory of the solar corona and the solar corpuscular radiation is to suppose that the temperature of the corona extends farther into space than $1/r$. With such an extended temperature, which declines asymptotically less rapidly than $1/r$ as $r \to \infty$, it is readily seen that $\lim N(r)\, T(r) > 0$. A finite pressure must be exerted inward on the atmosphere to satisfy the conditions for static equilibrium. Such pressures are lacking. Hence no static equilibrium exists for such an atmosphere.* So we must admit the possibility of radial motion $v(r)$.

Stationary equilibrium requires conservation of momentum**

$$NMv\frac{dv}{dr} + \frac{d}{dr}2NkT + \frac{GM_*MN}{r^2} = 0 \tag{3}$$

and conservation of mass

$$Nvr^2 = N_0 v_0 a^2. \tag{4}$$

Too little is known about the heat input to the solar corona to write a definite energy equation, so $T(r)$ is considered to be known from observation, etc., and the

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* A discussion of the necessary conditions for static equilibrium of an adiabatic atmosphere, as compared with the solar corona, have been given elsewhere (Parker, 1960).

**Viscosity has been omitted from this equation. Its effects are unimportant and will be discussed in section VII.

dependence of the dynamical properties of the atmosphere on the form of $T(r)$ will be enumerated. In the first theoretical demonstration of the dynamical properties described by (3) the extended temperature was represented by a simple step function, by putting $T(r) = T_0$ for some distance outward from the star, with $T(r) = 0$ beyond. The unique stationary expansion equilibrium of the atmosphere followed immediately, showing that the 1000 km/sec solar corpuscular radiation is the natural hydrodynamic equilibrium expansion of the solar atmosphere (Parker, 1958a). A more general discussion is preferable here.

The stationary equilibrium state of an atmosphere can be deduced from the general mathematical properties of (3) and (4). Use (4) to eliminate $N(r)$ from (3), obtaining

$$v \frac{dv}{dr}\left(1 - \frac{c^2}{v^2}\right) = R(r) \tag{5}$$

where the temperature $T(r)$ is expressed in terms of the characteristic thermal velocity $c(r) = [2kT(r)/M]^{1/2}$, the gravitational potential is denoted by the characteristic escape velocity $w = (GM_*/a)^{1/2}$, and $R(r)$ denotes the known function

$$R(r) \equiv -r^2 \frac{d}{dr}\left(\frac{c^2}{r^2}\right) - \frac{w^2 a}{r^2} . \tag{6}$$

It is assumed that the atmosphere is strongly bound to the star, $w^2 >> c_0^2 >> v_0^2$, where the subscript zero denotes the value at the base of the atmosphere, $r = a$. For simplicity suppose that

$T(r)$ declines monotonically to zero at $r = \infty$. It is pointed out above that, if $T(r)$ declines asymptotically with r faster than $1/r$, there exists a static equilibrium*. Consider, then, the extended temperature where $T(r)$ declines asymptotically less rapidly than $1/r$, i.e. there exists two positive numbers A and ϵ such that $T(r) > A/r^{1-\epsilon}$ for all $r > a$. The quantity $R(r)$ is negative at the base of the atmosphere because $w^2 >> c_0^2$. It is readily shown that if $T(r)$, of $c^2(r)$, declines less rapidly than $1/r$, the term $-r^2 d(c^2/r^2)/dr$ is positive and decreasing less rapidly than $1/r^2$, so that $R(r)$ becomes positive for large r. Hence $R(r)$ has a zero at some intermediate value r_c. The nature of the solutions of (5) is determined by the existence of r_c. The general topology of the solutions is sketched in Fig. 1.

To see how the form of the solutions comes about, note that at the base of the atmosphere v is small and both $1 - c^2/v^2$ and $R(r)$ are negative, making dv/dr positive. If v increases so rapidly with r that v becomes equal to c at some $r < r_c$, then dv/dr has a simple pole where $v = c$ and dv/dr goes to $+\infty$, and then $-\infty$ on the other side of the pole. Such solutions are without physical significance. On the other hand, if v increases so slowly that it does not reach c by the time $r = r_c$, then dv/dr becomes zero at r_c and negative beyond.

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*The static equilibrium may be subject to convective instability if the temperature declines more rapidly than the adiabatic lapse rate, of course.

The velocity passes through a maximum at $r = r_c$. The velocity cannot then surpass c beyond r_c because $dv/dr \rightarrow -\infty$ if v should approach c from below, quickly sending v to zero. Thus $v(r)$ declines to zero as $r \rightarrow \infty$. It is readily shown that a finite inward pressure from infinity is required to maintain this declining flow. To demonstrate this, integrate (3) from r_c to r, obtaining

$$p(r) = p(r_c) \exp\left[- I_1(r) + I_2(r)\right]$$

where

$$I_1(r) = w^2 a \int_{r_c}^{r} \frac{dr}{r^2 c^2(r)}, \quad I_2 = - \int_{r_c}^{r} dr \frac{v}{c^2} \frac{dv}{dr}. \tag{7}$$

It is evident that both $I_1(r)$ and $I_2(r)$ are positive for $r > r_c$, and that $I_1(r)$ is finite as $r \rightarrow \infty$. Hence $p(\infty)$ is finite.

There remains, then, the single solution for which v reaches c simultaneously with the vanishing of $R(r)$. This solution passes smoothly and continuously across the point* $(r_c, c(r_c))$ on the vr-plane and extends out to $r = \infty$ with $dv/dr > 0$. The density, according to (4) declines at least as fast as $1/r^2$ and the pressure required at $r = \infty$ to maintain the solution is zero. It follows, therefore, that <u>when the</u>

*The solutions in the neighborhood of the critical point $(r_c, c(r_c))$ can be found in the literature (see p. 830 of Parker, 1960; p. 59 of Parker, 1963; p. 89, Parker, 1964a).

temperature declines less rapidly than $1/r$ the only steady equilibrium state of an atmosphere surrounded by vacuum is expansion to supersonic velocity at large radial distance from the parent body. Any star with a corona such that $T(r) > A/r^{1-\epsilon}$ will have a stellar wind unless the corona is forcibly confined by external pressure.

Now, the discussion could go on to treat more general temperature distributions. For instance, it is not necessary to assume that $T(\infty) = 0$. If $T(\infty) > 0$, the expansion is enhanced, the isothermal corona being a simple case in point. The argument can be carried through along the simple lines given above for the case that $c^2(r)$ is a monotonic function of r, bounded above by $w^2/3$ if increasing, and below by $c_0^2 - \frac{1}{3}w^2(1 - a/r)$ if decreasing, etc. Nor is it necessary to suppose $T(r)$ to be a monotonic function of r. $T(r)$ may increase for some distance out from the star*, and then decrease at larger r. $R(r)$ will have a zero if $T(r)$ does not increase as fast as r^2, or does not decrease faster than $1/r$ from a point too near the star. Supersonic expansion occurs provided only that $T(r)$ extends to some distance r_2 sufficiently beyond the critical distance r_c. Beyond r_2, $T(r_2)$ may decline adiabatically** or vanish altogether *** without altering the supersonic character of the expansion. In

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*See p. 864, Parker (1960)

**See p. 850, 851, Parker (1960)

***See p. 667, Parker (1958a)

connection with the latter, note that, while a temperature declining asymptotically faster than $1/r$ permits a static atmosphere, it does not require a static atmosphere in all cases. To illustrate this theoretical fact suppose that the temperature is uniform out to some distance $r = b$, and vanishes beyond. Beyond $r = b$ the gas is coasting in the gravitational field so that

$$v^2(r) = v^2(b) - 2w^2 \frac{a}{b}\left(1 - \frac{b}{r}\right).$$

The gas reaches $r = \infty$ with finite, and therefore supersonic, velocity if $v^2(b) > 2w^2 a/b$. It is readily shown from the integration of (5) from a to b that this is achieved provided that $b > r_c\, 2^{-2/3} \exp 1 \cong 1.71\, r_c$, where $r_c = aw^2/2c^2$ in this case. Thus a static atmosphere would be a possible equilibrium, with supersonic expansion the other equilibrium. It should be pointed out, however, that the static equilibrium would be convectively unstable as a consequence of the rapid decline of $T(r)$ at $r = b$, so one would probably not expect to find the static equilibrium in nature.

Finally, it should be noted that if $T(r)$ declines asymptotically exactly as $1/r$, instead of faster or slower, the stationary equilibrium may be a subsonic expansion resembling the classical evaporation from the top of a static atmosphere. This case has been explored extensively by Chamberlain (1960, 1961) and conditions for achieving it are discussed in section IV B.

III. Stationary Expansion of an Extended Atmosphere with Known Temperature

Consider an atmosphere with an extended temperature $T(r)$ which is presumed to be a known function of r. The form $v(r)$ of the expansion can be computed by integrating (5). The solution of physical interest is the one beginning with low velocity at small r and giving vanishing pressure at $r = \infty$, which was shown to be the solution passing across the point $(r_c, c(r_c))$ on the rv-plane, where r_c is the zero of $R(r)$. This nonlinear eigenvalue problem can be solved analytically as follows (Parker, 1964a).

From the base of the atmosphere out to the critical distance r_c, the velocity of expansion is smaller than the thermal velocity, so that to a first approximation the term $v\, dv/dr$ can be neglected in (5). Integration then yields

$$v(r) = v_0 \left(\frac{a\, c(r)}{r\, c_0}\right)^2 \exp\left[+w^2 a \int_a^r \frac{dr}{r^2 c^2(r)}\right]. \tag{8}$$

The requirement that $v(r) = c(r_c)$ yields the eigenvalue

$$\frac{v_0}{c_0} = \frac{r_c^2\, c_0}{a^2\, c(r_c)} \exp\left[-w^2 a \int_a^{r_c} \frac{dr}{r^2 c^2(r)}\right] \tag{9}$$

and the eigenfunction

$$v(r) = \frac{c^2(r)\, r_c^2}{c(r_c)\, r^2} \exp\left[-w^2 a \int_r^{r_c} \frac{dr}{r^2 c^2(r)}\right]. \tag{10}$$

The next higher approximation is obtained by using this first approximation to estimate $v\, dv/dr$, etc. The iteration procedure converges and the first approximation (10) is not in error anywhere by more than about 50 percent. Physically the approximation is based on the fact that the density distribution in (a, r_c) is near the static equilibrium value given by (2) because the outward acceleration of the gas is small compared to the gravitational acceleration. The velocity follows from the density through (4), yielding (8).

Beyond r_c the equations may be integrated by neglecting the term $c^2\, d\ln v/dr$ on the left hand side of (5). Integrating (5) then gives

$$\frac{1}{2} v^2 + c^2 - \frac{w^2 a}{r} = \frac{3}{2} c^2(r_c) - \frac{w^2 a}{r_c} + 2\int_{r_c}^{r} \frac{dr}{r}\, c^2(r). \tag{11}$$

The second approximation consists in using the first approximation to estimate the neglected term, etc. It has been shown that the procedure converges and the first approximation yields a $v(r)$ which is not in error anywhere by more than about 50 percent for $w^2 >> c_0^2$.

The simple example where $T(r)$ is of the form $T_0(a/r)^\beta$ $(\beta < 1)$ has been worked out (Parker, 1964a) yielding

$$r_c = a\left(\frac{w^2}{(2+\beta)c_0^2}\right)^{1/(1-\beta)}, \tag{12}$$

$$v_0 \cong c_0 \left(\frac{w^2}{(2+\beta)\, c_0^2} \right)^{(4+\beta)/2(1-\beta)} \exp\left(\frac{2+\beta - w^2/c_0^2}{1-\beta} \right) \tag{13}$$

and

$$v(r) \cong c_0 \left(\frac{a}{r}\right)^{2+\beta} \left(\frac{w^2}{(2+\beta)\, c_0^2} \right)^{(4+\beta)/2(1-\beta)} \exp\left\{ \frac{w^2}{(1-\beta^2)\, c_0^2} \left[1 - \left(\frac{a}{r}\right)^{1-\beta} \right] \right\} \tag{14}$$

in $r \leq r_c$, with

$$v^2(r) \cong c_0^2 \left\{ \frac{4-\beta-2\beta^2}{\beta} \left[\frac{(2+\beta)\, c_0^2}{w^2} \right]^{\beta/(1-\beta)} \right. \tag{15}$$

$$\left. - \frac{2(\beta+2)}{\beta} \left(\frac{a}{r}\right)^{\beta} + \frac{2w^2}{c_0^2} \frac{a}{r} \right\}$$

in $r > r_c$. It is evident that

$$v^2(\infty) \cong c_0^2 \, \frac{4-\beta-2\beta^2}{\beta} \left[\frac{(2+\beta)\, c_0^2}{w^2} \right]^{\beta/(1-\beta)} . \tag{16}$$

The second approximation is easily worked out and is expressible in terms of the incomplete gamma function and an integral of an algebraic function of r and r^{β}. The example serves to illustrate the variation of the velocity and density of the wind with T_0 and β. The velocity $v(\infty)$ declines with increasing β, going to zero as $\beta \rightarrow 1$. The asymptotic density $N(r)r^2$ at large r first increases with increasing β and finally declines to zero as $\beta \rightarrow 1$. The increase of the density is brought about by the fact that $v(\infty)$ decreases more rapidly than v_0. The results are plotted in Fig. 2 for the special cases $w^2/c_0^2 = 10$, 5, and $2+\beta$. The value $w^2/c_0^2 = 2+\beta$ is the minimum value of w^2/c_0^2 for which $R(r)$ has a zero for $r > 2$. For larger T_0, the atmosphere is no longer tightly bound to the star. It is important to note from Fig. 2 that the density of the wind at large r declines with decreasing T_0 much more rapidly than $v(\infty)$. The reason is the exponential dependence of v_0 on T_0, in (9). The density of the wind at large can be increased (or decreased) enormously for a given value of $v(\infty)$ by suitably increasing (or decreasing) T_0 and β together.

The expansion of an isothermal atmosphere (Parker, 1958a; 1960) is particularly instructive because of its simplicity and its close approximation to the solar wind as presently observed at the orbit of Earth. The equations can then be integrated exactly yielding the eigenfunction $v(r)$ for vanishing pressure at infinity,

$$\frac{v^2}{c_0^2} - \ln\frac{v^2}{c_0^2} = -3 + 4\ln\frac{r}{r_c} + 4\frac{r_c}{r}, \tag{17}$$

where $r_c = a w^2 / 2 c_0^2$. Fig. 3 is a plot of $v(r)$ with $M_* = M_\odot$, $a = R_\odot$ for various temperatures. Fig. 4 is a plot of the velocity v_0 at the base of the corona, $v(r_E)$ at 1 a.u., and the density $N(r_E)$ at 1 a.u. assuming $N_0 = 10^8$ atoms/cm^3 at the base of the atmosphere.

Models for expansion in which the temperature is isothermal out to some distance $r = b$, and adiabatic beyond, and models in which $T(r)$ is taken to be of the form $N^{\alpha - 1}$ may be found in the literature (Parker, 1958a, 1960, 1963). See also the review by Lüst (1963). The case for $\beta = 2/7$ was computed numerically by deJager (1963). Models for $T(r)$ limited solely to thermal conductivity have been computed numerically by Noble and Scarf (1963). The present section has been concerned with the basic theoretical properties of the momentum equation for a tightly bound atmosphere, which in the next section, is considered in connection with the energy equation which determines $T(r)$.

IV. Stationary Expansion of an Extended Atmosphere with Temperature Determined by the Expansion

The ideal theoretical tool for treating the temperature $T(r)$ in an atmosphere would be the complete energy equation in which the divergence of the energy flux is set equal to the net energy loss. Unfortunately the heating of extended atmospheres, such as the solar corona, is understood only qualitatively. The reader is refered to the literature for some of the present ideas on coronal heating by wave dissipation, and the general quantitative uncertainty which exists (Parker,

1960, 1964d; Schatzman, 1961; Osterbrock, 1961; Lüst, et al, 1962; Billings, 1963; Whitaker, 1963; Bird, 1964; Moore and Spiegel, 1964; and references therein to earlier papers). About all that can be said with confidence is that radiative losses appear to be small compared to the energy transport by thermal conduction, except perhaps in the densest coronal regions (Chapman, 1957, 1959); the temperatures appear to lie in the range $1 - 4 \times 10^{6}$ °K and vary only slowly, if at all, with radial distance from the sun (van de Hulst, 1953; Dollfus, 1953, 1957; Billings, 1959, 1962; Pottasch, 1960; Burgess, 1960, 1964; Burke and Seator, 1961; Malik and Trefftz, 1961; Billings and Lilliequist, 1963). Nothing is really known about the temperatures of the coronas of other stars, though several classes of stars are expected for various reasons to have coronas (see discussion in Parker, 1958a, 1960, 1963; Wilson and Skumanich, 1964; Clayton, 1964). Consequently the present theoretical treatment of temperature in an expanding atmosphere is limited to hypothetical cases which illustrate the consequences of various possible conditions for energy transport in the coronas of the sun and other stars. The study is necessary to develop a theoretical understanding of the role of various kinds of energy transport, and in the end permits us to assert from observations of the solar wind that the corona of the sun is actively heated by wave dissipation for some considerable, but unspecified, distance into space.

A. Polytrope Temperature Dependence

The equilibrium states of atmospheres with various forms of temperature have been illustrated by a number of special examples. The isothermal atmosphere was the first to be worked out in the original demonstration of the existence of the

supersonic expansion equilibrium of a tightly bound atmosphere (Parker, (1958a). Later the consequences of an outward decline of temperature were investigated using the conventional artifice of the polytrope relation $T(r) \propto N^{\alpha-1}(r)$ (Parker, 1960, 1963). The method is sufficiently flexible as to permit an investigation of expansion which is not radial, but the reader interested in this generalization is refered to the literature (Parker, 1963). It is sufficient in the present exposition to consider only the case for purely radial flow, which arises in an atmosphere with spherical symmetry, etc. The mathematical treatment shows that the necessary and sufficient condition for the stationary equilibrium state to be supersonic expansion from quasi-static conditions at small r is that $\alpha < 3/2$ and

$$2\alpha < \frac{w^2}{c_0^2} < \frac{\alpha}{\alpha - 1} + \frac{v_0^2}{2c_0^2} . \tag{18}$$

The final velocity of expansion at $r = \infty$ is then

$$v^2(\infty) = v_0^2 + \frac{2\alpha}{\alpha - 1} c_0^2 - 2w^2 . \tag{19}$$

If the temperature T_0 at the base of the atmosphere becomes so high that $w^2/c_0^2 \geq 2\alpha$, the atmosphere is no longer strongly bound by the gravitational field. The atmosphere explodes violently away from the star with $v_0 = O(c_0, w)$. When T_0 is smaller than $w^2/c_0^2 = 2\alpha$, the atmosphere is tightly bound and $v_0^2 << c_0^2$. When T_0 becomes

so small that $w^2/c_0^2 > \alpha/(\alpha-1)$, the equilibrium is static, with the familiar static polytropic atmosphere

$$N(r) = N_0 \left[1 - \frac{\alpha-1}{\alpha}\frac{w^2}{c_0^2}\left(1 - \frac{a}{r}\right)\right]^{1/(\alpha-1)} \tag{20}$$

out to

$$r = a\left[1 - \frac{\alpha c_0^2}{w^2(\alpha-1)}\right]^{-1} \tag{21}$$

and zero beyond.

Outside the range $1 < \alpha < 3/2$ there is an expansion equilibrium $v = v_0(a/r)^{1/2}$ for $\alpha = 5/3$ when T_0 has precisely the value $c_0^2 = (2w^2 - v_0^2)/5$. This equilibrium corresponds to the classical evaporation of an otherwise static atmosphere, which Chamberlain (1960, 1961) has studied extensively with the hydrodynamic equations.

Dahlberg (1964) has rewritten the equations for $T \propto N^{\alpha-1}$ using the local Mach number and the local ratio of gravitational to thermal energy as an alternative set of variables. He points out a class of solutions for $3/2 < \alpha < 5/3$ with $2\alpha/(\alpha-1) < \lambda < 4\alpha$ going from subsonic velocity at the base of the corona to supersonic velocity at large r. The solutions do not appear in the context of the present problem because they start with large, though subsonic, velocity at a finite distance from the

origin and decellerate outward from the star. It would be interesting to work out what conditions the solution would fit to at the base of the corona where they start. The decelleration is less rapid than the temperature decline, so the solutions become supersonic at large r, with $v(r) \sim$ constant > 0. Decelleration equilibrium requires that $T(r)$ decline more rapidly than $1/r$, in this case as $1/r^{2(\alpha-1)}$. It is evident that the same $T(r)$ has a static equilibrium associated with it too. Unfortunately the asymptotic behavior of both $T(r)$ and $v(r)$ is only implicit in Dahlberg's formulation of the problem.

In a recent paper Carovillano and King (1965) have demonstrated that no solutions, subject to the boundary conditions of zero pressure at infinity and vanishing velocity at the origin, have been omitted by the original treatment of the problem. There are, of course, a variety of other solutions for $1 < \alpha < 3/2$, and for $\alpha < 1$, $\alpha > 3/2$, but they apply to circumstances other than the outward equilibrium extension of an atmosphere from a quasi-static star.

The equivalent energy transport for $T \propto N^{\alpha-1}$ has been worked out (Parker, 1963) and is not unlike that which one might expect for wave dissipation and/or thermal conduction. For instance, it is readily shown that, in order to maintain the polytrope α different from $5/3$ from the base of the atmosphere out to where the density has fallen to N_1 $(< N_0)$, the energy flux (ergs/sec steradian) carried by the propagation of waves, thermal conduction, etc. is

$$F(r) = a^2 N_0 v_0 k T_0 \frac{5-3\alpha}{\alpha-1} \left[\left(\frac{N}{N_0}\right)^{\alpha-1} - \left(\frac{N_1}{N_0}\right)^{\alpha-1} \right] \qquad (22)$$

in addition to the convection of thermal energy, kinetic energy, and gravitational energy. It is evident that $F(r)$ is a monotonically declining function of r.

B. Energy Transport Limited to Thermal Conduction

The minimum energy transport in a stellar corona is thermal conduction. Thermal conductivity is limited in most cases to the direction along the magnetic fields, but the expansion of the atmosphere causes the fields to be approximately radial (Parker, 1958a). Hence the conductivity is essentially uninhibited, for the most part, by such fields. One can, of course, imagine special circumstances in which this does not obtain, but they will not be taken up in the present theoretical discussion.

The first solutions of (3) and (4) with the minimum energy transport were numerical, consisting of some subsonic cases (Chamberlain, 1961), and some supersonic cases fitted to the observed wind at the orbit of Earth (Noble and Scarf, 1963). The present discussion will be concerned principally with the basic analytical and theoretical properties of the stellar corona with energy transported limited to thermal conduction, (Parker, 1964b, 1965) rather than with quantitative numerical models of the actual wind and corona. The atmosphere with minimum energy transport is an interesting hypothetical case because it can be solved by analytical methods which permit a detailed theoretical comparison of the subsonic and supersonic equilibria not possible with the simpler polytrope temperature dependence. Comparison of the theoretical calculations with the observed solar corona shows that minimum energy transport gives a bad fit and demonstrates that the

solar corona has more outward energy transport than thermal conduction.

The energy equation may be written in the form of the total energy flow per sec steradian,

$$F = -r^2 \kappa(T) \frac{dT}{dr} + N(r) v(r) r^2 \left(\frac{1}{2} M v^2 + 5kT - \frac{G M_* M}{r} \right), \tag{23}$$

where the thermal conductivity is given approximately as

$\kappa(T) \cong 6 \times 10^{-7} T^{5/2}$ ergs / cm² sec °K (Chapman, 1954).

The terms on the right hand side of the equation represent thermal conduction and convection of kinetic energy, enthalpy, and gravitational energy, respectively. In the absence of other sources or sinks, $dF/dr = 0$. The coefficient $N v r^2$ is also independent of r. The temperature must be determined by simultaneous solution of (5) and (23). The mathematical solution of (5) for a given $T(r)$ was discussed in section III. The next step is to consider the mathematical solution of (23). The mathematics illustrate so well the general properties of an atmosphere described by (5) and (23) that we shall outline the solution rather than merely quote the results. The discussion will be restricted to a tightly bound atmosphere $(w^2 \gg c_0^2)$.

The same approach can be used for (23) as was employed in the solution of (5). Between the base of the atmosphere $r = a$ and the critical point at r_c, the kinetic energy $\frac{1}{2} M v^2$ is small compared to $5kT$, so that to a first approximation

$$F \cong -r^2 \kappa(T) \frac{dT}{dr} + N_0 v_0 a^2 \left(5kT - \frac{GM_* M}{r} \right) \tag{24}$$

in (a, r_c) . But this equation is independent of $v(r)$ except for the eigenvalue v_0 . The equation can be integrated to give $T(r)$ directly, which then yields $v(r)$ from (5). The general integration of (24) has been given elsewhere (Parker, 1964b). It is sufficient for the present purposes to consider two special cases. Consider first the solution of (24) in the limit of small N_0 . Write $\kappa(T)$ as $\kappa_0 (c^2/c_0^2)^{5/2}$. The differential equation reduces to

$$\frac{F}{\kappa_0 a T_0} = -\frac{r^2}{a} \left(\frac{c^2}{c_0^2} \right)^{5/2} \frac{d}{dr} \left(\frac{c^2}{c_0^2} \right). \tag{25}$$

The solution of this equation subject to the boundary conditions that $c^2(a) = c_0^2$ and $c^2(\infty) = 0$ is the familiar Chapman solution

$$c^2(r) = c_0^2 \left(\frac{a}{r} \right)^{2/7}, \quad F = \frac{2}{7} \kappa_0 a T_0 \tag{26}$$

for a static atmosphere. The result is not surprising, of course, because the limit $N_0 \longrightarrow 0$ effectively eliminates the convective transport. Note that F is uniquely determined by the boundary conditions.

The next degree of approximation in integrating (24) is to include

N_o , but take advantage of the fact that $5kT$ is small compared to $GM_* M/r$ in $r < r_c$ for a tightly bound corona $(w^2 >> c_o^2)$.*

Then it is readily shown that

$$T(r) = T_o \left[\frac{a}{r} \left(\frac{1 + r_2/r}{1 + r_2/a} \right) \right]^{2/7} , \tag{27}$$

where

$$r_2 \equiv a \frac{N_o v_o a^2 M w^2}{2F} \tag{28}$$

and F is determined uniquely as

$$F = \frac{2}{7} \kappa_o a T_o - \frac{1}{2} N_o v_o a^2 M w^2 \tag{29}$$

by the boundary conditions. The energy flux F_c across the base of the atmosphere as a consequence of thermal conduction is

$$F_c = \frac{4}{7} \kappa_o a T_o - F \tag{30}$$

$$= \frac{2}{7} \kappa_o a T_o + \frac{1}{2} N_o v_o a^2 M w^2$$

---

*For $T \propto r^{-\beta}$ the critical distance r_c is given by (12) and $5kT$ divided by $GM_* M/r$ at r_c is $5/(2+\beta) = O(1)$, so that $5kT$ becomes comparable to $GM_* M/r$ only as r reaches r_c .

and is different from F because F includes convective transport. Note that $F \geq 0$, so that $F_c \leq 4 k_0 a T_0 / 7$, and hence

$$N_0 v_0 a^2 M w^2 \leq \frac{4}{7} k_0 a T_0 . \tag{31}$$

This inequality plays an important role in demonstrating the necessary and sufficient conditions for supersonic expansion.

Now consider the solution of the momentum equation associated with the temperature distributions (26) and (27). For very small N_0, $r_c \to 0$ and $T(r)$ is given by (26). The distance r_c to the critical point is then given by (12) with $\beta = 2/7$ as

$$r_c = a \left(\frac{7 w^2}{16 c_0^2} \right)^{7/5} \tag{32}$$

at which point

$$v(r_c) = c(r_c) = c_0 \left(\frac{16 c_0^2}{7 w^2} \right)^{1/5} . \tag{33}$$

The velocity v_0 is given by (13) as

$$v_0 \cong c_0 \left(\frac{7w^2}{16c_0^2}\right)^3 \exp\left(\frac{16}{5} - \frac{7w^2}{5c_0^2}\right) \tag{34}$$

Things remain in this state as N_0 increases until $N_0 v_0 a^2 M w^2$ cannot be neglected and (27) must be substituted for (26). Note that (27) gives $T \propto r^{-4/7}$ for $r < r_2$ and $T \propto r^{-2/7}$ for $r > r_2$. Thus there is no change in $T(r)$ until r_2 becomes comparable to a, at which point it is readily shown from (28) and (29) that

$$N_0 v_0 a^2 M w^2 = \frac{2}{7} k_0 a T_0 \quad , \quad F = \frac{1}{7} k_0 a T_0 \tag{35}$$

Since v_0 is still given approximately by (34) when $r_2 = a$, it follows at once that $r_2 = a$ when N_0 has increased to N_A, where, in order of magnitude,

$$N_A = O\left\{\frac{\frac{2}{7} k_0 a T_0}{a^2 M w^2 c_0^2} \left(\frac{16c_0^2}{7w^2}\right)^3 \exp\left(\frac{7w^2}{5c_0^2} - \frac{16}{5}\right)\right\}. \tag{36}$$

For $N_0 > N_A$, it follows that $r_2 > a$, so that the

temperature declines as $r^{-4/7}$ for some distance before going over to $r^{-2/7}$. The larger is N_0, the larger is the transition distance r_2 at which $r^{-4/7}$ goes over into $r^{-2/7}$. For sufficiently large N_0, say $N_0 = N_B$, r_2 becomes equal to r_c. When this occurs, $T \propto r^{-4/7}$ all the way to r_c, so that from (12)

$$r_c \cong a\left(\frac{7w^2}{18c_0^2}\right)^{7/3}, \quad c(r_c) = c_0\left(\frac{18c_0^2}{7w^2}\right)^{4/3}. \tag{37}$$

Note from this expression that, with $w^2 >> c_0^2$, r_c/a is a very large number. Write r_2 as

$$\frac{r_2}{a} = \frac{N_B v_0 a^2 M w^2}{\frac{4}{7}k_0 a T_0 - N_B v_0 a^2 M w^2}. \tag{38}$$

Requiring that $r_2 = r_c$ means that the denominator of (38) must be very small compared to the numerator,

$$N_B v_0 a^2 M w^2 \cong \frac{4}{7} k_0 a T_0 . \tag{39}$$

The velocity v_0 is given by (13), with the result that (39) can be solved for N_B to give

$$N_B = O\left\{ \frac{\frac{4}{7}\kappa_0 a T_0}{a^2 M w^2 c_0} \left(\frac{18 c_0^2}{7 w^2}\right)^{16/3} \exp\left(\frac{7}{3}\frac{w^2}{c_0^2} - 6\right)\right\} \tag{40}$$

Now consider what happens if N_0 is increased beyond N_B. The energy supply by thermal conduction from the base of the atmosphere is $4\kappa_0 a T_0 / 7$, which remains fixed. The energy consumed by the expanding atmosphere is not less than the amount $N_0 v_0 a^2 M w^2$ required to lift the expanding gases out of the gravitational field. Hence $N_0 v_0 a^2 M w^2$ can never exceed $4\kappa_0 a T_0/7$, which is the physical meaning of the inequality (31). Therefore, a further increase in N_0 beyond N_B must yield a corresponding decrease in v_0, which means that $v(r)$ falls below the critical point and the solution can no longer go supersonic. But the pressure must vanish at $r = \infty$, so the only solution satisfying the boundary conditions is $F = 0$. It is evident from (29) that $F \cong 0$ when $N_0 = N_B$. For $N_0 > N_B$, it follows that F must be identically zero, giving the evaporative subsonic expansion equilibrium pointed out by Chamberlain (1960, 1961) on the basis of numerical integrations of equations. The details of the formal analysis have been given elsewhere (Parker, 1965b), but the general outline is evident from the equations given here. The critical distance r_c remains approximately fixed for increase of N_0 beyond N_B, with $T(r)$ given by (27)

in (a, r_c) . The velocity v_0 varies inversely with N_0 , so that

$$v_0(N_0) = v_0(N_B)\,\frac{N_B}{N_0} \quad . \tag{41}$$

The velocity at r_c was $c(r_c)$ for $N_0 = N_B$ (given by (37). For larger N_0 the velocity $v(r_c)$ is down by about the same factor as v_0 . Beyond r_c , $v(r)$ varies approximately as $r^{-1/2}$ and $T(r)$ varies as $1/r$.

Now, as noted earlier, this analytical study of the hypothetical case of an atmosphere with heat transport limited to thermal conduction permits a demonstration of the continuous transition from supersonic to subsonic expansion equilibrium as the extension of the temperature out through the atmosphere is choked off. The choking off of the temperature is achieved by increasing the density N_0 at the base of the atmosphere, supplying more and more material to be heated by the limited conduction energy. For small N_0 , $T(r) \propto r^{-2/7}$ all the way to $r = \infty$. The expansion is supersonic. Increasing N_0 beyond N_A , given by (36), begins to cut down on the outward extension of $T(r)$ by introducing an initial temperature decline $r^{-4/7}$ out to r_2 ,with $r^{-2/7}$ beyond. Increasing N_0 beyond N_B extends the $r^{-4/7}$ dependence all the way to the critical point and introduces $T(r) \propto 1/r$ beyond, which causes the expansion to become subsonic. N_A and N_B are

plotted in Fig. 5 for a star with the mass and radius of the sun. To apply the result to other stars, note that for fixed binding w^2/c_0^2 = constant, both N_A and N_B vary as T_0^2/a, which may be very small for giant stars, etc.

Finally, a word should be said about $T(r)$ beyond r_c when $N_0 < N_B$ and the expansion is supersonic. It is obvious that when $N_0 << N_B$, the temperature is given by (26) as $r^{-2/7}$ for all r, for the simple reason that convection is negligible for all r. When N_0 is as large as N_A and $r_2 \cong a$ in (27), convection of gravitational energy is no longer negligible near the star, i.e. $N_0 v_0 a^2 M w^2 \cong \frac{2}{7} \kappa_0 a T_0$. The energy flow to $r = \infty$ is then given by (35) as $F = \kappa_0 a T_0/7$. The temperature is obviously below the value $T_0 (a/r)^{2/7}$ which obtains in the limit $N_0 \to 0$, so from (16) if follows that

$$\frac{v^2(\infty)}{w^2} < \frac{87}{16} \frac{a}{r_c} << 1 . \tag{42}$$

The energy consumed by the wind beyond r_c is not more than the sum of the final kinetic energy and the energy required to lift the gas out of the gravitational field at r_c. It follows from (42) that this energy is small compared to the total energy flow at $r = \infty$,

$$N_0 v_0 a^2 M \left[\frac{1}{2} v^2(\infty) + w^2 \frac{a}{r_c} \right] \ll F . \tag{43}$$

Hence, everywhere beyond r_2 convection is negligible and the energy transport is principally thermal conduction, so that $T \propto 1/r^{2/7}$. Hence the temperature is given approximately by (27) for all $r > a$.

The temperature given by (27) is correct for all $r > a$ until N_0 becomes comparable to N_B. Then the expansion of the gas beyond r_c consumes most, or all, of the energy conducted and convected past r_c, and $T(r)$ shifts from an asymptotic $r^{-2/7}$ to r^{-1}. The precise nature of the transition between the two asymptotic forms as $N_0 \rightarrow N_B$ cannot be shown by the simple analytical methods employed here. Only when N_0 is rather less, and rather greater, than N_B does the present analysis work. Numerical calculations are then available too: Chamberlain (1961) gives some cases for $N_0 > N_B$; deJager (1963) treats the expansion in the limit as $N_0 \rightarrow 0$; Noble and Scarf (1963) give examples for N_0 between N_A and N_B. At the present there is no evident way to investigate the transition between supersonic and subsonic expansion except by numerical methods.

Whang and Chang (1965) recently pointed out an isolated solution of (5) and (23) with the asymptotic form $T(r) \propto r^{-2/5}$. The solution was established by numerical methods from the asymptotic form with the boundary condition that thermal conduction vanish at $r = \infty$. The expansion is supersonic and exists when a precise condition between

N_o and T_o is satisfied. The condition is equivalent to

$$\frac{N_o v_o a^2 M w^2}{F} \frac{a}{r_c} = 0.511 .$$

In a strongly bound corona $a/r_c << 1$, requiring that $F << N_o v_o a^2 M w^2$, indicating that $N_o = O(N_B)$. Thus the solution of Whang and Chang lies somewhere in the region of the transition between subsonic and supersonic flow, though what relation it bears to the family of solutions discussed above is not known.

C. Energy Transport in the Solar Corona

It is interesting to compare the mathematical solution of (5) and (23) for the hypothetical conduction corona with the solar corona. The comparison shows that the solar corona is heated by wave dissipation, in addition to thermal conduction, for some distance into space, and that the conduction corona is probably not a very good model for the solar corona. Let us try to fit the observed fact (Bonetti, et al, 1963; Bridge, et al, 1964; Bridge, 1963; Neugebauer and Snyder, 1962; Snyder, et al, 1963; Snyder and Neugebauer, 1964) that the velocity of the solar wind at the orbit of Earth ranges upward from about 400 km/sec while the density is typically 4 ions/cm^3. Note that a hypothetical isothermal corona with $T_o = 1 \times 10^6$ °K gives a wind velocity $v \cong 500$ km/sec, with a density of 4/cm^3 at the orbit of Earth if $N_o = 2 \times 10^8$/cm^3 in the low corona near the sun in rough agreement with the observations. A higher temperature gives larger velocity and density. The conduction corona with $T_o = 1 \times 10^6$ °K

gives $N_A \cong 1 \times 10^8/cm^3$ so that N_o is perhaps a little larger than N_A. it follows that $T(r) \propto r^{-2/7}$ probably overestimates the outward extension of the temperature a little, and hence overestimates the wind velocity. With $\beta = 2/7$, (16) yields $v(\infty) = 330$ km/sec as an upper limit on v. But 330 km/sec is a little too low for most of the observed wind speeds. Choosing N_o smaller than $10^8/cm^3$ serves only to reduce the density of the wind without increasing the velocity above this limiting value. Increasing T_o gives the observed high velocities only if N_o is taken to be less than $10^8/cm^3$. For instance, if $N_o = 1 \times 10^7/cm^3$, then $T_o = 1.5 \times 10^{6o}K$ gives 400 km/sec with $N = 2/cm^2$ at the orbit of Earth, but $10^7/cm^3$ is in apparent conflict with observations of the coronal density. Raising T_o without reducing N_o cuts off $T(r)$ too quickly and the calculated wind velocity falls rapidly. For instance $T_o = 1.5 \times 10^{6o}K$ with $N_o = 2 \times 10^8/cm^3$ gives only 200 km/sec and about $10^2/cm^3$ at the orbit of Earth. The tendency for the conduction model of the corona to give too small a density at the corona and/or too small a velocity at the orbit of Earth was pointed out first by Nobel and Scarf (1963) from the ir numerical model. They noted that, if the discrepancy is real, it implies an additional heat source in the corona for at least one or two solar radii beyond the base. They started the calculation at the orbit of Earth with typical observed quiet-day wind velocities, densities, and hydrogen-helium ratios, and integrated inward toward the sun, where the calculated density consistently fell about a factor of ten below the observed value.

This discrepancy is even more striking when the active corona is considered. Active regions in the corona are believed to be 2×10^{6} °K and 10^{9} atoms/cm^{3}, and up. A glance at Fig. 5 shows that $N_0 > N_B$, showing that the calculated temperature cuts off so sharply that the equilibrium is subsonic expansion. Observations of the corona show no evidence for so rapid an outward decline of $T(r)$ from active regions, and observations of the wind in space (Biermann, 1957; Bridge, et al, 1964; Snyder and Neugebauer, 1964) indicate that the wind from active coronal regions is generally faster, not slower, than from quiet regions. The formula (40) for N_B is, of course, based upon the assumption that $w^2/c_0^2 >> 1$, so the approximation is not as good at 2×10^{6} °K as it is for lower temperatures. Further, it is to be expected that an active coronal region may expand laterally as it expands outward into space so that each tube of flow increases its cross-section faster than r^2 with distance from the sun. A cross-section increasing as rapidly as r^s does in fact increase N_B for a given T_0, but the effect is not enough, as may be seen from Fig. 5 for s = 2.5 and 3.0. Thus the model for the hypothetical conduction corona leads to a temperature falling too rapidly with radial distance from the sun, indicating that the actual solar corona is probably actively heated for some considerable distance by the dissipation of waves. The conduction corona is valuable for exploring and illustrating theoretical principles, but does not approximate very closely to the actual solar corona.

V. Stability of an Expanding Atmosphere

Having discussed the equilibrium of a tightly bound atmosphere with an extended temperature, consider now whether that equilibrium is stable. Generally speaking a static atmosphere is stable to all perturbations for which the temperature and density perturbations are related by

$$\delta c^2 = (\alpha - 1)\, c^2 \, \frac{\delta N}{N} \tag{44}$$

provided that α is both greater than zero and greater than $1 + d\ln c^2/d\ln N$. The question is whether the atmosphere with an extended temperature is stable under the same circumstances. Stability is expected on physical grounds, because the expanding atmosphere approximates to a static atmosphere in $r < r_c$ and to free expansion into a vacuum in $r > r_c$. Stability is easily demonstrated in a formal way from the general dynamical equations (Parker, 1965d).

Consider the simple case of an atmosphere with spherical symmetry, which is inviscid, field-free, etc. Denote the perturbations by $\delta v(r,t)$, $\delta c^2(r,t)$, $\delta N(r,t)$ about the equilibrium expansion described by (3) and (4). It is readily shown that the linearized equations can be written as

$$\left(\frac{\partial}{\partial t} + v\frac{\partial}{\partial r}\right)\frac{\delta N}{N} + v\frac{\partial}{\partial r}\,\frac{\delta v}{v} = 0\,, \tag{45}$$

$$\left(\frac{\partial}{\partial t} + v\frac{\partial}{\partial r}\right) v^2 \frac{\delta v}{v} + c^2 v \frac{\partial}{\partial r}\frac{\delta N}{N} + \frac{\alpha - 1}{N} v \frac{\partial}{\partial r} c^2 \delta N = 0 \qquad (46)$$

upon using (3) and (4) to simplify the coefficients and using (44) to express δc^2 in terms of δN. The system of equations (45) and (46) is totally hyperbolic. The characteristics are

$$\frac{dr}{dt} = v(r) \pm \alpha^{1/2} c(r) \qquad (47)$$

in which $\alpha^{1/2} c(r)$ is recognizable as the local speed of sound. The equilibrium expansion $v(r)$ is assumed to become supersonic at large r, so there exists some radial distance r_1 beyond which $v(r) > \alpha^{1/2} c(r)$. There are no inward progressing characteristics beyond r_1. The two families of characteristics are sketched in Fig. 6.

Consider the region $r < r_1$. The disturbance at any point P_1 in (a, r_1) is caused entirely by disturbances originating between the backward extension of the characteristics through P_1 (see, for instance, Courant and Hilbert, 1962). Nothing beyond r_1 contributes to the disturbance at P_1. The disturbances propagating along the characteristics have only a finite time before leaving the region*, so unlimited growth of a small

*The exception to this is the limiting characteristic at r_1, but it will be shown in a moment that waves propagating away from r_1 along the limiting characteristic decrease their amplitudes rather than increase them.

disturbance is not possible. It follows that the atmosphere is stable.

Further investigation shows that the only changes in amplitude of a disturbance propagating along a characteristic are just those to be expected from the changes in density, temperature, etc. along the characteristic.

Another way to exhibit the stability is to put $\alpha = 1$ so that the system (45) and (46) may be reduced to a single second order differential equation for $\delta v / v$. Then put

$$\frac{\delta v}{v} = f(r) \exp \frac{t}{\tau} . \tag{48}$$

The result is the ordinary second order differential equation

$$\left(1 - \frac{c^2}{v^2}\right) \frac{d^2 f}{ds^2} + \left(\frac{2}{\tau} + \frac{2}{v^2} \frac{dv}{ds}^2 - \frac{1}{c^2} \frac{dc}{ds}^2\right) \frac{df}{ds} \tag{49}$$

$$+ \left[\frac{1}{\tau^2} + \frac{1}{\tau} \left(\frac{2}{v^2} \frac{dv}{ds}^2 - \frac{1}{c^2} \frac{dc}{ds}^2 \right) + \left(\frac{1}{v^2} \frac{d^2 v}{ds^2}^2 - \frac{1}{v^2} \frac{dv}{ds}^2 \frac{1}{c^2} \frac{dc}{ds}^2 \right) \right] f = 0 ,$$

where the independent variable ds is defined as $dr / v(r)$.
It is evident that there is a regular singularity at $c^2 = v^2$,
viz. $r = r_1$. There is an irregular singularity at $r = \infty$,
but no disturbance from $r = \infty$ has any effect on conditions at finite r.

Now the boundary conditions at the base of the atmosphere for a

disturbance originating wholly within the atmosphere are $\delta v = \delta N = 0$. It is readily shown from (45) and (46) that this requires that $(\partial/\partial r)\delta v/v = 0$. But $r = a$ is a regular point of the differential equation (49). Hence if both f and df/ds vanish at $r = a$, it follows that f vanishes everywhere. QED. There can be no disturbances of the form (48) originating in the atmosphere.

Finally, consider the disturbance propagating along the limiting characteristic $dr/dt = v - c$ away from the singularily at $r = r_1$. There are two independent solutions of the differential equation, one regular at the singularity $v^2 = c^2$, the other with a pole there. The regular solution corresponds to a disturbance propagating outward along the characteristic $dr/dt = v + c$. The solution with a pole corresponds to the solution propagating away from the singularity along the limiting characteristic $dr/dt = v - c$. The amplitude of such a disturbance declines enormously as it recedes from the pole.

VI. Models and Coronal Filaments

A. Models

Comparison of a number of hypothetical coronal expansion models with the observed corona and wind suggests that the corona may be of a partially filamentary nature near the sun. The question of the fine filamentary structure of the corona is not new, going back several decades to the hand drawings of the corona made during the time of various early solar eclipses. The degree of filamentation is important to the theory of coronal heating and was believed

to be the basis for the traditional phenomenological picture of solar corpuscular radiation as a number of isolated narrow streams. Present observations of the wind near the orbit of Earth show no evidence of small-scale filamentary structure, or isolated streams. The observations (Snyder, et al, 1963; Dessler and Fejer, 1963; Ness, et al, 1964; Ness and Wilcox, 1964) show the broad variation of wind velocity expected (Parker, 1963; Sarabhai, 1963; Dessler and Fejer, 1963) from the general variation of the corona around the sun, giving details that could not be anticipated from theory. Unfortunately a failure to observe small scale filamentary structure at the orbit of Earth does not rule out its existence closer to the sun, as the theory (Parker, 1964c) outlined below will show.

Now it is well known that the solar corona forms long finely divided streamers, (see drawings and photographs, Astronomer Royal, 1927; van Biesbroeck, 1953; Kiepenheuer, 1953; Mustel, 1963; Vsekhsvjatsky, 1963), each streamer representing relatively dense coronal gas confined along the magnetic lines of force of the general solar field. Unfortunately it is not a simple matter observationally to disentangle the streamer and inter streamer corona and observe the density and temperature as a function of r in each separately. The evidence suggests (see van de Hulst, 1953; Newkirk, 1961; and Vsekhsvjatsky, 1963) that the temperature in the denser filaments may be little, if any, higher than between filaments, whereas the density may perhaps be as much as ten times higher. If this is really the situation, it follows that the expansions of the filament and inter-filament regions of the corona proceed much alike. For a given $T(r)$ the momentum equation (3) is homogeneous in $N(r)$, so a fixed factor in

$N(r)$ is immaterial. No qualitative effect on the dynamics of the expansion equilibrium is expected. Only if $T(r)$ should be seriously affected by the density is a large difference in the rate of expansion expected.

Now the steady improvement of optical observations of the corona shows radial filamentary structure down to the limit of resolution. Harvey (1965) has recently shown that the coronal filaments probably extend outward from plage regions. The radio observations of Hewish (1958, 1961) show the presence of very small-scale radial striations in the wind out to the limit of observation at 0.5 a.u. On the other hand, direct observations of the wind from space vehicles near the orbit of Earth show that the wind is continuous (although often irregular). Comet tails show no evidence of gaps in the wind. Thus, altogether, it is evident that we are dealing with the equilibrium of an atmosphere which may be filamentary to some degree near the sun, but is not evidently so at large distance.

The first point to be made here, before presenting an outline of the dynamical equilibrium of a filamentary corona, is that comparison of the simple theoretical non-filamentary corona - used so far to present the basic theory of atmospheric equilibrium - with the observations of corona and wind densities suggests that the corona may, in fact, be somewhat filamentary.

Consider, then, the problem of constructing a spherically symmetric model of the solar corona and wind, i.e. a model without any filamentary structure. One way to begin would be to require that the density match the observed average density of the solar corona for as far (about 20 $R_\odot$) as it is observed into space (van de Hulst, 1953; Michard, 1954; Blackwell, 1956; Klüber, 1958;

Newkirk, 1959; Högbom, 1960; Ney, et al, 1961). This has been done using the polytrope model of the corona (Parker, 1963) where three examples were presented which fitted the density within the limits of observational accuracy. The three models, and the resulting wind velocity $v(r_E)$ and density $N(r_E)$ at the orbit of Earth, $r = r_E$, are:

(a) $\alpha = 1$, $T_0 = 1.22 \times 10^{6}$ °K, $v(r_E) = 550$ km/sec, $N(r_E) = 20$ atoms/cm^3

(b) $\alpha = 1$, $T_0 = 1.9 \times 10^{6}$ °K, $v(r_E) = 730$ km/sec, $N(r_E) = 40$ atoms/cm^3

(c) $\alpha = 1.1$, $T_0 = 1.77 \times 10^{6}$ °K, $v(r_E) = 440$ km/sec, $N(r_E) = 40$/cm^3

It is evident from the differences between these possibilities that fitting the observed density of the corona does not define the situation very closely.

Billings and Lilliequist (1963) constructed a model of the corona by fitting the polytrope model to the observed $T(r)$ in the corona. The observations indicated that $T_0 = 2.6 \times 10^{6}$ °K with an outward decline of the order of 3 /km, yielding $\alpha \cong 1.2$. It is then a straightforward matter to calculate that the wind velocity at the orbit of Earth is $v(r_E) = 270$ km/sec. The density at the orbit of Earth is directly proportional to the value chosen for N_0, and is 10^2 atoms/cm^3 for $N_0 = 2 \times 10^8$ atoms/cm^3.

Another approach to the problem is to fit the observed velocity and density at the orbit of Earth. As already noted, an isothermal corona yielding

500 km/sec and 4 atoms/cm^3 at the orbit of Earth projects back to the sun to give $T_o = 1 \times 10^{6o}$K, $N_o = 2 \times 10^8$/cm^3. In view of the observational uncertainties, this is a satisfactory model.

As noted earlier Noble and Scarf (1963) extrapolated back to the sun from 352 km/sec, 2×10^{5o}K, and 6.75 atoms/cm^3 at the orbit of Earth using the coronal model with energy transport limited to thermal conduction. They obtained $T_o = 2 \times 10^{6o}$K and $N_o = 2 \times 10^7$ atoms/cm^3.

The lack of uniqueness of all these models is their most conspicuous feature. The interesting feature is that the models with coronal temperature declining outward all give too large a density ratio $N(r_E)/N_o$. Or to state it differently, it is necessary to employ a density N_o in the corona well below the observed $10^8 - 10^9$ atoms/cm^3 in order to fit both velocity and density at the orbit of Earth. The isothermal corona at 1×10^{6o}K comes closest to avoiding this difficulty, and a corona in which $T(r)$ increases for some distance would completely reconcile the N_o with $N(r_E)$. But we wonder if the resolution perhaps may not lie elsewhere, viz, in the filamentary structure of the corona. A filamentary structure gives a smaller value for $N(r_E)/N_o$, because the equation for conservation of mass in a steady expansion of the corona along a slender filament with cross section $A(r)$ is $N(r)\,v(r)\,A(r)$ = <u>constant</u>. It is shown below that the cross-sectional area of a relatively dense filament increases faster than the r^2 for radial flow, giving a correspondingly lower density at large r. There is no proof that filaments are the explanation for the observed $N(r_E)/N_o$

being smaller than the value calculated for radial flow. But there is enough direct observational evidence for some degree of filamentation in the corona as to justify exploration of the dynamical equilibrium of coronal filaments.

B. Lateral Equilibrium of Extended Coronal Filaments

It is generally believed the coronal filaments lie along the magnetic lines of force. The gas is ionized and therefore constrained to slide along the lines of force, which offer no resistance to the passage of the gas.

It is evident that only those fields which are weaker than the gas pressure are extended outward by the gas to form the observed radial filaments. The gas pressure in a dense coronal region may be estimated from observations to be as high as 1 dyne/cm^2 (10^9 atoms/cm^3 at 3×10^{6} ^{o}K) which is sufficient to extend fields of as much as 5 gauss. Thus the lines of force of the general solar magnetic field, and those lines from intense fields of active regions which extend so far out as to fall below the gas pressure, may be extended by coronal expansion, as illustrated in Fig. 7. For the simple case of a weak field with uniform coronal expansion everywhere around the sun, the extended field at large distance r is related to the radial field $B(a, \theta, \phi)$ at the reference level $r = a$ by

$$B_r(r, \theta, \phi) = B(a, \theta, \phi^*) \left(\frac{a}{r}\right)^2, \tag{50}$$

$$B_\theta(r,\theta,\phi) = 0 \quad ,$$

$$B_\phi(r,\theta,\phi) = B(a,\theta,\phi^*)\ \frac{a}{r}\ \frac{a\Omega}{v}\sin\theta,$$

where Ω is the angular velocity of the sun, v is the wind velocity, θ is the polar angle measured from the axis of rotation of the sun, ϕ is azimuth, and $\phi^* = \phi + r\Omega/v$ (Parker, 1958a, 1963). This spiral field is carried outward by the wind which fills interplanetary space. The general presence and character of the field, with its close connection to the fields on the sun, has now been confirmed by the magnetic observations from IMP-I by Ness et al (1964) and Ness and Wilcox (1964).

The equilibrium of a slender coronal filament requires that the total gas and magnetic pressure inside the filament equals the total pressure outside. If the subscripts one and two denote conditions inside and outside the filament, respectively, then

$$2N_1kT_1 + \frac{B_1^2}{8\pi} = 2N_2kT_2 + \frac{B_2^2}{8\pi} \ . \tag{51}$$

Let $A(r)$ represent the cross-sectional area of a tube of flow. Then conservation of mass and magnetic flux requires that

$$N_{1,2}(r)\, v_{1,2}(r)\, A_{1,2}(r) = N_{o1,2}\, v_{o1,2}\, A_{o1,2}, \tag{52}$$

$$B_{1,2}(r)\, A_{1,2}(r) = B_{o1,2}\, A_{o1,2}, \tag{53}$$

neglecting rotation of the sun for the present discussion. The important property of equations (51) - (53) is that radial extension reduces the magnetic pressure more rapidly than the gas pressure, which will be demonstrated shortly. The result is a tendency to equalize the gas pressures and give a more uniform wind density. Only on the broadest scales are large density variations possible.

To illustrate this effect, consider the extreme case that at large r the magnetic field is negligible within a filament and the gas pressure is negligible outside. Then

$$2 N_1 k T_1 \cong B_2^2/8\pi \tag{54}$$

for large r. Let the total solid angle occupied by A_1 and A_2 be ϵ and put $A_1(r) = r^2 \epsilon \nu(r)$ and $A_2(r) = r^2 \epsilon (1 - \nu(r))$. Then it is readily shown from (52) - (54) that

$$\frac{(1-\nu)^2 \nu_0}{(1-\nu_0)^2 \nu} = \frac{T_{01}}{T_1(r)} \frac{v_1(r)}{v_{01}} \frac{a^2}{r^2}$$

But $r^2 T(r) / v(r)$ increases without limit as $r \longrightarrow \infty$ in an atmosphere in supersonic expansion equilibrium, even if $T(r)$ declines adiabatically at large r. Thus $\nu(r) \longrightarrow 1$ at large r. The consequences of this are that (a) $N_1(r_E) / N_{01}$ is much smaller than for radial flow, (b) interfilament regions of low gas density have a much larger ratio $N_2(r_E) / N_{02}$, so that at large r, $N_2(r)$ becomes comparable to $N_1(r)$, and finally (c) the interfilament regions are, compared to the filaments, very small at large r.

C. Dynamic Equilibrium along Radial Filaments

The theoretical treatment of the dynamical equilibrium of a filamentary corona has been given elsewhere (Parker, 1964c). For a given $T(r)$ the equations have the same structure as for the entirely radial expansion discussed earlier. In terms of the solid angle $\nu(r)$ subtended by the gaseous streamers at a radial distance r, the momentum equation analogous to (5) is

$$\frac{dv}{dr}\left(v - \frac{c^2}{v}\right) = -A \frac{d}{dr} \frac{c^2}{A} - \frac{w^2 a}{r^2} . \tag{55}$$

The position of the critical point is the position at which the right hand side of the equation vanishes. In (a, r_c), the hydrostatic approximation for $N(r)$ gives

$$p_{1,2}(r) = 2N_{1,2}(r)\, k\, T_{1,2}(r) \cong 2N_{01,2}\, k\, T_{01,2} \exp\left(-w^2 a \int_a^r \frac{dr}{r^2 c_{1,2}^2}\right), \tag{56}$$

so that (51) may be written

$$p_1(r) + \frac{B_{01}^2}{8\pi}\left(\frac{A_{01}}{A_1}\right)^2 = p_2(r) + \frac{B_{20}^2}{8\pi}\left(\frac{\epsilon a^2 - A_{01}}{\epsilon r^2 - A_1(r)}\right)^2 \tag{57}$$

upon eliminating $A_2(r)$ with the condition that $A_1(r) + A_2(r) = \epsilon r^2$, where ϵ is the total solid angle. It is evident that (57) is a general quartic equation for $A_1(r)$. Once $A_1(r)$ is known, the critical radius can be computed from (55). The velocity follows from (52) and (57), with v_0 coming from the condition that $v = c$ at the critical radius. There is, then, no qualitative change in the structure of the equations.

Unfortunately the general simultaneous solution of (55) and (57) is straightforward in principle but algebraicly transcendental. But fortunately the solution is relatively simple in a number of the more instructive physical cases. For instance, in a tightly bound corona $(w^2 >> c_0^2)$ the gas pressure

$p(r)$, which dominates $B^2/8\pi$ at the base of the atmosphere and at large radial distance, declines so rapidly with height near the star that there is an extended intermediate region in which $p(r) << B^2/8\pi$. It follows at once from (57) that

$$A_1(r) = r^2 \frac{\epsilon B_{01} A_{01}}{B_{01}A_{01} + B_{02}A_{02}}, \quad A_2(r) = r^2 \frac{\epsilon B_{02} A_{02}}{B_{01}A_{01} + B_{02}A_{02}} \tag{58}$$

and radial flow obtains. The only difference is that the equation for conservation of mass has an additional factor and now reads

$$N_1(r)\, v_1(r)\, r^2 = N_0 v_0 a^2 \frac{B_{01}A_{01} + B_{02}A_{02}}{\epsilon a^2 B_{01}} \tag{59}$$

etc. instead of (4). The extra factor comes into the calculation of v_0 from r_c and $v(r_c) = c(r_c)$.

Another example of physical interest is $A_{01} << A_{02}$, the gaseous filaments slender and sparse. Then, for some distance out through the atmosphere, $A_1(r) << A_2(r)$, and $A_2(r) \cong \epsilon r^2$ so that (57) yields

$$A_1(r) \cong A_{01} \left[\frac{B_{01}^2/8\pi}{p_2(r) - p_1(r) + (B_{02}^2/8\pi)(a/r)^4} \right] \tag{60}$$

with $p_1(r)$ and $p_2(r)$ given by (56).

The extreme case that the magnetic field is <u>identically zero</u> inside the filament and the gas pressure is identically zero outside is singular but instructive. Then for all r, (57) reduces to (54).

To illustrate the consequences of this condition, use the approximate expression (56) for $p_1(r)$ and write $B_2(r)$ in terms of $A_2(r)$. Then since $p_{01} = B_{02}^2/8\pi$, it follows that

$$\exp\left(-\frac{w^2 a}{2}\int_a^r \frac{dr}{r^2 c_1^2}\right) = \frac{A_{02}}{A_2(r)} \tag{61}$$

giving a relation between $T_1(r)$ and $A_2(r)$, or between $T_1(r)$ and $A_1(r)$ since $A_1(r) = \epsilon r^2 - A_2(r)$. It is evident from this equation that, if w^2/c^2 is too large, the upper bound ϵr^2 on $A_2(r)$ may prevent the right hand side from decreasing as fast with r as the left hand side. Thus there may be no solution to (61). Physically this means that the barometric decline of pressure with height must not be too rapid or else the surrounding field will close off the gaseous filament, giving $B_2(r) \propto 1/r^2$ everywhere beyond. Quantitatively this means that

$$\exp\left(-\frac{w^2 a}{2}\int_a^r \frac{dr}{r^2 c_1^2}\right) \geq \frac{A_{20}}{\epsilon r^2} \tag{62}$$

if closing off the gaseous filament is to be avoided. It is sufficient to make

$$c_1^2(r) \geq \frac{w^2 a}{4r}, \tag{63}$$

as is readily shown by differentiation of the inequality. At the base of the solar corona (63) corresponds to $T_0 \geq 2.8 \times 10^6$ °K. If the temperature is less than (63), the field-free gaseous filament will be cut off by the surrounding field and prevented from expanding. It must be emphasized, however, that the cutting off is possible only because it was assumed that the magnetic field with the gaseous filament is <u>identically</u> zero. Any finite field, no matter how small, prevents the cutting off, so we would not expect to find the cutoff in nature. The closest approach to the condition of zero field is probably to be found in the region between two extended fields of opposite polarity*, indicated by the broken line AB in Fig. 7. The region of weak field will be relatively thin. The solution of the momentum equation is straightforward under this circumstance, for with $A_1 << A_2$ it follows that $A_2 \cong \epsilon r^2$ and $B_2 = B_{02}(a/r)^2$.

*The field need not pass through zero between opposite fields in a three dimensional space, but it may be weakest there.

Assuming that coronal heating and expansion of the gas regulate the temperature and pressure so as to satisfy (54), it follows that $d\ln p_a/dr = -4/r$, so that the momentum equation is

$$v\frac{dv}{dr} - \frac{4c^2}{r} + \frac{w^2 a}{r^2} = 0. \tag{64}$$

Integration gives

$$v^2(r) = v_0^2 - 2w^2\left(1 - \frac{a}{r}\right) + 8\int_a^r \frac{dr\, c^2(r)}{r}. \tag{65}$$

The sufficient condition (63) for non-closure of the filament makes dv/dr positive everywhere. The equilibrium expansion for uniform temperature is

$$v^2(r) = v_0^2 + 8c_0^2 \ln\frac{r}{a} - 2w^2\left(1 - \frac{a}{r}\right) \tag{66}$$

and for a temperature decline of the simple form $r^{-\beta}$ $(\beta < 1)$,

$$v^2(\infty) = v_0^2 + \frac{8c_0^2}{\beta} - 2w^2 \tag{67}$$

Further discussion and illustrative examples are given in the literature (Parker, 1964c). The instability of the equilibrium filaments is discussed there too. The instability is important as a means for further obliterating the thin interfilament regions. It is the relatively small solid angle occupied by the interfilament regions,

together with the possible instability, which served as a basis for the statement at the beginning of the section that a failure to find small-scale filament in the wind at the orbit of Earth cannot be taken as an argument that small-scale filaments do not exist closer to the sun. Instabilities are also important for generating small-scale fluctuations in the large-scale field (50). It is the small-scale fluctuations in the field which give the principal interaction with the interstellar cosmic ray gas and give a partial cosmic ray vacuum in the solar system (Parker, 1958c, 1963).

D. General Remarks

It is presently not known to what extent the corona is striated and filamentary. There are a number of observations which suggested that there may be considerable filamentation. The observational question of intense filamentation in the corona is of basic importance to the theory of coronal heating. Not only would intense filamentation require intense local heating, but in the presence of such intense local heating the observed lack of Doppler displacement of the coronal lines would be even more restrictive on heating theory than already pointed out by Billings (1963). The dynamics of the equilibrium expansion is essentially unchanged by a filamentary structure, the principal effect being a lowering of the calculated mean density at large radial distance. The calculations show that any filamentary structure which might exist in the corona dimenishes outward to large distances in the wind because of instabilities and because of the tendency for equalization of densities by lateral expansion of the denser filaments. Direct observation of the wind at the orbit of Earth shows no evidence

of filaments or isolated streams. Presumably the statistical studies of solar corpuscular radiation and geophysical effects, which have traditionally been interpreted with the phenomenological picture of isolated streams of corpuscular radiation, should be considered from some other viewpoint, such as isolated sheets of turbulence and other disturbances in an otherwise continuous wind (see for instance, Sarabhai, 1963, Dessler and Fejer, 1963 for a discussion of this problem).

VII. The Role of Viscosity

The role of viscosity in the equilibrium expansion of an atmosphere has been neglected because the effects are not large and are generally uninteresting. The basis for the neglect is simply that the Prandtl number is small of the order of 0.1 for ionized hydrogen (Parker, 1963). The Prandtl number is defined as the ratio of the viscosity η to the thermometric conductivity, κ / C_p, where C_p is the specific heat per unit mass. The smallness of the Prandtl number implies that the energy dissipated by viscosity is small compared to the energy supplied by thermal conductivity. The energy supply to coronal expansion is at least as large as thermal conduction, so viscosity is unimportant.

Recently, however Scarf and Noble (1964) wrote down the complete Navier-Stokes equation for viscous radial expansion of an atmosphere with energy supplied by thermal conduction. They solved the equations by numerical methods and found that, while viscosity had no sensible effect in the expansion of the atmosphere near the sun, viscosity had a profound effect as $r \rightarrow \infty$, viz. the velocity of expansion increased without bound. This computational result is contrary to physical fact. They pointed out that the usual transport coefficients,

such as conductivity and viscosity, are not really applicable at large radial distance where the mean free path becomes larger than the scale of the system. Thus the solution obtained from the usual viscosity $\eta = 10^{-16}\, T^{5/2}$ gm/cm sec (Chapman, 1954) is not expected to be physically realistic. They pointed out further that the interplanetary magnetic field (50), with its superimposed small scale irregularities, probably inhibits both η and κ so much that the flow is adiabatic and largely nonviscous at large r. It is interesting to explore the problem a little further here.

The momentum equation for radial flow is*

$$NMv\frac{dv}{dr} + \frac{d}{dr}2NkT + \frac{GM_{*}MN}{r^2} \tag{68}$$

$$= \frac{4}{3}\left[\eta\left(\frac{d^2v}{dr^2} + \frac{2}{r}\frac{dv}{dr} - \frac{2v}{r^2}\right) + \frac{d\eta}{dr}\left(\frac{dv}{dr} - \frac{v}{r}\right)\right].$$

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*The general vector form of the viscous term can be written as

$$\eta\left(\nabla^2\underset{\sim}{v} + \frac{1}{3}\nabla\nabla\cdot\underset{\sim}{v}\right) + \nabla\times(\underset{\sim}{v}\times\nabla\eta) - \underset{\sim}{v}\nabla^2\eta + \frac{1}{3}\nabla\eta\,\nabla\cdot\underset{\sim}{v} + \nabla(\underset{\sim}{v}\cdot\nabla\eta).$$

The energy equation is *

$$F = -r^2 \kappa \frac{dT}{dr} + N_0 v_0 a^2 \left[\tfrac{1}{2} M v^2 + 5kT - \frac{GM_* M}{r} \right] \tag{69}$$

$$- \tfrac{4}{3} \eta r^2 v \left(\frac{dv}{dr} - \frac{v}{r} \right).$$

The physical basis for the unbounded increase of velocity with r in the presence of viscosity is that the radial flow causes each element of fluid to elongate in the two directions perpendicular to the radial direction. The viscous stress tensor σ_{ij} has the form

$$\sigma_{ij} = \eta \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} - \frac{2}{3} \delta_{ij} \frac{\partial v_k}{\partial x_k} \right)$$

with the result that each element of fluid tends to elongate in the radial direction too, giving a continued increase in the radial velocity. To demonstrate this let $i = 1$ represent the radial direction. R epresent the elongation

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*The general expression for the energy flux transported by the viscous stresses σ_{ij} is $v_i \sigma_{ij}$ or $\eta \left[\nabla v^2 + (\nabla \times \underset{\sim}{v}) \times \underset{\sim}{v} - \frac{2}{3} \underset{\sim}{v} \nabla \cdot \underset{\sim}{v} \right]$. The local rate of viscous dissipation is $\eta \left[(\nabla \times \underset{\sim}{v})^2 + \frac{2}{3} (\nabla \cdot \underset{\sim}{v})^2 + 2 \underset{\sim}{v} \cdot \nabla^2 \underset{\sim}{v} - \nabla^2 v^2 \right]$ $= \frac{4}{3} \eta \left(dv/dr - vr \right)^2$, but this is converted directly into heat and transported as $3kT$, so it is not lost to the system.

of the fluid elements perpendicular to the radial direction by $\partial v_2/\partial x_2 = \partial v_3/\partial x_3 = \alpha > 0$, with the result that the radial viscous force is

$$\sigma_{11} = +\frac{4}{3}\eta\left(\frac{\partial v_1}{\partial x_1} - \alpha\right)$$

as compared to

$$\sigma_{22} = \sigma_{33} = -\frac{2}{3}\eta\left(\frac{\partial v_1}{\partial x_1} - \alpha\right).$$

Hence uniform radial flow, $\partial v_1/\partial x_1 = 0$, leads to tension in the perpendicular direction and pressure in the radial direction.

Consider the question of why the expansion really does not accelerate as a consequence of viscosity. Magnetic fields may, of course, reduce viscosity to negligible levels. However, it would be sufficient to reduce the thermal conductivity. Then assuming no additional source of heat at large radial distance, the temperature would then decline so rapidly with increasing r that η would have no great effect. The viscous forces and viscous heating would become negligible with increasing r. To demonstrate this suppose that the pressure gradient and gravitational forces can be neglected, so that the gas is coasting freely, except for the viscous stresses, at large r. For adiabatic

conditions beyond the distance* $r = r_3$, write $T \propto N^{2/3}$ so that with $\eta \propto T^{5/2}$ there results

$$\eta(r) = \eta(r_3)\left(\frac{N}{N_3}\right)^{5/3}.$$

Assume that the asymptotic form of $v(r)$ is

$$v(r) \sim v_3\left(1 + s\left(\frac{r_3}{r}\right)^{\gamma} + \dots\right), \quad \gamma > 0. \tag{70}$$

Then it follows from (4) that

$$N(r) \sim N_3\left(\frac{r_3}{r}\right)^2 \tag{71}$$

and

$$\eta(r) \sim \eta_3\left(\frac{r_3}{r}\right)^{10/3}$$

Substitution into (68) then yields $\gamma = 7/3$ and

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*The flow becomes adiabatic only some distance beyond the point $r = r_2$ where the heat supply is cut off because viscous heating continues for some distance. This is easily demonstrated from the equation $\frac{4}{3}\eta(dv/dr - v/r)^2 - Nvk[3dT/dr - 2kT\,d\ln N/dr] = 0$ for the deposition of heat. If the velocity were maintained constant, so that N were of the form (71), it is readily shown that $T(r) = T_2(r_2/r)^{4/3}\left[1 - (3/2C)(1 - r_2/r)\right]^{-2/3}$, where $C \equiv 9N_2kT_2r_2/4\eta_2v_2$, which takes up the adiabatic form only asymptotically. Reasons will be given later way $3/2C$ is probably less than one.

$$s = -\frac{16\eta_3}{21 N_3 M v_3 r_3} \quad . \tag{73}$$

It follows, then, that cutting off the heat supply (wave dissipation and/or thermal conduction) is sufficient to suppress the viscosity by adiabatic cooling. Perhaps a more general statement would be that if for any reason the viscosity declines as $r^{-\mu}$, then the asymptotic form (70) for $v(r)$ yields $\gamma = \mu - 1$ and

$$s = \frac{4(2-\mu)\eta_3}{3(\mu-1) N_3 M v_3 r_3} \tag{74}$$

Hence anything which reduces the viscosity faster than $\mu = 1$, leads to a constant expansion velocity as $r \rightarrow \infty$.

As a matter of fact, these simple arguments based on the suppression of viscosity and/or thermal conductivity do not really get to the heart of the matter. The real point seems to be that if the wind is sufficiently dense that the ordinary viscosity is applicable, then it has so much inertia that viscous forces are negligible. On the other hand, if the wind is sufficiently tenuous that viscous stresses might have been important, it turns out that the particles make essentially no collisions and the effective viscosity is reduced to negligible levels. So it would appear that, even in the absence of magnetic fields, viscous effects are negligible and the inclusion of the conventional viscosity leads to erroneous results. To demonstrate this consider the terms in the energy equation (69) at large radial distance.

Suppose that enough heat is supplied to the atmosphere near the star that the equilibrium expansion becomes supersonic at large r. Viscosity is unimportant near the star. At large r the important terms are thermal conduction $r^2 \kappa \, dT/dr$, of the order of $\kappa r T$; convection of kinetic energy $N v r^2 \frac{1}{2} M v^2$; and the work of the order of $\frac{4}{3} \eta r v^2$ done by the viscous stresses. It is evident that if the density NM is high, then for a given v at large r, both thermal conduction and viscous stresses are negligible compared to the convection of kinetic energy. The temperature declines approximately adiabatically and the velocity approaches a constant value as $r \rightarrow \infty$, as discussed above. Suppose, on the other hand, that the density is sufficiently low ($N < N_A$ given by (36)) that thermal conduction remains important as $r \rightarrow \infty$. How does the viscous stress compare with thermal conduction? It is readily shown from elementary kinetic theory that

$$\kappa r T \cong \frac{kT}{2M} N M u \left(\frac{M}{m}\right)^{1/2} \lambda r \tag{75}$$

where M is the ion mass, m is the electron mass, u is the ion thermal velocity, $u (M/m)^{1/2}$ is the electron thermal velocity, and λ is the mean free path for either ions or electrons. It is also readily shown that

$$\eta r v^2 \cong \frac{1}{3} v^2 N M u \lambda r. \tag{76}$$

Since $(M/m)^{1/2} \cong 40$ and v^2 is typically 10-10^2 times larger than kT/M, it is evident that the thermal conduction and viscous terms are roughly comparable in magnitude. They are both to be compared with the term

$$\frac{1}{2} v^2 N M u r^2 \tag{77}$$

for convection of kinetic energy. Comparing (76) and (77), it is evident that kinetic energy (and adiabatic cooling) dominates if $vr > u\lambda$. Viscosity could dominate the flow only if $\lambda >> r$. But if $\lambda \gtrsim r$, the viscosity would no longer be given by $\frac{1}{3} N M u \lambda$, as it is when $\lambda < r$. Rather the viscosity would then be of the order of $\frac{1}{3} N M u \lambda (r/\lambda)^2$, smaller by $(r/\lambda)^2$ *, giving the rate of energy transfer by viscous stresses of the order of

$$\eta r v^2 \cong \frac{1}{3} v^2 N M u r^2 \frac{r}{\lambda}$$

which is smaller than the rate of convection of kinetic energy by the factor $(u/v)(r/\lambda)$. Or, to put it another way, constant velocity at large r yields

---

*The thermal conductivity might not be so severely reduced by low densities, but in any case could not exceed something of the order of $(kT/2M) N M u (M/m)^{1/2} r^2$, equivalent to all the electrons moving with the electron thermal velocity in the direction of the temperature gradient. Cutting off thermal conductivity at large r has the general effect of a higher equilibrium expansion velocity (Parker, 1964b).

$N \propto 1/r^2$ so that $\eta \propto (v/r)(r/\lambda)$ which declines faster than $1/r$ and is consistent with $v \sim$ constant at large r.

VIII. Fluctuations and Wave Propagation

It is to be expected that fluctuations arise in the extended atmosphere of a star. The basic theory of the equilibrium expansion is, of course, in terms of a stationary state, but it is evident from observations of the solar atmosphere and wind that fluctuations about stable equilibrium state have some practical importance. The origins of fluctuations have been discussed elsewhere (Parker, 1963) and need not be repeated here at length. To mention a few, the sun is rotating, with the result that a corona which is stationary in time in the frame of reference rotating with the sun, but which is not uniform around the sun, is not stationary in the fixed frame. The hotter regions of the corona expand somewhat more rapidly than the cooler regions and consequently overtake the slower cooler wind at some distance in space, creating turbulence and perhaps shocks, as sketched in Fig. 8 (see other sketches in Parker, 1963). Dessler and Fejer (1963) and Sarabhai (1963) have emphasized the importance of such regions of disturbance in producing the classical cosmic ray and geomagnetic effects, particularly those effects which tend to recur at the 25 - 27 day intervals of solar rotation.

A variation of corona temperature with time on the sun leads to fluctuations in the wind. The case of greatest interest is the sudden enhancement of coronal temperature following violent solar activity, which yields a blast wave propagating outward from the sun through the quiet-day wind. The process was pointed out and illustrated some years ago (Parker, 1961a, 1963) by considering the extreme case that the enhancement of the corona was uniform around the sun and so violent that the velocity of the wave is large compared to both the thermal velocity and the expansion velocity of the quiet-day wind. This idealization permits the final asymptotic form of the wave to be computed from the hydrodynamic equations by employing the similarity principle $r^{\lambda}/t = \eta$ where η represents the phase of any point moving with the wave. The general method applicable to progressive waves is outlined by Courant and Friedrichs (1948). The interested reader is refered to the literature for the details of the calculation. The velocity of the shock transition at the head of the blast wave varies as $1/r^{1-\lambda}$ as the wave progresses outward from the sun. The blast wave itself is formed by the scooping up of the quiet-day wind ahead, whose density varied about as $1/r^{2}$ before the arrival of the blast wave. The total energy of the blast wave varies as $t^{3/\lambda - 2} \propto r^{3-2\lambda}$. The enhanced corona lies at the rear of the blast wave, where it pushes on the wave ahead. The value $\lambda = 1$ corresponds to the extreme case that the enhanced corona pushes so hard that the velocity of the wave is maintained constant, with with energy increasing linearly with time because of the continual acquisition of material into the front of the wave. The value $\lambda = 3/2$ corresponds to no

pressure on the rear of the wave, so that the energy is constant and the velocity declines like $1/r^{1/3}$. The density profiles through these idealized waves are shown in Fig. 9.

Blast waves from sudden coronal enhancements are responsible for the non-recurring magnetic storms and the non-recurring Forbush type decreases of the cosmic ray intensity. The quiet-day spiral magnetic field, given by (50), is distorted by the blast wave with the consequent cosmic ray effects pointed out elsewhere (Parker, 1961, 1963). The actual blast waves are undoubtedly not as simple as the formal illustration mentioned here, but they have the same general dynamical properties. The detailed structure of the actual blast waves must be supplied by observation. In this direction some preliminary study of shock waves from the sun has already been carried out (Sonett, et al, 1964). The most interesting theoretical question at the present time is the structure of the collisionless shock at the head of the blast wave. Observations of the collisionless bow shock upstream from the geomagnetic field indicate considerable chaos immediately behind the shock (Sonett, 1960; Ness, et al, 1964). One may conjecture that the same chaos may be produced in the blast waves, which, while it probably would not greatly modify the overall hydrodynamic structure of the wave, may have important consequences for modulation of the cosmic ray intensity by the wave (Parker, 1963) and perhaps in the production of fast particles (Parker, 1965a).

This brings us to the question of the propagation of small-scale disturbances in the expanding atmosphere. Small-scale disturbances are generated

in the transition between slow and fast regions of wind and in blast waves, as already noted. Small-scale disturbances must also be generated at the sun by the various activities observed there and by the motions which are responsible for heating the corona. The smallest disturbances are probably heavily damped and do not propagate far, but somewhat longer wavelenghts (depending upon the temperature* etc. Whitaker, 1963) may easily last long enough to be carried an astronomical unit or more in the wind. Presumably the fluctuations observed in the wind near the orbit of Earth (Coleman, et al, 1962; Neugebauer and Snyder, 1963, 1965; Bridge, et al, 1964; Ness, et al, 1964) are waves of this character, originating near the sun, in the unstable break up of filaments in space (see section VIII), or in the regions where the wind varies rapidly with solar longitude, as illustrated in Fig. 8.

Consider the radial propagation of sound waves in an equilibrium atmosphere with spherical symmetry. The waves are described by (45) and (46) which can be reduced to the single second order equation (49) for the special case that $\alpha = 1$. Write $i\omega$ in place of $1/\tau$ and consider sound waves whose wavelengths are short compared to the scale of variation of v^2 and c^2. The WKB approximation may then be employed, writing,

$$\delta v = v \exp i\omega \left[t + \sum_{n=0}^{\infty} R_n(s) \right]$$

---

*The decay time for a wavelength λ is of the general order of $Nk\lambda^2/4\pi^2\kappa$ and $N\mu\lambda^2/4\pi\eta$ as a consequence of the effective thermal conductivity κ and viscosity η.

where $R_n(s)$ is composed of the terms which are nth order in $\omega^{-1} d\ln v/ds$ $(ds = dr/v)$ and $\omega^{-1} d\ln c/ds$. It is readily shown (Parker, 1965d) that

$$\delta v = \epsilon (cv)^{\frac{1}{2}} \exp\left(-\int \frac{dv}{v \pm c}\right) \exp i\omega\left(t - \int \frac{dr}{v \pm c}\right)$$

where ϵ is an arbitrary constant. For an isothermal corona, $dc^2/dr = 0$, the solution reduces to

$$\delta v = \epsilon \frac{(cv)^{1/2}}{v \pm c} \exp i\omega\left(t - \int \frac{dr}{v \pm c}\right)$$

and

$$\frac{\delta N}{N} = \pm \epsilon \left(\frac{v}{c}\right)^{1/2} \frac{1}{v \pm c} \exp i\omega\left(t - \int \frac{dr}{v \pm c}\right).$$

To see how the amplitude of such a wave varies, consider an outward propagating wave starting in the corona near the sun where $v = v_0 \ll c$. At a large distance where $v \gg c$, the velocity amplitude has increased by $(c^2/v_0 v)^{1/2}$. It is evident from Fig. 4 that the factor $(c^2/v_0 v)^{1/2}$ typically lies between 1 and 10 for the solar corona. Thus, we should see sound waves from the sun with some modest increase in velocity amplitude at the orbit of Earth.

Consider the radial propagation of Alfven waves with the simplification that the star does not rotate so that the magnetic field (50) is purely radial. Suppose that the velocity amplitude of the waves is in the ϕ-direction. Denote the perturbation in the radial field $B(r) = B_0(a/r)^2$ by $\delta \mathbf{B} = \nabla \times \delta \mathbf{A}$ where $\delta \mathbf{B}$ is in the ϕ-direction and $\delta \mathbf{A}$ is in the θ-direction. The $\delta \mathbf{v}$, etc. are functions only of r and t. It is readily shown that the linearized momentum equation can be written

$$\left(\frac{\partial}{\partial t} + \frac{v}{r}\frac{\partial}{\partial r} r\right)\delta v = \frac{B(r)}{4\pi M N r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial \delta A}{\partial r}\right).$$

The hydromagnetic equation yields

$$\left(\frac{\partial}{\partial t} + \frac{v}{r}\frac{\partial}{\partial r} r\right)\delta A = B(r)\,\delta v.$$

These two equations can be combined to give

$$\left[\frac{\partial^2}{\partial t^2} + 2v\frac{\partial^2}{\partial v \partial t} + (v^2 - C^2)\frac{\partial^2}{\partial r^2} + \frac{4v}{r^2}\frac{\partial}{\partial t}\right.$$

$$\left. + \left(\frac{1}{2}\frac{dv^2}{dr} + \frac{4v^2}{r} - \frac{2C^2}{r}\right)\frac{\partial}{\partial r} + \left(\frac{1}{2r}\frac{dv^2}{dr} + \frac{2v^2}{r^2}\right)\right]\delta A = 0$$

where $C = B/(4\pi M N)^{1/2}$ is the local Alfven speed. For wavelengths small compared to the scale of variation of v^2 and C^2, the WKB approximation gives

$$\delta A = \epsilon \left(\frac{v}{v_0}\right)^{1/4} \left(\frac{a}{r}\right)^{3/2} \exp i\omega \left(t - \int \frac{dr}{v \pm C}\right)$$

where ϵ is again an arbitrary constant. The amplitude of δv and δB can be expressed in a number of ways using (4) and the fact that $C = C_0 (v/v_0)^{1/2} (a/r)$. It is convenient to write them as

$$\delta B(r,t) = \delta B_0 \left(\frac{N}{N_0}\right)^{1/4} \left(\frac{v_0/C_0 \pm 1}{v/C \pm 1}\right) \exp i\omega \left(t - \int \frac{dr}{v \pm C}\right)$$

and

$$\delta v(r,t) = \mp \delta v_0 \left(\frac{N_0}{N}\right)^{1/4} \left(\frac{v_0/C_0 \pm 1}{v/C \pm 1}\right) \exp i\omega \left(t - \int \frac{dr}{v \pm C}\right)$$

where $\delta v_0 \equiv \delta B_0 / (4\pi\rho_0)^{1/2}$.

It should be noted that the average energy density of an Alfven wave with velocity amplitude $|\delta v|$ is $\frac{1}{2}\rho |\delta v|^2$, so that the energy flux is $F = (v \pm C)\frac{1}{2}\rho |\delta v|^2$. The waves do work on the wind through which they are propagating because of the centrifugal force $\frac{1}{2}\rho |\delta v|^2 / r$

and because of the average additional pressure $\frac{1}{2}|\delta B|^2/8\pi$ which they exert on the radial expansion dv/dr. The total rate at which work is done per unit volume is

$$\mathcal{E} = \frac{1}{2}\left\{\frac{NM|\delta v|^2 v}{r} + \frac{|\delta B|^2}{8\pi}\frac{dv}{dr}\right\}$$

$$= \frac{(\delta B_0)^2}{8\pi}\left(\frac{v_0/C_0 \pm 1}{v/C \pm 1}\right)^2\left(\frac{N}{N_0}\right)^{1/2}\left(\frac{v}{r} + \frac{1}{2}\frac{dv}{dr}\right),$$

and the equation for conservation of energy is then

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2 F\right) + \mathcal{E} = 0.$$

The detectability of the waves depends upon $\delta B/B$ and $\delta v/v$ at the orbit of Earth. As an example consider an Alfven wave from the sun. Suppose that $N/N_0 \cong 10^{-8}$, $B/B_0 \cong \frac{1}{2}\times 10^{-4}$, $v/C \cong 10$, $v_0/C_0 << 1$. Then $\delta v \cong 10\,\delta v_0$ and $\delta B = 10^{-3}\,\delta B_0$. The detectability is

$$\frac{\delta v}{v} \cong 10 \frac{\delta v_0}{c_0} \frac{c_0}{v} \cong 3 \frac{\delta v_0}{c_0} \cong 3 \frac{\delta v_0}{C_0}$$

$$\frac{\delta B}{B} \cong 10^{-3} \frac{\delta B_0}{B_0} \frac{B_0}{B} \cong 20 \frac{\delta B_0}{B_0} .$$

It would appear that, with the assumption $c_0 \cong C_0$ in the corona, that the magnetic component of the wave is more detectable than the velocity component, because the velocity component of the wave must be picked out of the very large wind velocity v $(v >> C)$. The same is true for a sound wave (fast mode). A modest amplitude of $\delta B_0 = 0.05\ B_0$ near the sun leads to $\delta B / B \cong 1$ at the orbit of Earth.

The point of these calculations is to illustrate how the disturbances in the wind change their amplitudes during propagation. The calculations show that an observer in the wind near the orbit of Earth may "listen" to the disturbances generated elsewhere in the solar system (see the radial characteristics in Fig. 6), particularly in the corona near the sun. The magnetometer may be more sensitive than the plasma detector for this astronomical "listening", but both are needed to learn something of the mode of propagation. The Pioneer 5 magnetic data indicated considerable disturbance at times (Coleman, et al, 1960). The more recent IMP-I observations of Ness, et al (1964) show detailed wave forms, illustrated from the magnetometer data in Fig. 10 by causing the pen to move with constant speed across the paper (representing the ecliptic) in the instantaneous direction of the interplanetary field.

The plot shows the average theoretical spiral angle given by (50), but the point of real interest here is the evident disturbances. The scale of the disturbances is indicated by the segments representing the radii of gyration of 10^8, 10^9, and 10^{10} ev protons in a 5×10^{-4} gauss field, the radius being 10^6 km for the 10^9 ev proton. The figure shows that the disturbances are essentially always present, often with large amplitude. The sharpness of some of the wave forms may be the result of the nonlinear effects of large amplitude $\delta B/B \sim 1$ (Parker, 1958b; Petschek, 1958; Montgomery, 1959).

IX. Termination of the Solar Wind

It is evident from the asymptotic decrease of the solar wind density $N \sim 1/r^2$ that the wind must become too tenuous beyond some suitably large distance r_s to push back the ambient interstellar medium. Roughly speaking, the dynamic pressure of the wind is NMv^2, and the wind cannot force its way farther than to where NMv^2 falls to the level of the surrounding ambient pressure p_i. In terms of the density $N(r_E)$ at the orbit of Earth, this radial distance r_s is

$$\frac{r_s}{r_E} = O\left[\left(\frac{N(r_E)\,Mv^2}{p_i}\right)^{1/2}\right].$$

The interstellar pressure p_i effective in this problem is the sum of the magnetic, gas, and some portion of the cosmic ray pressure. If the interstellar medium has a

velocity V relative to the sun, the effective p_i may vary considerably around the sun, but presumably $p_i = O(10^{-12})$ dynes/cm^2. The density and velocity of the solar wind are known only near the ecliptic and for the present solar minimum, where $N(r_E) = 5$ atoms/cm^3 and $v = 500$ km/sec are not untypical. The result is a calculated termination of the wind at $r_s = O(10^2)$ a.u. for the estimated numbers mentioned.

On the basis of classical hydrodynamic theory, the wind goes through a shock transition to subsonic velocity at about the distance r_s. A number of illustrations of this have been worked out in the literature (Parker, 1961b). The shock transition would be collisionless, presumably with the same violent small-scale disturbances as are observed in the bow shock upstream from the magnetosphere. One would suppose, too, that the disturbances produce fast electrons and protons, as observed (Fan, et al, 1964; Freeman, 1964; Frank, et al, 1964) in association with the bow shock and the magnetosphere. Strong collisionless shocks in the inner solar system appear to give particles with energies above 10^6 ev (Parker, 1965a). Protons with energies of this order probably would not penetrate upstream from the terminal shock at r_s to reach the orbit of Earth, but electrons of the same energy might because of their high velocity and small radius of gyration. The possibility that the 10^6 - 10^7 ev electrons observed from IMP-I are from interstellar space, having penetrated in through the solar wind was pointed out by Cline, et al, (1964). Following their idea, we wonder if some portion of the electrons might not be from the terminal shock at r_s.

It has been pointed by Axford, et al, (1963) that a neutral interstellar

medium of the order of one hydrogen atom per cm^3 around the solar system may have important effects on the termination of the solar wind through charge exchange between the 500 km/sec ions of the wind and the neutral thermal interstellar atoms. Charge exchange yields 500 km/sec neutral atoms and thermal ions. For instance, a 10 km/sec interstellar wind would carry 10^6 neutral atoms/cm^2 sec into the terminal region of the solar wind. The flux in the solar wind is observed to be about 10^8 ions/cm^2 sec at the orbit of Earth, so it would be of the order of $10^6/cm^2$ sec at 10 a.u. and $10^4/cm^2$ sec at 10^2 a.u. Thus there should be plenty of neutral hydrogen atoms from interstellar space to exchange with the solar wind.

The characteristic life of a proton or neutral hydrogen atom in a background density of N neutral or ionized hydrogen atoms/cm^3, respectively, is $10^7/N$ sec at the energies involved here (Fite, et al, 1960). The charge exchange time for a solar wind proton in neutral interstellar gas of $N = 1$atom/cm^3 is typically 10^7 sec. If the proton had passed through the shock before the charge exchange took place, its velocity could be directed inward at the time of the exchange yielding a neutral atom moving inward at a few hundred km/sec. The charge exchange time for a neutral interstellar atom is rather longer because the solar wind density is small compared to $1/cm^3$ far beyond the orbit of Earth, with the result that some small fraction of the thermal neutral atoms may find their way into the inner solar system. Axford et al (1960) point out, and discuss briefly, some of the effects which may occur beyond the shock transition as a result of charge exchange and instability.

Patterson, et al (1963) in a recent paper consider the effects of charge

exchange in a more detailed way for the reason that the resonant scattering of solar L_α from the neutral hydrogen in interplanetary space has been observed by Morton and Purcell (1962). The L_α scattered from interplanetary hydrogen was distinguished observationally from L_α from telluric hydrogen by the greater Doppler displacement, in excess of $\pm$ 0.04 Å, from the line center. The observations give the absolute concentration of neutral hydrogen. Patterson et al illustrate the entrance of neutral hydrogen into the solar wind with a simple model assuming 10 km/sec thermal neutral hydrogen incident from interstellar space. They find that with a solar wind of 400 km/sec and 5 ions/cm^3 at the orbit of Earth, the terminal shock lies at about 20 a.u. and the density of neutral hydrogen atoms produced by charge exchange with the solar wind is about 0.02/cm^3 at the orbit of Earth and 0.03/cm^3 at large distance. The density of interstellar neutral atoms is very low at the orbit of Earth ($\sim 10^{-3}$/cm^3) but rises rapidly with distance from the sun, exceeding the neutral atoms from charge exchange beyond about 6 a.u. The calculations illustrate the importance of the L_α observations, which are at the present time the <u>only</u> information available on (a) the solar wind far beyond the orbit of Earth, (b) the interaction of the wind with the interstellar medium, and (c) the interstellar medium outside the solar system. The authors point out that refined observations give promise of a more concrete and quantitative picture of the interaction between the wind and interstellar medium.

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Figures

Fig. 1. Sketch of the one parameter family of solutions of (5) for the circumstance that $T(r)$ declines with increasing r less rapidly than $1/r$ so that $R(r)$ has one zero in the interval (a, ∞).

Fig. 2. A plot of the velocity v_0/c_0 at the base of the corona and the velocity $v(\infty)/c_0$ at large distance as a function of the temperature profile parameter $\beta = -d\ln T/d\ln r$. The asymptotic value of the density $n\xi^2 = Nr^2/N_0 a^2$ at large distance is shown. The values w^2/c_0^2 = 5 and 10 represent a relatively hot and a relatively cool corona, respectively; $w^2/c_0^2 = 2+\beta$ represents the maximum temperature for which $r_c > a$ and the corona may be considered as tightly bound to the sun.

Fig. 3. Plot of the expansion velocity (km/sec) of an isothermal corona as a function of radial distance (in units of 10^6 km) for various temperatures around a star with the mass of the sun.

Fig. 4. Plot of the logarithm to the base ten of expansion velocity in cm/sec at the base of the corona, v_0, at 1 a.u. $v(r_E)$, and the density at 1 a.u. assuming $N_0 = 1 \times 10^8$ atoms /cm^3 at the base of the corona, for an isothermal corona around a star with the mass and radius of the sun. Note the change in the velocity scale at 10^7 cm/sec, indicated by the broken line.

Fig. 5. A plot of the density $N_A(T_0)$ at which the temperature of a corona supplied only by thermal conduction begins to drop off as $r^{-4/7}$ near the base of the corona, and a plot of the density $N_B(T)$ at which

the equilibrium expansion changes from supersonic to subsonic. The broken line represents a second computation of N_B (Parker, 1965a) to show the degree of uncertainty in the result. The curves labeled $s = 2.5$ and $s = 3.0$ represent N_B for nonradial flow in which the cross section increases as r^s.

p.6 Sketch of the characteristics of the equations for radial perturbations of an expanding atmosphere. The distance r_1 represents the point beyond which there are no inward propagating characteristics. The heavy lines represent $dr/dt = v + \alpha^{1/2} c$ and the light lines $dr/dt = v - \alpha^{1/2} c$.

Fig. 7. A sketch of the outward extension of the general magnetic field of the sun, as well as the peripheral lines of force from an active region, by the expansion of the solar corona (indicated by the short arrows). The broken line segment AB represents the region where the field density is expected to be lowest.

Fig. 8. A sketch of the compression (cross hatch) and rarefaction (stipple) arising in the solar wind as a consequence of variations in coronal temperature around the sun. Turbulence may result, particularly in the compression, as a consequence of the shear and/or a shock wave.

Fig. 9. A plot of the asymptotic density profile in the quiet-day interplanetary material scooped up into a blast wave by the more rapid expansion of the enhanced corona. The cases $\lambda = 3/2, 24/17, 4/3, 6/5, 1$ correspond to waves pushed by the enhanced corona so that the energy of the wave increases as the 0, 1/8, 1/4, 1/2, 1 power of t. The curve labelled $\rho_0(r)$ represents the undisturbed density ahead of the blast wave.

Fig. 10. The direction of the interplanetary magnetic field in the plane of the ecliptic plotted from the data of Ness, et al (1964). The direction of the line at any point indicates the direction of the field at that time.

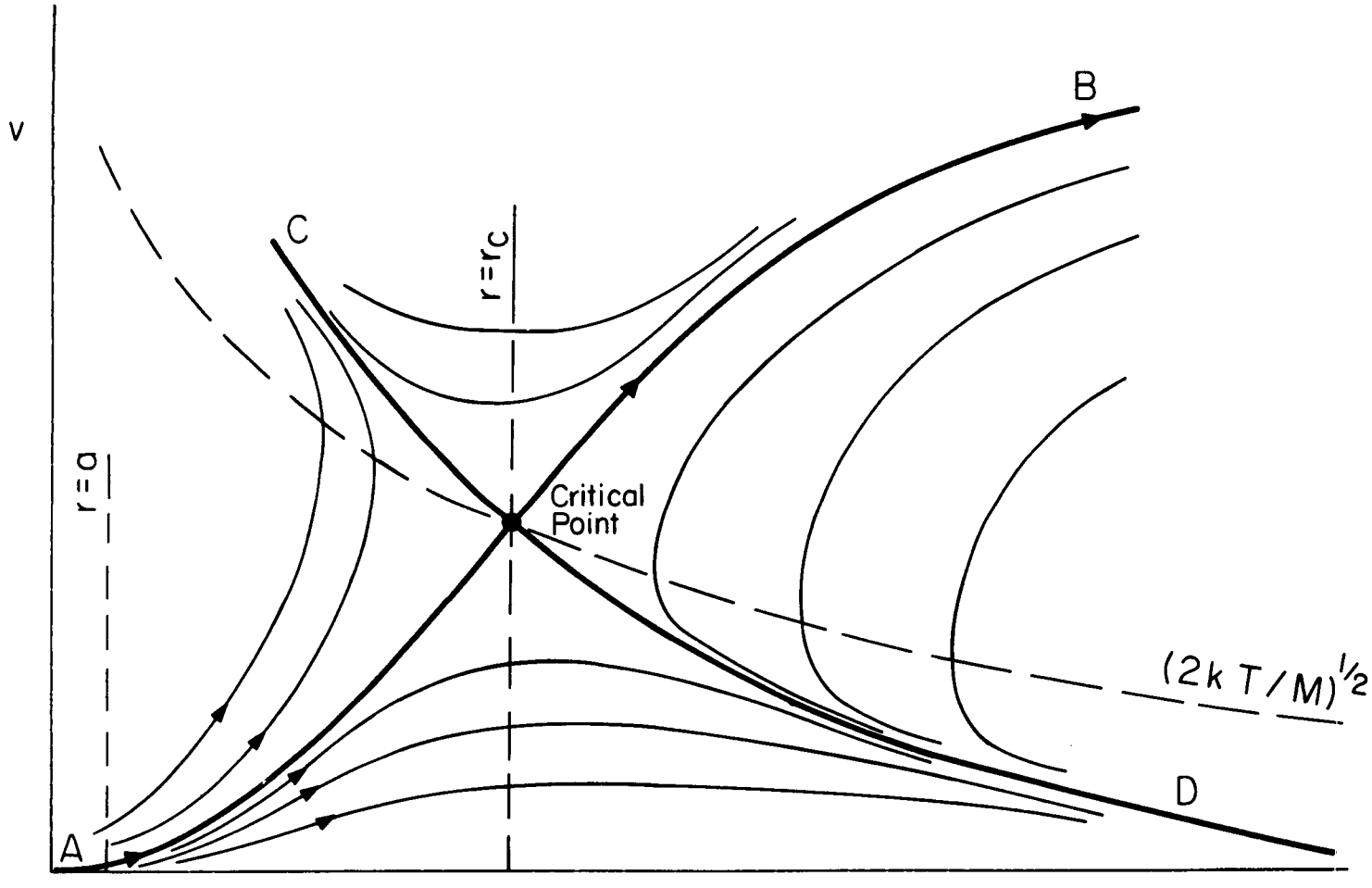

Fig. I

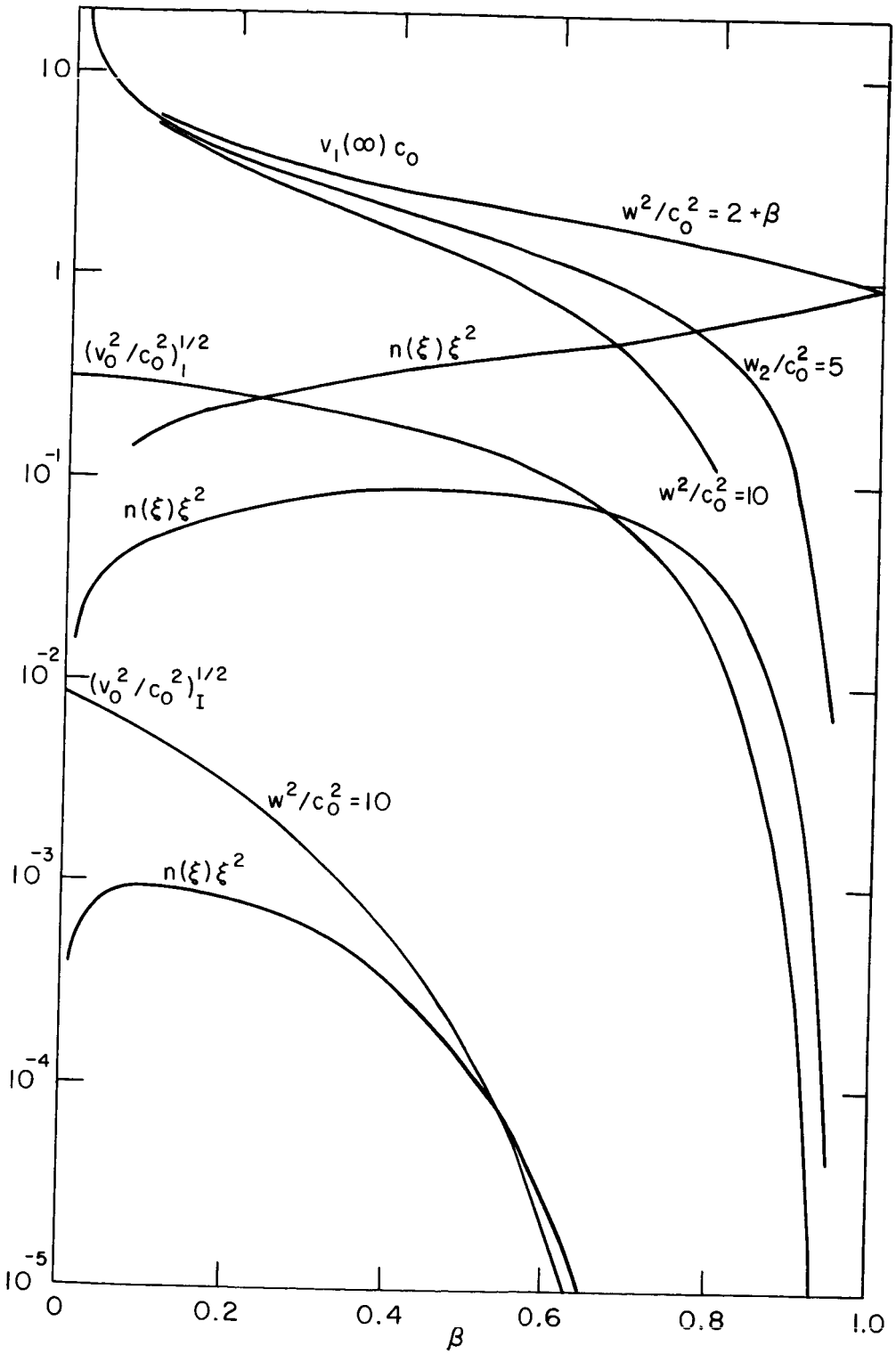
10
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
0
0.2
0.4
0.6
0.8
1.0
β
$v_1(\infty)\, c_0$
$w^2/c_0^2 = 2+\beta$
$(v_0^2/c_0^2)_I^{1/2}$
$n(\xi)\xi^2$
$w_2/c_0^2 = 5$
$w^2/c_0^2 = 10$
$n(\xi)\xi^2$
$(v_0^2/c_0^2)_I^{1/2}$
$w^2/c_0^2 = 10$
$n(\xi)\xi^2$

Fig. 2

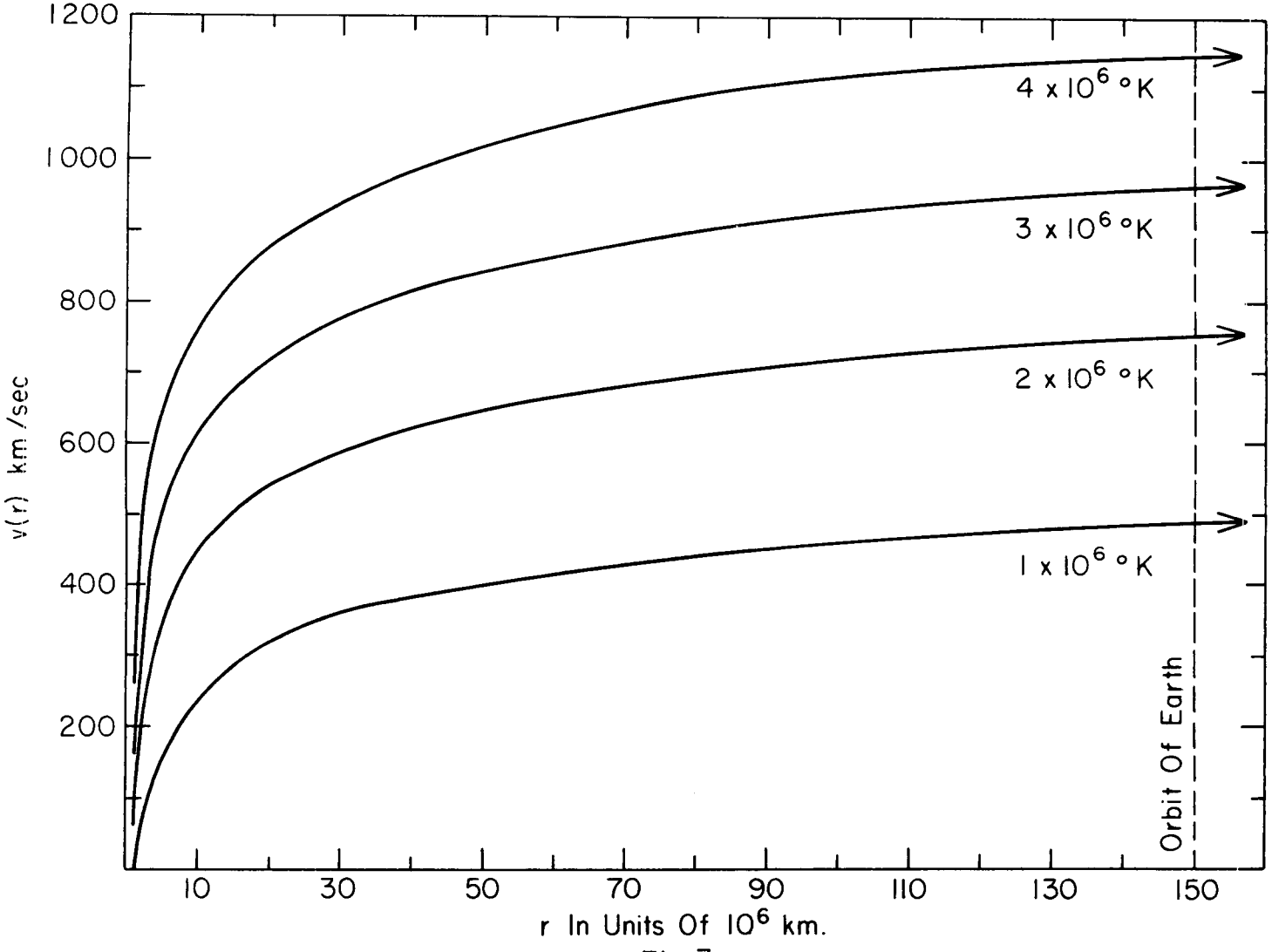
1200
1000
800
600
400
200
v(r) km/sec
4 x 10^6 °K
3 x 10^6 °K
2 x 10^6 °K
1 x 10^6 °K
Orbit Of Earth
10
30
50
70
90
110
130
150
r In Units Of 10^6 km.

Fig. 3

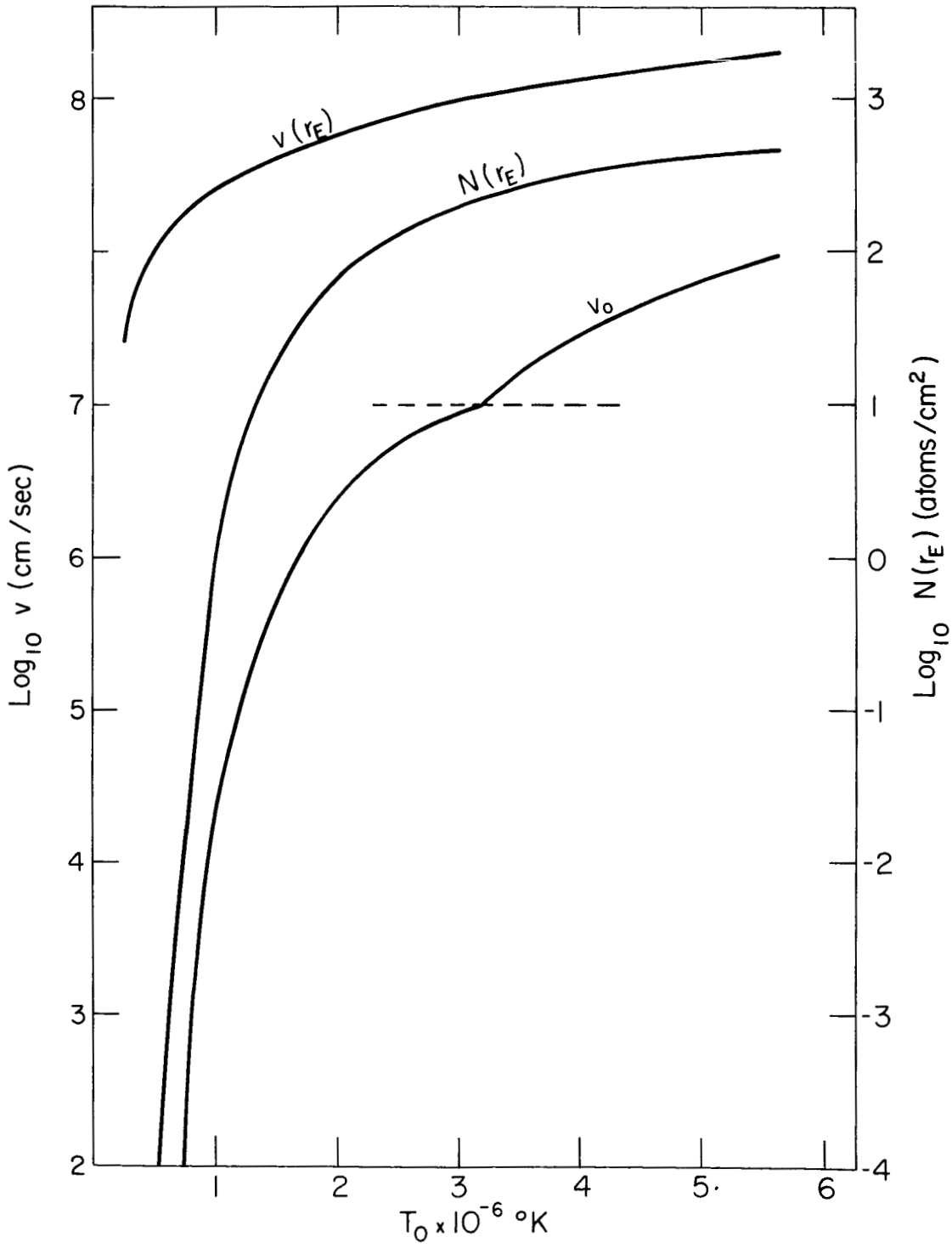
v (r_E)
N (r_E)
v_0
Log_10 v (cm/sec)
Log_10 N(r_E) (atoms/cm²)
T_0 x 10^-6 °K
8
7
6
5
4
3
2
3
2
1
0
-1
-2
-3
-4
1
2
3
4
5
6

Fig. 4

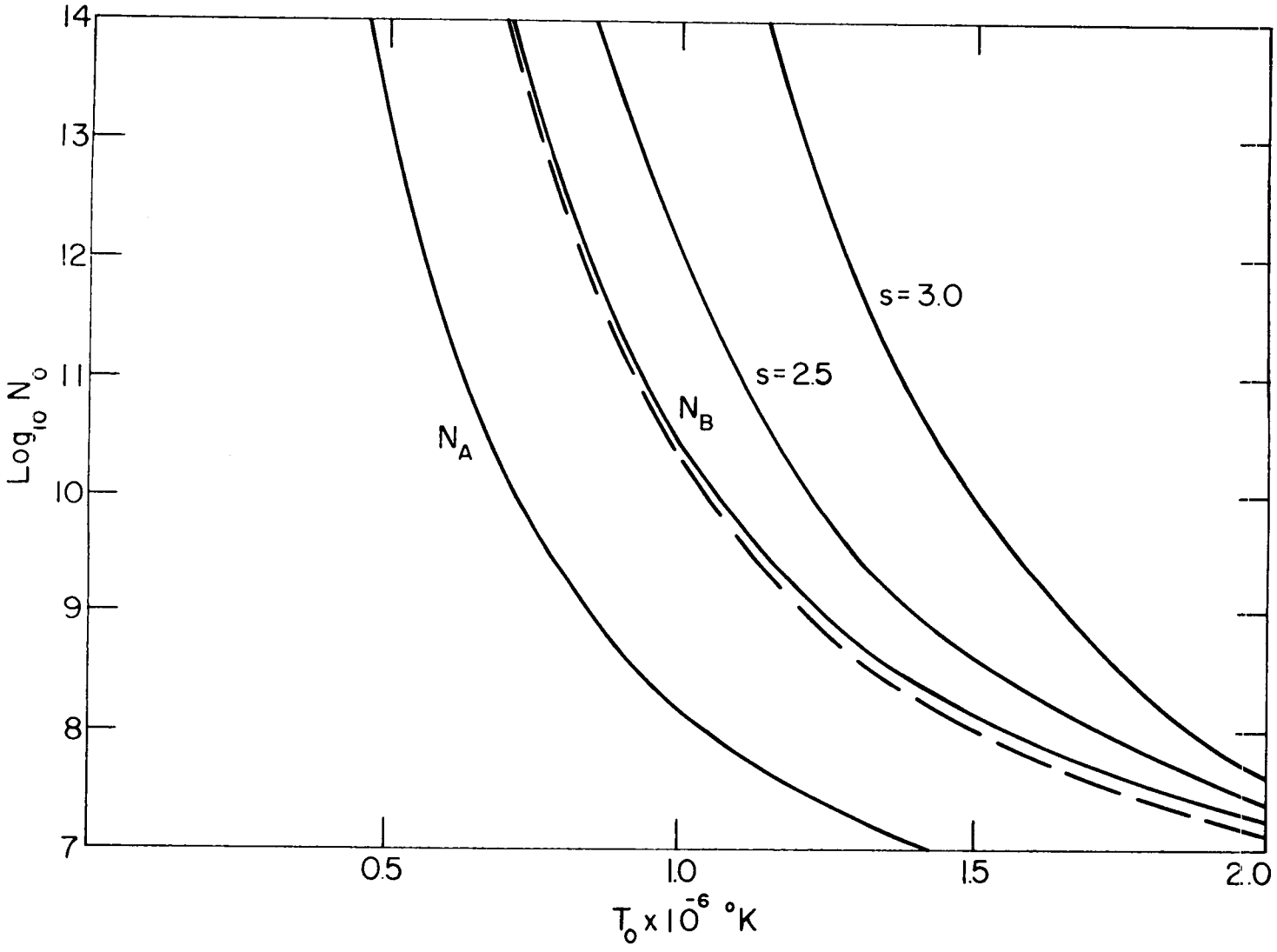

Fig.5

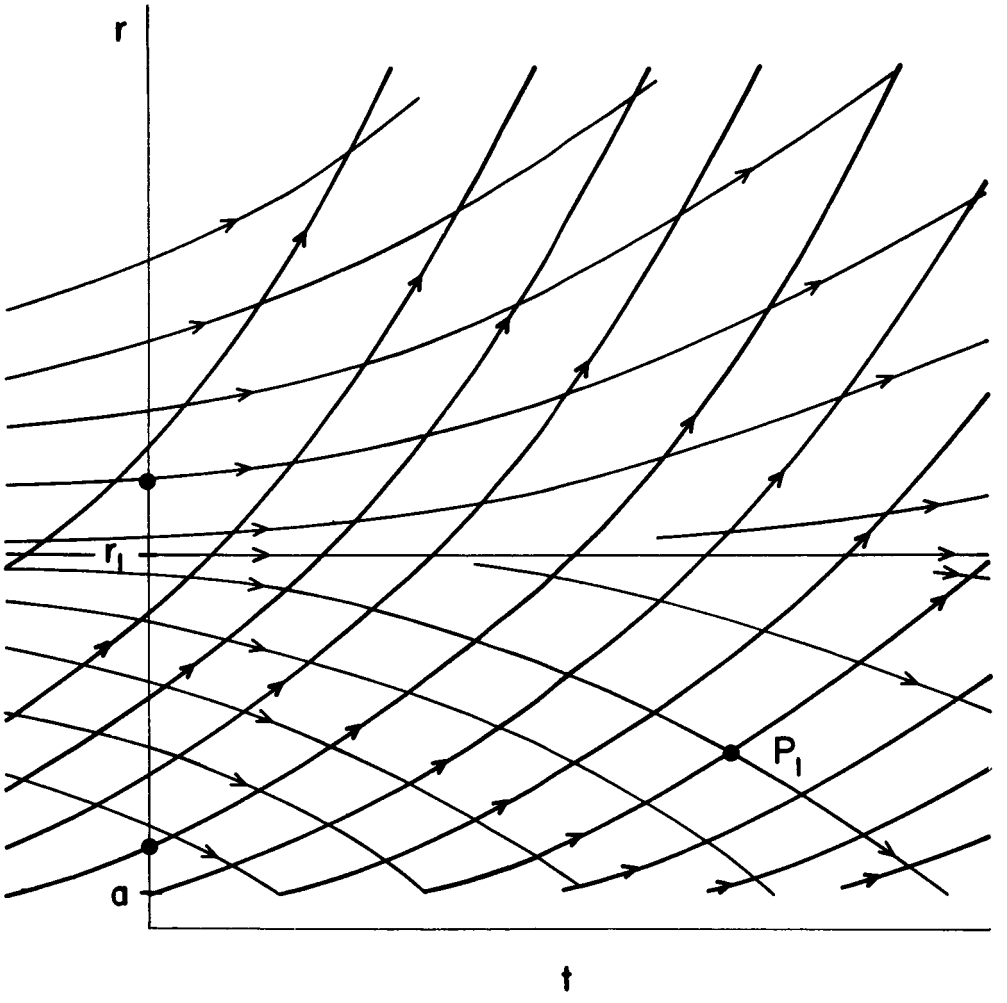

Fig. 6

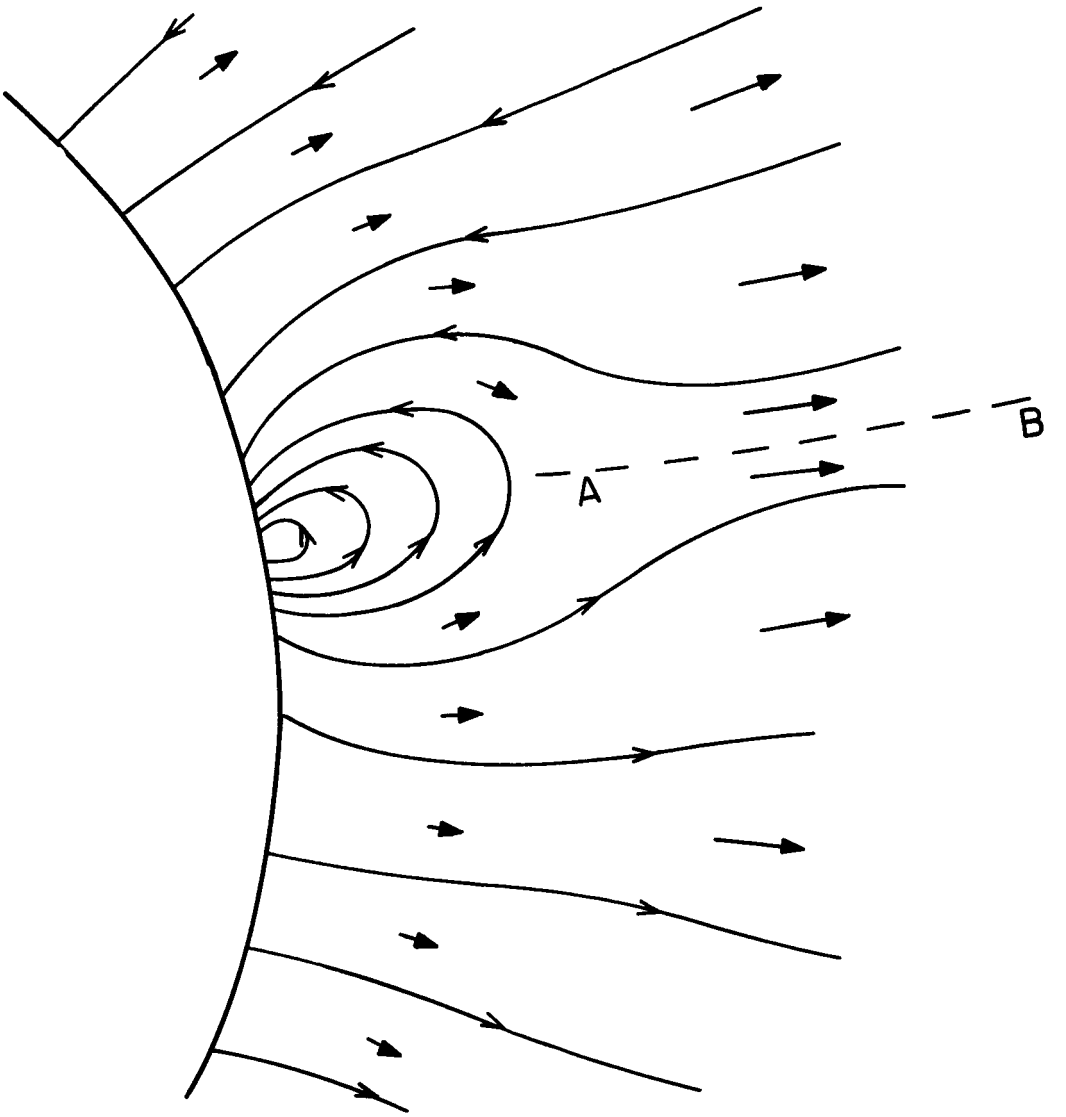

Fig.7

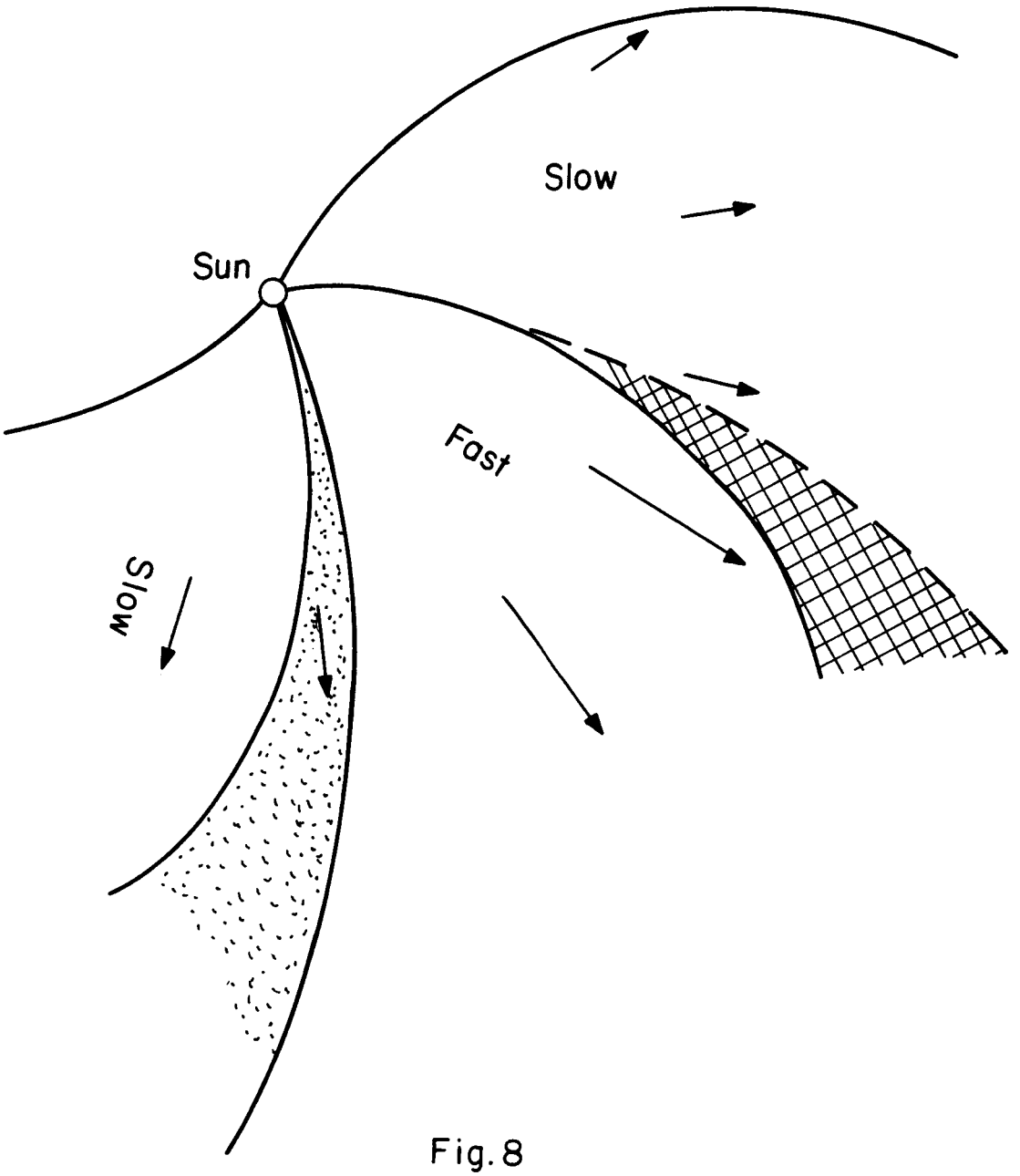

Fig. 8

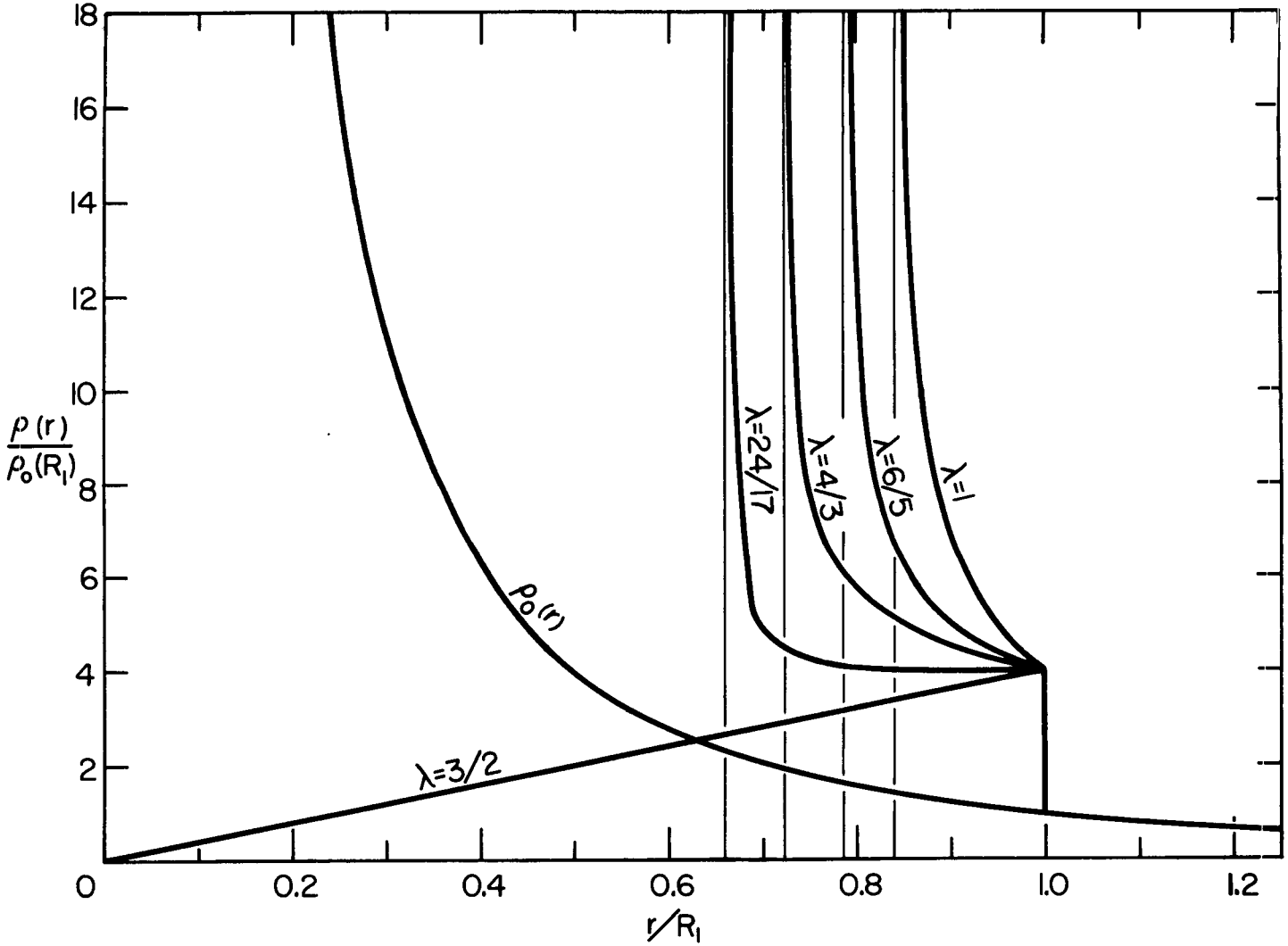
18
16
14
12
10
8
6
4
2
0
0.2
0.4
0.6
0.8
1.0
1.2
$\frac{\rho(r)}{\rho_0(R_1)}$
r/R_1
$\rho_0(r)$
$\lambda=3/2$
$\lambda=24/17$
$\lambda=4/3$
$\lambda=6/5$
$\lambda=1$

Fig. 9

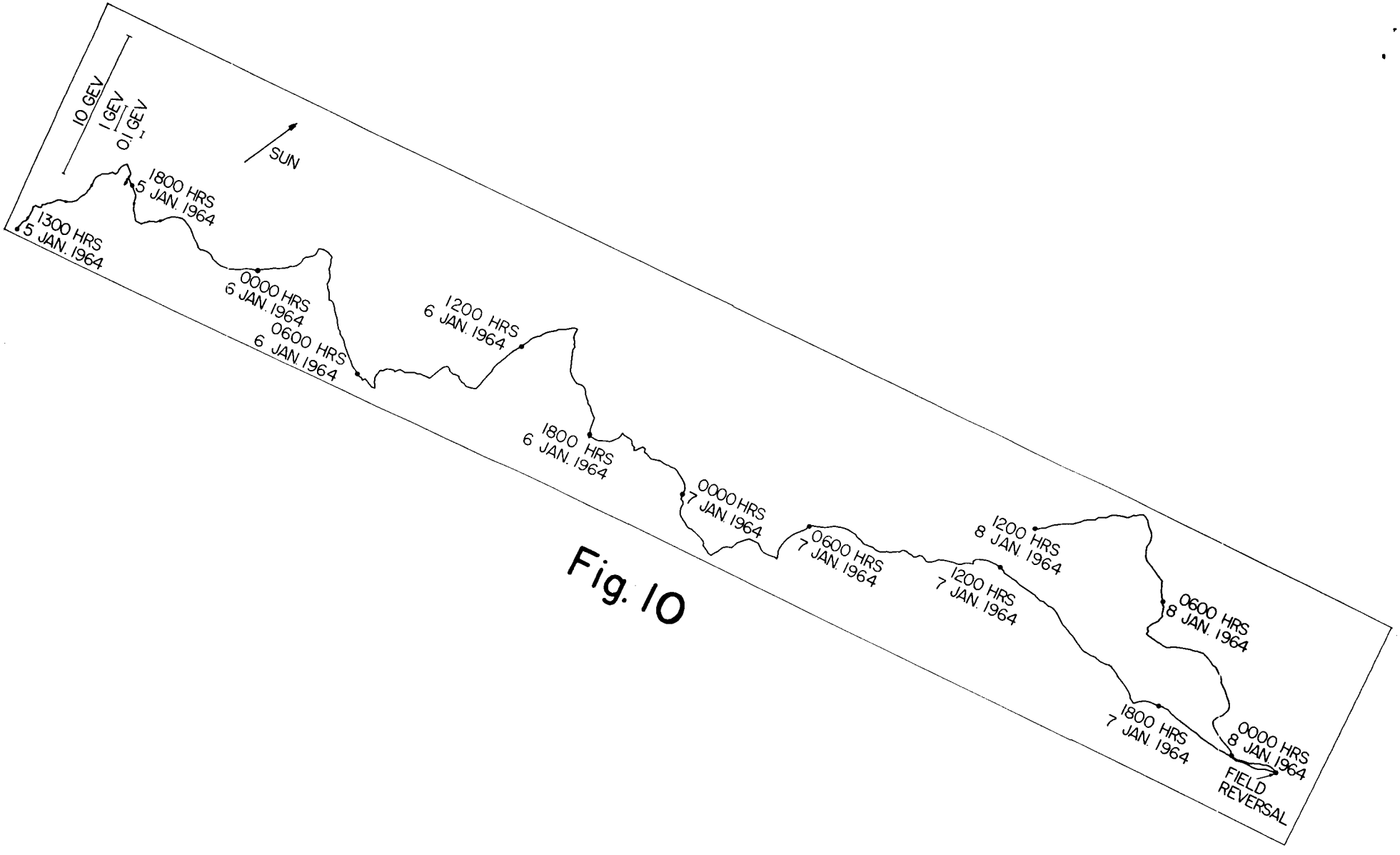
10 GEV
1 GEV
0.1 GEV
SUN
1300 HRS
5 JAN. 1964
1800 HRS
5 JAN. 1964
0000 HRS
6 JAN. 1964
0600 HRS
6 JAN. 1964
1200 HRS
6 JAN. 1964
1800 HRS
6 JAN. 1964
0000 HRS
7 JAN. 1964
0600 HRS
7 JAN. 1964
1200 HRS
7 JAN. 1964
1800 HRS
7 JAN. 1964
1200 HRS
8 JAN. 1964
0600 HRS
8 JAN. 1964
0000 HRS
8 JAN. 1964
FIELD
REVERSAL

Fig. 10