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CALCULATIONS OF MASS-FLOW AND THRUST PRODUCED
FOR A TWO-PHASE FLUID MIXTURE PASSING THROUGH A
CHOKED NOZZLE

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ABSTRACT

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Analytical models are presented for calculating mass flow and thrust produced for a two-phase fluid mixture passing through a choked orifice or a converging-diverging nozzle. The solutions for both frozen and shifting equilibrium flow assumptions give comparable results, yet overcome difficulties encountered with previously used models. These solutions, purely analytical, are applicable for any liquid-vapor fluid mixture and require only nozzle inlet conditions for making calculations.

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TABLE OF CONTENTS

	Page
SUMMARY.....	1
INTRODUCTION.....	1
THEORY	2
General Flow Equations.....	2
Calculations for Frozen Equilibrium Model.....	4
Calculations for Separate-Phase Shifting Equilibrium Model.....	5
DISCUSSION OF RESULTS.....	7
CONCLUSIONS.....	9
APPENDIX	16
REFERENCES	21

LIST OF ILLUSTRATIONS

Figure	Title	Page
1	Comparison of Reported Experimental and Analytical Critical Two-Phase Flow Data for Steam-Water Mixtures.....	10
2	Two-Phase Critical Flow Parameter.....	11
3	Fluid Velocities Along Nozzle Axis.....	12
4	Mass Flow Conditions Along Nozzle Axis.....	13
5	Comparison of Experimental Critical Two-Phase Flow Data with Results from Homogeneous and Presented Analytical Models..	14
6	Comparison of Experimental Critical Two-Phase Flow Data with Results from Homogeneous and Presented Analytical Models..	15
A-1	Fluid Velocity Along Nozzle Axis	19
A-2	Nozzle Mass Flow Conditions.....	20

DEFINITION OF SYMBOLS

Symbol	Definition
A	Passage cross sectional area (ft^2)
B	Ratio of gas to liquid velocity, U_g/U_l
C_F	Nozzle thrust coefficient
C_C	Contraction coefficient, A_c/A_t
F	Thrust (lbf)
G	Mass flow rate per unit area, $W/A \left(\frac{\text{lb m}}{\text{sec ft}^2} \right)$
g	Gravitational constant (32.2), $\left(\frac{\text{ft lb m}}{\text{sec}^2 \text{ lbf}} \right)$
H	Enthalpy (Btu/lb m)
I	Specific impulse $\left(\frac{\text{lbf sec}}{\text{lb m}} \right)$
J	Mechanical equivalent of heat (778), $\left(\frac{\text{ft lbf}}{\text{Btu}} \right)$
P	Static pressure $\left(\frac{\text{lbf}}{\text{ft}^2} \right)$
Q	Fluid mixture quality, W_g/W_m
R	Gas constant $\left(\frac{\text{ft lbf}}{\text{lb m}^\circ \text{R}} \right)$
S	Entropy $\left(\frac{\text{Btu}}{\text{lb m}^\circ \text{R}} \right)$
T	Absolute temperature ($^\circ \text{R}$)
U	Velocity (ft/sec)

DEFINITION OF SYMBOLS (CONT.)

Symbol	Definition
W	Mass flow rate $\left(\frac{\text{lb m}}{\text{sec}}\right)$
γ	Specific heat ratio, C_p/C_v , of vapor phase at inlet plane
ρ	Fluid density $\left(\frac{\text{lb m}}{\text{ft}^3}\right)$
ϵ	Nozzle expansion ratio, A_x/A_t
ψ	Two-phase critical flow parameter, equation (16)

Subscripts

a	Ambient
c	Vena contracta plane
e	Nozzle exit plane
g	Designates mass of fluid that is gas at the inlet plane
i	Nozzle inlet plane
l	Designates mass of fluid that is liquid at the inlet plane
m	Total two-phase fluid mixture
t	Nozzle throat plane
x	Station along nozzle axis
Ho	Designates calculations were based on homogeneous shifting equilibrium conditions

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CALCULATIONS OF MASS-FLOW AND THRUST PRODUCED FOR A TWO-PHASE FLUID MIXTURE PASSING THROUGH A CHOKED NOZZLE

SUMMARY

Analytical models are presented for calculating mass flow and thrust produced for a two-phase fluid mixture passing through a choked orifice or a converging-diverging nozzle. The solutions for both frozen and shifting equilibrium flow assumptions give comparable results, yet overcome difficulties encountered with previously used models. These solutions purely analytical, are applicable for any liquid-vapor fluid mixture and require only nozzle inlet conditions for making calculations.

INTRODUCTION

Cryogenic propellant tanks coasting in space must be vented periodically to prevent overpressurization. Since the vehicle is under a zero or near-zero gravity condition, it is possible for a two-phase fluid mixture to enter the vent system and be discharged overboard. The variations of flow and thrust produced when such a mixture passes through the discharge ports must be considered in the vent system design.

From a literature survey made by R. V. Smith (Ref. 4), it was found that no satisfactory solution exists for calculating choked two-phase flow rates for the full range of fluid qualities. FIGURE 1 shows the available experimental data and the results of various analytical calculations surveyed by Smith. It is evident from this figure that none of the reported analytical solutions are applicable for the entire fluid quality range. Other difficulties, such as the use of empirical constants, requirements of known throat conditions, and difficulties of incorporating thrust calculations, make the use of the reported analytical solutions undesirable. The methods for calculating two-phase flow described herein overcome the above difficulties and give comparable results with previously used models.

It can be seen from FIG 1 that a considerable variation in experimental mass flow occurs for a given fluid mixture quality. This flow variation is greatest for low quality fluid conditions, therefore, it is thought to be caused by the rate at which the fluid properties shift toward equilibrium conditions during the expansion process. In an attempt to account for this flow variation, two approaches for calculating mass flow are taken in this analysis. The first flow model assumes that the fluid properties remain under frozen equilibrium conditions during the expansion process while the second model assumes that the fluid properties immediately shift to equilibrium conditions. The flow models described herein should predict the upper and lower mass flow limits and the thrust produced when a two-phase fluid mixture passes through a choked orifice or a converging-diverging nozzle.

THEORY

General Flow Equations

The following basic equations are used in calculating the two-phase mass flow and nozzle thrust for both the frozen-equilibrium and separate-phase shifting equilibrium flow models and are given to define clearly the symbols used in this report. The quality, Q_i , of the two-phase fluid entering the nozzle is the ratio of vapor to total mass flow and is defined by

$$Q_i \equiv \frac{W_{gi}}{W_{li} + W_{gi}} \quad (1)$$

The flow passage cross sectional area, A_x , at a given plane location, x , along the nozzle axis is equal to the sum of the cross sectional areas occupied by the liquid and vapor phases.

$$A_x = A_{gx} + A_{lx} \quad (2)$$

From the continuity equation, the mass flow per unit area for each phase is

$$G_{lx} \equiv \frac{W_{li}}{A_{lx}} = \rho_{lx} U_{lx} \quad (3)$$

$$G_{gx} \equiv \frac{W_{gi}}{A_{gx}} = \rho_{gx} U_{gx} \cdot \quad (4)$$

By combining equations (1), (2), (3) and (4) and simplifying, the total mass flow per unit area is obtained.

$$G_{mx} = \frac{1}{\frac{1 - Q_i}{G_{lx}} + \frac{Q_i}{G_{gx}}} \quad (5)$$

The total thrust produced by a nozzle with the exit plane at any station, x , equals the summation of the thrust produced by the separate phases.

$$F_{mx} = F_{lx} + F_{gx} \quad (6)$$

The thrust produced by each phase is calculated from the momentum equation.

$$F_{lx} = \frac{U_{lx} W_{lx}}{g} + (P_{lx} - P_a) A_{lx} \quad (7)$$

$$F_{gx} = \frac{U_{gx} W_{gx}}{g} + (P_{gx} - P_a) A_{gx} \quad (8)$$

Since it is assumed that the static pressures of the separate phases along the nozzle wall are equal ($P_{lx} = P_{gx} = P_x$), equations (1), (2), (6), (7) and (8) can be combined and simplified to give the overall nozzle thrust coefficient for the two-phase fluid mixture.

$$C_{F_{mx}} = \frac{F_{mx}}{A_t P_i} = \frac{G_{mt}}{P_i g} [(1 - Q_i) U_{lx} + Q_i U_{gx}] + \frac{(P_x - P_a) \epsilon_x}{P_i} \quad (9)$$

where

$$\epsilon_x \equiv \frac{A_x}{A_t} = \frac{G_{mt}}{G_{mx}} \cdot \quad (10)$$

Knowing the thrust coefficient, one may calculate the specific impulse for the two-phase fluid.

$$I_{mx} = \frac{C_{Fmx} P_i}{G_{mt}} \quad (11)$$

Calculations for Frozen Equilibrium Model

The following assumptions describe the nozzle or orifice flow system and are implied when referring to "Frozen Equilibrium Conditions."

- a. The system is frictionless with no interaction or energy exchange between the liquid and vapor phases.
- b. Metastable, isentropic, one-dimensional expansion occurs, i. e., no evaporation or condensation occurs during the expansion process and the vapor phase acts as a perfect gas.
- c. The static pressures of the separate phases are equal at any plane along the nozzle axis.
- d. The fluid velocity at the nozzle inlet plane equals zero.

The velocity of the liquid and vapor phases at any station along the nozzle axis may be calculated from the general energy and perfect gas equations respectively.

$$U_{lx} = \sqrt{2g(P_i - P_x)/\rho_{li}} \quad (12)$$

$$U_{gx} = \sqrt{2gRT_i \left(\frac{\gamma}{\gamma - 1} \right) \left[1 - (P_x/P_i)^{\frac{\gamma - 1}{\gamma}} \right]} \quad (13)$$

By combining equations (12) and (13) with the continuity equation, the mass flow per unit area at any nozzle station can be calculated.

$$G_{lx} = \frac{W_{li}}{A_{lx}} = \sqrt{2g(P_i - P_x) \rho_{li}} \quad (14)$$

$$G_{gx} = \frac{W_{gi}}{A_{gx}} = P_i \left(\frac{P_x}{P_i} \right)^{\frac{1}{\gamma}} \sqrt{\frac{2g}{RT_i} \left(\frac{\gamma}{\gamma - 1} \right) \left[1 - \left(\frac{P_x}{P_i} \right)^{\frac{\gamma - 1}{\gamma}} \right]} \quad (15)$$

The nozzle throat or maximum flow conditions can now be determined by combining equations (14) and (15) with (5), differentiating G_{mx} with respect to P_x/P_i and equating to zero.

$$\frac{dG_{mx}}{d(P_x/P_i)} = 0 = \psi - \frac{\left(1 - \frac{P_t}{P_i}\right)^{1.5} \left(\frac{\gamma+1}{\gamma}\right) \left[\left(\frac{2}{\gamma+1}\right) \left(\frac{P_t}{P_i}\right)^{\frac{2-\gamma}{\gamma}} - \left(\frac{P_t}{P_i}\right)^{\frac{1}{\gamma}} \right]}{\left(\frac{\gamma}{\gamma-1}\right)^{0.5} \left[\left(\frac{P_t}{P_i}\right)^{\frac{2}{\gamma}} - \left(\frac{P_t}{P_i}\right)^{\frac{\gamma+1}{\gamma}} \right]^{1.5}} \quad (16)$$

where

$$\psi = \left(\frac{1 - Q_i}{Q_i} \right) \sqrt{\frac{\rho_{gi}}{\rho_{li}}}$$

Using equation (16), the critical two-phase flow parameter, ψ , was calculated and plotted on FIG 2 versus critical pressure ratio, P_t/P_i , for various vapor phase specific heat ratios, γ .

Knowing fluid properties at the nozzle inlet plane, one can readily calculate ψ and read P_t/P_i from FIG 2. After obtaining P_t/P_i , the critical mass flow, G_{mt} , can be calculated using equations (5), (14) and (15). Substituting fluid inlet properties and G_{mt} into the necessary equations (5 through 15), one can now calculate the nozzle thrust coefficient, expansion ratio, and specific impulse at selected pressure intervals in the expansion process. In some problems where nozzle thrust must be calculated at various nozzle expansion ratios for different inlet qualities, it may be desirable to use a graphical solution similar to the one employed in the separate-phase and homogeneous models for calculating nozzle performance.

Calculations for Separate-Phase Shifting Equilibrium Model

The following assumptions describe the nozzle flow system and are implied when referring to "Separate-Phase Shifting Equilibrium Conditions."

a. The system is frictionless with no interaction or energy exchange between the inlet liquid and vapor masses.

b. Isentropic one-dimensional expansion occurs with fluid properties of the individual phase shifting to equilibrium conditions at all points during the expansion process.

c. The portion of liquid that vaporizes or gas that condenses during the expansion process moves at the same velocity as its respective initial phase.

d. The static pressures of the separate phases are equal at any plane along the nozzle axis.

e. The fluid velocity at the nozzle inlet plane equals zero.

Since there are no simple equations for determining transport properties for a saturated fluid expanding along an isentrope, it is necessary to employ a graphical solution to simplify the critical flow and nozzle thrust calculations. The velocity at given points in the expansion process can be determined for both the liquid and vapor phases from the general energy equation.

$$U_{lx} = \sqrt{2gJ(H_{li} - H_{lx})} \quad (17)$$

$$U_{gx} = \sqrt{2gJ(H_{gi} - H_{gx})} \quad (18)$$

The mass flow rate per unit area for the separate phases can be determined from equations (17), (18) and the continuity equations (equations (3) and (4)). The values of enthalpy, H , and fluid density, ρ , for the separate phases can be obtained by starting from a known inlet condition on a pressure, P , versus entropy, S , chart and reading the required values at selected pressure intervals along an isentrope. The velocity and mass flow per unit area for both the liquid and gas phases can be calculated for the selected pressure intervals (FIGS 3 and 4). Using equations (3), (4), (5), (17) and (18), one can now determine the two-phase mass flow at selected pressure intervals for a given inlet fluid quality. By plotting the calculated two-phase flow per unit area versus pressure (similar to FIG 4), the maximum mass flow for the mixture, G_{mt} , and throat static pressure, P_t , are obtained.

Knowing G_{mt} and calculating G_{mx} from equation (10) one can select the static pressure, P_x , from the mass flow plot (FIG 4) for desired expansion ratios. Using G_{mt} , P_x , Q_i and selecting the phase

velocities from a plot similar to FIG 3, one can calculate the nozzle thrust coefficient and fluid specific impulse from equations (9) and (11), respectively.

DISCUSSION OF RESULTS

The greatest difficulty in making two-phase flow calculations is determining the ratio of liquid to gas velocities. In the homogeneous shifting equilibrium model (Appendix A), it is assumed that both the liquid and vapor travel at the same velocity. This assumption of equal velocity simplifies the solution, but the results do not agree well with experimental data; therefore, a velocity variation between the liquid and vapor appears to exist. To determine the separate-phase velocities in both the frozen equilibrium and separate-phase shifting equilibrium models, it was assumed that the initial liquid and gaseous masses act independently and are accelerated by their own available energy. This assumption permits a purely analytical method for determining phase velocities for both the converging and diverging nozzle sections.

Other assumptions used in describing the flow systems, such as the frictionless isentropic expansion, have produced results which are in reasonably good agreement with experimental data when used in making single phase critical flow calculations. Also, the one-dimensional expansion assumption is valid for properly designed converging-diverging nozzles and for well-rounded orifices. For a sharp-edged orifice, this assumption becomes less valid, and a contraction coefficient, C_c , should be used for determining the effective critical flow area. If a contraction coefficient is employed in determining critical mass flow, care should be taken to insure proper use of this coefficient in the thrust equations.

The assumption of zero fluid velocity at the nozzle inlet plane was made to simplify the flow equations. In most instances the effect of inlet velocities on nozzle performance is negligible, however, a correction for inlet velocities other than zero may be made by using total rather than static fluid properties at the nozzle inlet plane.

Since the homogeneous flow model assumptions permit the most straightforward solution to two-phase flow problems, it was used by

Smith (Ref. 4) and in this report as a basis for comparing the results of experimental and other analytical solutions. Critical mass flow calculations for various quality steam-water mixtures were made using the presented models, and the results are compared with experimental data on FIG 5. FIGURE 6 is a similar plot showing the calculated results for a two-phase hydrogen mixture.

It may be noted on FIG 5 that the separate-phase shifting equilibrium model gives better agreement for the lower limit of experimental data than does the homogeneous model. This indicates that, when shifting equilibrium conditions occur, the mass flow should be calculated using the separate-phase rather than the previously used homogeneous model. For metastable flow conditions, Smith recommends using the vapor choking model for calculating critical mass flow. The results obtained from this model agree with results from the frozen equilibrium model for high quality-steam-water mixtures (FIGS 1 and 5). However, for low quality mixtures and for fluids other than steam-water, the vapor choking and frozen equilibrium models do not agree. The ratio of mass flow calculated from the vapor choking model to that calculated from the homogeneous model is essentially the same for fluids such as water, hydrogen, oxygen, and nitrogen (Ref. 4); the results using the frozen equilibrium model vary with different fluids (compare FIGS 5 and 6).

Until additional experimental critical two-phase flow data are available for fluids other than steam-water and for various nozzle configurations, it will be difficult to predict which theoretical model gives better results. Recent experimental data from the National Bureau of Standards (Ref. 1), indicate that saturated liquid hydrogen or liquid nitrogen expanding through a sharp-edged orifice follows the frozen equilibrium flow assumptions. However, other data (Ref. 2) obtained by expanding saturated gaseous hydrogen through a converging-diverging nozzle, indicate that shifting equilibrium conditions prevail. These experimental results show that either shifting or frozen equilibrium flow may occur, depending on the fluid properties and nozzle configuration.

CONCLUSIONS

Theoretical models presented give purely analytical solutions to two-phase critical flow problems. These solutions offer the following advantages over previously used methods.

- a. The solutions permit calculations to be made knowing only nozzle inlet conditions and fluid properties.
- b. The models allow calculations of not only critical mass flow but nozzle performance and fluid conditions at any point in the expansion process as well.
- c. The solutions are applicable for the entire range of fluid qualities.
- d. Although this report deals with two phases of a single fluid (i. e., steam-water, $\text{GH}_2\text{-LH}_2$, etc.), the presented solutions may be equally applied to unlike fluids (air-water, GHe-LH_2 , etc.)

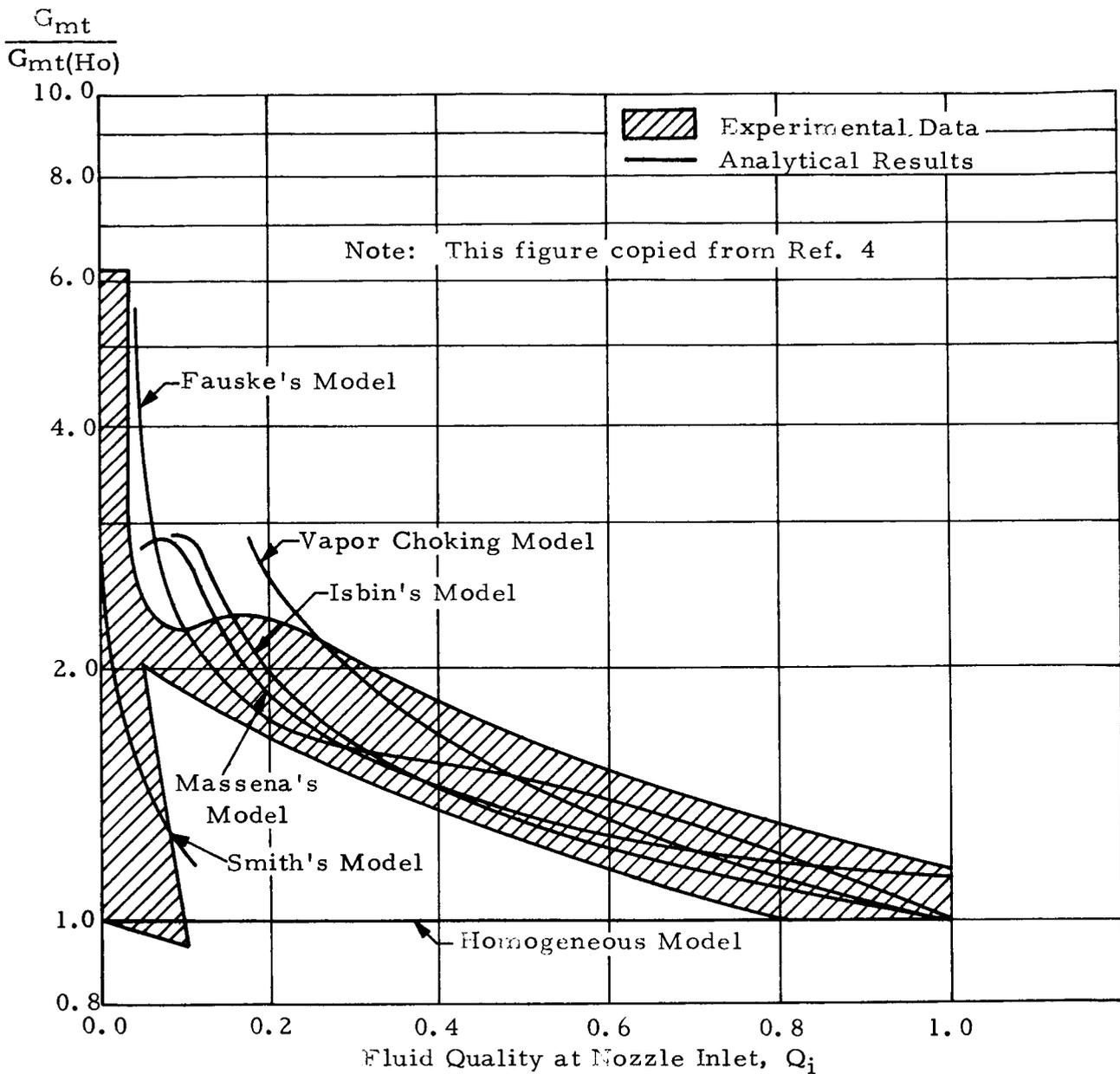


FIGURE 1 COMPARISON OF REPORTED EXPERIMENTAL AND ANALYTICAL CRITICAL TWO-PHASE FLOW DATA FOR STEAM-WATER MIXTURES

$$\text{Two-Phase Critical Flow Parameter, } \psi = \left(\frac{1 - Q_i}{Q_i} \right) \sqrt{\frac{\rho g i}{\rho l i}}$$

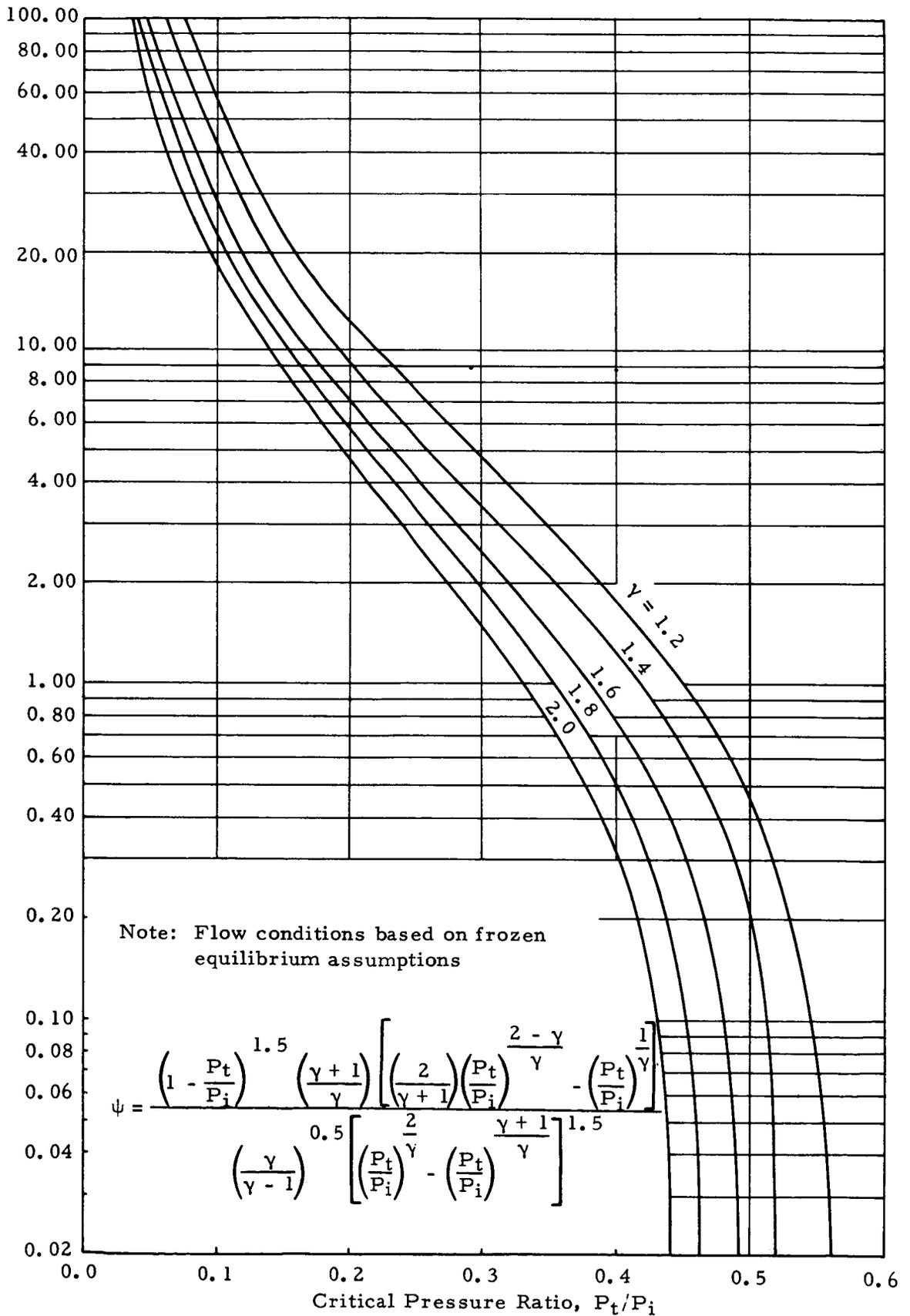


FIGURE 2 TWO-PHASE CRITICAL FLOW PARAMETER

(ft/sec) Velocity of Initial Phase

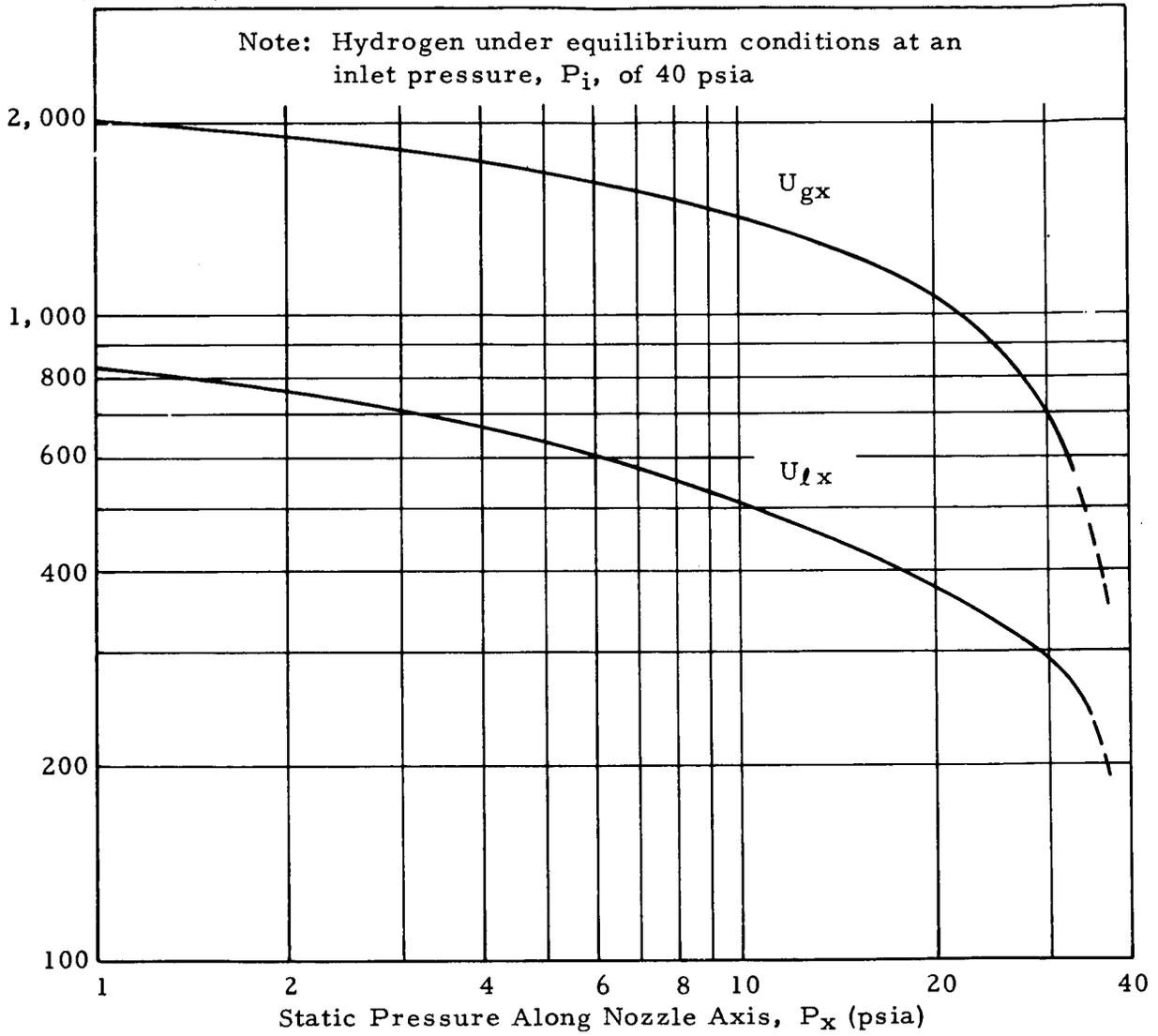


FIGURE 3 FLUID VELOCITIES ALONG NOZZLE AXIS

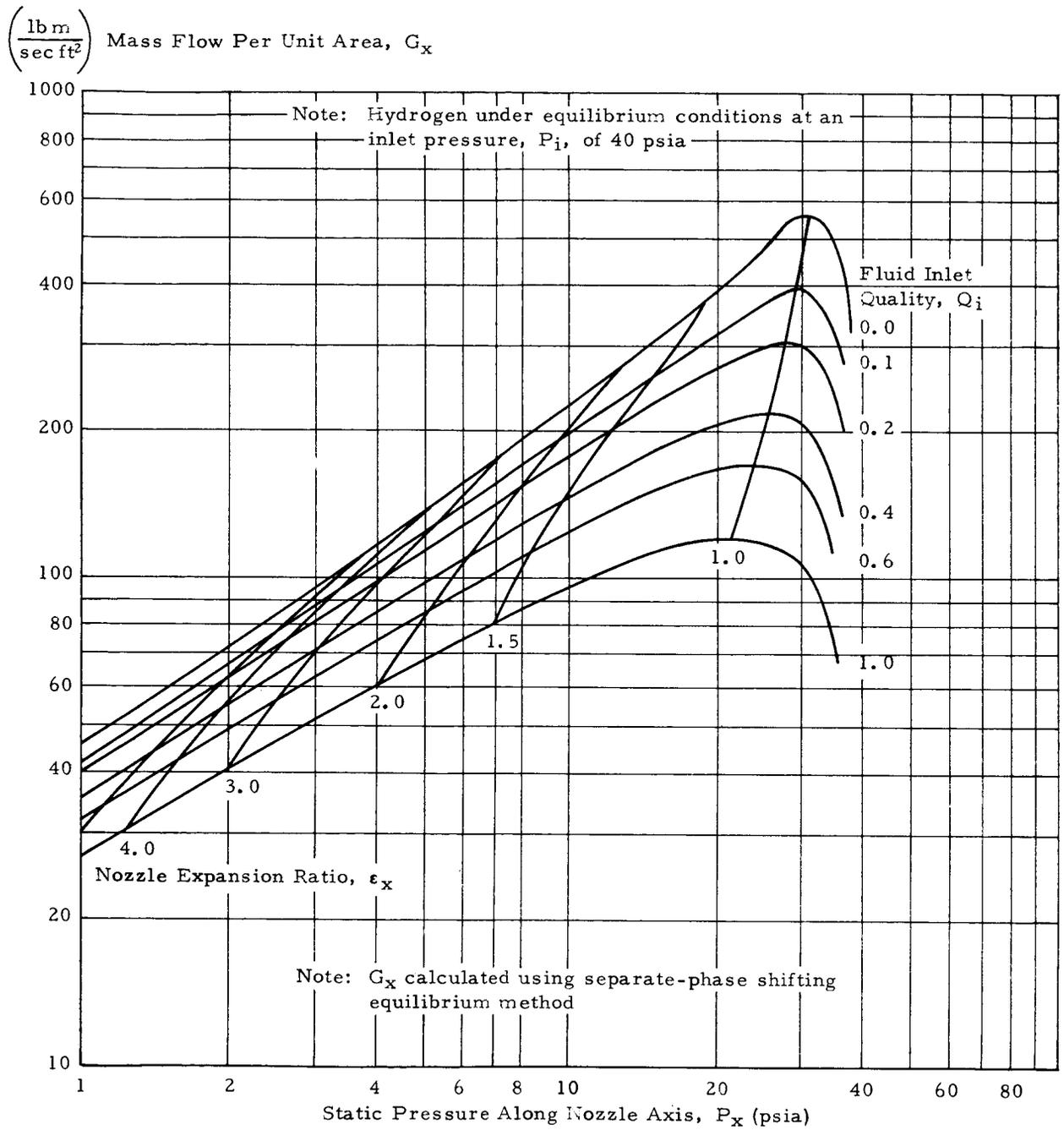


FIGURE 4 MASS FLOW CONDITIONS ALONG NOZZLE AXIS

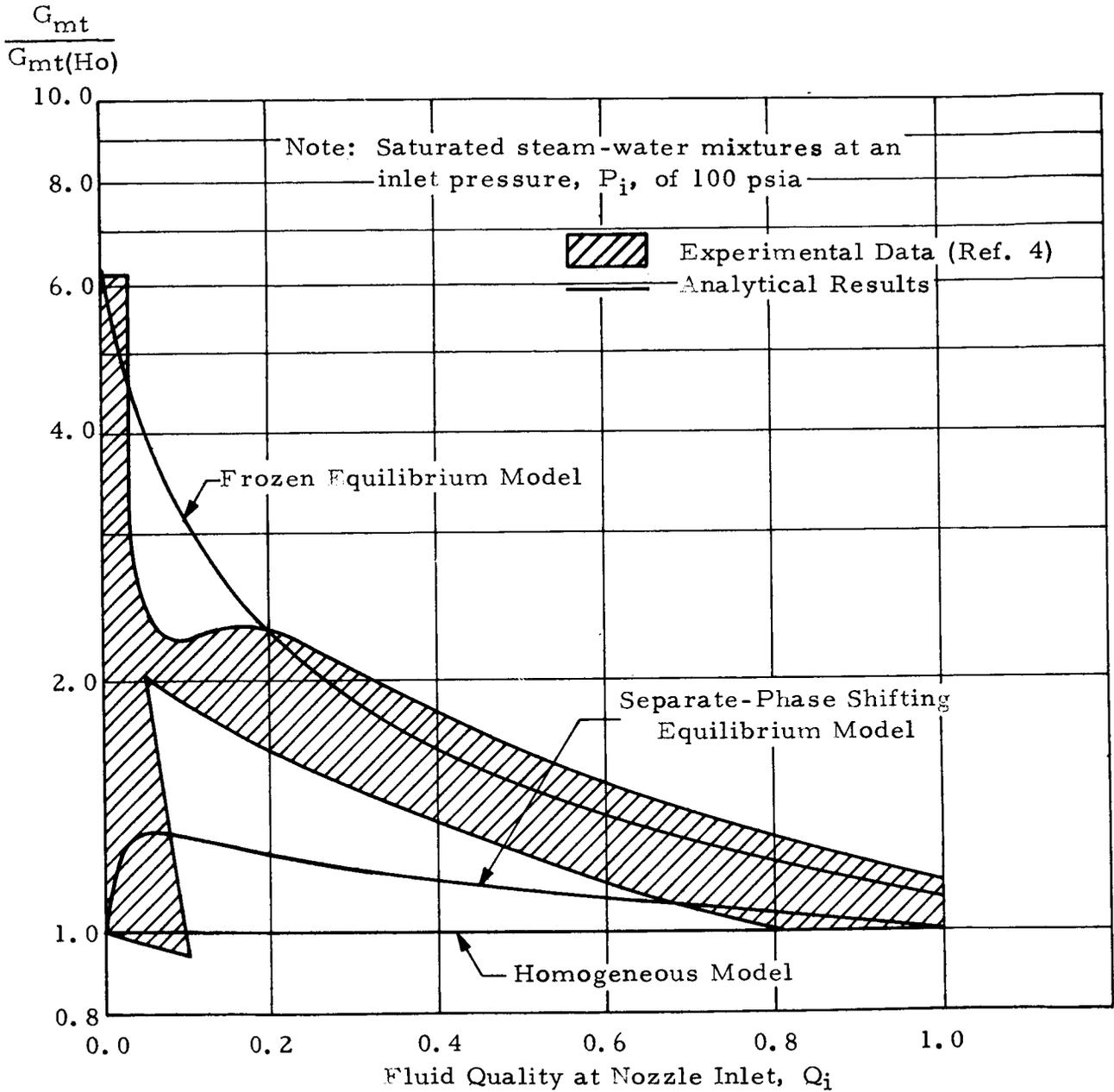


FIGURE 5 COMPARISON OF EXPERIMENTAL CRITICAL TWO-PHASE FLOW DATA WITH RESULTS FROM HOMOGENEOUS AND PRESENTED ANALYTICAL MODELS

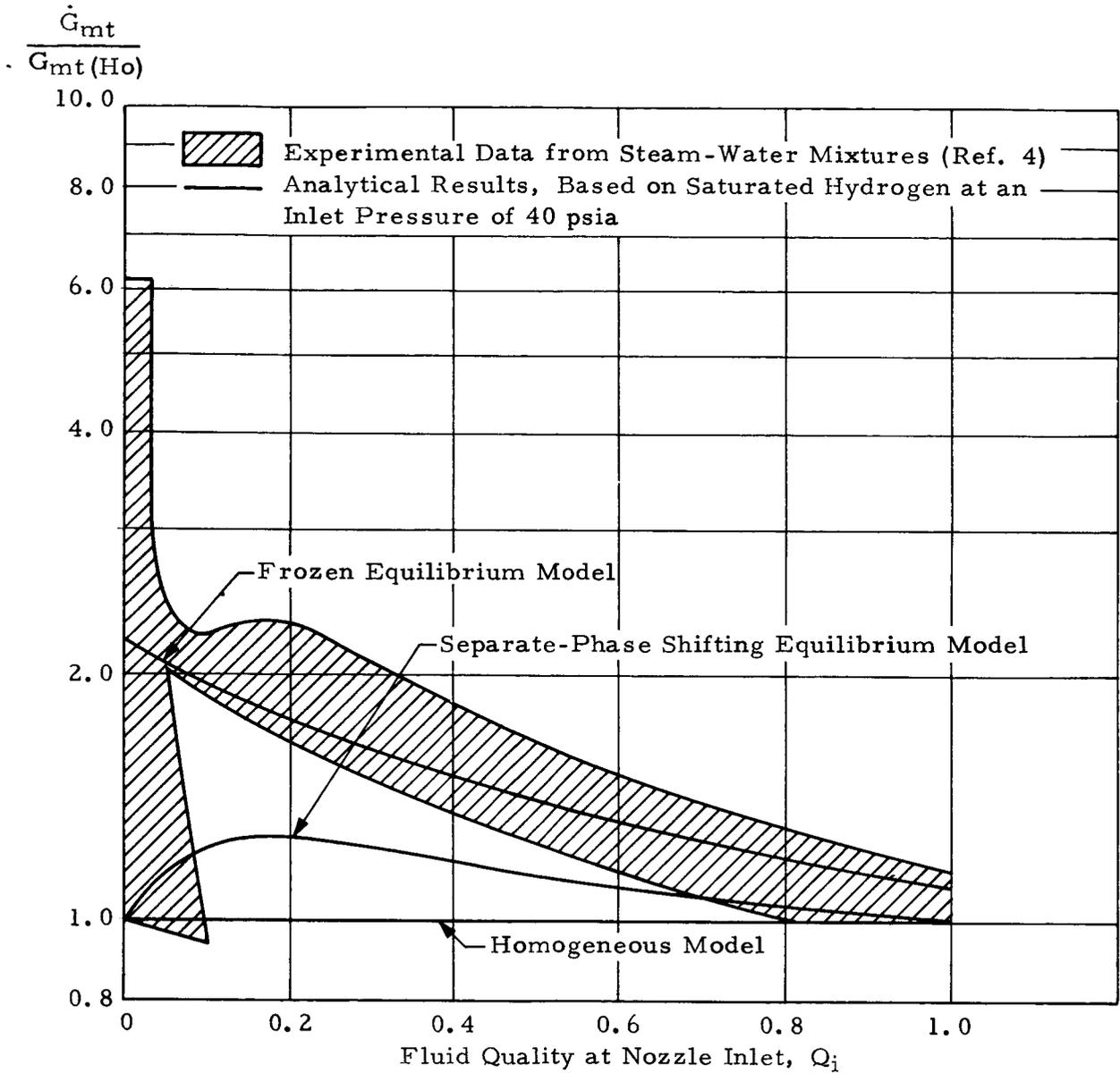


FIGURE 6 COMPARISON OF EXPERIMENTAL CRITICAL TWO-PHASE FLOW DATA WITH RESULTS FROM HOMOGENEOUS AND PRESENTED ANALYTICAL MODELS

APPENDIX

HOMOGENEOUS SHIFTING EQUILIBRIUM MODEL

Since the homogeneous shifting equilibrium assumption permits the simplest solution to two-phase flow problems, it is generally used as a basis for comparing results of experimental and other analytical methods. A number of analytical approaches have been proposed for determining the critical flow rate assuming homogeneous conditions (Ref. 4). Essentially the same results are obtained when using any of the proposed solutions, therefore, the best approach depends upon the available information.

If calculations are to be made using the separate-phase shifting equilibrium model, it is recommended that the method described in this appendix be used for determining the homogeneous flow since the required data are the same. The following assumptions are implied when referring to "Homogeneous Shifting Equilibrium Conditions."

- a. Isentropic one-dimensional expansion occurs, and the system is frictionless.
- b. The fluid mixture shifts to equilibrium conditions at all planes along the nozzle axis (free exchange of energy).
- c. The liquid and vapor phases travel at the same velocity.
- d. The fluid velocity at the nozzle inlet plane equals zero.

Mass Flow Equations

Knowing the inlet conditions, one can calculate the velocity of the mixture, U_{mx} , at various pressure intervals along an isentrope.

$$U_{mx} = \sqrt{2gJ(H_{mi} - H_{mx})} \quad (A-1)$$

If the enthalpy of the liquid, H_{lx} , and the vapor, H_{gx} , phases are known, the enthalpy of the mixture may be calculated.

$$H_{mx} = \frac{W_{gx}H_{gx} + W_{lx}H_{lx}}{W_{mx}} = Q_i H_{gx} + (1 - Q_i)H_{lx} \quad (A-2)$$

Combining equations (A-1) and (A-2) and simplifying, one can obtain the mixture velocity from the individual phase velocities used in the separate-phase shifting equilibrium method.

$$U_{mx} = U_{lx} \sqrt{B^2 Q_i + (1 - Q_i)} \quad (A-3)$$

where

$$B = \frac{U_{gx}}{U_{lx}} .$$

Using the above equation and velocities calculated by the separate-phase shifting equilibrium model (FIG 3), one can calculate values of mixture velocity at various pressure intervals and mixture qualities. FIGURE A-1 is a typical plot showing mixture velocities along the nozzle axis. The mass flow rate of the two-phase mixture is obtained from the continuity equation.

$$G_{mx} = U_{mx} \rho_{mx} \quad (A-4)$$

By solving for the mixture density, ρ_{mx} , in terms of the separate-phase velocities and mass flow rates, the equation for the mixture mass flow rate becomes

$$G_{mx} = \frac{\sqrt{B^2 Q_i + (1 - Q_i)}}{\frac{1 - Q_i}{G_{lx}} + \frac{Q_i B}{G_{gx}}} . \quad (A-5)$$

A plot of mass flow rate versus pressure similar to FIG A-2 can be made by using equation (A-5). From these curves the critical mass flow rate of the mixture, G_{mt} , can be obtained.

Thrust Equations

The thrust produced when a homogeneous two-phase mixture passes through a converging-diverging nozzle may be calculated from the following equations.

$$F_{mx} = \frac{W_{mt} U_{mx}}{g} + (P_x - P_a) A_x \quad (A-6)$$

By modifying equation (A-6) the thrust coefficient may be calculated.

$$C_{F_{mx}} = \frac{F_{mx}}{P_i A_t} = \frac{G_{mt} U_{mx}}{g P_i} + \frac{(P_x - P_a) \epsilon_x}{P_i} \quad (A-7)$$

where

$$\epsilon_x = \frac{A_x}{A_t} = \frac{G_{mt}}{G_{mx}}$$

Using equation (A-7) and selecting required values from figures similar to (A-1) and (A-2), one can determine the thrust coefficient for various mixture qualities and nozzle expansion ratios. The specific impulse may now be determined.

$$I_{mx} = \frac{C_{F_{mx}} P_i}{G_{mt}} \quad (A-8)$$

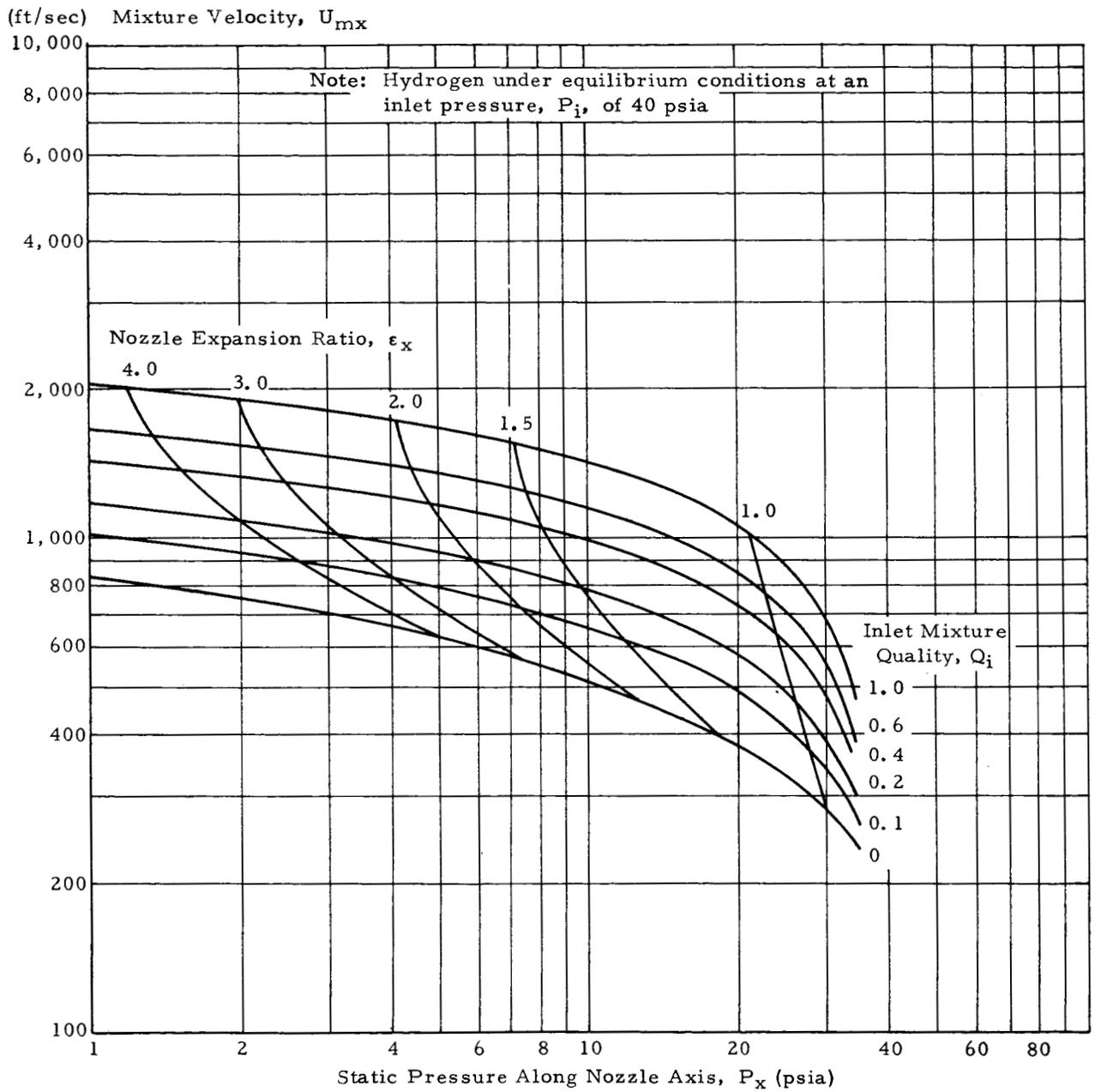


FIGURE A-1 FLUID VELOCITY ALONG NOZZLE AXIS

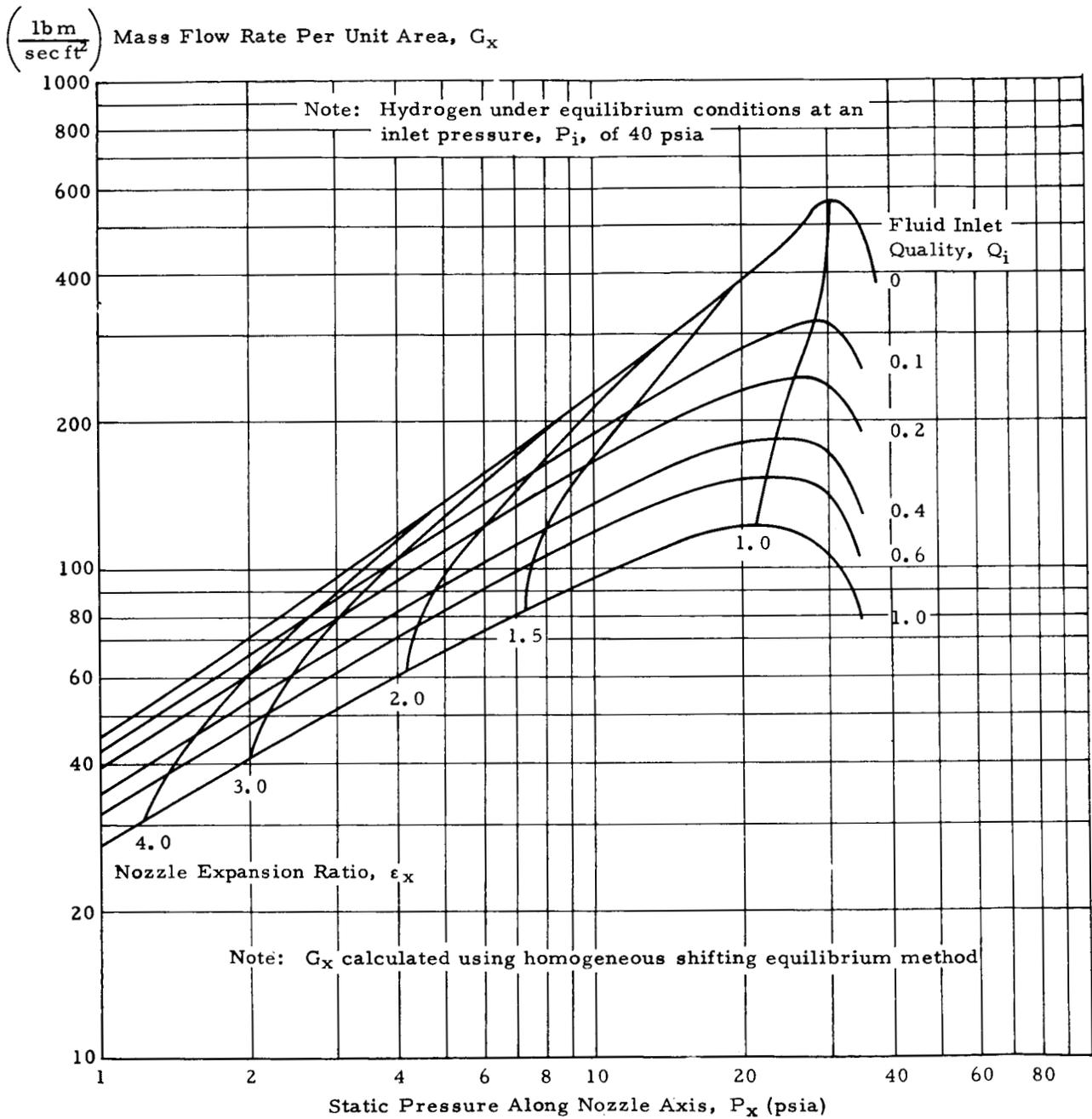


FIGURE A-2 NOZZLE MASS FLOW CONDITIONS

REFERENCES

1. Brennan, J. A., "A Preliminary Study of the Orifice Flow Characteristics of LN_2 and LH_2 Discharging into a Vacuum," Paper No. E-6, Presented at the 1963 Cryogenic Engineering Conference, August 19-21, 1963.
2. Millett, I. and R. C. Nelson, "Experimental Evaluation of Cryogenic Gaseous Hydrogen as a Monopropellant," Lockheed Missile and Space Company, Report No. LMSC-A707769, SP-4500-64-22, October 2, 1964.
3. Shaffer, A. and J. Rousseau, "Thermodynamic Properties of 20.4 degree K-Equilibrium Hydrogen," ASD Technical Report 61-360, October 1961.
4. Smith, R. V., "Choking Two-Phase Flow Literature Summary and Idealized Design Solutions for Hydrogen, Nitrogen, Oxygen, and Refrigerants 12 and 11," NBS Technical Note No. 179, August 3, 1963.

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APPROVAL

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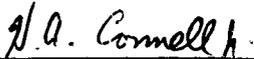
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The information in this report has been reviewed for security classification. Review of any information concerning Department of Defense or Atomic Energy Commission programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

This document has also been reviewed and approved for technical accuracy.



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