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Langley Research Center
Langley Station, Hampton, Va.

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SUMMARY

The dynamics of a manned rotating space-station configuration have been investigated both analytically and experimentally. The solutions for the angular body motions due to external and internal disturbances were obtained by an exact computer method and by an approximate method. The approximate method was developed through the linearization of the equations of motion by using small-angle theory. In addition, an existing scale model of an inflatable space-station configuration was used to obtain experimental results for applied disturbances on the station. The disturbances considered for this analytical and experimental analysis included externally applied torques and static and transient products of inertia.

INTRODUCTION

In the past, the effects of external and internal disturbances on a rotating body in space have been determined by numerical integration of the equations of motion with the aid of a digital computer. However, the computer solutions can be obtained only for specific configurations and disturbances, and they yield little general information. Thus a general closed-form solution which would allow a better insight into the effects of the various disturbance and configuration parameters would be of considerable value. Furthermore, a closed-form method would yield an approximate solution to the equations of motion without the aid of computers, even though the closed-form equations could also be programed to permit a solution in very short computer time.

The present report presents the development, results, and use of a closed-form solution based on small-angle theory. This solution can be used to predict the uncontrolled motion of a spinning vehicle under various disturbances. The results obtained from this solution are compared with results obtained from a scaled dynamic model which represents a typical space-station configuration. In addition, these results are checked with the exact solution of the equations of motion, as solved by numerical integration with the IBM 7094 electronic data processing system.

SYMBOLS

a	acceleration of moving mass, ft/sec^2
I	moment or product of inertia, $slug\text{-}ft^2$
m	mass of moving body, slugs
r	radius of moving mass from body center of gravity, ft
M	external torque, ft-lb
t	time, sec
t_a	actual time, sec
t_∞	anytime after t_s, sec
t^*	relative time, sec
V	velocity of moving mass, ft/sec
X,Y,Z	orthogonal body-axis system
x,y,z	position coordinates of moving mass in body-axis system with origin at center of gravity, ft
ν	angular velocity of moving mass, radians/sec
ϕ,θ,ψ	Euler angles, deg or radians
ω	angular velocity, radians/sec
ω_z	spin rate, radians/sec

Subscripts:

b	body
f	final value
fs	fixed space
i	integer
max	maximum
o	initial value
s	value at the time at which disturbance is removed

x,y,z	component for X-, Y-, or Z-axis
xy,xz,yz	component for XY-, XZ-, or YZ-plane
1,2,3,4	component for mass 1, 2, 3, or 4

A dot over a symbol denotes the derivative with respect to time.

ANALYSIS

Presentation of Approximate Solution, Restrictions, and Procedure for Use

The procedure for obtaining an analytical solution for the uncontrolled body motion of a rotating spacecraft is developed in the appendix. The effects of transient and static product-of-inertia disturbances as well as externally applied torques on the body can be analyzed by this procedure. However, in order to obtain acceptable accuracy with this solution, certain limitations are imposed on the use of the linearized equations of motion. As can be seen in the appendix, small-angle theory is used in the development of the solution. Thus, the modified Euler angles* produced by a disturbance must be less than 15°. Therefore, in order to apply the approximate solution to a particular problem, it must be ascertained that only small angles exist in the problem. The maximum angular displacements resulting from a constant product-of-inertia disturbance can be approximated by the following equations from reference 1:

$$\left.\begin{aligned} \phi_{max} &= \tan^{-1} \frac{2I_{yz}}{I_z - I_y} \\ \theta_{max} &= \tan^{-1} \frac{2I_{xz}}{I_z - I_x} \end{aligned}\right\} \qquad (1)$$

In a manner similar to that of reference 1, the maximum angular displacement resulting from a constant applied torque can be approximated by

$$\left.\begin{aligned} \phi_{max} &= \tan^{-1} \frac{2M_x}{(I_z - I_y)\omega_z^2} \\ \theta_{max} &= \tan^{-1} \frac{2M_y}{(I_z - I_x)\omega_z^2} \end{aligned}\right\} \qquad (2)$$

---

*The modified Euler angles used for this analysis are the same as those used by Kurzhals in reference 1. As shown in figure 1, the body position is defined by three consecutive angles of rotation, ψ, θ, and ϕ.

Equations (1) and (2) are applicable for body configurations with inertia ratios I_X/I_Z in the interval from 0.5 to 1.0 and zero initial transverse body angular velocities and attitude angles.

The solution presented herein is an example solution and it assumes special cases of disturbances. However, most of the basic disturbances that would be experienced by manned rotating space stations may be analyzed by this solution. For example, assumptions made in the development of the approximate solution include a constant velocity of the moving masses, a constant applied disturbing torque, and nearly constant spin rates. These assumptions are realistic, but are limiting in certain instances. A more general solution which considers all possible disturbances can be determined by a method similar to that presented in the appendix.

If a body in space is initially spinning about the axis of maximum inertia (the Z-axis) and is disturbed from the steady-spin condition by an externally applied moment or an internal mass movement, the motion of the body can be determined as shown in the following sections.

<u>Body motion during the application of disturbance</u>.- For the development of the approximate solution, a relative time t^* is defined as $t^* = t_a - t_o$ for the interval $0 \leqq t_a \leqq t_s$ (where t_s is the time at which the disturbance is removed). If the disturbance is applied to the body at zero time, the value of t_o is zero and t^* is therefore equal to the actual time t_a for the interval during which the disturbance acts. For this period, the body angular velocities about the body X- and Y-axes are

$$\omega_X = \left(\omega_{X,o} - \frac{A_1}{I_X\lambda^2}\right)\cos \lambda t^* + \left(\dot{\omega}_{X,o} - \frac{B_1}{I_X\lambda^2}\right)\frac{\sin \lambda t^*}{\lambda} + \frac{B_1 t^*}{I_X\lambda^2} + \frac{A_1}{I_X\lambda^2} \tag{3}$$

$$\omega_Y = \left(\omega_{Y,o} - \frac{A_2}{I_Y\lambda^2}\right)\cos \lambda t^* + \left(\dot{\omega}_{Y,o} - \frac{B_2}{I_Y\lambda^2}\right)\frac{\sin \lambda t^*}{\lambda} + \frac{B_2 t^*}{I_Y\lambda^2} + \frac{A_2}{I_Y\lambda^2} \tag{4}$$

where

$$\lambda_X = \frac{I_Z - I_X}{I_X}\omega_Z \tag{5a}$$

$$\lambda_Y = \frac{I_Z - I_Y}{I_Y}\omega_Z \tag{5b}$$

$$\lambda^2 = \lambda_X\lambda_Y \tag{5c}$$

$$A_1 = -\left[\lambda_Y M_Y + \dot{I}_{YZ}\omega_Z\left(\omega_Z + \lambda_Y\right) + I_{XZ,o}\lambda_Y\omega_Z^{\,2}\right] \tag{5d}$$

$$B_1 = -\dot{I}_{xz}\lambda_y\omega_z^2 \tag{5e}$$

$$A_2 = \lambda_x M_x + \dot{I}_{xz}\omega_z\left(\omega_z + \lambda_x\right) - I_{yz,o}\lambda_x\omega_z^2 \tag{5f}$$

$$B_2 = -\dot{I}_{yz}\lambda_x\omega_z^2 \tag{5g}$$

and, for initial spin about the inertial reference axis,

$$\left.\begin{aligned}
\omega_{x,o} &= 0 \\
\omega_{y,o} &= 0 \\
\dot{\omega}_{x,o} &= \frac{M_x}{I_x} + \frac{\left(\dot{I}_{xz,o} - I_{yz,o}\omega_z\right)\omega_z}{I_x} \\
\dot{\omega}_{y,o} &= \frac{M_y}{I_y} + \frac{\left(\dot{I}_{yz,o} + I_{xz,o}\omega_z\right)\omega_z}{I_y}
\end{aligned}\right\} \tag{6}$$

The constants λ, A_1, B_1, A_2, B_2, $\dot{\omega}_{x,o}$, and $\dot{\omega}_{y,o}$ are dependent only on the disturbance applied and the initial characteristics of the body. Therefore, the body angular velocities can be solved for directly as a function of time for the period during which the disturbance acts.

Similarly, for the interval $0 \leqq t^* \leqq t_s$, the modified Euler reference angles can be found from the following equations:

$$\begin{aligned}
\phi = {} & \phi_o \cos \omega_z t^* + \frac{\dot{\phi}_o}{\omega_z} \sin \omega_z t^* + \frac{C_1}{\omega_z^2 - \lambda^2}\left(\cos \lambda t^* - \cos \omega_z t^*\right) \\
& + \frac{D_1\lambda}{\omega_z^2 - \lambda^2}\left(\frac{\sin \lambda t^*}{\lambda} - \frac{\sin \omega_z t^*}{\omega_z}\right) + \frac{E_1}{\omega_z^2}\left(1 - \cos \omega_z t^*\right) \\
& + \frac{F_1}{\omega_z^3}\left(\omega_z t^* - \sin \omega_z t^*\right)
\end{aligned} \tag{7}$$

$$\theta = \theta_o \cos \omega_z t^* + \frac{\dot{\theta}_o}{\omega_z} \sin \omega_z t^* + \frac{C_2}{\omega_z^2 - \lambda^2}\left(\cos \lambda t^* - \cos \omega_z t^*\right)$$

$$+ \frac{D_2 \lambda}{\omega_z^2 - \lambda^2}\left(\frac{\sin \lambda t^*}{\lambda} - \frac{\sin \omega_z t^*}{\omega_z}\right) + \frac{E_2}{\omega_z^2}\left(1 - \cos \omega_z t^*\right)$$

$$+ \frac{F_2}{\omega_z^3}\left(\omega_z t^* - \sin \omega_z t^*\right) \tag{8}$$

$$\psi = \omega_z t^* \tag{9}$$

where the constants are defined as

$$\left.\begin{aligned}
C_1 &= \dot{\omega}_{x,o} + \omega_z \omega_{y,o} - \frac{B_1}{I_x \lambda^2} - \frac{A_2 \omega_z}{I_y \lambda^2} \\
D_1 &= \frac{\omega_z \dot{\omega}_{y,o}}{\lambda} - \lambda \omega_{x,o} - \frac{B_2 \omega_z}{I_y \lambda^3} + \frac{A_1}{I_x \lambda} \\
E_1 &= \frac{B_1}{I_x \lambda^2} + \frac{A_2 \omega_z}{I_y \lambda^2} \\
F_1 &= \frac{B_2 \omega_z}{I_y \lambda^2} \\
C_2 &= \dot{\omega}_{y,o} - \omega_z \omega_{x,o} - \frac{B_2}{I_y \lambda^2} + \frac{A_1 \omega_z}{I_x \lambda^2} \\
D_2 &= -\left(\frac{\omega_z \dot{\omega}_{x,o}}{\lambda} + \lambda \omega_{y,o} - \frac{B_1 \omega_z}{I_x \lambda^3} - \frac{A_2}{I_y \lambda}\right) \\
E_2 &= \frac{B_2}{I_y \lambda^2} - \frac{A_1 \omega_z}{I_x \lambda^2} \\
F_2 &= -\frac{B_1 \omega_z}{I_x \lambda^2}
\end{aligned}\right\} \tag{10}$$

and, for initial spin about the inertial reference axis,

$$\left.\begin{aligned} \phi_o &= 0 \\ \theta_o &= 0 \\ \dot{\phi}_o &= \omega_{x,o} + \omega_z\theta_o = 0 \\ \dot{\theta}_o &= \omega_{y,o} - \omega_z\phi_o = 0 \end{aligned}\right\} \tag{11}$$

<u>Body motion after cessation of disturbance</u>.- In the development of the solution for the body motion after the disturbance ceases, the same definition of the relative time t^* was used. Since, by definition, $t^* = t_a - t_o$, then at the time the disturbance ceases $t_o = t_s = t_a$, and t^* in the following solution is again initially zero.

The body angular velocities for any time after time t_s are

$$\omega_x = \omega_{x,s} \cos \lambda t^* + \dot{\omega}_{x,s} \frac{\sin \lambda t^*}{\lambda} + \frac{G_1}{\lambda^2}\left(1 - \cos \lambda t^*\right) \tag{12}$$

$$\omega_y = \omega_{y,s} \cos \lambda t^* + \dot{\omega}_{y,s} \frac{\sin \lambda t^*}{\lambda} + \frac{G_2}{\lambda^2}\left(1 - \cos \lambda t^*\right) \tag{13}$$

where

$$\left.\begin{aligned} G_1 &= -\frac{\lambda_y\omega_z^2}{I_{x,f}} I_{xz,f} \\ G_2 &= -\frac{\lambda_x\omega_z^2}{I_{y,f}} I_{yz,f} \end{aligned}\right\} \tag{14}$$

and where λ_x, λ_y, and λ^2 have to be recalculated using the final values of the body moments of inertia $I_{x,f}$, $I_{y,f}$, and $I_{z,f}$. These inertias can easily be computed if the final position of the moving mass is known. The values of $\dot{\omega}_{x,s}$ and $\dot{\omega}_{y,s}$ may be found from the following equations by using known values of $\omega_{x,s}$ and $\omega_{y,s}$ computed at time t_s:

$$\left.\begin{aligned} \dot{\omega}_{x,s} &= -\frac{\left(I_{z,f} - I_{y,f}\right)\omega_{y,s}\omega_z + I_{yz,f}\omega_z^2}{I_{x,f}} \\ \dot{\omega}_{y,s} &= \frac{\left(I_{z,f} - I_{x,f}\right)\omega_{x,s}\omega_z + I_{xz,f}\omega_z^2}{I_{y,f}} \end{aligned}\right\} \tag{15}$$

The modified Euler angles after time t_s are

$$\phi = \phi_s \cos \omega_z t^* + \frac{\dot{\phi}_s}{\omega_z} \sin \omega_z t^* + \frac{H_1}{\omega_z^2 - \lambda^2}\left(\cos \lambda t^* - \cos \omega_z t^*\right)$$

$$+ \frac{J_1\lambda}{\omega_z^2 - \lambda^2}\left(\frac{\sin \lambda t^*}{\lambda} - \frac{\sin \omega_z t^*}{\omega_z}\right) + \frac{K_1}{\omega_z^2}\left(1 - \cos \omega_z t^*\right) \tag{16}$$

$$\theta = \theta_s \cos \omega_z t^* + \frac{\dot{\theta}_s}{\omega_z} \sin \omega_z t^* + \frac{H_2}{\omega_z^2 - \lambda^2}\left(\cos \lambda t^* - \cos \omega_z t^*\right)$$

$$+ \frac{J_2\lambda}{\omega_z^2 - \lambda^2}\left(\frac{\sin \lambda t^*}{\lambda} - \frac{\sin \omega_z t^*}{\omega_z}\right) + \frac{K_2}{\omega_z^2}\left(1 - \cos \omega_z t^*\right) \tag{17}$$

$$\psi = \omega_z\left(t^* + t_s\right) \tag{18}$$

where ϕ_s and θ_s are already known and

$$\left.\begin{aligned} \dot{\phi}_s &= \omega_{x,s} + \omega_z\theta_s \\ \dot{\theta}_s &= \omega_{y,s} - \omega_z\phi_s \end{aligned}\right\} \tag{19}$$

The constants in equations (16), (17), and (18) for ϕ, θ, and ψ are

$$H_1 = \dot{\omega}_{x,s} + \omega_z\omega_{y,s} - \frac{G_2\omega_z}{\lambda^2} \tag{20a}$$

$$J_1 = \frac{G_1}{\lambda} - \lambda\omega_{x,s} + \frac{\omega_z\dot{\omega}_{y,s}}{\lambda} \tag{20b}$$

$$K_1 = \frac{G_2\omega_z}{\lambda^2} \tag{20c}$$

$$H_2 = \dot{\omega}_{y,s} - \omega_z\omega_{x,s} + \frac{G_1\omega_z}{\lambda^2} \tag{20d}$$

$$J_2 = \frac{G_2}{\lambda} - \lambda\omega_{y,s} - \frac{\omega_z\dot{\omega}_{x,s}}{\lambda} \tag{20e}$$

$$K_2 = -\left(\frac{G_1\omega_z}{\lambda^2}\right) \tag{20f}$$

Experimental Model Description

An experimental model subjected to typical simulated disturbances was analyzed by both the approximate solution and an exact computer solution. The rotating model is a 1/3-scale model of an inflatable, torus-shaped, space-station concept. The full-scale station, shown in figure 2, would be 30 feet in diameter and would rotate about the axis of maximum inertia at $6\frac{2}{3}$ rpm to produce a 0.2g environment for the two crew members. The products of inertia about the transverse axes of the space station with a mass of 400 slugs would be 11 200 slug-ft^2 about the X-axis and 12 400 slug-ft^2 about the Y-axis. The product of inertia about the spin axis would be 16 300 slug-ft^2.

The 10-foot-diameter model is constructed of a three-ply neoprene with an aluminum hub and has products of inertia about the X-, Y-, and Z-axes of 46, 51, and 67 slug-ft^2, respectively. The model rotates about the Z-axis at a rate of 20 rpm. A 4-inch air bearing supports the model and allows $\pm 30^\circ$ of movement about the transverse axes and complete freedom about the spin axis. Disturbance torques are applied to the model by the firing of one of four nitrogen jets located on the outer periphery of the model, and crew motions or cargo shifts are simulated by masses which move in tubes as shown in figure 3. The position of these tubes can be changed to yield radial or chordwise mass motions at constant speeds. For chordwise motion, as shown in figure 3, either of the two opposite pairs of masses $\left(m_1 \text{ and } m_2 \text{ or } m_3 \text{ and } m_4\right)$ are moved at the same time in such a manner that the crew motion or cargo shift is simulated while the center-of-gravity position of the model remains fixed.

The model is rotated to the desired spin rate by a spin-up motor and shaft supported by a boom assembly. The assembly is lifted away from the model after the rotational speed has been obtained. The juncture of the spin-up shaft and the model is a cone-frustum arrangement which allows the boom to separate from the model with little or no tipoff error.

Instrumentation on the model includes an onboard recorder which traces the time history of the body position with respect to inertial space. Onboard electrical and nitrogen supplies are also included, and a programing mechanism is used to initiate various combinations of disturbance inputs. The position of the model is determined by a solar-cell arrangement located on top of the hub structure. These solar cells sense the angular deviation of the model with respect to a fixed simulated sun on the laboratory ceiling. Angles obtained by this arrangement are essentially the small angular displacements of the model about the X- and Y-axes. These angular displacements are approximately the modified Euler angles, ϕ and θ, previously discussed.

RESULTS AND DISCUSSION

The approximate, exact, and experimental results are compared in figures 4 to 8. These comparisons were made for several disturbances acting on the example space station. Disturbances considered included externally applied torques and static and transient products of inertia.

Externally Applied Torque

Figure 4 shows the motion of the model station due to an externally applied torque. The disturbance on the model was produced by firing a jet of 1-pound thrust at a moment arm of 5 feet from the model center of gravity for 5 seconds. The motion of the model after the torque is removed could simulate the response of the full-scale station to the docking impact of a 7000-pound Gemini vehicle. Figure 4(a) shows the time histories of the body angular velocities ω_x and ω_y about the transverse body axes and the modified Euler reference angles ϕ and θ. It should be noted that in figure 4(a) is shown a comparison between the exact computer solution and the approximate solution, whereas in figure 4(b) is shown a comparison between the approximate solution and the experimental results obtained from the model. The body angular velocities were not obtained from the experimental results since no rate-measuring devices were included in the model instrumentation. As can be seen in figure 4, the motions of the model station determined by the approximate solution, the exact solution, and the experiments are in very good agreement for the applied-torque disturbance, which resulted in a maximum angular displacement of 4.75°.

Static Product of Inertia

In figure 5 are shown the results of a simulated static or "step" product-of-inertia disturbance. Two crewmen are assumed to be located at extreme positions in the space station to produce the dynamic unbalance. It should be noted that no impulse to simulate the two men instantaneously moving from the center of gravity to the extreme positions was imparted to the vehicle. Such a movement would have been difficult to simulate on the experimental model. However, the case presented represents a typical example of the motion of the station due to a crew dynamic unbalance. On the experimental model two 0.111-slug masses were located at the following coordinates: $x = 3.92$ feet, $y = 0$, and $z = 1.42$ feet for mass 1 and $x = -3.92$ feet, $y = 0$, and $z = -1.42$ feet for mass 2. Again, good agreement was obtained from both solutions and the experimental results; the maximum angular displacement for the unbalance was approximately 2.5°.

Transient Chordwise Products of Inertia

Figures 6 and 7 illustrate the motion of the station due to transient chordwise product-of-inertia disturbances both in the direction of rotation and in the direction opposite to the rotation of the station. Movement of the masses in the direction of rotation resulted in a slightly larger maximum angular displacement than obtained for movement of the masses in the direction opposite to the rotation (approximately 1.7° as compared with 1.4°). It was assumed that on the full-scale station one 6-slug crewmember moved in a chordwise direction at a rate of 0.6 foot per second for 10 seconds. Shown in figures 6 and 7 are the time histories of the body angular velocities and Euler angles of the 1/3-scale model for the chordwise movement of two 0.111-slug masses at 0.6 foot per second for $3\frac{1}{2}$ seconds. The initial coordinates for the location of mass 1 were $x_o = 3.92$ feet, $y_o = 0$, and $z_o = 1.42$ feet; those for mass 2, $x_o = -3.92$ feet, $y_o = 0$, and $z_o = -1.42$ feet. Each mass moved in its respective Z-plane with absolute component velocities of 0.141 foot per second in the x-direction and of 0.583 foot per second in the y-direction. (See fig. 3.)

The station motions as calculated by the exact and approximate solutions were in very good agreement for both types of chordwise mass movements. Although there is a small phase shift between the approximate solution and the experimental results in the Euler angle plots, the frequencies and amplitudes compare very favorably. A possible explanation of these phase shifts is that the inflatable model is not a rigid body. A comparison of the flexible-body motions with a rigid-body solution would be expected to result in similar discrepancies.

Transient Radial Product of Inertia

In figure 8 are compared the results obtained for the motion of the model station due to a transient radial product-of-inertia disturbance. It was assumed that on the full-scale station one 6-slug crewman moved radially outward from the station Z-axis, parallel to and at a constant distance from the X-axis. On the model, the two 0.111-slug masses moved parallel to the X-axis at a rate of 0.6 foot per second from initial positions $x_o = 1.62$ feet, $y_o = 0$, and $z_o = 1.42$ feet for m_1 and $x_o = -1.62$ feet, $y_o = 0$, and $z_o = -1.42$ feet for m_2. Both masses moved for 3.1 seconds. Again, as in the previous cases of mass-motion disturbances, the frequencies and amplitudes of the station motion are in good agreement for both analytical solutions and the experimental data obtained. However, the maximum angular deviation experienced by the station due to the radial mass movement (approximately 1.1°) is slightly smaller than that due to the chordwise mass movement.

CONCLUDING REMARKS

An approximate solution to the equations of motion for a spinning body in space has been developed by using small-angle theory to analyze the effects of various disturbances on the body. Results for the approximate solution are compared with experimental data obtained from a space-station model subjected to simulated disturbances and with the results obtained from an exact computer solution. These comparisons have shown that extremely good correlation is obtainable between the approximate solution and the exact solution. In addition, the results of both analytical solutions have been verified by the results of tests performed on the dynamic model.

The closed-form solution developed in this study may thus be used to investigate the effects of both station body characteristics and disturbance parameters on the resulting body motion. These investigations will yield reasonable results for small body angles and rates.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Station, Hampton, Va., May 21, 1965.

APPENDIX

DEVELOPMENT OF THE LINEARIZED EQUATIONS OF MOTION

In order to develop the approximate solution, a detailed analysis was performed on the exact equations of motion. An example disturbance was used to investigate the order of magnitude of the various terms in the equations. The particular disturbance considered was the scaled mass of one crewman moving in a chordwise direction on the model station as shown in figure 3. The 6-slug crewman was assumed to produce a typical transient product-of-inertia disturbance by moving at a constant velocity in the chordwise direction for a total of 2 seconds. The scaled mass on the model was divided into two smaller masses, m_1 and m_2, so that the analysis would be simplified by keeping the center-of-gravity location stationary. Two other masses, m_3 and m_4, were present only for initial balancing purposes. This type of disturbance analysis also allowed easy substantiating of analytical data with results obtained from tests on the existing experimental space-station model, the characteristics of which were used for this analysis.

The rotational equations of motion for a spinning body in space, as derived in references 1 and 2, are

$$M_X = I_X\dot{\omega}_X - I_{XY}\dot{\omega}_Y - I_{XZ}\dot{\omega}_Z + \dot{I}_X\omega_X - \dot{I}_{XY}\omega_Y - \dot{I}_{XZ}\omega_Z$$
$$- \omega_Z\left(I_Y\omega_Y - I_{YZ}\omega_Z - I_{XY}\omega_X\right) + \omega_Y\left(I_Z\omega_Z - I_{XZ}\omega_X - I_{YZ}\omega_Y\right) \qquad \text{(A1a)}$$

$$M_Y = I_Y\dot{\omega}_Y - I_{YZ}\dot{\omega}_Z - I_{XY}\dot{\omega}_X + \dot{I}_Y\omega_Y - \dot{I}_{YZ}\omega_Z - \dot{I}_{XY}\omega_X$$
$$- \omega_X\left(I_Z\omega_Z - I_{XZ}\omega_X - I_{YZ}\omega_Y\right) + \omega_Z\left(I_X\omega_X - I_{XY}\omega_Y - I_{XZ}\omega_Z\right) \qquad \text{(A1b)}$$

$$M_Z = I_Z\dot{\omega}_Z - I_{XZ}\dot{\omega}_X - I_{YZ}\dot{\omega}_Y + \dot{I}_Z\omega_Z - \dot{I}_{XZ}\omega_X - \dot{I}_{YZ}\omega_Y$$
$$- \omega_Y\left(I_X\omega_X - I_{XY}\omega_Y - I_{XZ}\omega_Z\right) + \omega_X\left(I_Y\omega_Y - I_{YZ}\omega_Z - I_{XY}\omega_X\right) \qquad \text{(A1c)}$$

By definition, the space-station configuration under consideration rotates about the axis of maximum inertia (Z-axis) at a spin rate ω_Z. The position of a moving mass on the body may be defined at any time by (see ref. 1)

$$\left.\begin{aligned} x &= \tfrac{1}{2}a_{x,o}t^2 + V_{x,o}t + x_o + r\cos\nu t \\ y &= \tfrac{1}{2}a_{y,o}t^2 + V_{y,o}t + y_o + r\sin\nu t \\ z &= \tfrac{1}{2}a_{z,o}t^2 + V_{z,o}t + z_o \end{aligned}\right\} \tag{A2}$$

It is not unrealistic to assume that the velocity of the motion of the crew is constant, since a crewman walking will accelerate and decelerate almost instantaneously. In order to correlate the approximate solution developed herein with the results from the experimental model in which linear motion of the crew is very closely approximated, the position of the moving mass is defined simply as

$$\left.\begin{aligned} x &= x_o + V_{x,o}t \\ y &= y_o + V_{y,o}t \end{aligned}\right\} \tag{A3a}$$

and, inasmuch as the motion of the crew is always at the same z-coordinate,

$$z = z_o \tag{A3b}$$

If the positions of any number of masses m_i at any time do not affect the center-of-gravity position of the overall body, the products of inertia for a body are defined as

$$I_{xz} = \sum m_i x_i z_i \tag{A4a}$$

$$I_{yz} = \sum m_i y_i z_i \tag{A4b}$$

$$I_{xy} = \sum m_i x_i y_i \tag{A4c}$$

In addition, for any number of masses m_i, the body moments of inertia about the X-, Y-, and Z-axes are defined as

$$\left.\begin{aligned} I_x &= I_{x,b} + \sum m_i\left(y_i^2 + z_i^2\right) \\ I_y &= I_{y,b} + \sum m_i\left(x_i^2 + z_i^2\right) \\ I_z &= I_{z,b} + \sum m_i\left(x_i^2 + y_i^2\right) \end{aligned}\right\} \tag{A5}$$

Equations (A5) also apply only if there is no change in the center-of-gravity position, as in the case of the experimental model used in the present investigation.

In order to investigate the order of magnitude of the terms of equations (A1), the values of the body moments of inertia and products of inertia were calculated by substituting the characteristics of the model and of the transient product-of-inertia disturbance into equations (A3), (A4), and (A5). The values of these moments of inertia and the derivatives of these inertias were then substituted into equations (A1) so that the relative magnitudes of the terms of each equation could be analyzed. This analysis was performed at the time the masses began to move and again at the time the masses stopped and the body disturbance ceased. The effect of the time-varying disturbance on the inertia terms were analyzed by using this procedure. Equations (A1a) and (A1b) were reduced by assuming that any external torque would be applied only about the X- or Y-axis and that the values of ω_x and ω_y are much smaller than the value of the spin rate ω_z. Eliminating all second-order terms then yielded

$$\left.\begin{aligned} M_x &= I_x\dot{\omega}_x + \left(I_z - I_y\right)\omega_z\omega_y - \left(\dot{I}_{xz} - I_{yz}\omega_z\right)\omega_z \\ M_y &= I_y\dot{\omega}_y - \left(I_z - I_x\right)\omega_z\omega_x - \left(\dot{I}_{yz} + I_{xz}\omega_z\right)\omega_z \end{aligned}\right\} \tag{A6}$$

and from equation (A1c) the spin speed was found to be approximately constant. In addition, the change in the values of the body moments of inertia due to the mass movement was negligible.

Differentiating equations (A6) and assuming I_x, I_y, I_z, and ω_z constant yields

$$\left.\begin{aligned} \dot{M}_x &= I_x\ddot{\omega}_x + \left(I_z - I_y\right)\omega_z\dot{\omega}_y - \left(\ddot{I}_{xz} - \dot{I}_{yz}\omega_z\right)\omega_z \\ \dot{M}_y &= I_y\ddot{\omega}_y - \left(I_z - I_x\right)\omega_z\dot{\omega}_x - \left(\ddot{I}_{yz} + \dot{I}_{xz}\omega_z\right)\omega_z \end{aligned}\right\} \tag{A7}$$

and combining equations (A6) and (A7) yields

$$\left.\begin{aligned}
\ddot{\omega}_x + \lambda^2\omega_x &= \frac{1}{I_x}\left[\dot{M}_x - \lambda_y M_y + \ddot{I}_{xz}\omega_z - \dot{I}_{yz}\omega_z\left(\omega_z + \lambda_y\right) - I_{xz}\lambda_y{\omega_z}^2\right] \\
\ddot{\omega}_y + \lambda^2\omega_y &= \frac{1}{I_y}\left[\dot{M}_y + \lambda_x M_x + \ddot{I}_{yz}\omega_z + \dot{I}_{xz}\omega_z\left(\omega_z + \lambda_x\right) - I_{yz}\lambda_x{\omega_z}^2\right]
\end{aligned}\right\} \tag{A8}$$

where

$$\left.\begin{aligned}
\lambda_x &= \frac{I_z - I_x}{I_x}\omega_z \\
\lambda_y &= \frac{I_z - I_y}{I_y}\omega_z \\
\lambda^2 &= \lambda_x\lambda_y
\end{aligned}\right\} \tag{A9}$$

Substituting equations (A3), which represents the positions of the moving masses on the model, into equations (A4a) and (A4b) yields

$$\left.\begin{aligned}
I_{xz} &= mz_o\left(x_o + V_{x,o}t\right) \\
I_{yz} &= mz_o\left(y_o + V_{y,o}t\right)
\end{aligned}\right\} \tag{A10}$$

and the derivatives of these equations are

$$\left.\begin{aligned}
\dot{I}_{xz} &= mz_oV_{x,o} \\
\dot{I}_{yz} &= mz_oV_{y,o} \\
\ddot{I}_{xz} &= 0 \\
\ddot{I}_{yz} &= 0
\end{aligned}\right\} \tag{A11}$$

APPENDIX

The products of inertia can be written as

$$\left.\begin{aligned} I_{xz} &= I_{xz,o} + \dot{I}_{xz}t \\ I_{yz} &= I_{yz,o} + \dot{I}_{yz}t \end{aligned}\right\} \tag{A12}$$

Also, the disturbing torques for the experimental model of the present investigation are not time-varying; therefore $\dot{M}_x = \dot{M}_y = 0$, and, with equations (A12), equations (A8) become

$$\left.\begin{aligned} \ddot{\omega}_x + \lambda^2\omega_x &= \frac{1}{I_x}\left[-\lambda_y M_y - \dot{I}_{yz}\omega_z\left(\omega_z + \lambda_y\right) - \dot{I}_{xz,o}\lambda_y\omega_z^{\,2} - \dot{I}_{xz}\lambda_y\omega_z^{\,2}t\right] \\ \ddot{\omega}_y + \lambda^2\omega_y &= \frac{1}{I_y}\left[\lambda_x M_x + \dot{I}_{xz}\omega_z\left(\omega_z + \lambda_x\right) - I_{yz,o}\lambda_x\omega_z^{\,2} - \dot{I}_{yz}\lambda_x\omega_z^{\,2}t\right] \end{aligned}\right\} \tag{A13}$$

If a relative time t^* is chosen to be $t^* = t_a - t_o$ for the interval $0 \leqq t_a \leqq t_s$ (where t_s is the time at which the mass stops moving or the torque is removed), then, when $t_o = 0$, $t^* = t_a$. If this definition of t^* is used, the body angular velocities ω_x and ω_y during this interval are found from the solution of the second-order differential equations (A13) (assuming a constant spin rate) and are

$$\left.\begin{aligned} \omega_x &= \left(\omega_{x,o} - \frac{A_1}{I_x\lambda^2}\right)\cos\lambda t^* + \left(\dot{\omega}_{x,o} - \frac{B_1}{I_x\lambda^2}\right)\frac{\sin\lambda t^*}{\lambda} + \frac{B_1 t^*}{I_x\lambda^2} + \frac{A_1}{I_x\lambda^2} \\ \omega_y &= \left(\omega_{y,o} - \frac{A_2}{I_y\lambda^2}\right)\cos\lambda t^* + \left(\dot{\omega}_{y,o} - \frac{B_2}{I_y\lambda^2}\right)\frac{\sin\lambda t^*}{\lambda} + \frac{B_2 t^*}{I_y\lambda^2} + \frac{A_2}{I_y\lambda^2} \end{aligned}\right\} \tag{A14}$$

where

$$\left.\begin{aligned} A_1 &= -\left[\lambda_y M_y + \dot{I}_{yz}\omega_z\left(\omega_z + \lambda_y\right) + I_{xz,o}\lambda_y\omega_z^{\,2}\right] \\ B_1 &= -\dot{I}_{xz}\lambda_y\omega_z^{\,2} \\ A_2 &= \lambda_x M_x + \dot{I}_{xz}\omega_z\left(\omega_z + \lambda_x\right) - I_{yz,o}\lambda_x\omega_z^{\,2} \\ B_2 &= -\dot{I}_{yz}\lambda_x\omega_z^{\,2} \end{aligned}\right\} \tag{A15}$$

If the body is initially spinning about the Z-axis only, then $\omega_{x,o} = \omega_{y,o} = 0$, and from equations (A6)

$$\left.\begin{aligned} \omega_{x,o} &= \frac{M_x}{I_x} + \frac{\left(\dot{I}_{xz,o} - I_{yz,o}\omega_z\right)\omega_z}{I_x} \\ \omega_{y,o} &= \frac{M_y}{I_y} + \frac{\left(\dot{I}_{yz,o} + I_{xz,o}\omega_z\right)\omega_z}{I_y} \end{aligned}\right\} \qquad \text{(A16)}$$

In reference 1, it is shown that the body motion may be related to a set of modified Euler angles. The equations involving these angles and the body rates are

$$\left.\begin{aligned} \dot{\phi} &= \omega_x + \omega_y \tan\theta \sin\phi + \omega_z \tan\theta \cos\phi \\ \dot{\theta} &= \omega_y \cos\phi - \omega_z \sin\phi \\ \dot{\psi} &= \omega_z \cos\phi \sec\theta + \omega_y \sin\phi \sec\theta \end{aligned}\right\} \qquad \text{(A17)}$$

Now, if θ and ϕ are assumed to be small, then the sine and tangent of θ and ϕ can be approximated by the angles themselves and the cosine and secant of θ and ϕ can be approximated by unity. By using this small-angle approximation and assuming that $\theta\phi \ll 1$, $\omega_y \ll \omega_z$, and $\omega_x \ll \omega_z$, equations (A17) reduce to

$$\dot{\phi} = \omega_x + \omega_z\theta \qquad \text{(A18)}$$

$$\dot{\theta} = \omega_y - \omega_z\phi \qquad \text{(A19)}$$

$$\dot{\psi} = \omega_z \qquad \text{(A20)}$$

The time histories of these modified Euler angles must now be obtained. These angles, illustrated in figure 1, define the position of the body with respect to a set of fixed-space axes at any time and are necessary to determine the body motion completely. Differentiating equation (A18), solving for $\dot{\theta}$, and substituting into equation (A19) yields

$$\ddot{\phi} + \omega_z^2\phi = \dot{\omega}_x + \omega_y\omega_z \qquad \text{(A21)}$$

and in a similar manner the following may be obtained from equations (A19) and (A18):

$$\ddot{\theta} + \omega_z{}^2\theta = \dot{\omega}_y - \omega_x\omega_z \tag{A22}$$

The equation for ψ can be found by direct integration of equation (A20). After substitution for ω_x, ω_y, $\dot{\omega}_x$, and $\dot{\omega}_y$, equations (A21) and (A22) are solvable differential equations. If the same definition of t^* as before, during the interval $0 \leqq t^* \leqq t_s$ is used, these solutions yield

$$\begin{aligned}\phi = {} & \phi_o \cos \omega_z t^* + \frac{\dot{\phi}_o}{\omega_z} \sin \omega_z t^* + \frac{C_1}{\omega_z{}^2 - \lambda^2}\left(\cos \lambda t^* - \cos \omega_z t^*\right) \\ & + \frac{D_1\lambda}{\omega_z{}^2 - \lambda^2}\left(\frac{\sin \lambda t^*}{\lambda} - \frac{\sin \omega_z t^*}{\omega_z}\right) + \frac{E_1}{\omega_z{}^2}\left(1 - \cos \omega_z t^*\right) \\ & + \frac{F_1}{\omega_z{}^3}\left(\omega_z t^* - \sin \omega_z t^*\right)\end{aligned} \tag{A23}$$

$$\begin{aligned}\theta = {} & \theta_o \cos \omega_z t^* + \frac{\dot{\theta}_o}{\omega_z} \sin \omega_z t^* + \frac{C_2}{\omega_z{}^2 - \lambda^2}\left(\cos \lambda t^* - \cos \omega_z t^*\right) \\ & + \frac{D_2\lambda}{\omega_z{}^2 - \lambda^2}\left(\frac{\sin \lambda t^*}{\lambda} - \frac{\sin \omega_z t^*}{\omega_z}\right) + \frac{E_2}{\omega_z{}^2}\left(1 - \cos \omega_z t^*\right) \\ & + \frac{F_2}{\omega_z{}^3}\left(\omega_z t^* - \sin \omega_z t^*\right)\end{aligned} \tag{A24}$$

$$\psi = \omega_z t^* \tag{A25}$$

where

$$C_1 = \dot{\omega}_{x,o} + \omega_z\omega_{y,o} - \frac{B_1}{I_x\lambda^2} - \frac{A_2\omega_z}{I_y\lambda^2}$$

$$D_1 = \frac{\omega_z\dot{\omega}_{y,o}}{\lambda} - \lambda\omega_{x,o} - \frac{B_2\omega_z}{I_y\lambda^3} + \frac{A_1}{I_x\lambda}$$

$$E_1 = \frac{B_1}{I_x\lambda^2} + \frac{A_2\omega_z}{I_y\lambda^2}$$

$$F_1 = \frac{B_2\omega_z}{I_y\lambda^2}$$

$$C_2 = \dot{\omega}_{y,o} - \omega_z\omega_{x,o} - \frac{B_2}{I_y\lambda^2} + \frac{A_1\omega_z}{I_x\lambda^2}$$

$$D_2 = -\left(\frac{\omega_z\dot{\omega}_{x,o}}{\lambda} + \lambda\omega_{y,o} - \frac{B_1\omega_z}{I_x\lambda^3} - \frac{A_2}{I_y\lambda}\right)$$

$$E_2 = \frac{B_2}{I_y\lambda^2} - \frac{A_1\omega_z}{I_x\lambda^2}$$

$$F_2 = -\frac{B_1\omega_z}{I_x\lambda^2}$$

A new set of equations are now needed for the station motion after time t_s when the disturbance is removed. At time t_s equations (A13) becomes

$$\left.\begin{aligned} \ddot{\omega}_x + \lambda^2\omega_x &= G_1 \\ \ddot{\omega}_y + \lambda^2\omega_y &= G_2 \end{aligned}\right\} \tag{A26}$$

where

$$G_1 = -\frac{\lambda_y\omega_z^2}{I_{x,f}}\left(I_{xz,o} + \dot{I}_{xz}t_s\right) = -\frac{\lambda_y\omega_z^2}{I_{x,f}}I_{xz,f}$$

$$G_2 = -\frac{\lambda_x\omega_z^2}{I_{y,f}}\left(I_{yz,o} + \dot{I}_{yz}t_s\right) = -\frac{\lambda_x\omega_z^2}{I_{y,f}}I_{yz,f}$$

If the time t^* is again defined as $t^* = t_a - t_o$, where now $t_o = t_s$, then at the time the disturbance is removed, $t_a = t_s$ and the solutions to equations (A26) for ω_x and ω_y in the interval $t_s \leqq t_a \leqq t_\infty$ or $0 \leqq t^* \leqq t$ are:

$$\left.\begin{aligned}\omega_x &= \omega_{x,s}\cos\lambda t^* + \dot{\omega}_{x,s}\frac{\sin\lambda t^*}{\lambda} + \frac{G_1}{\lambda^2}\left(1 - \cos\lambda t^*\right)\\ \omega_y &= \omega_{y,s}\cos\lambda t^* + \dot{\omega}_{y,s}\frac{\sin\lambda t^*}{\lambda} + \frac{G_2}{\lambda^2}\left(1 - \cos\lambda t^*\right)\end{aligned}\right\} \tag{A27}$$

In the equations for ω_x and ω_y, the values of $\omega_{x,s}$ and $\omega_{y,s}$ have previously been determined, and from equations (A6),

$$\left.\begin{aligned}\dot{\omega}_{x,s} &= -\left[\frac{\left(I_{z,f} - I_{y,f}\right)\omega_{y,s}\omega_z + I_{yz,f}\omega_z^2}{I_{x,f}}\right]\\ \dot{\omega}_{y,s} &= \frac{\left(I_{z,f} - I_{x,f}\right)\omega_{x,s}\omega_z + I_{xz,f}\omega_z^2}{I_{y,f}}\end{aligned}\right\} \tag{A28}$$

Substitution of equations (A27) and the derivatives of equations (A27) into equations (A21) and (A22) yields two additional second-order differential equations which, when solved, are the equations for the modified Euler angles ϕ and θ after time t_s. These solutions are

$$\begin{aligned}\phi = \phi_s\cos\omega_z t^* &+ \frac{\dot{\phi}_s}{\omega_z}\sin\omega_z t^* + \frac{H_1\lambda}{\omega_z^2 - \lambda^2}\left(\cos\lambda t^* - \cos\omega_z t^*\right)\\ &+ \frac{J_1\lambda}{\omega_z^2 - \lambda^2}\left(\frac{\sin\lambda t^*}{\lambda} - \frac{\sin\omega_z t^*}{\omega_z}\right) + \frac{K_1}{\omega_z^2}\left(1 - \cos\omega_z t^*\right)\end{aligned} \tag{A29}$$

$$\begin{aligned}\theta = \theta_s\cos\omega_z t^* &+ \frac{\dot{\theta}_s}{\omega_z}\sin\omega_z t^* + \frac{H_2}{\omega_z^2 - \lambda^2}\left(\cos\lambda t^* - \cos\omega_z t^*\right)\\ &+ \frac{J_2\lambda}{\omega_z^2 - \lambda^2}\left(\frac{\sin\lambda t^*}{\lambda} - \frac{\sin\omega_z t^*}{\omega_z}\right) + \frac{K_2}{\omega_z^2}\left(1 - \cos\omega_z t^*\right)\end{aligned} \tag{A30}$$

where

$$H_1 = \dot{\omega}_{x,s} + \omega_z \omega_{y,s} - \frac{G_2 \omega_z}{\lambda^2}$$

$$J_1 = \frac{G_1}{\lambda} - \lambda \omega_{x,s} + \frac{\omega_z \dot{\omega}_{y,s}}{\lambda}$$

$$K_1 = \frac{G_2 \omega_z}{\lambda^2}$$

$$H_2 = \dot{\omega}_{y,s} - \omega_z \omega_{x,s} + \frac{G_1 \omega_z}{\lambda^2}$$

$$J_2 = \frac{G_2}{\lambda} - \lambda \omega_{y,s} - \frac{\omega_z \dot{\omega}_{x,s}}{\lambda}$$

$$K_2 = -\frac{G_1 \omega_z}{\lambda^2}$$

Again at time $t^* = 0$, $t_a = t_s$ and the values of θ_s and ϕ_s have already been determined. The values of $\dot{\phi}_s$ and $\dot{\theta}_s$ can be found from equations (A18) and (A19) where, now,

$$\left.\begin{aligned} \dot{\phi}_s &= \omega_{x,s} + \omega_z \theta_s \\ \dot{\theta}_s &= \omega_{y,s} - \omega_z \phi_s \end{aligned}\right\} \tag{A31}$$

If the same definition of relative time after time t_s is used, the third modified Euler angle ψ, from integration of equation (A20), becomes

$$\psi = \omega_z \left(t^* + t_s\right) \tag{A32}$$

REFERENCES

1. Kurzhals, Peter R.; and Keckler, Claude R.: Spin Dynamics of Manned Space Stations. NASA TR R-155, 1963.

2. Thomson, William Tyrrell: Introduction to Space Dynamics. John Wiley & Sons, Inc., c.1961.

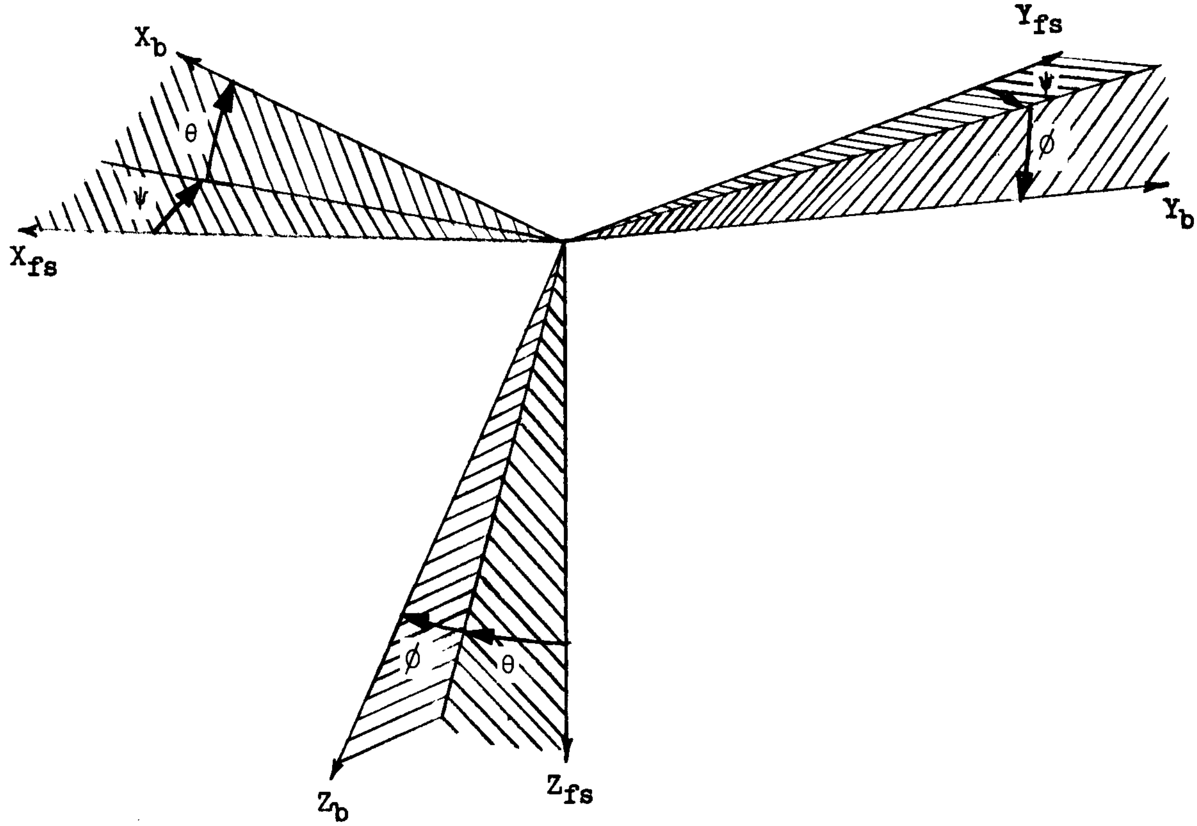


Figure 1.- Orientation of body axes with respect to fixed-space axes.

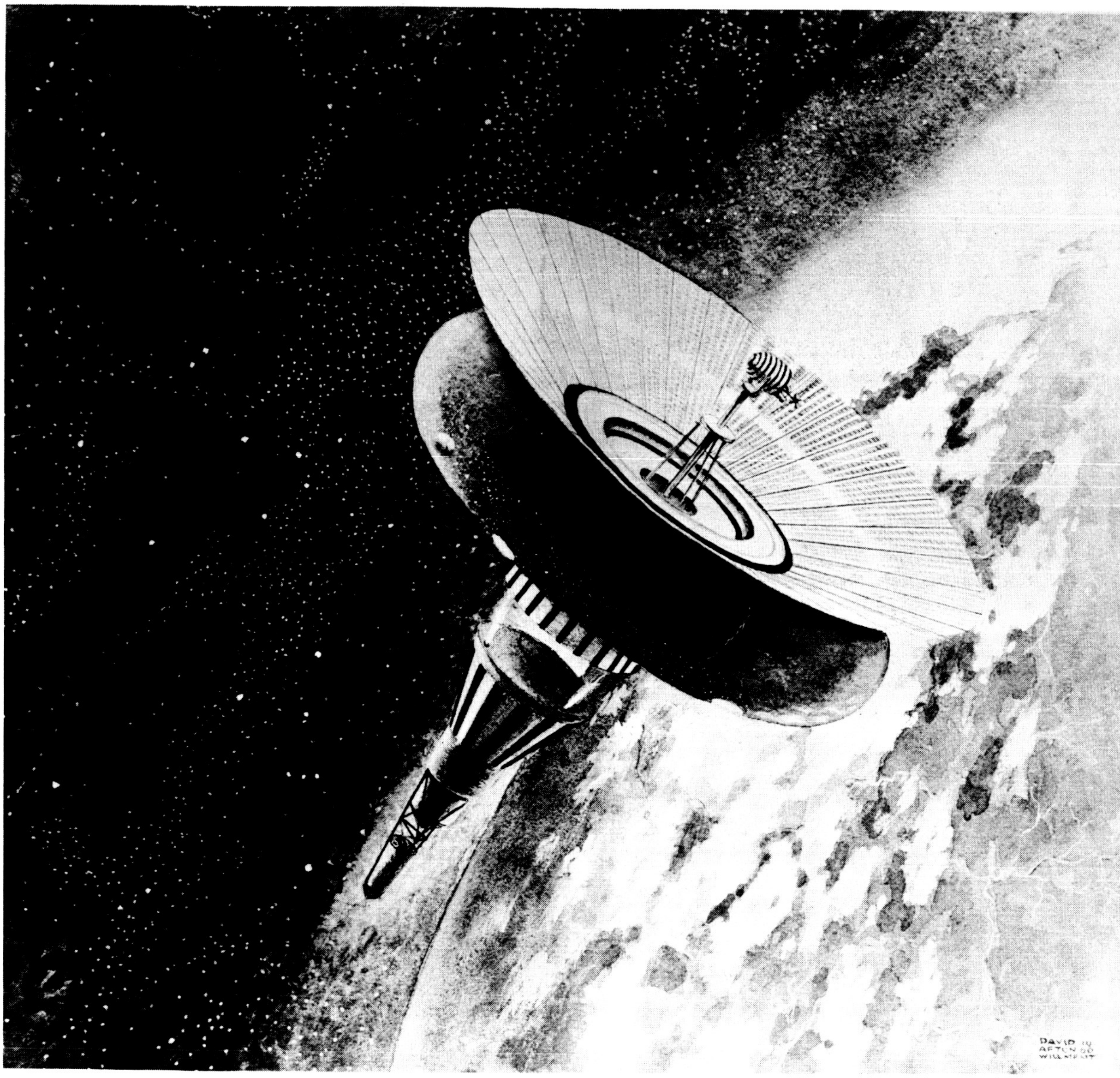
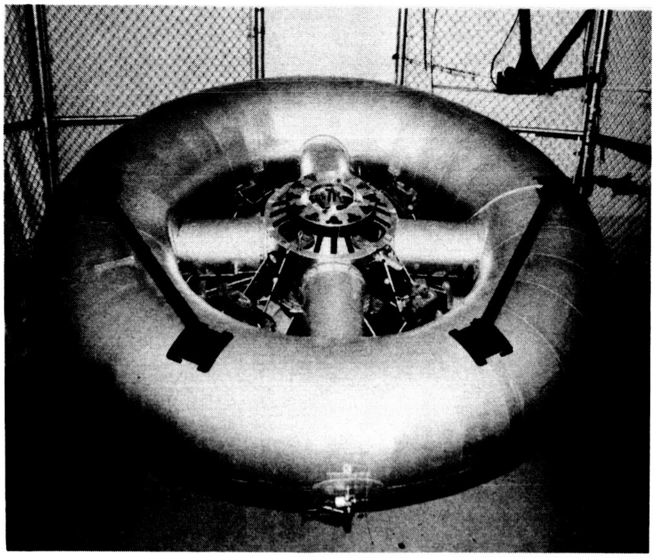
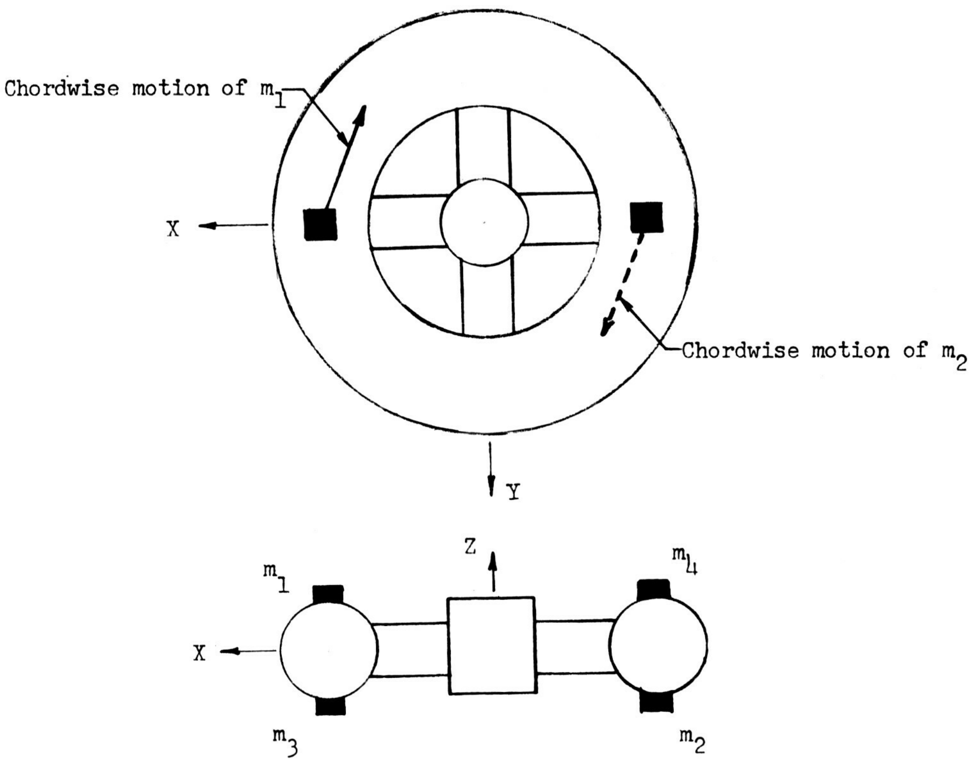


Figure 2.- Sketch of inflatable space station. L-60-7389

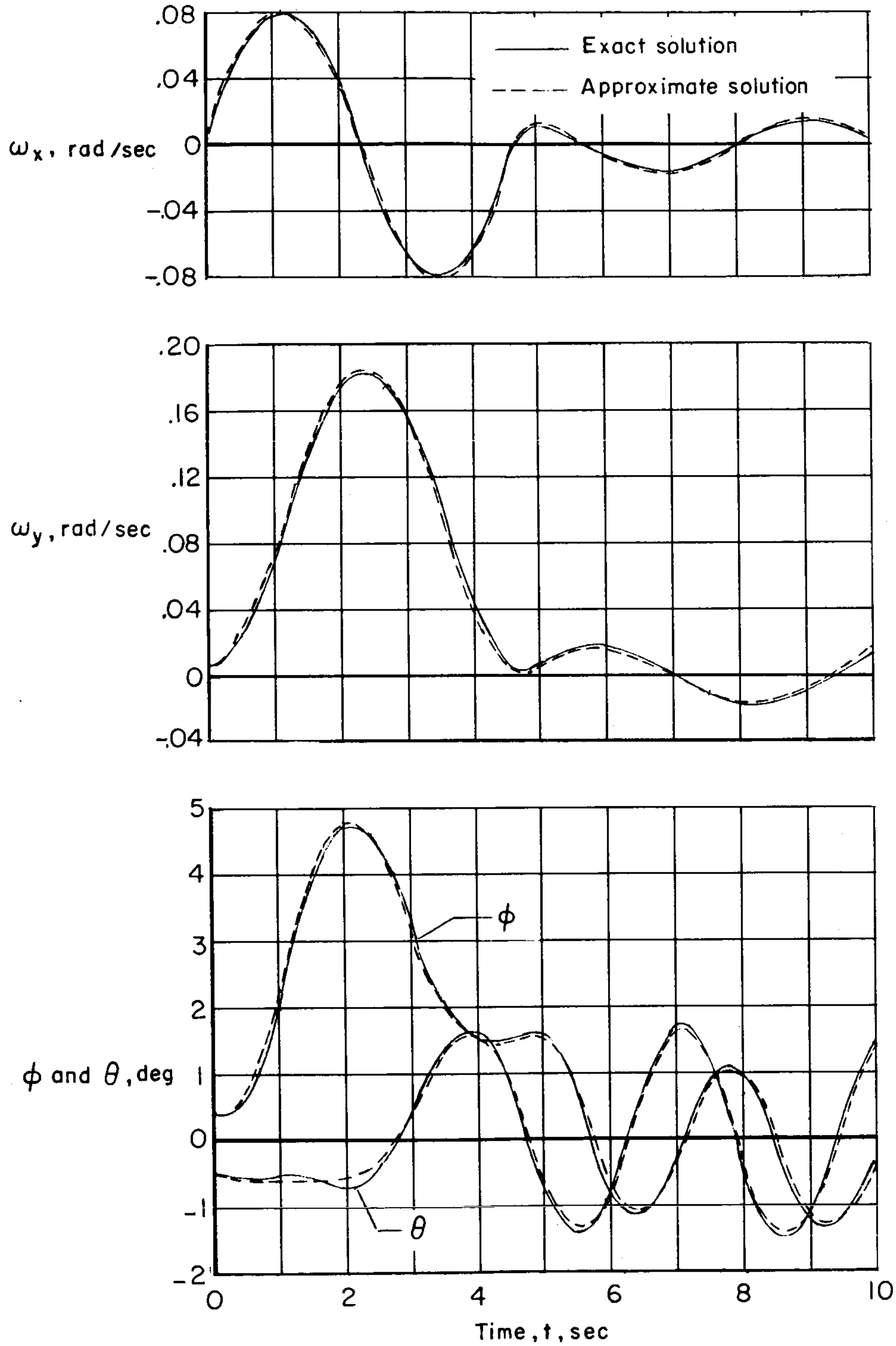


(a) Photograph of experimental model. L-65-124



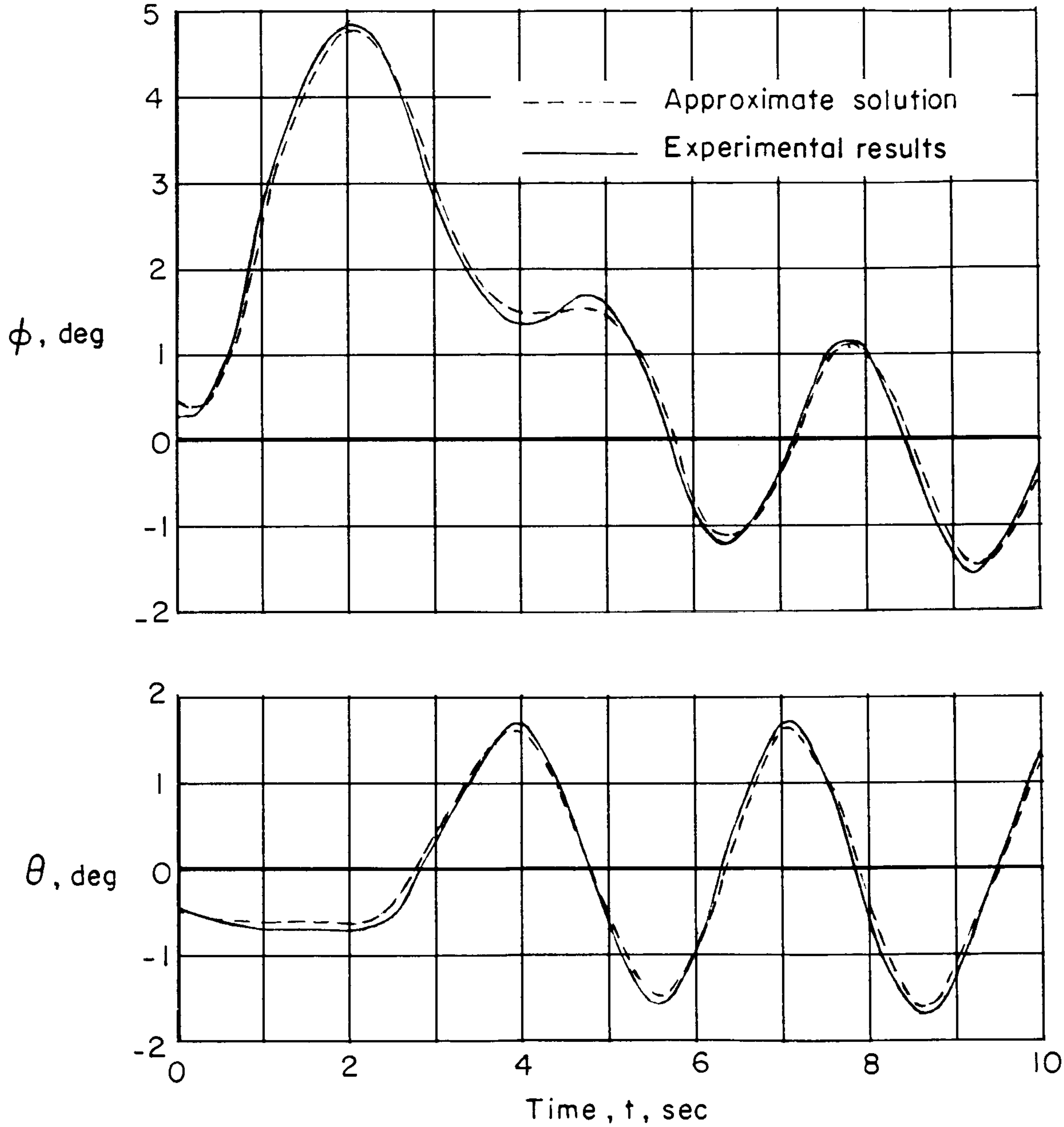
(b) Schematic drawing of model and moving masses.

Figure 3.- Characteristics of experimental space-station model.



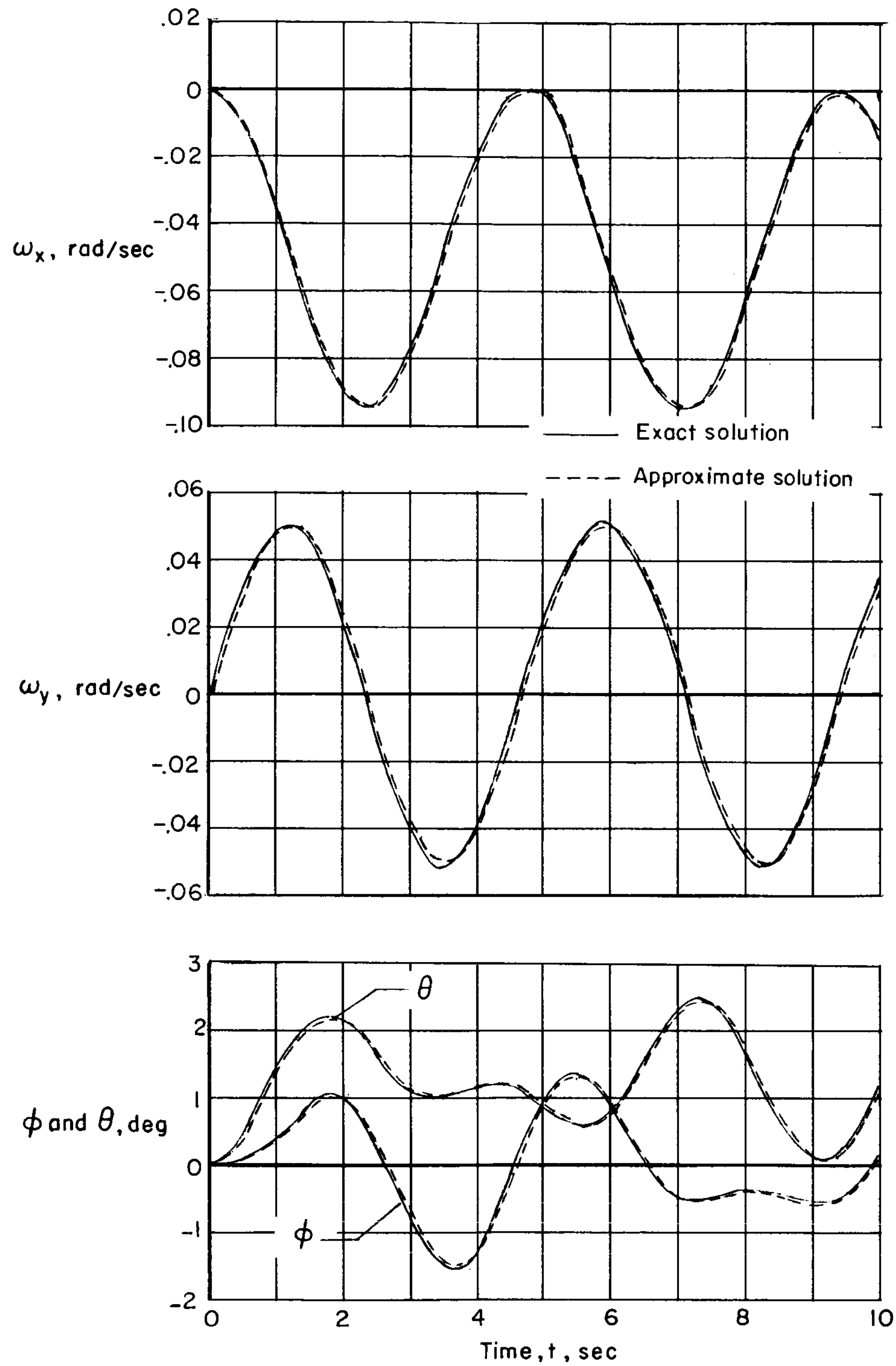
(a) Comparison of exact solution and approximate solution.

Figure 4.- Motion of example station due to an externally applied torque.



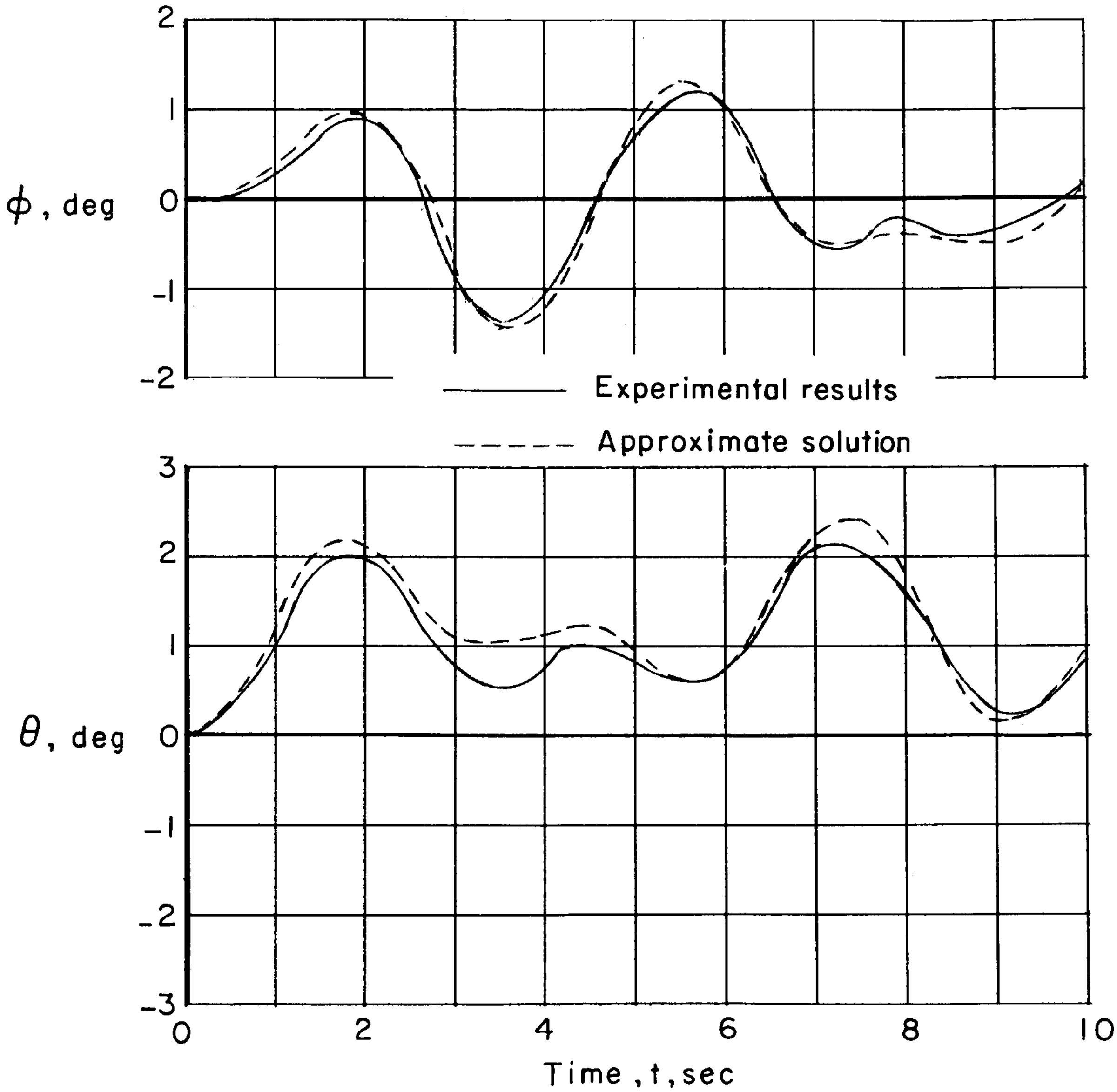
(b) Comparison of approximate solution and experimental results.

Figure 4.- Concluded.



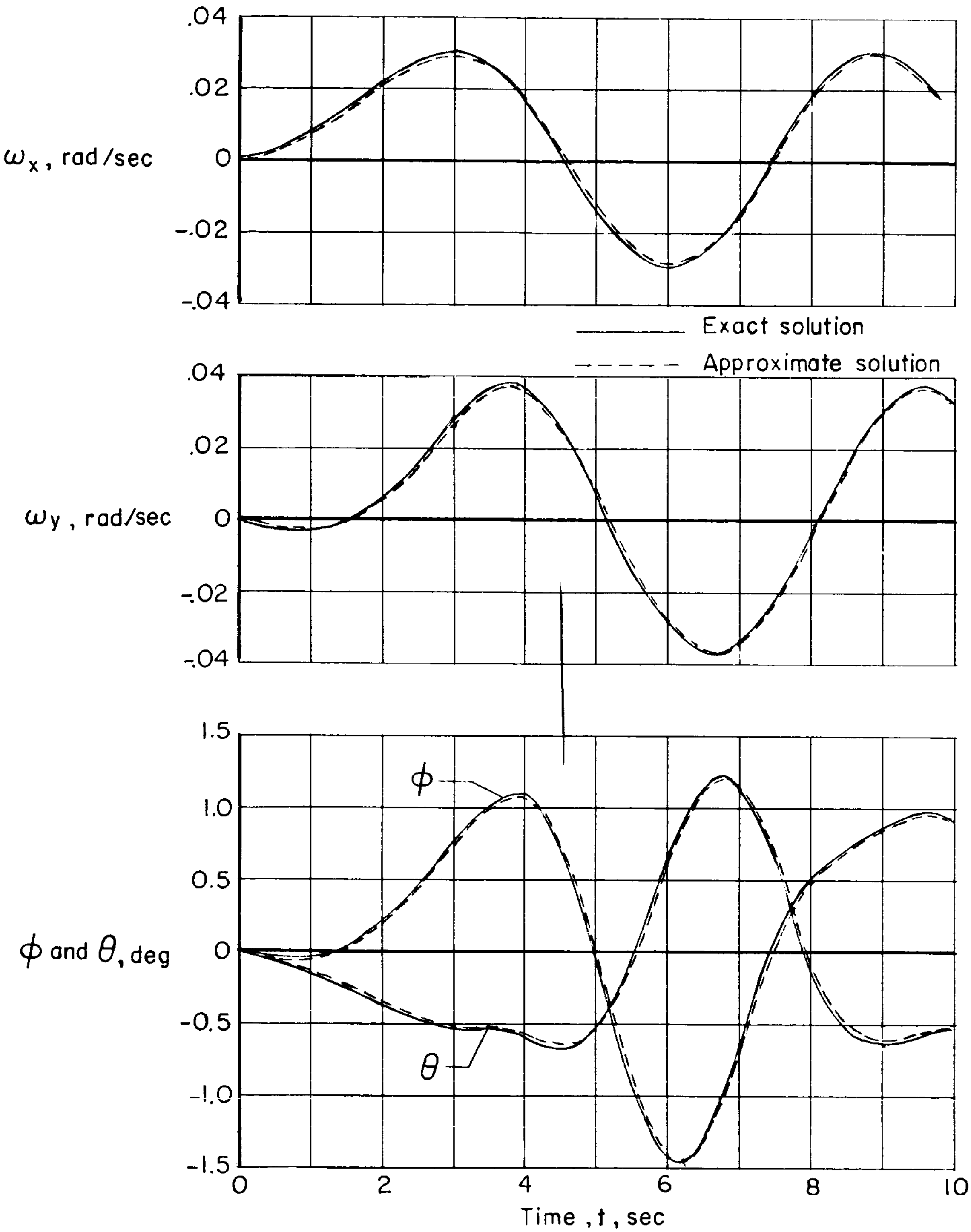
(a) Comparison of exact solution and approximate solution.

Figure 5.- Motion of example station due to a static product-of-inertia disturbance I_{xz}.



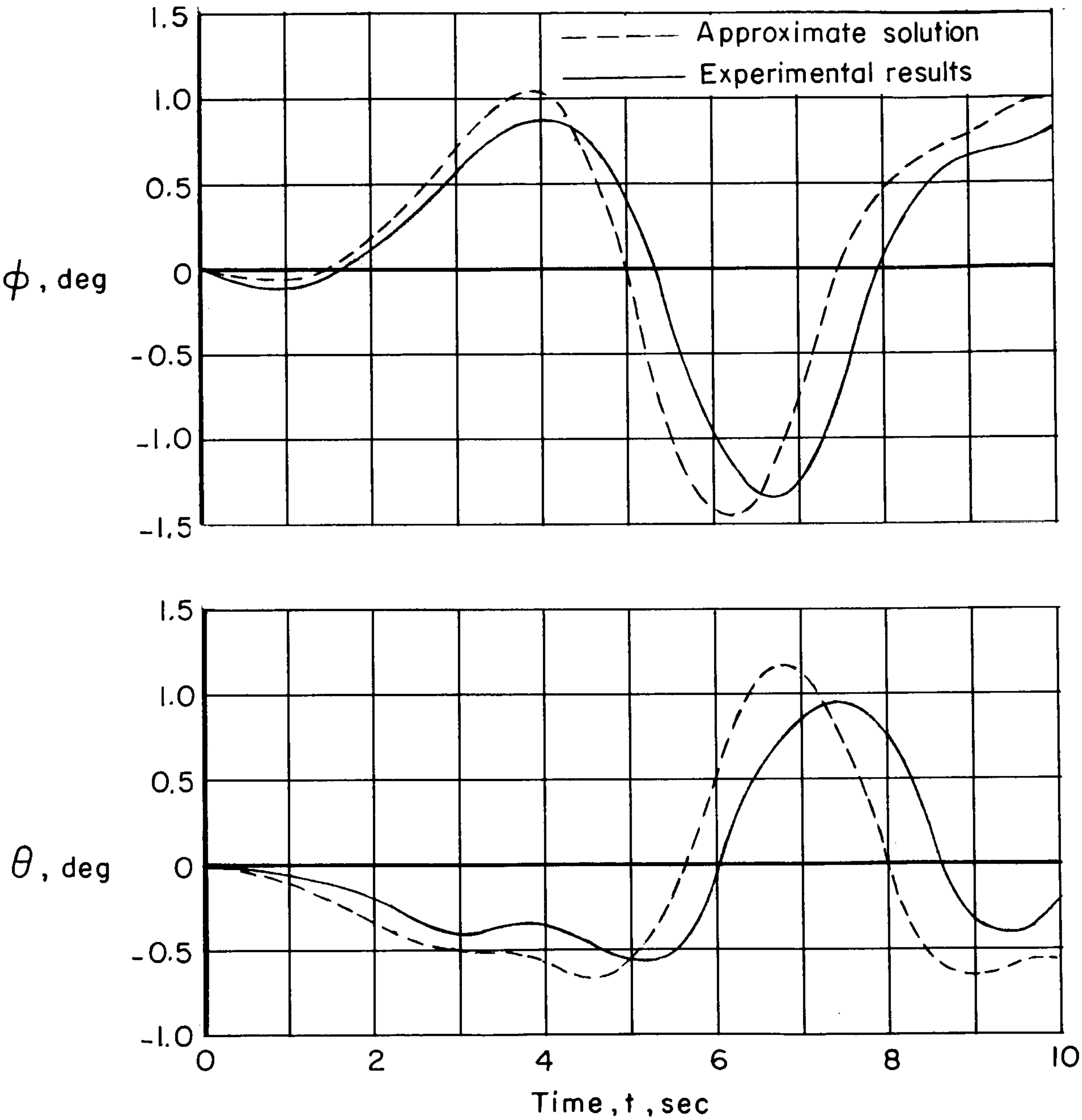
(b) Comparison of experimental results and approximate solution.

Figure 5.- Concluded.



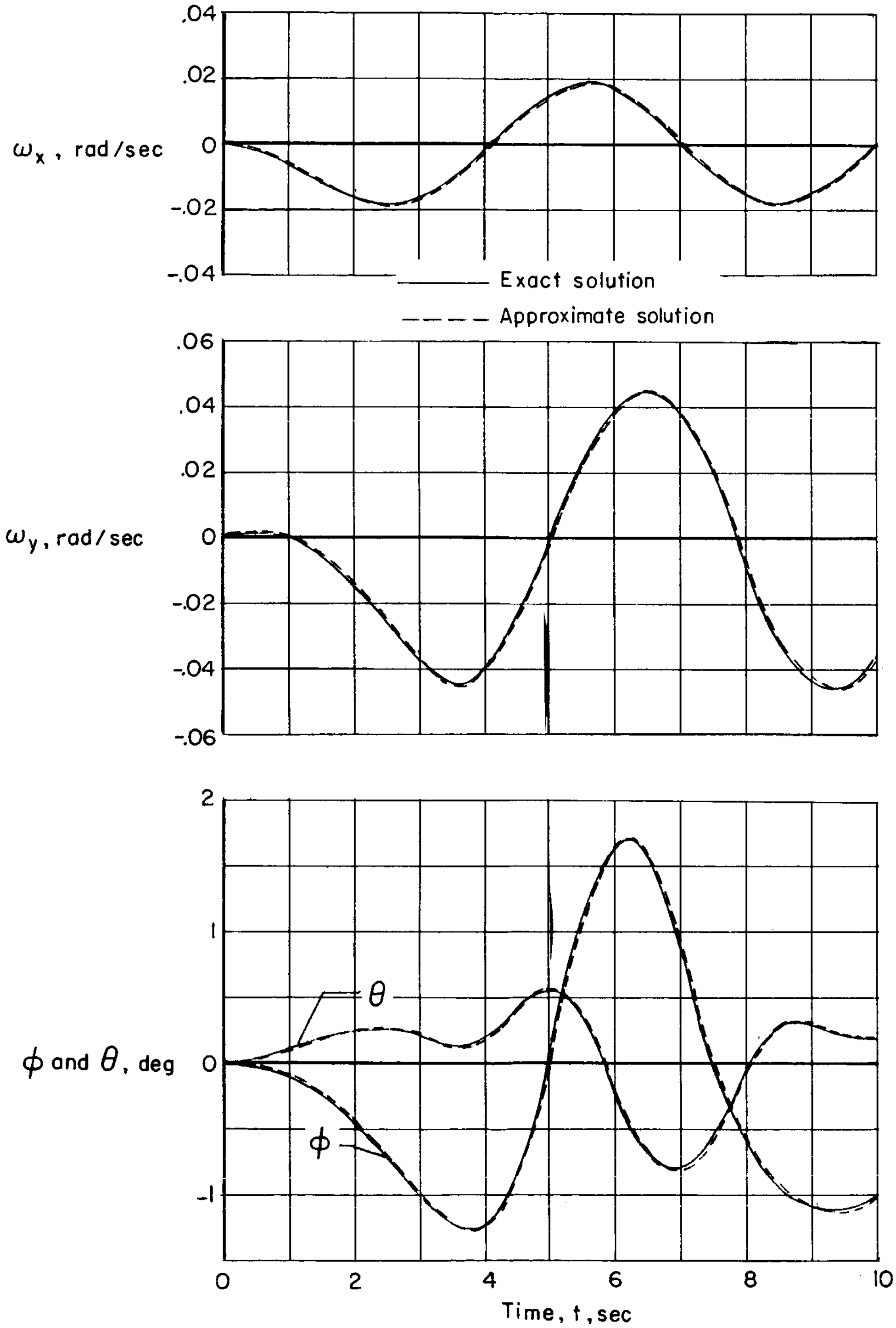
(a) Comparison of exact solution and approximate solution.

Figure 6.- Motion of example station due to a chordwise mass movement opposite to station rotation.



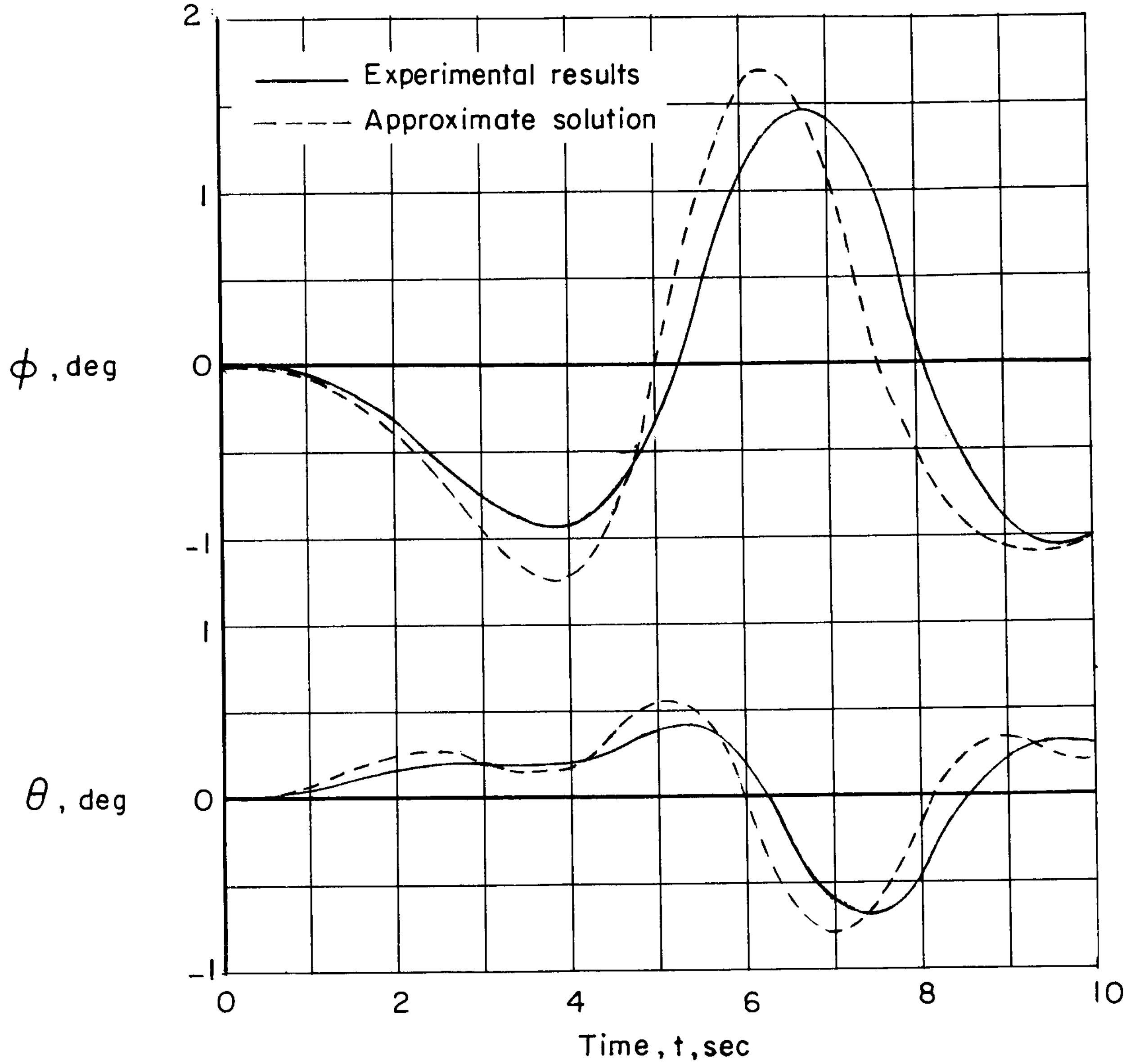
(b) Comparison of approximate solution and experimental results.

Figure 6.- Concluded.



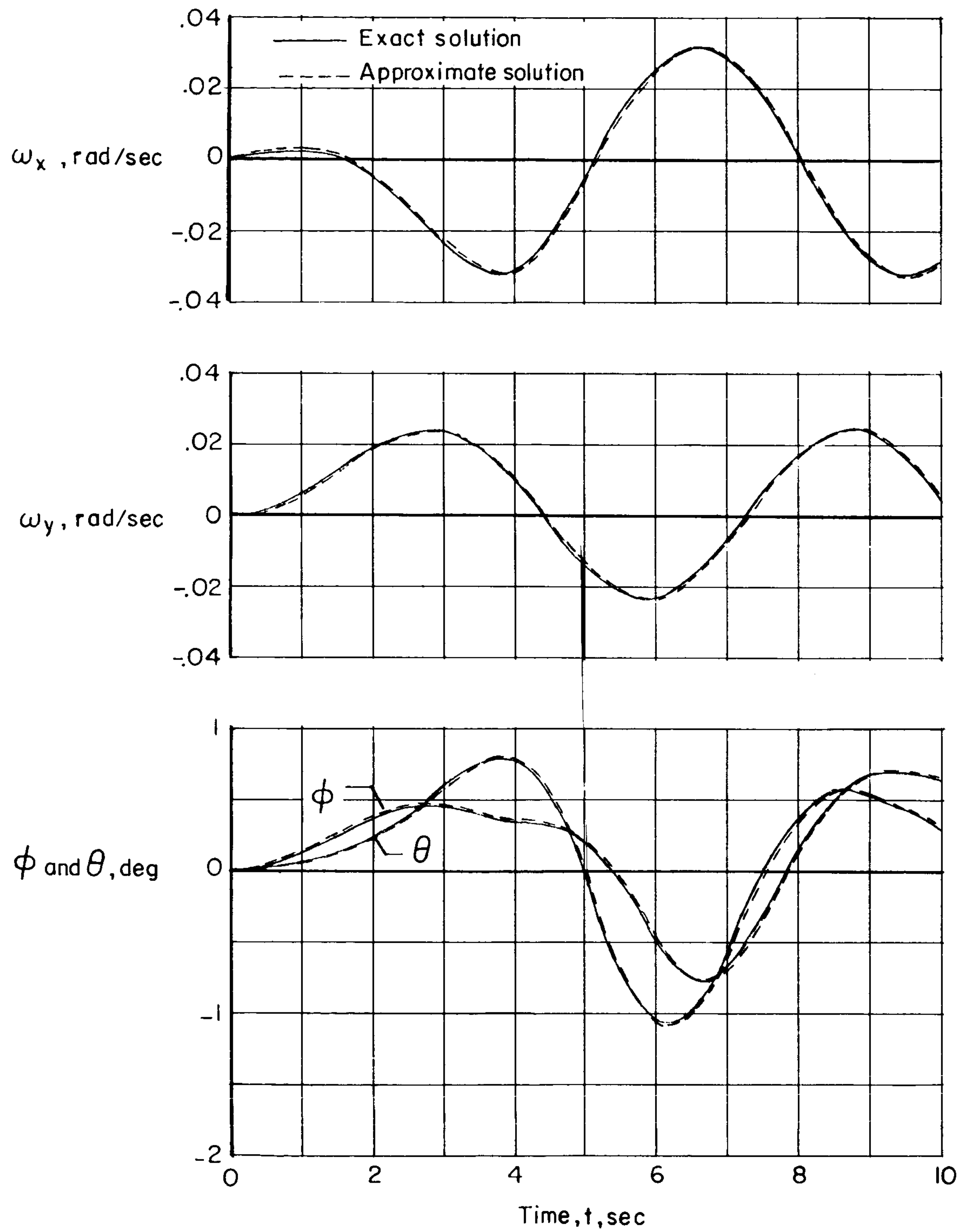
(a) Comparison of exact solution and approximate solution.

Figure 7.- Motion of example station due to a chordwise mass movement in the direction of station rotation.



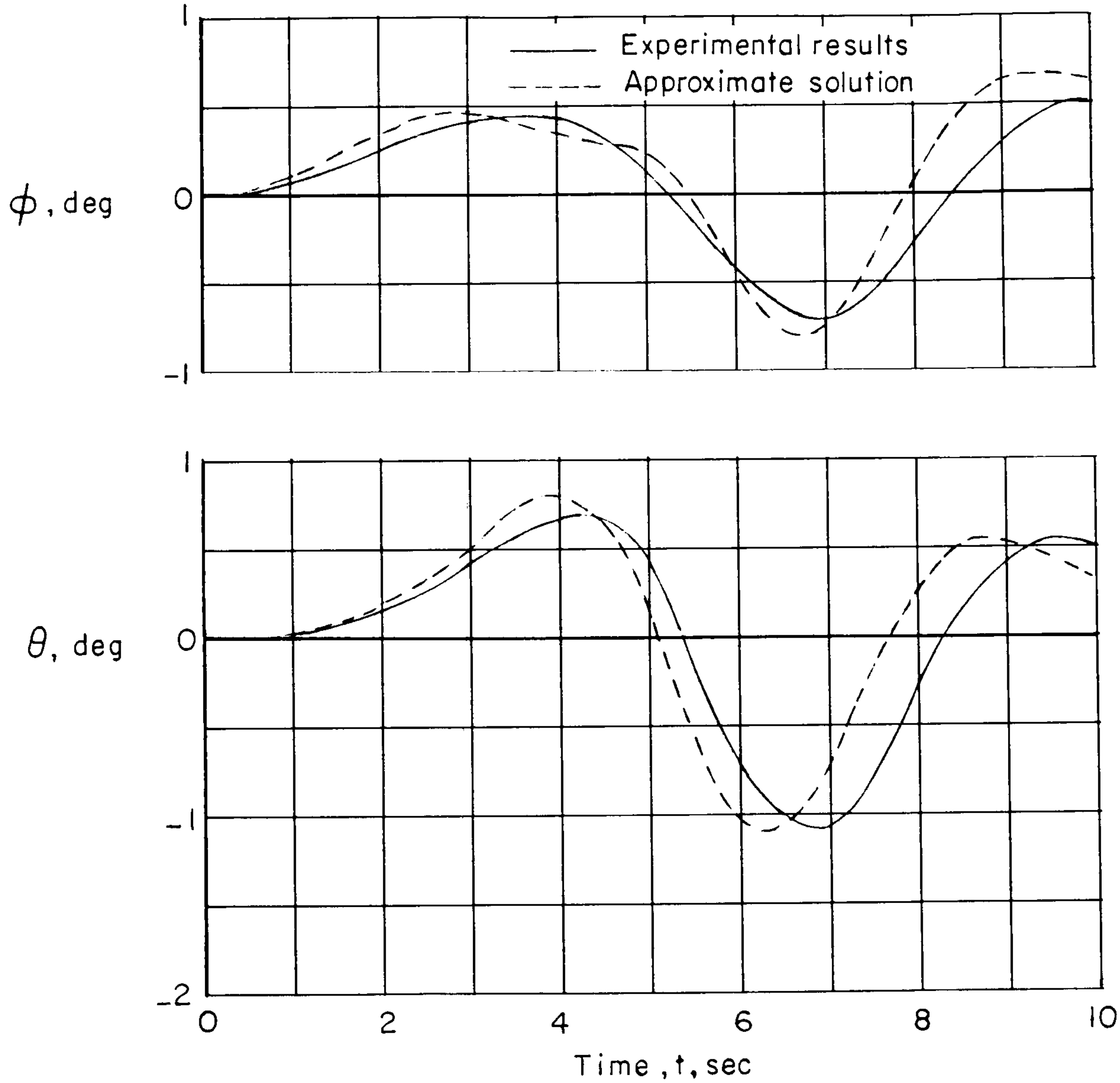
(b) Comparison of experimental results with approximate solution.

Figure 7.- Concluded.



(a) Comparison of exact solution and approximate solution.

Figure 8.- Motion of example station due to a radial mass movement.



(b) Comparison of experimental results with approximate solution.

Figure 8.- Concluded.

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—NATIONAL AERONAUTICS AND SPACE ACT OF 1958

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