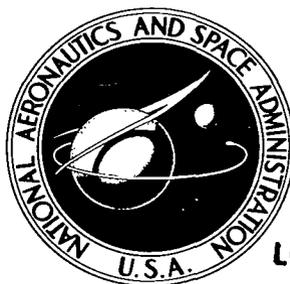


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# TWO-STREAM INSTABILITY IN GRAVITATING PLASMAS WITH MAGNETIC FIELD AND ROTATION

*by S. P. Talwar*

*Goddard Space Flight Center  
Greenbelt, Md.*





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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

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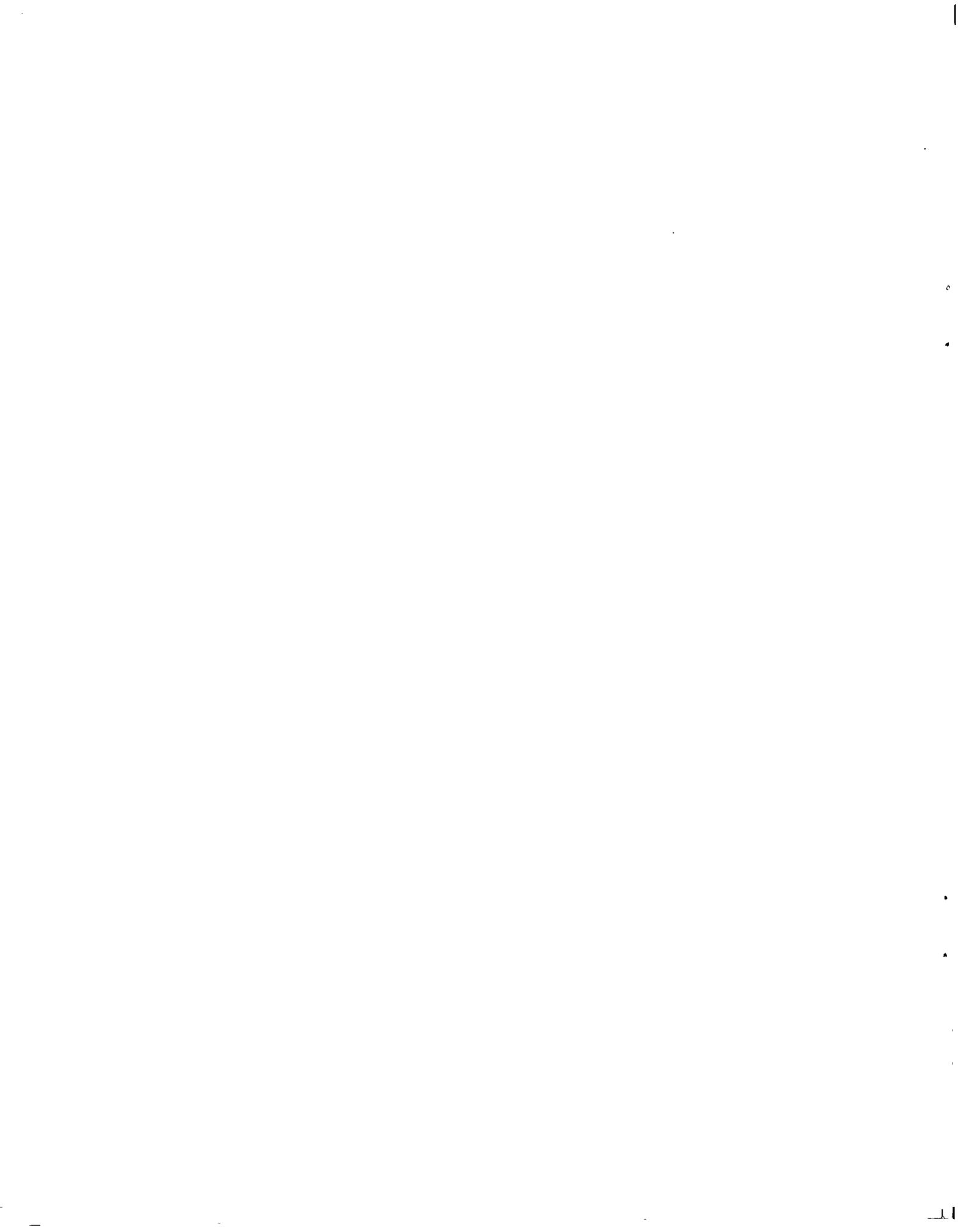
by

S. P. Talwar

*Goddard Space Flight Center*

## **SUMMARY**

The problem of instability in contraststreaming streams of plasma or self-gravitating gas clouds is investigated for general propagation direction, using moment equations. A uniform rotation is also included in view of its astrophysical importance. Conditions for instability (monotonic or growing wave) are derived. It is found that the classical Jeans wavelength for fragmentation of interstellar medium is considerably diminished due to interstreaming speeds. For a non-gravitating plasma it is concluded that perturbations propagating normal to the interstreaming direction lead to a monotonic instability. Though characterized by a small growth rate, it should be possible to observe this instability in laboratory plasmas if dimensions are suitably chosen to eliminate the conventional electrostatic two-stream instability.



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# TWO-STREAM INSTABILITY IN GRAVITATING PLASMAS WITH MAGNETIC FIELD AND ROTATION

by

S. P. Talwar\*

*Goddard Space Flight Center*

## INTRODUCTION

Contraststreaming plasmas are a common occurrence in nature, for example, in colliding galaxies, plasma streams from M-regions, and solar flares in the background solar wind. This makes the investigation of the possibility of excitation of stable or unstable oscillations due to collective interactions in contraststreaming plasmas an important question. Various investigations of the two-stream instability in collisionless plasmas, both cold and warm, have been reported (for example, Haeff, Pierce, Kahn, Parker, Buneman, and Jackson (References 1 through 6)), but they have been restricted to *electrostatic perturbations in non-gravitating plasmas*. The electrostatic instability arises due to electrostatic interactions arising from a charge separation produced by wave propagation *along* the streaming direction so that the magnetic field remains unperturbed. In general, the system is subject to a perturbation propagating at any angle to the streaming motion. It is of interest, therefore, to explore whether electromagnetic interactions due to a perturbed magnetic field can lead to an instability in contraststreaming plasmas. Again in astrophysical situations (e.g., interpenetrating star streams) the streams are self-gravitating and endowed with a large-scale galactic rotation. The two problems (namely contraststreaming instability in collisionless plasmas and stellar streams) are essentially alike except for the important difference that gravitational interactions are always attractive (as against attractive and repulsive forces in charges constituting a plasma), and that the self-gravitational field is not neutralized as in an ionized gas which is electrically neutral. It may be mentioned that cooperative phenomena in collisionless stellar streams (in the absence of rotation and magnetic field) have recently been studied by Sweet (Reference 7).

The purpose of the present paper is to present a unified treatment of two-stream instability for general perturbations for ionized streams or self-gravitating streams of unionized gas including the effect of a uniform rotation and prevailing uniform interstellar magnetic field. We shall make use of the moment equations for a warm, collisionless plasma. These equations will naturally preclude phenomena like Landau damping. The results obtained, though exact for cold configurations, would, it is hoped, represent reasonably well the situations including thermal effects.

\*On leave from the Department of Physics and Astrophysics, University of Delhi, India.

## INITIAL STATE

Consider the two unbounded, homogeneous, plasma streams interpenetrating with equal and opposite speeds  $U_0$ ,  $-U_0$ . The ions and electrons of either stream will be assumed to move together so that there is no initial electric current in either medium, and will be characterized by equal temperatures. The two streams will be supposed to be self-gravitating and subject to the simultaneous effect of a homogeneous rotation and magnetic field. In homogeneous, isothermal streams, we are required, as shown below, to take the prevailing magnetic field  $\vec{B}_0$ , the rotation vector  $\vec{\Omega}$ , and the streaming motion  $\vec{U}_0$ , all parallel to one another in order that the steady-state equations be consistently satisfied for both streams.

The initial state is governed by the following equations with respect to a rotating frame of reference,

### Beam 1

$$-\frac{1}{N_{01}} \nabla P_1^{(e)} - e \left[ \vec{E}_0 + \frac{\vec{U}_0 \times \vec{B}_0}{c} \right] + 2m_e (\vec{U}_0 \times \vec{\Omega}) + m_e \nabla \Phi_0 - m_e \vec{\Omega} \times (\vec{\Omega} \times \vec{r}) = 0 \quad (\text{electrons}) . \quad (1)$$

$$-\frac{1}{N_{01}} \nabla P_1^{(i)} + e \left[ \vec{E}_0 + \frac{\vec{U}_0 \times \vec{B}_0}{c} \right] + 2m_i (\vec{U}_0 \times \vec{\Omega}) + m_i \nabla \Phi_0 - m_i \vec{\Omega} \times (\vec{\Omega} \times \vec{r}) = 0 \quad (\text{ions}) . \quad (2)$$

### Beam 2

$$-\frac{1}{N_{02}} \nabla P_2^{(e)} - e \left[ \vec{E}_0 - \frac{\vec{U}_0 \times \vec{B}_0}{c} \right] - 2m_e (\vec{U}_0 \times \vec{\Omega}) + m_e \nabla \Phi_0 - m_e \vec{\Omega} \times (\vec{\Omega} \times \vec{r}) = 0 \quad (\text{electrons}) , \quad (3)$$

and

$$-\frac{1}{N_{02}} \nabla P_2^{(i)} + e \left[ \vec{E}_0 - \frac{\vec{U}_0 \times \vec{B}_0}{c} \right] - 2m_i (\vec{U}_0 \times \vec{\Omega}) + m_i \nabla \Phi_0 - m_i \vec{\Omega} \times (\vec{\Omega} \times \vec{r}) = 0 \quad (\text{ions}) , \quad (4)$$

$$\nabla^2 \Phi_0 = -4\pi G \sum m_j N_{0j} , \quad (5)$$

$$\nabla \cdot \vec{B}_0 = 0 , \quad \nabla \times \vec{B}_0 = 0 , \quad (6)$$

and

$$\nabla \times \vec{E}_0 = 0 , \quad \nabla \cdot \vec{E}_0 = 0 , \quad (7)$$

where the symbols have their usual meaning. For gravitating streams of un-ionized gas, Equations 6 and 7 have no meaning and we have Equation 5 together with two equations selected from Equations 1 through 4 without the electromagnetic quantities. Setting Equations 1 and 3 equal to

each other, we obtain

$$-\frac{1}{N_{01}} \nabla P_1^{(e)} + 4m_e \vec{U}_0 \times \left[ \vec{\Omega} + \frac{\vec{\Omega}_e}{2} \right] = -\frac{1}{N_{02}} \nabla P_2^{(e)} , \quad (8)$$

which must be satisfied for the initial state. Thus, for beams characterized by homogeneous pressures we must have

$$\vec{U}_0 \times \left[ \vec{\Omega} + \frac{\vec{\Omega}_e}{2} \right] = 0 \quad (9)$$

so that the three vectors  $\vec{U}_0$ ,  $\vec{\Omega}$ ,  $\vec{\Omega}_j$ , (Larmor frequency,  $e_j \vec{B}_0/m_j c$ , for  $j^{\text{th}}$  particle (electron or ion)) are parallel.

Again, for a gravitating gas stream we need to satisfy

$$\nabla \Phi_0 = \vec{\Omega} \times (\vec{\Omega} \times \vec{r}) \quad (10)$$

for each beam. This leads to the equilibrium relation

$$\Omega^2 = 2\pi G \sum m_j N_{0j} . \quad (11)$$

## PERTURBATION EQUATIONS

The moment equations defining the time-dependent perturbed state of the streaming plasma are written as,

$$m_j \left[ \frac{\partial \vec{v}_j}{\partial t} + (\vec{v}_j \cdot \nabla) \vec{v}_j \right] = -\frac{\nabla P_j}{N_j} + e_j \left[ \vec{E} + \frac{\vec{v}_j \times \vec{B}}{c} \right] + 2m_j (\vec{v}_j \times \vec{\Omega}) + m_j \nabla \Phi - m_j \vec{\Omega} \times (\vec{\Omega} \times \vec{r}) , \quad (12)$$

$$\frac{\partial N_j}{\partial t} + \nabla \cdot (N_j \vec{v}_j) = 0 , \quad (13)$$

$$\nabla^2 \Phi = -4\pi G \sum m_j N_j , \quad (14)$$

$$\frac{d}{dt} P_j = S_j^2 \frac{d}{dt} \rho_j , \quad (15)$$

$$\nabla \cdot \vec{B} = 0 , \quad (16)$$

$$\nabla \times \vec{B} = \frac{4\pi}{c} \vec{J} + \frac{1}{c} \frac{\partial}{\partial t} \vec{E} = z \frac{4\pi}{c} \sum N_j e_j \vec{v}_j + \frac{1}{c} \frac{\partial}{\partial t} \vec{E} , \quad (17)$$

$$\nabla \times \vec{E} = - \frac{1}{c} \frac{\partial}{\partial t} \vec{B} , \quad (18)$$

and

$$\nabla \cdot \vec{E} = 4\pi \sum e_j N_j . \quad (19)$$

Here

$$\vec{B} = \vec{B}_0 + \vec{b} ,$$

$$\vec{E} = \delta \vec{E} ,$$

$$\vec{v}_j = \vec{U}_{0j} + \vec{u}_j , \quad (20)$$

$$\vec{J} = \vec{J} ,$$

and

$$\Phi = \Phi_0 + \phi .$$

The equilibrium quantities are shown with a subscript '0' and the corresponding perturbations are denoted by small letters. In Equations 14, 17 and 19, summation includes ions and electrons for both the beams. The quantity  $S_j$  stands for the characteristic sound speed ( $= \sqrt{\gamma k T_j / m_j}$ ) for the  $j^{\text{th}}$  particle.

From Equations 17 and 18 we obtain

$$\nabla \nabla \cdot \delta \vec{E} - \nabla^2 \delta \vec{E} + \frac{4\pi}{c^2} \frac{\partial}{\partial t} \vec{J} + \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \delta \vec{E} = 0 \quad (21)$$

Assuming the perturbations to be of the form

$$e^{i(\vec{k} \cdot \vec{r} - \omega t)} , \quad (22)$$

we write Equation 21 as,

$$(\omega^2 - c^2 k^2) \delta \vec{E} + c^2 \vec{k} (\vec{k} \cdot \delta \vec{E}) + 4\pi i \omega \sum e_j (N_{0j} \vec{u}_j + n_j \vec{U}_{0j}) = 0 . \quad (23)$$

The expression for  $n_j$  is obtained from Equation 13 as

$$n_j = \frac{N_{0j} \vec{k} \cdot \vec{u}_j}{(\omega - \vec{k} \cdot \vec{U}_{0j})} . \quad (24)$$

The Poisson Equation (Equation 14) yields

$$\phi = \frac{4\pi G}{k^2} \sum \frac{m_j N_{0j} \vec{k} \cdot \vec{u}_j}{(\omega - \vec{k} \cdot \vec{U}_{0j})} . \quad (25)$$

The equation of motion (Equation 12) can be rewritten, using Equations 24 and 25 as

$$\begin{aligned} (-i\omega + i\vec{k} \cdot \vec{U}_{0j}) \vec{u}_j + \frac{S_j^2 i\vec{k} \vec{k} \cdot \vec{u}_j}{(\omega - \vec{k} \cdot \vec{U}_{0j})} + (\vec{\Omega}_j + 2\vec{\Omega}) \times \vec{u}_j \\ + \frac{4\pi G}{ik^2} \vec{k} \sum \frac{m_j N_{0j} \vec{k} \cdot \vec{u}_j}{(\omega - \vec{k} \cdot \vec{U}_{0j})} = \frac{e_j}{m_j} \left[ \delta \vec{E} + \frac{\vec{U}_{0j} \times \vec{b}}{c} \right] . \end{aligned} \quad (26)$$

Using Equation 18 we rewrite Equation 26 as

$$\begin{aligned} (-i\omega + i\vec{k} \cdot \vec{U}_{0j}) \vec{u}_j + \frac{S_j^2 i\vec{k} \vec{k} \cdot \vec{u}_j}{(\omega - \vec{k} \cdot \vec{U}_{0j})} + (\vec{\Omega}_j + 2\vec{\Omega}) \times \vec{u}_j \\ + \frac{4\pi G}{ik^2} \vec{k} \sum \frac{m_j N_{0j} \vec{k} \cdot \vec{u}_j}{(\omega - \vec{k} \cdot \vec{U}_{0j})} = \frac{e_j}{m_j \omega} [\omega - \vec{k} \cdot \vec{U}_{0j} + \vec{k} \cdot \vec{U}_{0j} \cdot] \delta \vec{E} . \end{aligned} \quad (27)$$

Equation 23 is rewritten as,

$$(\omega^2 - c^2 k^2) \delta \vec{E} + c^2 \vec{k} (\vec{k} \cdot \delta \vec{E}) + 4\pi i \omega \sum e_j \left( N_{0j} \vec{u}_j + N_{0j} \vec{U}_{0j} \frac{\vec{k} \cdot \vec{u}_j}{\omega - \vec{k} \cdot \vec{U}_{0j}} \right) = 0 . \quad (28)$$

Equations 27 and 28 constitute the coupled set of equations for the problem under investigation. Clearly, for gravitating interpenetrating streams the perturbation equation is Equation 27 with the right hand side set equal to zero. It may also be noted that the rotation vector occurs in the perturbation equation along with the Larmor frequency term, and we can therefore speak of an effective Larmor frequency vector,  $\vec{a} = (\vec{\Omega}_j + 2\vec{\Omega})$ , in the presence of rotation.

Let us now fix the direction of the wave number vector,  $\vec{k}$ , as the x-axis and assume the parallel vectors  $\vec{U}_{0j}$  and  $\vec{a}$  to have two components, for generality, in the x and z directions. We may now eliminate  $\delta\vec{E}$  from Equations 27 and 28 and write the final perturbation equations in component form as

$$(\omega^2 - c^2 k^2) \left[ u_{jx} - \frac{ia_z u_{jy}}{(\omega - \vec{k} \cdot \vec{U}_{0j})} \right] - \frac{4\pi e_j}{m_j} \sum N_{0j} e_j \left( u_{jx} + \frac{kU_{0jx} u_{jz}}{(\omega - \vec{k} \cdot \vec{U}_{0j})} \right) = 0, \quad (29)$$

$$(\omega^2 - c^2 k^2) \left[ u_{jy} + \frac{i(a_z u_{jx} - a_x u_{jz})}{(\omega - \vec{k} \cdot \vec{U}_{0j})} \right] - \frac{4\pi e_j}{m_j} \sum e_j N_{0j} u_{jy} = 0, \quad (30)$$

and

$$\left[ kU_{0jx} \left( u_{jx} - \frac{ia_z u_{jy}}{\omega - \vec{k} \cdot \vec{U}_{0j}} \right) - (\omega - \vec{k} \cdot \vec{U}_{0j}) u_{jz} - ia_x u_{jy} + \frac{S_j^2 k^2 u_{jz}}{\omega - \vec{k} \cdot \vec{U}_{0j}} - 4\pi G \sum \frac{m_j N_{0j} u_{jz}}{\omega - \vec{k} \cdot \vec{U}_{0j}} \right] + 4\pi \frac{e_j}{m_j} \sum N_{0j} e_j \frac{u_{jz}}{\omega - \vec{k} \cdot \vec{U}_{0j}} = 0. \quad (31)$$

Equations 29 through 31 constitute a set of twelve equations (three for electrons and three for ions) for both beams taken together. The summation is over both electrons and ions in the two beams and would thus consist of four terms. For gravitating un-ionized streams there will be, in all, six equations (three for each beam) with each summation having only two terms. The self-gravitation term occurs only in Equation 31, which on comparison with the last summation term reveals that the contribution from self-gravitation in an ionized gas is negligible compared to the contribution from the charged particles as  $\frac{\omega_p^2}{4\pi G N_0 m_{i,e}}$  independently of charged particle density, is always much greater than unity. Thus, so far as we are dealing with ionized streams, the self-gravitation effect is entirely negligible. We may, therefore, discuss the case of un-ionized gravitating gas separately from the ionized (plasma) streams.

## ELECTRON OSCILLATIONS IN PLASMA STREAMS

Having seen that the self-gravitational effects are negligible so long as the plasma frequency is not zero, we may, for simplicity, consider the case of electron oscillations only in interpenetrating

plasma streams. This approximation is reasonable in view of the large mass of the ions which can, therefore, be regarded as unperturbed unless the frequency of oscillation is small.

### Field-Free Non-Rotating Plasma Streams

For a configuration of interpenetrating plasma streams in the absence of magnetic field and rotation, it is easy to obtain the dispersion relation for electron oscillations from Equations 29 through 31. This is written as

$$\left[ -(\omega - \vec{k} \cdot \vec{U}_0)^2 + k^2 S_1^2 + \frac{\omega_{p_1}^2}{\omega^2 - c^2 k^2 - \left( \frac{\omega_{p_1}^2}{\omega_{p_1}^2} + \frac{\omega_{p_2}^2}{\omega_{p_2}^2} \right)} \right] + \omega_{p_2}^2 \left[ 1 - \frac{k^2 U_{0x}^2}{\omega^2 - c^2 k^2 - \left( \frac{\omega_{p_1}^2}{\omega_{p_1}^2} + \frac{\omega_{p_2}^2}{\omega_{p_2}^2} \right)} \right] \left[ \begin{array}{l} -(\omega - \vec{k} \cdot \vec{U}_0)^2 + k^2 S_1^2 + \frac{2k^2 U_{0x}^2 \omega_{p_1}^2}{\omega^2 - c^2 k^2 - \left( \frac{\omega_{p_1}^2}{\omega_{p_1}^2} + \frac{\omega_{p_2}^2}{\omega_{p_2}^2} \right)} \\ -(\omega + \vec{k} \cdot \vec{U}_0)^2 + k^2 S_2^2 + \frac{2k^2 U_{0x}^2 \omega_{p_2}^2}{\omega^2 - c^2 k^2 - \left( \frac{\omega_{p_1}^2}{\omega_{p_1}^2} + \frac{\omega_{p_2}^2}{\omega_{p_2}^2} \right)} \end{array} \right] = 0 \quad (32)$$

Here subscripts 1 and 2 refer to the two beams, and all quantities  $U_0$ ,  $S$ , and  $\omega_p$  refer to electrons.

For the particular case of parallel propagation ( $k \parallel U_0$ ), we put  $U_{0x} = 0$  and obtain

$$\frac{\omega_{p_1}^2}{(\omega - \vec{k} \cdot \vec{U}_0)^2 - k^2 S_1^2} + \frac{\omega_{p_2}^2}{(\omega + \vec{k} \cdot \vec{U}_0)^2 - k^2 S_2^2} = 1 \quad (33)$$

This is the well-known dispersion relation for electrostatic instability in contraststreaming field-free plasmas. For identical plasma streams ( $\omega_{p_1} = \omega_{p_2}$ ,  $S_1 = S_2$ ) the configuration is stable for all wave lengths of perturbation to the extent that the streaming velocity is less than the thermal speed. In case  $U_0 > S$ , the configuration is monotonically unstable for  $k < k_*$  where  $k_*$  is given by

$$k_*^2 = \frac{2\omega_p^2}{U_0^2 - S^2} \quad (34)$$

In the unstable range of wavelengths there exists a mode of maximum instability defined by,

$$\begin{aligned} n_m^2 &= \frac{\beta^2 - 1}{\beta^2 + 1} \omega_p^2 \left[ 1 - \frac{(\beta^2 - 1)}{2} \left\{ \frac{1}{\beta} \left( \frac{\beta + 1}{\beta - 1} \right)^{1/2} - 1 \right\} \right] \\ &= \frac{\omega_p^2}{4} \quad \text{for } S = 0 \end{aligned} \quad (35)$$

and

$$\begin{aligned}
 k_m^2 &= \frac{\omega_p^2}{2S^2} \left[ \frac{\beta^2 + 1}{\beta (\beta^2 - 1)^{1/2}} - 1 \right] \\
 &= \frac{3\omega_p^2}{4U_0^2} \quad \text{for } S = 0,
 \end{aligned} \tag{36}$$

where  $\beta = U_0/S$ . The quantities  $\alpha_m$  and  $k_m$ , respectively, denote the growth rate and the wave number of the mode of maximum instability.

To see whether instability may arise when the wave propagation vector  $\vec{k}$  is inclined to the streaming direction, let us consider the particular case of transverse propagation  $k \perp U_0$ . For this case we put  $\vec{k} \cdot \vec{U}_0 = 0$  in Equation 32 and obtain for identical interpenetrating plasma streams, the dispersion relation as,

$$\omega^4 - \omega^2 \left[ (c^2 + S^2) k^2 + 2\omega_p^2 \right] + k^2 S^2 (c^2 k^2 + 2\omega_p^2) - 2k^2 U_{0x}^2 \omega_p^2 = 0 \tag{37}$$

Equation 37 shows that the configuration of field-free plasma streams are unstable monotonically for propagation normal to the streaming if the following inequality,

$$k < \frac{2\omega_p^2 (U_0^2 - S^2)}{S^2 c^2}, \tag{38}$$

is satisfied.

Clearly, the streams characterized by  $U_0 \leq S$  are stable as was the case for  $k \parallel U_0$ . For a pressureless configuration (cold streams) it may be noted that there is instability for all wavelengths transverse to streaming motion. Thus, even those wavelengths which were stable for the wave vector along streaming are, strictly speaking, unstable when they propagate normal to the streaming direction. We conclude, therefore, that cold interpenetrating streams are, in general, unstable for all values of  $k$ . Again there also exists a mode of maximum instability in this case, defined by

$$\begin{aligned}
 \alpha_m^2 &= \frac{2\omega_p^2 (U_0^2 - S^2) - 2k^2 c^2 S^2}{S^2 + c^2} \\
 &= \frac{2\omega_p^2 U_0^2}{c^2} \quad \text{for } S = 0,
 \end{aligned} \tag{39}$$

and

$$k_m^2 = \frac{2\omega_p^2 (c^2 + 2U_0^2)}{(c^2 - S^2)^2} \left[ \left\{ 1 + \left( \frac{U_0^2}{S^2} - 1 \right) \left( 1 - \frac{S^2}{c^2} \right)^2 \left( 1 + \frac{U_0^2}{c^2} \right) \right\}^{1/2} \right. \\ \left. - 1 \right] \\ \simeq \frac{2\omega_p^2 (U_0 - S)}{c^2 S} \quad \text{for } S \rightarrow 0 . \quad (40)$$

The electromagnetic instability ( $k \perp U_0$ ) is characterized by a very small growth rate and so should be masked by the electrostatic instability when both are simultaneously present. It should, however, be possible to observe  $k \perp U_0$  instability by a proper choice of plasma dimensions, thus getting rid of the electrostatic instability.

### Plasma Streams with Field and Rotation

When the interpenetration of plasma streams takes place along the direction of external uniform magnetic field, the results are well-known for the case of parallel propagation, i.e.,  $k \parallel U_0$ . We may recapitulate by noting that the longitudinal oscillations (involving the perturbation velocity component parallel to the magnetic field) are unaffected by the presence of the magnetic field, leading to the same dispersion equation as Equation 33. In addition to a longitudinal mode, we obtain a mixed transverse mode in the presence of a uniform magnetic field, the results for which are also well-known (Reference 8).

Let us now investigate in particular the case of propagation normal to the streaming motion ( $k \perp U_0$ ) in the presence of a uniform magnetic field (and/or rotation). After some simplifications on Equations 29, 30, and 31 we obtain the dispersion relation as

$$\left( -\omega^2 + k^2 S_1^2 + \omega_{p_1}^2 \right) \left( \omega^2 - c^2 k^2 - \left( \omega_{p_1}^2 + \omega_{p_2}^2 \right) \right) + \left[ k^2 U_{0x}^2 \omega_{p_1}^2 + a_x^2 \left( \omega^2 - c^2 k^2 - \omega_{p_2}^2 \right) \right] \\ + \left[ \omega_{p_2}^2 \left( \omega^2 - c^2 k^2 - \left( \omega_{p_1}^2 + \omega_{p_2}^2 \right) \right) - \omega_{p_2}^2 \left( a_x^2 + k^2 U_{0x}^2 \right) \right] \left[ \begin{array}{c} \omega^2 - k^2 S_1^2 - a_x^2 - \omega^2 - c^2 k^2 - \left( \omega_{p_1}^2 + \omega_{p_2}^2 \right) \\ \omega^2 - k^2 S_2^2 - a_x^2 - \omega^2 - c^2 k^2 - \left( \omega_{p_1}^2 + \omega_{p_2}^2 \right) \end{array} \right] = 0 . \quad (41)$$

For identical plasma streams Equation 41 leads to

$$\omega^4 - \omega^2 \left[ k^2 (c^2 + S^2) + a_x^2 + 2\omega_p^2 \right] + (k^2 S^2 + a_x^2)(c^2 k^2 + 2\omega_p^2) - 2k^2 U_{0x}^2 \omega_p^2 = 0 . \quad (42)$$

Equation 42 shows that the overstability (growing wave instability) is absent, but that the configuration of *cold* plasmas is monotonically unstable if the wave number of transverse perturbation exceeds a certain critical value given by,

$$k_* = \frac{a_x \omega_p}{\sqrt{\omega_p^2 U_{0x}^2 - \frac{a_x^2}{2} c^2}} . \quad (43)$$

Thus contrastreaming *cold* plasmas are completely stabilized by a sufficiently strong prevailing magnetic field (or rotation) defined by,

$$a_x^* = \frac{\sqrt{2} \omega_p U_{0x}}{c} . \quad (44)$$

For warm plasma streams we conclude that there is no overstability possible and that the configuration is stable for all values of  $k$  when  $U_0 \leq S$ , or the field is stronger than that defined by,

$$a_x^* = \frac{\sqrt{2} \omega_p (U_0 - S)}{c} , \quad (45)$$

in the case where  $U_0 > S$ . The configuration shows monotonic instability only in the case where  $a_x^2 < 2\omega_p^2 (U_0 - S)^2 / c^2$  for a range of wave numbers defined by,

$$2c^2 S^2 k_{1,2}^2 = \left[ a_x^2 c^2 - 2\omega_p^2 (U_0^2 - S^2) \right] \left[ -1 \pm \left\{ 1 - \frac{8a_x^2 \omega_p^2 c^2 S^2}{\left[ a_x^2 c^2 - 2\omega_p^2 (U_0^2 - S^2) \right]^2} \right\}^{1/2} \right] . \quad (46)$$

## GRAVITATING STREAMS WITH ROTATION

For interpenetrating streams of un-ionized gas characterized by self-gravitation, Equation 27 is relevant with its right-hand side set equal to zero. The summation is over the particles (or stars in stellar streams) constituting the two streams. The dispersion relation, as obtained by simplifying

Equation 27, is finally written as,

$$\left[ (\omega - \vec{k} \cdot \vec{U}_0)^2 \left\{ 1 - \frac{a_x^2}{(\omega - \vec{k} \cdot \vec{U}_0)^2 - a_z^2} \right\} - k^2 S_1^2 \right] + 4\pi G m_1 N_{01} \left[ 1 + \frac{m_2}{m_1} \frac{N_{02}}{N_{01}} \left\{ \begin{array}{l} k^2 S_1^2 - (\omega - \vec{k} \cdot \vec{U}_0)^2 + \frac{a_x^2 (\omega - \vec{k} \cdot \vec{U}_0)^2}{(\omega - \vec{k} \cdot \vec{U}_0)^2 - a_z^2} \\ k^2 S_2^2 - (\omega + \vec{k} \cdot \vec{U}_0)^2 + \frac{a_x^2 (\omega + \vec{k} \cdot \vec{U}_0)^2}{(\omega + \vec{k} \cdot \vec{U}_0)^2 - a_z^2} \end{array} \right\} \right] = 0 \quad (47)$$

To digress, let us consider for the general dispersion equation (Equation 47), the special cases of parallel propagation ( $\vec{k} \parallel U_0 \parallel \Omega$ ) and perpendicular propagation ( $\vec{k} \perp U_0, \Omega$ ).

### Parallel Propagation ( $\vec{k} \parallel U_0$ )

In this case  $a_x = 0$ , and the dispersion relation (Equation 47) yields

$$\begin{aligned} & (\omega^2 - k^2 U_0^2)^2 - k^2 S_1^2 (\omega + kU_0)^2 - k^2 S_2^2 (\omega - kU_0)^2 + k^4 S_1^2 S_2^2 \\ & + 4\pi G m_1 N_{01} \left[ \omega^2 \left( 1 + \frac{m_2}{m_1} \frac{N_{02}}{N_{01}} \right) + 2\omega k U_0 \left( 1 - \frac{m_2}{m_1} \frac{N_{02}}{N_{01}} \right) + k^2 U_0^2 \left( 1 + \frac{m_2}{m_1} \frac{N_{02}}{N_{01}} \right) - k^2 (S_1^2 + S_2^2) \right] = 0 \end{aligned} \quad (48)$$

which is a fourth degree polynomial in  $\omega$ , the parameter deciding the question of stability of the configuration. For streams of identical stellar masses and number densities and having the same thermal velocities, Equation 48 reduces to

$$\omega^4 - 2\omega^2 \left[ k^2 (U_0^2 + S^2) - 4\pi G m N_0 \right] + k^2 (U_0^2 - S^2) \left[ k^2 (U_0^2 - S^2) + 8\pi G m N_0 \right] = 0 \quad (49)$$

This expression, independent of rotation, reduces to the well-known Jeans criterion for fragmentation of interstellar gas for the case,  $U_0 = 0$ . The analysis of Equation 49 reveals that two interpenetrating gravitating streams do not show any overstability if  $U_0 \leq S$ . In this case the configuration is monotonically unstable for

$$k^2 < \frac{8\pi G m N_0}{(S^2 - U_0^2)} \quad (50)$$

and stable for

$$k^2 > \frac{8\pi G m N_0}{(S^2 - U_0^2)} .$$

The significance of this result (Equation 50) is in the modification of the Jeans criterion for fragmentation due to the interpenetrating interstellar clouds. The classical Jeans theory ( $U_0 = 0$ ) led to a critical size so large that it is impossible for stars to be formed with masses less than about 500 times the solar mass. This prompted various workers (Reference 9) to think of some operative mechanisms which could result in star condensations of much smaller masses. In Equation 50 we find that the critical wavelength,  $\lambda^* (= 2\pi/k^*)$ , (above which there is monotonic instability) is reduced by the presence of interpenetrating speeds and goes down to zero for  $U_0 = S$ . The interstellar medium is by no means quiescent and such interpenetrating speeds are quite likely to occur. We may, therefore, surmise that the essential condition for monotonic instability ( $U_0 \leq S$ ) is likely to be satisfied in various regions of interstellar gas and would thus play a part in fragmentations leading to star formation of much smaller masses than given by the classical Jeans theory. Similar results were obtained by Sweet (Reference 7) although the question of whether instability would lead to fragmentation or only to a conversion of the initial streaming energy to disordered energy till the linearized theory breaks down is still open.

Again the streaming motion may exceed the thermal velocities ( $\sim 1$  km/sec.) for the interstellar conditions. In that case Equation 49 predicts a growing wave instability (overstability) for a range of wave numbers given by,

$$k_{1,2}^2 = \frac{2\pi G m N_0}{S^2} \left[ 1 \pm \left\{ 1 - \frac{S^2}{U_0^2} \right\}^{1/2} \right] . \quad (51)$$

If we neglect the thermal effects altogether, the corresponding conditions for instability to appear are,

$$k^2 > \frac{\pi G m N_0}{U_0^2} \quad (\text{overstability})$$

and

$$k^2 < \frac{\pi G m N_0}{U_0^2} \quad (\text{monotonic instability}) . \quad (52)$$

Thus, we may say that for cold gravitating stellar systems, the situation is unstable for all relative streaming: through overstability for high values of streaming velocities

(or wave number for a given  $U_0$ ) and through monotonic increase of amplitude for slow streaming.

### Perpendicular Propagation ( $\mathbf{k} \perp \mathbf{U}_0$ )

In this case  $\vec{k} \cdot \vec{U}_0 = 0$ , and the dispersion relation (Equation 47) gives

$$\begin{aligned} \omega^4 - \omega^2 \left[ 2a_x^2 + k^2 (S_1^2 + S_2^2) - 4\pi G (m_1 N_{01} + m_2 N_{02}) \right] + a_x^2 \left[ a_x^2 + k^2 (S_1^2 + S_2^2) - 4\pi G (m_1 N_{01} + m_2 N_{02}) \right] \\ + k^4 S_1^2 S_2^2 - 4\pi G (m_1 N_{01} S_2^2 + m_2 N_{02} S_1^2) = 0 . \end{aligned} \quad (53)$$

This equation for identical gravitating streams leads to

$$(\omega^2 - k^2 S^2 - 4\Omega^2) \left[ \omega^2 - (k^2 S^2 + 4\Omega^2 - 8\pi G m N_0) \right] = 0 . \quad (54)$$

With equilibrium rotation as defined by Equation 11, Equation 54 gives two stable modes.

### ACKNOWLEDGMENTS

This work was done while the author held a Senior Resident Research Associateship of the National Academy of Sciences—National Research Council.

(Manuscript received May 3, 1965)

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