DATA REDUCTION FOR
FILTER SPECTRORADIOMETRY

by Ralph E. Wagoner and John L. Pollack

Lewis Research Center
Cleveland, Ohio
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SUMMARY

Two methods are presented for determining spectra from filter spectroradiometer data. The first method uses only data from the filter spectroradiometer, while the second combines these data with monochromator radiometer data. The result, in either case, is a computed spectral curve that is consistent with the experimental data. To illustrate the data reduction techniques, sample spectra from a carbon arc solar simulator are given. The experiment is performed to determine the degree of agreement of measurements on the same source by the monochromator and filter instruments.

INTRODUCTION

Measurement of the spectral irradiance of solar simulator systems is a current field of interest. The solar simulator, as a part of a space environment facility, is intended to duplicate the solar spectrum. Because the spectrum is only approximated, the difference must be found and evaluated to avoid error.

In optical work, the filter method is of interest because the compactness and simplicity of the equipment allow field measurements, as opposed to laboratory measurements, to be made with relative ease. In addition, this method can provide an independent check of measurements made by a prism or grating monochromator. A monochromator is large and cumbersome, and the fact that the geometry and spectrum of the source being measured is different from the calibrated standard lamp used for comparison also causes difficulties. A summary of the difficulties encountered when the monochromator method is used may be found in reference 1. This situation motivated Wyszecki (ref. 2) to use filters for determining the spectral sensitivity of a photoelectric detector. The same method also reported by references 3 and 4 is applied to spectro-radiometry in this report.

The filter factor method of data reduction described in the open literature (e.g., ref. 5) was felt to be inadequate. The statements of the method found there are incomplete in that they do not explain how to determine the spectrum of an unknown source. The filter factor method requires that the bandwidth limits of a filter be defined arbitrarily.
The problem presented herein is confined to determining a best estimate of the actual spectrum, based on the data available, within the framework of the arbitrary choices that have to be made in selecting a reduction method. No consideration is given to determining the number of filters and the transmission characteristics required for measuring a spectrum.

Two principal methods of data reduction are presented. Each requires a large amount of computation, which necessitates the use of a computer. Neither method gives a truly unique solution; hence, any number of equally valid solutions could be found by imposing different arbitrary conditions on the solution.

The first method of data reduction is the "linear equivalent spectrum" method. It is so named because the calculated spectrum, which consists of a series of straight line segments, is equivalent to the actual spectrum in the sense that it would give exactly the same experimental results for the filter measurements as would the actual spectrum, if such an experiment could be performed. The resolution of this spectrum will be limited by the number of filters used. This method treats filter spectroradiometry as a completely independent method of measurement and utilizes no prior knowledge about the spectrum in the data reduction process. It does not, however, rule out the possibility that the choice of filters may have been made with some knowledge of the spectrum to be measured.

The second or "composite spectrum" method uses both filter measurements and high resolution spectral measurements obtained by means of a monochromator; hence, this method utilizes prior knowledge about the spectrum. The procedure is to apply a polynomial correction to the monochromator data in order to achieve a best fit of the filter data. The final spectrum has the same resolution as the monochromator data.

Spectra irradiances measurements from a carbon arc solar simulator are presented for the purpose of illustrating the data reduction methods introduced.

ANALYSIS

The required equipment for filter spectroradiometry consists of the following: (1) a set of narrow band-pass filters, each covering a different spectrum region (regions may or may not overlap); (2) a detector to receive the radiation passed by the filter; and (3) an amplifier and voltmeter to measure the electrical output of the detector.

The output voltage of the spectroradiometer with the \( i \)th filter in position \( V_i \) is given by

\[
V_i = \int_{0}^{\infty} H_\lambda(\lambda) T_i S(\lambda) d\lambda
\]  

(1)
(Symbols are defined in appendix A.) The $H_\lambda(\lambda)$ is the irradiance produced by the source at the point of measurement (may be expressed in mW/(cm$^2$)(µ)) and is a function of the wavelength $\lambda$. The $T_i$ is the transmission of the $i^{th}$ filter, and $S(\lambda)$ is the sensitivity of the detector. If the detector is a blackened thermopile, the sensitivity is assumed to be constant with wavelength. Equation (1) also assumes the detector output to be linear with flux level. If these are not the case, the experimental voltages must be corrected to give values that are proportional to spectral sensitivity and flux level in order to apply the theory that follows.

The data reduction problem is that the $V_i$ are known but that the $H_\lambda(\lambda)$ is not. The method of this analysis is to construct an assumed irradiance function $H_\lambda, a(\lambda)$ such that when substituted for $H_\lambda(\lambda)$ in equation (1) an output voltage $V_{a, \lambda}$ results which agrees with the $V_i$ value obtained experimentally. The $H_\lambda, a(\lambda)$ must satisfy all $nV_i$ values simultaneously, where $n$ is the number of filters. This report presents two general methods of accomplishing this. The first, an independent method, utilizes only the data from the filter spectroradiometer. This method will result in a $H_\lambda, a(\lambda)$ curve that will, in general, lack the detailed structure of the actual $H(\lambda)$ curve. The second, a dependent method, combines filter and monochromator data. The filters provide a polynomial correction term for the monochromator result. The $H_\lambda, a(\lambda)$ curve then has the same resolution as the monochromator curve.

**Linear Equivalent Spectrum Method**

The assumed spectrum $H_\lambda, a(\lambda)$ consists of $n$ points, one for each filter. Each point is defined by the pair of values $H_\lambda, a(\lambda)$ and $\lambda_i$. The wavelength value $\lambda_i$ chosen for each point is the center of gravity of the corresponding filter's transmission curve. It is defined by

$$\lambda_i = \frac{\int_0^\infty \lambda T_i(\lambda)d\lambda}{\int_0^\infty T_i(\lambda)d\lambda} \quad (2)$$

Each irradiance value $H_\lambda, a, i$ is taken to be unity as a starting assumption, although any arbitrary set of values, each greater than zero, would serve as well. The spectrum $H_\lambda, a(\lambda)$ is thus defined by the series of straight line segments connecting the $n$ assumed points (and extending beyond them in the case of the first and last points). The spectrum $H_\lambda, a(\lambda)$ thus consists of $n - 1$ regions. In each region it is defined by a straight line:
\[ H_{\lambda, a} = \frac{H_{\lambda, a, i+1} - H_{\lambda, a, i}}{\lambda_{i+1} - \lambda_i} + H_{\lambda, a, i} \]  \hspace{1cm} (3)

In the event equation (3) would require a negative value, \( H_{\lambda, a}(\lambda) \) is taken to be zero. A region, of course, extends from the center of gravity of one filter's transmission curve to the center of gravity of the next, except for the first and last regions, which extend indefinitely beyond the first and last filters.

The iteration scheme for the solution \( H_{\lambda, a}(\lambda) \) is now given. The starting assumption is

\[ H_{\lambda, a, i} = 1 \]  \hspace{1cm} (4)

for each \( n \). This expression defines the trial spectrum \( H_{\lambda, a}(\lambda) \) everywhere in accordance with equation (3). The \( n \) values, each a calculated voltage from the assumed irradiance, can be found by solving

\[ V_{a, i} = \int_0^\infty H_{\lambda, a}(\lambda) T_1(\lambda) S(\lambda) d\lambda \]  \hspace{1cm} (5)

The new value assumed by each \( H_{\lambda, a, i} \) is

\[ \left( H_{\lambda, a, i}\right)_{\text{new}} = \frac{V_i}{V_{a, i}} \left( H_{\lambda, a, i}\right)_{\text{old}} \]  \hspace{1cm} (6)

The process is repeated from equation (5) on until

\[ \left| \frac{V_i}{V_{a, i}} - 1 \right| \leq 0.0001 \]  \hspace{1cm} (7)

for each and every \( n \). A limit of 100 iterations was permitted. In most cases convergence is obtained within this arbitrary limit. When convergence is completed, the assumed irradiance function \( H_{\lambda, a}(\lambda) \) is the solution to the problem, and when it is used in place of \( H_{\lambda}(\lambda) \) in equation (1) it is equivalent in the sense that it gives the same \( V_1 \) values as were obtained experimentally.

Once a measurement is performed and the data reduced, the \( n \) ratios can be found:
On any subsequent measurement of a similar radiation source, an approximate solution

\[ R_i = \frac{H_{\lambda, a, i}}{V_i} \]  

(8)

can quickly be obtained.

An example of the spectrum produced by this method is the solid line curve of figure 1. This result was obtained on a carbon arc solar simulator. The instrument used was the Eppley Laboratory's Mark IV Radiometer equipped with a set of 12 filters, which the manufacturer chose for carbon arc spectra and designated as "C" filters.

Applications of Linear Equivalent Spectrum Method

It is now possible to compare the linear equivalent spectrum to a higher resolution spectrum such as might be obtained on a monochromator. The comparison can be made by converting the monochromator result to what the filter method would produce. The \( V_i \) is calculated according to equation (1) where the \( H_{\lambda}(\lambda) \) is the actual high resolution monochromator spectrum. The calculation proceeds as though the \( V_i \) had been obtained

\[ H_{\lambda, a, i} = R_i V_i \]  

(9)

Figure 1. - Spectroradiometric measurements of carbon arc solar simulator.
experimentally with a filter radiometer, and the equivalent spectrum is derived by equations (4) to (7). This equivalent filter spectrum would have been obtained by measuring an actual source with the spectrum indicated by the monochromator.

An an example of the conversion of a high resolution spectrum by use of the equivalent spectrum method form, figure 2 shows a grating monochromator measurement of the same carbon arc solar simulator. The measurement was made in the same location as the filter measurement. The bandwidth of the instrument was as low as 0.0008 micron in the 0.31 to 0.7 micron range. The data were taken at 0.01-micron intervals. The transmission curves of the same set of 12 filters of the Eppley Laboratory's Mark IV radiometer were used with this spectrum.

The equivalent spectrum values at the appropriate center wavelengths are plotted in figure 2. Linear interpolation between these values generates the equivalent spectrum. Both the high resolution spectrum and its equivalent spectrum yield the same detector voltages if calculated by equation (1). Since the same set of filters were used to calculate the equivalent spectrum of the monochromator measurement and for the actual filter measurements, direct comparison of the two instruments is possible.

The equivalent spectrum of the monochromator is plotted as a dashed line in figure 1 for comparison to the filter measurement. The difference between these two curves indicates possible accuracy of the instruments and can give a better estimate of the true spectrum.
A second attractive application of the equivalent spectrum method is to determine the difference between the zero air mass spectrum of solar irradiance (Johnson spectrum, ref. 6) and the spectrum of a solar simulator. The equivalent spectrum of the Johnson spectrum was calculated as previously described for the monochromator data. It is plotted in figure 3 and compared to the filter radiometer equivalent spectrum of the carbon arc simulator shown in figure 1 (p. 5). The difference of the curves is the deviation of the simulator from the ideal spectrum. The equivalent of the Johnson curve was plotted with its peak at 100. The area of all curves illustrated in this report is equal to the area under the Johnson curve between 0.25 and 2.2 microns.

Composite Spectrum Method

The linear equivalent spectrum method made use of no prior knowledge about the spectrum. A composite reduction method has also been developed that utilizes the strong points of both monochromator and filter radiometer measurements. (Differences between the monochromator and filter radiometer results are shown in fig. 1.) The monochromator is quite reliable for giving the relative intensities of two spectral regions close together in wavelength. The characteristics of the instrument vary, however, as the wavelength is changed so that there is less confidence in the relative intensities as the wavelength becomes more separated. The filter radiometer because of its simplicity is less prone to this type of error. Thus it appears attractive to use the monochromator for its
high resolution content and supplement it with filter measurements. A method generally applicable is to seek a solution of the form

$$H_{\lambda, a}(\lambda) = H_{\lambda, m}(\lambda) \sum_{j=0}^{k-1} P_j \lambda^j$$

(10)

where \( k \leq n \) and \( H_{\lambda, m}(\lambda) \) is a high resolution spectral curve resulting from a monochromator measurement. The set of \( P_j \) values is then found that gives the best agreement between the calculated \( V_{a, i} \) from equation (5) and the experimental \( V_1 \) indicated in equation (1). This method has the effect of using the monochromator to define the spectrum over the narrow range and the filters to define broad spectrum regions with respect to one another. The filter measurements determine the polynomial correction term to be applied to the monochromator data.

It is possible to find the solution of the form indicated in equation (10) by an analytical least squares method, which requires no iterative process. This is shown in appendix B. As a special case of this procedure, if \( H_{\lambda, m} = 1 \), a polynomial solution for \( H_{\lambda, a} \) can be obtained that makes no use of prior knowledge of the spectrum.

The monochromator and filter measurements of figures 1 and 2 were used as an example of the composite method. A polynomial of second order \((P_0 + P_1 \lambda + P_2 \lambda^2)\) was used in equation (10). The value of \( \epsilon \) (appendix B) was chosen as 1 percent of the maximum \( V_1 \) obtained experimentally. The calculation yielded a composite spectrum of high

![Figure 4. Filter measurement and filter equivalent of composite spectrum.](image-url)
TABLE I. - COMPARISON OF MONOCHROMATOR AND FILTER MEASUREMENTS

<table>
<thead>
<tr>
<th>Center wavelength of filter</th>
<th>Deviation of composite spectrum using second-degree polynomial, percent (fig. 4)</th>
<th>Deviation of uncorrected monochromator spectrum, percent (fig. 1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.264</td>
<td>-2.8</td>
<td>-1.7</td>
</tr>
<tr>
<td>.320</td>
<td>10.9</td>
<td>12.7</td>
</tr>
<tr>
<td>.355</td>
<td>-10.7</td>
<td>-9.0</td>
</tr>
<tr>
<td>.406</td>
<td>-.3</td>
<td>1.9</td>
</tr>
<tr>
<td>.459</td>
<td>-1.8</td>
<td>.7</td>
</tr>
<tr>
<td>.548</td>
<td>3.9</td>
<td>6.7</td>
</tr>
<tr>
<td>.643</td>
<td>3.8</td>
<td>6.7</td>
</tr>
<tr>
<td>.753</td>
<td>-6.3</td>
<td>-3.9</td>
</tr>
<tr>
<td>.880</td>
<td>-6.0</td>
<td>-4.3</td>
</tr>
<tr>
<td>1.167</td>
<td>5.4</td>
<td>4.6</td>
</tr>
<tr>
<td>1.543</td>
<td>.7</td>
<td>-6.0</td>
</tr>
<tr>
<td>2.082</td>
<td>-1.2</td>
<td>-16.4</td>
</tr>
<tr>
<td>Root-mean-square deviation</td>
<td>5.6</td>
<td>7.6</td>
</tr>
</tbody>
</table>

The biggest problem in using this composite method is obtaining a good monochromator curve to use with the filter data. The two sets of data should be obtained under as nearly identical conditions as possible. It is not proper, for example, to take a monochromator curve on a bare carbon arc and combine this with filter data taken on a solar simulator. The spectrum is modified in passing through the optical system of the simulator, but even more important is the fact that the simulator may utilize only a portion of the source or perhaps a different portion of the source than is seen by the monochromator in making its measurement on the bare arc.

CONCLUSIONS

The first data reduction method - the linear equivalent spectrum method - yields a spectrum that uses only voltage outputs of a detector and the transmission characteristics of the filters. No prior knowledge of the source is required. The method is mathematically consistent in the sense that the computed spectrum yields the same radiometer resolution as in figure 2, but the correction applied by the filter data caused a low resolution slowly varying deviation. In order to compare the results of the composite method with the filter radiometer alone, the composite spectrum was converted by use of the equivalent spectrum method previously applied in figure 2.

The equivalent spectrum by the composite method is plotted as a dashed line in figure 4 for comparison to the equivalent spectrum of the filter radiometer alone. The deviations, at the center wavelength points, between the monochromator measured curves of figures 4 and 1 and the filter radiometer measured spectrum are tabulated in table I. The difference of the deviations in the two columns when plotted against wavelength form a second-degree polynomial. The root-mean-square deviation of the composite spectrum has been reduced from 7.6 to 5.6 percent. The correction is only partial with the second-degree polynomial.
voltage readings as were obtained by measuring the incident radiation. This condition is necessary but not sufficient to obtain the best estimate of the actual spectrum with a given set of filters. Two other necessary conditions not investigated in this report are being investigated at Lewis Research Center. One involves the optimum method of interpolation. The other is the choice of filter shape and bandwidth which affect the uniqueness of the spectrum in the sense that a given set of filters if displaced equally in wavelength will produce a different spectrum.

It was shown how to compare high resolution spectra of a monochromator with the low resolution of a filter radiometer. The comparison was applied to determine the deviation of measurements of the two instruments from each other. It also enabled a better estimate of the spectrum of a source, and led to the composite method of superimposing the high resolution of a monochromator spectrum on the filter radiometer spectrum. The accuracy of the filter radiometer and the resolution of the monochromator were retained during this comparison.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, July 22, 1965.
APPENDIX A

SYMBOLS

\( G \) error term

\( H_\lambda \) irradiance (spectral)

\( K_{j, i} \) \( \int_0^\infty H_\lambda, m T_i S^j \, d\lambda \)

\( P_j \) coefficient in monochromator correction polynomial

\( R_i \) ratio of assumed irradiance to filter detector voltage

\( S \) detector sensitivity

\( T_i \) filter transmission

\( V \) detector output voltage

\( X_i \) \( \frac{V_i - V_{a, i}}{V_i + \epsilon} \)

\( \epsilon \) number

\( \lambda \) wavelength

\( \text{Subscripts:} \)

\( a \) assumed

\( i \) integer referring to particular filter

\( j \) integer referring to term in monochromator correction polynomial

\( k \) number of terms in monochromator correction polynomial

\( m \) monochromator

\( n \) number of filters
APPENDIX B

ANALYTICAL LEAST SQUARES FIT OF FILTER DATA

It is desired to find a solution of the form

\[ H_{\lambda, a} (\lambda) = H_{\lambda, m}(\lambda) \sum_{j=0}^{k-1} P_j \lambda^j \]  

(10)

where \( k \leq n \). The \( H_{\lambda, a} \) must agree with the data from the \( n \) filters; that is, the

\[ V_{a, i} = \int_{0}^{\infty} H_{\lambda, a}(\lambda) T_i(\lambda) S(\lambda) d\lambda \]  

(5)

must agree with the \( V_i \) measured experimentally:

\[
\begin{align*}
V_{a, i} &= \int_{0}^{\infty} H_{\lambda, a}(\lambda) T_i(\lambda) S(\lambda) d\lambda = \int_{0}^{\infty} H_{\lambda, m} \left( P_0 + P_1 \lambda + \cdots + P_{k-1} \lambda^{k-1} \right) T_i S d\lambda \\
&= \int_{0}^{\infty} H_{\lambda, m} P_0 T_i S d\lambda + \int_{0}^{\infty} H_{\lambda, m} P_1 \lambda T_i S d\lambda + \cdots + \int_{0}^{\infty} H_{\lambda, m} P_{k-1} \lambda^{k-1} T_i S d\lambda \\
&= \sum_{j=0}^{k-1} \int_{0}^{\infty} H_{\lambda, m} P_j \lambda^j T_i S d\lambda \\
&= \sum_{j=0}^{k-1} V_{j, i}
\end{align*}
\]

(B1)

where in general

\[ K_{j, i} = \int_{0}^{\infty} H_{\lambda, m}(\lambda) T_i(\lambda) S(\lambda) \lambda^j d\lambda \]

Then,

\[ V_{a, i} = \sum_{j=0}^{k-1} P_j K_{j, i} \]
with the coefficients $K_0, i, K_1, i \ldots K_{k-1}, i$ evaluated once by numerical integration. Let

$$X_i = \frac{V_i - V_{a, i}}{V_i + \epsilon} \quad (B2)$$

and define

$$G = \sum_{i=1}^{n} X_i^2 \quad (B3)$$

If $\epsilon = 0$, $X_i$ is the relative error of $V_{a, i}$ compared to $V_i$. Some small value for $\epsilon$ is necessary in the event one of the $V_i$ is zero. The $G$ is the sum of the squares of all the individual errors.

The smaller $G$ is, the better the $H_{\lambda, a}$ solution. The problem is to find that set of $P_j$ values which makes $G$ a minimum. The term $G$ constitutes a merit function that tells how good any trial solution is. Once one has taken a measurement, obtained the monochromator data $H_{\lambda, m}$, and arbitrarily selected $\epsilon$ and the number of $P_j$'s the solution is to have, $G$ is found to be a function of only the $P_j$'s, or $G(P_0, P_1 \ldots P_{k-1})$. A necessary condition for $G$ to be a minimum is that

$$\frac{\partial G}{\partial P_0} = \frac{\partial G}{\partial P_1} = \ldots = \frac{\partial G}{\partial P_{k-1}} = 0 \quad (B4)$$

$$\frac{\partial G}{\partial P_0} = \sum_{i=1}^{n} 2X_i \frac{\partial X_i}{\partial P_0} = \sum_{i=1}^{n} 2X_i \left( \frac{-K_{0, i}}{V_i + \epsilon} \right) \quad (B5)$$

$$\frac{\partial G}{\partial P_0} = -2 \sum_{i=1}^{n} \frac{K_{0, i}}{(V_i + \epsilon)^2} (V_i - P_0 K_{0, i} - P_1 K_{1, i} - \ldots - P_{k-1} K_{k-1, i}) \quad (B6)$$

Rearranging the expression gives
\[
\frac{\partial G}{\partial P_0} = -2 \left[ -P_0 \sum_{i=1}^{n} \frac{K_{0,1}K_{0,1}}{(V_1 + \epsilon)^2} - P_1 \sum_{i=1}^{n} \frac{K_{0,1}K_{1,1}}{(V_1 + \epsilon)^2} \ldots - P_{k-1} \sum_{i=1}^{n} \frac{K_{0,1}K_{k-1,1}}{(V_1 + \epsilon)^2} + \sum_{i=1}^{n} \frac{K_{0,1}V_i}{(V_1 + \epsilon)^2} \right]
\]  

(B7)

Similar expressions can be found for \( \partial G/\partial P_1 \), \( \partial G/\partial P_2 \), and so forth. Each is then set equal to zero, which gives

\[
\begin{align*}
P_0 \sum_{i=1}^{n} \frac{K_{0,1}K_{0,1}}{(V_1 + \epsilon)^2} + P_1 \sum_{i=1}^{n} \frac{K_{0,1}K_{1,1}}{(V_1 + \epsilon)^2} \ldots + P_{k-1} \sum_{i=1}^{n} \frac{K_{0,1}K_{k-1,1}}{(V_1 + \epsilon)^2} &= \sum_{i=1}^{n} \frac{K_{0,1}V_i}{(V_1 + \epsilon)^2} \\
P_0 \sum_{i=1}^{n} \frac{K_{1,1}K_{0,1}}{(V_1 + \epsilon)^2} + P_1 \sum_{i=1}^{n} \frac{K_{1,1}K_{1,1}}{(V_1 + \epsilon)^2} \ldots + P_{k-1} \sum_{i=1}^{n} \frac{K_{1,1}K_{k-1,1}}{(V_1 + \epsilon)^2} &= \sum_{i=1}^{n} \frac{K_{1,1}V_i}{(V_1 + \epsilon)^2} \\
\ldots \\
P_0 \sum_{i=1}^{n} \frac{K_{k-1,1}K_{0,1}}{(V_1 + \epsilon)^2} + P_1 \sum_{i=1}^{n} \frac{K_{k-1,1}K_{1,1}}{(V_1 + \epsilon)^2} \ldots + P_{k-1} \sum_{i=1}^{n} \frac{K_{k-1,1}K_{k-1,1}}{(V_1 + \epsilon)^2} &= \sum_{i=1}^{n} \frac{K_{k-1,1}V_i}{(V_1 + \epsilon)^2}
\end{align*}
\]  

(B8)

These \( k \) simultaneous linear equations in \( k \) unknowns can then be solved for the \( P_i \)'s, which completes the solution.

When this method is applied, judgment must be exercised in choosing the number of \( P_i \)'s used. In no case can it exceed the number of filters. The fewer \( P_i \)'s used, the more the data is smoothed and the less resolution obtained in the correction term. In practice, the number of \( P_i \)'s that may be used is limited by round-off errors in the computation process.

The choice of \( \epsilon \) determines the type of merit function. If \( \epsilon \) is much smaller than the largest \( V_i \) value, equal weights are given to equal percentage errors in matching the computed \( V_{a,1} \) to the experimental \( V_i \). In this case the \( \epsilon \) merely prevents division by zero in the event a \( V_i \) is zero. But if \( \epsilon \) is much larger than the largest \( V_i \), equal weights are given to equal absolute differences between the \( V_i \) and \( V_{a,1} \). Here again judgment must be exercised.
REFERENCES


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—National Aeronautics and Space Act of 1958

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