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# PERFORMANCE ANALYSIS OF CHEMICAL STAGES IN THE 300 TO 400 SECOND SPECIFIC IMPULSE RANGE FOR INTERPLANETARY MISSIONS

*by Walter H. Stafford and Carmen R. Catalfamo  
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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

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## DEFINITION OF SYMBOLS

Symbol	Definition
F	Thrust, N
$F/W_o$	Initial thrust-to-weight ratio (based on weight at Earth sea level)
f	Stage mass fraction, $(W_P/W_A)$
g	Gravitational acceleration, $m/sec^2$
H	Energy
h	Altitude, km
$\Delta h$	Altitude change, $(h - h_o)$ , km
$I_{sp}$	Specific impulse, sec
m	Mass, kg
r	Radius, km
R	Planet radius, km
$r_o$	$(h_o + R)$ , km
$t_B$	Burning time, sec
V	Velocity
V*	Comparative velocity
$V_1$	Stage characteristic velocity
$V_\infty$	Hyperbolic excess velocity
$W_A$	Stage weight, $(W_o - W_L)$ , N
$W_L$	Payload weight, N
$W_o$	Gross weight, N

## DEFINITION OF SYMBOLS (Continued)

Symbol	Definition
$W_P$	Propellant weight, N
X	Range, km
$\alpha$	Angle of attack (angle between velocity vector and thrust vector), deg
$\xi$	Propellant mass fraction, $(W_P/W_O)$
$\vartheta$	Flight path angle from vertical, deg
$\mu$	Gravitational constant
$\psi$	Central angle, deg
<b>Subscripts</b>	
C	Burnout
esc	Escape
ex	Exhaust
f	Final
id	ideal
K	Circular
o	Initial
P	Propellant
<b>Abbreviations</b>	
kg	Kilogram
km	Kilometer
m	Meter

DEFINITION OF SYMBOLS (Concluded)

Symbol	Definition
sec	Second
N	Newton

# PERFORMANCE ANALYSIS OF CHEMICAL STAGES IN THE 300 TO 400 SECOND SPECIFIC IMPULSE RANGE FOR INTERPLANETARY MISSIONS

## SUMMARY

The effect of thrust-to-weight ratios and specific impulses on trajectory parameters has been investigated for hyperbolic escape from, and braking into, an orbit about Earth. For the escape trajectories, the thrust vector was applied tangential to the flight path and in the direction of the velocity vector. For the braking-in trajectories, the thrust vector was directed against the velocity vector. Thrust-to-weight ratios of 0.2 to 1.0 and specific impulses of 300 to 400 seconds were used.

The results of the study are presented graphically.

## INTRODUCTION

A study of the trajectory requirements is of fundamental importance in planning interplanetary round-trip missions. Preliminary analysis of the mission requires a rapid method of sufficient accuracy for determining the trajectory parameters. The sizing of boost vehicles is dependent, to a large extent, on the velocity requirements of the particular trajectory chosen.

The purpose of this study is to present a method for determining the trajectory parameters and vehicle mass characteristics for a specific mission, when the hyperbolic excess velocity is known.

The approach used for escape from orbit was to determine the trajectory parameters at burnout, convert the characteristic velocity to a hyperbolic excess velocity, and then present the data graphically. The approach used for braking into orbit was to determine the arrival velocity for a given transfer trajectory and initiate burning such that circular orbit conditions are attained at burnout. The equations of motion were integrated on a digital computer, using a Runge-Kutta numerical integration procedure.

## ASSUMPTIONS

The following is a summary of the basic assumptions used in this study:

1. For escape -- Acceleration of a single stage from a reference orbit about Earth, using a constant thrust directed along the velocity vector.
2. For braking -- Deceleration of a single stage from an interplanetary transfer trajectory to a reference orbit about Earth, using a constant thrust directed against the velocity vector.
3. Reference orbit about the Earth was circular with a radius of 6556 km and a velocity of 7798 m/sec.
4. Constant specific impulse values from 300 to 400 seconds.
5. The thrust-to-weight ratios used were varied parametrically from 0.2 to 1.0.
6. Mean spherical Earth:
 
$$\mu = 398606.6 \text{ km}^3/\text{sec}^2$$

$$R = 6371.27 \text{ km}$$

## ANALYSIS

For interplanetary flight, the ideal<sup>1</sup> total energy that must be imparted to the spacecraft is the ideal energy required to escape the gravitational field of the planet, plus the energy required to change its path about the Sun. The ideal energy required to escape the gravitational attraction of a planet can be determined from two-body mechanics to be  $H_{\text{ESC}} = \mu/r$  and the energy needed to alter the flight path about the Sun,  $H_{\infty}$ , is determined by the characteristics of the interplanetary trajectory.

For determining the vehicle size necessary to inject the spacecraft into the interplanetary trajectory, it is convenient to express the ideal total energy,  $H = H_{\text{ESC}} + H_{\infty}$ , in terms of a burnout velocity. This produces equations of the following forms:

$$V_C = \sqrt{H_{\text{ESC}} + H_{\infty}} \tag{1}$$

---

<sup>1</sup> The term "ideal" refers to an instantaneous change of energy.

or

$$V_C = \sqrt{(V_{esc})^2 + (V_\infty)^2} \quad (2)$$

When considering finite vehicle systems there is an additional energy requirement,  $H_{loss}$ , which results from expending the propellants at different energy levels. Therefore, the total velocity increment for the injecting stage is now

$$V_1 = V_C - V_0 + V_{loss} \quad (3)$$

where  $V_0$  is the initial velocity. The vehicle mass characteristics can be determined from the equation

$$\frac{W_0}{W_C} = e^{\frac{V_1}{V_{ex}}} \quad (4)$$

To accomplish this study, the two-degree-of-freedom equations of motion were numerically integrated.

Referring to the sketches, "a" for escaping from and "b" for braking into, computations were made for a point mass moving in a plane using the following equations of motion:

$$\dot{V} = \frac{F \cos \alpha}{m} - \frac{\mu}{r^2} \cos \vartheta \quad (5)$$

$$V \dot{\vartheta} = \frac{F \sin \alpha}{m} + \left( \frac{\mu}{r^2} - \frac{V^2}{r} \right) \sin \vartheta \quad (6)$$

$$\dot{r} = V \cos \vartheta \quad (7)$$

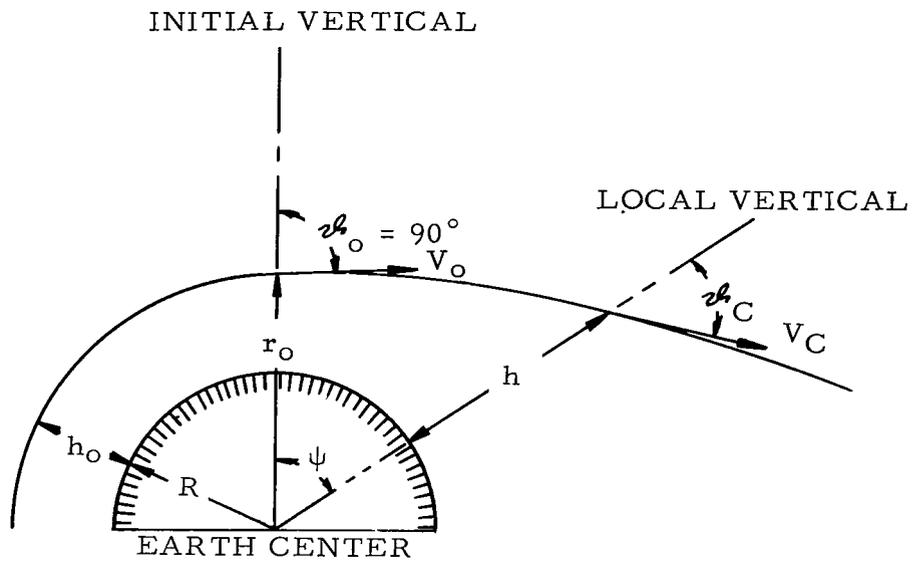
$$\dot{\psi} = \frac{V \sin \vartheta}{r} \quad (8)$$

where

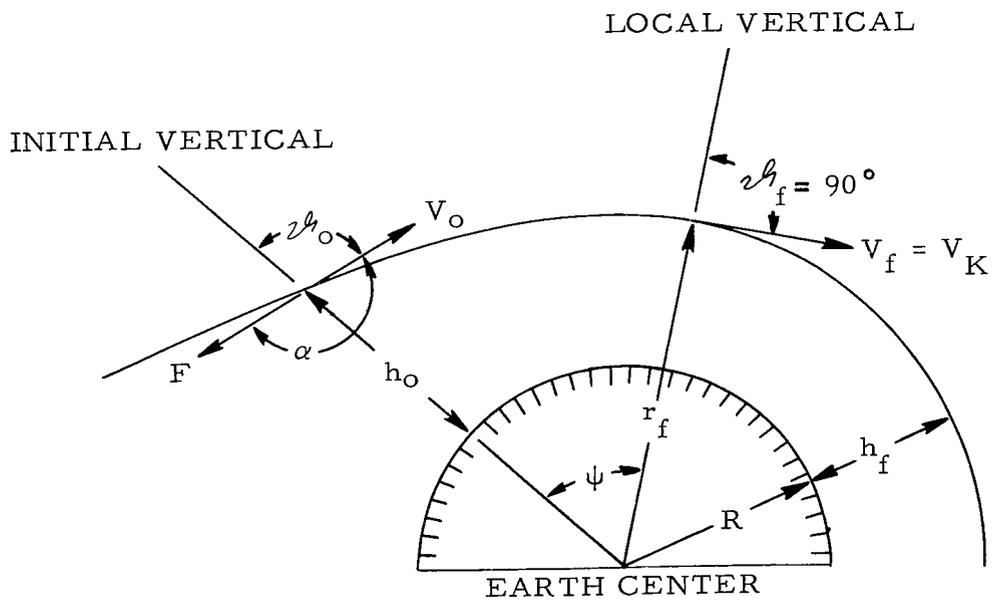
$$m = m_0 + \int \dot{m} dt \quad (9)$$

and

$$\dot{m} = - \frac{F}{V_{ex}} \quad (10)$$



SKETCH a. ESCAPE FROM ORBIT



SKETCH b. BRAKE TO ORBIT

The velocity and flight path angle may be obtained by integrating the equations of motion

$$V = V_o + \int \dot{V} dt \quad (11)$$

$$\vartheta = \vartheta_o + \int \dot{\vartheta} dt \quad (12)$$

The range and altitude can be calculated by the relations

$$X = X_o + \int V \sin \vartheta dt \quad (13)$$

$$h = h_o + \int \dot{h} dt \quad (14)$$

The central angle is

$$\psi = \psi_o + \int \frac{\dot{X}}{r} dt \quad (15)$$

The initial weight of the stage is

$$W_o = W_C + W_P \quad (16)$$

The velocity expended by the stage is the characteristic velocity, or

$$V_1 = V_{ex} \ln \left( \frac{1}{1 - \zeta} \right) \quad (17)$$

Then the velocity losses are the difference between the characteristic velocity and the change in comparative velocity, or

$$V_{loss} = V_1 - \Delta V^* \quad (18)$$

where the comparative velocity is

$$V^* = \sqrt{V^2 + 2\mu \left( \frac{1}{R} - \frac{1}{r} \right)} \quad (19)$$

For ascent from  $r = r_o$  to  $r = r_f$ , the change in comparative velocity is

$$\Delta V^* = \sqrt{V_f^2 + 2\mu \left( \frac{1}{r_o} - \frac{1}{r_f} \right)} - V_o \quad (20)$$

and the velocity loss due to gravity is

$$V_{\text{loss}} = V_{\text{ex}} \ln\left(\frac{1}{1 - \zeta}\right) - \left[ \sqrt{V_f^2 + 2\mu \left(\frac{1}{r_o} - \frac{1}{r_f}\right)} - V_o \right] \quad (21)$$

For descent from  $r = r_o$  to  $r = r_f$ , the change in comparative velocity is

$$\Delta V^* = \sqrt{V_o^2 + 2\mu \left(\frac{1}{r_f} - \frac{1}{r_o}\right)} - V_f \quad (22)$$

and the velocity loss due to gravity is

$$V_{\text{loss}} = V_{\text{ex}} \ln\left(\frac{1}{1 - \zeta}\right) - \left[ \sqrt{V_o^2 + 2\mu \left(\frac{1}{r_f} - \frac{1}{r_o}\right)} - V_f \right] \quad (23)$$

## DISCUSSION OF RESULTS

The results of this investigation are presented graphically in Figures 1 through 11 for each of the two areas considered.<sup>2</sup> The stage characteristic velocity,  $V_1$ , is plotted versus hyperbolic excess velocity, with thrust-to-weight ratios as a parameter, in Figures 1 through 5.

Figures 6 and 7 show the velocity losses,  $V_{\text{loss}}$ , due to gravity for specific impulse values of 300 seconds and 400 seconds, respectively. The flight path angle at burnout for departure trajectories and at initiation of burning for braking trajectories is shown versus hyperbolic excess velocity in Figure 8.

Figure 9 gives the change in altitude. This change is the difference between the altitude of the reference orbit about the planet and the altitude at burnout for the departure trajectories, and the difference between the altitude of the reference orbit and the altitude at initiation of burning for braking trajectories. The change in other trajectory parameters is shown in Figures 10 and 11.

The vehicle mass characteristics can be determined from Figures 12 and 13 in the Section entitled "Brake to Earth Orbit."

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<sup>2</sup> Figure references apply to each of the two areas considered.

## CONCLUSIONS

From these parametric analyses, sufficient data are provided to enable the designer to make a preliminary design of a stage that will accomplish any one of the mission areas studied when that particular mission's requirements are defined.

George C. Marshall Space Flight Center,  
National Aeronautics and Space Administration,  
Huntsville, Alabama, July 12, 1965.



## GRAPHIC PRESENTATION

### Departure From Earth Orbit

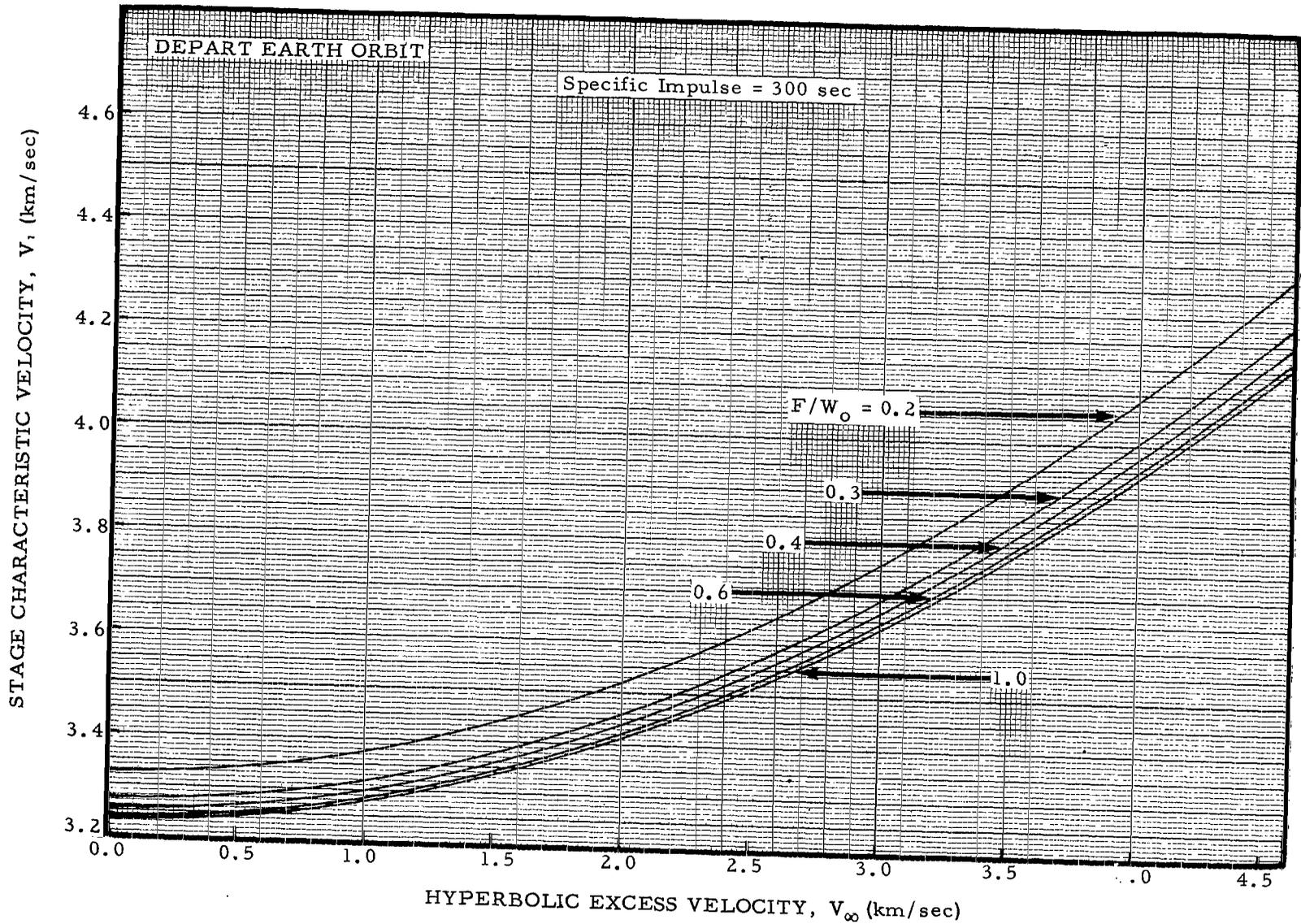


FIGURE 1a. CHARACTERISTIC VELOCITY VERSUS HYPERBOLIC EXCESS VELOCITY WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 300 SECONDS FOR HYPERBOLIC EXCESS VELOCITIES OF 0.0 THROUGH 4.6 KM/SEC

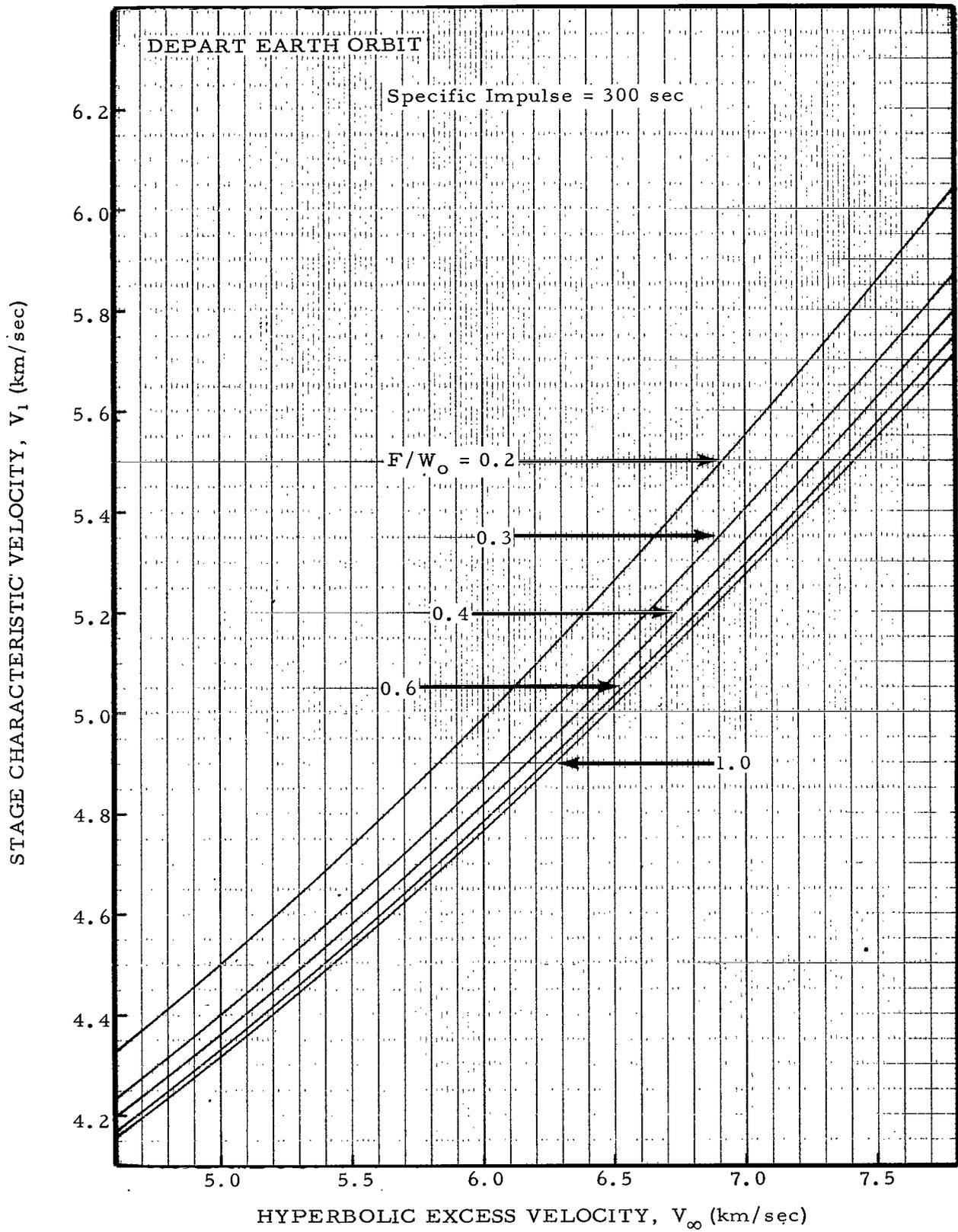


FIGURE 1b. CHARACTERISTIC VELOCITY VERSUS HYPERBOLIC EXCESS VELOCITY WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 300 SECONDS FOR HYPERBOLIC EXCESS VELOCITIES OF 4.6 THROUGH 7.8 KM/SEC

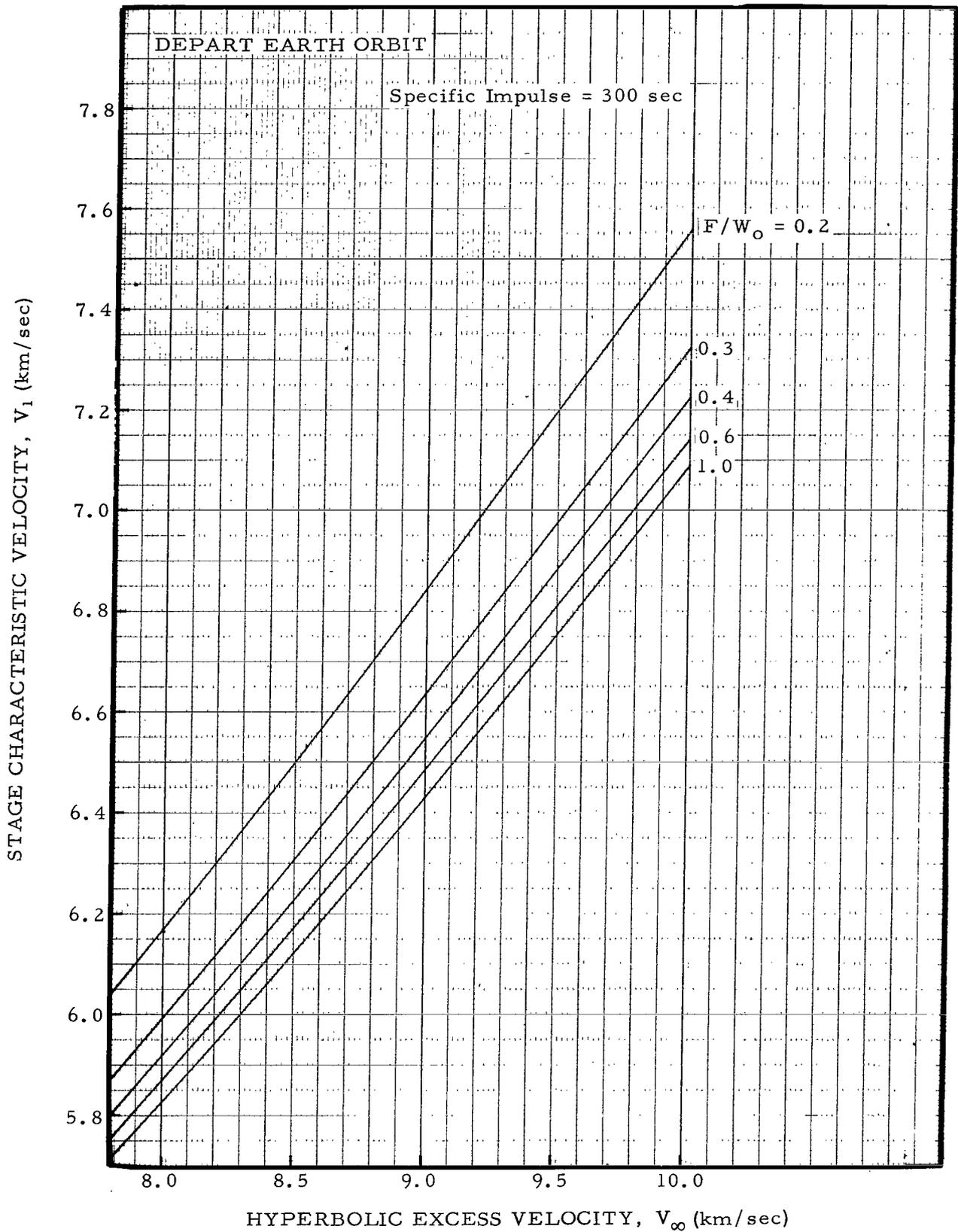


FIGURE 1c. CHARACTERISTIC VELOCITY VERSUS HYPERBOLIC EXCESS VELOCITY WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 300 SECONDS FOR HYPERBOLIC EXCESS VELOCITIES OF 7.8 THROUGH 10.0 KM/SEC

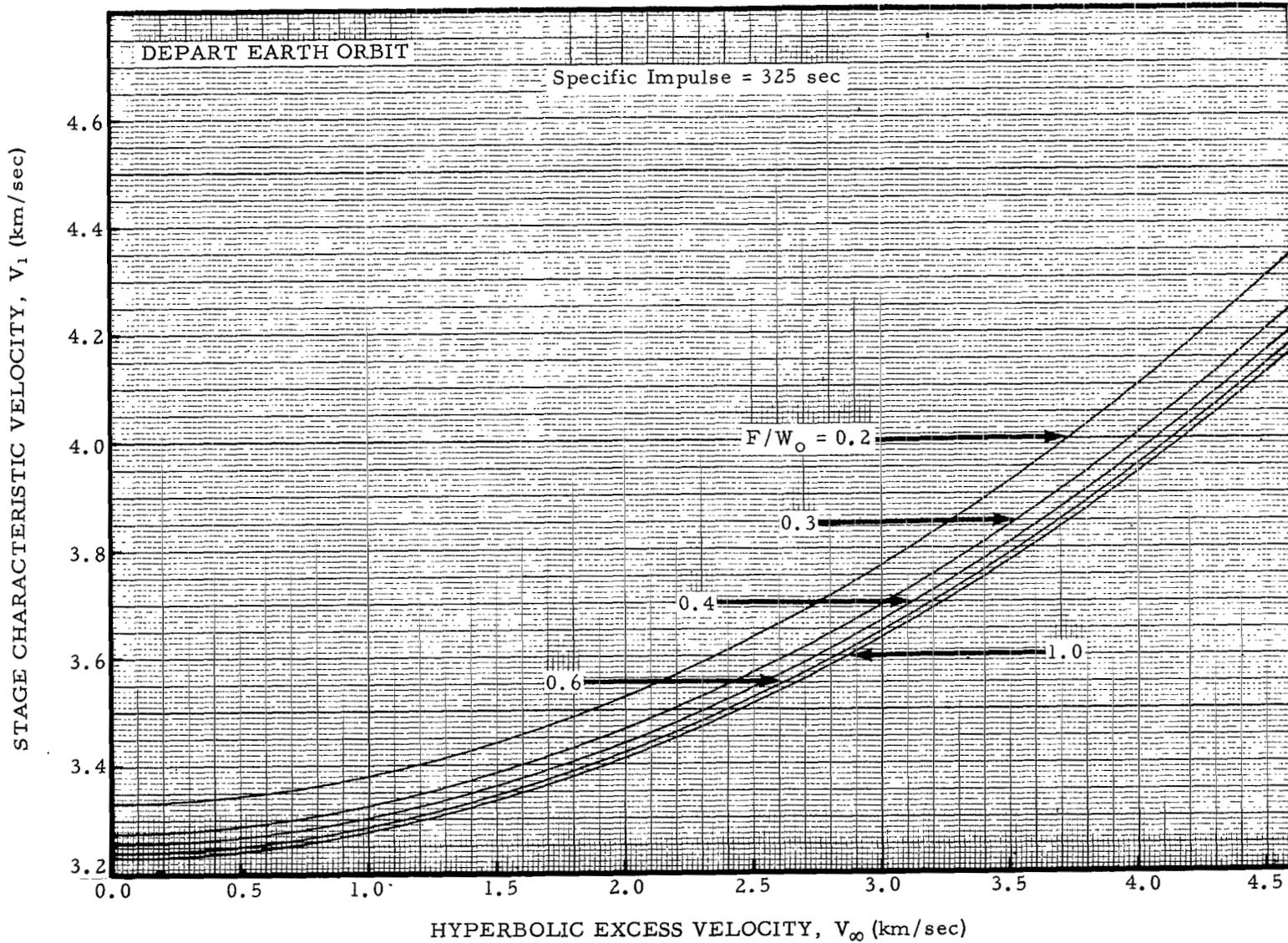


FIGURE 2a. CHARACTERISTIC VELOCITY VERSUS HYPERBOLIC EXCESS VELOCITY WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 325 SECONDS FOR HYPERBOLIC EXCESS VELOCITIES OF 0.0 THROUGH 4.6 KM/SEC

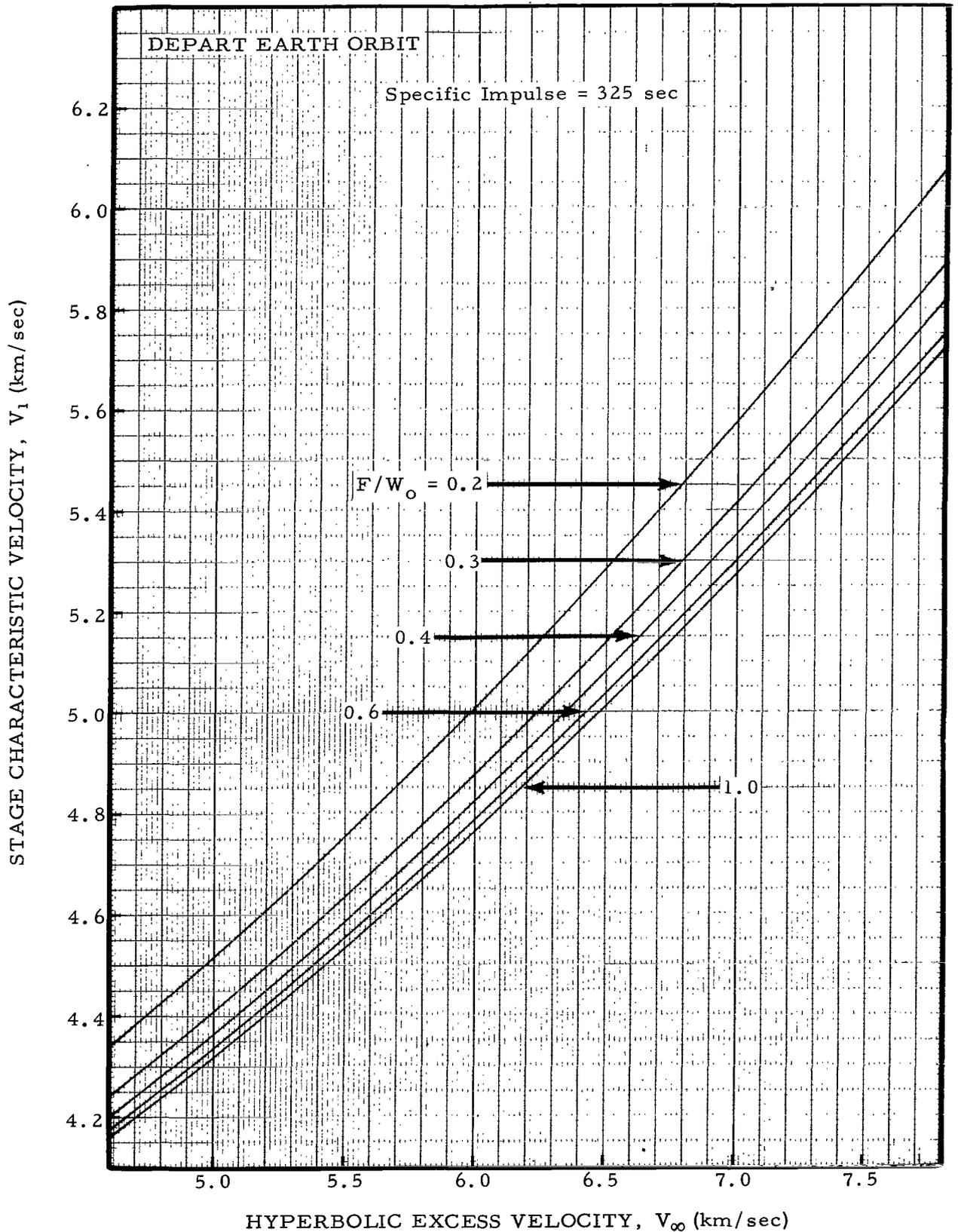


FIGURE 2b. CHARACTERISTIC VELOCITY VERSUS HYPERBOLIC EXCESS VELOCITY WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 325 SECONDS FOR HYPERBOLIC EXCESS VELOCITIES OF 4.6 THROUGH 7.8 KM/SEC

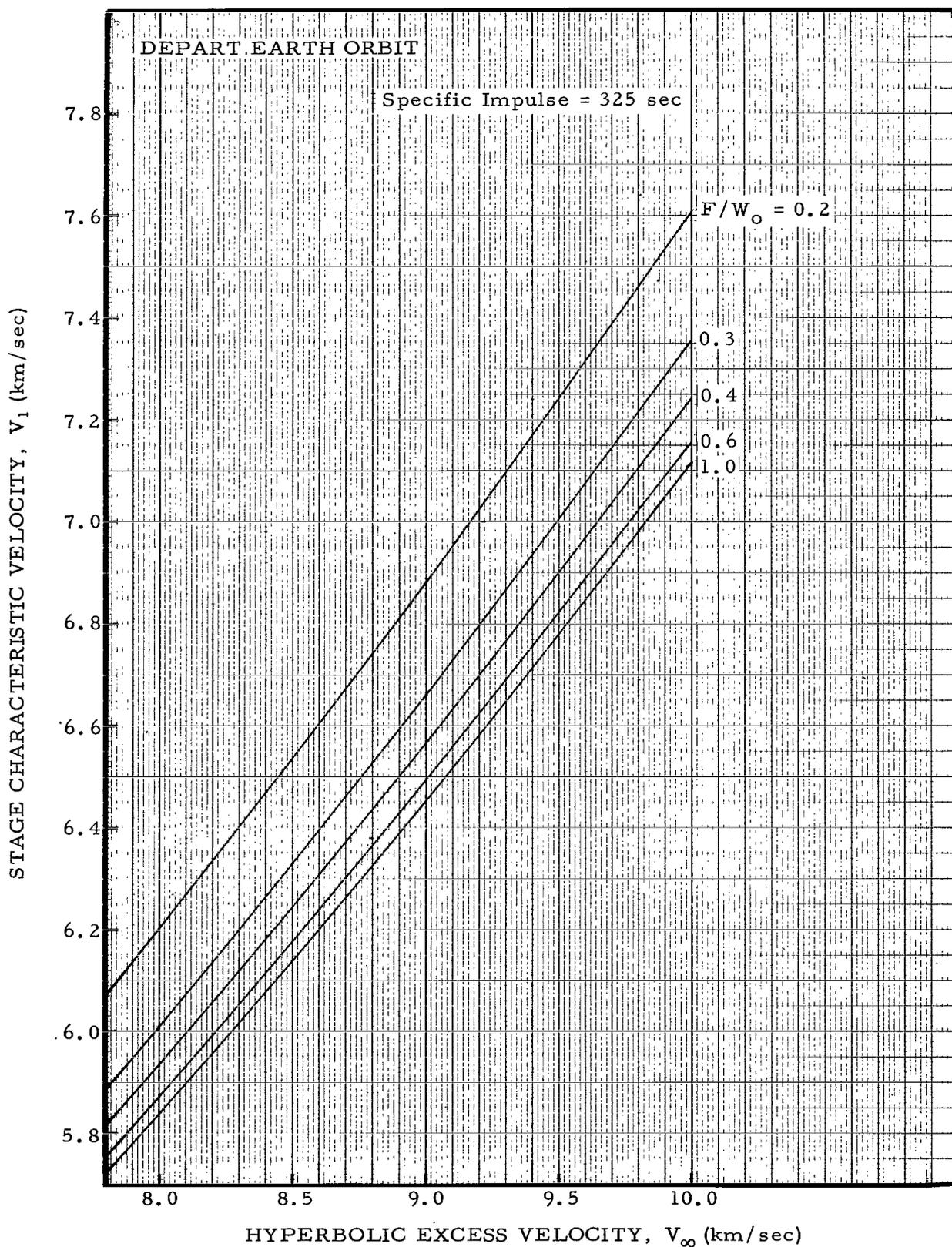


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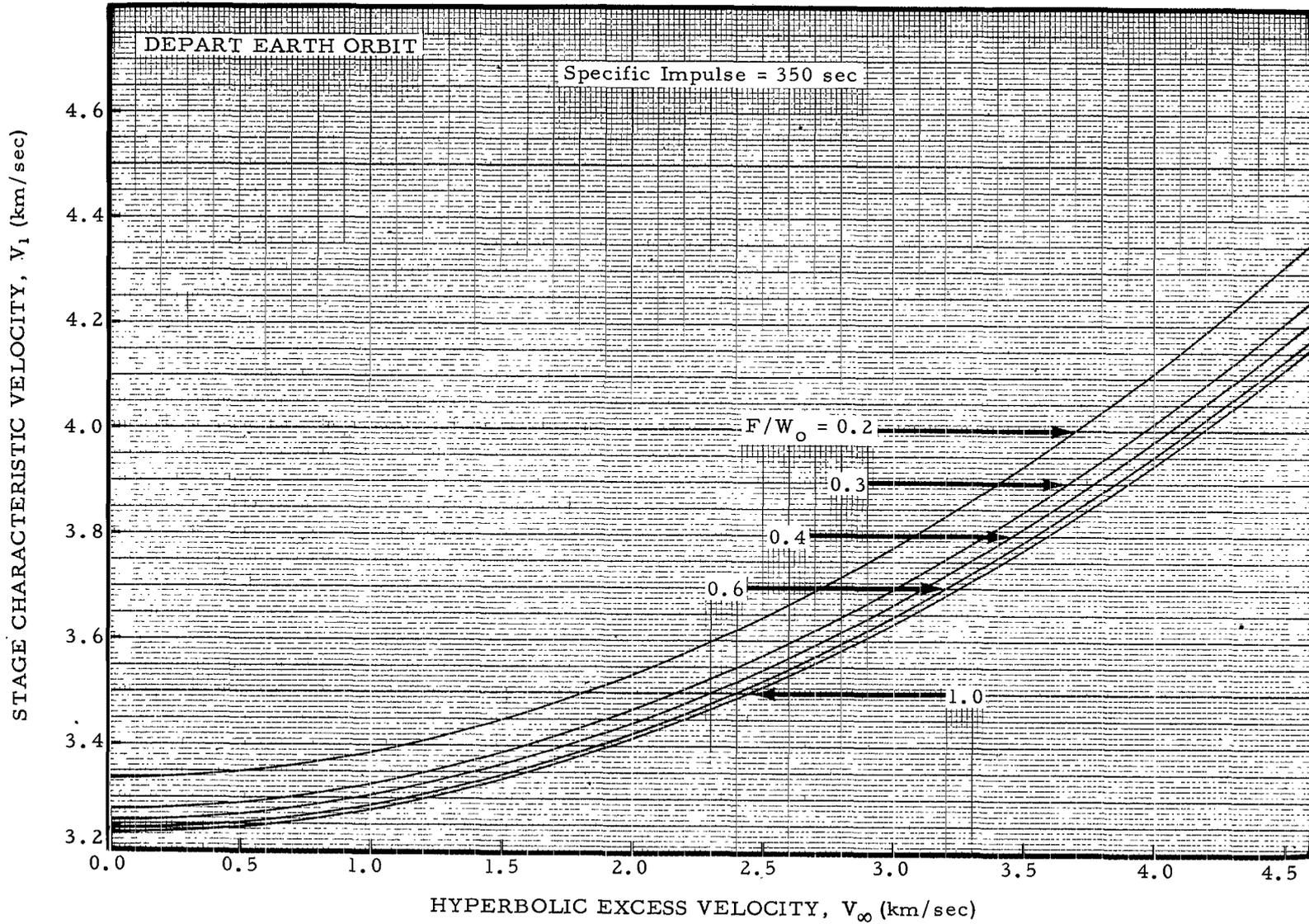


FIGURE 3a. CHARACTERISTIC VELOCITY VERSUS HYPERBOLIC EXCESS VELOCITY WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 350 SECONDS FOR HYPERBOLIC EXCESS VELOCITIES OF 0.0 THROUGH 4.6 KM/SEC

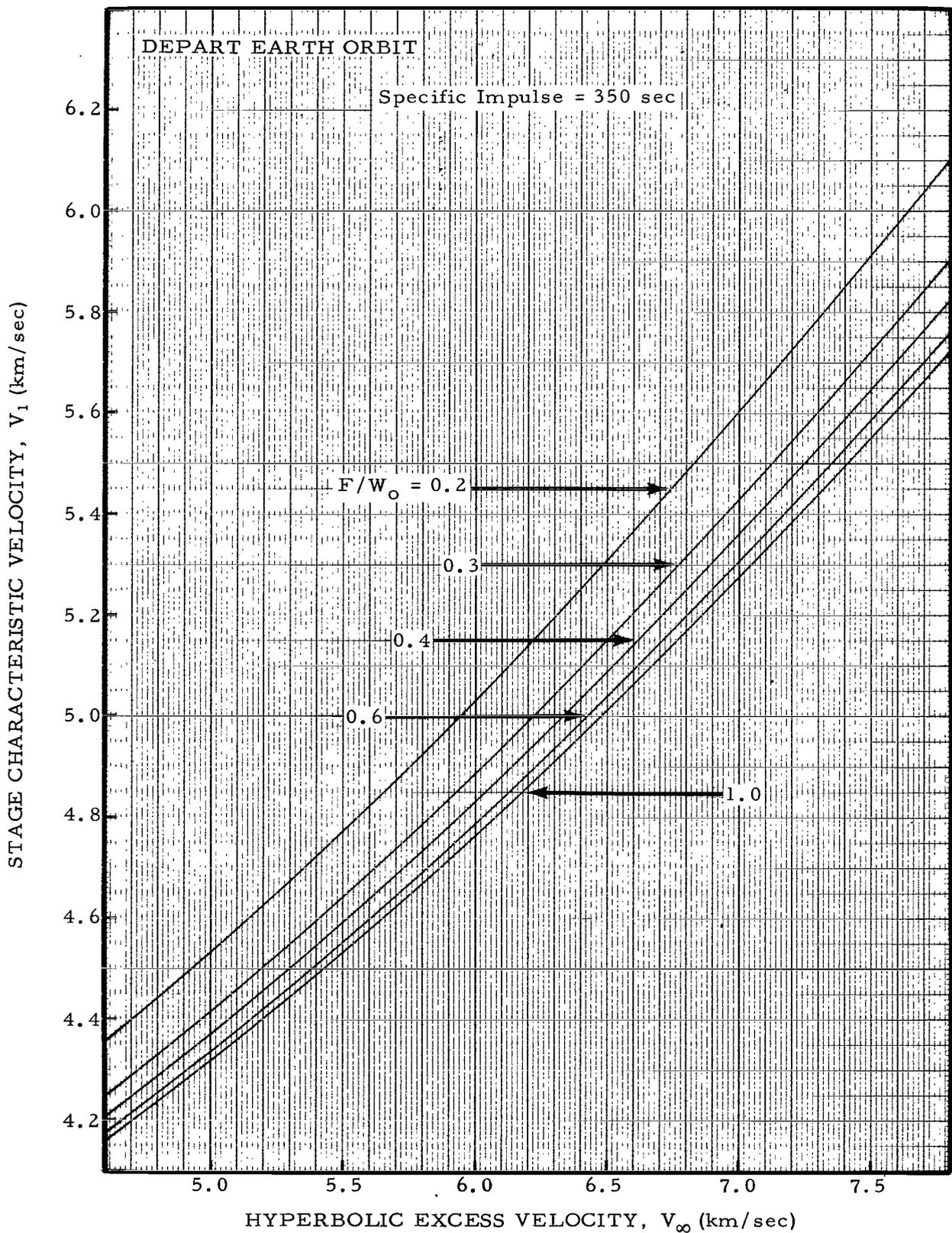


FIGURE 3b. CHARACTERISTIC VELOCITY VERSUS HYPERBOLIC EXCESS VELOCITY WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 350 SECONDS FOR HYPERBOLIC EXCESS VELOCITIES OF 4.6 THROUGH 7.8 KM/SEC

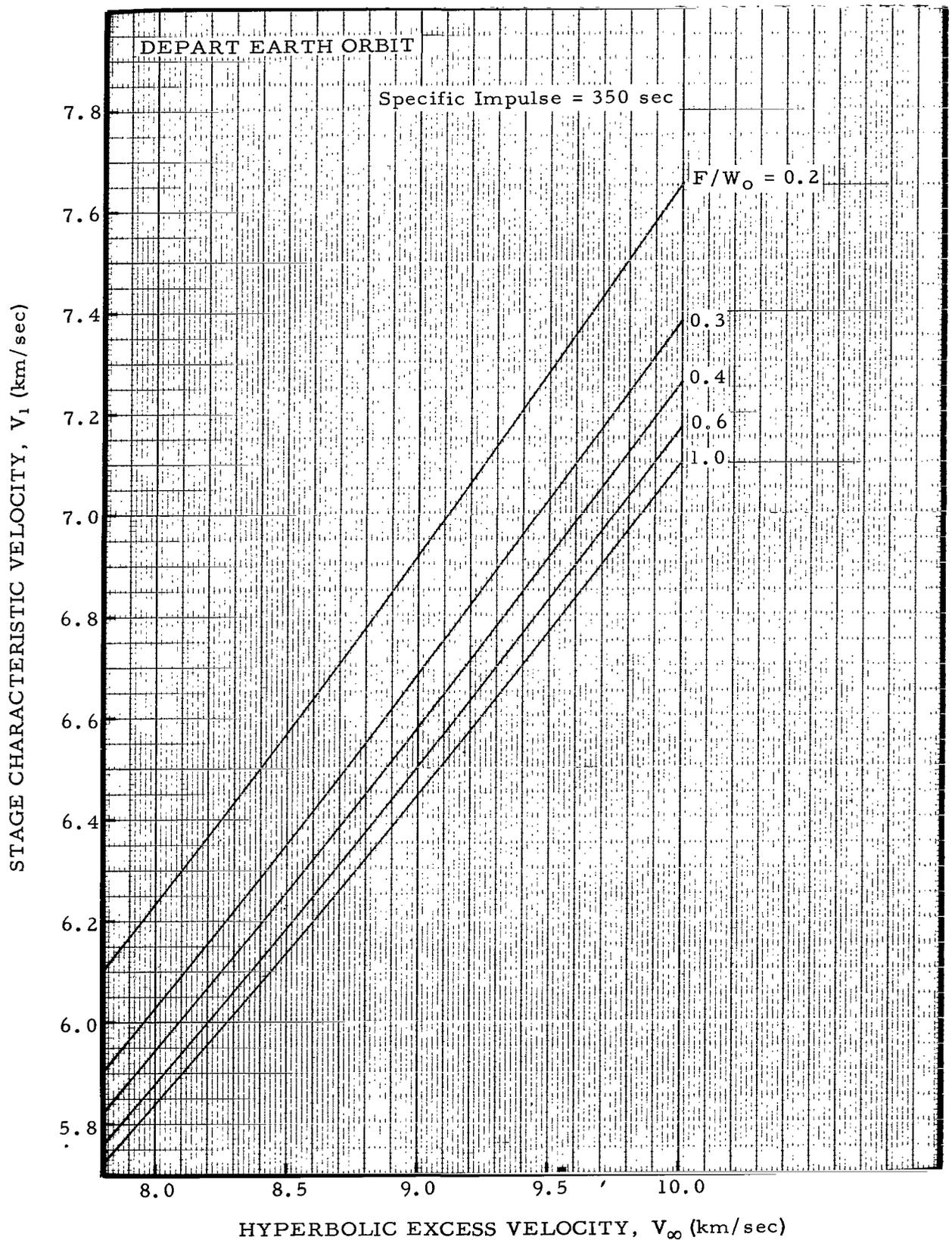


FIGURE 3c. CHARACTERISTIC VELOCITY VERSUS HYPERBOLIC EXCESS VELOCITY WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 350 SECONDS FOR HYPERBOLIC EXCESS VELOCITIES OF 7.8 THROUGH 10.0 KM/SEC

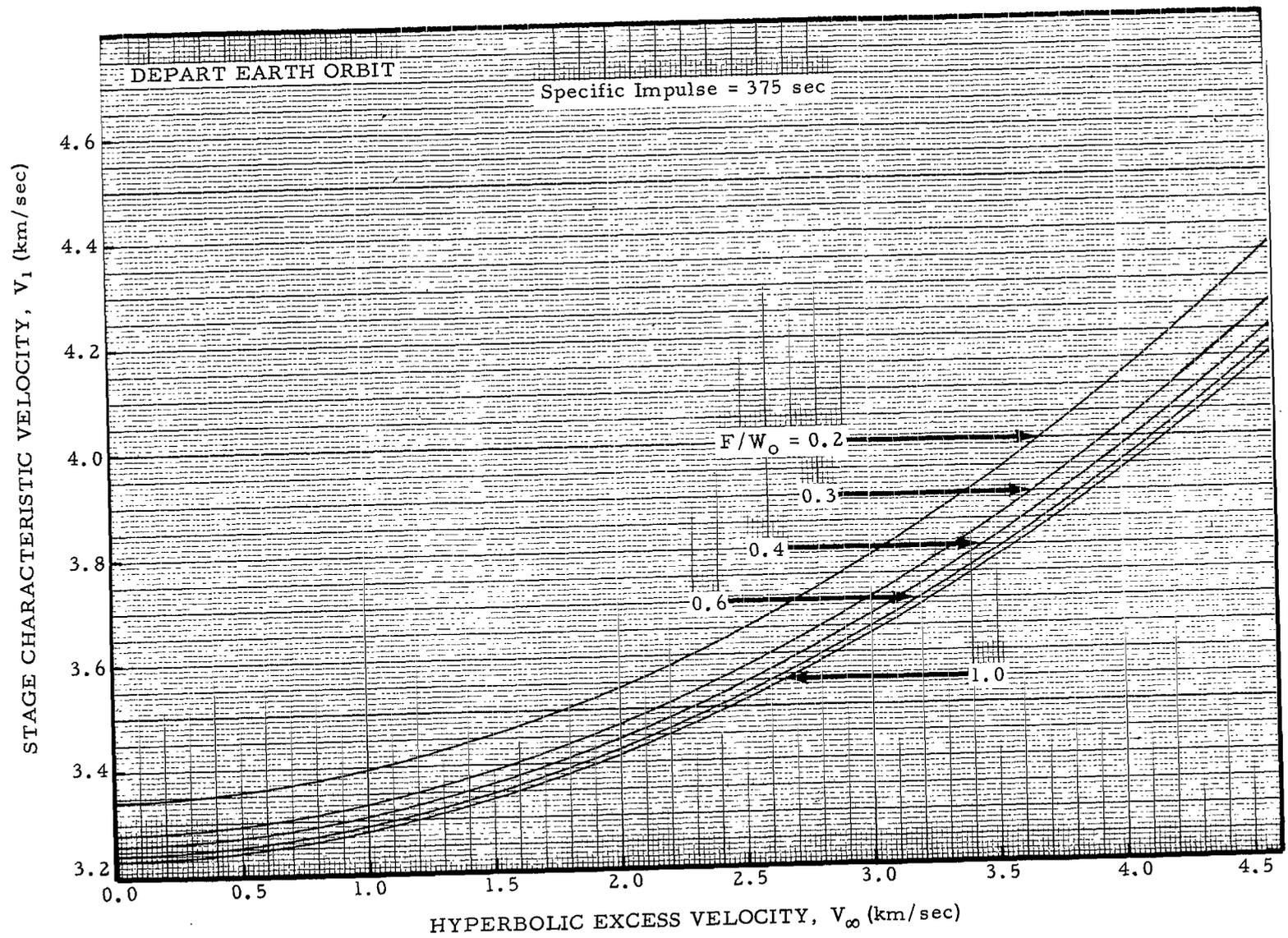


FIGURE 4a. CHARACTERISTIC VELOCITY VERSUS HYPERBOLIC EXCESS VELOCITY WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 375 SECONDS FOR HYPERBOLIC EXCESS VELOCITIES OF 0.0 THROUGH 4.6 KM/SEC

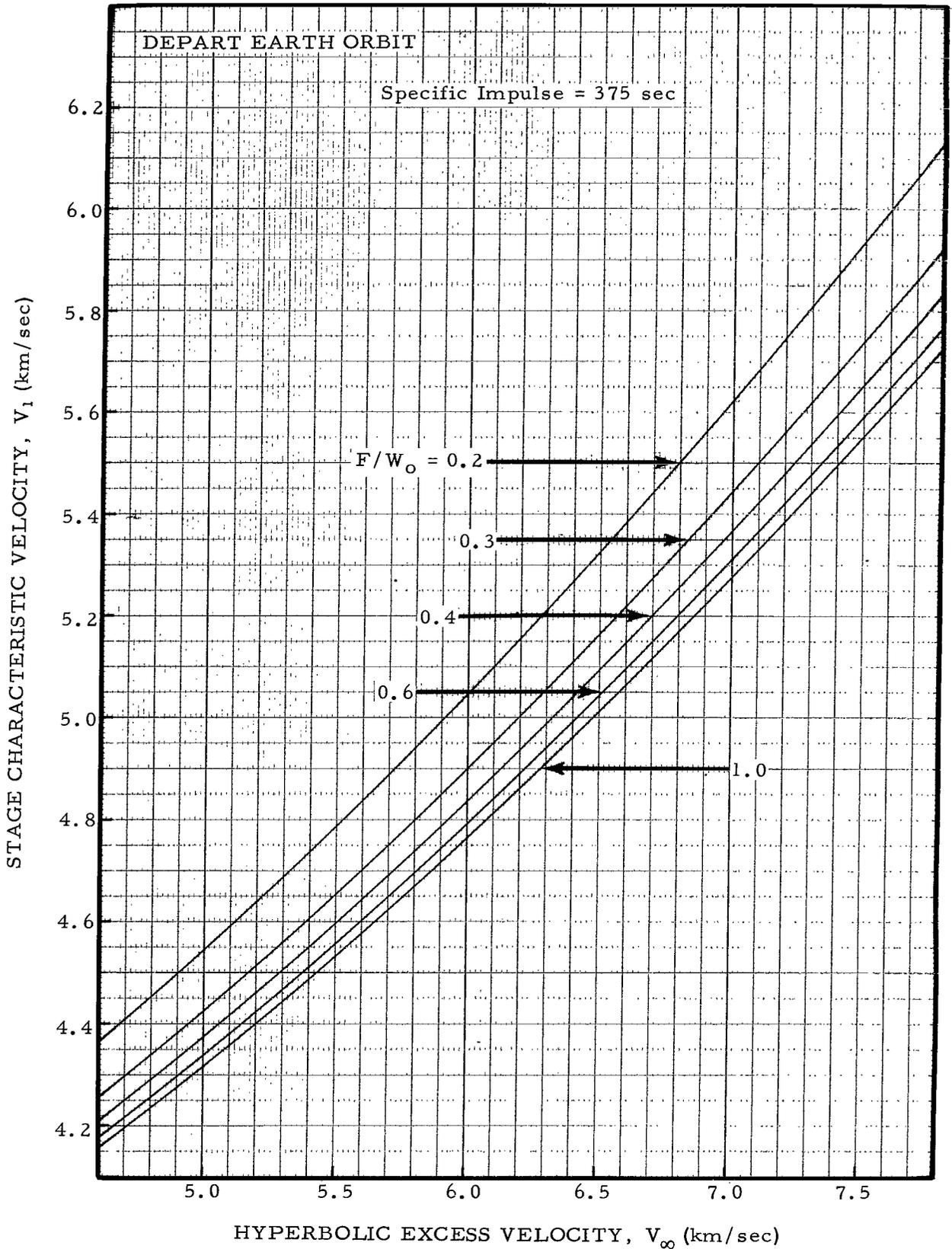


FIGURE 4b. CHARACTERISTIC VELOCITY VERSUS HYPERBOLIC EXCESS VELOCITY WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 375 SECONDS FOR HYPERBOLIC EXCESS VELOCITIES OF 4.6 THROUGH 7.8 KM/SEC

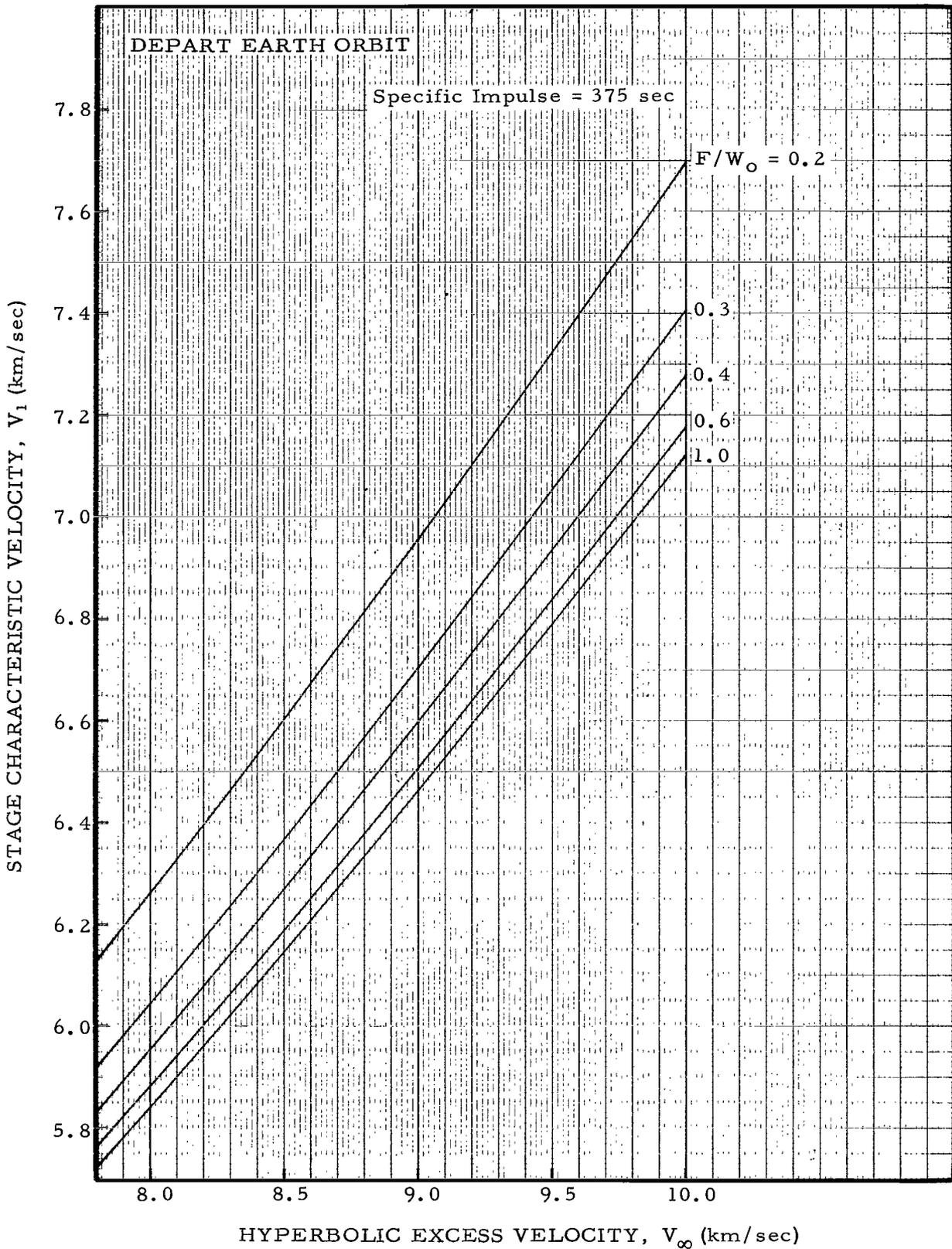


FIGURE 4c. CHARACTERISTIC VELOCITY VERSUS HYPERBOLIC EXCESS VELOCITY WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 375 SECONDS FOR HYPERBOLIC EXCESS VELOCITIES OF 7.8 THROUGH 10.0 KM/SEC

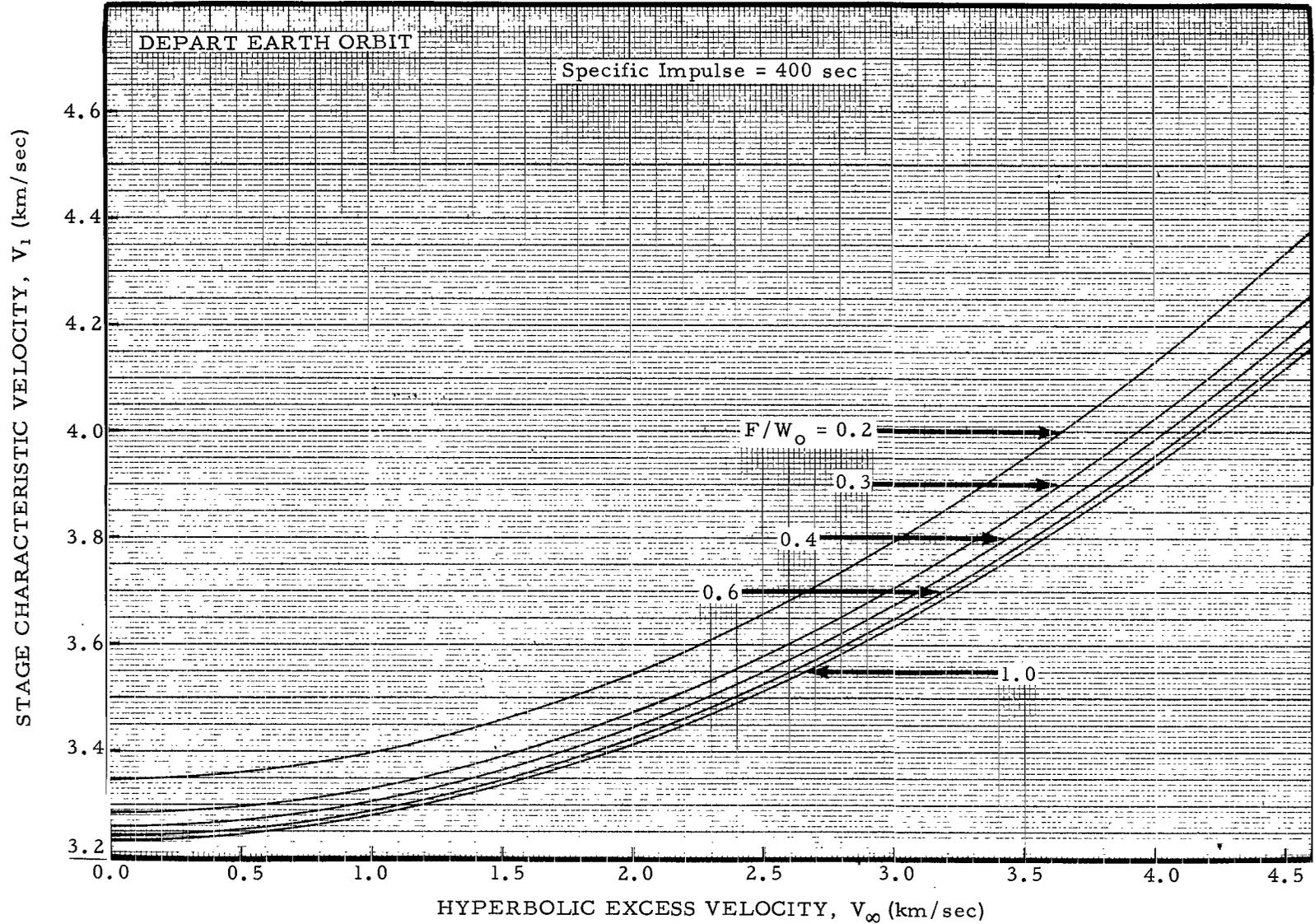


FIGURE 5a. CHARACTERISTIC VELOCITY VERSUS HYPERBOLIC EXCESS VELOCITY WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 400 SECONDS FOR HYPERBOLIC EXCESS VELOCITIES OF 0.0 THROUGH 4.6 KM/SEC

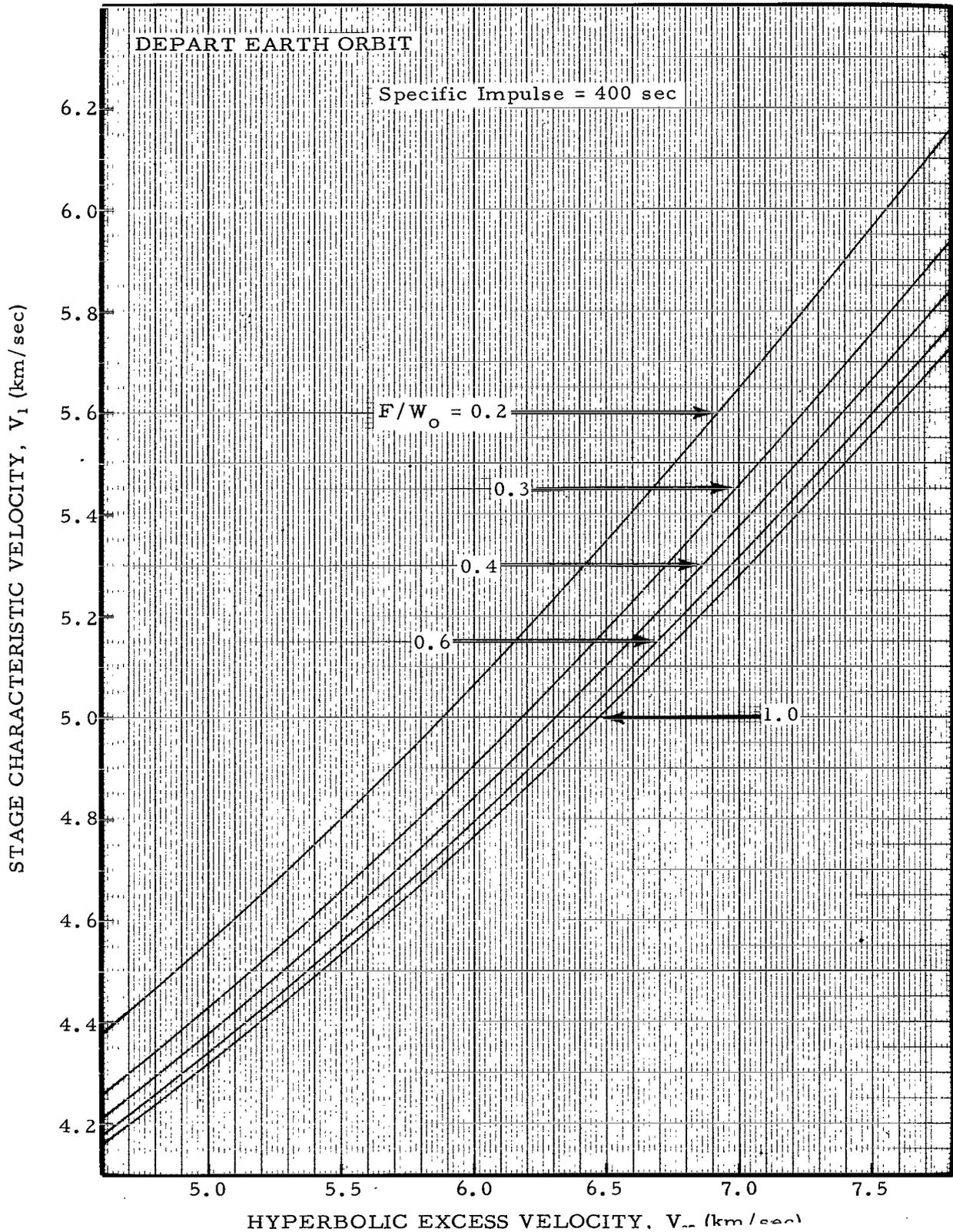


FIGURE 5b. CHARACTERISTIC VELOCITY VERSUS HYPERBOLIC EXCESS VELOCITY WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 400 SECONDS FOR HYPERBOLIC EXCESS VELOCITIES OF 4.6 THROUGH 7.8 KM/SEC

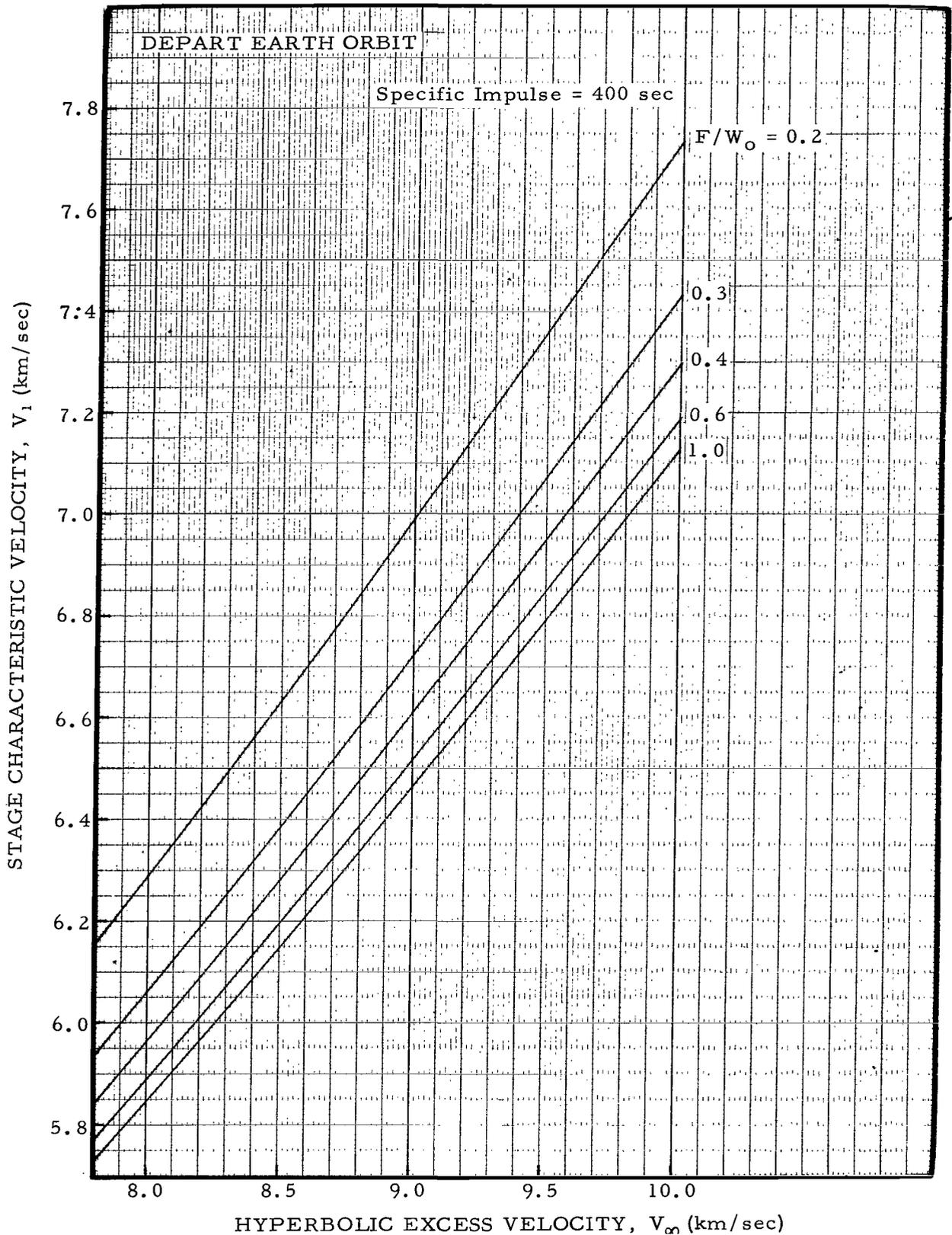


FIGURE 5c. CHARACTERISTIC VELOCITY VERSUS HYPERBOLIC EXCESS VELOCITY WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 400 SECONDS FOR HYPERBOLIC EXCESS VELOCITIES OF 7.8 THROUGH 10.0 KM/SEC

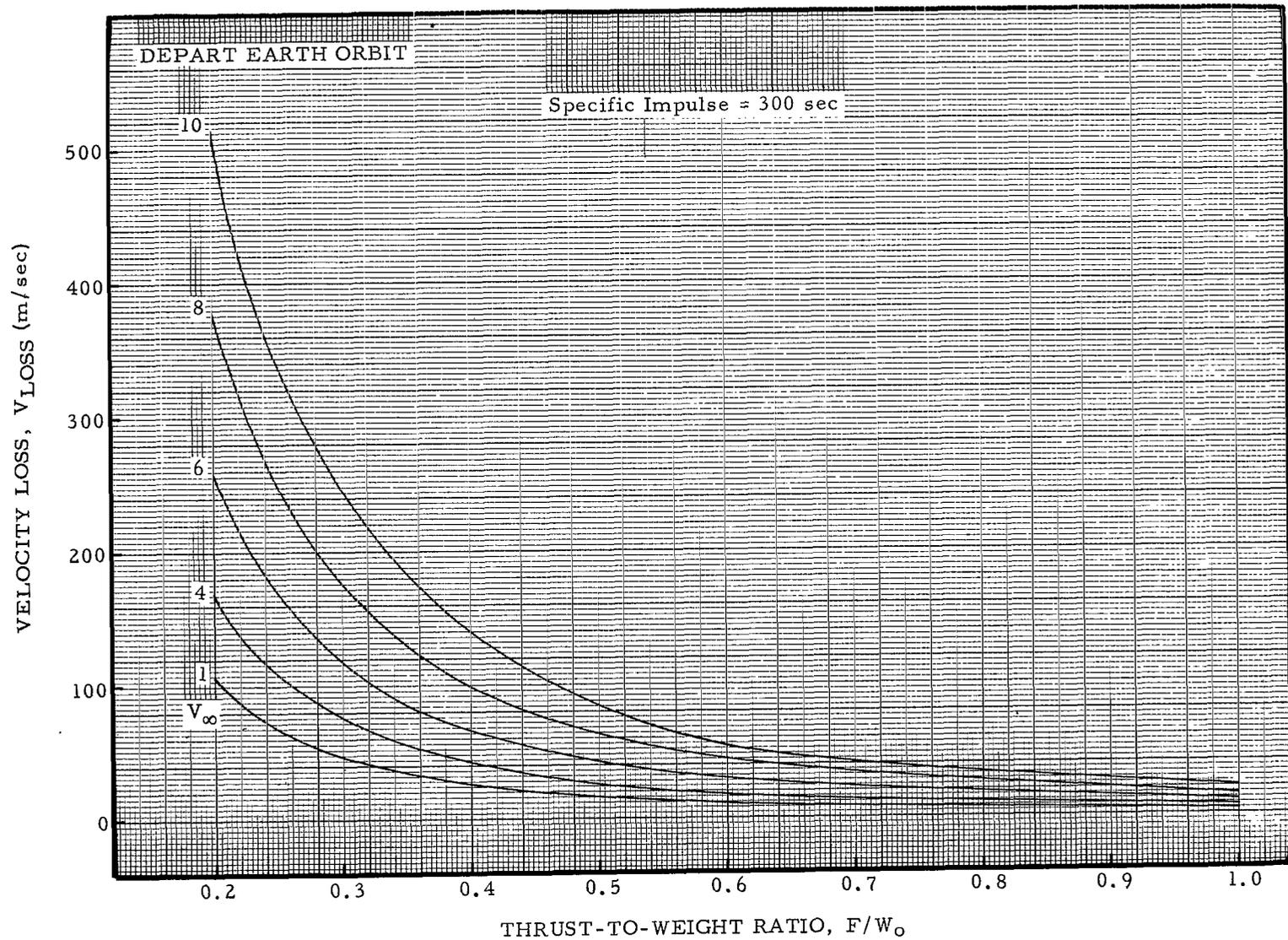


FIGURE 6. VELOCITY LOSS DUE TO GRAVITY VERSUS THRUST-TO-WEIGHT RATIO WITH HYPERBOLIC EXCESS VELOCITY AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 300 SECONDS

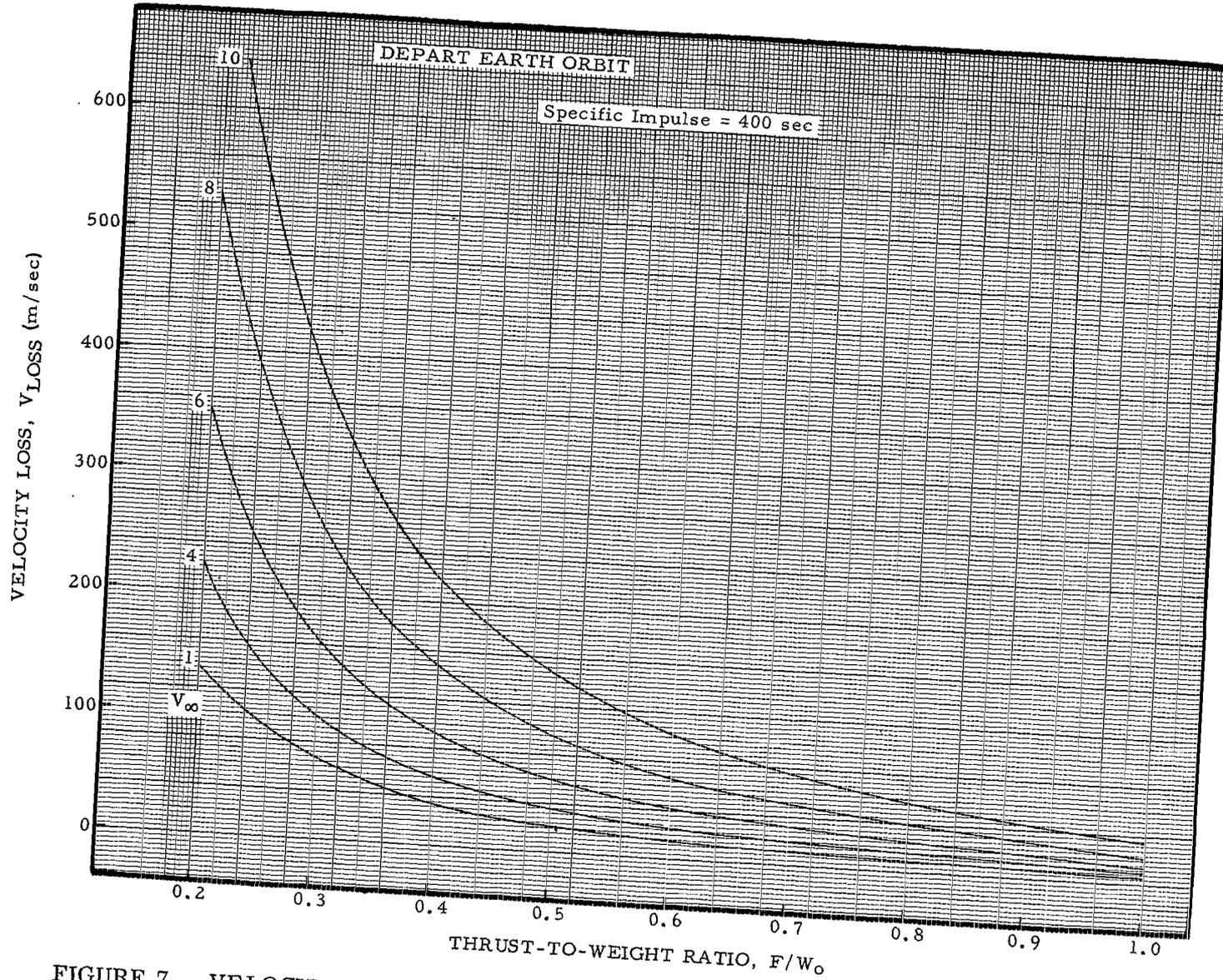


FIGURE 7. VELOCITY LOSS DUE TO GRAVITY VERSUS THRUST-TO-WEIGHT RATIO WITH HYPERBOLIC EXCESS VELOCITY AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 400 SECONDS

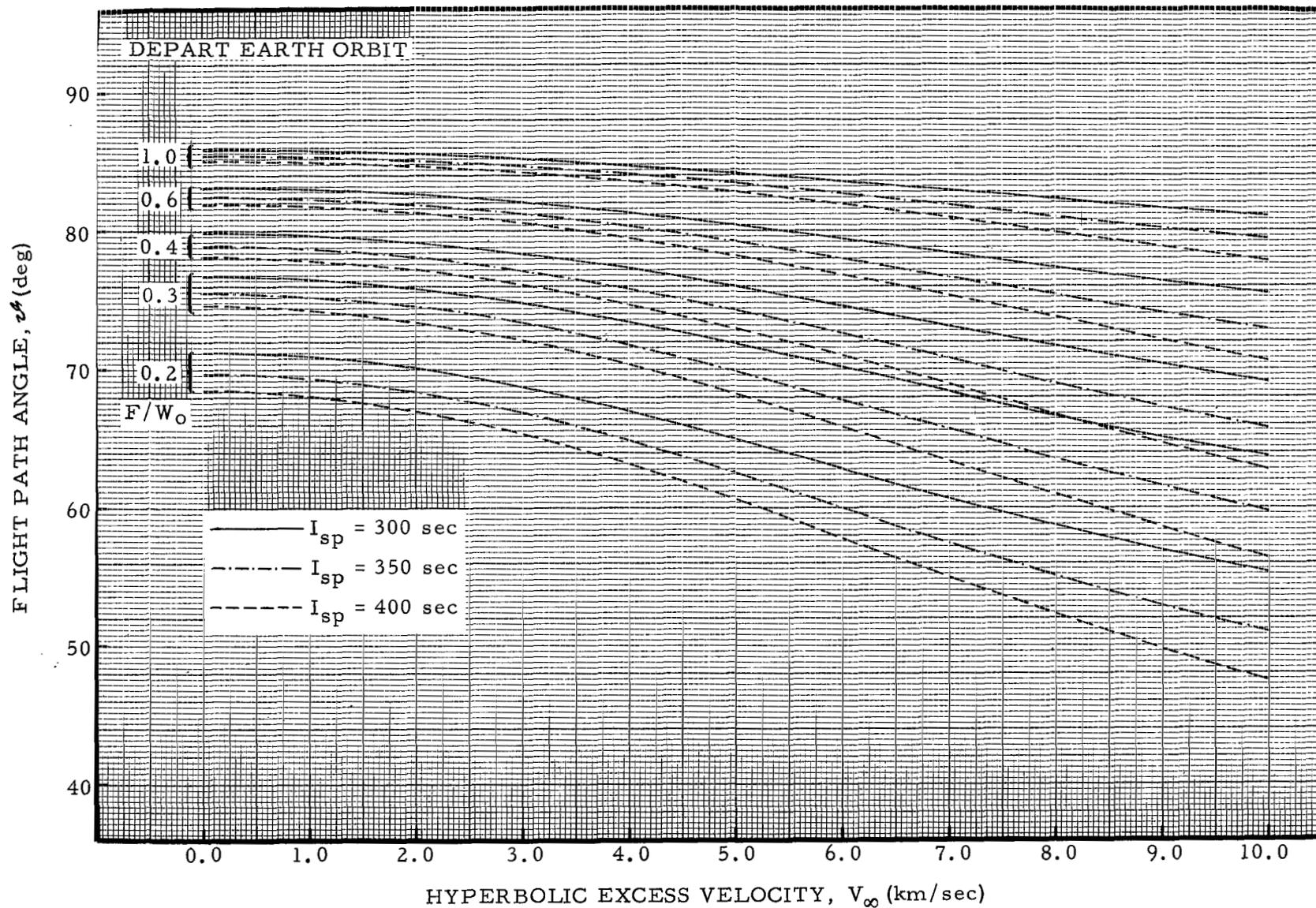


FIGURE 8. FLIGHT PATH ANGLE VERSUS HYPERBOLIC EXCESS VELOCITY WITH THRUST-TO-WEIGHT RATIO FOR SPECIFIC IMPULSES OF 300, 350, and 400 SECONDS AS A PARAMETER

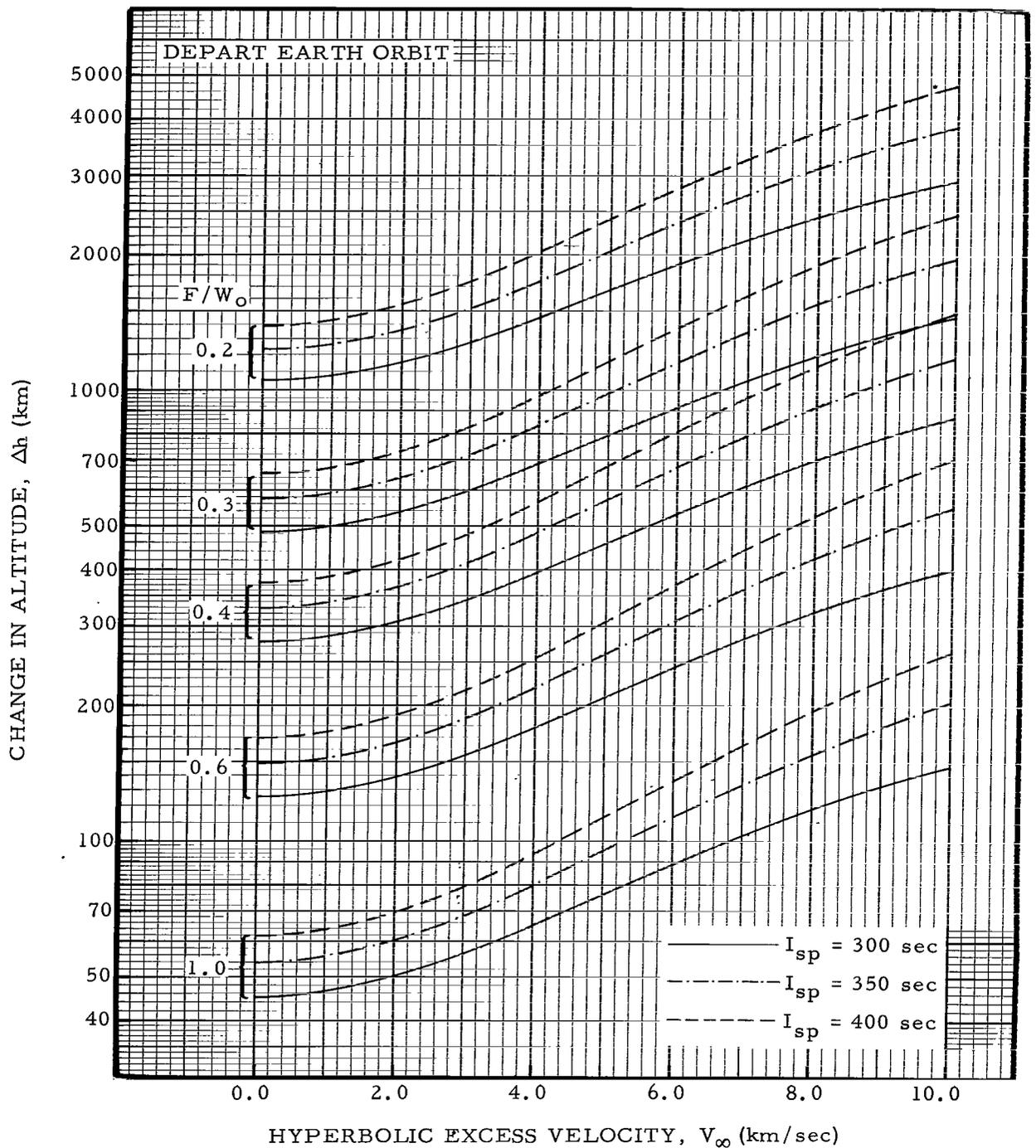


FIGURE 9. CHANGE IN ALTITUDE VERSUS HYPERBOLIC EXCESS VELOCITY WITH THRUST-TO-WEIGHT RATIO FOR SPECIFIC IMPULSES OF 300, 350, AND 400 SECONDS AS A PARAMETER

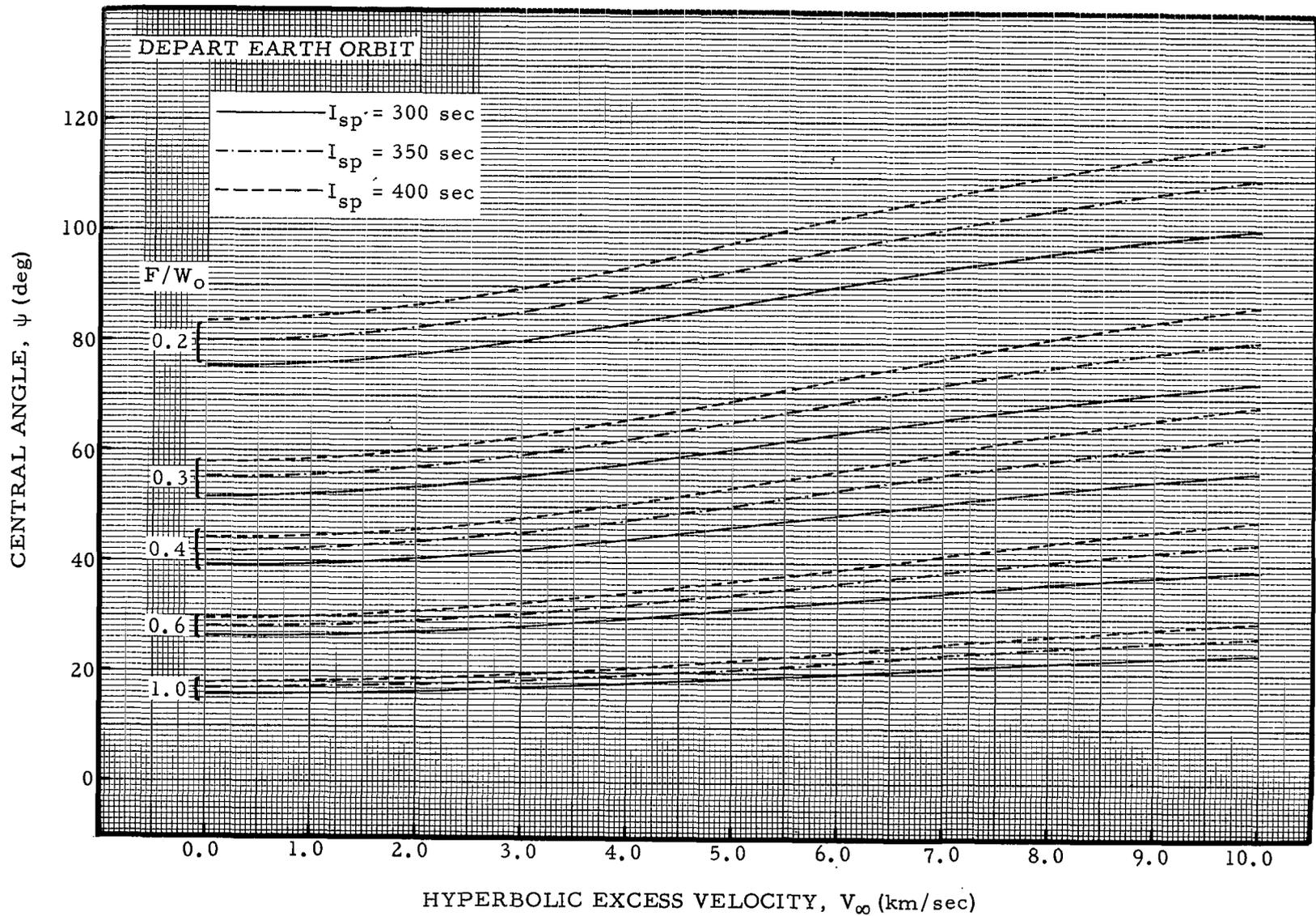


FIGURE 10. CENTRAL ANGLE VERSUS HYPERBOLIC EXCESS VELOCITY WITH THRUST-TO-WEIGHT RATIO FOR SPECIFIC IMPULSES OF 300, 350, AND 400 SECONDS AS A PARAMETER

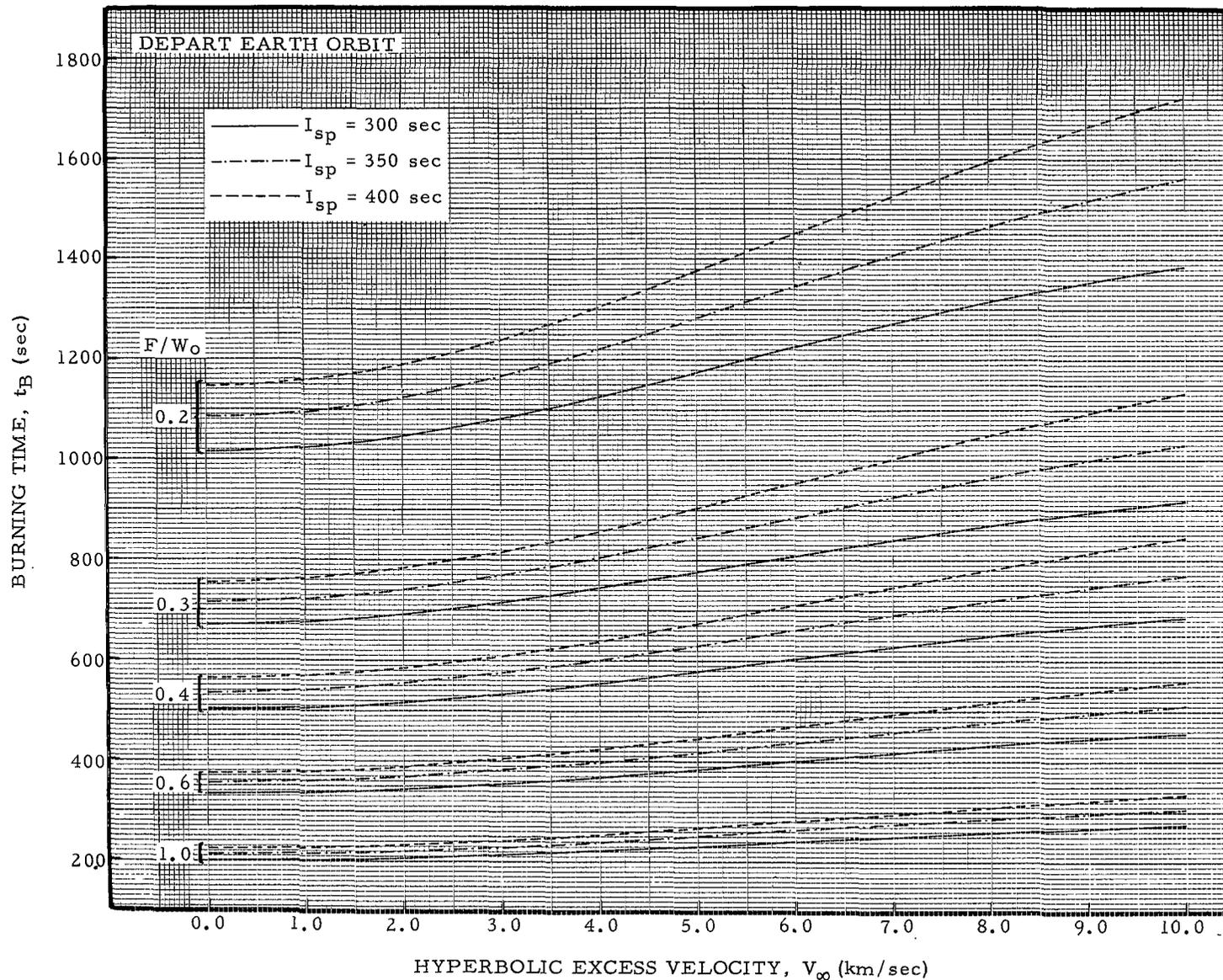


FIGURE 11. BURNING TIME VERSUS HYPERBOLIC EXCESS VELOCITY WITH THRUST-TO-WEIGHT RATIO FOR SPECIFIC IMPULSES OF 300, 350, AND 400 SECONDS AS A PARAMETER

Brake to Earth Orbit

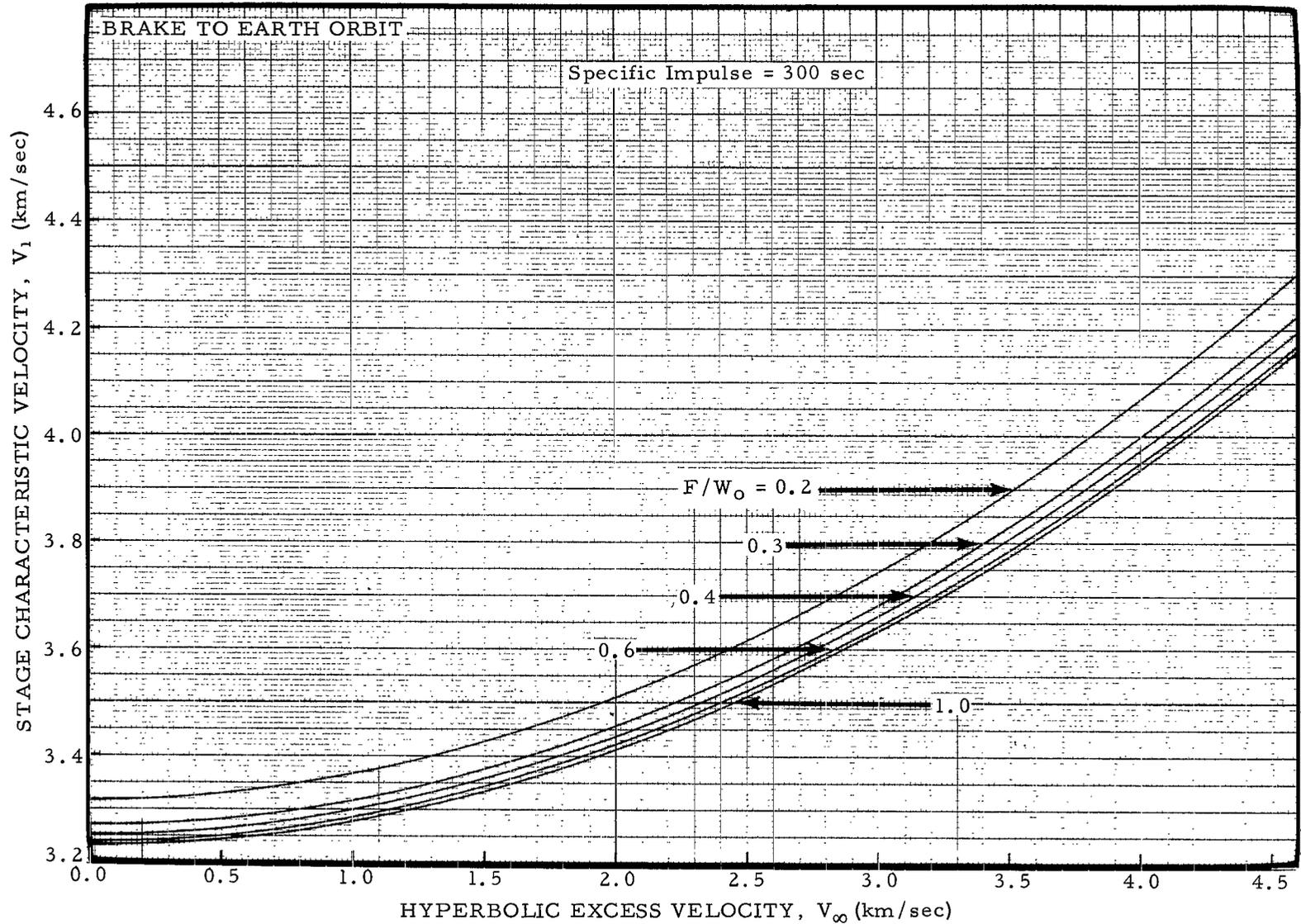


FIGURE 1a. CHARACTERISTIC VELOCITY VERSUS HYPERBOLIC EXCESS VELOCITY WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 300 SECONDS FOR HYPERBOLIC EXCESS VELOCITIES OF 0.0 THROUGH 4.6 KM/SEC

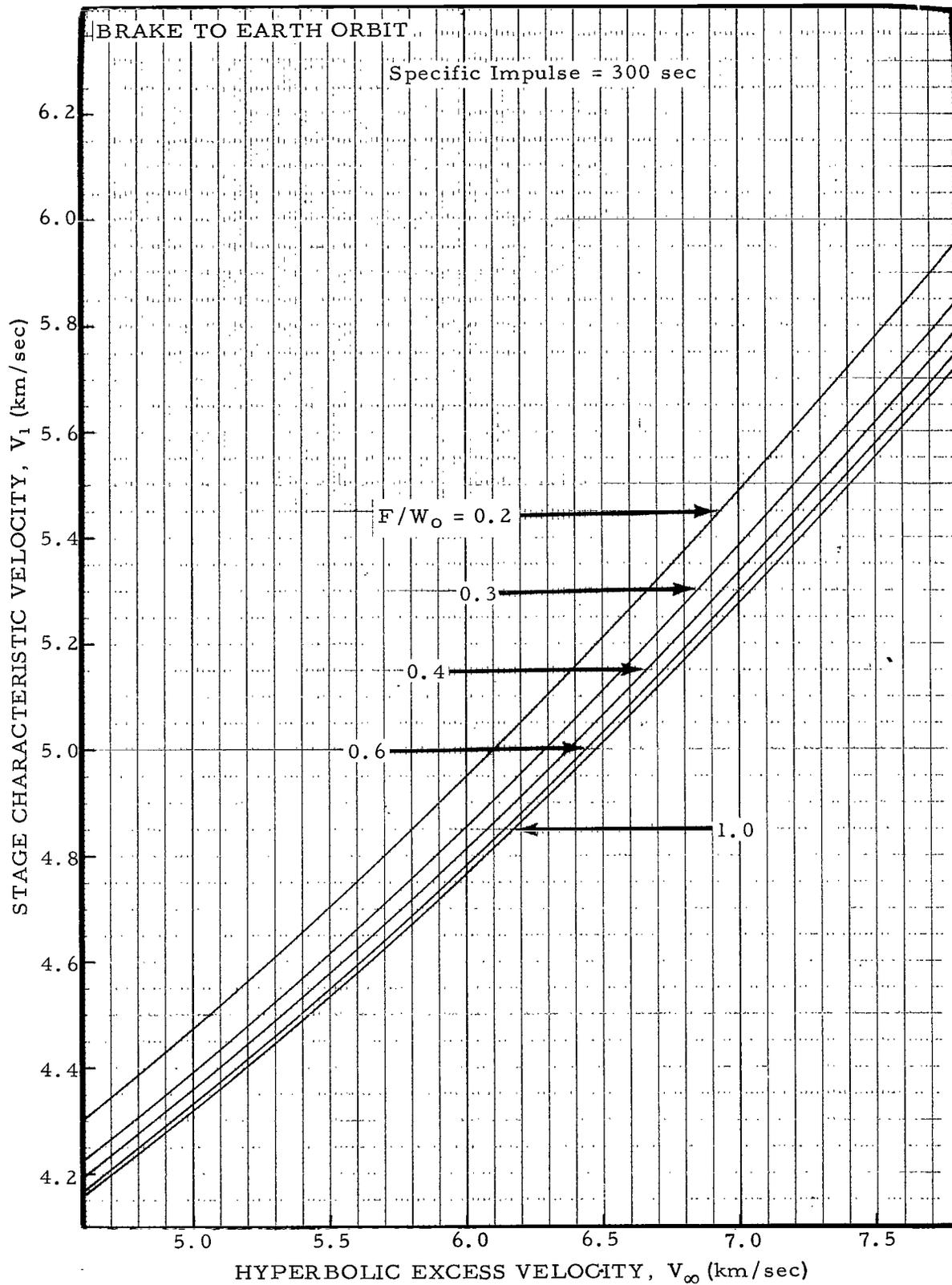


FIGURE 1b. CHARACTERISTIC VELOCITY VERSUS HYPERBOLIC EXCESS VELOCITY WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 300 SECONDS FOR HYPERBOLIC EXCESS VELOCITIES OF 4.6 THROUGH 7.8 KM/SEC

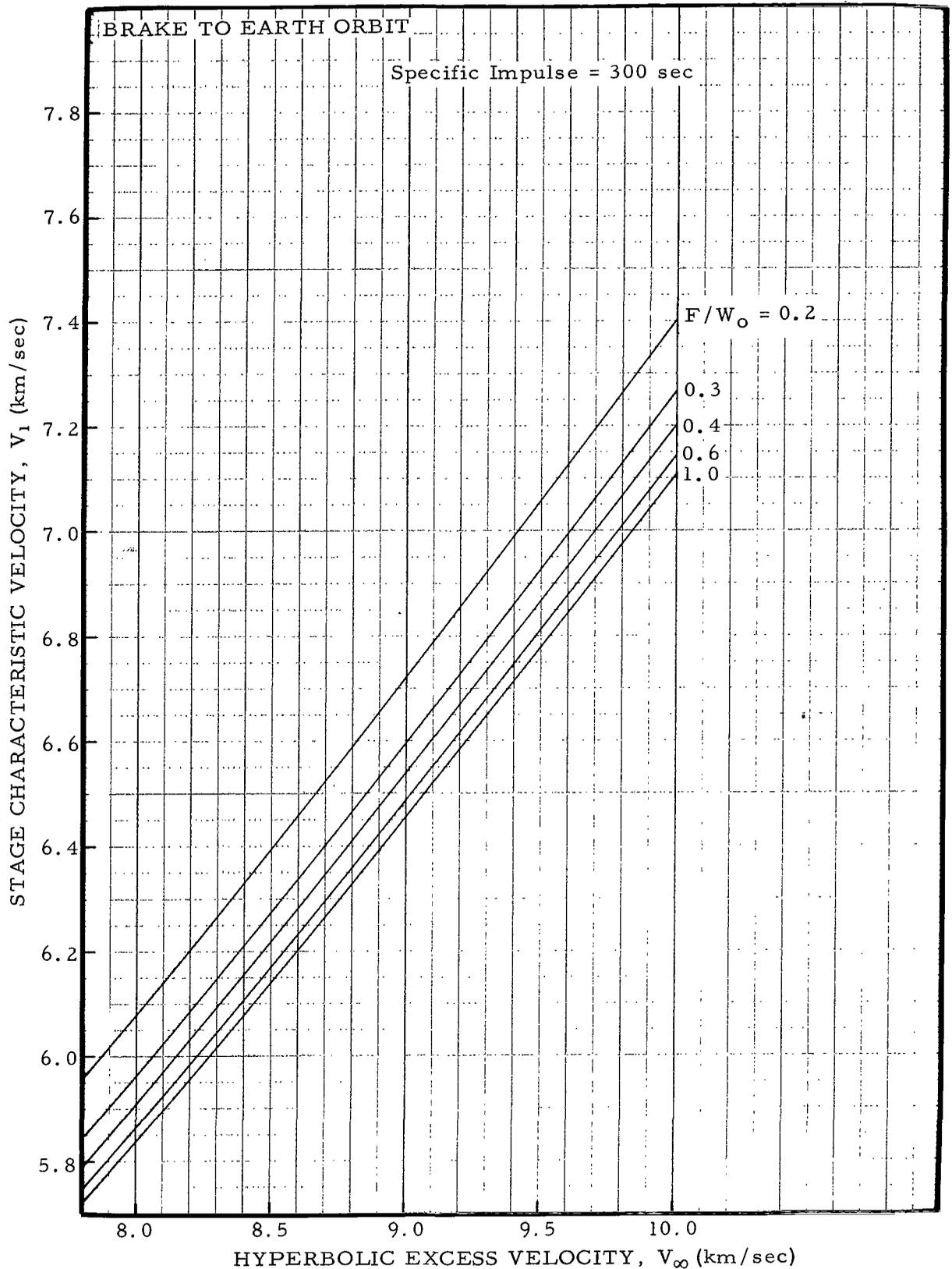


FIGURE 1c. CHARACTERISTIC VELOCITY VERSUS HYPERBOLIC EXCESS VELOCITY WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 300 SECONDS FOR HYPERBOLIC EXCESS VELOCITIES OF 7.8 THROUGH 10.0 KM/SEC

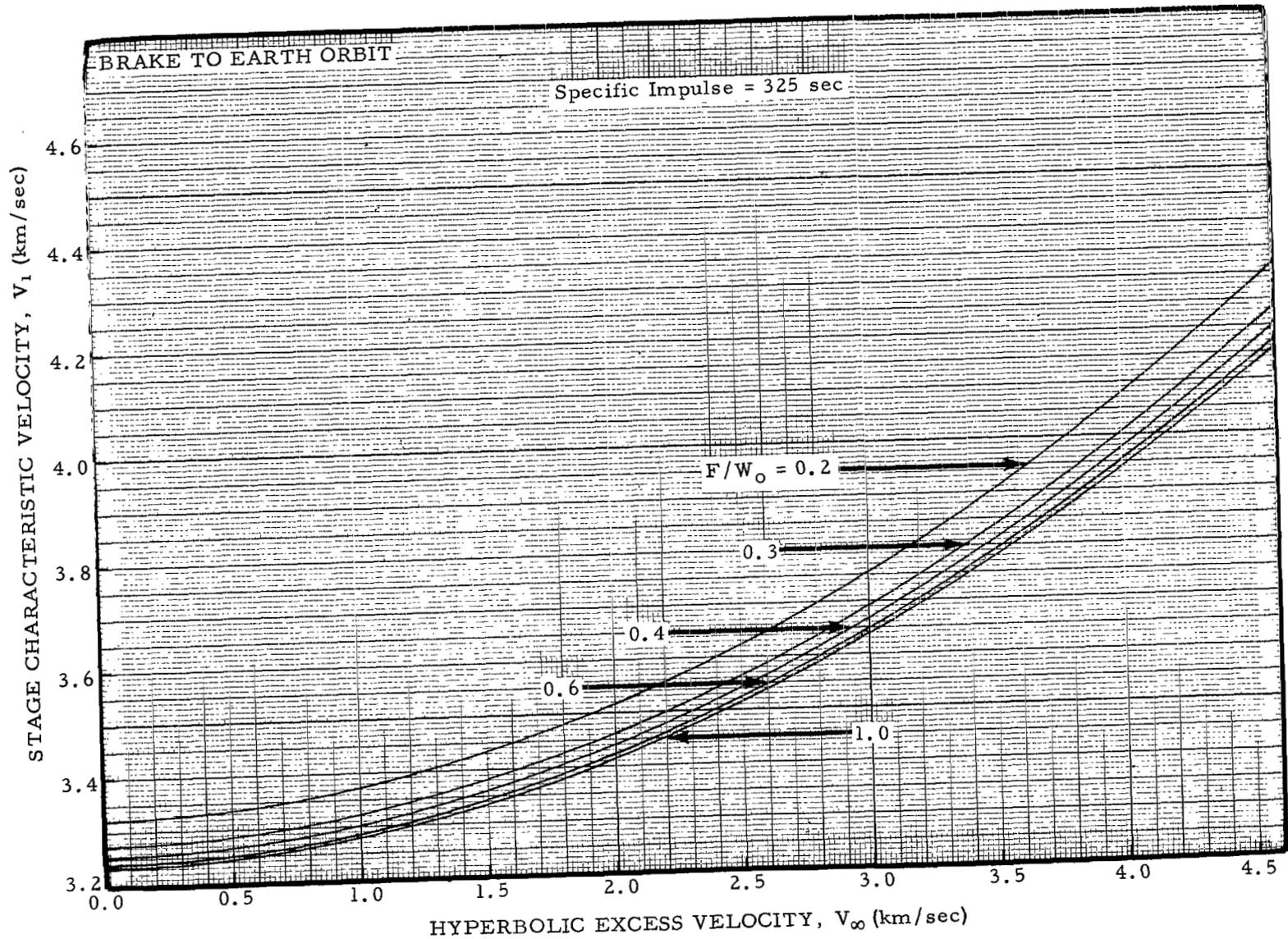


FIGURE 2a. CHARACTERISTIC VELOCITY VERSUS HYPERBOLIC EXCESS VELOCITY WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 325 SECONDS FOR HYPERBOLIC EXCESS VELOCITIES OF 0.0 THROUGH 4.6 KM/SEC

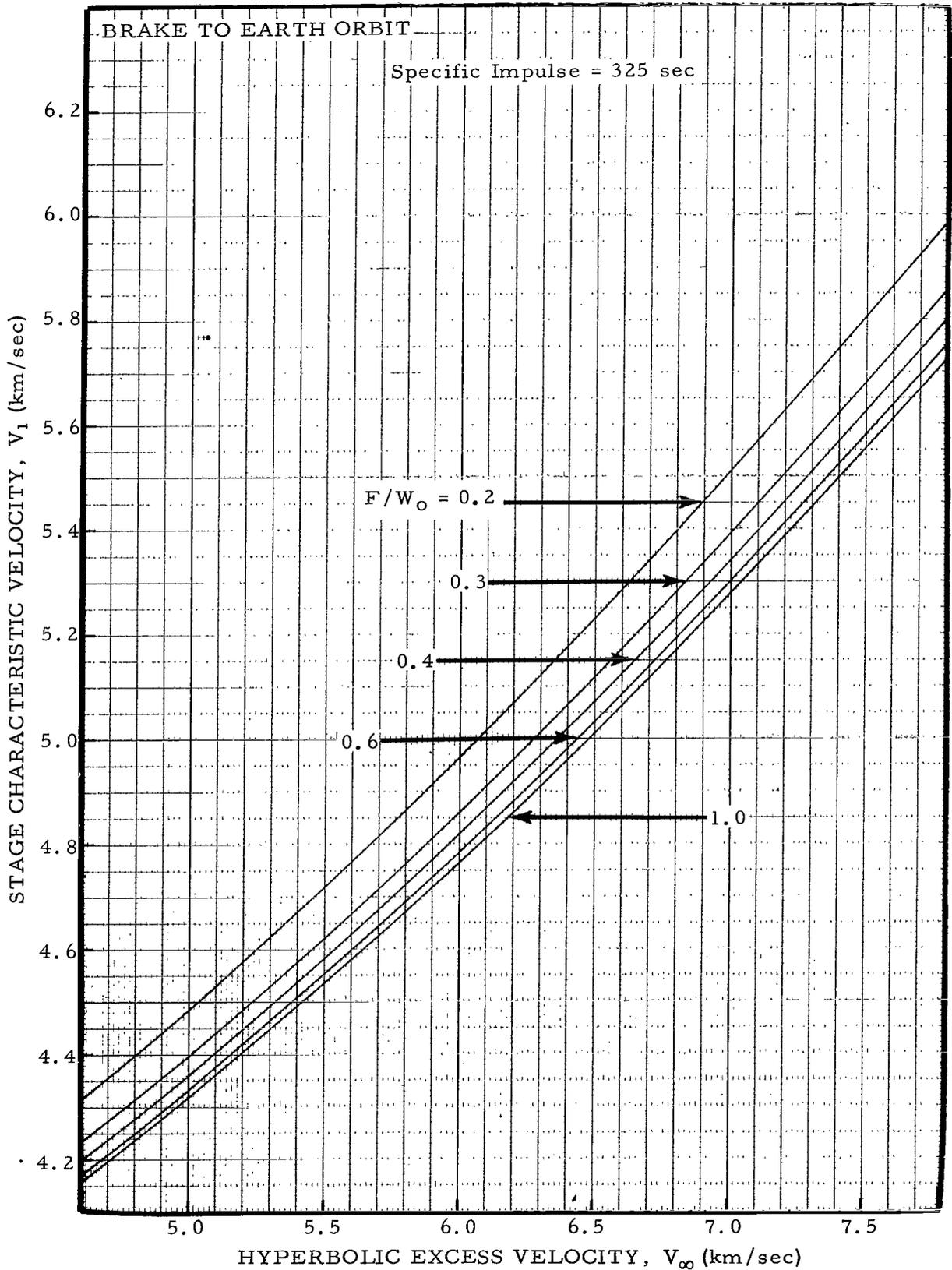


FIGURE 2b. CHARACTERISTIC VELOCITY VERSUS HYPERBOLIC EXCESS VELOCITY WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 325 SECONDS FOR HYPERBOLIC EXCESS VELOCITIES OF 4.6 THROUGH 7.8 KM/SEC

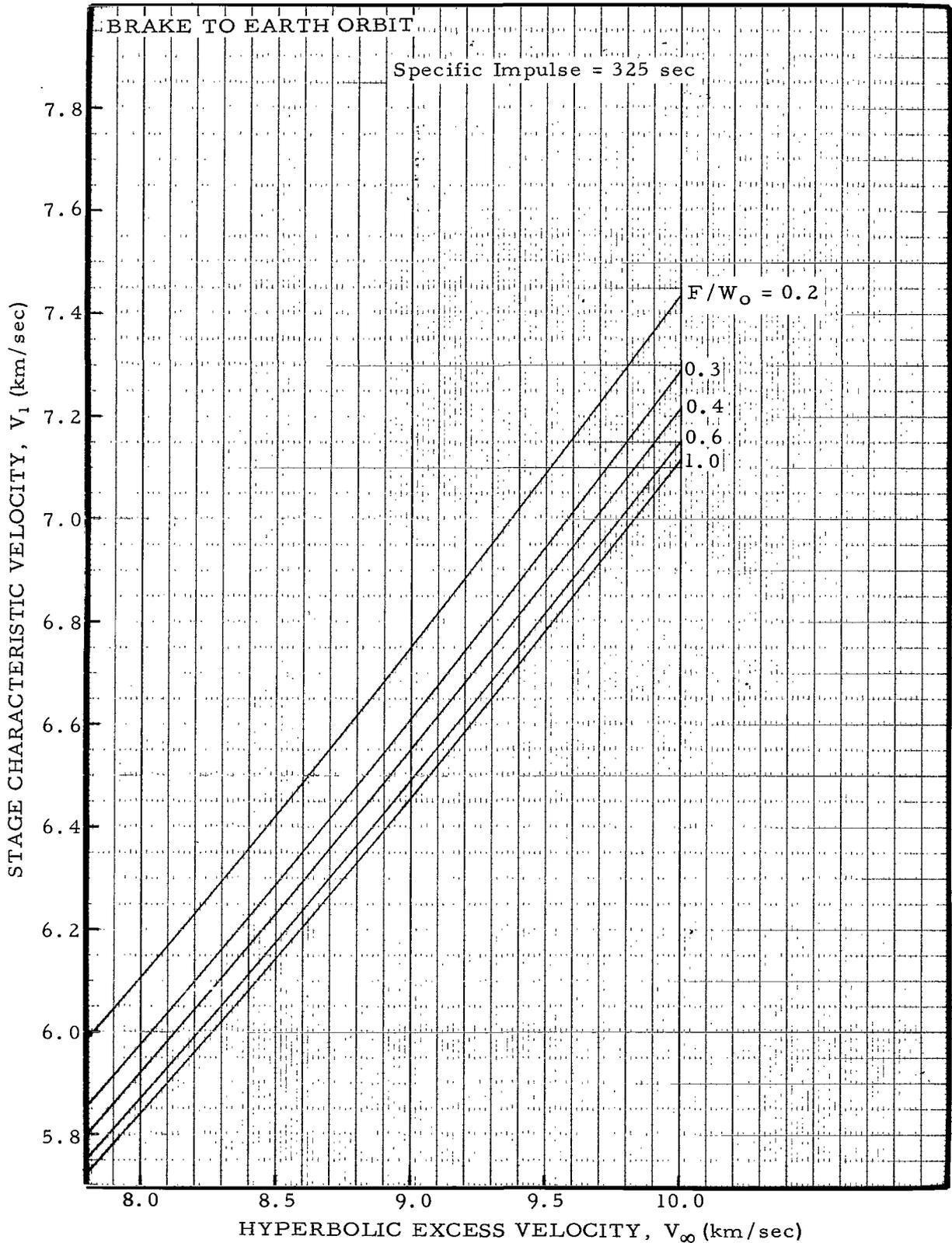


FIGURE 2c. CHARACTERISTIC VELOCITY VERSUS HYPERBOLIC EXCESS VELOCITY WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 325 SECONDS FOR HYPERBOLIC EXCESS VELOCITIES OF 7.8 THROUGH 10.0 KM/SEC

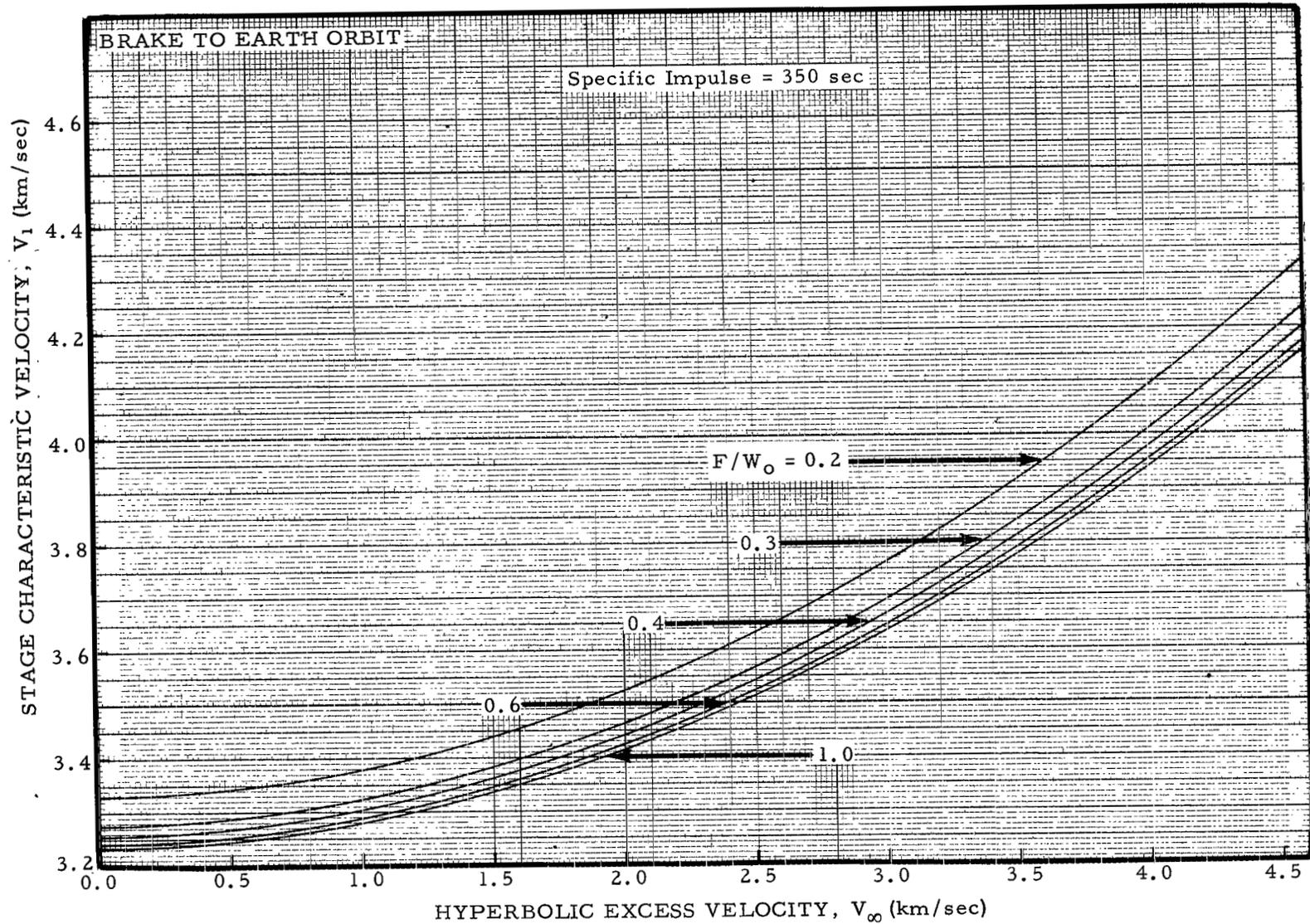


FIGURE 3a. CHARACTERISTIC VELOCITY VERSUS HYPERBOLIC EXCESS VELOCITY WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 350 SECONDS FOR HYPERBOLIC EXCESS VELOCITIES OF 0.0 THROUGH 4.6 KM/SEC

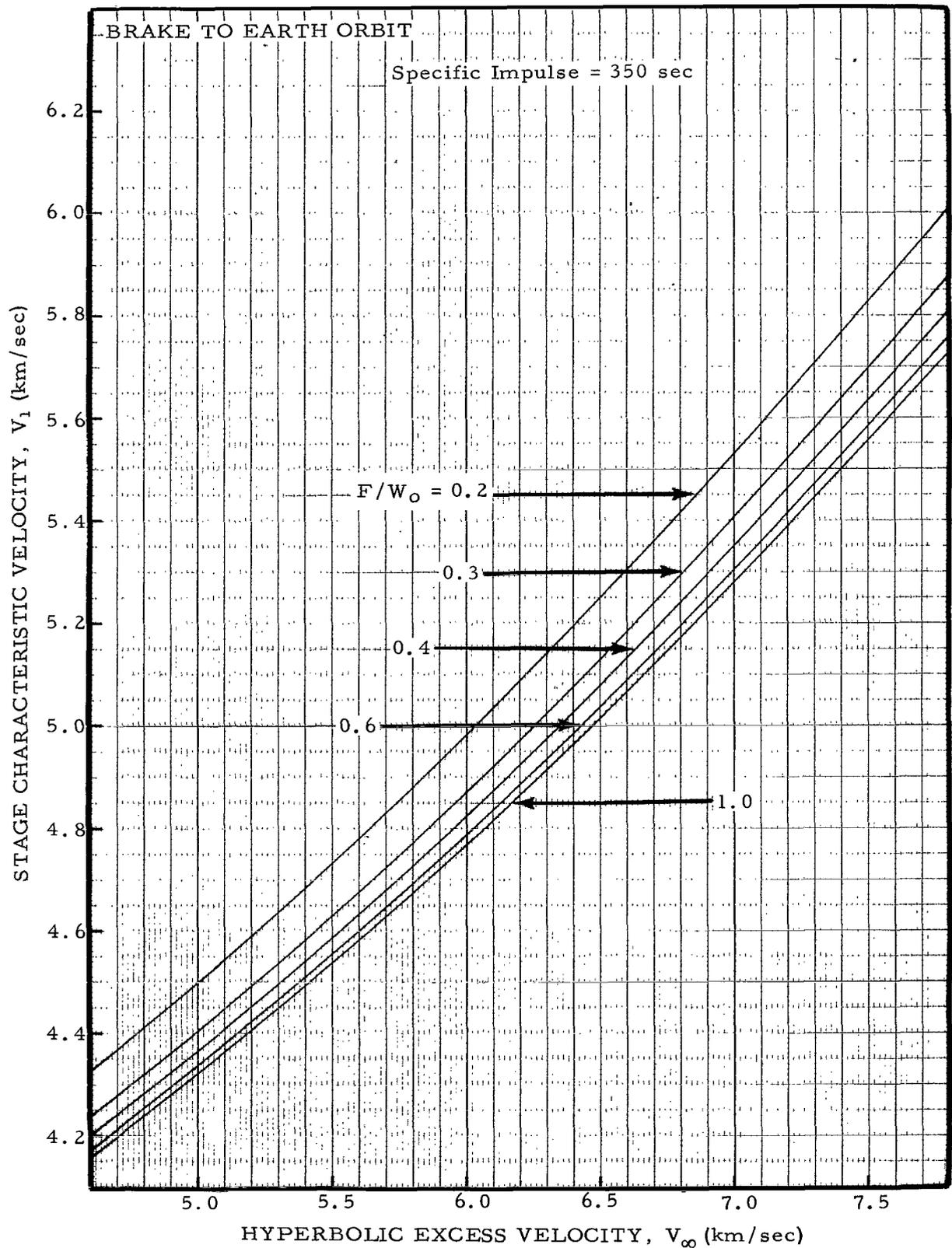


FIGURE 3b. CHARACTERISTIC VELOCITY VERSUS HYPERBOLIC EXCESS VELOCITY WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 350 SECONDS FOR HYPERBOLIC VELOCITIES OF 4.6 THROUGH 7.8 KM/SEC

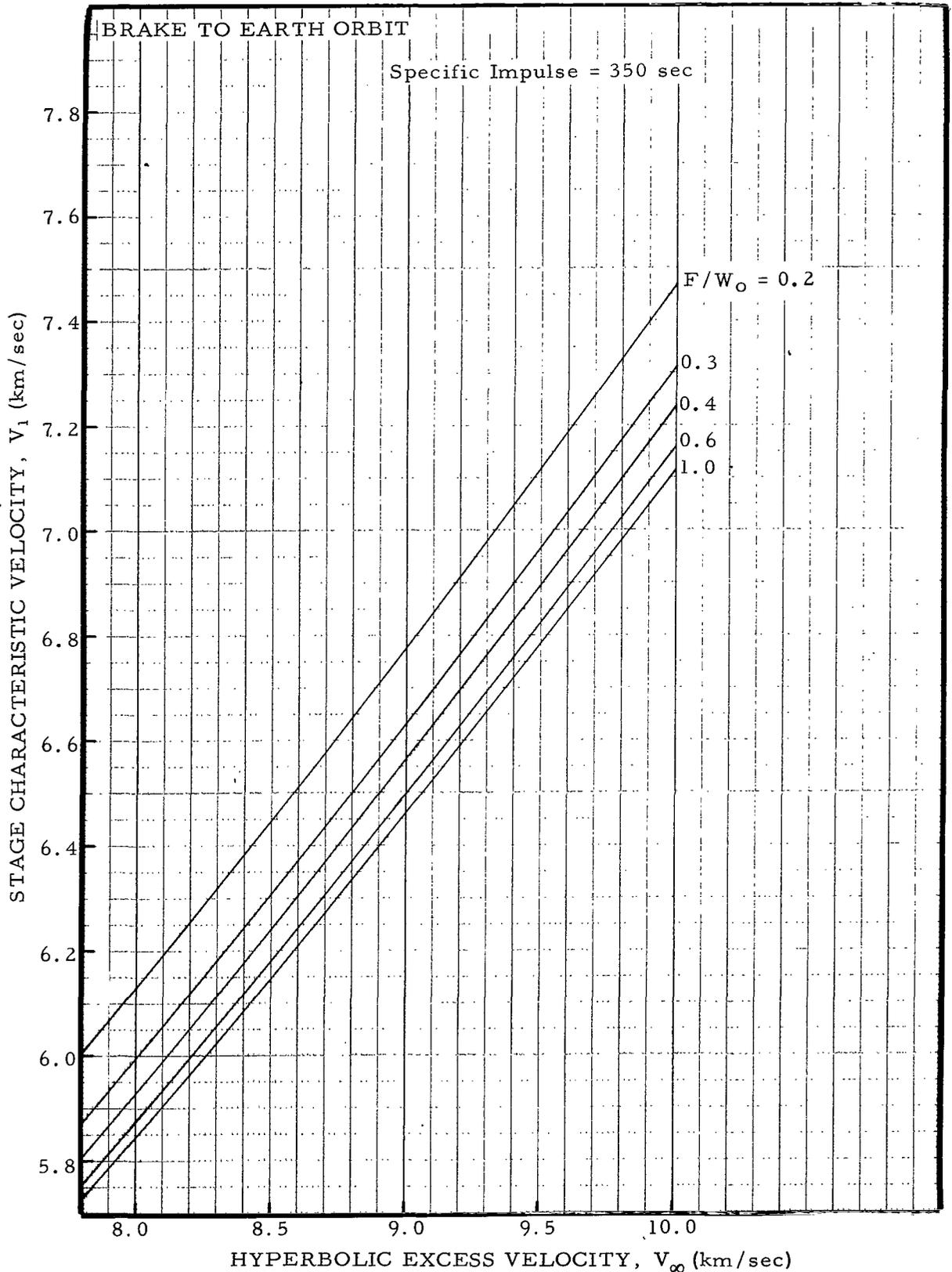


FIGURE 3c. CHARACTERISTIC VELOCITY VERSUS HYPERBOLIC EXCESS VELOCITY WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 350 SECONDS FOR HYPERBOLIC EXCESS VELOCITIES OF 4.6 THROUGH 7.8 KM/SEC

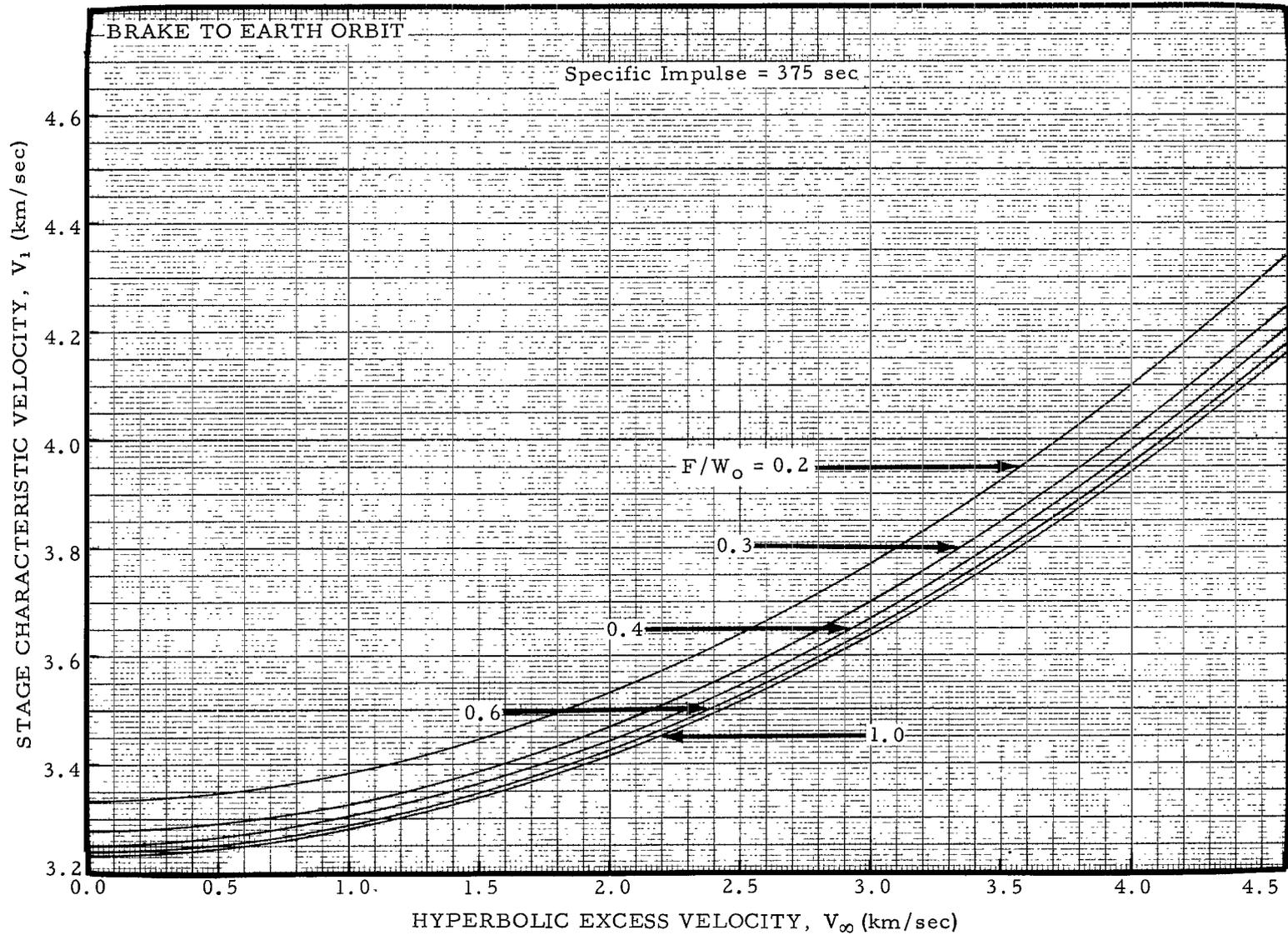


FIGURE 4a. CHARACTERISTIC VELOCITY VERSUS HYPERBOLIC EXCESS VELOCITY WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 375 SECONDS FOR HYPERBOLIC EXCESS VELOCITIES OF 0.0 THROUGH 4.6 KM/SEC

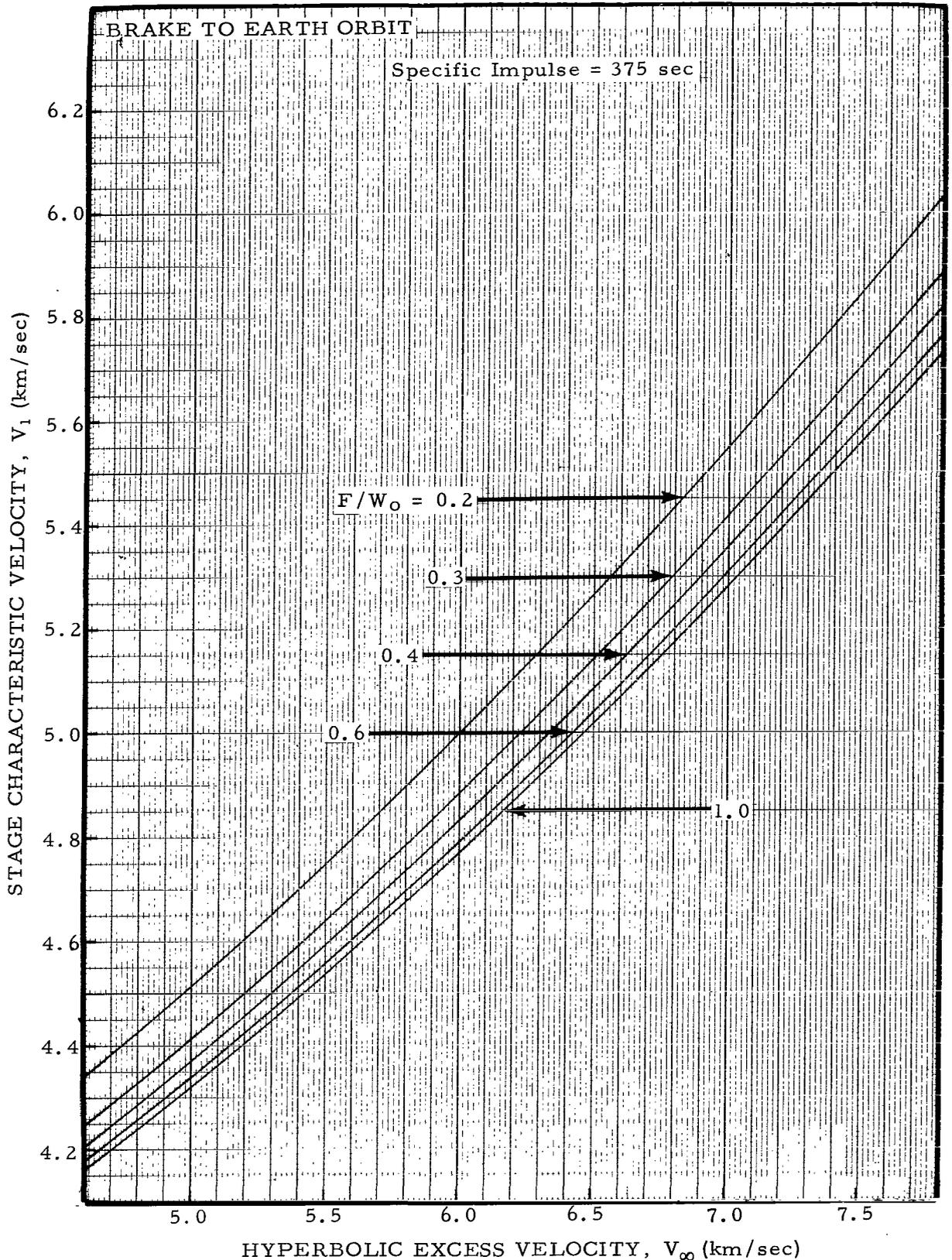


FIGURE 4b. CHARACTERISTIC VELOCITY VERSUS HYPERBOLIC EXCESS VELOCITY WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 375 SECONDS FOR HYPERBOLIC EXCESS VELOCITIES OF 4.6 THROUGH 7.8 KM/SEC

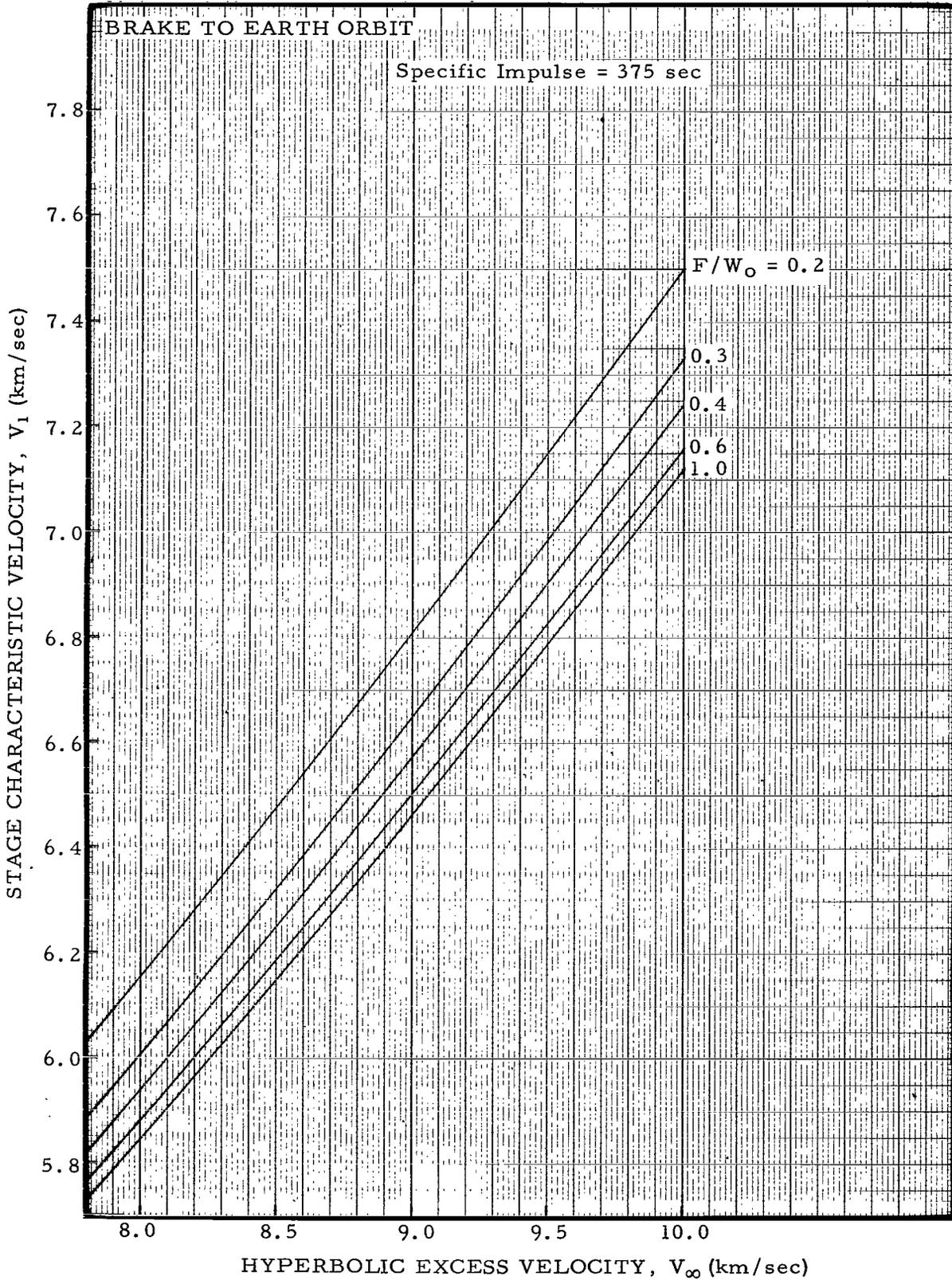


FIGURE 4c. CHARACTERISTIC VELOCITY VERSUS HYPERBOLIC EXCESS VELOCITY WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 375 SECONDS FOR HYPERBOLIC EXCESS VELOCITIES OF 7.8 THROUGH 10.0 KM/SEC

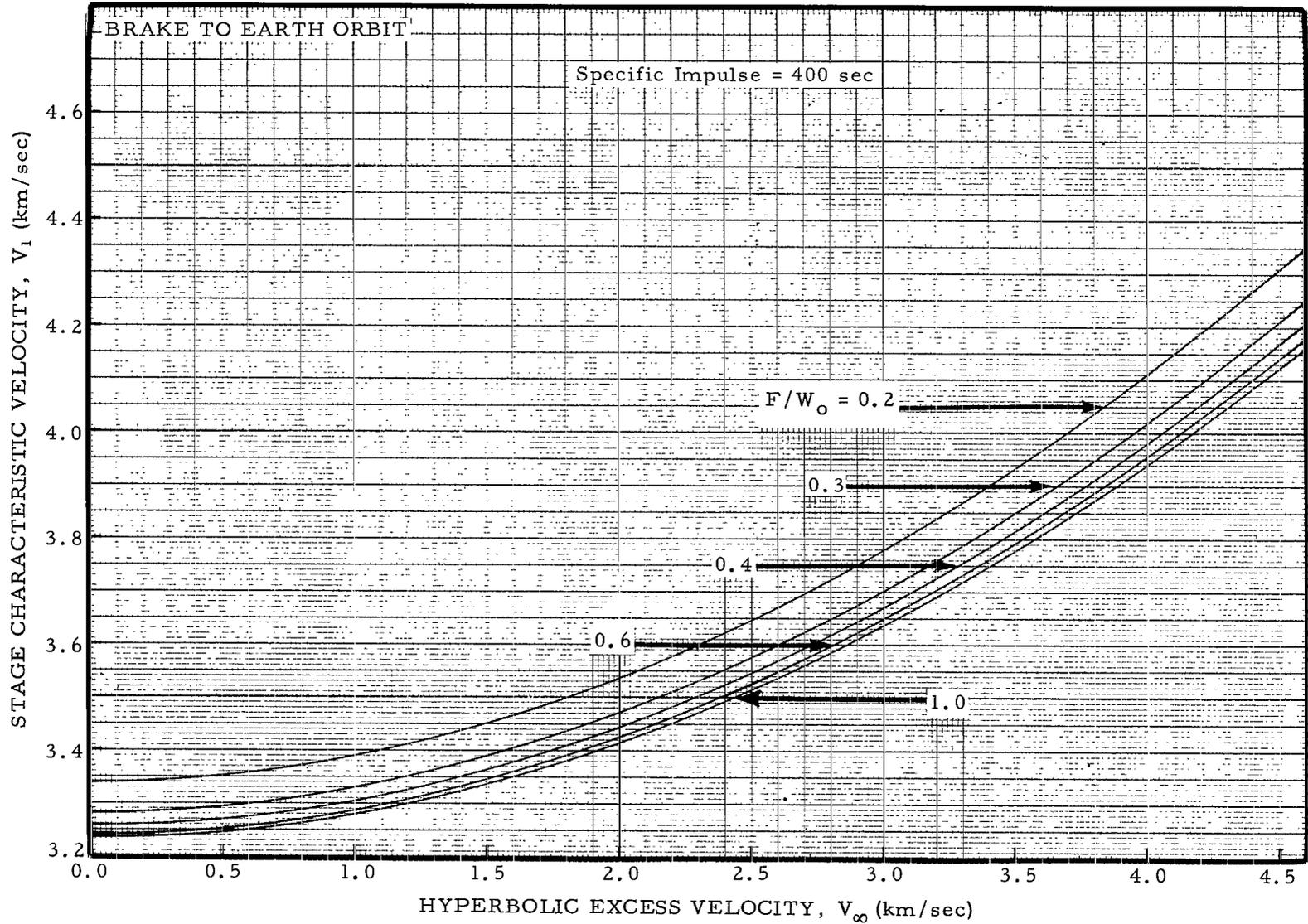


FIGURE 5a. CHARACTERISTIC VELOCITY VERSUS HYPERBOLIC EXCESS VELOCITY WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 400 SECONDS FOR HYPERBOLIC EXCESS VELOCITIES OF 0.0 THROUGH 4.6 KM/SEC

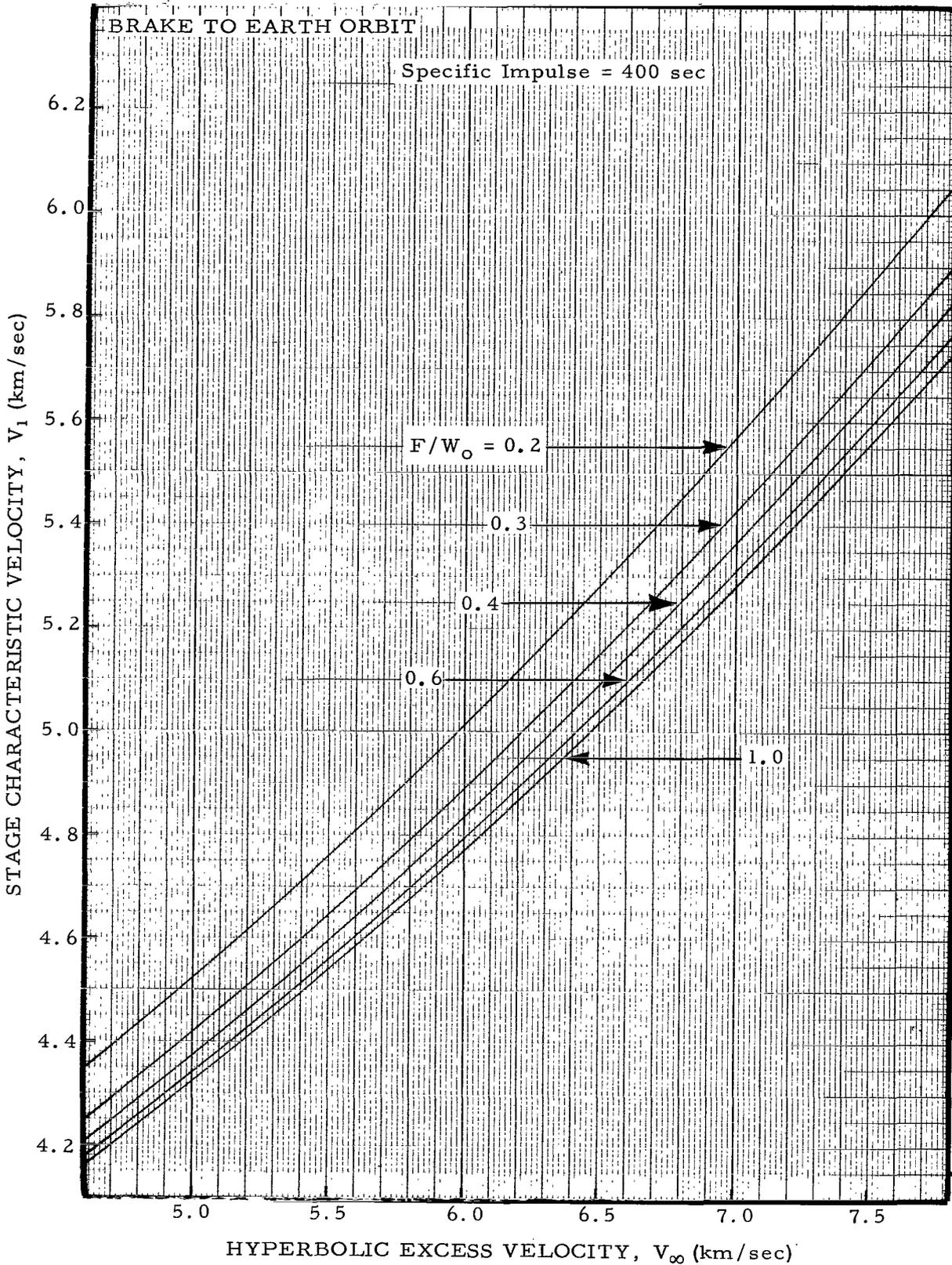


FIGURE 5b. CHARACTERISTIC VELOCITY VERSUS HYPERBOLIC EXCESS VELOCITY WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 400 SECONDS FOR HYPERBOLIC EXCESS VELOCITIES OF 4.6 THROUGH 7.8 KM/SEC

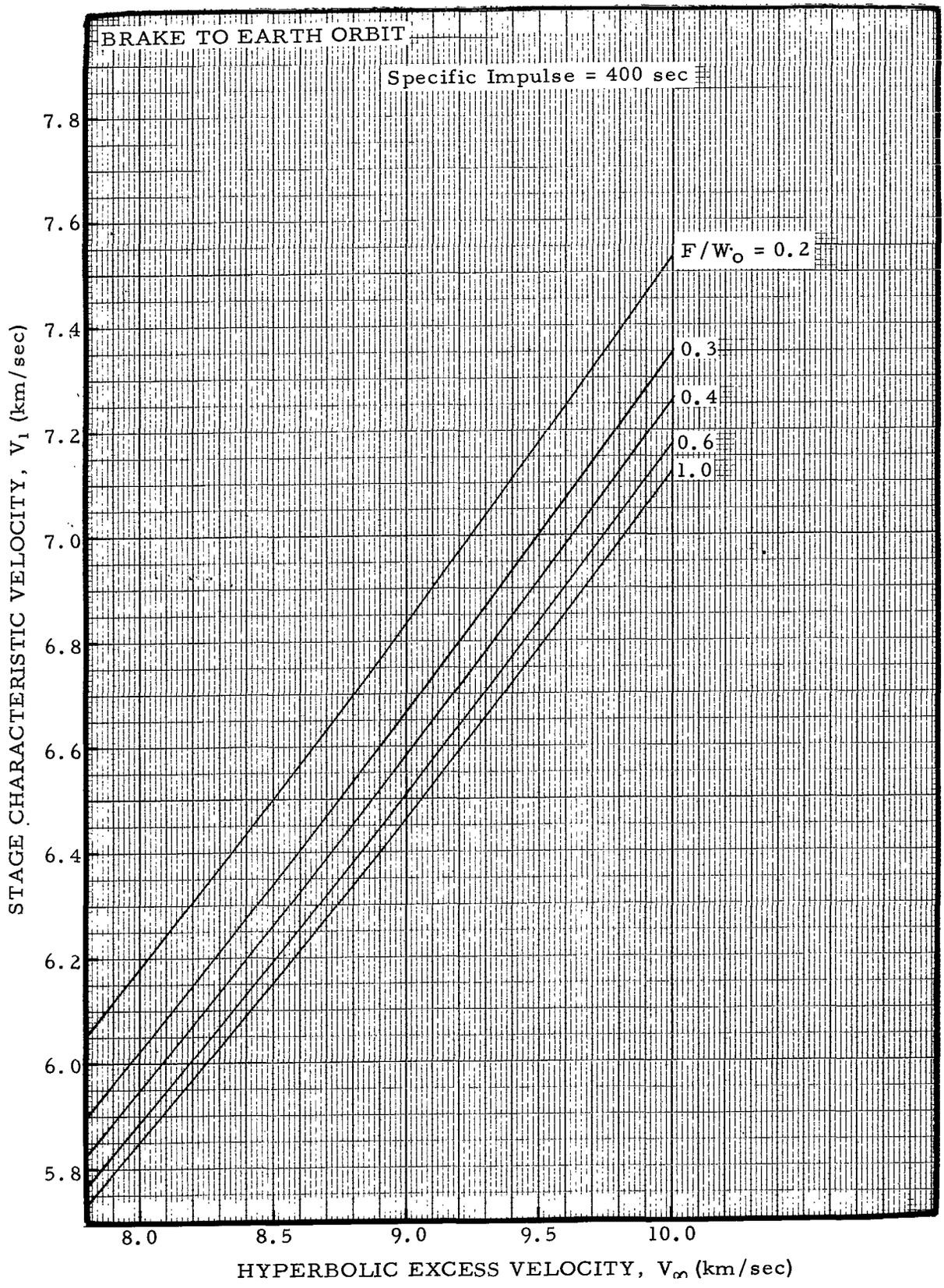


FIGURE 5c. CHARACTERISTIC VELOCITY VERSUS HYPERBOLIC EXCESS VELOCITY WITH THRUST-TO-WEIGHT RATIO AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 400 SECONDS FOR HYPERBOLIC VELOCITIES OF 7.8 THROUGH 10.0 KM/SEC

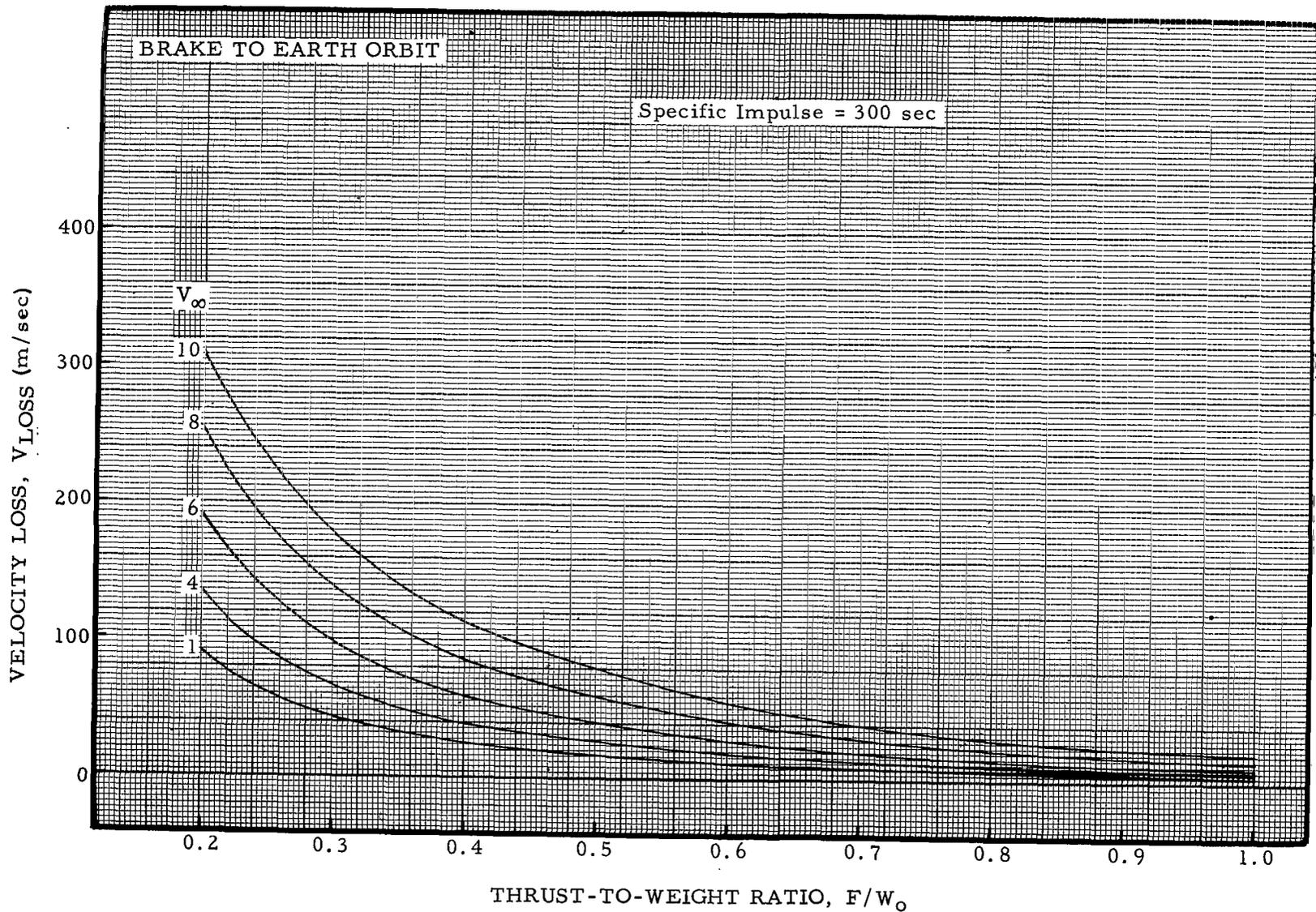


FIGURE 6. VELOCITY LOSS DUE TO GRAVITY VERSUS THRUST-TO-WEIGHT RATIO WITH HYPERBOLIC EXCESS VELOCITY AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 300 SECONDS

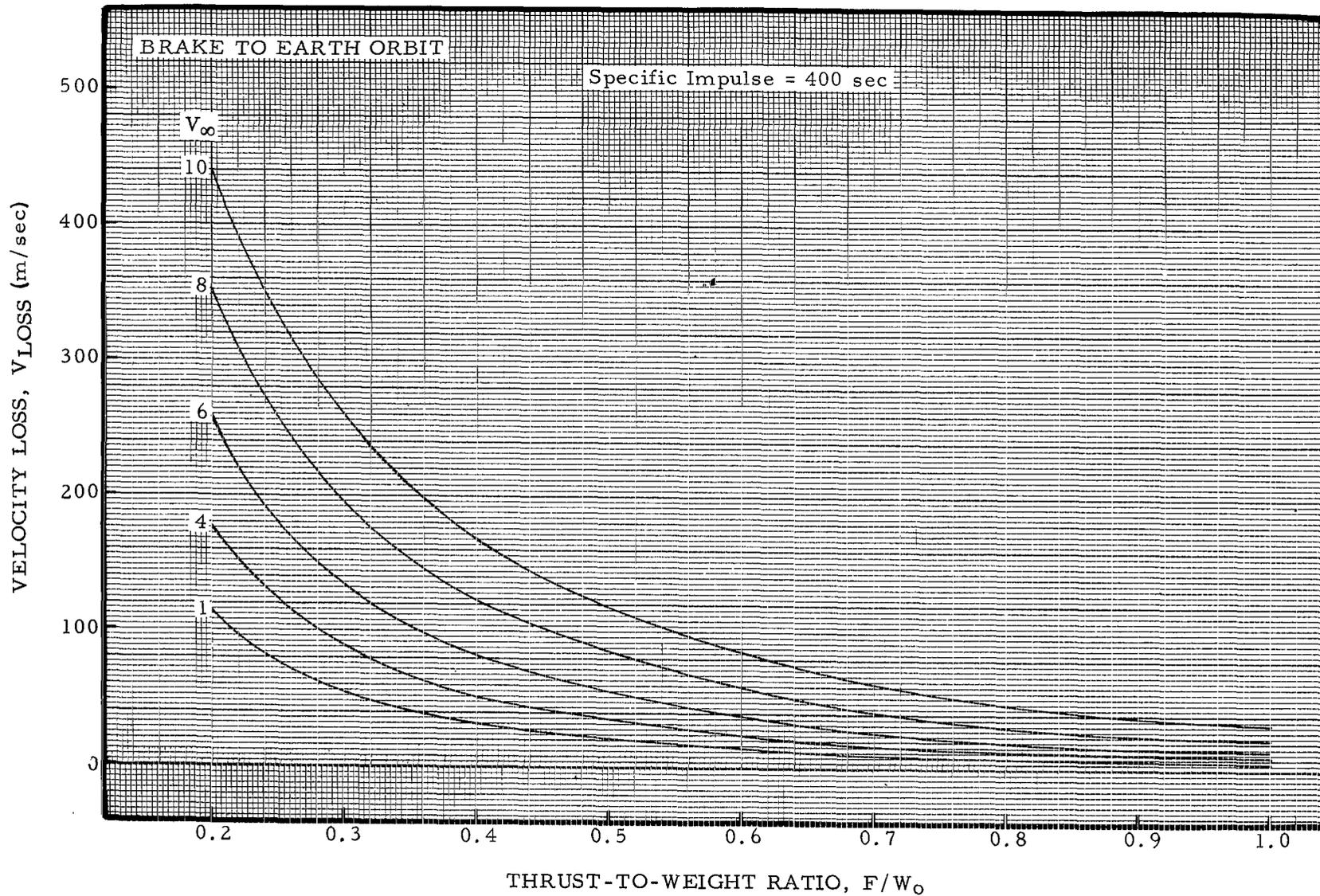


FIGURE 7. VELOCITY LOSS DUE TO GRAVITY VERSUS THRUST-TO-WEIGHT RATIO WITH HYPERBOLIC EXCESS VELOCITY AS A PARAMETER FOR A CONSTANT SPECIFIC IMPULSE OF 400 SECONDS

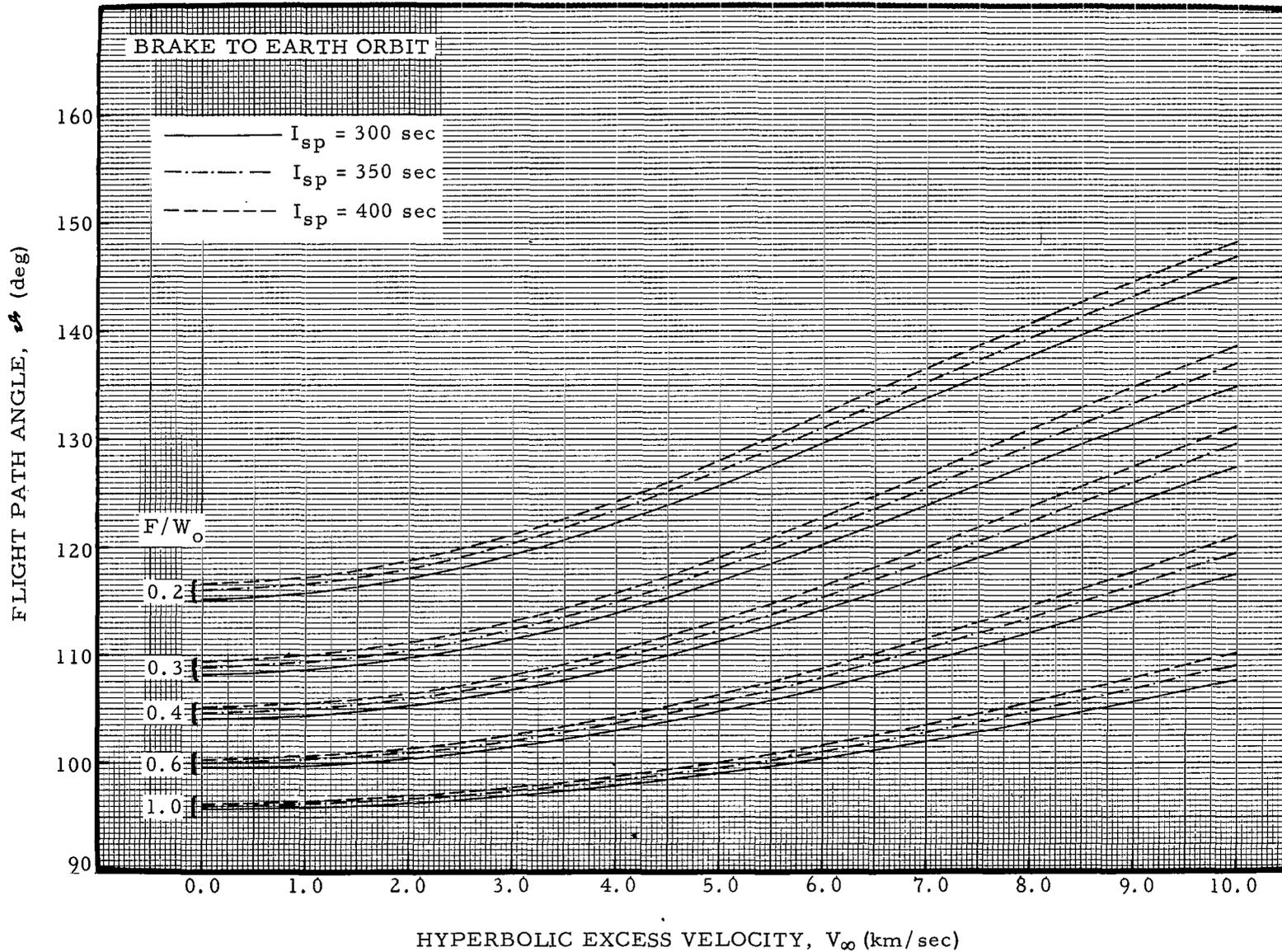


FIGURE 8. FLIGHT PATH ANGLE VERSUS HYPERBOLIC EXCESS VELOCITY WITH THRUST-TO-WEIGHT RATIO FOR SPECIFIC IMPULSES OF 300, 350, AND 400 SECONDS AS A PARAMETER

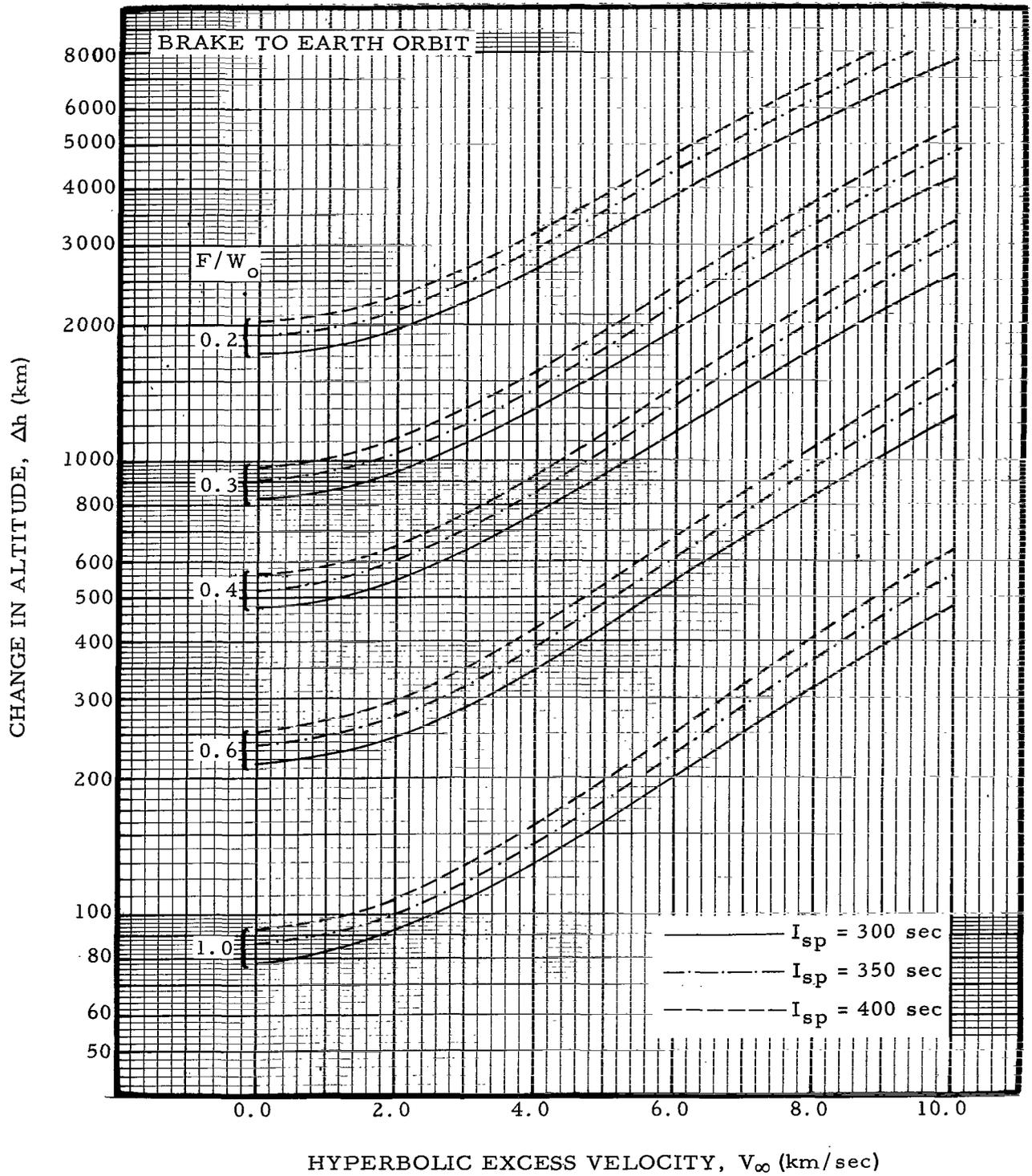


FIGURE 9. CHANGE IN ALTITUDE VERSUS HYPERBOLIC EXCESS VELOCITY WITH THRUST-TO-WEIGHT RATIO FOR SPECIFIC IMPULSES OF 300, 350, AND 400 SECONDS AS A PARAMETER

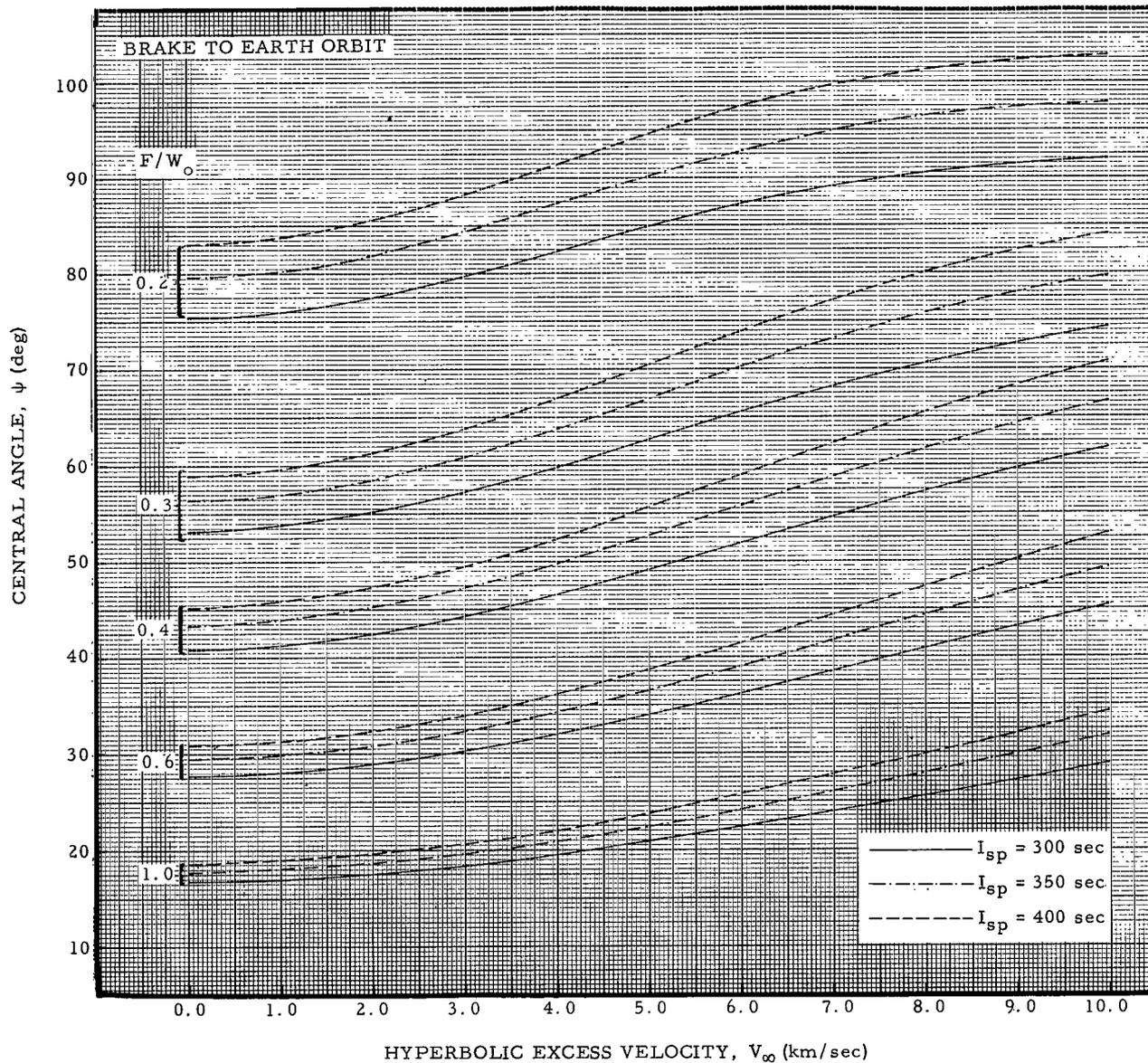


FIGURE 10. CENTRAL ANGLE VERSUS HYPERBOLIC EXCESS VELOCITY WITH THRUST-TO-WEIGHT RATIO FOR SPECIFIC IMPULSES OF 300, 350, AND 400 SECONDS AS A PARAMETER

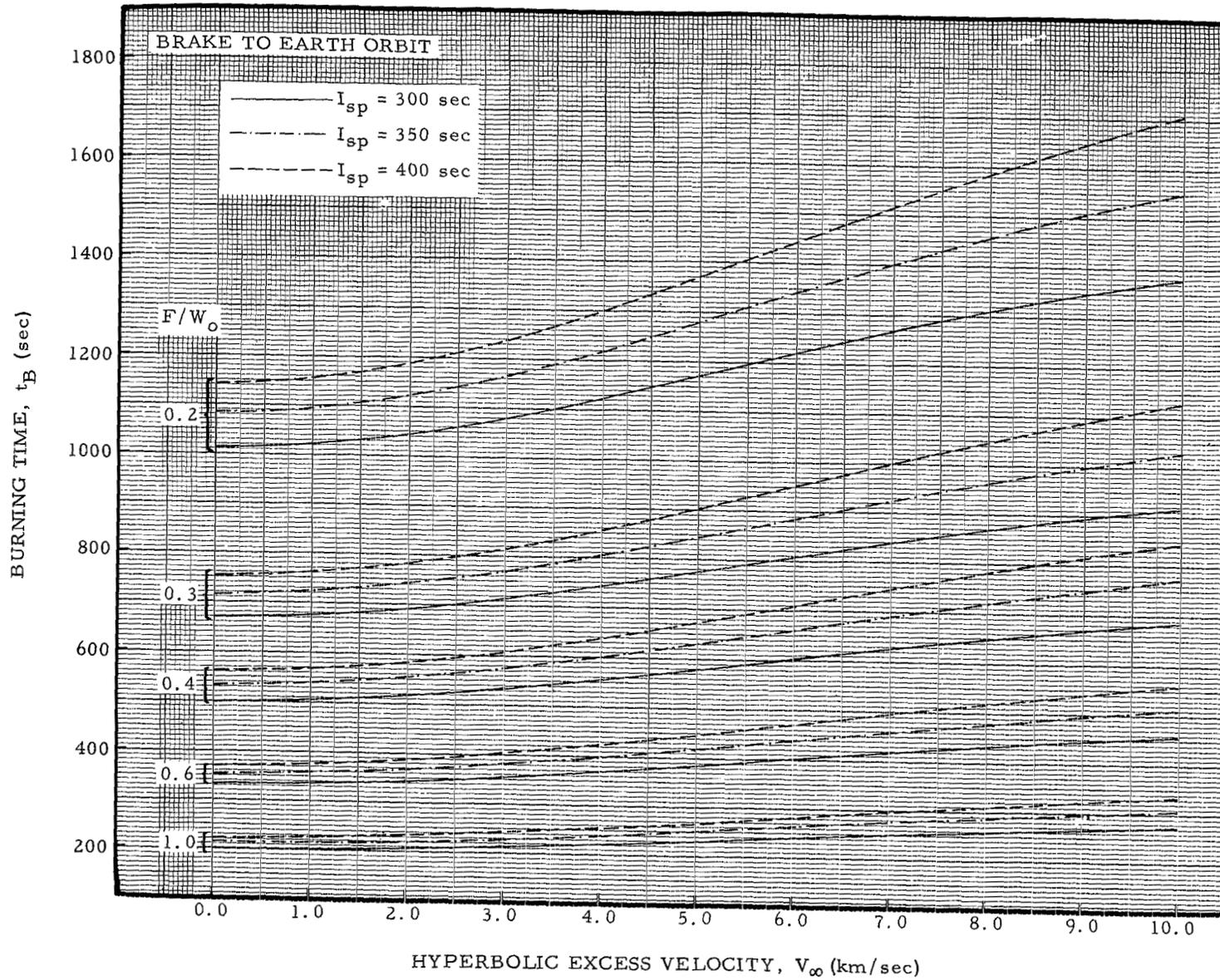


FIGURE 11. BURNING TIME VERSUS HYPERBOLIC EXCESS VELOCITY WITH THRUST-TO-WEIGHT RATIO FOR SPECIFIC IMPULSES OF 300, 350, AND 400 SECONDS AS A PARAMETER

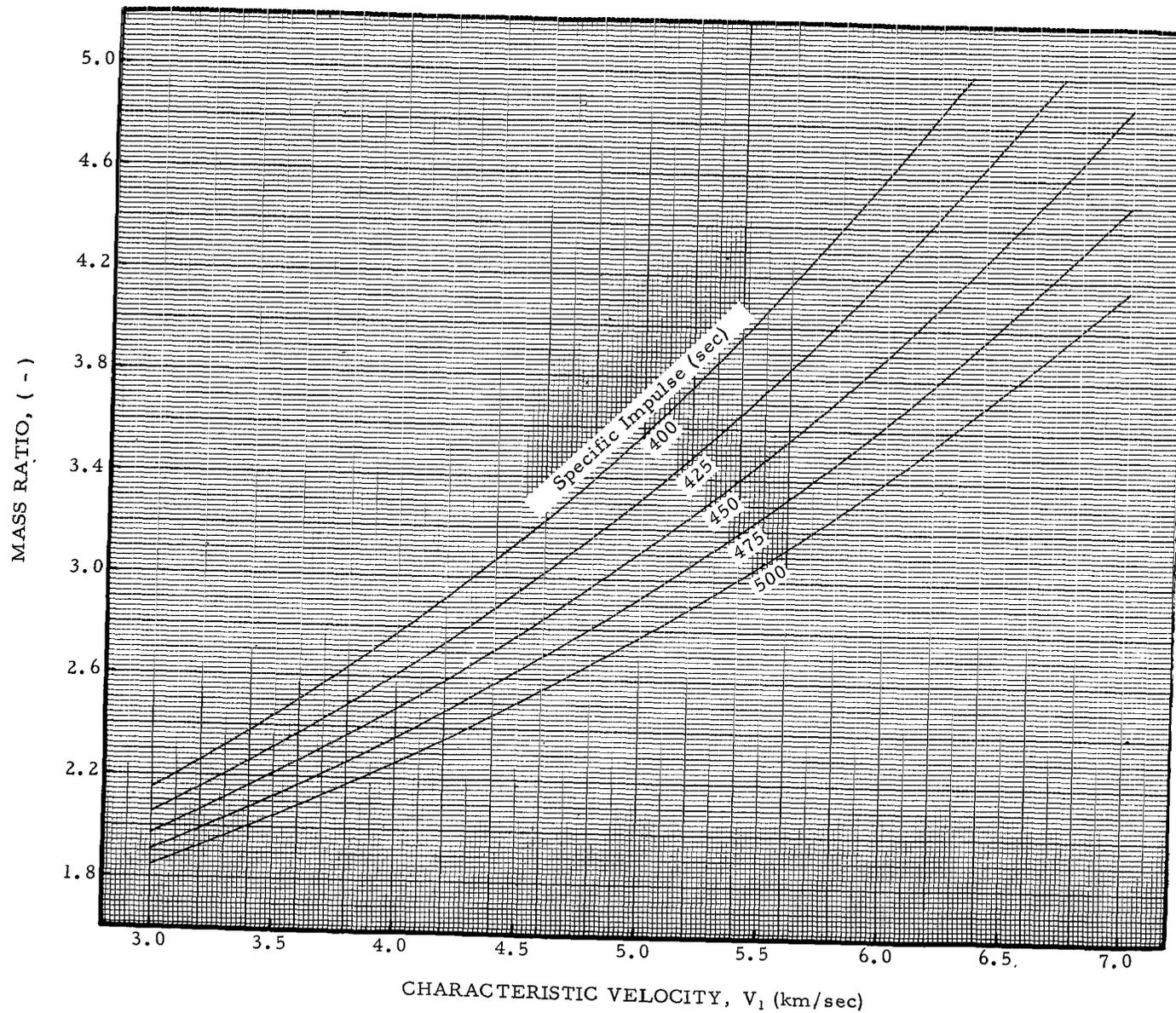


FIGURE 12. MASS RATIO VERSUS CHARACTERISTIC VELOCITY WITH SPECIFIC IMPULSE AS A PARAMETER

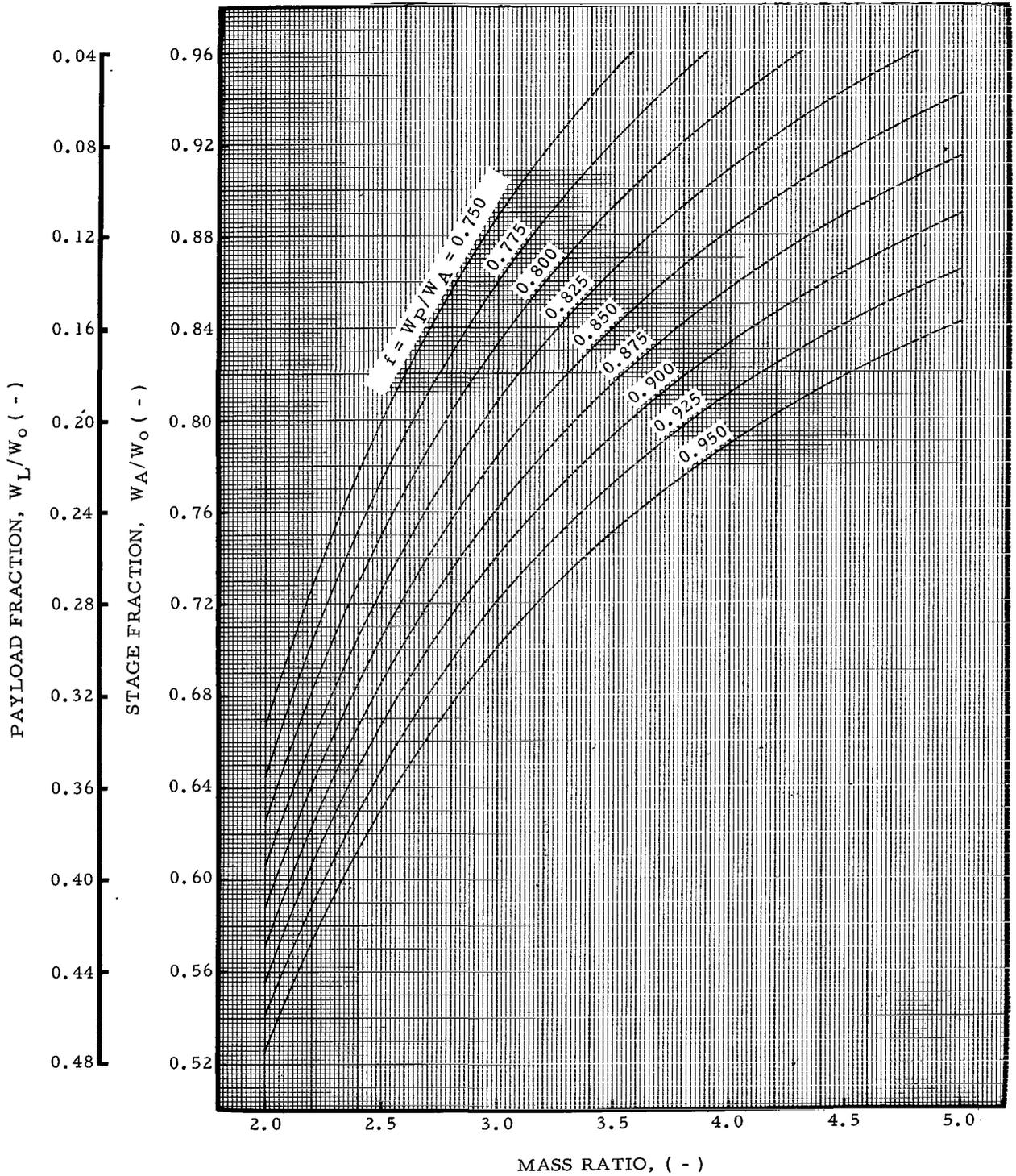


FIGURE 13. PAYLOAD FRACTION AND STAGE FRACTION VERSUS MASS RATIO WITH STAGE MASS FRACTION AS A PARAMETER

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