A Calculation of Proton-Induced Damage in Germanium

Edward A. Saunders* and Gerald P. Calame
Department of Nuclear Science and Engineering,
Rensselaer Polytechnic Institute, Troy, N.Y.

Experimental work of Corelli and Fischer involving high-energy proton irradiation of germanium shows disagreement between observed results and earlier damage calculations. This work accounts for the observed results by means of a complete optical-model calculation of elastic and inelastic cross sections and the use of the "channeling" theory of Oen and Robinson in the treatment of the displacement cascade. The model of Seitz and Koehler is used to account for the transition between elastic and inelastic reactions involving the high-energy primary knockons. Calculations were run for proton energies from 5 Mev to 140 Mev. The results...
show that agreement with experiment is obtained using the 
sharp cutoff of Seitz and Koehler between elastic and in-
elastic reactions provided the nonhomogeneity of the crystal 
is taken into account through the use of the Oen and Robinson 
channeling model.

I. INTRODUCTION

Interest in the problem of radiation damage to semiconductor devices 
has received new impetus in recent years by virtue of the extensive space 
exploration program. In particular, the effects of electron and proton 
radiations became of primary concern with the discovery that space vehicles 
would be exposed to such radiations in greater intensities and at higher 
energies than had been anticipated.

At the same time, extensive experimental work\textsuperscript{1,2,3} has been initiated 
in an attempt to understand the fundamental phenomena involved in radiation 
damage to semiconductor materials and devices. Concurrently, various 

\textit{Attempts have been made to fit currently accepted damage theory to the} 

experimental results.\textsuperscript{1,4,5,6} When one considers the variation of damage to 

silicon and germanium with the energy of incident protons, three major 

areas of disagreement between experiment and theory are apparent. First, 
the slope of the damage vs. energy curves for proton-irradiated semi-

conductor materials is much less than that predicted by current theory\textsuperscript{1,2,3}. 

For example, some of the experimental results for germanium using change 
in minority carrier lifetime as a damage probe show a damage curve which
is essentially independent of proton energy for energies from about 50 Mev to about 110 Mev. Second, the falloff in damage with energy (reduction in conductivity or decrease in minority carrier lifetime) is much less experimentally than that predicted by current calculations. Finally, there is a uniform tendency for the calculated results to be lower than experimental results for proton energies above 50 Mev.

In this work, an attempt was made, with some success, to predict the variation of damage to semiconductor materials as a function of the energy of incident protons. The recent availability of experimental results for proton damage on germanium has made it apparent that a really meaningful calculation of damage requires the inclusion of inelastic effects. One possible approach to such a calculation is that of Simon, et al., in which they assumed a reasonable reaction cross section and then proceeded to calculate numbers of displacements. Another possible approach has been suggested by Corelli whereby one uses the nuclear optical model to predict cross sections for germanium and silicon based upon experimentally determined cross sections for elements such as copper, zinc, nickel, and aluminum.

It is the suggestion of Corelli which furnished the basic concept for this work. The discussions of this article are restricted to proton damage on germanium. Calculations pertaining to silicon were made but are not discussed for the following reasons: (1) The methods used and the general trends of the results do not differ from those pertaining to germanium; (2) The greater portion of the experimental data on silicon pertains to
devices (silicon solar cells) rather than to the intrinsic material; (3) The
effects of spallation occur at a much lower energy in silicon than in ger-
manium and are not specifically accounted for in these calculations.

Proton energies from 5 Mev to 140 Mev were considered. The upper
limit was dictated by the size of the digital computer used for the calcula-
tions. However, since the majority of the available experimental data are
below 140 Mev, this is not considered a serious limitation. It must be
emphasized that the calculations are for intrinsic germanium and that they
cannot be considered applicable to situations in which the effects of doping
or other impurities are dominant.

Any calculation of radiation-induced displacements entails, of course,
two major portions. One of these is the question of secondary displace-
ments produced by energetic primary knockons. The other is the determi-
nation, either by experiment or calculation, of the differential cross section
for displacement of the primary knockon as a function of the energy of the
primary knockon \( \frac{d\sigma(E)}{dE} \) (which is, of course, also a function of the
energy of the incident proton). These questions are treated respectively
in Sections II and III. Section IV is a summary of the results of these
calculations together with the methods used to calculate total number of
displacements. The calculated results are compared with the available
experimental results in Section V. Section VI outlines certain conclusions
which one can draw from these results and points out several critical
problem areas in which additional work is required.
II. THE DISPLACEMENT CASCADE MODEL

One of the earliest, and probably still the most widely used, of the displacement cascade models is that of Kinchin and Pease.\textsuperscript{12} When this model is generalized to include the effect of inelastic or ionizing interactions of the high-energy recoiling atoms, it becomes\textsuperscript{12,13}

\[
g(E) = 0 \quad E \leq E_d \\
g(E) = 1 \quad E < E \leq 2E_d \\
g(E) = E/2E_d \quad 2E_d < \leq E_{10a} \\
g(E) = E_{10a}/2E_d \quad E_{10a} < E
\]

in which:

\( E \) = the recoil energy of the primary knockon.

\( g(E) \) = the number of secondary displacements arising from a primary knockon of energy, \( E \).

\( E_d \) = the threshold energy for the displacement of an atom from its position in the crystal lattice.

\( E_{10a} \) = the energy at which the transition from elastic (displacement) to inelastic (ionization) reaction may be assumed to occur.

In addition to its widespread usage, the Kinchin and Pease model also has served as the basis for several somewhat more sophisticated models\textsuperscript{4,7}, all of which are characterized by their treatment of the crystal as a homogeneous mass. In contrast with these models, extensive work has been done in the last several years in an attempt to predict the effect of the highly-ordered arrangement of crystals on the displacement problem. Of particular significance is the Oen and Robinson "channeling" model\textsuperscript{14,15,16}. 

5
and the supporting calculations of Lehman and Leibfried. The "channeling" model predicts a significantly smaller number of displacements per primary knockon than does the Kinchin and Pease model. Of even more significance is the fact that the percentage decrease becomes greater with increasing primary knockon energies. When a cutoff energy ($E_{\text{ion}}$) between elastic and inelastic interactions is included, the displacement model according to Oen and Robinson becomes

\begin{align}
  g(E) &= 0 \quad \text{for} \quad E \leq E_d \\
  g(E) &= 1 \quad \text{for} \quad E_d < E \leq 2E_d \\
  g(E) &= \frac{1}{1-2P} \left[ (1-P) \left( \frac{E}{2E_d} \right)^{1-2P} - P \right] \quad \text{for} \quad 2E_d < E \leq E_{\text{ion}} \\
  g(E) &= \frac{1}{1-2P} \left[ (1-P) \left( \frac{E_{\text{ion}}}{2E_d} \right)^{1-2P} - P \right] \quad \text{for} \quad E_{\text{ion}} < E
\end{align}

In the above, $P$ is a parameter associated with the probability that a given atom will be "channeled" along a line of voids in the crystal such that the energy dissipated per interaction is insufficient to cause secondary displacements. For "diamond" crystals such as germanium and silicon, $P$ is expected to have values between 0.07 and 0.10. All other parameters are as previously defined.

At the heart of any displacement calculation is the problem of the transition from elastic to inelastic collisions between atoms in the crystal; i.e., the specification of $E_{\text{ion}}$. Seitz and Koehler first defined this transition in terms of the atom's velocity relative to its electron binding energy. They also assumed that the transition was adequately approximated
by the use of a sharp-cutoff energy. The use of a sharp cutoff between
elastic and inelastic reactions ($E_{\text{ion}}$) has been extended by both Snyder and
Neufeld, and Baicker and his coworkers. This approach has yielded
reasonable agreement with experiment as long as the energy of the incident
protons was comparatively low ($< 30$ to $40$ Mev).

Attempts to obtain agreement with experiment for damage by protons
of energies above about $30$ Mev using a sharp-cutoff value for $E_{\text{ion}}$ have not
been very satisfactory. Simon, Denney, and Downing concluded from their
work on silicon that the use of a sharp-cutoff value for $E_{\text{ion}}$ was completely
unsatisfactory. Instead, they used the theory of Lindhard and Scharff in
which the portion of the primary knockon energy dissipated through elastic
vs inelastic reactions is a smoothly varying function of the primary knockon
velocity. Very recently Wikner, Horiye, and Nichols have reported work,
also on silicon, from which they too have concluded that the Lindhard model
provides the best fit with experimental data.

None of the aforementioned work has made any attempt to include the
effect of the regularity of the crystal arrangement. In this work, the non-
homogeneity of the crystal is considered the use of the Oen and
Robinson channeling model in connection with the use of $E_{\text{ion}}$ provides agreement with the exper-
imental data comparable to that obtained through the use of the Lindhard
model alone.
III. THE DIFFERENTIAL CROSS SECTION FOR THE FORMATION OF PRIMARY KNOCKONS.

Recently, two independent groups have attempted to include the effect of the reaction cross section in the calculation of displacements. Simon, Denney, and Downing\(^6\) assumed a reasonable reaction cross section (1 barn) for the silicon-proton interaction. They have pointed out that the major uncertainty in their calculations is the use of an average momentum for the recoil nuclei. They also calculated that the displacements resulting from product nucleons at all energies is less than 1% of those due to the recoil of the residual nuclei. They have reported work on silicon only.

Wikner, Horiye, and Nichols\(^3\) have studied the displacement problem in both silicon and germanium as it arises from \(\gamma\) and neutron irradiation. They applied the theory of Montalbetti, et al,\(^2\) to predict the cross sections for the Ge (\(\gamma, n\)) reactions. A ratio of approximately 1:1 between elastic and inelastic (Ge (\(n, n'\))) reactions was used for the case of 14 Mev neutrons. The energy ranges considered were 15-30 Mev \(\gamma\) (bremsstrahlung), and reactor spectrum neutrons plus monoenergetic 14 Mev neutrons.

The most successful model for the calculation of nuclear cross sections is the "cloudy crystal ball" or optical model of Feshbach, Porter and Weisskopf\(^2\). It has also been shown by Hauser and Feshbach\(^3\) that the parameters (specifically, the penetrabilities) calculated from the optical model can be used to calculate a meaningful inelastic differential cross section for any or all of the excited levels of a target nucleus. Although such a calculation accounts for all inelastic reactions as (\(p, p'\)) reactions, it should still
provide meaningful results for the calculation of displacements because the recoil nuclei account for such a large percentage of the displacements\textsuperscript{6}. Hence, a complete optical model differential elastic cross section calculation plus a calculation of the differential inelastic cross sections combined with those cascade models of equations (1) and (2) should result in a useful prediction of radiation-induced displacements. The optical model calculation was accomplished using the method of partial waves. The numerical methods and techniques used are essentially those of the ABACUS-II digital computer program developed by Auerbach, Francis, Goldman, and Lubitz\textsuperscript{24,25,26} as reprogrammed for use on the GE-225 digital computer at the United States Military Academy, West Point, New York.

Since it is known that a unique set of optical model parameters does not exist, consideration must be given to selection of a combination of such parameters best suited to the purpose of these calculations; that is, the estimation of total numbers of displacements. This implies the best possible fit to the elastic cross section in the lower range of energies ($< \sim 20$ Mev), even at the expense of introducing errors into the reaction cross section. On the other hand, parameters which will optimize the reaction cross section accuracies are desired at high energies. The problem is further complicated by the lack of experimental data on germanium for use as check points on the calculations.

Certain parameters are comparatively standard in optical model work and can be considered to remain constant from nucleus to nucleus and energy
to energy. Most workers\textsuperscript{27,28,30,31,32,33} have considered the nuclear radius and the real and imaginary diffusion parameters to be among these. These have been set at $R = 1.25$ Å, $a = 0.65$ and $b = 0.98$ A, all in fermis, following Perey\textsuperscript{30} and Bjorkland\textsuperscript{29}. Furthermore, the imaginary spin-orbit potential is set to zero throughout these calculations since it adds little to the results. The Woods-Saxon real potential form\textsuperscript{34} and the Gaussian surface imaginary form are commonly used with good results and are employed herein.

The parameters remaining to be selected are the depth of the real potential well ($V_o$), the depth of the imaginary potential well ($W_o$), and the depth of the real spin-orbit potential ($V_{so}$). Perey\textsuperscript{30} showed that one can obtain good fits to experimental data up to 22 Mev by using a smooth variation with energy for these parameters. He has developed empirical relations for $V_o$ and $W_o$ but warns against extrapolation to higher energies.

Two rather complete compilations of optical model parameters at high energies have been prepared by Bjorkland\textsuperscript{29} and Bowen, et al\textsuperscript{39}. The major differences between these compilations and an extrapolation of Perey's parameters are that the former have (1) lower values for the real potential at high energies, (2) higher values for the imaginary potential at high energies, and (3) a spin-orbit potential which decreases with energy. Bjorkland has pointed out that the elastic cross section at large angles is significantly too low with these parameters (and this was confirmed in these calculations) but that good agreement with reaction cross sections is obtained. The general variation of the potentials with energy as quoted by Bjorkland
and Bowen are very similar and differ only in the combination of values used (e.g., Bowen quotes a lower value of $W_0$ but a higher value of $V_0$ than does Bjorkland). It should be noted that the use of an imaginary potential well diffusion parameter $\delta_0$ of 1.2 fermis with the Bowen parameters will yield results very similar to those using $\delta = 0.98$ fermis with the Bjorkland parameters.

The work quoted above indicates that the best parameters are those of Perey below $\approx 20$ Mev and those of Bjorkland above $\approx 50$ Mev. The approach used was to plot each of these in the appropriate energy ranges and to use a smooth variation for the transition region. The resulting optical model parameters for germanium as used in these calculations are listed in Table I.

In addition to the selection of optical model parameters discussed above, there is the problem of handling the data for the excited levels of the nuclei. Not only must one consider a large number of excitation levels but also one must consider all the various isotopes of naturally occurring germanium. Again, it must be kept in mind that the purpose of this work is to estimate total numbers of displacements. Hence, the accuracy of the cross sections at a given energy level is not of primary importance so long as the total effective cross section for displacements remains within acceptable bounds of accuracy.
### TABLE I

Optical Model Parameters* Used for Calculations on Germanium

<table>
<thead>
<tr>
<th>Energy</th>
<th>$V_0$</th>
<th>$W_0$(surface)</th>
<th>$W_0$(volume)</th>
<th>$V_{so}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>54.3</td>
<td>8.2</td>
<td></td>
<td>7.0</td>
</tr>
<tr>
<td>10</td>
<td>52.5</td>
<td>10.4</td>
<td></td>
<td>7.5</td>
</tr>
<tr>
<td>15</td>
<td>50.6</td>
<td>12.0</td>
<td></td>
<td>8.0</td>
</tr>
<tr>
<td>20</td>
<td>48.5</td>
<td>13.5</td>
<td></td>
<td>8.5</td>
</tr>
<tr>
<td>30</td>
<td>41.0</td>
<td>15.9</td>
<td></td>
<td>8.0</td>
</tr>
<tr>
<td>40</td>
<td>34.8</td>
<td>11.7</td>
<td>6.0</td>
<td>5.5</td>
</tr>
<tr>
<td>50</td>
<td>28.9</td>
<td></td>
<td>19.2</td>
<td>5.25</td>
</tr>
<tr>
<td>60</td>
<td>25.0</td>
<td></td>
<td>20.0</td>
<td>5.0</td>
</tr>
<tr>
<td>70</td>
<td>22.6</td>
<td></td>
<td>20.6</td>
<td>4.75</td>
</tr>
<tr>
<td>80</td>
<td>18.8</td>
<td></td>
<td>21.1</td>
<td>4.5</td>
</tr>
<tr>
<td>90</td>
<td>16.1</td>
<td></td>
<td>21.6</td>
<td>4.25</td>
</tr>
<tr>
<td>100</td>
<td>14.0</td>
<td></td>
<td>22.0</td>
<td>4.0</td>
</tr>
<tr>
<td>110</td>
<td>12.1</td>
<td></td>
<td>22.3</td>
<td>3.75</td>
</tr>
<tr>
<td>120</td>
<td>10.6</td>
<td></td>
<td>22.6</td>
<td>3.5</td>
</tr>
<tr>
<td>130</td>
<td>9.2</td>
<td></td>
<td>22.8</td>
<td>3.25</td>
</tr>
<tr>
<td>140</td>
<td>8.0</td>
<td></td>
<td>23.0</td>
<td>3.0</td>
</tr>
</tbody>
</table>

* All parameters are in MeV.
In order to determine the effect of the energy levels, displacement calculations were made on silicon because the knowledge on the excited levels of silicon is better than that for germanium. Initially, runs to calculate total displacements were made at two energies (20 Mev and 100 Mev for Si using all twelve known energy levels). Spin and parity for the first four levels are accurately known, and reasonable values were assumed for the other levels. The number of levels was reduced successively to nine, seven, and five and the displacements calculated for each. The shapes of the differential reaction cross sections are unchanged by these reductions. The total number of displacements for twelve levels vs five levels is reduced by about 20% at 20 Mev and about 30% at 100 Mev. This change is a direct result of the change in the calculated reaction cross section which results from the reduction in levels.

Test runs were also made for germanium under several different conditions. In this case the excited levels are not so well known as in the case of Si with fewer known for the other isotopes. Runs were made for Ge using seven, six, and five levels. Again the shapes of the cross sections were unchanged, and in this case the reduction in number of displacements resulting in the change from seven to five levels was less than 4% for all energies. Further runs were made using the levels for Ge. The results for total displacements were within 1% of those of Ge with comparable numbers of energy levels.
It was concluded from the above that the calculated numbers of displacements are remarkably insensitive to the exact number and values of the energy levels used so long as the incident energy exceeds that of the most highly excited level and the total reaction cross section is kept constant by the appropriate selection of optical model parameters. Thus, one can assume that reasonable results are obtained by retaining the angular distribution (which is not dependent on the number of levels) and the total cross sections. As a result of these findings and the uncertainties in the values for the higher energy levels and their associated spins and parities, these calculations were made using five levels for Ge\textsuperscript{72}. The uncertainty introduced thereby approaches the uncertainty of the reaction cross section at the extreme high-energy end and is correspondingly less for lower energies. Since this uncertainty is really that of the reaction cross sections as evidenced by the results from the silicon tests, availability of experimental results for these reaction cross sections would permit the selection of a "best" combination of optical model parameters and energy levels.

The calculations as outlined are an attempt to account for all reactions in terms of (p, p') reactions. This, of course, can introduce significant errors since it is known that the majority of the reactions in the energy range of concern are (p, 2n) and (p, pn)\textsuperscript{38} which have a much higher absorption of kinetic energy (Q value) than do the (p, p') reactions. A simple calculation yields a value for Q for the (p, 2n) reaction of between 11 and 14 Mev (depending upon the germanium isotope used). This is in good agreement
with the results of Ghoshal\textsuperscript{38} on copper and provides a reasonable basis for a correction to the Ge (p, p') reactions. It turns out that if one uses five energy levels for germanium in the Hauser-Feshbach model, the contribution of the fifth level to the reaction cross section varies from about 15% at 20 Mev to about 20% at 140 Mev. The corresponding contribution to the number of displacements is somewhat less than 15% at 20 Mev and roughly 20% at 140 Mev (since the energy of the fifth level is a significant fraction of 20 Mev but is $\ll$ 140 Mev). Thus, the effect of the contribution of a single level on the number of displacements is both the same order of magnitude and varies in the same direction as the effect of the increased $Q$ value of the true reactions over that of the (p, p') reactions. Therefore, the contribution of the reaction cross section for the fifth level was not included in calculating numbers of displacements. This is a very conservative correction and results in calculated numbers of displacements that can be considered as a lower limit on the true values for the following reasons:

(1) As shown above, the $Q$ value accounts for only 10% of the incident energy at 140 Mev as opposed to the 20% reaction cross section omitted. (2) The reduction in energy of the high-energy recoil nuclei by the $Q$ value does not result in a comparable reduction in the number of displacements because much of the additional energy would have been dissipated through inelastic collisions. (3) All displacements resulting from secondary reactions (the product neutrons, gamma and beta recoils) have been neglected.
The total reaction cross sections for protons incident on germanium as calculated herein are plotted in Figure 1. The calculated reaction cross sections are 1.098 barns at 20 Mev and 1.14 barns at 24 Mev. Compared to these, Ghoshal has reported experimental values for copper of 0.94 and 1.15 barns respectively. Also shown in the figure for comparison purposes are cross sections as calculated using the results of Shapiro.

IV. CALCULATION OF DISPLACEMENTS.

Once one has the complete differential elastic and inelastic cross sections, it should be possible to draw a parallel between the calculated displacements and observed damage. It was pointed out in Part II that the cascade model of Oen and Robinson is the only one which attempts to account for crystal orientation. Because of this unique attribute plus the fact that no complete displacement calculations using this model have been reported, the "channeling" phenomenon of Oen and Robinson was applied.

To calculate total numbers of displacements, one uses the equation:

\[ N_d = 2\pi N_o \int_{\theta_d}^{\pi} g(\theta) \frac{d\sigma(\theta)}{d\omega} \sin \theta \, d\theta \]  

in which

- \( N_d \) = number of displacements per cm of proton path.
- \( N_o \) = number of germanium nuclei per cm\(^3\).
- \( \theta_d \) = the proton angle of deflection (center-of-mass) at which the threshold energy, \( E_d \), is imparted to the germanium nucleus.
- \( g(\theta) \) = number of secondary displacements resulting from a proton reaction at angle \( \theta(c.m.) \).
\( \frac{d\sigma(\theta)}{d\omega} \) the differential cross section for the proton-germanium reaction of interest.

The calculation of \( N_\theta \) must be made separately for each set of differential cross sections. In this work, five sets were used: the shape elastic, the compound elastic, and three sets of inelastic. \( g(\theta) \) is rewritten from \( g(E) \) as given, equations (2a) thru (2d). For the Oen and Robinson model, this is

\[
g(\theta) = 0 \quad \theta \leq \theta_d \quad (4a)
\]

\[
g(\theta) = 1 \quad \theta_d < \theta \leq \theta' \quad (4b)
\]

\[
g(\theta) = 1 - 2P \left[ (1-P) \left( \frac{E_a \sin^2 \theta/2}{2 E_d} \right)^{1-2P} - P \right] \quad \theta' < \theta \leq \theta'' \quad (4c)
\]

\[
g(\theta) = \frac{1}{1-2P} \left[ (1-P) \left( \frac{E_{10a}}{2 E_d} \right)^{1-2P} - P \right] \quad \theta'' < \theta \leq \tau \quad (4d)
\]

In the above, \( E_a \) is the maximum energy which can be transferred to a target nucleus in a single collision. For the case of elastic collisions, this is just

\[
E_a = \frac{4 M_p M_{Ge}}{(M_p + M_{Ge})^2} \cdot E_p
\]

where

\[ M_p = \text{proton mass}. \]

\[ M_{Ge} = \text{germanium mass}. \]

\[ E_p = \text{proton incident energy}. \]

For the inelastic reactions, the complete kinematics equations must be used to account for the \( Q \) of the reactions\(^{40}\).

\( E_d \) and \( E_{10a} \) are respectively the displacement threshold energy and the cutoff energy between elastic and inelastic secondary interactions discussed earlier.
\( \theta_0, \theta', \) and \( \theta'' \) are respectively the c.m. angle of proton deflection at which energies \( E_d, 2E_d, \) and \( E_{\text{ion}} \) are transferred to the struck nucleus. Since for the elastic case the energy of the struck nucleus is given by

\[
E = E_n \sin^2 \theta/2
\]

\( \theta_d \) may be found from

\[
\theta_d = 2 \arcsin \left( \frac{E_d}{E_n} \right)^{1/2}
\]

Then \( \theta' \) and \( \theta'' \) may be evaluated in a like manner.

For the inelastic reaction, \( \theta_0, \theta', \) and \( \theta'' \) must be evaluated from the complete kinematics equation\(^{26}\).

Three parameters must be selected if one is to use the Oen and Robinson cascade model. These are the displacement threshold energy \( (E_d) \), the ionization cutoff energy \( (E_{\text{ion}}) \), and the probability parameter \( (P) \). The value of \( E_d \) was set at 15 ev following the work of Loferski and Rappaport\(^{41}\). In any case, a variation of \( E_d \) over the range from 15 ev to 30 ev has no effect on the shape of the displacements vs energy curves.

The value for \( E_{\text{ion}} \) can vary over a wide range depending upon what one uses for the binding energy of the least tightly bound electron in the germanium crystal. If this is assumed to be the band-gap energy, the theory of Seitz and Koehler\(^{13}\) yields a value of about 0.1 Mev for \( E_{\text{ion}} \). If one uses the ionization potential for the free germanium atom, the same theory yields a value of about 1.2 Mev. Calculations were made over the entire range, \( 0.1 \text{ Mev} < E_{\text{ion}} < 1.5 \text{ Mev} \), in order to determine which value resulted in the best agreement with experiment.
Oen and Robinson have concluded that reasonable values for the probability parameter \((P)\) in a "diamond" crystal such as germanium or silicon lie between 0.07 and 0.114\(^{14,16}\) and that a value of 0.07 yields results in agreement with the ratio of numbers of displacements in copper determined experimentally by Coltman and others\(^{14,16}\). As a consequence, the values selected for \(P\) in these calculations are 0.05, 0.07, 0.10 and 0.20. One expects the best results from the 0.07 and 0.10 values while the values of 0.2 and 0.05 are included as extreme cases for purposes of comparison.

Equation (3) was integrated numerically for the five sets of differential cross sections. These integrations yield a calculated value for the number of displacements per centimeter of proton path in germanium. Figures 2 through 5 are a graphical representation of the results of these calculations. Figures 2 and 3 show the variation in the displacement vs. energy curves as a function of the probability parameter \((P)\) for two specific values of the ionization cutoff energy \((E_{\text{ion}})\). Figures 4 and 5 represent the variation of the displacement vs. energy curves as a function of \(E_{\text{ion}}\) with \(P\) being held constant.

V. COMPARISON WITH EXPERIMENT.

Corelli and his coworkers have reported extensive experimental work on proton damage to germanium\(^{2,3}\). They have used both change in conductivity\(^{3}\) and change in minority carrier lifetime\(^{3}\) as measures of the radiation damage. Their experimental results are reproduced in Figures 6 and 7. The curves as shown in the figures are those of the experimental team.
A few comments with respect to these curves are in order before attempting to compare them to the calculated results herein. In the case of Figure 6 the relative damage is the value of \((\sigma_0 - \sigma) / \sigma_0\) where \(\sigma_0\) is conductivity before bombardment and \(\sigma\) is conductivity after bombardment. Furthermore, the proton flux on the 10 \(\Omega\)-cm sample was four times that on the 1 \(\Omega\)-cm sample. Hence, the relative positions of the curves are not significant for the purposes of this work. In a like manner, the curves of Figure 7 are plots of the rate of change of \(1/\tau - 1/\tau_0\) with proton energy (where \(\tau_0\) and \(\tau\) are respectively minority carrier lifetimes before and after bombardment). Thus, as in the case of the curves of Figure 6, the relative positions of these curves are functions of the initial state of the samples \((\tau_0)\). Therefore, it hardly seems justified to draw any conclusions from the relative positions of the curves for the various samples. However, it should be emphasized that the work herein is pertinent to pure (intrinsic) samples. Therefore, any deviations in the experimental results arising from the presence of impurities in the samples cannot be expected to appear in these calculations. As a consequence, one would expect the best agreement between these calculations and the lower two or three curves of Figure 7 (those for 40 \(\Omega\)-cm, 25 \(\Omega\)-cm, and 20 \(\Omega\)-cm materials).

The experimental results reproduced in Figures 6 and 7 have not been corrected for the effects of sample thickness. Such a correction yields about 12% fewer displacements per cm for a 20-Mev proton in an infinitely thin sample than is the case for the samples used in the experiments.
Similarly, the reduction at 50 Mev is about 5.3% and at 130 Mev is less than 1%. The experimental points in Figure 8 have been corrected for the effect of sample thickness.

Consideration of Figure 6 and 7 indicates little, if any, difference in the trend of the experimental results from minority-carrier-lifetime measurements and those from conductivity measurements. The fact that the two sets of results are so similar substantiates the contention of Loferski and Rappaport that changes in both conductivity and minority carrier lifetime are manifestations of the same phenomena.

In Figure 8, all the experimental points from Figures 6 and 7 have been replotted after normalization and correction for sample thickness. The normalization has been accomplished by taking the value of each of the curves of Figures 6 and 7 at 100 Mev and comparing it to the value of Curve A of Figure 6. The resulting ratio is used to normalize each of the experimental points pertaining to that particular curve. Plotted on the same figure are the calculated curves using a probability parameter of 0.10 with an ionization cutoff energy of 1.5 Mev, and a probability parameter of 0.07 with an ionization cutoff energy of 1.0 Mev.

VI. DISCUSSION OF RESULTS.

From Figure 8 it is seen that these calculations yield results well within the dispersion of the available experimental results. There are, of course, too many assumptions and uncertainties in such calculations to assert either that they confirm the Oen and Robinson model or that they
refute the Lindhard model. The calculations do show, however, that one can obtain agreement with experiment using a sharp cutoff between elastic and inelastic reactions provided the nonhomogeneity of the crystal is taken into account through the use of the Oen and Robinson channeling model.

The two greatest sources of uncertainty in these calculations are the accuracy of the calculated reaction cross sections (Figure 1) and the approximations involved in using \((p, p')\) reactions in lieu of the \((p, n)\), \((p, 2n)\), and \((p, pn)\) reactions. Some of the uncertainty arising from the latter approximation can be removed by more detailed calculations. Unfortunately, the uncertainty of the reaction cross section calculations can be resolved only by the availability of experimentally determined reaction cross sections at two or three energies between 20 Mev and 150 Mev. Until such data are available there is little point in attempting to improve the momentum transfer aspects of the calculations.

It is likely that the best possible calculation of displacements would result from combining the Oen and Robinson channeling model with the broad transition energy region of Lindhard. However, as seen in this work and the work of Wikner, et al., the present uncertainties in damage calculations and the dispersion in experimental results are such that a comparison between calculation and experiment cannot distinguish between the effectiveness of (1) the Oen and Robinson channeling model with sharp cutoff, (2) the Lindhard model, or (3) a combination of the Oen and Robinson model and the Lindhard model.
ACKNOWLEDGEMENT

We would like to thank Professors J. Corelli and N. Francis of Rensselaer Polytechnic Institute for helpful discussions, ideas, and assistance in the preparation of this work. Also of great value were discussions with Mr. J. Fischer of Rensselaer Polytechnic Institute. We thank the personnel of Knolls Atomic Power Laboratory for making available to us their ABACUS-II program as a pattern for the optical model calculations herein.

FOOTNOTE CONSOLIDATION

* From part of a thesis submitted by E. Saunders in partial fulfillment of the requirements for the degree of Doctor of Philosophy, Department of Nuclear Science and Engineering, Rensselaer Polytechnic Institute. This work was supported in part by NASA Research Grant NsG-371.

† Present address: Professor of Electricity, United States Military Academy, West Point, New York.


4. J. A. Baicker, Appl Phys Ltrs, 2, 104 (1963)
23. W. Hauser and H. Feshbach, Phys Rev, 87, 366 (1952)
27. F. Bjorkland and S. Fernbach, Phys Rev, 109, 1295 (1958)
33. R. M. Sternheimer, Phys Rev, 100, 886 (1955)
34. R. D. Woods and D. S. Saxon, Phys Rev, 95, 577 (1954)
35. P. M. Endt and J. C. Kluver, Rev Mod Phys, 26, 95 (1954)
38. S. N. Ghoshal, Phys Rev, 80, 939 (1950)
43. From informal discussions with Dr. Corelli, Rensselaer Polytechnic Institute

FIGURE CAPTIONS

Figure 1: Proton-Germanium Reaction Cross Sections:
(a) As calculated in this work from the full optical model.
(b) From the theory of Shapiro (Ref. 47).

Figure 2: Number of Displacements per Cm of Proton Path in Germanium for an Ionization Cutoff Energy of 1.5 Mev.
Figure 3: Number of Displacements per Cm of Proton Path in Germanium for an Ionization Cutoff Energy of 1.9 Mev.

Figure 4: Number of Displacements per Cm of Proton Path in Germanium for a Probability Parameter ($P$) of 0.1.

Figure 5: Number of Displacements per Cm of Proton Path in Germanium for a Probability Parameter ($P$) of 0.07.

Figure 6: Relative Damage in Germanium from Change in Conductivity (from Ref. 2).

Figure 7: Relative Damage in Germanium from Change in Minority Carrier Lifetime (from Ref. 3).

Figure 8: Comparison of Calculated Curves with Experimental Points. (Both curves and all experimental points are normalized to a common value of 100 Mev.).
1.4
1.2
1.0
0.8
0.6
0.4
0.2

A
B

PROTON ENERGY (MEV)

REACTION CROSS SECTION (BARNES)

20 40 60 80 100 120 140

PROTON ENERGY (MEV)
Author: E. A. Saunders, Figure 2

Reduce to $\frac{1}{3}$.
Authors: E. A. Saunders, Figure 3

PROTON ENERGY (MEV)

DISPLACEMENTS PER CM

P = 0.05
P = 0.07
P = 0.10
P = 0.20

Reduce to 1/2.
Displacements per cm

Proton Energy (MeV)

E_{ion} = 1.5, 1.0, 0.5, 0.1
Author: E. A. Saunders, Figure 6

Reduce to $\frac{1}{2}$.

PROTON ENERGY (MEV)

RELATIVE DAMAGE

CURVE A: 10 OHM-CM
CURVE B: 1 OHM-CM
SOLID: $E = 1.5$, $P = 0.10$

DOTTED: $E = 1.0$, $P = 0.07$