PLASMA PRESSURES IN THE MAGNETOSPHERE

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Abstract

By using a model that has the solar wind as the source for protons in the outer radiation belt, plasma energy densities near the equatorial magnetopause have been estimated to be comparable to the magnetic field energy density there.

The inflation of the geomagnetic field due to the presence of this plasma is discussed. The presence of high energy density plasma near the magnetic equator in the tail suggests that closed field lines exist at times out to 17 earth radii.
Introduction

Well before satellites were available, Dungey [1955, 1958] considered the effects of internal plasma pressures on the deformation of the magnetosphere. Using electron densities from whistlers and upper ionosphere temperatures, he found that the pressure of the thermal ionization was less than the magnetic pressure for magnetic fields greater than 5 gammas. He thus concluded that internal plasma pressures should have little effect on the shape of the magnetosphere.

With the advent of satellites and the discovery of the Van Allen belts, a source of plasma pressure that had not been anticipated must now be considered. Of particular importance are the outer belt protons that were discovered by Davis and Williamson [1963], since their energy density seems to be greater than that of any other component.

In this report, measured energy densities of these protons are combined with a model that appears to account for their origin to estimate plasma pressures in the outer radiation belt. Since these estimated plasma pressures are comparable to magnetic pressures there, some consequences are discussed.
Plasma Energy Density

Kellogg [1959] first suggested the model for the explanation of the outer belt protons which is the most successful one today. In this model solar particles are injected at the magnetopause and are transported across L shells through the violation of the third adiabatic invariant for trapped particles but without violation of the magnetic moment and longitudinal invariants. Dungey et al. [1965] applied this model to explain the spectral changes of the protons with L and equatorial pitch angle and found remarkable agreement with experiment. By using a diffusion process that was developed by Parker [1960], Herlofson [1960], Davis and Chang [1962], Dungey [1965], and Falthammer [1965], Tverskoy [1965] and Nakada and Mead [1965] have found good agreement between experiment and theory although the latter authors find some uncertainty as to whether sudden commencements and sudden impulses may be sufficient to account for the diffusion.

With the adoption of this model, it is possible to make good estimates of particle fluxes below the energy threshold of present measurements. This is accomplished through the use of Liouville's theorem. In this application, the theorem states that in the presence of loss mechanisms the density in phase space as one follows a particle
can only decrease away from the source. To apply this theorem it is necessary to neglect time variations in the source and work with average values for the various quantities. Angular dependences of particle fluxes are neglected, since pitch angle changes with $r$ at large $r$ have been found to be small if the magnetic moment and longitudinal invariants are preserved [Dungey et al., 1965].

The energy density of the protons is

$$\overline{PE} = \int \int \frac{1}{\gamma} EdEd\Omega \times \sqrt{\frac{m}{2}} \int \int \frac{1}{E} E^{3/2} dEd\Omega$$

where $j$ is the directional flux in no $(\text{cm}^2\text{-sec-ster-Mev})^{-1}$ and the integration is over all angles and energies. As a proton is followed in $L$ space, the density in phase space is $j/E$ where the energy and angle changes according to preservation of the magnetic moment and longitudinal invariants. If $j/E$ is constant in $L$ for any particular values of the magnetic moment and longitudinal invariants, then $\overline{PE}$ is proportional to $B^{5/2}$, since both $E$ and $dE$ are proportional to $B$. Changes due to integration over solid angle are small and are neglected although this integration tends to increase $\overline{PE}$ at small $B$ if the angular distributions show a maximum perpendicular to $B$. Thus

$$\beta(B) = \frac{\overline{PE}}{B^2/8\pi} \propto \sqrt{B} \quad (1)$$
Hoffman and Bracken [1965] have evaluated $\beta$ from measurements and find that $\beta = 0.165$ at the equator at $r = 3.9$ earth radii for protons with energy greater than 100 kev.

Dungey et al. [1965] have evaluated $j/E$ from experimental data and find that it increases out to $L = 6$. Hess [1965] has made a more thorough study of the changes in measured $j/E$ for a range of magnetic moments and longitudinal invariants. He finds that relative changes with $L$ in $j/E$ are independent of values of the invariants and that $j/E$ increases by about a factor 5 from $L = 3.9$ to $L = 7$. Using these experimental results and equation (1), $\beta$ at $L = 7$ near the equator is approximately:

$$\beta \text{ (L = 7)} = 0.165 \times 5 \times \left(\frac{2.2}{7}\right)^{3/2} = 0.343$$

If $j/E$ is constant beyond $L = 7$, then

$$\beta = 0.343 \sqrt{\frac{B}{E \text{ (L = 7)}}} = 0.036 \sqrt{B}$$

with $B$ in gammas.

If $B$ is about 70 gammas just inside the magnetopause, then $\beta = 0.3$. This does not include the effects of protons which are in
the magnetopause but which are lost before being transferred to $L = 7$ or which have such low energy that they would be below the 100 kev threshold of the detector at $L = 3.9$. For near equatorial protons, this equivalent threshold energy, $E_{th}$, for constant magnetic moment is $0.19B$ kev with $B$ in gammas. Just within the magnetopause where $B$ is about 70G, $E_{th}$ is 13 kev. If the transition zone is the source of these protons, this proton energy is quite large compared with most of the protons there. Unless there is a large change in energy during injection or the injection process has an energy threshold, it seems likely that protons just within the magnetosphere but below 13 kev have energy densities a few times that for protons above 13 kev.

These results imply that $\beta$ must be almost 1 just within the sunward magnetopause. Measurements by Freeman [1964] with his Cds-B detector offer partial confirmation of this implication. This detector measured the total proton energy flux for proton energies between 1 kev and 10 Mev. Because of a magnetic broom, only electrons with energies above 250 kev were detected. Just within the sunward magnetosphere, this detector measured average fluxes of 5.5 erg/cm$^2$-sec-ster. This flux may be converted to energy density
by dividing by an average velocity. If the average proton velocity is given by an energy as high as 40 kev, this flux corresponds to a magnetic field energy density of 70V for eachsteradian of flux. If there is at least one steradian of protons, these measurements indicate a $\beta$ near 1.

Freeman's CdSTE detector, which measured electrons above 200 ev as well as protons above 1 kev, gave energy fluxes that were about a factor of 5 greater than CdSB fluxes near the magnetopause. Since electrons at the same density and average energy as protons should give 43 times (ratio of $\sqrt{m_p/m_e}$) as much energy flux, the measurements indicate that the contribution of electrons to the energy density at the magnetopause is small.

Large $\beta$ values within the magnetosphere imply little change in energy density across the magnetopause; this may be derived from pressure balance conditions for plasma and magnetic fields. Freeman's measurements indicate almost no change in energy fluxes for protons; so if average proton velocities are comparable, proton energy densities should also be comparable.

It is interesting that pressure balance at the sunward magnetopause does not forbid $\beta > 1$ within the magnetosphere.
Discussion

In the previous section, it was demonstrated that $\beta$ is likely to be at least 0.3 near the equator from 7 to 10 earth radii from the earth and may even be near 1. Some consequences of such large $\beta$ values are discussed here.

An important implication of large $\beta$ within the magnetosphere is that the concept of a magnetopause that tends to exclude external plasma may not be valid. Somehow external plasma enters the magnetosphere rather easily. Perhaps orbits through the magnetosphere as have been suggested by Wentworth [1965] and Fejer [1965] are the cause. These orbits are clearly demonstrated in Hones' [1963] earlier study. Perhaps instabilities arising from turbulent plasma in the transition region are more important.

The presence of this internal plasma may necessitate a change in the understanding of the theoretical causes for the shape of the magnetosphere since; (a) pressure balance conditions will be modified and (b) internal boundary currents may be as important as external boundary currents.
The high β plasma at $R > 7$ earth radii will cause an inflation of the geomagnetic field. Hoffman's and Bracken's [1965] calculations of the effects of trapped plasma indicate that this plasma will produce a westward current which tends to extend field lines away from the earth near the equatorial plane. (See also Akasofu, Cain and Chapman [1961].) On the sunward side, this inflation is likely to be masked by solar plasma pressure effects. On the night side, where solar plasma pressure effects are much smaller, the inflation is likely to be pronounced and may account for the effects that Cahill [1964] has observed.

The inflation and lengthening of magnetic lines of force in the tail can also produce the day-night asymmetry of low altitude but high latitude energetic trapped electrons that has been discussed by Williams and Mead [1965].

Electron ($E > 200$ ev) fluxes in the tail region were discovered by Gringauz et al. [1961] who found intensities of $2 \times 10^8$ (cm$^2$-sec)$^{-1}$ out to $R = 13$ earth radii at low geomagnetic latitudes. Measurements by Freeman [1964] with his CdSTE detector indicate that energy fluxes in the tail to $R = 11$ earth radii are comparable to those in the sunward magnetosphere.
Recent measurements by the Vela satellites at $R = 17$ earth radii have been reported by Bame [1965] and indicate electron fluxes similar to those found by Gringauz et al. The Vela measurements find (a) electron temperatures of a few kev; (b) no streaming of this plasma; (c) that this plasma is detectable only within about 6 earth radii of the magnetic equator; (d) that the plasma is there much of the time.

A flux of $2 \times 10^8 \text{ (cm}^2\text{-sec)}^{-1}$ at 2.5 kev with protons at the same temperature as electrons gives a particle energy density that is equivalent to a magnetic field energy density for a field of 11Y. If these particles at 17 earth radii in the tail are related to those in the forward magnetosphere where proton energy densities are greater than electron energy densities, then the particle energy densities in the tail may be larger than the above estimate.

Since measured fields in the tail are not much larger than the above estimate, it appears that $\beta$ is of the order of 0.5 at 17 earth radii near the magnetic equator. A possible interpretation of the existence of large $\beta$ plasma in the tail is that the magnetic field lines are closed and thus contains the plasma. It seems that the magnetic field configuration with open field lines in the tail would
have difficulties in explaining the presence of this plasma since plasma would escape at about the thermal velocity of the protons, which is 6 earth radii/minute for a 2.5 kev proton. Perhaps what has been interpreted as a neutral sheet in the open field configuration [Ness, 1965] at less than 17 earth radii are instead time dependent changes due to the arrival of energetic electrons with energy densities comparable to the magnetic field energy density [Anderson, 1965] to a region where high $\beta$ thermal plasma already exists.

Summary

An examination of proton plasma in the magnetosphere leads to the conclusion that the energy density of the plasma is comparable to the energy density of the magnetic field beyond 7 earth radii near the magnetic equator. Since there is little change in the energy flux at the sunward magnetopause it is concluded that the magnetopause is quite porous.

This large $\beta$ plasma should inflate the magnetosphere; although inflation may not be noticeable in the solar direction, it is likely to be pronounced in the anti-solar direction. Because of the presence of high $\beta$ plasma at 17 earth radii near the magnetic equator in the tail, it is suggested that closed field lines exist at times to 17 earth radii.
Beyond about 6 earth radii from the magnetic equator, the absence of plasma with properties similar to that found near the magnetic equator in the tail may indicate that these field lines are open or that solar plasma does not enter the polar region easily.


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