

HEAT TRANSFER FOR LIQUID METAL FLOW IN RECTANGULAR
CHANNELS WITH PRESCRIBED WALL HEAT FLUXES
AND HEAT SOURCES IN THE FLUID STREAM

by Robert M. Inman

Lewis Research Center
Cleveland, Ohio

TECHNICAL PAPER proposed for presentation at
Fluid Mechanics and Heat Transfer Institute
Santa Clara, California, June 22-24, 1966

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

HEAT TRANSFER FOR LIQUID METAL FLOW IN RECTANGULAR
CHANNELS WITH PRESCRIBED WALL HEAT FLUXES
AND HEAT SOURCES IN THE FLUID STREAM

by Robert M. Inman

Lewis Research Center
National Aeronautics and Space Administration
Cleveland, Ohio

ABSTRACT

As a step towards a better understanding of turbulent liquid-metal heat transfer in rectangular ducts with heat generation in the fluid stream, an analysis is performed for forced-convection heat transfer to slug flow in rectangular channels with aspect ratios from 1 to ∞ and with heat sources in the fluid. The channel walls are uniformly heated, and the heat flux on the short sides is considered an arbitrary fraction or multiple of the heat flux on the broad sides. The analysis is based on the additional assumptions that (1) heat transport by eddy conduction is negligible compared with molecular conduction, (2) internal heat generation is spatially uniform, and (3) fluid properties are invariant with temperature.

The temperature distributions are determined by utilizing the method of superposition, and the required eigenvalues and constants are determined analytically. The results obtained apply in the thermal entrance region of the channel as well as far downstream from the entrance.

The effects of (1) the ratio which determines the relative role of internal heat generation to that of wall heat transfer, (2) specified aspect ratio, and (3) specified heat fluxes around the channel periphery on temperature distributions are investigated. Numerical results for wall temperatures and bulk-mean fluid temperatures are presented graphically. The solutions point out the locations of maximum temperature.

The results are useful in estimating local heat-transfer characteristics in turbulent heat-generating liquid metal flow in rectangular ducts when the Prandtl and Reynolds moduli are low.

INTRODUCTION

Recent technological developments in space power generation have stimulated interest in the problem of forced-convection heat transfer to liquid metal flow in passages with internal heat generation in the fluid stream. This system has applications, for example, in the design of

liquid metal-fuel reactors, electromagnetic pumps and flowmeters, and liquid metal MHD generators. The fluid in these devices will be heated by radioactive fission products or by an electric current flowing through the fluid. A factor of importance for the proper operation of these devices is maintaining a satisfactory temperature distribution along the passage walls. The designer, therefore, must be able to compute the temperature distribution along the walls and to know how much heat must be removed to cool the walls to prevent temperatures from exceeding design limits. The problem involves studying liquid-metal duct flow with combined internal heat sources and wall heat transfer. Turbulent flows are very often encountered in practice, and it is this flow regime which is of concern here.

This investigation is concerned with hydrodynamically developed turbulent flow of a liquid metal in rectangular ducts with aspect ratios from 1 to ∞ and with uniform internal heat generation. The channel walls are considered uniformly heated, but the heat flux on the short sides of the channel is an arbitrary fraction or multiple of the heat flux on the long sides. Such a heat transfer situation, for example, may be the result of unwanted heat leakage or addition through insulation.

Attention is focused here on the rectangular duct because of its increasing use in the applications mentioned. Within the knowledge of the author, the experimental and analytical studies of turbulent liquid-metal duct flows with internal heat generation have been confined to elementary geometries such as the circular tube (refs. 1 to 3, and 5) and the parallel-plate channel (refs. 4 and 5). In contrast to this moderate amount of information, turbulent-flow heat transfer to a heat-generating liquid metal in a rectangular duct has apparently received little analytical and no experimental work. A few studies related to the problem considered in the present investigation are noted. In the absence of internal heat generation in the fluid, there have been developed solutions which approximate situations which might occur with liquid metals in turbulent flow through rectangular ducts. Fully developed slug-flow Nusselt numbers and wall-temperature distributions have been presented in references 6 to 8. In these references, the solution of the problem was obtained by assuming, in addition to a uniform velocity throughout the duct, that turbulent eddy-ing does not contribute to conduction of heat within the fluid. The results pertain to systems characterized by low Reynolds and Prandtl moduli and to the portion of rectangular duct beyond the thermal entrance region. In the discussion of reference 9, Hoagland discusses work done on the thermal entrance region for laminar slug flow in rectangular ducts, and reports some numerical results. Reference 10 has examined forced-convection heat transfer to laminar slug flow in a rectangular channel for the boundary condition of a duct wall temperature both peripherally and axially uniform. This analysis was carried out under the restriction of no internal heat sources.

It is the purpose of this investigation to study forced-convection heat transfer in a heat-generating liquid metal flowing in a rectangular channel where heating occurs on all four walls. The uniform heat flux on the short walls is assumed an arbitrary fraction or multiple of the flux on the broad walls. For the sake of completeness, the converse condition is also considered; namely, that the heat flux on the broad walls is any fraction or multiple of the flux on the short walls. Aspect ratios from 1 to ∞ are considered for the rectangular channel, and the aim of the following analysis is the determination of the axial and peripheral temperature distribution and heat-transfer characteristics in the channel. The findings of the analysis should be applicable along the entire length of the ducts, that is, in thermal entrance as well as fully developed regions.

The present very limited knowledge of turbulent liquid-metal flow and eddy-diffusivity-variation in noncircular passages make theoretical progress very unlikely without appeal to simplified models. It does not seem likely, moreover, that any single model (for which a mathematical analysis is feasible) will prove adequate for all Reynolds and Prandtl moduli. Therefore, in order to gain some understanding of the complex problem of turbulent liquid-metal flow in rectangular ducts with wall heat transfer and internal heat sources, consideration will be given to a simplified but representative model; this specific model not only retains many of the physical characteristics of turbulent liquid metal duct flow, but also leads to a tractable mathematical problem. This model should, therefore, provide information on the temperature distribution and heat-transfer characteristics for such flows in rectangular passages.

The idealized system assumed to approximate the forced convection system under consideration is based on the following postulates: (1) The established turbulent velocity profile is represented by a uniform distribution; (2) the thermal eddy diffusivity is small compared to the thermal molecular diffusivity and is neglected; (3) longitudinal heat conduction is small compared to longitudinal convection and transverse conduction and is neglected; and (4) the internal heat generation is spatially uniform. It is pointed out (e.g., refs. 6 and 11) that the blunt-nosed turbulent velocity distribution for a liquid metal system can be represented satisfactorily by a uniform distribution. The second postulate implies (ref 12) that the thermal solution pertains to systems characterized by low Prandtl moduli and low and intermediate Reynolds moduli. The third postulate has been shown in reference 13 to introduce a negligible error for Peclet moduli equal to, or greater than, approximately 100. References 6 to 8 point out that turbulent-flow heat transfer to liquid metals may be estimated, at least in the absence of internal heat generation, by the use of the slug-flow solutions for molecular conduction.

In this investigation numerical results are provided for the case of internal heat sources which are uniform across the duct cross section and along the duct length. The results can undoubtedly be extended, however, to include sources which vary in the transverse and longitudinal directions (refs. 14 and 15).

ANALYSIS

The rectangular channel and its coordinate system are shown in figure 1. Turbulent velocity is postulated to be fully established at $x = 0$. The fluid temperature at the channel entrance is uniform across the section at a value t_e . Within the channel a heating process takes place that includes a uniform heat generation within the liquid metal and a uniform heat transfer at the channel walls. The fluid is assumed to have constant physical properties, and only steady-state heat transfer is investigated.

The established turbulent velocity profile is represented by

$$u = U \quad (1)$$

The differential equation describing convective heat transfer for the idealized system takes the form

$$\frac{\partial t}{\partial x} = \frac{\kappa}{\rho c_p} \left(\frac{\partial^2 t}{\partial y^2} + \frac{\partial^2 t}{\partial z^2} \right) + \frac{Q}{\rho c_p} \quad (2)$$

The linearity of the energy equation (2) suggests that the temperature $t(x, y, z)$ be written as the sum of two parts,

$$t(x, y, z) = t_Q(x, y, z) + t_q(x, y, z) \quad (3)$$

in which t_Q corresponds to the situation where a heat-generating fluid with an entering temperature of zero flows through a channel with insulated walls ($q_B = q_S = 0$) and t_q corresponds to the situation where a nongenerating fluid entering at temperature t_e flows through a channel with heat transfer q_B and q_S at the walls. The general solution can then be obtained by superposition of the two simpler solutions in accordance with equation (3), since if the individual temperature fields satisfy the linear energy equation, then their sum does also.

At this stage, it is convenient to employ dimensionless coordinates; the dimensionless equations and boundary conditions used to determine t_Q and t_q are then, respectively:

$$\frac{\partial t_Q}{\partial x'} = \frac{\partial^2 t_Q}{\partial y'^2} + \frac{\partial^2 t_Q}{\partial z'^2} + \frac{Qa^2}{\kappa} \quad (4a)$$

$$\frac{\partial t_Q}{\partial y'} = 0 \quad \text{at } y' = 0 \quad \text{and } 1 \quad (\text{Insulated walls}) \quad (4b)$$

$$\frac{\partial t_Q}{\partial z'} = 0 \quad \text{at } z' = 0 \quad \text{and } \sigma \quad (\text{Insulated walls}) \quad (4c)$$

$$t_Q(0, y', z') = 0 \text{ (Entrance condition)} \quad (4d)$$

and

$$\frac{\partial t_q}{\partial x'} = \frac{\partial^2 t_q}{\partial y'^2} + \frac{\partial^2 t_q}{\partial z'^2} \quad (5a)$$

$$\frac{\partial t_q}{\partial y'} = -\frac{q_B a}{\kappa} = -\frac{\tilde{q}}{2\kappa(\sigma + \alpha)} \text{ at } y' = 0 \text{ (Specified wall heat flux)} \quad (5b)$$

$$\frac{\partial t_q}{\partial y'} = \frac{\tilde{q}}{2\kappa(\sigma + \alpha)} \text{ at } y' = 1 \text{ (Specified wall heat flux)} \quad (5c)$$

$$\frac{\partial t_q}{\partial z'} = -\frac{q_S a}{\kappa} = -\frac{\alpha \tilde{q}}{2\kappa(\sigma + \alpha)} \text{ at } z' = 0 \text{ (Specified wall heat flux)} \quad (5d)$$

$$\frac{\partial t_q}{\partial z'} = \frac{\alpha \tilde{q}}{2\kappa(\sigma + \alpha)} \text{ at } z' = \sigma \text{ (Specified wall heat flux)} \quad (5e)$$

$$t_Q(0, y', z') = t_e \text{ (Entrance condition)} \quad (5f)$$

These two problems will be treated separately and the results combined to yield information for the general situation. There is first considered the problem of a heat-generating fluid flowing in an insulated channel. The problem of wall heat transfer to a nongenerating flowing fluid is then considered.

Internal Heat Generation with Channel Insulated

The solution for t_Q is found most easily by separate consideration of the fully developed and entrance regions. There is first considered the fully developed temperature which applies in the region downstream of the entrance region. The temperature $t_{Q,d}$ satisfies equation (4a)

$$\frac{\partial t_{Q,d}}{\partial x'} = \frac{\partial^2 t_{Q,d}}{\partial y'^2} + \frac{\partial^2 t_{Q,d}}{\partial z'^2} + \frac{Qa^2}{\kappa} \quad (6)$$

The fully developed situation for uniform internal heat generation is characterized by the fact that

$$\frac{\partial t_{Q,d}}{\partial x} = \frac{Q}{\rho U c_p} \quad (7)$$

Equation (7) states that the temperature at all positions in the rectangular cross section rises in the same linear fashion along the channel length. For the fully developed situation the boundary condition at the entrance of the channel ($x = 0$) need not be considered, since it is accounted for by the entrance region solution, and equation (7) may be rephrased as

$$\frac{t_{Q,d}}{Qa^2/k} = x' + f(y', z') \quad (8)$$

The function $f(y', z')$ is found by inserting equation (8) into the differential equation (6). This leads to the equation for $f(y', z')$ as

$$\frac{\partial^2 f}{\partial y'^2} + \frac{\partial^2 f}{\partial z'^2} = 0 \quad (9)$$

The boundary conditions on $f(y', z')$ are determined from the thermal boundary conditions (eqs. (4b) and (4c)), so that

$$\frac{\partial f}{\partial y'} = 0 \quad \text{at } y' = 0 \text{ and } 1 \quad (10a)$$

$$\frac{\partial f}{\partial z'} = 0 \quad \text{at } z' = 0 \text{ and } \sigma \quad (10b)$$

Taking a solution of equation (9) in the form

$$f(y', z') = a_0 y'^2 + a_1 y' + a_2 z'^2 + a_3 z' + a_4 \quad (11)$$

in which a_0, a_1, \dots are constants chosen to satisfy the boundary conditions (eqs. (10a) and (10b)), there is obtained the result

$$f(y', z') = a_4 = \text{constant} \quad (12a)$$

The constant can be evaluated from an overall energy balance on the fluid for the length of channel from 0 to x :

$$\begin{aligned}
 t_{Q,b} &= \frac{Q}{\rho U c_p} x = \frac{\int_0^\sigma \int_0^1 U t_{Q,d} dy' dz'}{\int_0^\sigma \int_0^1 U dy' dz'} \\
 &= \frac{1}{\sigma} \int_0^\sigma \int_0^1 t_{Q,d} dy' dz'
 \end{aligned}$$

This leads to the condition from equation (8):

$$\int_0^\sigma \int_0^1 f(y', z') dy' dz' = 0 \quad (12b)$$

from which the constant is evaluated. The resulting expression for $f(y', z')$ is found to be

$$f = 0 \quad (13)$$

The final expression for the temperature distribution $t_{Q,d}$ is given by

$$\frac{t_{Q,d}}{Qa^2/\kappa} = x' \quad (14)$$

which applies only in the fully developed region downstream of the thermal entrance region.

To determine the temperatures in the entrance region, it is convenient to introduce an entrance temperature t_Q^* so that

$$t_Q = t_{Q,d} + t_Q^* \quad (15)$$

From the linearity of the energy equation (4a), it is found that t_Q^* satisfies the equation

$$\frac{\partial t_Q^*}{\partial z'} = \frac{\partial^2 t_Q^*}{\partial y'^2} + \frac{\partial^2 t_Q^*}{\partial z'^2} \quad (16a)$$

with the boundary conditions

$$\frac{\partial t_Q^*}{\partial y'} = 0 \quad \text{at } y' = 0 \text{ and } 1 \quad (16b)$$

$$\frac{\partial t_Q^*}{\partial z'} = 0 \quad \text{at } z' = 0 \text{ and } \sigma \quad (16c)$$

At $x' = 0$, the condition is

$$t_Q(0, y', z') = 0 = t_{Q,d}(0, y', z') + t_Q^*(0, y', z')$$

or, by rearranging:

$$\frac{t_Q^*(0, y', z')}{Qa^2/\kappa} = -f(y', z') = 0 \quad (16d)$$

The solution of equation (16a) which will satisfy equations (16b) to (16d) can be found by using a product solution which leads to a separation of variables. This will have the form

$$\frac{t_Q^*}{Qa^2/\kappa} = \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} a_{ij} Y(y') Z(z') e^{-\lambda_{ij}^2 x'} \quad (17)$$

in which the functions $Y(y')$ and $Z(z')$ satisfy the differential equations

$$\frac{d^2 Y}{dy'^2} + \alpha_i^2 Y = 0 \quad (18a)$$

$$\frac{d^2 Z}{dz'^2} + \beta_j^2 Z = 0 \quad (18b)$$

with the respective boundary conditions

$$\frac{dY}{dy'} = 0 \quad \text{at } y' = 0 \text{ and } 1 \quad (18c)$$

$$\frac{dZ}{dz'} = 0 \quad \text{at } z' = 0 \text{ and } \sigma \quad (18d)$$

and $\lambda_{ij}^2 = \alpha_i^2 + \beta_j^2$. Equations (18a) and (18b) with their boundary conditions belong to the well-known class of differential equations of the Sturm-Liouville type. Solutions are possible only for discrete, though infinite, sets of α_i and β_j values.

It may be seen readily that the differential equations (18a) and (18b) and the boundary conditions are satisfied if there are taken for the functions $Y(y')$ and $Z(z')$ the expressions

$$Y = \cos(i\pi y'); \quad i = 0, 1, \dots \quad (19a)$$

$$Z = \cos\left(\frac{j\pi z'}{\sigma}\right); \quad j = 0, 1, \dots \quad (19b)$$

The eigenvalue λ_{ij} is then given by the following expression:

$$\lambda_{ij}^2 = \pi^2 \left(i^2 + \frac{j^2}{\sigma^2} \right) \quad (19c)$$

The coefficients a_{ij} in equation (17) are evaluated to satisfy the entrance boundary condition equation (16d). This gives the condition

$$\sum_{i=0}^{\infty} \sum_{j=0}^{\infty} a_{ij} \cos(i\pi y') \cos\left(\frac{j\pi z'}{\sigma}\right) = \frac{t_Q(0, y', z')}{Qa^2/\kappa} = 0 \quad (20a)$$

It follows immediately that

$$a_{ij} = 0 \quad \text{for all } i \text{ and } j \quad (20b)$$

This result indicates that there is no entrance region solution; that is, $t_Q^*/(Qa^2/\kappa) = 0$.

Now that $t_{Q,d}$ and t_Q^* are known, they can be superposed as in equation (15) to obtain the solution that applies over the entire length of the channel, which is simply

$$\frac{t_Q}{Qa^2/\kappa} = x' \quad (21)$$

The local bulk fluid temperature $t_{Q,b}(x')$ along the channel length, for the uniform heat source, is given by

$$t_{Q,b} = \frac{Q}{\rho U c_p} x$$

or

$$t_{Q,b}(x') = \frac{Qa^2}{\kappa} x' \quad (22)$$

Wall Heat Transfer Without Internal Sources

There is considered next the situation where there are prescribed wall heat fluxes q_B and $q_S = \alpha q_B$ at the channel walls (fig. 1) but no internal heat sources. The temperature $t_Q(x', y', z')$ is the solution to

equation (5). To obtain a solution for t_q that will apply over the entire length of the channel, it is convenient to break t_q into two parts. The first part is $t_{q,d}$, the fully developed solution. The second part is t_q^* , an entrance region solution that is added to $t_{q,d}$ to obtain temperatures in the region near the entrance of the channel. The temperature t_q is then given by

$$t_q = t_{q,d} + t_q^*$$

From the linearity of the energy equation, $t_{q,d}$ and t_q^* have to each satisfy equations (5).

The fully developed solution is considered first. Far from the entrance of the rectangular channel the temperature rises linearly in the axial direction because of the uniform, but unequal, heat inputs at the channel walls. From a heat balance on the fluid, the temperature gradient in the fully developed region must be

$$\frac{\partial t_{q,d}}{\partial x} = \frac{\tilde{q}}{\rho U c_p a^2 \sigma} = \text{constant} \quad (24)$$

The temperature distribution is then given by the equation

$$\frac{t_{q,d} - t_e}{\tilde{q}/\kappa} = \frac{1}{\sigma} [x' + F(y', z')] \quad (25)$$

An equation for the function $F(y', z')$ is found by substituting equation (25) into equation (5a). This gives

$$\frac{\partial^2 F}{\partial y'^2} + \frac{\partial^2 F}{\partial z'^2} = 1 \quad (26a)$$

The boundary conditions on $F(y', z')$ are determined from the thermal boundary conditions (eqs. (5b) to (5e)) so that

$$\left. \begin{aligned} \frac{\partial F}{\partial y'} &= -\frac{\sigma}{2(\alpha + \sigma)} \text{ at } y' = 0 \\ \frac{\partial F}{\partial y'} &= \frac{\sigma}{2(\alpha + \sigma)} \text{ at } y' = 1 \end{aligned} \right\} \quad (26b)$$

$$\left. \begin{aligned} \frac{\partial F}{\partial z'} &= -\frac{\alpha\sigma}{2(\alpha + \sigma)} \text{ at } z' = 0 \\ \frac{\partial F}{\partial z'} &= \frac{\alpha\sigma}{2(\alpha + \sigma)} \text{ at } z' = \sigma \end{aligned} \right\} \quad (26c)$$

The solution of equation (25a) is taken in the form

$$F(y', z') = b_0 y'^2 + b_1 y' + b_2 z'^2 + b_3 z' + b_4 \quad (27)$$

The constants b_0 , b_1 , b_2 , and b_3 can be found from the conditions given by equations (26b) and (26c) and which yield

$$b_0 = \frac{\sigma}{2(\alpha + \sigma)}, \quad b_1 = -b_0, \quad b_2 = \frac{\alpha}{2(\alpha + \sigma)}, \quad b_3 = -\sigma b_2$$

Hence equation (27) can be represented in the following form:

$$F(y', z') = \frac{\sigma}{2(\alpha + \sigma)} \left[y'^2 - y' + \frac{\alpha}{\sigma} z'^2 - \alpha z' \right] + b_4 \quad (28a)$$

The remaining constant b_4 can be evaluated from an overall energy balance on the fluid for the length of channel from 0 to x :

$$\int_0^\sigma \int_0^1 (t_{q,d} - t_e) dy' dz' = \frac{\tilde{q}}{\rho U c_p a^2} x$$

This leads to the condition from equation (25):

$$\int_0^\sigma \int_0^1 F(y', z') dy' dz' = 0 \quad (28b)$$

from which the constant is evaluated. The expression is then inserted into equation (25) to give the final expression for the fully developed temperature distribution:

$$\frac{t_{q,d} - t_e}{\tilde{q}/\kappa} = \frac{1}{\sigma} x' + \frac{1}{2(\alpha + \sigma)} \left[y'^2 - y' + \frac{\alpha}{\sigma} z'^2 - \alpha z' + \frac{1}{6} (1 + \alpha\sigma) \right] \quad (29a)$$

This can be rephrased in an equivalent form

$$\frac{t_{q,d} - t_e}{q_B a / \kappa} = 2 \left(1 + \frac{\alpha}{\sigma} \right) x' + y'^2 - y' + \frac{\alpha}{\sigma} z'^2 - \alpha z' + \frac{1}{6} (1 + \alpha\sigma) \quad (29b)$$

Equation (29b) applies only in the fully developed region.

To determine the temperatures in the thermal entrance region the function t_q^* is needed. From the linearity of the energy equation, the equation for t_q^* is the same as equation (5a):

$$\frac{\partial t_q^*}{\partial x'} = \frac{\partial^2 t_q^*}{\partial y'^2} + \frac{\partial^2 t_q^*}{\partial z'^2} \quad (30a)$$

Since the wall heat additions have already been accounted for in the fully developed solution, the boundary conditions for t_q^* are that no heat is transferred at the walls:

$$\frac{\partial t_q^*}{\partial y'} = 0 \text{ at } y' = 0 \text{ and } 1 \quad (30b)$$

$$\frac{\partial t_q^*}{\partial z'} = 0 \text{ at } z' = 0 \text{ and } \sigma \quad (30c)$$

At the channel entrance ($x' = 0$) the condition is

$$t_q^*(0, y', z') = t_e = t_{q,d}(0, y', z') + t_q^*(0, y', z')$$

or by rearranging,

$$t_q^*(0, y', z') = - \left(\frac{q_{B^a}}{\kappa} \right) \left[y'^2 - y' + \frac{\alpha}{\sigma} z'^2 - \alpha z' + \frac{1}{6} (1 + \sigma\alpha) \right] \quad (30d)$$

It will now be convenient to represent the entrance region temperature t_q^* of the fluid by two functions $\theta(x', y')$ and $\psi(x', z')$ such that

$$\frac{t_q^*}{q_{B^a}/\kappa} = \theta(x', y') + \psi(x', z') \quad (31)$$

In terms of θ the energy equation (30a) becomes

$$\frac{\partial \theta}{\partial x'} = \frac{\partial^2 \theta}{\partial y'^2} \quad (32)$$

for which the boundary conditions will be taken as

$$\frac{\partial \theta}{\partial y'} = 0 \text{ at } y' = 0 \text{ and } 1$$

$$\theta(0, y') = - \left[y'^2 - y' + \frac{1}{6} (1 + \sigma\alpha) \right]$$

Correspondingly, the function ψ is given by:

$$\frac{\partial \psi}{\partial x'} = \frac{\partial^2 \psi}{\partial z'^2} \quad (33)$$

with the boundary conditions

$$\frac{\partial \psi}{\partial z'} = 0 \text{ at } z' = 0 \text{ and } \sigma$$

$$\psi(0, z') = - \left(\frac{\alpha}{\sigma} z'^2 - \alpha z' \right)$$

A solution for the function θ is taken in the form

$$\theta = \sum_{m=2,4,\dots}^{\infty} A_m e^{-\gamma_m^2 x'} \cos(\gamma_m y') \quad (34)$$

in which the eigenvalues γ_m are given by

$$\gamma_m = m\pi \quad (35)$$

In a similar manner the function ψ has the solution

$$\psi = \sum_{n=2,4,\dots}^{\infty} B_n e^{-\delta_n^2 x'} \cos(\delta_n z') \quad (36)$$

where the eigenvalues δ_n are given by

$$\delta_n = \frac{n\pi}{\sigma} \quad (37)$$

The coefficients A_m of equation (34) and the coefficients B_n of equation (36) are evaluated to satisfy the respective entrance boundary conditions. This gives the conditions

$$\sum_{m=2,4,\dots}^{\infty} A_m \cos(m\pi y') = - \left[y'^2 - y' + \frac{1}{6} (1 + \alpha\sigma) \right] \quad (38a)$$

$$\sum_{n=2,4,\dots}^{\infty} B_n \cos\left(n\pi \frac{z'}{\sigma}\right) = - \left(\frac{\alpha}{\sigma} z'^2 - \alpha z' \right) \quad (38b)$$

According to Sturm-Liouville theory the coefficients A_m and B_n given by

$$A_m = - \frac{\int_0^1 \left[y'^2 - y' + \frac{1}{6} (1 + \alpha\sigma) \right] \cos(m\pi y') dy'}{\int_0^1 \cos^2(m\pi y') dy'} \quad (39a)$$

$$B_n = - \frac{\int_0^\sigma \left(\frac{\alpha}{\sigma} z'^2 - \alpha z' \right) \cos \left(n\pi \frac{z'}{\sigma} \right) dz'}{\int_0^\sigma \cos^2 \left(n\pi \frac{z'}{\sigma} \right) dz'} \quad (39b)$$

The integral appearing in the numerator of equation (39a) may be written as

$$\int_0^1 \left[y'^2 - y' + \frac{1}{6} (1 + \alpha\sigma) \right] \cos (m\pi y') dy' = \frac{2}{m^2 \pi^2}$$

where m is an even integer. If m is an odd number, the integral yields a value of zero. The series coefficients are thus

$$A_m = - \frac{4}{m^2 \pi^2}; \quad m = 2, 4, \dots \quad (40a)$$

In a similar manner the coefficients B_n are obtained as

$$B_n = - \frac{4\alpha\sigma}{n^2 \pi^2}; \quad n = 2, 4, \dots \quad (40b)$$

Now that $t_{q,d}$ and t_q^* are known, they can be superposed as in equation (23) to obtain the solution which applies over the entire length of the channel. This is

$$\begin{aligned} \frac{t_q - t_e}{q_B a / K} &= 2 \left(1 + \frac{\alpha}{\sigma} \right) x' + y'^2 - y' + \frac{\alpha}{\sigma} z'^2 - \alpha z' + \frac{1}{6} (1 + \alpha\sigma) \\ &- \sum_{m=2,4,\dots}^{\infty} \frac{4}{m^2 \pi^2} \cos (m\pi y') e^{-m^2 \pi^2 x'} \\ &- \alpha\sigma \sum \frac{4}{n^2 \pi^2} \cos \left(n\pi \frac{z'}{\sigma} \right) e^{-(n^2 \pi^2 x' / \sigma^2)} \quad (41) \end{aligned}$$

The local bulk mean temperature $t_{q,b}$ along the channel is given by

$$\begin{aligned}
 t_{q,b} &= t_e + \frac{\tilde{q}}{\rho U c_p \sigma a^2} x \\
 &= t_e + \frac{\tilde{q}}{\sigma k} x'
 \end{aligned} \tag{42a}$$

Substituting $\tilde{q} = 2(\sigma + \alpha)(q_B a)$ into equation (42a) gives

$$\frac{t_{q,b} - t_e}{q_B a / k} = 2 \left(1 + \frac{\alpha}{\sigma} \right) x' \tag{42b}$$

The analysis of the preceding paragraphs led to the determination of the temperature distribution for the condition where the heat flux on the short sides is an arbitrary fraction or multiple of the heat flux on the broad sides, or more simply, where $q_s = \alpha q_B$. There may arise, however, practical applications where it is desirable to express the heat flux on the broad walls as $q_B = \bar{\alpha} q_s$. The temperature distribution in this situation is then obtained from equation (41) by substituting $\alpha = 1/\bar{\alpha}$ and $q_B = \bar{\alpha} q_s$:

$$\begin{aligned}
 \frac{t_q - t_e}{q_s a / k} &= 2 \left(\bar{\alpha} + \frac{1}{\sigma} \right) x' + \bar{\alpha} (y'^2 - y') + \frac{1}{\sigma} z'^2 - z' + \frac{1}{6} (\bar{\alpha} + \sigma) \\
 &\quad - \bar{\alpha} \sum_{m=2, \dots}^{\infty} \frac{4}{m^2 \pi^2} \cos(m\pi y') e^{-m^2 \pi^2 x'} \\
 &\quad - \sigma \sum_{n=2, \dots}^{\infty} \frac{4}{n^2 \pi^2} \cos \left(n\pi \frac{z'}{\sigma} \right) e^{-n^2 \pi^2 x' / \sigma^2}
 \end{aligned} \tag{43}$$

The local bulk mean temperature is given by

$$\frac{t_{q,b} - t_e}{q_s a / k} = 2 \left(\bar{\alpha} + \frac{1}{\sigma} \right) x' \tag{44}$$

Combined Internal Heat Generation and Wall Heat Transfer

Results for the situation where internal heat generation and wall heat transfer are occurring simultaneously are found by combining the solutions for t_Q and t_q in accordance with equation (3). For side wall heat transfer given by $q_s = \alpha q_B$, the solution can be written as

$$\begin{aligned}
\frac{t - t_e}{q_B a / K} &= \left[2 \left(1 + \frac{\alpha}{\sigma} \right) + R \right] x' + y'^2 - y' + \frac{\alpha}{\sigma} z'^2 - \alpha z' + \frac{1}{6} (1 + \sigma \alpha) \\
&- \sum_{m=2,4,\dots}^{\infty} \frac{4}{m^2 \pi^2} \cos(m\pi y') e^{-m^2 \pi^2 x'} \\
&- \alpha \sigma \sum_{n=2,4,\dots}^{\infty} \frac{4}{n^2 \pi^2} \cos\left(n\pi \frac{z'}{\sigma}\right) e^{-n^2 \pi^2 z' / \sigma^2} \quad (45)
\end{aligned}$$

where

$$R = \frac{Qa}{q_B} \quad (46)$$

the temperature distribution may also be expressed in terms of side wall heat transfer by substituting $1/\alpha$ for α , $\bar{\alpha}q_S$ for q_B , and replacing the parameter R by the parameter S , defined as

$$S = \frac{Qa}{q_S} \quad (47)$$

The parameters R and S are the ratios of internal heat evolution to the heat transferred at the channel walls and give a measure of the relative importance, in connection with temperature development, of internal heat generation in the presence of wall heat transfer.

Heat Transfer Results

From the temperature distribution given by equation (45), various quantities of engineering interest can be determined. In the developments to follow, the analytical results will be expressed in terms of the broad wall heat transfer q_B .

Results of practical interest are the wall temperature variations corresponding to prescribed wall heat transfer and internal heat generation. The local temperatures $t_w(x', z') \equiv t_{wB}$ along the broad walls can be found from equation (45) by evaluating it at $y' = 0$ or at $y' = 1$:

$$\begin{aligned}
\frac{t_{wB} - t_e}{q_B a / k} &= \left[2 \left(1 + \frac{\alpha}{\sigma} \right) + R \right] x' + \frac{\alpha}{\sigma} z'^2 - \alpha z' + \frac{1}{6} (1 + \alpha \sigma) \\
&- \sum_{m=2, \dots}^{\infty} \frac{4}{m^2 \pi^2} e^{-m^2 \pi^2 x'} \\
&- \alpha \sigma \sum_{n=2, \dots}^{\infty} \frac{4}{n^2 \pi^2} \cos \left(n \pi \frac{z'}{\sigma} \right) e^{-(n^2 \pi^2 x' / \sigma^2)} \quad (48)
\end{aligned}$$

The local temperatures $t_w(x', y') \equiv t_{ws}$ along the side walls are found in a similar manner from equation (45) by evaluating it at $z' = 0$ or at $z' = \sigma$

$$\begin{aligned}
\frac{t_{ws} - t_e}{q_B a / k} &= \left[2 \left(1 + \frac{\alpha}{\sigma} \right) + R \right] x' + y'^2 - y' + \frac{1}{6} (1 + \alpha \sigma) \\
&- \sum_{m=2, \dots}^{\infty} \frac{4}{m^2 \pi^2} \cos (m \pi y') e^{-m^2 \pi^2 x'} \\
&- \alpha \sigma \sum_{n=2, \dots}^{\infty} \frac{4}{n^2 \pi^2} e^{-(n^2 \pi^2 x' / \sigma^2)} \quad (49)
\end{aligned}$$

Another form of these equations which is more convenient is obtained by introducing the bulk mean temperature t_b . For a uniform heat source and uniform wall heat transfer, the bulk temperature is given by

$$\frac{t_b - t_e}{q_B a / k} = \left[2 \left(1 + \frac{\alpha}{\sigma} \right) + R \right] x' \quad (50)$$

Then the local wall- to bulk-temperature difference along the broad walls is given by

$$\begin{aligned} \frac{t_{wB} - t_b}{q_B a / \kappa} &= \frac{\alpha}{\sigma} z'^2 - \alpha z' + \frac{1}{6} (1 + \alpha\sigma) - \sum_{m=2, \dots}^{\infty} \frac{4}{m^2 \pi^2} e^{-m^2 \pi^2 x'} \\ &- \alpha\sigma \sum_{n=2, \dots}^{\infty} \frac{4}{n^2 \pi^2} \cos\left(n\pi \frac{z'}{\sigma}\right) e^{-(n^2 \pi^2 x' / \sigma^2)} \end{aligned} \quad (51)$$

while the temperature difference along the side walls is

$$\begin{aligned} \frac{t_{ws} - t_b}{q_B a / \kappa} &= y'^2 - y' + \frac{1}{6} (1 + \alpha\sigma) - \sum_{m=2, \dots}^{\infty} \frac{4}{m^2 \pi^2} \cos(m\pi y') e^{-m^2 \pi^2 x'} \\ &- \alpha\sigma \sum_{n=2, \dots}^{\infty} \frac{4}{n^2 \pi^2} e^{-(n^2 \pi^2 x' / \sigma^2)} \end{aligned} \quad (52)$$

A noteworthy feature of equations (51) and (52) is that the variation of the local wall- to bulk-temperature differences is independent of the internal heat generation rate. The differences between the fully developed wall and bulk temperatures are

$$\frac{(t_{wB} - t_b)_d}{q_B a / \kappa} = \frac{\alpha}{\sigma} z'^2 - \alpha z' + \frac{1}{6} (1 + \alpha\sigma) \quad (53a)$$

$$\frac{(t_{ws} - t_b)_d}{q_B a / \kappa} = y'^2 - y' + \frac{1}{6} (1 + \alpha\sigma) \quad (53b)$$

The ratios of local to fully developed temperature differences along the broad walls or side walls at any location in the channel is found from equations (51) to (53) as

$$\frac{(t_{wB} - t_b)}{(t_{wB} - t_b)_d} = 1 - \frac{\sum_{m=2, \dots}^{\infty} \frac{4}{m^2 \pi^2} e^{-m^2 \pi^2 x'} + \alpha\sigma \sum_{n=2, \dots}^{\infty} \frac{4}{n^2 \pi^2} \cos\left(n\pi \frac{z'}{\sigma}\right) e^{-(n^2 \pi^2 x' / \sigma^2)}}{\frac{\alpha}{\sigma} z'^2 - \alpha z' + \frac{1}{6} (1 + \alpha\sigma)} \quad (54a)$$

$$\frac{(t_{ws} - t_b)}{(t_{ws} - t_b)_d} = 1 - \frac{\sum_{m=2, \dots}^{\infty} \frac{4}{m^2 \pi^2} \cos(m\pi y') e^{-m^2 \pi^2 x'} + \alpha \sigma \sum_{n=2, \dots}^{\infty} \frac{4}{n^2 \pi^2} e^{-(n^2 \pi^2 x' / \sigma^2)}}{y'^2 - y' + \frac{1}{6} (1 + \alpha \sigma)} \quad (54b)$$

It may be observed from the foregoing results that a prescribed heat flow from the duct surfaces to the fluid produces, at a given axial location, a local wall temperature or wall- to bulk-temperature difference which varies around the periphery of the duct and which also varies with duct aspect ratio.

Results and Discussion

To illustrate the effects of internal heat generation and prescribed wall heat transfer on wall temperature distributions, local bulk mean temperature, and thermal entry lengths for liquid metal flow in rectangular passages, a number of solutions have been obtained for various combinations of aspect ratio σ , wall heat flux ratio α or $\bar{\alpha}$, and heat flux ratio R or S . For the sake of brevity, only some of these results are included in this paper. In particular, values of the parameters α and $\bar{\alpha}$ chosen for the computations correspond to the following cases: (a) $\alpha = -1$, in which the broad walls are heated and the side walls are cooled; (b) $\alpha = 0$, in which the side walls are insulated; (c) $\alpha = 1$, in which uniform heating (or cooling) takes place all around the duct periphery; and (d) $\bar{\alpha} = 0$, in which the broad walls are insulated.

Designers of channels for the applications previously mentioned are interested in the temperatures achieved by the walls, and in particular the peripheral location where the wall temperature will assume its highest value for a known wall heat input and internal heat generation rate. From an examination of the analytical results, it is to be expected that the peak temperatures will occur either in the corner of a duct or else at the centerline of the broad wall or short wall, depending upon the wall heat flux ratio α . Therefore, knowledge of temperature conditions in such regions is of special interest. The wall temperatures are given here relative to the bulk temperature, since the heat flux ratio R or S is then eliminated as a parameter.

The longitudinal variation of the dimensionless wall temperatures were evaluated from the analytical solution and are presented in figures 2 for duct aspect ratios of 1, 4, and 10. For the special case of side walls insulated ($\alpha = 0$) the solution is independent of the duct aspect ratio, and therefore, the result applies for all aspect ratios. At some places along a wall, the wall- to bulk-temperature difference

may be negative. This is understandable, however, if it is recalled that t_w is a local value along the wall, while t_b is an average value over the entire cross section. When the broad walls are heated and the side walls are cooled, the peak temperatures occur along the centerline of the broad walls at all axial positions for the aspect ratios shown (fig. 2(a)). With heating from only the broad walls (fig. 2(b)), these walls assume a uniform temperature for all aspect ratios which is higher than the temperatures along the insulated side walls. For a rectangular duct with uniform heat flux all around the periphery (fig. 2(c)), the peak temperatures for all σ 's occur at the duct corners. Finally, when heat is transferred from only the short walls (fig. 2(d)), these walls attain a uniform temperature which is higher than the temperatures along the adiabatic broad walls.

Another quantity which is of practical interest is the bulk mean temperature variation along the length of the channel. The bulk temperature is given here relative to the temperature of the fluid at the entrance to the channel. The dimensionless bulk temperatures are presented in figures 3 for duct aspect ratios of 1, 20, and ∞ (parallel-plate channel) with the heat flux ratio appearing as a family parameter.

Positive and negative values of the parameters R and S are considered in the figures. It is supposed that Q is positive (a heat source). A positive value of R , therefore, implies that q_B is positive, that is, that heat is being transferred from the broad walls to the fluid. A negative value of R , on the other hand, implies that q_B is negative or that heat is being transferred from the fluid to the broad walls. For positive R , therefore, internal heat generation and broad-wall heat transfer reinforce one another to produce a bulk temperature larger than that obtained in the absence of internal heat generation. Conversely, for negative R , the broad wall heat transfer opposes internal heat generation in the bulk temperature development. Similar arguments apply in connection with side wall heat transfer q_S and heat flux ratio S .

For very small values of $|R|$ or $|S|$, wall heat transfer dominates the bulk temperature development, while for large values of $|R|$ or $|S|$, the effects of internal heat generation dominate. This accounts for the variety of trends that are evident in each of the figures. It is also observed that the bulk temperature development is, in general, slightly affected by duct aspect ratio σ to about 20 and insignificantly thereafter. For a duct with insulated side walls, however, the bulk temperature development is independent of the duct aspect ratio.

The foregoing presentation of results has been concerned with wall temperatures in the thermal entrance region. As a matter of general interest the wall temperature variation around the periphery of the duct in the fully developed region is considered. The wall temperatures are again given relative to the bulk temperature, since in the fully developed region the temperature difference $(t_w - t_b)_d$ is independent of x . In addition, as noted earlier, the temperature difference $t_w - t_b$ is independent of the heat flux ratio R at all axial positions.

Fully developed wall temperatures are presented in figures 4 for values of α and $\bar{\alpha}$ considered earlier and for various aspect ratios. It is worth while to recall that t_w is a local value along the wall, while t_b is an average value over the entire cross section. Therefore, at some places along the wall, the temperature difference $(t_w - t_b)_d$ is negative, which means that t_b is larger than t_w . The hot spots are strikingly displayed in these figures, appearing in the corners, or at the broad wall or short wall midpoints. It is also seen that the aspect ratio has a profound effect on the temperature distribution.

Of considerable practical importance to the designer is the knowledge of the conditions under which entrance effects must be accounted for in heat-transfer calculations. The approach of a local wall- to bulk-temperature difference to the fully developed value is, in theory, asymptotic. Consequently, it is difficult to identify a specific length of channel as a thermal entrance length. It is practice to define a thermal entrance length in terms of the downstream distance $x/aRePr$ at which the temperature difference approaches to within 5 percent of the fully developed value. The variation with dimensionless axial distance of local to fully developed wall temperatures at the duct corner and at the broad wall and short wall centerlines has been evaluated from the analytical expressions. The results thus obtained have been plotted in figures 5 for values of $\alpha = -1$ and $\bar{\alpha} = 0$, for parametric aspect ratio values. It should be noted that the corner temperature ratio is not shown for the square duct when $\alpha = -1$, since for this situation the corner temperature difference is zero at all axial positions. Lines delineating the condition $(t_w - t_b)/(t_w - t_b)_d = 0.95$ have been drawn in the figures to facilitate determining the thermal entrance length.

From an inspection of the graphs, it is seen that for the wall heating conditions represented, the wall temperature profiles become fully developed at widely different distances from the entrance to the heating section, depending upon the particular wall location chosen for consideration. It is also evident that there is a strong influence of the side walls of the ducts on the temperature developments, with the thermal entrance lengths increased with increased aspect ratio.

It is practice in reporting heat transfer connected with flow through noncircular passages to present average heat transfer coefficients or Nusselt numbers based upon a heat flow averaged around the duct periphery and on an average wall temperature. This practice has utility when the wall temperature remains constant everywhere, or is at least constant around the periphery of a duct at a given axial position. In the present situation where the wall boundary condition is one of peripherally and axially uniform heat input, however, it is apparent that the knowledge of the resulting local wall-temperature distribution is of more importance to the designer than the Nusselt numbers or even average thermal entry lengths. Therefore, no attempt has been made to determine these quantities. It is felt that the results are presented in a form more convenient for engineering calculations.

CONCLUDING REMARKS

Results have been presented for heat transfer in rectangular channels with prescribed wall heat fluxes and heat sources uniformly distributed in the fluid. The various effects considered have thrown some light on the axial and peripheral temperature distributions as well as on the heat transfer characteristics.

The present results are strictly valid only for the slug-flow velocity distribution with heat transfer only by molecular conduction. The simplification provided by these assumptions, however, has made it possible to obtain exact mathematical solutions to the governing energy equation. Of greater importance is the fact that these temperature distributions approximate conditions to be expected for turbulent liquid-metal flow in rectangular ducts for relatively low Prandtl and Reynolds moduli. The results, in addition, point out the locations of maximum temperatures.

For improved heat transfer calculations, velocity-profile and eddy diffusivity variations would have to be taken into account to provide a more realistic description. Turbulent velocity profiles for circular pipe and parallel plate duct systems have been satisfactorily represented by power-law expressions. Within the knowledge of the author, velocity-profile and eddy diffusivity distributions for turbulent flow in rectangular ducts have received little theoretical consideration. Reference 16 presents a variational method for determining velocity distributions for flow of a power law fluid in cylindrical ducts. The results might prove useful for heat transfer studies.

In closing, it should be mentioned that the slug-flow, molecular-conduction analysis can be used to treat other wall boundary conditions, such as a specified axial and peripheral wall temperature distribution.

NOMENCLATURE

A_m	coefficient in series for temperature distribution with wall heat transfer and $Q = 0$
a	length of short side
a_{ij}	coefficient in series for temperature distribution with internal heat generation and $q_B = q_S = 0$
B_n	coefficient in series for temperature distribution with wall heat transfer and $Q = 0$
b	length of broad side
c_p	specific heat of fluid at constant pressure

$F(y', z')$	transverse temperature distribution in fully developed region with wall heat transfer and $Q = 0$
$f(y', z')$	transverse temperature distribution in fully developed region with internal heat generation and $q_B = q_S = 0$
Pr	Prandtl number, $\mu c_p / \kappa$
Q	rate of internal heat generation per unit volume
q	heat addition per unit wall area
\tilde{q}	heat addition per unit channel length, equal to $2bq_B + 2aq_S$
R	heat flux ratio, Qa/q_B
Re	Reynolds number, $\rho Ua/\mu$
S	heat flux ratio, Qa/q_S
t	temperature
U	velocity taken as uniform over duct cross-section
u	local fluid velocity
x	coordinate measured along the axial direction
x'	dimensionless axial coordinate, $x/aRePr$
$Y(y')$	eigenfunction for case of internal heat generation and $q_B = q_S = 0$
y	coordinate measured along short side
y'	dimensionless coordinate, y/a
$Z(z')$	eigenfunction for case of internal heat generation and $q_B = q_S = 0$
z	coordinate measured along long side
z'	dimensionless coordinate, z/a
z''	dimensionless coordinate, $z/b = z'/\sigma$
α	wall heat flux ratio, q_S/q_B
$\bar{\alpha}$	wall heat flux ratio, $q_S/q_B = 1/\alpha$
α_i	eigenvalues for case of internal heat generation and $q_B = q_S = 0$

β_j	eigenvalues for internal heat generation and $q_B = q_S = 0$
γ_m	eigenvalues for wall heat transfer and $Q = 0$
δ_n	eigenvalues for wall heat transfer and $Q = 0$
$\theta(x', y')$	function defined by equation (32)
κ	thermal conductivity of fluid
λ_{ij}^2	eigenfunction, $\alpha_i^2 + \beta_j^2$
μ	fluid viscosity
ρ	fluid density
σ	aspect ratio, b/a
$\psi(x', z')$	function defined by equation (33)

Subscripts

B	refers to broad wall
b	bulk mean value
d	fully developed
e	entrance value
Q	insulated wall, internal heat generation
q	wall heat flux, no internal generation
s	refers to side wall
w	value at wall

Superscript

*	entrance region
---	-----------------

REFERENCES

1. Muller, George L: Experimental Forced Convection Heat Transfer With Adiabatic Walls and Internal Heat Generation in a Liquid Metal. ASME Paper No. 58-HT-17.

2. Petukhov, B. S.; and Genin, L. G.: Heat Transfer in Tubes With Internal Heat Sources in the Fluid Stream. *Int. Chem. Eng.*, vol. 3, no. 4, Oct 1963, pp. 383-386.
3. Kutateladze, S. S.: *Fundamentals of Heat Transfer*. Academic Press, Inc., 1963, pp. 217-220.
4. Blanco, Jose A., et al.: The Effects of Dissipation, Compression Work, Unsymmetrical Boundary Conditions and Variable Wall Temperatures on Turbulent Heat Transfer. NSF-GP-1137, Syracuse Univ., N. Y., 1964.
5. Inman, Robert M.: Heat Transfer to Liquid Metal Flow in a Round Tube or Flat Duct With Heat Sources in the Fluid Stream. (Unpublished work).
6. Clairborne, H. C.: Heat Transfer in Noncircular Ducts, pt. I. ORNL-985, May 14, 1951.
7. *Liquid Metals Handbook*, June 1952 (AEC).
8. Hartnett, J. P.; and Irvine, T. F., Jr.: Nusselt Values for Estimating Turbulent Liquid Metal Heat Transfer in Noncircular Ducts. *A. I. Ch. E. Jour.*, vol. 3, no. 3, Sept. 1957, pp. 313-317.
9. Eckert, E. R. G.; and Irvine, T. F., Jr.: Pressure Drop and Heat Transfer in a Duct With Triangular Cross Section. *Jour. Heat Trans.*, May 1960, pp. 125-138.
10. Maier, K. H.: Kühlung und Erwärmung von Stoffen in Rechteckigen Kanälen Bei Kolbenströmung (Cooling and Heating of Fluids in Rectangular Channels with Rodlike Flow). *Kalte-Technik*, vol. 16, no. 4, April 1964, pp. 98-103.
11. Hall, W. B.: *Reactor Heat Transfer*. Temple Press, Ltd., 1958.
12. Poppendiek, H. F.: Liquid-Metal Heat Transfer. *Heat Transfer Symposium*, Engr. Research Inst., University of Michigan, 1953, pp. 77-100.
13. Schneider, P. J.: Effect of Axial Fluid Conduction on Heat Transfer in the Entrance Regions of Parallel Plates and Tubes. *Trans. ASME*, vol. 79, no. 4, May 1957, pp. 765-773.
14. Sparrow, E. M.; and Siegel, R.: Laminar Tube Flow With Arbitrary Internal Heat Sources and Wall Heat Transfer. *Nucl. Sci. Engr.*, vol. 4, no. 2, August 1958, pp. 239-254.
15. Sparrow, E. M., et al.: Laminar Flow of a Heat-Generating Fluid in a Parallel-Plate Channel. *A.I.Ch.E. Jour.* vol. 9, no. 6, Nov. 1963, pp. 797-804.
16. Schechter, R. S.: On the Steady Flow of a Non-Newtonian Fluid in Cylindrical Ducts. *A.I.Ch.E. Jour.*, vol. 7, no. 3, Sept. 1961, pp. 445-448.

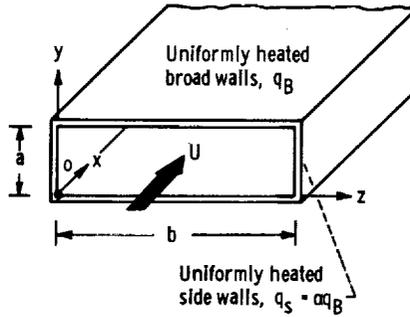
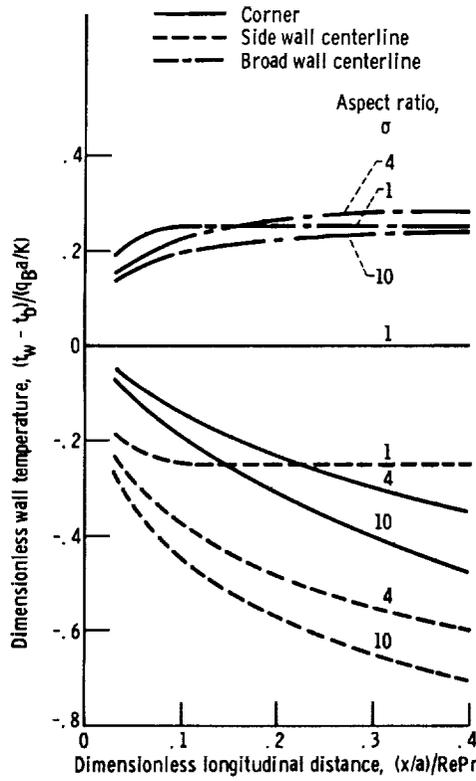
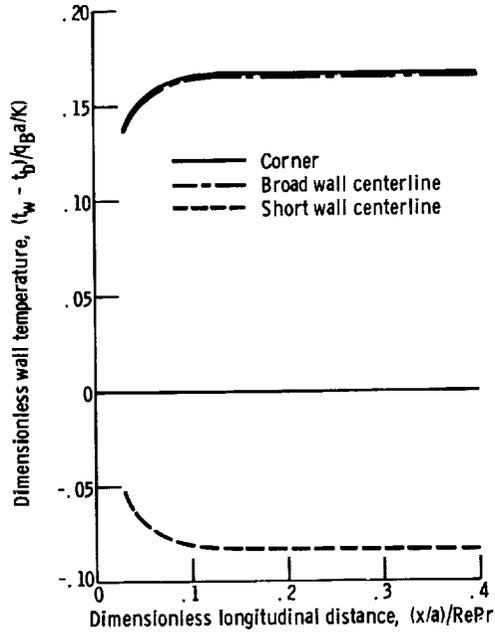


Figure 1. - Coordinate system for rectangular channel with different uniform heating on each pair of opposite walls.



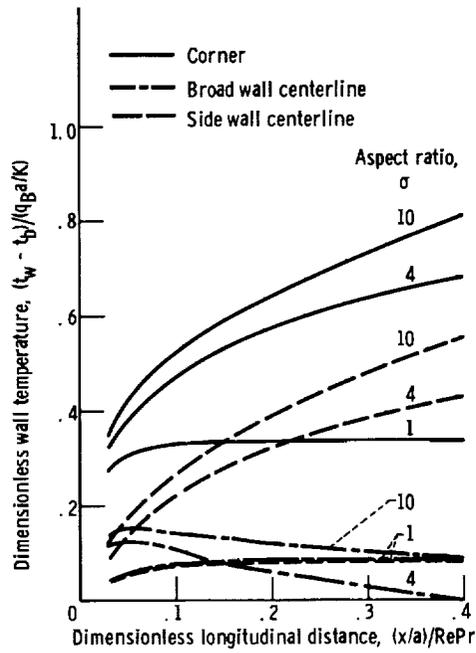
(a) Uniform heating of broad walls and uniform cooling of side walls ($\alpha = -1$).

Figure 2. - Wall temperature development in thermal entrance region of rectangular ducts.



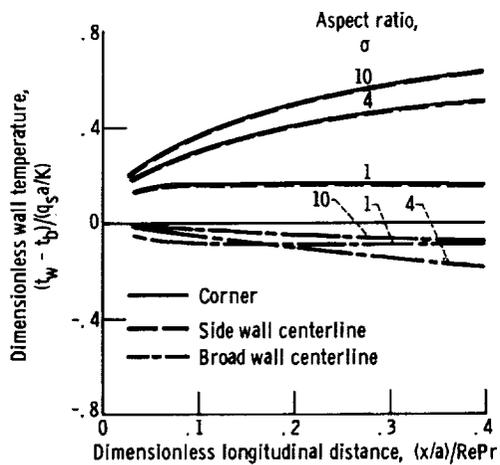
(b) Uniform heat transfer from broad walls only ($\alpha = 0$). Duct aspect ratio, $1 \leq \sigma \leq \infty$.

Figure 2. - Continued.



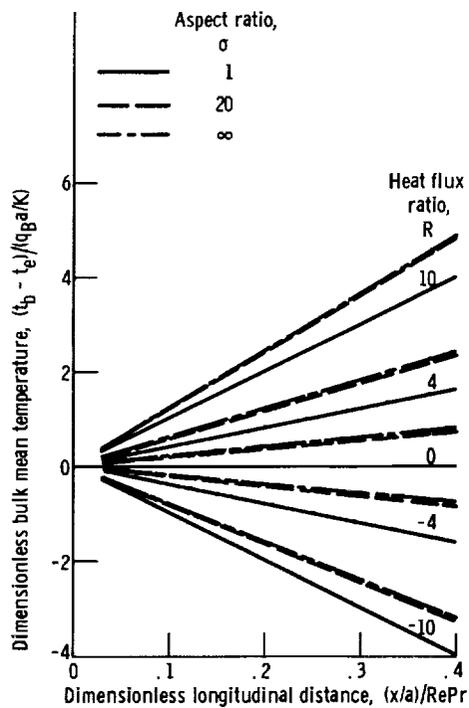
(c) Uniform heating of four walls ($\alpha = 1$).

Figure 2. - Continued.



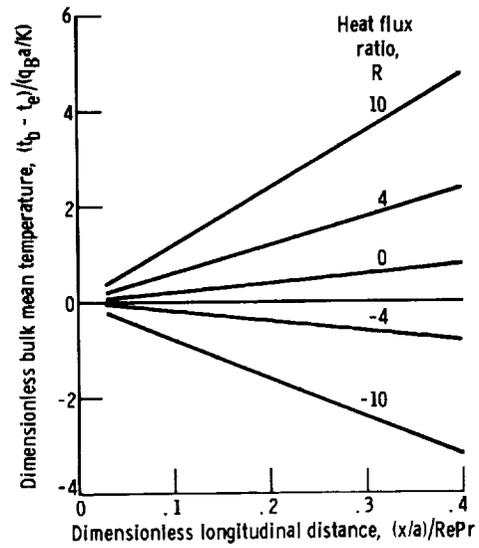
(d) Uniform heating of short walls ($\alpha = 0$).

Figure 2. - Concluded.



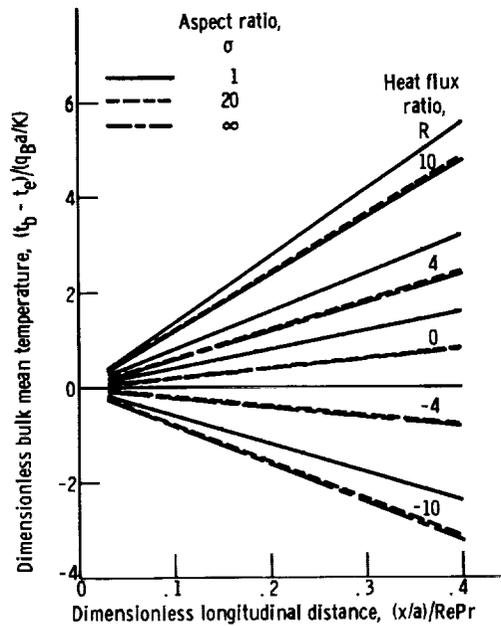
(a) Uniform heating of broad walls and uniform cooling of side walls ($\alpha = -1$).

Figure 3. - Longitudinal variation of bulk mean temperature for various values of heat flux ratio R .



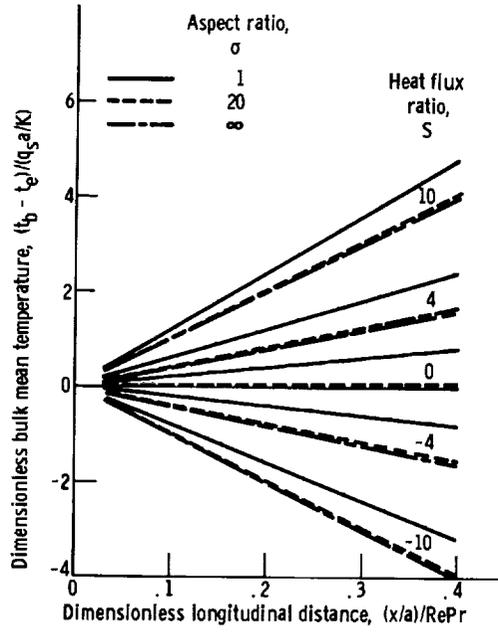
(b) Uniform heating of broad walls ($\alpha = 0$). Duct aspect ratio, $1 \leq \sigma \leq \infty$.

Figure 3. - Continued.



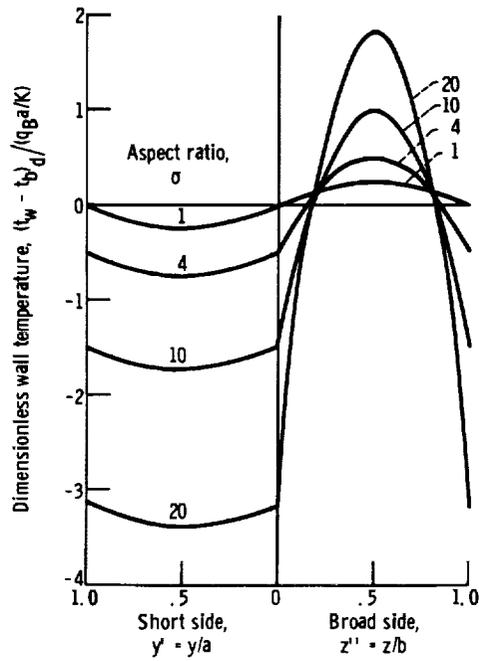
(c) Uniform heating of four walls ($\alpha = 1$).

Figure 3. - Continued.



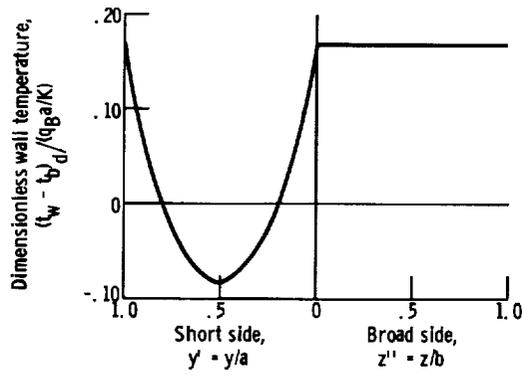
(d) Uniform heating of side walls ($\alpha = 0$).

Figure 3. - Concluded.



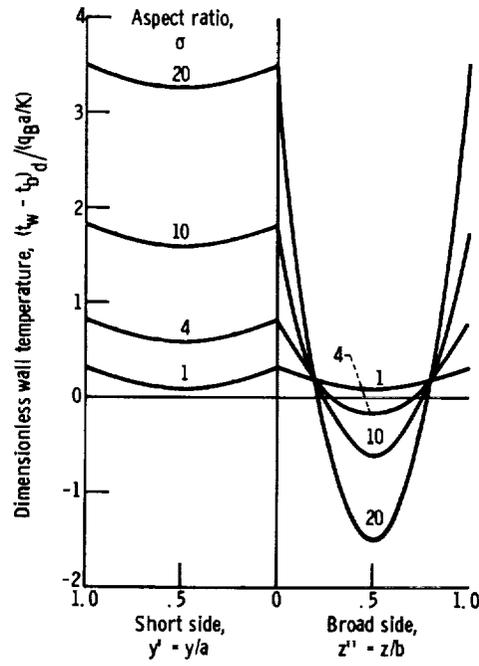
(a) Uniform heating of broad walls and uniform cooling of side walls ($\alpha = -1$).

Figure 4. - Fully developed wall temperatures of rectangular channels.



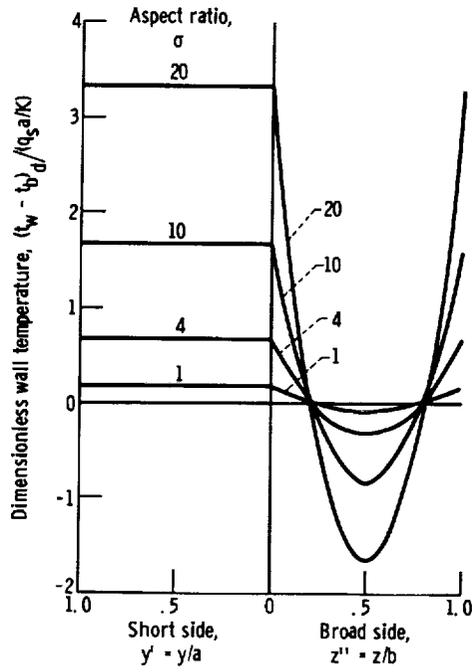
(b) Uniform heating of broad walls only ($\alpha = 0$).
Duct aspect ratio, $1 \leq \sigma \leq \infty$.

Figure 4. - Continued.



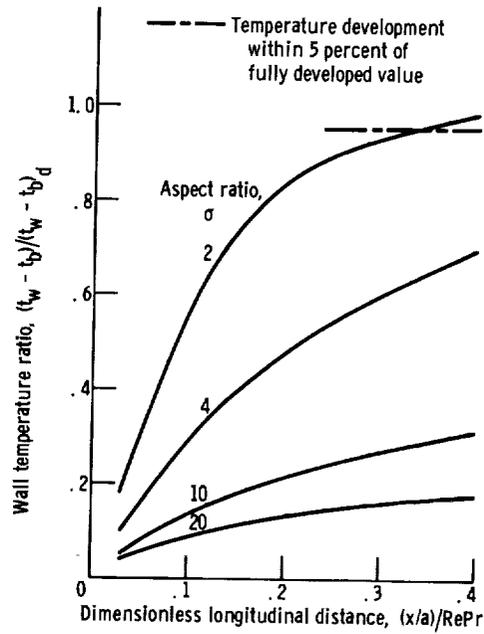
(c) Uniform heating of four walls ($\alpha = 1$).

Figure 4. - Continued.



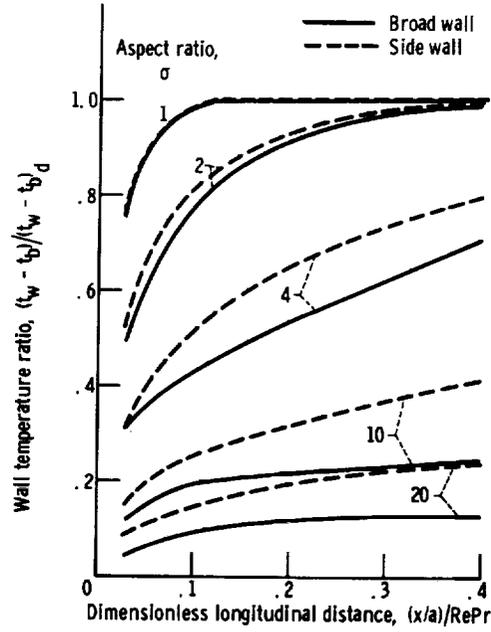
(d) Uniform heating of side walls only ($\alpha = 0$).

Figure 4. - Concluded.



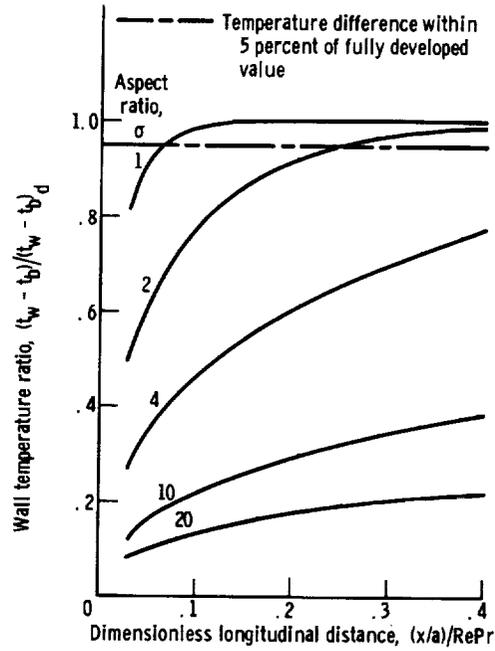
(a) Along duct corner in thermal entrance region with uniform heating of broad walls and uniform cooling of side walls ($\alpha = -1$).

Figure 5. - Wall temperature ratios.



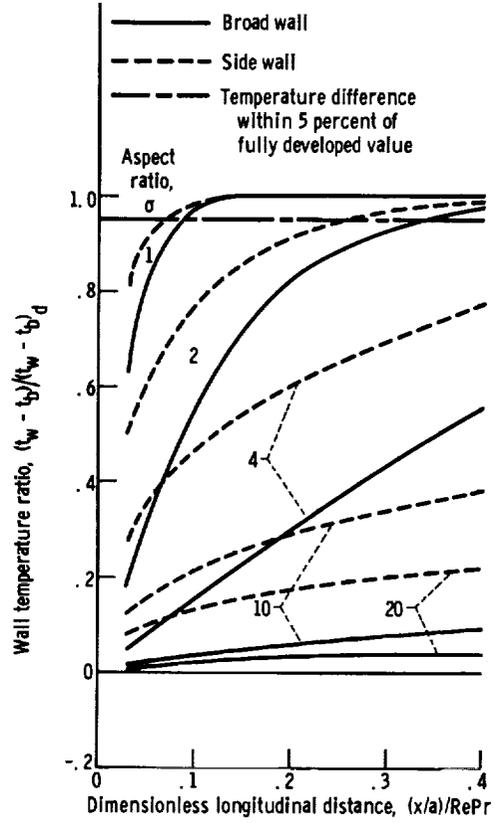
(b) Along wall centerlines in thermal entrance region with uniform heating of broad walls and uniform cooling of side walls ($\alpha = -1$).

Figure 5. - Continued.



(c) Along duct corner in thermal entrance region with heat transferred from short walls ($\bar{\alpha} = 0$).

Figure 5. - Continued.



(d) Along wall centerlines in thermal entrance region with heat transferred from short walls ($\alpha = 0$).

Figure 5. - Concluded.