FOREWORD

This report documents work accomplished under a contract at the Massachusetts Institute of Technology.

This study was principally concerned with test investigations required to evaluate the performance of an electronic control programmer into the control flight control system under contract NASA-4247.
SUMMARY

This paper describes several methods of designing switching circuits and optimizing these designs, using a digital computer as a design tool. A method of examining the on-off equation plot of an inverter switching circuit is discussed; and a technique for maximizing the output load current and the fan-out factor for an inverter circuit is also presented.

A tabulation of existing computer programs for transistor resistor logic circuits is given.

In addition, an analysis of a linear circuit is made to determine the parametric changes affecting the d-c bias stability and the manner in which this, in turn, affects the linearity and gain.
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INTRODUCTION

1.1 GENERAL

This report describes work which is being accomplished in the area of worst case and optimized circuit design by the General Dynamics/Astronautics-Centaur flight control group. Particular areas of development are engineering aids to:

a. Predict the optimum design.

b. Find the maximum stress that a circuit may withstand without reduced operability.

These aids are primarily oriented towards new design such that improved devices may be efficiently and carefully utilized by executing an analog model of the circuits. Mathematical models are primarily based on on-off equations for digital circuits and nodal equations for linear circuits.

All programs produced for circuit evaluation are written in Fortran to be operated on the IBM-7094 computer configuration. These programs are available and other engineering groups are invited to use and add to the library as the need arises.

This report will be revised to include new solutions and methods of analysis switching circuit and analog problems as they occur.

The services and assistance of the scientific data processing logic systems programming group Dept 158-1 are utilized in the programming of the worst case and optimized design solutions.

1.2 PURPOSE

The purpose of this program is to develop a library of computer programs to predict optimum design configurations and evaluate maximum capability of a particular configuration. These programs will provide design assurance essential to high reliability goals of the Centaur program. All programs are designed for flexibility and ease of operation such that an engineer may modify input data and receive stress data in minimum time at minimum cost. The goal is to provide these data at a cost less than that required to manually evaluate the circuit.
1.3 SCOPE

Programs have been developed which evaluate resistor-transistor logic schemes (NOR-LOGIC) without clamping diodes. This type of logic is presently being used in the Centaur upper stage for simplicity and reliability. Programs also exist to evaluate linear circuits with ten nodes. These programs include special-purpose matrix solutions, whose input constants may be varied, and modifications of the IBM-Share library models oriented to the Centaur servo and gyro amplifier configurations.

1.4 UTILIZATION

The designer desiring to utilize these programs will face an indoctrination period to familiarize himself with the techniques and tools available at his disposal. It is expected that the average engineer would become efficient after performing one or two designs by this technique. In addition, new design configurations may be analyzed with minor modification of input parameters. This allows the designer to readily implement new devices, changes, and suggestions with minimum modification design time.
SECTION II
SWITCHING CIRCUITS

2.1 DESIGN CONSIDERATIONS

2.1.1 TRL CIRCUITS. The optimized design of TRL circuits is based on the following considerations.

a. Upper and lower temperature limits.
b. Transistor parameter variations.
c. Component variations.
d. Power supply variations.

2.1.2 CIRCUIT CONFIGURATIONS. The following circuit configurations have been considered for analysis at this time.

a. TRL nor gate.
b. Relay driver.
c. Inverter.
d. Set reset – bistable.

Optimized design circuits can be designed to meet several objectives. A table of possible design objectives together with the constraining condition is shown in Table 2-1. Also shown in Table 2-1 is the circuit variable that can be adjusted to obtain an optimum design. Which design parameter to be optimized is a design choice or is dictated by the design specification.

2.2 DESIGN PROCEDURES

A worst case design can be accomplished by different methods of analysis. Several methods of attack are described below and examples given of each.

2.2.1 ON-OFF EQUATION PLOT. Given the "on" equation and the "off" equation of a simple saturated inverter, the "on" and "off" equations can be plotted on the same coordinate axes. The area enclosed by the "on" equation and the "off" equation and the X axis represents solutions satisfying both "on" and "off" conditions. The resistor tolerances and power supply tolerances are to be included in the "on" and "off" equations. The centroid of the area enclosed by the "on" and "off" curves is the best solution. This solution does not give the maximum fan out for the circuit constraints, but does give the maximum safe operating point. The analysis of the circuit is given in Paragraph 2.2.3.
TABLE 2-1. POSSIBLE DESIGN OBJECTIVES

<table>
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<td>Fan Out – N maximum</td>
<td>R_B, R_K, R_C</td>
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<tr>
<td>NOR-RTL</td>
<td>Fan In – M maximum</td>
<td>R_B, R_K, R_C</td>
<td>V_BB, V_CC, I_D2</td>
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<td>Fan Out – N maximum</td>
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<td>V_BB, V_CC, I_B2</td>
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4.5 EXAMPLE

4.5.1 PROBLEM. Find optimum values of R_1 and R_2. Given a fan out of 1 and base current I_B2 of Q_2.

Figure 2-1. Optimum Values of R_1 and R_2
2.3.1.1 Design Constants

\( V_{cc} \) = collector supply voltage

\( V_{BE\text{on}} \) = base to emitter voltage — forward bias of Q2

\( N \) = fan out

\( V_{BE\text{off}} \) = reverse bias voltage — design option

\( V_{CE\text{S}} \) = saturation voltage Q1

\( I_C \) = base current Q2

\( t \) = resistor tolerance (decimal fraction)

\( R_C \) = collector resistance of Q1

\( I_{cbo} \) = collector leakage current at maximum operating temperature.

\( R_L \) = Q2 load

The "on" equation for Q2 is as follows:

\[
I_B = \frac{V_{CC} - V_{BE\text{on}} - \frac{I_{cbo} R_C}{R_C (1+t) + R_1 (1+t)}}{R_2 (1-t)} - \frac{V_{BB} + V_{BE\text{on}}}{R_1 (1+t)}
\]

(2.1)

where

\[
\overline{I_B} = \frac{V_{CC}}{R_C (1-t)} B
\]

where \( B \) = minimum \( H_{FE} \) of Q2.  

(2.2)

Subscripts and superscripts indicate a minimum and maximum value respectively. Placing the value of \( I_B \) obtained in equation (2.2) in equation (2.1). Setting the resulting equation equal to zero and collecting terms.

\[
R_1 V_B (1 + t) + R_1 R_2 I_B (1 - t^2) + R_2 \left[ - V_A (1 - t) + I_B R_C R_2 (1 - t)^2 \right] + V_B R_C (1 + t) = 0
\]

(2.3)
\[ V_A = V_{cc} - \frac{V_{BE\,on}}{R_c} - I_{cbo} R_c (1 + t) \]
\[ V_B = \frac{V_{BB}}{} + \frac{V_{BE\,on}}{} \]

where

\[ A_1 = V_B (1 + t) \]  \hspace{1cm} (2.4)
\[ B_1 = I_B (1 - t^2) \]  \hspace{1cm} (2.5)
\[ C_1 = -V_A (1 - t) + I_B R_c R_2 (1 - t)^2 \]  \hspace{1cm} (2.6)
\[ D_1 = V_B R_c (1 + t) \]  \hspace{1cm} (2.7)

which is an equation of the form

\[ A_1 R_1 + B_1 R_1 R_2 + C_1 R_2 + D_1 = 0 \]  \hspace{1cm} (2.8)

Solving for \( R_2 \)

\[ R_2 = \frac{-AR_1 + D_1}{C_1 + B_1 R_1} \] \hspace{1cm} (2.9)

\[ R_2 = \frac{(V_{BB} + V_{BE\,on}) (1 + t) [R_c + R_1]}{(1-t) \left[ V_{BE\,on} \left( V_{cc} + I_{cbo} R_c \right) + (1-t)^2 I_B R_1 + I_B (1-t^2) R_c \right]} \] \hspace{1cm} (2.10)

the "off" equation is

\[ V_{BE\,off} = \frac{V_{ces} R_2 (1 + t) - V_{BB} (1 - t) R_1 + I_{cbo} R_1 R_2 (1 - t^2)}{R_1 (1 - t) + R_2 (1 + t)} \] \hspace{1cm} (2.11)

Collecting terms and setting equation (2.11) equal to zero

\[ R_1 \left[ V_{BB} (1 - t) - V_{BE\,off} (1 - t) \right] + R_2 \left[ V_{ces} (1 + t) - V_{BE\,off} (1 + t) \right] \]
\[ + R_1 R_2 \left( (1 - t^3) I_{cbo} \right) = 0 \] \hspace{1cm} (2.12)

which is an equation of the form

\[ A_2 R_1 + B_2 R_1 R_2 + C_2 R_2 = 0 \]
where

\[ A_2 = (1-t)(-V_{BB} - V_{BE\text{off}}) \]  

(2.13)

\[ B_2 = (1+t)(V_{CES} - V_{BE\text{off}}) \]  

(2.14)

\[ C_2 = (1-t^2)I_{\text{cbo}} \]  

(2.15)

solving for \( R_2 \)

\[ R_2 = \frac{-A_2 R_1}{B_2 R_1 + C_2} \]  

(2.16)

\[ R_2 = \frac{(1-t)(-V_{BB} - V_{BE\text{off}})}{(1+t)(V_{CES} - V_{BE\text{off}}) R_1 + (1-t)^2 I_{\text{cbo}}} \]  

(2.17)

The two resulting expressions for the "on" equation and the "off" equation

\[ R_2 = f_1(R_1) \quad \text{"on" equation} \]  

(2.18)

\[ R_2 = f(R_1) \quad \text{"off" equation} \]  

(2.19)

are both expressed as functions of \( R_1 \). A graph of \( R_2 = f(R_1) \) "ON" \( R_2 = \) \( f(R_1) \) "OFF" is plotted. Figure 2-2 shows a plot of the "on" and "off" equations. The \( I_{\text{cbo}} \) term is neglected in these plots.

![Figure 2-2. On-Off Equations](Image)
The area bounded by the "on" and "off" equations is an acceptable solution satisfying both "on" and "off" conditions.

A computer program exists for obtaining the "ON" and "OFF" equation. The existing program has the $I_{CBO}$ terms set equal to zero since for the types of transistors used, the collector leakage terms made negligible contributions to the $V_{BEoff}$ and $I_B$ terms.

The "on" equation may be modified to include a fan out factor $N$, where $N$ is the number of stages being supplied a base current of $I_B$ from $Q_1$. Equation (2.1) becomes

$$I_B = \frac{V_{CC} - V_{BEon} - I_{CBO}R_C}{N R_C (1+t) + R_2 (1+t)} - \frac{V_{BB} + V_{BEon}}{R_2 (1-t)}$$

(2.20)

Equation (2.20) does not take into account current hogging which results in differences in $R_1$ and $V_{BEon}$ between stages of inverters being supplied by $Q_1$. When current hogging is taken into account, the fan out is reduced to $N_O$ where $N_O$ is given by:

$$N_O = (N-1) K + 1$$

(2.21)

$$K = \frac{\left(\frac{V_{CC} - I_{CBO} R_1 (1+t) - V_{BEon}}{V_{CC} - V_{BEon}}\right)}{\left(\frac{R_1 (1-t) + N R_L (1-t)}{R_K (1+t) + N R_L (1+t)}\right)}$$

(2.22)

The circuit may be optimized as shown in Table 2-1 by maximizing $R_C$ or $N$ with respect to $R_1$. The method of attack in optimizing equation (2.1) is to substitute equation (2.2) into (2.1). Find $dR_1/dR_1$ or $dN/dR_1$ set resulting equation equal to zero and solve resulting equations for $R_1$, $R_2$, and $R_C$ using equation (2.2) as an auxiliary equation. This particular example is not optimized. The following example is optimized for a maximum $R_C$ given a base current $I_B$, and also for a maximum $N_1$ given a base current $I_B$.

2.4 INVERTER CIRCUIT

Consider the inverter circuit shown in Figure 2-3 with back bias provided with an emitter supply voltage $V_E$ instead of the conventional back bias supply $V_{BE}$. The $I_{CBO}$ terms have been neglected in this case but can be easily inserted into the on off equation if desired. A program has been written to find the minimum collector current $I_c$ for the driver stage $Q_1$ by finding $dR_1/dR_1$ and solving for $R_1$, $R_2$ and $R_C$. A second program number of stages of inverters that may be connected to $Q_1$ each inverter requiring a base current $I_B$. 

2-6
Figure 2-3. Inverter Circuit

$V_{cc} =$ collector supply voltage

$N =$ fan out

$V_{e} =$ emitter supply voltage

$V_{BEoff} =$ Reverse bias voltage - design option

$V_{ces} =$ saturation voltage $Q_1$

$I_B =$ base current $Q_2$ design constant

$t =$ resistor tolerance -(decimal fraction)

$R_L =$ collector load resistor $Q_1$

$\overline{V_{BEon}} = \overline{V_{e}} + V_f$

$V_f =$ Base to emitter voltage forward bias of $Q_2$
From Figure 2-3 the "on" equation can be written as:

\[
\frac{V_{CE}}{(1+t)(R_K + N R_L)} - \frac{V_{BEon}}{R_B(1-t)} = I_B
\]

(2.23)

and the "off" equation as:

\[
V_{PFO} = \frac{V_O R_B(1-t)}{(1+t)(R_K + R_B(1-t))} - \frac{V_E}{v_E}
\]

(2.24)

where

\[
V_O = \frac{V_E}{v_E} + \frac{V_{CES}}{v_E}
\]

Solving equation (2.24) for \( R_B \)

\[
\left(\frac{1+t}{R_K + R_B(1-t)}\right) V_{BEO} - \frac{V_O R_B (1-t) + V_E (1+t) R_K + V_F (1-t) R_B}{v_E} = 0
\]

(2.25)

\[
V_{BEO} (1+t) R_K + V_{BFO} (1-t) R_B - V_O (1-t) + V_E (1+t) R_K + V_F (1-t) R_B = 0
\]

(2.26)

\[
R_B = R_K \left[ \frac{(1+t)}{(1-t)} \left( \frac{V_{BEO} + V_F}{V_{BEO} - V_E + V_{CES}} \right) \right]
\]

(2.27)

(2.28)

\[
R_B = R_K \left[ \frac{(1+t)}{(1-t)} \left( \frac{V_{BEO} + V_F}{V_{BEO} - V_E + V_{CES}} \right) \right]
\]

(2.29)

\[
R_B = R_K \left[ \frac{(1+t)}{(1-t)} \left( \frac{V_{BEO} + V_E}{V_{BEO} - V_{CES}} \right) \right]
\]

(2.30)

\[
R_B = R_K G
\]

(2.31)

where

\[
G = \frac{1+t}{1-t} \left( \frac{V_{BEO} + V_F}{V_{BEO} - V_{CES}} \right)
\]

(2.32)
substituting equation (2.31) into equation (2.23)

\[
\frac{V_{CC} - V_{BE on}}{(1 + t) (R_K + NR_L)} - \frac{V_{BE on}}{R_K' C (1 - t)} = I_B
\]  

(2.33)

\[
R_L = R_K^2 \left( \frac{-I_B (1-t^2) C + R_K' \left( C (V_{CC} - V_{BE on}) \right)}{R_K' \left( I_B (1-t^2) C N \right) + V_{BE} \cdot N} \right)
\]  

(2.34)

\[
R_L = \frac{C_1 R_K^2 + C_2 R_K}{C_3 R_K + C_4}
\]  

(2.35)

where

\[
C_1 = I_B (1-t^2)
\]  

(2.36)

\[
C_2 = C (V_{CC} - V_{BE on}) - V_{BE on}
\]  

(2.37)

\[
C_3 = I_B (1-t^2) C \cdot N
\]  

(2.38)

\[
C_4 = V_{BE on} N
\]  

(2.39)

\[
\frac{d R_L}{d R_K} = \frac{(C_3 R_K + C_4) (2C_1 R_K + C_2) - [C_1 R_K^2 + C_2 R_K] C_3}{[C_3 R_K + C_4]^2} = 0
\]  

(2.40)

collecting terms

\[
C_1 C_3 R_K^2 + 2C_4 C_1 R_K + C_4 C_2 = 0
\]  

(2.41)

and solving the quadratic equation for \(R_K\)

\[
R_K = \frac{-V_{BE on} \pm \sqrt{C V_{BE on} (V_{CC} - V_{BE on})}}{I_B (1-t^2) C}
\]  

(2.42)
The positive root of (2.42) is then substituted into equation (2.24) to obtain $R_B$. $R_L$ is obtained by substituting $R_K$ and $R_B$ into equation (2.23). The computer program is set up to provide print out of solutions for $R_K$, $R_F$, and $R_L$ for an N and I_B as variables. If it is desired to obtain $dN/R_K$ equation (2.33) is solved for N instead of $dR_K$. Equation (2.33) becomes

$$N = \frac{R_K^{\frac{2}{3}} \left[ -I_B (1-t^2) C \right] + R_K \left[ \left( V_{CE} - V_{BE \text{on}} \right) C - V_{BE \text{on}} \right]}{R_K \left[ I_B (1-t^2) R_L C \right] + V_{BE \text{on}} R_L}$$  \hspace{1cm} (2.43)

$$N = \frac{R_K^{\frac{2}{3}} \left[ -I_B (1-t^2) C \right] + R_K \left[ V_{CC} - V_{BE} \right] C - V_{BE}}{R_K \left[ I_B (1-t^2) R_L C \right] + V_{BF} R_L}$$  \hspace{1cm} (2.44)

where

$$G_1 = -I_B (1-t^2) C$$  \hspace{1cm} (2.45)

$$G_2 = \left( V_{CC} - V_{BE} \right) C - V_{BE}$$  \hspace{1cm} (2.46)

$$G_3 = I_B (1-t^2) R_L C$$  \hspace{1cm} (2.47)

$$G_4 = V_{BF} R_L$$  \hspace{1cm} (2.48)

$$C = \frac{(1+t)}{(1-t)} \left( \frac{V_{BE \text{on}} + V_I}{V_{BE \text{on}} - V_{BE \text{on}}} \right)$$  \hspace{1cm} (2.49)

Taking the derivative of $N$ with respect to $R_K$

$$\frac{dN}{dR_K} = \frac{(C_3 R_K + C_4) (2 C_1 R_K + C_2) - (C_3 R_K^2 + C_2 R_K) C_3}{(C_3 R_K + C_4)^2}$$  \hspace{1cm} (2.50)

Setting equation (2.50) equal to zero and collecting terms

$$R_K^{\frac{2}{3}} C_1 C_3 + 2 C_4 C_1 R_K + C_4 C_2 = 0$$  \hspace{1cm} (2.51)
Solving equation (2.46) for $R_K$

$$R_K = \frac{C_4 C_1 \pm \sqrt{C_4^2 C_1^2 - C_1 C_3 C_4 C_2}}{C_1 C_3}$$

(2.52)

$$[-I_B (1-t^2) V_{BE} R_L] \pm \sqrt{V_{BE}^2 R_L (1-t^2) C^2 + [I_B (1-t^2) R_L C] \cdot \sqrt{[V_{BE} R_L] [V_{CC} - V_{BE}] C - V_{BE}}$$

$$\frac{[-I_B (1-t^2) C]}{[I_B (1-t^2) C R_L]}$$

(2.53)

$$R_K = -\frac{V_{BE} \pm \sqrt{V_{BE} \cdot C (V_{CC} - V_{BE})}}{I_B (1-t^2) C}$$

(2.54)

$R_B$ is obtained by substituting equation (2.50) into equation (2.24) and in turn $R_B$ and $R_K$ substituted into equation (2.23) for solution for $N$. In this case the computer program will provide solutions for $R_K$, $R_B$ and $N$ given $R_L$ and $I_B$ as variables.
SECTION III
LINEAR CIRCUITS

3.1 CIRCUIT ANALYSIS

The analysis of linear amplifier circuits used in the gyro and servo units is formulated to consider worst case parametric data as indicated previously.

The parametric changes affect the dc biasing stability which in turn affect the maximum linear output and gain. The dc equations show the relative sensitivity of the individual parameters which may be minimized by proper selection of components. The equations were derived by writing the network loop equations for each complete dc circuit, such as for $Q_7$ and $Q_8$ or $Q_9$ and $Q_{10}$ transistor pairs in the 55-40305 gyro signal amplifier.

The solution for the quiescent bias points $V_{ca}$ and $V_{c10}$ permits evaluation of the maximum ac linear (unclipped) output swing, versus parametric data. The equations were formulated for exactness and ease of handling by the Data Processing Groups of GD/A Department 158-1.

3.2 CIRCUIT EVALUATION

The computer solution of the general equation was handled by separate equations for each of the parametric variables as stored inputs.

The equation for $V_{ce}$ ($Q_9$ collector voltage) of the gyro signal amplifier is shown in Figure 3-1.

3.3 COMPUTER SOLUTION

3.3.1 An example of a solution for the variation of $V_{ce}$ of Figure 3-1 versus transistor common emitter current gains ($B_7$ or $B_8 = \frac{a}{1-a}$) is shown by the computer graph of Figure 3-2. Other data are obtained by variation of each parameter in the equation of Figure 3-1 leading to a solution of total effect of all variations on circuit output.

\[
\begin{align*}
V_{ce} &= \frac{\alpha_9 \left( V_{CC} - V_{be7} \left[ 1 + \frac{R_{17}}{R_{24}} (1-\alpha_8) \right] - V_{be8} \right) - V_{be8}}{\frac{R_{22} + (1-\alpha_7)R_{23}}{KR_{24}} + \frac{V_{be7}}{R_{24} + S_{1c07}}} \\
V_{CC} - R_{18} \left[ \frac{\alpha_9 \left( V_{CC} - V_{be7} \left[ 1 + \frac{R_{17}}{R_{24}} (1-\alpha_8) \right] - V_{be8} \right) - V_{be8}}{\frac{R_{22} + (1-\alpha_7)R_{23}}{KR_{24}} + \frac{V_{be7}}{R_{24} + S_{1c07}}} \right]
\end{align*}
\]
\[ S^* = \text{stability} = \left[ 1 + \frac{\alpha_8}{\alpha_7 R_{24}} \right] \left( R_{22} + (1 - \alpha_7) R_{23} \right) \]

\[ \left( K + \frac{\alpha_7 R_{23}}{K} \left( 1 + \frac{R_{17}}{R_{24}} (1 - \alpha_7) \right) + \frac{\alpha_7 R_{17}}{R_{24}} \right) - \alpha_7 \frac{R_{23}}{R_{24}} = 0 \]

\[ K = \alpha_7 R_{17} \left( R_{22} + (1 - \alpha_7) R_{23} \right) \left( 1 + \frac{R_{17}}{R_{24}} (1 - \alpha_7) \right) \]

*assume \( I_{GB07} = I_{GB08} \)

**Figure 3-1. Variation Solution**

\[ \text{where:} \]

\[ V_{cc} = \text{dc supply} \]

\[ V_{CQ7}, V_{Q8} = Q_7 \text{ or } Q_8 \text{ Collector Volts} \]

\[ V_{VCC7} = 0.58 - 2 \times 10^{-3} (T-25) \text{ volts} = Q_7 \text{ base to emitter volts} \]

\[ V_{VCC8} = 0.62 - 2 \times 10^{-3} (T-25) \text{ volts} = Q_8 \text{ base to emitter volts} \]
\[ \alpha_7, \alpha_8 = Q_7 \text{ or } Q_8 \text{ current gain at } T^\circ \text{C (hFB)} \]

\[ I_{CB07}, I_{CB08} = \text{common base collector leakage current} \]

\[ S = \frac{\Delta I_{C8}}{\Delta f(I_{CB07}, I_{CB08})} = V_{C8} \text{ stability versus } I_{CB07} \text{ and } I_{CB08} \]

\[ T = \text{Degrees centigrade} \]
## SECTION IV

### AVAILABLE PROGRAM FOR SWITCHING CIRCUITS

#### 4.1 CIRCUITRY AVAILABLE

<table>
<thead>
<tr>
<th>Circuit Code</th>
<th>Circuit Type</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>3606</td>
<td>RTL Nor Circuit and RS FLIP-FLOP</td>
<td>Provides maximum fan out for given fan in and load resistor $R_L$. Provides solutions for $N$, $R_{K'}$, and $R_B$.</td>
</tr>
<tr>
<td>3614 A</td>
<td>Inverter Circuit</td>
<td>Provides maximum fan out $M$ and $R_B$ and solutions for $R_B$ and $R_K$ given a load resistor $R_L$.</td>
</tr>
<tr>
<td>3614 B</td>
<td>Inverter Circuit</td>
<td>Provides minimum collector current of driver transistor given load and fan out requirements.</td>
</tr>
<tr>
<td>3596</td>
<td>RTL Nor Circuit</td>
<td>Provides solutions for $R_{B_{on}}$ and $R_{B_{off}}$ given $M$, $N$, and load.</td>
</tr>
<tr>
<td>3581 A/B</td>
<td>Inverter Circuit</td>
<td>Provides plots of &quot;on&quot; and &quot;off&quot; equation given load current and $V_{BE_{off}}$.</td>
</tr>
<tr>
<td>3595 A</td>
<td>Unijunction Time Delay Circuit</td>
<td>Provides optimum solution of circuit parameter.</td>
</tr>
<tr>
<td>3594 B</td>
<td>Linear Amplifier Circuit</td>
<td>Provides minimum permissible $B$ values over given temperature range.</td>
</tr>
<tr>
<td>3261</td>
<td>RTL Nor Circuit</td>
<td>Provides solutions for fan out given 50 different loud resistors.</td>
</tr>
</tbody>
</table>
SECTION V

REFERENCES


