

GPO PRICE \$ \_\_\_\_\_

X-614-65-171

CFSTI PRICE(S) \$ \_\_\_\_\_

Hard copy (HC) \$2.00

Microfiche (MF) .50-

NASA TMX-55254

# 653 July 65

# HIGH-RESOLUTION LUNAR-BASED TELESCOPES FOR COSMOLOGICAL STUDIES

BY  
KENNETH L. HALLAM

FACILITY FORM 602	N66-19502	
	(ACCESSION NUMBER)	(THRU)
	27	1
	(PAGES)	(CODE)
<del>TMX 50418</del>	30	
(NASA CR OR TMX OR AD NUMBER)	(CATEGORY)	
TMX-55254		

APRIL 1965



GODDARD SPACE FLIGHT CENTER  
GREENBELT, MARYLAND

~~ALL INFORMATION CONTAINED  
HEREIN IS UNCLASSIFIED~~

GODDARD X-614-65171  
NASA TMX-55254

HIGH-RESOLUTION LUNAR-BASED TELESCOPES FOR  
COSMOLOGICAL STUDIES

By

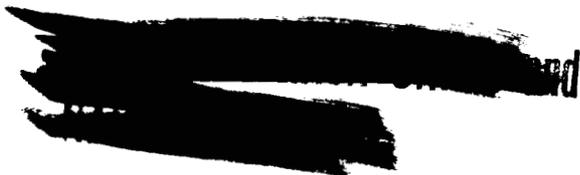
Kenneth L. Hallam  
Goddard Space Flight Center  
Greenbelt, Maryland

ABSTRACT

19502

Optical observations from lunar-based high-resolution telescopes are shown to be required to provide many of the necessary observational discriminants between various cosmological models. Unsurmountable observational limitations set by the earth's atmosphere are discussed. Basic instrumental requirements for lunar-based telescopes are developed, with emphasis on their application to the solution of the cosmological problem.

author



## I. INTRODUCTION

Present cosmological studies are supported observationally by 1) the largest reflecting telescopes in dark-sky locations, 2) the largest Schmidt telescopes in dark-sky locations, and 3) large, high spatial resolution radio telescopes. The last two types of instruments are of particular value in finding profitable objects to study in greater detail with large reflectors.

With the optical instruments, the ultimate limitations to their ability to probe space is a combination of circumstances inherent with observing from the ground on the earth. These are 1) night sky background illumination, 2) atmospheric induced image smearing, 3) limited periods and regions of observing dictated by diurnal and seasonal geometry. The first two phenomena are responsible for inducing loss of contrast, or decreased signal-to-noise ratio in the desired observations.

The optical discovery of extragalactic nebulae is typically made by inspection of large Schmidt telescope plates. The limiting faintness attainable in this way occurs for some ratio of surface brightness of the object  $B_o$  to the surface brightness of the background  $B_B$ . The background brightness is due to light from the sky and to detector noise. In the case of the photographic plate, the detector noise is due to chemical fog and plate grain. The object surface brightness is determined by the resolution limit in the focal surface of the telescope. Random variations in the central position and in the pattern of light

distribution from a point source as transferred to the focal plane of a telescope are caused by uncontrollable fluctuations of the refractive index in the earth's atmosphere (seeing). The theoretical resolution of a ground based telescope is never realized for this reason. The best resolution obtainable from the ground for exposures longer than a fraction of a second in time is about 0.5 arc second. More often it is 1 arc second, whereas the theoretical capability of a resolution limited 200 inch diameter telescope should be about .025 arc second. The light from a point source is therefore spread out over an area 1600 times greater than theoretical, and the contrast ratio  $\frac{B_o}{B_B}$  goes down accordingly.

At the same time, the atomic and molecular constituents of the atmosphere contribute a somewhat variable background emission radiation consisting of lines, bands, and continua, amounting to the equivalent of 70 10th mag. stars per square degree for the photographic continua near the Zenith (Allen, 1963), which is equal to about one 23.3 magnitude star per square second of arc. In addition, there is an irreducible contribution from the Zodiacal cloud itself and from atmospheric scattering of light from all the stars and the Zodiacal cloud. This adds an unavoidable contribution to the mean surface brightness of the sky, with the result that the minimal value of sky brightness obtainable from the ground under any circumstances is equivalent to one 22.7 mpg star per square second of arc.

Now from above the earth's atmosphere, the minimal sky background consists essentially of the minimal value for the Zodiacal light, which is one  $24.6 m_{pg}$  star per square second of arc. Hence the reduction in background brightness achieved by getting outside the earth's atmosphere is at least  $1.9 m_{pg}$ , while at the same time the achievement of theoretical resolution would give an increase in effective surface brightness for detail at the limit of resolution of 8.0 magnitudes for a 200-inch telescope. The combination of these two factors shows that an increase in signal-to-noise ratio of as much as 9.9 magnitudes over the best present ground-based situation might be achieved by the use of a 200-inch telescope completely outside the earth's atmosphere and with adequate stabilization and diffraction limited optics. Similarly, a diffraction limited 40-inch telescope outside the atmosphere would permit an improvement in signal-to-noise over a 200-inch ground telescope of as much as 6.4 magnitudes.

It is the precept of observational cosmology to address itself to the study of mass-systems at the limit of instrumental capability. Here we have a clear-cut case where proper use of extra-terrestrial observing sites will ensure great advances in a most fundamental problem in astronomy - that of cosmology.

The attack on the problems of observational cosmology have been made most successfully by the largest telescopes. It was hoped with the advent of the Palomar 200-inch telescope that the

first definitive answer to the geometry of the universe could be found. This has proven to be not quite possible, by any approach attempted to date (Minkowski, 1962). Sandage (1961) has outlined four tests for discriminating between different world models. These include the magnitude-red shift relation, the galaxy count-magnitude relation, the angular diameter-red shift relation, and the Hubble time scale. The present state of observational evidence is unsatisfactory to make any definitive choices. All methods require information concerning fainter galaxies, clusters of galaxies, or details of galaxies that is not currently available. Sandage (1961) has shown that the observational discriminants sought from studies of extra-galactic systems have observational errors attached to them which mask their meaning from the standpoint of cosmology. These errors, which in the studies show up as errors of distance, of brightness, and of identification, may all be traced back to limitations essentially imposed by the adverse effects of the earth's atmosphere including direct photography, photographic and photoelectric photometry and spectrographic observations.

Basic to the observational problem also is the ability to discover and select the proper objects for study. A large part of this burden has fallen upon the sky-survey performed by the Palomar 48-inch Schmidt. That is to say, a sky survey capability matched to the potential of the 200-inch prime instrument has been required to provide the requisite objects for study. This

capability will also be needed for an extra-terrestrial program of observational cosmology, but again it must be performed outside the earth atmosphere to provide the correct match for the increased potentiality.

## II. THE MAGNITUDE-REDSHIFT RELATION AS A COSMOLOGICAL MODEL DISCRIMINANT

An improvement of several magnitudes in surface brightness signal-to-noise ratio over that provided by the Palomar 200-inch should be adequate to provide the needed observational discriminants to select or discard certain world-models by means of the classical extra-galactic systems. Minkowski (1960) has stated that spectrographic redshifts greater than  $\frac{\Delta\lambda}{\lambda} = 0.2$  of normal galaxies with absorption features are not possible because of obliteration by sky background. Discovery of clusters of galaxies with redshifts greater than 0.35 is also beyond the capability of the 48-inch Schmidt telescope. By use of radio sources, galaxies in emission may be found which are yet further distant. Such galaxies with spectral emission features might permit redshift determinations up to 0.6, according to Minkowski (1962). Thus far, only one such distant galaxy has been found - 3C295, with a redshift of 0.46. But the total number of those with redshifts near the limit 0.6 within reach of the 200-inch Palomar telescope, has been estimated by Minkowski to be no more than 10 in the whole sky. It is evident that these radio galaxies will probably not be very useful in the long run for unravelling

the cosmological problem, since the statistics obtained through so few objects of this rare class would be poor.

If we restrict our consideration to cosmological models with a range of values for the deceleration parameter  $g_0$  between +1 and -1, we find by using Table 1 of Sandage (1961), the difference in bolometric magnitude separating models of differing  $g_0$  increases substantially with increasing redshift,  $z$ , as shown in Table 1 below.

TABLE 1  
Cosmological Model Discrimination From Redshifts

$z = \frac{\Delta\lambda}{\lambda}$	$-(\Delta m_{\text{bol}}/\Delta g_0) z$
0.35	0.32
0.60	0.50
1.00	0.75

By extending the magnitude-redshift measurements of normal galaxies from 0.35 to 1.0, a gain in the magnitude discrimination between cosmological models by a factor greater than two may be obtained. Depending on the model, this would represent an increase of up to 3.2 bolometric magnitudes in faintness. Any improvement greater than 4 magnitudes would be difficult (but not impossible) to use on distant galaxies with presently available detectors because the peak of the energy distribution would then be red-shifted beyond  $1\mu$ .

Nevertheless, additional improvement would be useful, due to the comprehensive nature of the cosmological problem. For instance, one would be able to derive directly the quantity  $dL/dt$ , the evolutionary change of galactic luminosities, which must now be inferred from theory alone, if spectrographic redshifts of fainter objects were combined with extended multicolor photometry similar to that of Baum (1962).

This would represent a truly significant advance for it has direct bearing itself on possible cosmological models and postulates. A more technical, but nevertheless important obstacle to present progress is that of the aperture effect, which deals with the uncertainty attached to measures of the total light from a given galaxy, due to the gradual fade-off of the galactic light into the background near its outer limit (Whitford, 1962; Humason, Mayall, and Sandage, 1956). The increased signal-to-noise attainable above the earth's atmosphere should allow this problem to be solved.

The recent identification of certain radio sources as suspected extra-galactic systems (quasars) with different characteristics of mass, luminosity distribution, and energy content than classical systems, might provide greatly increased potentiality to observational studies of cosmology, once their extra-galactic identification were certain, and if homogeneous sets could be established. Because their small apparent sizes leave most of the quasars featureless to ground based telescopes,

we again see the vast importance to cosmology of the higher resolution work obtainable from suitably designed and situated space telescopes.

### III. NEBULAR SPECTROGRAPH DESIGN REQUIREMENTS

Great efforts have been made toward construction of fast nebular spectrographs for the 200-inch and other telescopes. Their designs are based on the characteristics of the photographic emulsion (reciprocity failure, contrast, efficiency, grain size) and on the desirability to shorten and optimize the use of telescope observing time, consistent with the requirement of extracting only sufficient spectral information for the purpose of red-shift determinations of the fainter objects of ever decreasing effective surface brightness. If the object's angular dimension  $\alpha_0$  is less than the "seeing" limit  $\alpha_s$  set by the atmosphere, the effective surface brightness  $B_e$  in the focal plane of a telescope is decreased with respect to the brightness  $B_0$  of a perfect image.

$$B_e = \left(\frac{\alpha_0}{\alpha_s}\right)^2 B_0, \quad \alpha_0 < \alpha_s . \quad (1)$$

The function of a telescope may be treated in terms of differential operations. Thus, an object of incremental angle  $d\alpha$  is transformed at the focal surface to an incremental distance  $ds$ . The ratio  $P = d\alpha/ds$  is known as the plate scale. Similarly, the dispersion element of a spectrograph (prism or grating) performs the transform  $\frac{d\theta}{d\lambda}$  on its exiting rays. The spectrograph camera next transfers

the angular dispersion to a linear dimension, by the operator  $\frac{d\sigma}{d\theta}$ . The product  $L = \frac{d\theta}{d\lambda} \frac{d\sigma}{d\theta} = \frac{d\sigma}{d\lambda}$  is the linear dispersion of the spectrograph. Once the minimal working spectral resolution is chosen (which may be, for instance, in the case of red-shift spectra the ability to separate H and K), optimization of the system with respect to the ratio  $\frac{dI}{dt}$ , where I is the retrievable information content transferred to the emulsion, is customarily performed by adjusted matching of the emulsion resolving power to the linear dispersion L.

The customary determinants are these:

1. Entrance slit width adjusted to satisfy
  - a. Minimal admittance of sky background, i-e. maximize as much as is consistent with other requirements the ratio  $B_e/B_B$ , where  $B_e$  is computed over the width of the slit. This requirement pertains to objects with sky background limitation.
  - b. For brighter objects, maximize admittance of object flux through entrance slit. Open slit at least as wide as seeing disk  $\alpha_s \frac{ds}{d\alpha}$ , providing this is consistent with spectral resolution limit.
2. Collimator focal length chosen sufficient to ensure angular subtense of entrance slit small enough so as not to degrade required spectral resolution. Focal length must also be great enough to provide sufficient beam diameter at dispersing element to allow required spectral resolving power.

3. Dispersing element aperture sufficient to use all of light in the beam and to provide required spectral resolving power. Angular dispersion of element must be consistent with total spectral range to be covered and field of view of camera.
4. Camera aperture sufficient to collect all light dispersed from dispersing element. Useful field of view of camera needs to be adequate also.
5. Focal length of camera chosen by selecting minimal linear dispersion sufficient for information retrieval required.
6. Resolving power of film depends on length of spectral lines, which depends in turn upon whether or not the spectrum is widened by drifting the object along the entrance slit. It should be noted that since the minimal length is that of the seeing disk diameter, and if no drifting along the slit to "widen" the spectrum is to be allowed, that the stated resolving power of the emulsion must be accordingly degraded in practice, if the projected length of the seeing disk is less than that used for determining the resolving power of the emulsion. By widening the spectrum, a proportionally greater number of photographic grains are exposed per wavelength interval, with a resulting statistical increase in signal-to-noise

for the spectrum proportional to the square root of the slit length used to widen the spectrum. If drifting is employed to widen the spectrum, greater vertical masking must be employed if the ultimate limit in faintness is to be reached.

If some image detector other than the direct photographic emulsion were employed, the boundary conditions for the spectrograph design would be set by some equivalent to the resolution of the emulsion.

The requirements for the design of a nebular spectrograph may now be formulated in the following manner:

The practical photographic resolving power is

$$R_P = R_E \frac{L}{L_E}, \quad (2)$$

where it is assumed grains are randomly distributed, and where  $L_E$  is the length of the test lines used for the stated resolving power  $R_E$ , Denoting the required spectral resolution for red-shift studies as  $R_\lambda$ , the first optimization condition requires

$$\Delta\sigma = \frac{1}{R}, \quad \Delta\lambda = R_\lambda \quad (3)$$

$$L = \frac{\Delta\sigma}{\Delta\lambda} = \frac{1}{R_P R_\lambda} \quad (4)$$

Now the angular subtense of the resolution element from the camera lens is

$$d\theta = \frac{1/R_P}{F_2} = \frac{1}{F_2 R_2}, \quad (5)$$

where  $F_2$  is the focal length of the camera.

The required angular dispersion of the dispersing element is

$$\frac{d\theta}{d\lambda} = \frac{1}{F_2 R_P R_\lambda}, \quad (6)$$

and the required resolving power of the dispersing element itself is

$$\frac{\lambda}{\Delta\lambda} = \frac{\lambda}{R_\lambda} \quad (7)$$

In order to attain this resolving power, the required projected grating height,  $H'$ , must be

$$H' = \frac{\lambda}{R_\lambda} \frac{d'}{m}, \quad (8)$$

where  $d'$  is the projected groove spacing and  $m$  is the spectral order number.

Assuming the practicable grating height does not limit the collimator focal length,  $F_1$ , the required  $F_1$  to match  $F_2$  is set by the entrance slit width,  $S = \alpha_s F$ , where  $F$  is focal length of telescope and  $\alpha_s$  is the seeing disk diameter:

$$F_1 = G F_2 S R_P. \quad (9)$$

G is a factor determined by the grating geometry, and is given by

$$G = \frac{d\theta}{d\phi} \approx 1 \quad (10)$$

where  $d\phi$  is the incident angular increment on the dispersing element and  $d\theta$  is the dispersed angular increment for monochromatic light.

The diameter of the collimator aperture is simply

$$D_1 = F_1 f, \quad (11)$$

where  $f$  is the focal-ratio of the telescope.

The diameter of the camera aperture,  $D_2$ , must be sufficiently larger than  $D_1$  to incorporate the dispersion of the beam.

The total wavelength range is determined by the detector range, which is typically  $2000\text{\AA}$  or less. The angular dispersion of the dispersing element must be such that the camera can accept all detectable wavelengths. If we choose an acceptance angle of  $10^\circ$  for a typical Schmidt camera, then we must have

$$\frac{\Delta\theta}{\Delta\lambda} \leq \frac{10^\circ}{2000\text{\AA}} \quad (12)$$

Another constraint on  $F_2$  is provided by realizing that  $F_2$  be greater than that value which is just possible for a particular  $D_2$ , as determined by the fastest camera focal ratio  $f_2^0$  that can be built:

$$f_2^0 \leq \frac{F_2}{D_2} \quad (13)$$

From these criteria, we are able to determine the focal length of the collimator  $F_1$  and camera  $F_2$ , the diameter of the grating  $D_1$ , and the width of the slit  $S$  for a fast nebular spectrograph.

$$\begin{aligned}
 S &= F\alpha \\
 F_2 &= \frac{1}{R_\lambda R_P \frac{d\theta}{d\lambda}} \\
 GR_P SF_2 \leq F_1 &= \frac{f F_2}{f_2} \\
 D_2 \approx D_1 &= \frac{F_1}{f}
 \end{aligned}
 \tag{14}$$

The formulae derived above still apply to requirements for a nebular spectrograph of a space telescope with diffraction limited optics. The only change in useage is the angular size of the object, which is no longer seeing limited:  $S = F\alpha_0$ , where  $\alpha_0$  is the angular diameter of a desired feature of the object, or is the diffraction limit. Since  $\alpha_0 < \alpha_s$ , then the inequality of equations (14) will still be satisfied since  $S_0 < S_s$ . The spectra would look much the same as those from the ground, since the detail of spectral features of the plate are limited by the photographic emulsion and never by the entrance slit size. The ability to close down the slit for the same amount of light will result in less background darkening on the emulsion and accordingly better contrast and information transfer, which will

show up most markedly for objects at the present limit of ground observations and for those objects still fainter which are now impossible to reach at all. The requisite exposure times for entire galaxies which are smaller than the seeing disk would not be changed by the new instrument. To record stellar-like features, the exposure time will have to be increased, but the resulting spectra will be of higher quality, because of higher surface brightness of the feature, and the simultaneous exclusion of surrounding background illumination by means of a narrower slit.

As the slit width is narrowed, the amount of background continuum falling on the spectrogram is proportionately reduced. At the same time, in the vertical direction, a stellar-like feature will not be smeared over so much of the background, hence there is another proportionate gain here. But with the photographic spectra taken with cameras where the emulsion structure sets the resolution limit, a similar limit is set for the vertical direction, which in turn re-defines in this case the meaning of stellar-like. That is, for features less than

$$S_V = \frac{F_1}{F_2 R_P} \quad (15)$$

in vertical height at the slit, no such gain in signal-to-background may be obtained. Hence,  $F_1$  should be kept as small as possible, relative to  $F_2$ , which in turn implies  $F_2$  should be made as large as the inequality of (14) will allow. Then we find longer

exposures will be necessary since the transferred surface brightness to the emulsion will be accordingly decreased. Whether or not this is permissible depends among other factors, on the reciprocity characteristics of the film. However, it will be noted that this automatically lengthens the projection of the feature in the direction of slit height on the emulsion plane, hence helps to increase resolution by contributing more photographic grains for the information transfer in spectral-space.

In any case, there is a gain in S/N at least in the width dimension, and possibly also available in the height dimension. Hence, for spectra, the improvement in S/N is

$$\left(\frac{B_B^S}{B_B^O}\right) \left(\frac{\alpha_S}{\alpha_O}\right) \leq \frac{(S/N)_O}{(S/N)_S} \leq \left(\frac{\alpha_S}{\alpha_O}\right)^2 \left(\frac{B_B^S}{B_B^O}\right) \quad (16)$$

where  $\alpha_S$  is the seeing-limited dimension from the ground,  $\alpha_O$  is the diffraction limited dimension of the actual feature from space,  $B_B^S$  is the background surface brightness from the ground, and  $B_B^O$  is that from space.

In the case of direct photometry or photography, we can use a similar argument to find that the gain in S/N is

$$\frac{(S/N)_O}{(S/N)} = \left(\frac{\alpha_S}{\alpha_O}\right)^2 \left(\frac{B_B^S}{B_B^O}\right) \quad (17)$$

As was shown previously, the gain from the first factor on the right hand side of equation (17) would typically be 8.0 magnitudes

for a 200-inch telescope, and for the second factor, at least 1.9 photographic magnitudes. If space were simply Euclidian, we see that the possible depth of penetration would be increased by up to a factor of one-hundred under these circumstances for any given class of objects!

For stellar-like features, a 200-inch telescope above the atmosphere working at the diffraction limit would require nearly identical exposure times as that presently required for seeing-limited stellar images of the same source recorded by the ground based 200-inch Palomar telescope, disregarding atmospheric absorption. The reason for this is that in each case the plate scale would presumably be nearly ideally matched to the resolution capabilities of the photographic emulsion, consequently with nearly equal photon densities in the image pattern. The same conclusion may be drawn regarding slit spectra, providing the spectrograph designs are optimized to match the emulsion and to pass all the light of the desired object or feature through the entrance slit. The advantages that are then gained by a diffraction limited space telescope are increased spatial resolution, and increased signal-to-noise ratio in the image pattern, with a corresponding increase in the limiting magnitude.

Similar arguments hold regardless of whether the image detection system is photographic or includes electronic aids, providing the characteristic parameters of merit are wholly analogous. The principal gains at present from such devices are increased speed, resulting from higher intrinsic quantum

efficiency, and in some cases the capacity for linear response. Continuing work will no doubt result in enhanced signal-to-noise values that would be of significance to our requirements.

#### IV. LUNAR-BASED TELESCOPES FOR COSMOLOGICAL STUDIES

This present study demonstrates that the cosmological problem will not be solved by merely building larger telescopes, but that it arises from atmospheric effects which cannot be overcome by any techniques from the ground. High resolution, faint-working space telescopes will be needed to further the study of cosmology. Wide-angle searches for more distant clusters of galaxies than those discoverable from the ground are urgently needed. Redshift spectra and accurate magnitudes of these galaxies will be needed for comparison with theoretical values of various cosmological models, based upon the fundamental postulates and theories of the universe. Acquisition of these data will require both a high-resolution wide-angle telescope, such as a Schmidt, for survey purposes, as well as a high resolution telescope suitable for nebular spectroscopy and photometry. The successful use of this scheme by the Palomar 48-inch Schmidt and 200-inch main telescope indicates the need for its continuance for fainter work from space.

Photographic exposures of many hours duration are required for spectra of the faintest objects obtainable with the 200-inch telescope, some lasting an entire night. When the fainter objects which will be made available by space-situated telescopes are

sought, even longer exposures will be required. Their duration will also depend upon the aperture of the telescope. Such long observing periods would be very annoying from a low orbiting telescope, but would be nicely facilitated by the very long uninterrupted exposures possible from a lunar-based telescope. These considerations alone strongly indicate the unsuitability of earth orbit for space telescopes designed for high resolution faint-object research. Another very serious drawback arises from the large and constantly varying thermal inputs during orbit from the sun and the earth, which would require enormous efforts to overcome for large instruments designed to operate at their diffraction limit. Added to these great problems are those of guidance, which must also work to the diffraction limit capability of the telescope. In low earth orbit, the disturbing torques due to atmospheric drag, magnetic field, and gravitational gradient are large, and would require an inordinate amount of engineering effort to compensate adequately to the very high constant precision required. This may be judged from that which has been required for suitable OAO stabilization development, where the orbital torques are considerably less than those which would be generated in the lower earth orbits suitable for human survival. Higher orbits reduce the night-time conditions necessary for observing faint objects, and increase the problems from trapped radiation enormously.

The only suitable alternative to earth orbit for placement of high resolution faint-object astronomical telescopes within the

solar system is the moon. Here we are indeed fortunate to have an admirably suitable situation for such telescopes. Many advantages are immediately apparent. The large lunar mass and low rotation rate give assurance that adequate stabilization could be achieved. The long lunar night of 13.6 days would allow the long uninterrupted exposures to be made that are required for very faint objects. Thermal variations during this period would occur slowly and predictably. Such telescopes would be essentially automatic. Man would be required to initiate and monitor their operation during observing periods. Man would also be necessary for proper site selection, installation, and maintenance. Such a site would be an adjunct to a pre-established lunar base.

While the most fundamental problem of astronomy is cosmology, and while that alone could provide an inexhaustible supply of objects to be studied, telescopes suitable for pursuing cosmological studies of the type that have been examined here are necessary to help put the whole of the astronomers' house in order. Parallax determinations; stellar mass determinations from multiple star systems; discovery of faint companions or planets; investigations of luminosity functions of the solar neighborhood, of compact or faint clusters, and of extragalactic systems; delineation of nebular structure - these represent a partial list of problems requiring a fresh attack which can come only from the availability of large high resolution lunar-based telescopes capable of working several magnitudes fainter than that obtainable from the earth.

## REFERENCES

Allen, C. W. 1963, Astrophysical Quantities (London: Athlone Press), p. 135

Baum, W. A., 1962, Problems of Extra-galactic Research, ed. by G. C. McVittie (N. Y.: MacMillan), p. 390.

Humason, M. L., Mayall, N.U., and Sandage, A. R., 1956, A.J. 61, 97

Minkowski, R., 1960, Ap. J. 132, 908

Minkowski, R., 1962, Problems of Extra-galactic Research, ed. by G. C. McVittie (N.Y.: MacMillan), p. 379

Sandage, A., 1961, Ap. J. 133, 355

Whitford, A. E., 1962, Problems of Extra-galactic Research, ed. by G. C. McVittie (N.Y.: MacMillan), p. 27