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SUMMARY

The basic linearized equations for orthotropic panel flutter are solved by the Galerkin method for simply supported leading and trailing edges and various side-edge support conditions. In addition, allowance is made for the finite transverse shear stiffness of simply supported panels. These qualitative results show that both transverse shear stiffness and side-edge boundary conditions can be extremely important in determining the flutter boundaries for orthotropic panels.

INTRODUCTION

Panel flutter has received much attention in the past few years (refs. 1 to 8), the analysis of uniform isotropic rectangular simply supported panels receiving the most attention (refs. 1 to 4). However, aircraft and spacecraft are now being constructed of various built-up panels (corrugation-stiffened skin panels or sandwich core panels, for example), and hence an understanding of the flutter behavior of such orthotropic panels is necessary. Several recent papers (refs. 7 and 8) have initiated investigation into orthotropic panel flutter. A theoretical investigation was undertaken of some effects not considered in these references, but thought to be of importance in the prediction of the flutter characteristics of orthotropic panels.

As the influence of core transverse shear stiffness has been shown to be of importance for uniform isotropic sandwich-core panels (ref. 6), the general case of a panel with two arbitrary transverse shear stiffnesses $\left(D_{Q_x}\text{ and }D_{Q_y}\right)$ has been investigated. The results of the investigation reported herein indicate that these stiffnesses can have a large influence on the flutter phenomena for orthotropic panels.

Experimental results on orthotropic panels (ref. 9) have shown that the panel side-edge boundary conditions (along the edges parallel to the airflow) have a significant effect on their flutter characteristics. In order to investigate the importance of these boundary conditions, a Galerkin solution was used to obtain the flutter equation for flat rectangular orthotropic panels with general boundary conditions. Specific solutions are also presented for simply supported, clamped, and free boundary conditions along the side edges, and it is shown that free boundaries can significantly increase the flutter susceptibility of an orthotropic panel, especially where the bending stiffness in the crossflow direction is much higher than the bending stiffness in the flow direction (as is often the case).

SYMBOLS

a_{pn}	series coefficients appearing in expansion of w
a,b	x- and y-dimensions of orthotropic panel (see fig. 1)
$\bar{A}_{qp}^{(r)},\bar{B}_{qp}^{(r)}$	flutter parameters for general orthotropic panels, defined by equation (5)
$\bar{A}(r),\bar{B}(r)$	flutter parameters for simply supported orthotropic panels including transverse shear stiffness, defined by equation (25)
$A_1(p,n),A_2(p,n),A_3(p,n)$	coefficients in free-free mode shape expression (see eq. (17))
$B_1(p,n),B_2(p,n)$	coefficients in free-free mode shape expression (see eq. (17))
$\bar{C}_x$	transverse shearing stiffness parameter in the x-direction, $\frac{\pi^2}{2a^2}\frac{D_x}{D_{Q_x}}$
$\bar{C}_y$	transverse shearing stiffness parameter in the y-direction, $\frac{\pi^2}{2b^2}\frac{D_y}{D_{Q_y}}$
D_x,D_y	bending stiffnesses of orthotropic panel
D_{Q_x},D_{Q_y}	transverse shearing stiffnesses of orthotropic panel
D_{xy}	twisting stiffness of orthotropic panel
$F_{pn}(y)$	assumed pth mode shape in the y-direction (with p half-waves)
$G_{np}(x)$	assumed nth mode shape in the x-direction (with n half-waves)

$I_{qp}^{(r)}(m,n)$	cross-stream mode shape integral expression, $b^{r-1} \int_0^b F_{qm}(y) \frac{d^r}{dy^r} F_{pn}(y)\, dy$
i,j,m,n,p,q,r,s	mode shape numbers (number of half-waves)
$J_{mn}^{(r)}(q,p)$	streamwise mode shape integral expression, $a^{r-1} \int_0^a G_{mq}(x) \frac{d^r}{dx^r} G_{np}(x)\, dx$
$\bar{k}$	frequency factor, $\sqrt{\frac{m\omega^2 a^4}{\pi^4 D_x}}$
$K_1(j),K_2(j)$	factors defined by equation (25)
$\bar{L},\bar{K}$	constants defined by equation (19b)
m	mass of orthotropic panel per unit area
M_x,M_y	applied bending moments per unit length
M	Mach number of airflow
N_x,N_y	applied in-plane forces per unit length for orthotropic panel (positive in compression)
N_{xy}	in-plane shearing force per unit length
q	dynamic pressure
$\bar{R}_x,\bar{R}_y$	in-plane loading factors, $\frac{N_x a^2}{\pi^2 D_x}$ and $\frac{N_y a^2}{\pi^2 D_x}$, respectively
V_y	vertical shearing force per unit length
w	lateral deflection of orthotropic panel
x,y	coordinate axes (see fig. 1)
α_n	set of transverse shearing stiffness constants (see eq. (20))
$\bar{\alpha}_n$	nondimensional set of α_n constants (see eq. (23))

$\beta = \sqrt{M^2 - 1}$

γ boundary condition parameter, $\mu_x + \frac{D_{xy}}{D_y}$

δ_{qp} Kronecker delta

$\bar{\theta} = \mu_x \pi^2 \frac{b^2}{a^2}$

η_m constants defined in equation (19)

λ dynamic-pressure parameter, $\frac{2qa^3}{\beta D_x}$

$\lambda_{cr}(\infty)$ critical dynamic-pressure parameter for panels with infinite shear stiffness

μ_x,μ_y Poisson's ratios

$\bar{\varphi} = 1 + \frac{D_{xy}}{\mu_x D_y}$

ω circular frequency

THE INFLUENCE OF SIDE-EDGE BOUNDARY CONDITIONS

General Galerkin Solutions for Panels Simply Supported at Their Leading and Trailing Edges

The orthotropic panels under consideration have a width b (in the y-direction) and a length a (in the x-direction), and are subjected to a supersonic airflow of Mach number M flowing over the upper surface in the x-direction. (See fig. 1.) These panels were assumed to be simply supported at both the leading and trailing edges and subjected to in-plane loadings N_x and N_y considered positive in compression. If simple static strip-theory (Ackeret) aerodynamics are used and simple harmonic motion is assumed, as discussed in reference 1, the differential equation for the lateral deflection w of the orthotropic panel becomes

$$\frac{\partial^4 w}{\partial x^4} + 2\left(\frac{D_{xy}}{D_x} + \mu_y\right)\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{D_y}{D_x}\frac{\partial^4 w}{\partial y^4} + \frac{\pi^2 \bar{R}_x}{a^2}\frac{\partial^2 w}{\partial x^2} + \frac{\pi^2 \bar{R}_y}{a^2}\frac{\partial^2 w}{\partial y^2} + \frac{\lambda}{a^3}\frac{\partial w}{\partial x} - \frac{\bar{k}^2 \pi^4}{a^4} w = 0 \tag{1}$$

where

$$\bar{R}_x = \frac{N_x a^2}{\pi^2 D_x}$$

$$\bar{R}_y = \frac{N_y a^2}{\pi^2 D_x}$$

$$\lambda = \frac{2qa^3}{\beta D_x}$$

$$\bar{k}^2 = \frac{m\omega^2 a^4}{\pi^4 D_x}$$

$$\mu_x \mu_y \ll 1$$

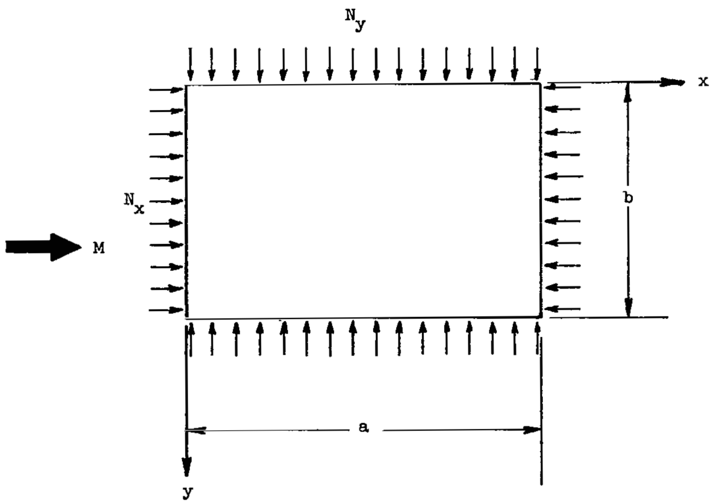


Figure 1.- Loadings and dimensions for a typical orthotropic panel.

If the product $\mu_x\mu_y$ is not very small with respect to 1, this equation is still valid if D_x and D_y are redefined by dividing them both by $1 - \mu_x\mu_y$. Generally, the μ_y-factor in the second term of equation (1) is neglected.

In the appendix the solution of equation (1) is determined by the Galerkin method for general boundary conditions. To illustrate the influence of side-edge boundary conditions, however, several explicit examples are computed in this section. The Galerkin method is used here for generality, even though in a few cases an exact solution of equation (1) is possible.

For panels simply supported at their leading and trailing edges (the boundaries parallel to the y-axis) the deflection must satisfy

$$\left.\begin{aligned} w\Big]_{x=0,a} &= 0 \\ M_x\Big]_{x=0,a} &= -D_x\left(\frac{\partial^2 w}{\partial x^2} + \mu_y \frac{\partial^2 w}{\partial y^2}\right)\Bigg]_{x=0,a} = 0 \end{aligned}\right\} \tag{2}$$

To satisfy these boundary conditions, the deflection was selected as

$$w = \sum_n a_{pn} \sin \frac{n\pi x}{a} F_{pn}(y) \tag{3}$$

Only a one-term expansion in the y-direction has been assumed in this case and, thus, a summation with respect to p does not appear. The function $F_{pn}(y)$ has the subscript n because of possible coupling of the x and y functions through the particular side-edge boundary conditions used in some cases. The Galerkin procedure is applied by substituting equation (3) into equation (1), multiplying the resulting equation by $\sin\frac{m\pi x}{a} F_{pm}(y)$, and integrating over the area of the plate. The following equations then result:

$$\left(m^4 - \bar{A}_{pp}^{(m)}m^2 - \bar{B}_{pp}^{(m)}\right)a_{pm} + \frac{2\lambda}{\pi^4} \sum_{\substack{n \\ (n\neq m)}} a_{pn}\left(\frac{mn}{m^2 - n^2}\right)\left[(-1)^{m+n} - 1\right]\frac{I_{pp}^{(0)}(m,n)}{I_{pp}^{(0)}(m,m)} = 0 \tag{4}$$

where

$$\left.\begin{aligned}
I_{qp}^{(r)}(m,n) &= b^{r-1}\int_0^b F_{qm}(y)\,\frac{d^r}{dy^r}F_{pn}(y)\,dy \\
\bar{A}_{qp}^{(r)} &= \bar{R}_x + \frac{2}{\pi^2}\left(\frac{D_{xy}}{D_x} + \mu_y\right)\frac{a^2}{b^2}\,\frac{I_{qp}^{(2)}(r,r)}{I_{qp}^{(0)}(r,r)} \\
\bar{B}_{qp}^{(r)} &= \bar{k}^2 - \bar{R}_y\,\frac{1}{\pi^2}\,\frac{a^2}{b^2}\,\frac{I_{qp}^{(2)}(r,r)}{I_{qp}^{(0)}(r,r)} - \frac{1}{\pi^4}\,\frac{D_y}{D_x}\,\frac{a^4}{b^4}\,\frac{I_{qp}^{(4)}(r,r)}{I_{qp}^{(0)}(r,r)}
\end{aligned}\right\} \tag{5}$$

For any specific boundary conditions in the crossflow direction, then, the functions $F_{pn}(y)$ must be chosen to satisfy all the y boundary conditions and the integrals $I_{qp}^{(r)}(m,n)$ must be evaluated. Substituting these expressions into equations (4) yields a set of homogeneous linear algebraic equations for the coefficients a_{pn} and the condition for a nontrivial solution is that the determinant of the coefficients be zero. This determinant yields an algebraic equation that must be solved for the dynamic pressure parameter λ.

These general equations (eqs. (4)) are examined for the special case of only two terms in the series expansion for w. Although this two-term solution is not exact, it does yield the qualitative results necessary for the evaluation of the effect of crossflow boundary conditions. If only a two-term expansion (in which $m + n$ is odd) is used, the condition for a nontrivial solution yields for the flutter parameter λ

$$\lambda = \frac{\pi^4}{4mn}\left|n^2 - m^2\right| \sqrt{\left(m^4 - m^2\bar{A}_{pp}^{(m)} - \bar{B}_{pp}^{(m)}\right)\left(-n^4 + n^2\bar{A}_{pp}^{(n)} + \bar{B}_{pp}^{(n)}\right)} \sqrt{\frac{I_{pp}^{(0)}(m,m)\ I_{pp}^{(0)}(n,n)}{I_{pp}^{(0)}(m,n)\ I_{pp}^{(0)}(n,m)}} \tag{6}$$

The critical value of λ is determined by solving for the lowest value of λ at which the frequencies for the two modes coalesce. Thus, $\frac{\partial\lambda}{\partial\omega} = 0$ $\left(\text{since the frequency appears only in } \bar{k}^2\text{, this relation is equivalent to } \frac{\partial\lambda^2}{\partial\bar{k}^2} = 0\right)$ yields the critical value

$$\lambda_{cr} = \frac{\pi^4}{8mn}\left|m^2 - n^2\right|\left(m^4 - n^4 - m^2\bar{A}_{pp}^{(m)} + n^2\bar{A}_{pp}^{(n)} + \bar{B}_{pp}^{(n)} - \bar{B}_{pp}^{(m)}\right) \sqrt{\frac{I_{pp}^{(0)}(m,m)\ I_{pp}^{(0)}(n,n)}{I_{pp}^{(0)}(m,n)\ I_{pp}^{(0)}(n,m)}} \tag{7}$$

If the boundary conditions in the y-direction are such that the function $F_{pn}(y)$ is independent of the mode number n in the x-direction, the critical flutter parameter becomes the simple expression

$$\lambda_{cr} = \frac{\pi^4}{8mn}\left(m^2 - n^2\right)^2\left(m^2 + n^2 - \bar{A}_{pp}\right) \tag{8}$$

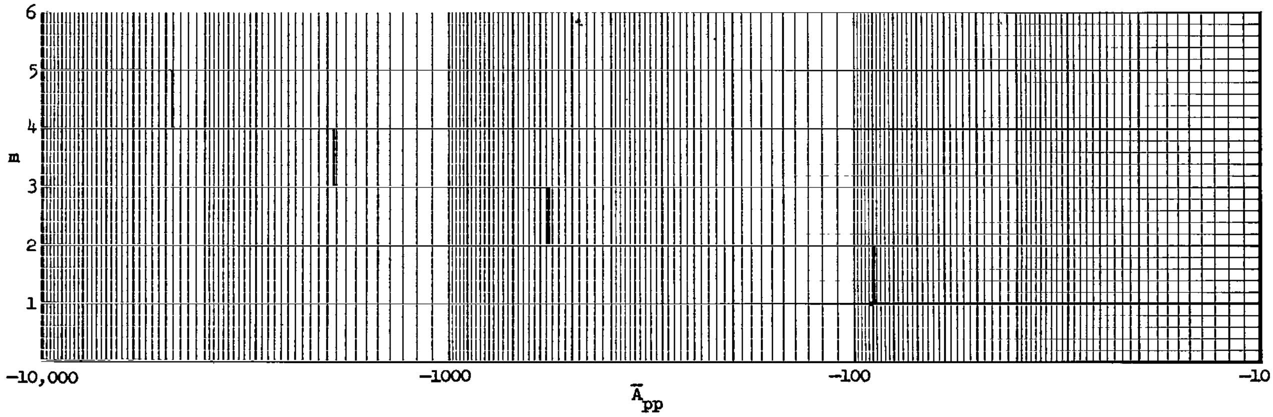


Figure 2.- The value of m for minimum λ_{cr} as a function of $\bar{A}_{pp}$ for panels simply supported at the leading and trailing edges (Galerkin solution using m and $m + 1$ half-waves).

If, in addition, two consecutive values of m and n are chosen $(n = m + 1)$, the minimum λ_{cr} is found at one of the two integer values m nearest the value $\frac{1}{2}\left(-1 + \sqrt{1 + \sqrt{2 - 2\bar{A}_{pp}}}\right)$. A plot of m against $\bar{A}_{pp}$ is shown in figure 2, and values of λ_{cr} for several values of m and $\bar{A}_{pp}$ are given in table I. Table I indicates that $m = 1$ does not give the minimum for this process for large negative values of $\bar{A}_{pp}$. However, a difference of only 10 percent is incurred in the range investigated by using $m = 1$.

TABLE I.- VALUES OF λ_{cr} FOR A PANEL SIMPLY SUPPORTED AT THE LEADING AND TRAILING EDGES AND EITHER SIMPLY SUPPORTED OR CLAMPED ON THE SIDE EDGES (TWO-MODE GALERKIN SOLUTION)

m,n	Values of λ_{cr} for values of $\bar{A}_{pp}$ of -			
	-10	-100	-1000	-10 000
1,2	0.822×10^3	5.753×10^3	5.507×10^4	5.482×10^5
2,3	1.167	5.733	5.139	5.080
3,4	1.740	6.215	5.096	4.984
4,5	2.515	6.953	5.134	4.952
5,6	3.487	7.907	5.211	4.941
6,7	-----------	-----------	-----------	4.941

Since end fixity is to be investigated, only the modes $m = 1$ and $n = 2$ are used (even though it is possible that other modes might be critical in certain situations). The cases of simply supported, clamped, and free boundary conditions along the edges parallel to the flow are discussed.

Panels with the side edges simply supported.- The boundary conditions in the y-direction are satisfied by $F_{pn}(y) = \sin\frac{p\pi y}{b}$. Thus, the cross-stream integral expressions are independent of n and can be written as

$$I_{qp}^{(r)} = (-1)^{r/2}p^r\pi^r\left(\frac{1}{2}\right)$$

since $p = q$ and r is even. The parameter $\bar{A}_{pp}$ (for the single pth term in the y-direction) becomes

$$\bar{A}_{pp} = \bar{R}_x - 2\left(\frac{D_{xy}}{D_x} + \mu_y\right)\frac{a^2}{b^2}p^2 \tag{9}$$

The value of λ_{cr} is obtained by substitution of $\bar{A}_{pp}$ into equation (8) and selection of $m = 1$ and $n = 2$. The results are the same as those of reference 1, where the lowest term in the y-direction $(p = 1)$ was used for a uniform plate. These results are shown in figure 3, where the critical flutter parameter λ_{cr} is plotted against $\bar{A}_{11}$.

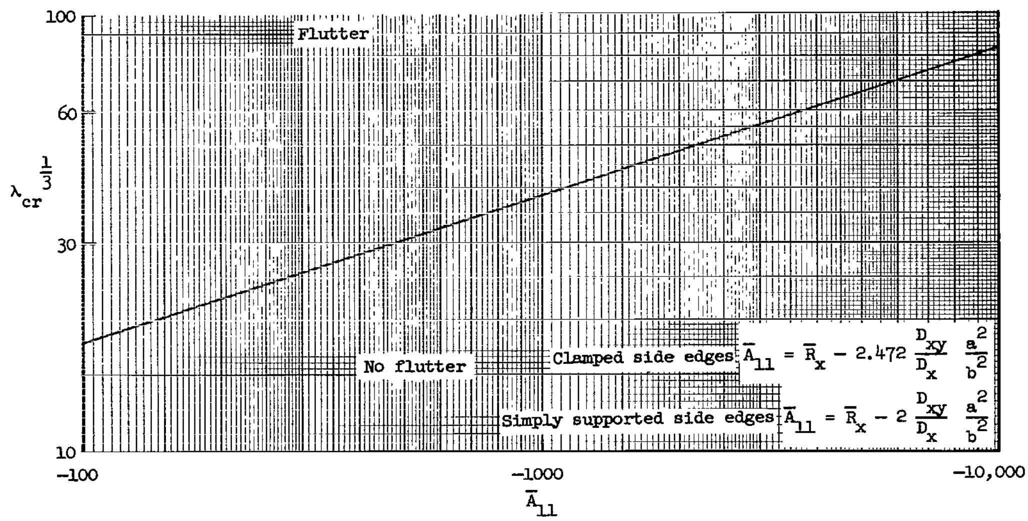


Figure 3.- Flutter results from a two-mode Galerkin solution (lowest two terms) for orthotropic panels simply supported at their leading and trailing edges.

Panels with side edges clamped.- In the case of clamped panels, several functions for $F_{pn}(y)$ could be used to satisfy the clamped boundary conditions

$$\left.\begin{aligned} w\Big]_{y=0,b} &= 0 \\ \frac{\partial w}{\partial y}\Big]_{y=0,b} &= 0 \end{aligned}\right\} \tag{10}$$

The Iguchi function (ref. 10) is assumed here and is defined as

$$F_{pn}(y) = \frac{y}{b}\left(\frac{y}{b} - 1\right)^2 + (-1)^p\left(\frac{y}{b}\right)^2\left(\frac{y}{b} - 1\right) - \frac{\sin \frac{p\pi y}{b}}{p\pi} \tag{11}$$

Therefore the integrals become (see ref. 11) when $q = p$

$$I_{qp}^{(0)} = \frac{1}{30} - \frac{8}{\pi^4}\left(\frac{1}{q^4}\right) + \frac{1}{2q^2\pi^2} \qquad (q \quad \text{odd})$$

$$I_{qp}^{(0)} = \frac{1}{210} - \frac{24}{\pi^4}\left(\frac{1}{q^4}\right) + \frac{1}{2q^2\pi^2} \qquad (q \quad \text{even})$$

$$I_{qp}^{(2)} = -\frac{1}{3} + \frac{8}{\pi^2}\left(\frac{1}{q^2}\right) - \frac{1}{2} \qquad (q \quad \text{odd})$$

$$I_{qp}^{(2)} = -\frac{1}{5} + \frac{24}{\pi^2}\left(\frac{1}{q^2}\right) - \frac{1}{2} \qquad (q \quad \text{even})$$

$$I_{qp}^{(4)} = -4 + \frac{q^2\pi^2}{2} \qquad (q \quad \text{odd})$$

$$I_{qp}^{(4)} = -12 + \frac{q^2\pi^2}{2} \qquad (q \quad \text{even})$$

With these functions, if the lowest term in the y-direction with $p = 1$ is considered

$$\overline{A}_{11} = \overline{R}_x - 2.472\,\frac{a^2}{b^2}\left(\frac{D_{xy}}{D_x} + \mu_y\right) \tag{12}$$

The critical values of the flutter parameter are given by equation (8) by redefining $\overline{A}_{11}$ according to equation (12). The flutter boundary which has been plotted on figure 3 for simply supported side edges is identical to the flutter boundary for these clamped side edges.

Panels with the side edges free.- Panels having free edges in the cross-flow direction must have mode shapes satisfying the following boundary conditions:

$$M_y\Big]_{y=0,b} = -D_y\left(\frac{\partial^2 w}{\partial y^2} + \mu_x \frac{\partial^2 w}{\partial x^2}\right)\Bigg]_{y=0,b} = 0$$

$$V_y\Big]_{y=0,b} = -D_y\left[\frac{\partial^3 w}{\partial y^3} + \left(\mu_x + \frac{D_{xy}}{D_y}\right)\frac{\partial^3 w}{\partial x^2 \partial y}\right]\Bigg]_{y=0,b} = 0$$

and since $\frac{\partial^2 w}{\partial x^2} = -\frac{n^2\pi^2}{a^2} w$, the function of y must satisfy the boundary conditions:

$$\left.\begin{aligned} \left[\frac{d^2F_{pn}(y)}{dy^2} - \frac{n^2\pi^2}{a^2}\mu_x F_{pn}(y)\right]\Bigg]_{y=0,b} &= 0 \\ \left[\frac{d^3F_{pn}(y)}{dy^3} - \frac{n^2\pi^2}{a^2}\gamma\frac{dF_{pn}(y)}{dy}\right]\Bigg]_{y=0,b} &= 0 \end{aligned}\right\} \tag{13}$$

where

$$\gamma = \mu_x + \frac{D_{xy}}{D_y}$$

In order to satisfy boundary conditions (13), the general function of y $\left(F_{pn}(y)\right)$ must of necessity be a function of n (the number of waves in the x-direction). The function selected herein is

$$F_{pn}(y) = \cos\frac{p\pi y}{b} + \left[1 + (-1)^p\right]\left[A_1 + A_2\left(\frac{y}{b} - \frac{1}{2}\right)^2 + A_3\left(\frac{y}{b} - \frac{1}{2}\right)^4\right]$$
$$+ \left[1 - (-1)^p\right]\left[B_1\left(\frac{y}{b} - \frac{1}{2}\right) + B_2\left(\frac{y}{b} - \frac{1}{2}\right)^3\right] \tag{14}$$

where

$$A_1(p,n) = \frac{-p^2a^2}{2b^2\mu_x n^2} - \frac{1}{2} + \frac{3a^2A_3(p,n)}{\pi^2b^2n^2\mu_x}\left(\frac{2}{3} - \frac{\mu_x}{\gamma} + \frac{8a^2}{n^2\pi^2b^2\gamma} + \frac{b^2\mu_x n^2\pi^2}{48a^2}\right)$$

$$A_2(p,n) = \frac{\left(12 - \frac{\gamma b^2n^2\pi^2}{2a^2}\right)A_3(p,n)}{\frac{\gamma b^2n^2\pi^2}{a^2}}$$

$$B_1(p,n) = \frac{6B_2(p,n)a^2}{\gamma n^2\pi^2b^2}\left(1 - \frac{\gamma b^2n^2\pi^2}{8a^2}\right)$$

$$B_2(p,n) = \frac{-\gamma\left(p^2\pi^2 + \mu_x\frac{b^2n^2\pi^2}{a^2}\right)}{6\mu_x\left(\frac{\gamma}{\mu_x} - 1 + \frac{\gamma b^2n^2\pi^2}{12a^2}\right)}$$

for any arbitrary $A_3(p,n)$. For even values of p with $A_3(p,n) = 0$, the slope at the free ends would become zero, an unnatural condition; however, a slight curvature of the panels at the side edges is caused by using a value for $A_3(p,n)$ in such cases. The integrals then become

$$I_{qp}^{(0)}(m,n) = \frac{1}{2}\delta_{qp} - \frac{4}{q^2\pi^2}B_1(p,n) - \frac{4}{q^4\pi^4}\left(\frac{3}{4}q^2\pi^2 - 6\right)B_2(p,n) - \frac{4}{p^2\pi^2}B_1(q,m)$$

$$+ \frac{B_1(q,m)\ B_2(p,n)}{20} + \frac{B_2(q,m)\ B_1(p,n)}{20} - \frac{4}{p^4\pi^4}\left(\frac{3}{4}p^2\pi^2 - 6\right)B_2(q,m)$$

$$+ \frac{B_2(q,m)\ B_2(p,n)}{112} + \frac{1}{3}B_1(q,m)\ B_1(p,n) \qquad (q \text{ and } p \text{ both odd})$$

$$I_{qp}^{(0)}(m,n) = \frac{1}{2}\,\delta_{qp} + \frac{4}{q^4\pi^4}\left(\frac{q^2\pi^2}{2} - 12\right)A_3(p,n) + \frac{4}{p^4\pi^4}\left(\frac{p^2\pi^2}{2} - 12\right)A_3(q,m) + \frac{4A_2(p,n)}{q^2\pi^2}$$

$$+ \frac{A_3(p,n) + A_3(q,m)}{576} + \frac{4A_2(q,m)}{p^2\pi^2} + \frac{A_3(p,n)\,A_3(q,m) + A_2(q,m)\,A_3(p,n)}{112}$$

$$+ \frac{A_2(q,m)\,A_2(p,n)}{20} + 4A_1(q,m)\,A_1(p,n)$$

$$+ \frac{A_1(q,m)\,A_3(p,n) + A_1(p,n)\,A_3(q,m)}{20}$$

$$+ \frac{A_2(p,n)\,A_1(q,m) + A_1(p,n)\,A_2(q,m)}{3} \qquad \text{(q and p both even)}$$

$$I_{qp}^{(0)}(m,n) = 1 + \frac{A_3(0,n) + A_3(0,m)}{40} + \frac{A_3(0,n)\,A_3(0,m)}{576} + 2\left[A_1(0,n) + A_1(0,m)\right]$$

$$+ \frac{A_2(0,m) + A_2(0,n)}{6} + \frac{A_1(0,n)\,A_3(0,m) + A_1(0,m)\,A_3(0,n)}{20}$$

$$+ \frac{A_3(0,m)\,A_2(0,n) + A_3(0,n)\,A_2(0,m)}{112} + 4A_1(0,m)\,A_1(0,n)$$

$$+ \frac{A_2(0,m)\,A_2(0,n)}{20} + \frac{A_1(0,m)\,A_2(0,n) + A_1(0,n)\,A_2(0,m)}{3}$$

(q and p both zero)

$$I_{qp}^{(0)}(m,n) = 0 \qquad \text{(For other combinations of q and p)}$$

$$I_{qp}^{(2)}(m,n) = -\frac{p^2\pi^2}{2}\delta_{qp} + 4B_1(q,m) - \frac{24B_2(p,n)}{q^2\pi^2} - \frac{24B_2(q,m)}{p^2\pi^2} + 3B_2(q,m)$$

$$+ 2B_1(q,m)\,B_2(p,n) + \frac{3}{10}B_2(q,m)\,B_2(p,n) \qquad (q \text{ and } p \text{ both odd})$$

$$I_{qp}^{(2)}(m,n) = -\frac{p^2\pi^2}{2}\delta_{qp} + 48\left[\frac{A_3(q,m)}{p^2\pi^2} + \frac{A_3(p,n)}{q^2\pi^2}\right] + 4A_1(q,m)\,A_3(p,n)$$

$$+ \frac{1}{10}A_2(p,n)\,A_3(q,m) + \left[\frac{3}{28}A_3(p,n) - 2\right]A_3(q,m)$$

$$+ \left[\frac{3}{5}A_3(p,n) - 4\right]A_2(q,m) + 8A_2(p,n)\left[A_1(q,m) + \frac{1}{12}A_2(q,m)\right]$$

$$(q \text{ and } p \text{ both even})$$

$$I_{qp}^{(2)}(m,n) = 2A_3(0,n)\left[1 + 2A_1(0,m) + \frac{3}{10}A_2(0,m) + \frac{3}{56}A_3(0,m)\right]$$

$$+ 4A_2(0,n)\left[1 + \frac{1}{40}A_3(0,m) + 2A_1(0,m) + \frac{A_2(0,m)}{6}\right]$$

$$(q \text{ and } p \text{ both zero})$$

$$I_{qp}^{(2)}(m,n) = 0 \qquad (\text{For other combinations of } q \text{ and } p)$$

$$I_{qp}^{(4)}(m,n) = \frac{p^4\pi^4}{2}\delta_{qp} - 4p^2\pi^2B_1(q,m) - \left(3p^2\pi^2 - 24\right)B_2(q,m)$$

$$(q \text{ and } p \text{ both odd})$$

$$I_{qp}^{(4)}(m,n) = \frac{p^4\pi^4}{2}\delta_{qp} + 2A_3(q,m)\left[p^2\pi^2 - 24 + \frac{3}{5}A_3(p,n)\right]$$

$$+ 4A_2(q,m)\left[p^2\pi^2 + 2A_3(p,n)\right] + 96A_1(q,m)\,A_3(p,n)$$

$$(q \text{ and } p \text{ both even})$$

$$I_{qp}^{(4)}(m,n) = 48 + \frac{6}{5} A_3(0,m)\, A_3(0,n) + 96A_3(0,n)\, A_1(0,m) + 8A_2(0,m)\, A_3(0,n)$$

(q and p both zero)

$$I_{qp}^{(4)}(m,n) = 0 \qquad \text{(For other combinations of } q \text{ and } p\text{)}$$

where

$$\delta_{qp} = 0 \qquad (p \neq q)$$

$$\delta_{qp} = 1 \qquad (p = q)$$

These integrals are seen to be functions of the mode numbers m,n of the two terms in the x-direction. In all factors except the aerodynamic loading factor, m equals n; whereas in the aerodynamic loading factor m is not equal to n, since an odd derivative appears only in that term. Thus, the distinction between m and n need only be made for the expression $I_{qp}^{(0)}(m,n)$ that multiplies the λ-factor. If a two-mode solution is considered, the dynamic-pressure parameter can be written, in general, as given in equation (6) (if $m + n$ is odd), and the critical value of the flutter parameter becomes either

$$\lambda_{cr} = \frac{\pi^4\left(m^2 - n^2\right)\left(m^4 - n^4 + n^2\bar{A}_{pp}^{(n)} - m^2\bar{A}_{pp}^{(m)} - \bar{B}_{pp}^{(m)} + \bar{B}_{pp}^{(n)}\right)}{8mn\sqrt{\dfrac{I_{pp}^{(0)}(m,n)\, I_{pp}^{(0)}(n,m)}{I_{pp}^{(0)}(m,m)\, I_{pp}^{(0)}(n,n)}}} \qquad (m > n) \qquad (15)$$

or upon expanding

$$\lambda_{cr} = \frac{\pi^4(m^2 - n^2)}{8mn\sqrt{\dfrac{I_{pp}^{(0)}(m,n)\, I_{pp}^{(0)}(n,m)}{I_{pp}^{(0)}(m,m)\, I_{pp}^{(0)}(n,n)}}}\left\{m^4 - n^4 + n^2\bar{A}_{pp}^{(n)} - m^2\bar{A}_{pp}^{(m)} - \frac{a^2\bar{R}_y}{\pi^2 b^2}\left[\frac{I_{pp}^{(2)}(n,n)}{I_{pp}^{(0)}(n,n)} - \frac{I_{pp}^{(2)}(m,m)}{I_{pp}^{(0)}(m,m)}\right] - \frac{a^4}{\pi^4 b^4}\frac{D_y}{D_x}\left[\frac{I_{pp}^{(4)}(n,n)}{I_{pp}^{(0)}(n,n)} - \frac{I_{pp}^{(4)}(m,m)}{I_{pp}^{(0)}(m,m)}\right]\right\} \qquad (m > n) \qquad (16)$$

Rewriting the constants A_1, A_2, B_1, and B_2 appearing in the integral expressions in terms of only two parameters yields

$$\left.\begin{aligned}
A_1(p,n) &= -\frac{p^2\pi^2}{2n^2\bar{\theta}} - \frac{1}{2} + \frac{3A_3(p,n)}{n^2\bar{\theta}\bar{\varphi}}\left(\frac{2}{3}\bar{\varphi} - 1 + \frac{8}{n^2\bar{\theta}} + \frac{n^2\bar{\theta}\bar{\varphi}}{48}\right) \\
A_2(p,n) &= \left(\frac{12}{n^2\bar{\theta}\bar{\varphi}} - \frac{1}{2}\right)A_3(p,n) \\
B_1(p,n) &= B_2(p,n)\left(-\frac{3}{4} + \frac{6}{n^2\bar{\theta}\bar{\varphi}}\right) \\
B_2(p,n) &= -\frac{\bar{\varphi}}{6}\left(\frac{p^2\pi^2 + n^2\bar{\theta}}{\bar{\varphi} - 1 + \frac{n^2\bar{\theta}\bar{\varphi}}{12}}\right)
\end{aligned}\right\} \tag{17}$$

where

$$\bar{\theta} = \mu_x \frac{\pi^2 b^2}{a^2}$$

and

$$\bar{\varphi} = \frac{\gamma}{\mu_x} = 1 + \frac{D_{xy}}{\mu_x D_y}$$

If the zeroeth term in the y-direction $(p = 0)$ and the lowest two terms in the x-direction are considered, λ_{cr} is given by either equation (15) or (16).

As an example of the effect of free edges on the flutter boundary, the flutter conditions for the following set of panels (typical of corrugation-stiffened panels with stiffeners running perpendicular to the airflow direction) will be computed:

$$N_x = N_y = 0$$

$$\bar{\theta} = 0.01$$

$$\bar{\varphi} = 10\ 000$$

$$A_3(p,n) = 1$$

then

$$\lambda_{cr} = 18.264\left[15 - \left(7.147 \times 10^{-3}\right)\bar{A}_{00}^{(r)} \frac{I_{00}^{(0)}(r,r)}{I_{00}^{(2)}(r,r)}\right] \tag{18}$$

where

$$\bar{A}_{00}^{(r)} \frac{I_{00}^{(0)}(r,r)}{I_{00}^{(2)}(r,r)} = - \frac{2}{\pi^2} \frac{a^2}{b^2}\left(\frac{D_{xy}}{D_x} + \mu_y\right)$$

A comparison of the critical dynamic-pressure parameter λ_{cr} for panels with free side edges (eq. (18)) with the corresponding expressions for simply supported side edges (using eq. (9)) and clamped side edges (using eq. (12)) is shown in figure 4. The lowest two terms in the x-direction and the lowest

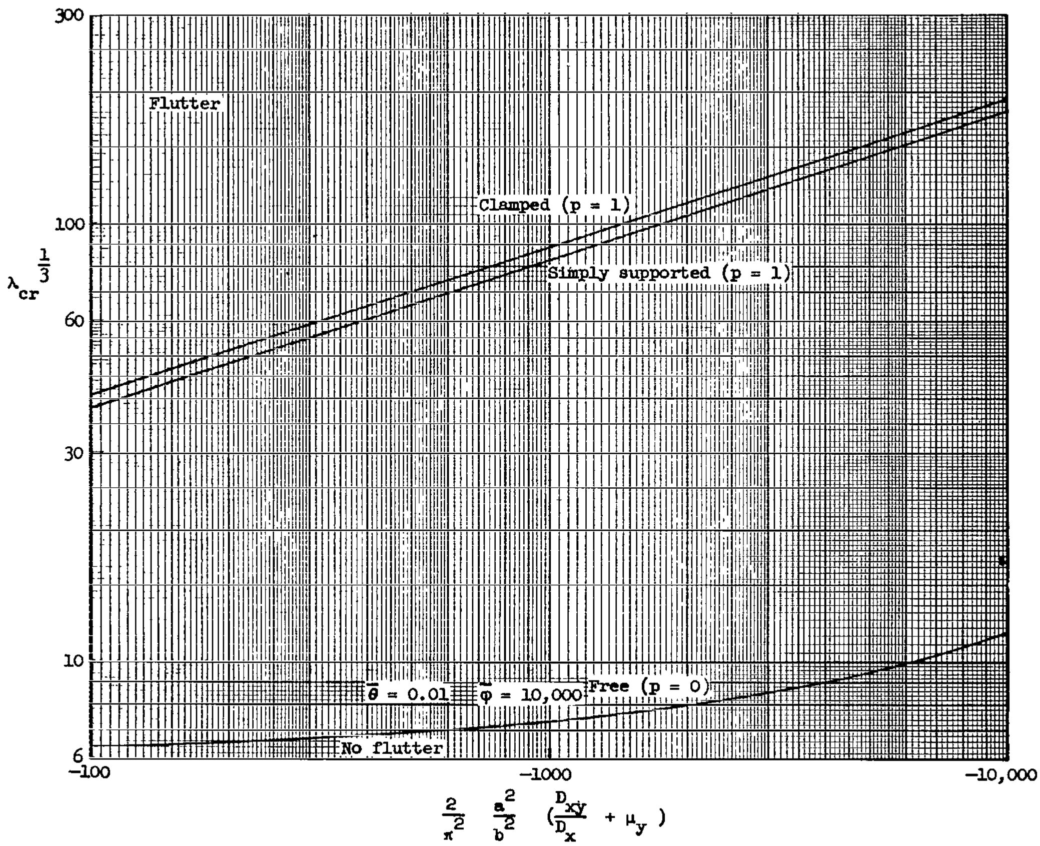


Figure 4.- The influence of side-edge boundary conditions on the flutter of orthotropic panels simply supported at their leading and trailing edges (two-term Galerkin solution). $N_x = N_y = 0$. First two terms in the x-direction; lowest pth term in the y-direction.

allowable term in the y-direction have been used in all cases. The allowance of a rigid-body or "zeroeth" mode shape for the free edges is the governing condition and can cause the lowering of the flutter dynamic pressure q by as much as an order of magnitude for the example series shown in figure 4. Some of this difference might be caused by the low value of μ_x $\left(\text{or the relatively large value of } \frac{D_y}{D_x}\right)$ selected for this example, but free side edges can cause significant flutter differences as shown.

Four-Mode Galerkin Solutions for Panels Simply Supported in the Flow Direction

The general Galerkin equations (eqs. (4)) can be examined not only for the two-term expansion but for the special case of a four-term expansion as well. Even though this solution is not exact, it yields more accurate results than the two-term expansion shown in the previous section.

For simply supported and clamped side-edge boundary conditions, a comparison between the two- and four-mode solutions and an "exact" solution (presented later) indicates that convergence is still not accomplished, and some caution should be exercised in using these values for design. For the free side-edge boundary conditions, however, the results of this analysis may be more useful.

By still selecting only a single p term in the y-direction, and four terms (m, $m + 1$, n, $n + 1$) in the x-direction, the flutter parameter becomes

$$\lambda = \frac{\pi^4}{4}\sqrt{\overline{K}\left(-1 + \sqrt{1 - \frac{\eta_m\eta_n\eta_{m+1}\eta_{n+1}}{\overline{L}\overline{K}^2}}\right)} \tag{19a}$$

where

$$\overline{L} = \frac{1}{\left[(m+1)^2 - m^2\right]^2\left[(n+1)^2 - n^2\right]^2} \frac{I_{pp}^{(0)}(m+1,m)\, I_{pp}^{(0)}(m,m+1)\, I_{pp}^{(0)}(n,n+1)\, I_{pp}^{(0)}(n+1,n)}{I_{pp}^{(0)}(m+1,m+1)\, I_{pp}^{(0)}(m,m)\, I_{pp}^{(0)}(n,n)\, I_{pp}^{(0)}(n+1,n+1)}$$

$$+ \frac{1}{\left(n^2 - m^2\right)^2\left[(n+1)^2 - (m+1)^2\right]^2} \frac{I_{pp}^{(0)}(m,n)\, I_{pp}^{(0)}(n,m)\, I_{pp}^{(0)}(m+1,n+1)\, I_{pp}^{(0)}(n+1,m+1)}{I_{pp}^{(0)}(m,m)\, I_{pp}^{(0)}(n,n)\, I_{pp}^{(0)}(m+1,m+1)\, I_{pp}^{(0)}(n+1,n+1)}$$

$$- \frac{I_{pp}^{(0)}(m,n)\, I_{pp}^{(0)}(m+1,m)\, I_{pp}^{(0)}(n+1,m+1)\, I_{pp}^{(0)}(n,n+1) + I_{pp}^{(0)}(m+1,n+1)\, I_{pp}^{(0)}(n+1,n)\, I_{pp}^{(0)}(n,m)\, I_{pp}^{(0)}(m,m+1)}{\left(n^2 - m^2\right)\left[(m+1)^2 - m^2\right]\left[(n+1)^2 - n^2\right]\left[(n+1)^2 - (m+1)^2\right] I_{pp}^{(0)}(m+1,m+1)\, I_{pp}^{(0)}(m,m)\, I_{pp}^{(0)}(n+1,n+1)\, I_{pp}^{(0)}(n,n)}$$

$$(m + n \quad \text{odd})$$

(Equation continued on next page)

$$
\begin{aligned}
\bar{L} = {} & \frac{1}{\left[(n+1)^2 - m^2\right]^2\left[n^2 - (m+1)^2\right]^2} \frac{I_{pp}^{(0)}(n+1,m)\, I_{pp}^{(0)}(m,n+1)\, I_{pp}^{(0)}(n,m+1)\, I_{pp}^{(0)}(m+1,n)}{I_{pp}^{(0)}(n+1,n+1)\, I_{pp}^{(0)}(m,m)\, I_{pp}^{(0)}(n,n)\, I_{pp}^{(0)}(m+1,m+1)} \\
& + \frac{1}{\left[(m+1)^2 - m^2\right]^2\left[(n+1)^2 - n^2\right]^2} \frac{I_{pp}^{(0)}(m,m+1)\, I_{pp}^{(0)}(m+1,m)\, I_{pp}^{(0)}(n,n+1)\, I_{pp}^{(0)}(n+1,n)}{I_{pp}^{(0)}(m,m)\, I_{pp}^{(0)}(m+1,m+1)\, I_{pp}^{(0)}(n,n)\, I_{pp}^{(0)}(n+1,n+1)} \\
& + \frac{I_{pp}^{(0)}(m+1,m)\, I_{pp}^{(0)}(m,n+1)\, I_{pp}^{(0)}(n,m+1)\, I_{pp}^{(0)}(n+1,n) + I_{pp}^{(0)}(m,m+1)\, I_{pp}^{(0)}(n+1,m)\, I_{pp}^{(0)}(m+1,n)\, I_{pp}^{(0)}(n,n+1)}{\left[(m+1)^2 - m^2\right]\left[(n+1)^2 - n^2\right]\left[n^2 - (m+1)^2\right]\left[(n+1)^2 - m^2\right] I_{pp}^{(0)}(m,m)\, I_{pp}^{(0)}(m+1,m+1)\, I_{pp}^{(0)}(n,n)\, I_{pp}^{(0)}(n+1,n+1)} \\
& \hspace{30em} (m + n \quad \text{even}) \\[2ex]
\bar{K} = {} & \frac{1}{2\bar{L}} \left\{ \frac{\eta_m \eta_{m+1}}{\left[(n+1)^2 - n^2\right]^2} \frac{I_{pp}^{(0)}(n,n+1)\, I_{pp}^{(0)}(n+1,n)}{I_{pp}^{(0)}(n,n)\, I_{pp}^{(0)}(n+1,n+1)} + \frac{\eta_m \eta_n}{\left[(n+1)^2 - (m+1)^2\right]^2} \frac{I_{pp}^{(0)}(n+1,m+1)\, I_{pp}^{(0)}(m+1,n+1)}{I_{pp}^{(0)}(n+1,n+1)\, I_{pp}^{(0)}(m+1,m+1)} \right. \\
& \left. + \frac{\eta_n \eta_{n+1}}{\left[(m+1)^2 - m^2\right]^2} \frac{I_{pp}^{(0)}(m+1,m)\, I_{pp}^{(0)}(m,m+1)}{I_{pp}^{(0)}(m+1,m+1)\, I_{pp}^{(0)}(m,m)} + \frac{\eta_{m+1} \eta_{n+1}}{\left(n^2 - m^2\right)^2} \frac{I_{pp}^{(0)}(m,n)\, I_{pp}^{(0)}(n,m)}{I_{pp}^{(0)}(m,m)\, I_{pp}^{(0)}(n,n)} \right\} \qquad (m + n \quad \text{odd}) \\[2ex]
\bar{K} = {} & \frac{1}{2\bar{L}} \left\{ \frac{\eta_m \eta_{m+1}}{\left[(n+1)^2 - n^2\right]^2} \frac{I_{pp}^{(0)}(n,n+1)\, I_{pp}^{(0)}(n+1,n)}{I_{pp}^{(0)}(n,n)\, I_{pp}^{(0)}(n+1,n+1)} + \frac{\eta_m \eta_{n+1}}{\left[(m+1)^2 - n^2\right]^2} \frac{I_{pp}^{(0)}(m+1,n)\, I_{pp}^{(0)}(n,m+1)}{I_{pp}^{(0)}(m+1,m+1)\, I_{pp}^{(0)}(n,n)} \right. \\
& \left. + \frac{\eta_n \eta_{n+1}}{\left[(m+1)^2 - m^2\right]^2} \frac{I_{pp}^{(0)}(m+1,m)\, I_{pp}^{(0)}(m,m+1)}{I_{pp}^{(0)}(m+1,m+1)\, I_{pp}^{(0)}(m,m)} + \frac{\eta_{m+1} \eta_n}{\left[(n+1)^2 - m^2\right]^2} \frac{I_{pp}^{(0)}(n+1,m)\, I_{pp}^{(0)}(m,n+1)}{I_{pp}^{(0)}(n+1,n+1)\, I_{pp}^{(0)}(m,m)} \right\} \qquad (m + n \quad \text{even}) \\[2ex]
\eta_i = {} & i^2 - \bar{A}_{pp}^{(i)} - \frac{\bar{B}_{pp}^{(i)}}{i^2}
\end{aligned}
\tag{19b}
$$

If, for some particular boundary conditions under investigation, the y-expression $F_{pn}(y)$ is not an explicit function of the x-direction mode number n, the integral expressions $I_{pp}^{(r)}(m,n)$ are also independent of m and n, and all the integral ratios used in equations (19) become 1. For example, the expression for $\bar{L}$ then becomes simply

$$\bar{L} = \left\{\frac{1}{(2m + 1)(2n + 1)} + \frac{1}{\left[n^2 - (m + 1)^2\right]\left[(n + 1)^2 - m^2\right]}\right\}^2 \qquad (m + n \quad \text{even})$$

$$\bar{L} = \left\{\frac{1}{(2m + 1)(2n + 1)} - \frac{1}{\left[(n + 1)^2 - (m + 1)^2\right]\left(n^2 - m^2\right)}\right\}^2 \qquad (m + n \quad \text{odd})$$

and $\bar{A}_{pp}^{(r)}$ and $\bar{B}_{pp}^{(r)}$ become $\bar{A}_{pp}$ and $\bar{B}_{pp}$, respectively. Since $F_{pn}(y)$ is often independent of n (for example, simply supported or clamped side-edge boundary conditions), this case is discussed in some detail below.

For a given value of $\bar{A}_{pp}$ the condition $\frac{\partial\lambda}{\partial\bar{B}_{pp}} = 0$ determines the value of λ_{cr}. A plot of λ against $\bar{B}_{pp}$ for a selected value of $\bar{A}_{pp}$ must be made, from which the lowest value of $\bar{B}_{pp}$ can be found at which $\frac{\partial\lambda}{\partial\omega} = 0$. This value of $\bar{B}_{pp}$ is substituted back into equations (19) to determine λ_{cr} for that value of $\bar{A}_{pp}$. If the problem is to be calculated on a computer, it is helpful to start $\bar{B}_{pp}$ at the value $\frac{m^4 + n^4}{2} - \frac{(m^2 + n^2)}{2}\bar{A}_{pp}$ and decrease $\bar{B}_{pp}$ by $\bar{A}_{pp} \times 10^{-2}$ until the value of λ starts to decrease. All that needs to be known are the assumed mode shapes for particular cases.

The lowest two terms in the x-direction for a two-mode solution were selected, and the resulting critical flutter parameter was plotted as a function of $\bar{A}_{11}$ in figure 5. Also the lowest four terms in the x-direction were used in a four-mode solution as noted in figure 5. The "exact" solution curve is the value to which the Galerkin solution would limit if the number of x-terms became very large. This solution is determined from the solution of reference 1 for simply supported plates and is based on the fact that the flutter boundary for panels with cross-stream boundary conditions other than simply supported can be computed from simply supported results by merely redefining $\bar{A}_{pp}$ and $\bar{B}_{pp}$. Thus, these exact results are not strictly exact, since only a single approximate mode shape term was used in the y-direction. It can be seen from figure 5 that the two- and four-term solutions agree fairly well with each other, but a large discrepancy exists between these solutions and the exact solution and indicates that these solutions are not well converged with only four terms.

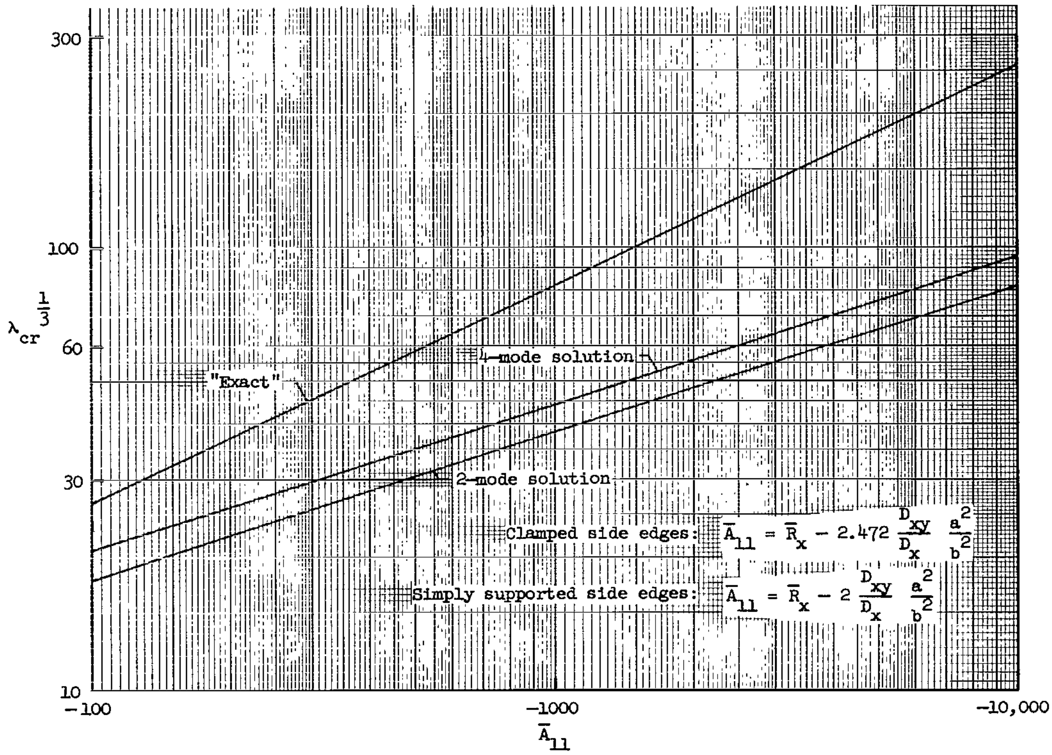


Figure 5.- Accuracy of the Galerkin approximation for the flutter of panels simply supported at their leading and trailing edges. Lowest pth term in the y-direction.

Basic Equations - General Case of Simply Supported Panels

The two transverse shearing stiffnesses D_{Q_x} and D_{Q_y} of the panels shown in figure 1 can be included in the differential equation for deflection, provided the equation is expanded to the sixth-order (see ref. 12) equation:

$$\alpha_1 \frac{\partial^6 w}{\partial x^6} + \alpha_2 \frac{\partial^6 w}{\partial x^4 \partial y^2} + \alpha_3 \frac{\partial^6 w}{\partial x^2 \partial y^4} + \alpha_4 \frac{\partial^6 w}{\partial y^6} - D_x \frac{\partial^4 w}{\partial x^4} - 2\left[D_{xy}\left(1 - \mu_x\mu_y\right) + \frac{1}{2}\mu_x D_y\right.$$

$$\left. + \frac{1}{2}\mu_y D_x\right]\frac{\partial^4 w}{\partial x^2 \partial y^2} - D_y \frac{\partial^4 w}{\partial y^4} - \alpha_5\left(N_x \frac{\partial^6 w}{\partial x^6} + N_y \frac{\partial^6 w}{\partial x^4 \partial y^2} + 2N_{xy} \frac{\partial^6 w}{\partial x^5 \partial y}\right) - \alpha_6\left(N_x \frac{\partial^6 w}{\partial x^4 \partial y^2}\right.$$

$$\left. + N_y \frac{\partial^6 w}{\partial x^2 \partial y^4} + 2N_{xy} \frac{\partial^6 w}{\partial x^3 \partial y^3}\right) - \alpha_7\left(N_x \frac{\partial^6 w}{\partial x^2 \partial y^4} + N_y \frac{\partial^6 w}{\partial y^6} + 2N_{xy} \frac{\partial^6 w}{\partial x\, \partial y^5}\right) + \alpha_8\left(N_x \frac{\partial^4 w}{\partial x^4}\right.$$

$$\left. + N_y \frac{\partial^4 w}{\partial x^2 \partial y^2} + 2N_{xy} \frac{\partial^4 w}{\partial x^3 \partial y}\right) + \alpha_9\left(N_x \frac{\partial^4 w}{\partial x^2 \partial y^2} + N_y \frac{\partial^4 w}{\partial y^4} + 2N_{xy} \frac{\partial^4 w}{\partial x\, \partial y^3}\right)$$

$$- \left(1 - \mu_x\mu_y\right)\left(N_x \frac{\partial^2 w}{\partial x^2} + N_y \frac{\partial^2 w}{\partial y^2} + 2N_{xy} \frac{\partial^2 w}{\partial x\, \partial y}\right) - \frac{2q}{\beta}\left[\alpha_5 \frac{\partial^5 w}{\partial x^5} + \alpha_6 \frac{\partial^5 w}{\partial x^3 \partial y^2} + \alpha_7 \frac{\partial^5 w}{\partial x\, \partial y^4}\right.$$

$$\left. - \alpha_8 \frac{\partial^3 w}{\partial x^3} - \alpha_9 \frac{\partial^3 w}{\partial x\, \partial y^2} + \left(1 - \mu_x\mu_y\right)\frac{\partial w}{\partial x}\right] + m\omega^2\left[\alpha_5 \frac{\partial^4 w}{\partial x^4} + \alpha_6 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \alpha_7 \frac{\partial^4 w}{\partial y^4}\right.$$

$$\left. - \alpha_8 \frac{\partial^2 w}{\partial x^2} - \alpha_9 \frac{\partial^2 w}{\partial y^2} + \left(1 - \mu_x\mu_y\right)w\right] = 0 \qquad (20)$$

where N_x and N_y are in-plane loads that are positive in compression, and α_n are functions of the shear stiffnesses; specifically,

$$\alpha_1 = \frac{1}{2} \frac{D_{xy} D_x}{D_{Q_y}}$$

$$\alpha_2 = \alpha_1 \frac{D_{Q_y}}{D_{Q_x}} + \frac{\alpha_{10}}{D_{Q_y}}$$

$$\alpha_3 = \alpha_1 \frac{D_y}{D_x} + \frac{\alpha_{10}}{D_{Q_x}}$$

$$\alpha_4 = \frac{1}{2} \frac{D_{xy} D_y}{D_{Q_x}}$$

$$\alpha_5 = \frac{\alpha_1}{D_{Q_x}}$$

$$\alpha_6 = \frac{\alpha_{10}}{D_{Q_x} D_{Q_y}}$$

$$\alpha_7 = \frac{\alpha_4}{D_{Q_y}}$$

$$\alpha_8 = \frac{1}{2} \frac{D_{xy}}{D_{Q_y}}\left(1 - \mu_x \mu_y\right) + \frac{D_x}{D_{Q_x}}$$

$$\alpha_9 = \frac{1}{2} \frac{D_{xy}}{D_{Q_x}}\left(1 - \mu_x \mu_y\right) + \frac{D_y}{D_{Q_y}}$$

$$\alpha_{10} = D_x D_y - \frac{1}{2} D_{xy}\left(D_x \mu_y + D_y \mu_x\right)$$

The shear stiffnesses can be determined by a method of approach similar to the method shown in reference 13. The boundary conditions for simple supports (in which all points along an edge are restricted from moving parallel to the edge) are

$$
\left.\begin{aligned}
&w\Big]_{x=0,a} = 0 \\
&\frac{Q_y}{D_{Q_y}}\Bigg]_{x=0,a} = 0 \\
&M_x\Big]_{x=0,a} = -\frac{D_x}{1-\mu_x\mu_y}\left(\frac{\partial^2 w}{\partial x^2} + \mu_y \frac{\partial^2 w}{\partial y^2} - \frac{1}{D_{Q_x}}\frac{\partial Q_x}{\partial x} - \frac{\mu_y}{D_{Q_y}}\frac{\partial Q_y}{\partial y}\right)\Bigg]_{x=0,a} = 0 \\
&w\Big]_{y=0,b} = 0 \\
&\frac{Q_x}{D_{Q_x}}\Bigg]_{y=0,b} = 0 \\
&M_y\Big]_{y=0,b} = -\frac{D_y}{1-\mu_x\mu_y}\left(\frac{\partial^2 w}{\partial y^2} + \mu_x \frac{\partial^2 w}{\partial x^2} - \frac{1}{D_{Q_y}}\frac{\partial Q_y}{\partial y} - \frac{\mu_x}{D_{Q_x}}\frac{\partial Q_x}{\partial x}\right)\Bigg]_{y=0,b} = 0
\end{aligned}\right\} \quad (21)
$$

These boundary conditions are satisfied by the assumptions

$$
Q_x = \sum_r \sum_s \bar{a}_{rs} \cos\frac{r\pi x}{a} \sin\frac{s\pi y}{b}
$$

$$
Q_y = \sum_m \sum_n \bar{\bar{a}}_{mn} \sin\frac{m\pi x}{a} \cos\frac{n\pi y}{b}
$$

(where $\bar{a}_{rs}$ and $\bar{\bar{a}}_{mn}$ are constants) and finally

$$
w = \sum_i \sum_j a_{ij} \sin\frac{i\pi x}{a} \sin\frac{j\pi y}{b} \tag{22}
$$

Application of the Galerkin method yields the following set of homogeneous equations

$$a_{rs}\left\{-\alpha_1 \frac{r^6\pi^6}{a^6} - \alpha_2 \frac{r^4s^2\pi^6}{a^4b^2} - \alpha_3 \frac{r^2s^4\pi^6}{a^2b^4} - \alpha_4 \frac{s^6\pi^6}{b^6}\right.$$

$$- D_x \frac{r^4\pi^4}{a^4} - 2\left[D_{xy}\left(1 - \mu_x\mu_y\right) + \frac{\mu_y D_x + \mu_x D_y}{2}\right]\frac{r^2s^2\pi^4}{a^2b^2}$$

$$- D_y \frac{s^4\pi^4}{b^4} + \alpha_5\left(N_x \frac{r^6\pi^6}{a^6} + N_y \frac{r^4s^2\pi^6}{a^4b^2}\right) + \alpha_6\left(N_x \frac{r^4s^2\pi^6}{a^4b^2} + N_y \frac{r^2s^4\pi^6}{a^2b^4}\right)$$

$$+ \alpha_7\left(N_x \frac{r^2s^4\pi^6}{a^2b^4} + N_y \frac{s^6\pi^6}{b^6}\right) + \alpha_8\left(N_x \frac{r^4\pi^4}{a^4} + N_y \frac{r^2s^2\pi^4}{a^2b^2}\right)$$

$$+ \alpha_9\left(N_x \frac{r^2s^2\pi^4}{a^2b^2} + N_y \frac{s^4\pi^4}{b^4}\right) + \left(1 - \mu_x\mu_y\right)\left(N_x \frac{r^2\pi^2}{a^2} + N_y \frac{s^2\pi^2}{b^2}\right)$$

$$\left.+ m\omega^2\left(\alpha_5 \frac{r^4\pi^4}{a^4} + \alpha_6 \frac{r^2s^2\pi^4}{a^2b^2} + \alpha_7 \frac{s^4\pi^4}{b^4} + \alpha_8 \frac{r^2\pi^2}{a^2} + \alpha_9 \frac{s^2\pi^2}{b^2} + 1 - \mu_x\mu_y\right)\right\}$$

$$+ \frac{8q}{a\beta} \sum_{\substack{i \\ i+r\ \mathrm{odd}}} \frac{i r a_{is}}{\left(i^2 - r^2\right)}\left[\alpha_5 \frac{i^4\pi^4}{a^4} + \alpha_6 \frac{i^2s^2\pi^4}{a^2b^2} + \alpha_7 \frac{s^4\pi^4}{b^4} + \alpha_8 \frac{i^2\pi^2}{a^2} + \alpha_9 \frac{s^2\pi^2}{b^2}\right.$$

$$\left.+ \left(1 - \mu_x\mu_y\right)\right] - N_{xy} \frac{16}{ab} \sum_{\substack{i \\ i+r\ \mathrm{odd}}} \sum_{\substack{j \\ j+s\ \mathrm{odd}}} \frac{i j r s a_{ij}}{\left(i^2 - r^2\right)\left(j^2 - s^2\right)}\left(\alpha_5 \frac{i^4\pi^4}{a^4} + \alpha_6 \frac{i^2j^2\pi^4}{a^2b^2}\right.$$

$$\left.+ \alpha_7 \frac{j^4\pi^4}{b^4} + \alpha_8 \frac{i^2\pi^2}{a^2} + \alpha_9 \frac{j^2\pi^2}{b^2} + 1 - \mu_x\mu_y\right) = 0 \tag{23}$$

where r and s take on all values. If the usual case where $N_{xy} = 0$ is considered and the constants are redefined as

$$\bar{\alpha}_1 = \frac{\alpha_1 \pi^2}{a^2 D_x} \qquad \bar{\alpha}_8 = \frac{\alpha_8 \pi^2}{a^2}$$

$$\bar{\alpha}_2 = \frac{\alpha_2 \pi^2}{b^2 D_x} \qquad \bar{\alpha}_9 = \frac{\alpha_9 \pi^2}{b^2}$$

$$\bar{\alpha}_3 = \frac{\alpha_3 \pi^2}{b^2 D_x} \qquad \bar{R}_x = \frac{N_x a^2}{D_x \pi^2}$$

$$\bar{\alpha}_4 = \frac{\alpha_4 \pi^2}{b^2 D_x} \qquad \bar{R}_y = \frac{N_y a^2}{D_x \pi^2}$$

$$\bar{\alpha}_5 = \alpha_5 \frac{\pi^4}{a^4} \qquad \lambda = \frac{2 q a^3}{\beta D_x}$$

$$\bar{\alpha}_6 = \frac{\alpha_6 \pi^4}{a^2 b^2} \qquad \bar{k}^2 = \frac{m \omega^2 a^4}{D_x \pi^4}$$

$$\bar{\alpha}_7 = \frac{\alpha_7 \pi^4}{b^4}$$

the following set of homogeneous equations appears:

$$\begin{aligned}
a_{rs} \Bigg\{ & -r^6 \bar{\alpha}_1 - r^4 s^2 \bar{\alpha}_2 - r^2 s^4 \frac{a^2}{b^2} \bar{\alpha}_3 - s^6 \frac{a^4}{b^4} \bar{\alpha}_4 - r^4 - 2 r^2 s^2 \frac{a^2}{b^2} \left[\left(1 - \mu_x \mu_y\right) \frac{D_{xy}}{D_x} + \frac{\mu_y}{2} + \frac{\mu_x D_y}{2 D_x} \right] - s^4 \frac{a^4}{b^4} \frac{D_y}{D_x} \\
& + \bar{\alpha}_5 \left(r^6 \bar{R}_x + r^4 s^2 \frac{a^2}{b^2} \bar{R}_y \right) + \bar{\alpha}_6 \left(r^4 s^2 \bar{R}_x + r^2 s^4 \frac{a^2}{b^2} \bar{R}_y \right) + \bar{\alpha}_7 \left(r^2 s^4 \bar{R}_x + s^6 \frac{a^2}{b^2} \bar{R}_y \right) + \bar{\alpha}_8 \left(r^4 \bar{R}_x + r^2 s^2 \frac{a^2}{b^2} \bar{R}_y \right) \\
& + \bar{\alpha}_9 \left(r^2 s^2 \bar{R}_x + s^4 \frac{a^2}{b^2} \bar{R}_y \right) + \left(1 - \mu_x \mu_y\right) \left(r^2 \bar{R}_x + s^2 \frac{a^2}{b^2} \bar{R}_y \right) + \bar{k}^2 \left(r^4 \bar{\alpha}_5 + r^2 s^2 \bar{\alpha}_6 + s^4 \bar{\alpha}_7 + r^2 \bar{\alpha}_8 + s^2 \bar{\alpha}_9 + 1 - \mu_x \mu_y \right) \Bigg\} \\
& + \frac{4\lambda}{\pi^4} \sum_{\substack{i \\ i+r \text{ odd}}} \frac{i r a_{is}}{i^2 - r^2} \left(\bar{\alpha}_5 i^4 + \bar{\alpha}_6 i^2 s^2 + \bar{\alpha}_7 s^4 + \bar{\alpha}_8 i^2 + \bar{\alpha}_9 s^2 + 1 - \mu_x \mu_y \right) = 0
\end{aligned} \tag{24}$$

where r and s take on all values. To determine the effect of the shear stiffnesses, a two-mode solution was completed and compared with the two-mode solution with infinite shear stiffnesses. Selecting a single term s in the y-direction and any two terms i,r in the x-direction (one even and one odd), and setting the determinant of equations (24) equal to zero yields the following expression for the flutter parameter λ:

$$\lambda = \frac{\pi^4}{4}\left|i^2 - r^2\right| \sqrt{\left\{\frac{-K_2(r) + r^2\overline{A}(r) + \overline{B}(r)}{r^2\left[1 + K_1(r)\right]}\right\}\left\{\frac{K_2(i) - i^2\overline{A}(i) - \overline{B}(i)}{i^2\left[1 + K_1(i)\right]}\right\}} \tag{25}$$

where

$$K_1(j) = j^4\bar{\alpha}_5 + j^2s^2\bar{\alpha}_6 + s^4\bar{\alpha}_7 + j^2\bar{\alpha}_8 + s^2\bar{\alpha}_9 - \mu_x\mu_y$$

$$K_2(j) = j^6\bar{\alpha}_1 + j^4s^2\bar{\alpha}_2 + j^2s^4\frac{a^2}{b^2}\bar{\alpha}_3 + s^6\frac{a^4}{b^4}\bar{\alpha}_4 + j^4$$

$$\overline{A}(j) = \left[1 + K_1(j)\right]\overline{R}_x - 2s^2\frac{a^2}{b^2}\left[\frac{D_{xy}}{D_x}\left(1 - \mu_x\mu_y\right) + \frac{\mu_y}{2} + \frac{\mu_x D_y}{2D_x}\right]$$

$$\overline{B}(j) = \left[1 + K_1(j)\right]\left(s^2\frac{a^2}{b^2}\overline{R}_y + \overline{k}^2\right) - s^4\frac{a^4}{b^4}\frac{D_y}{D_x}$$

The critical value of λ is found by setting the partial derivative of λ with respect to ω^2 equal to zero and solving for the value of ω^2 at λ_{cr}, and then resubstituting this value into equation (25) to yield

$$\lambda_{cr} = \frac{\pi^4}{8ir}\left|i^2 - r^2\right|\left\{\frac{r^2\overline{A}(r) - K_2(r)}{1 + K_1(r)} + \frac{K_2(i) - i^2\overline{A}(i)}{1 + K_1(i)} + \frac{K_1(r) - K_1(i)}{\left[1 + K_1(i)\right]\left[1 + K_1(r)\right]}\frac{D_y}{D_x}\frac{s^4a^4}{b^4}\right\} \qquad (i + r \text{ odd}) \tag{26}$$

The value of λ_{cr} for the case of infinite shear stiffnesses is

$$\lambda_{cr}(\infty) = \frac{\pi^4\left(i^2 - r^2\right)^2\left(i^2 + r^2 - \bar{A}\right)}{8ir} \qquad (i + r \quad \text{odd}) \tag{27}$$

where $\bar{A}(i)$ and $\bar{A}(r)$ both become

$$\bar{A} = \bar{R}_x - 2s^2 \frac{a^2}{b^2}\left(\frac{D_{xy}}{D_x} + \mu_y\right)$$

and the assumption $\mu_x\mu_y \ll 1$ and the fact $\mu_y D_x \equiv \mu_x D_y$ have both been used.

To illustrate the influence these transverse shear stiffnesses can have on panel flutter, the flutter conditions for the following series of panels are computed:

$$N_x = 0$$

$$\frac{a}{b} = 1$$

$$\frac{D_y}{D_x} = 1{,}000$$

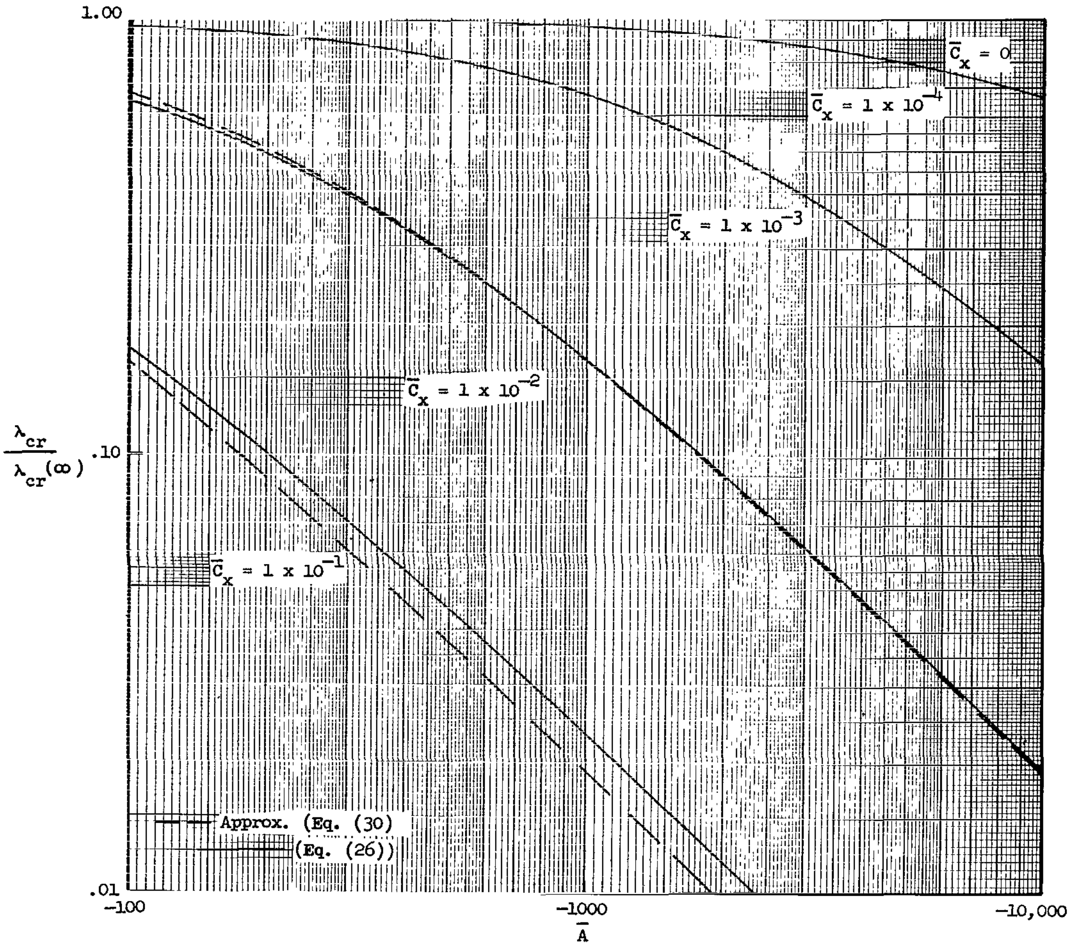
$$\mu_y = 0.15$$

$$i = 1$$

$$r = 2$$

$$s = 1$$

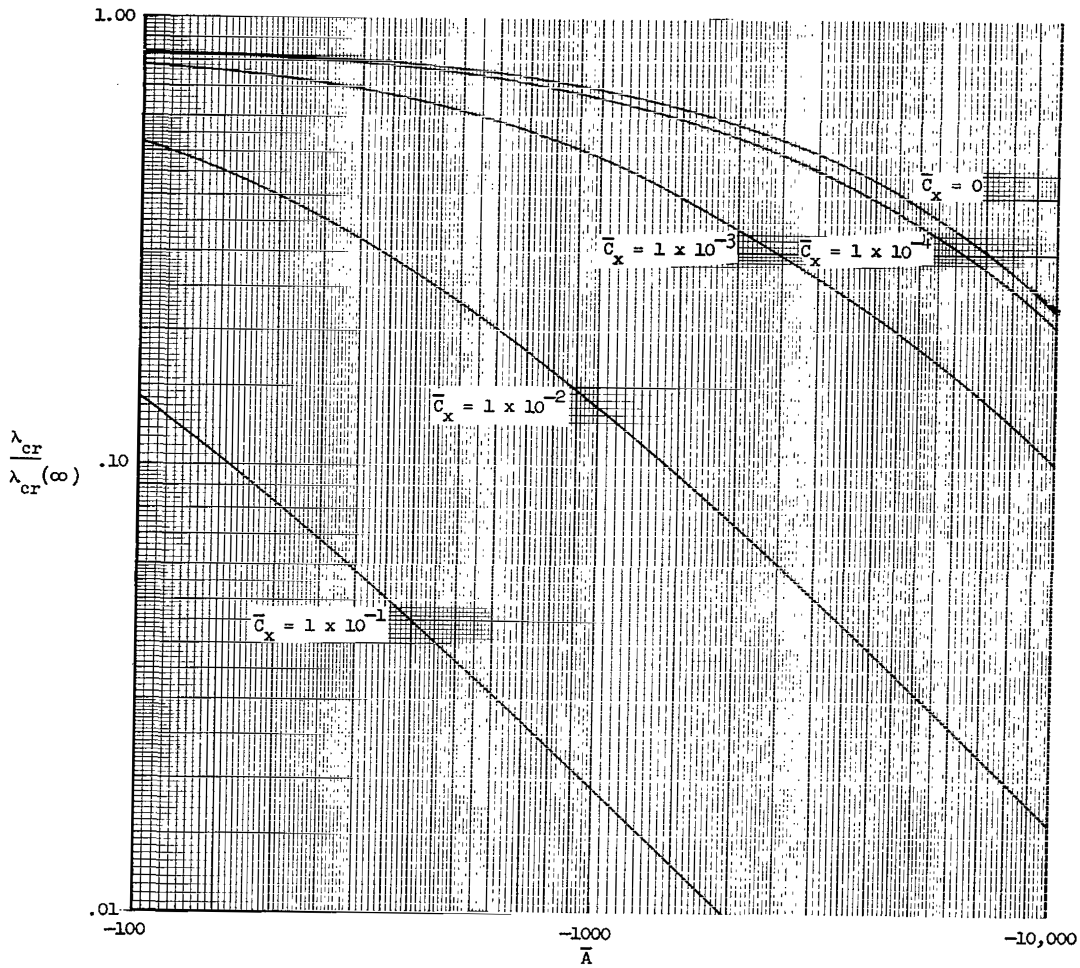
These conditions correspond to a square panel (typical of the corrugation-stiffened panels of ref. 7) without in-plane loading. A plot of $\dfrac{\lambda_{cr}}{\lambda_{cr}(\infty)}$ against $\bar{A}$ for several values of $\bar{C}_x$ $\left(\bar{C}_x = \dfrac{\pi^2}{2a^2}\dfrac{D_x}{D_{Q_x}}\right)$ and $\bar{C}_y$ $\left(\bar{C}_y = \dfrac{\pi^2}{2b^2}\dfrac{D_y}{D_{Q_y}}\right)$ is shown by the solid curves in figure 6, and the values used in this plot are also presented in table II.



(a) Infinite D_{Q_y} stiffness. $\overline{C}_y = 0$.

Figure 6.- The influence of transverse shear stiffnesses on the flutter of representative orthotropic panels simply supported on all sides. Two-mode Galerkin solution: $N_x = 0$; $\frac{a}{b} = 1$; $\frac{D_y}{D_x} = 1000$; $i,r,s = 1,2,1$ and $\mu_y = 0.15$.

From figure 6 several findings are evident. For very small shearing stiffnesses (large $\overline{C}_x$ values), the flutter parameter can drop to less than 1 percent of the value for infinite shear stiffnesses and, thus, can have an extremely important role in panel flutter. It can also be seen by comparing figures 6(a) and 6(b) that, for large values of $\overline{C}_x$, the influence of $\overline{C}_y$ is negligible, particularly for small negative values of $\overline{A}$. Thus, $\overline{C}_x$ is considered to be of more importance than $\overline{C}_y$, and a simplified analysis including only $\overline{C}_x$ is presented in the next section of this report.



(b) Small D_{Q_y} stiffness. $\bar{C}_y = 1 \times 10^{-1}$.

Figure 6.- Concluded.

Infinite Shear Stiffness, D_{Q_y}

It can be seen from figure 6 that $\bar{C}_x$ is generally of more importance than $\bar{C}_y$ (at least for nearly square panels of this stiffness ratio). By considering the D_{Q_x} stiffness only, a relatively simple relationship can be derived for the critical flutter parameter for panels with relatively small D_x stiffness. If $D_{Q_y} = \infty$, the constants in equation (25) become

TABLE II.- THE EFFECT OF TRANSVERSE SHEARING STIFFNESSES ON THE FLUTTER OF SIMPLY SUPPORTED PANELS FOR AN EXAMPLE SERIES (TWO-MODE GALERKIN SOLUTION)

$-\bar{A}(r)$	$\frac{\lambda_{cr}}{\lambda_{cr}(\infty)}$: Eq. (26): $\frac{a}{b} = 1$; $\mu_y = 0.15$; $i,r,s = 1,2,1$; $\frac{D_y}{D_x} = 1000$; $\bar{C}_y = 0$; $N_x = 0$	Eq. (30): $\bar{C}_y = 0$	Eq. (26): $\frac{a}{b} = 1$; $\mu_y = 0.15$; $i,r,s = 1,2,1$; $\frac{D_y}{D_x} = 0.001$; $\bar{C}_y = 0.1$, $N_x = 0$	Eq. (29): $\bar{C}_y = 0$; $i,r = 1,2$; $\bar{R}_x = 1$
	$\bar{C}_x = 0$			
100	1.0000	1.0000	0.8314	1.0000
300	1.0000	1.0000	.7938	1.0000
1 000	1.0000	1.0000	.6929	1.0000
3 000	1.0000	1.0000	.4987	1.0000
10 000	1.0000	1.0000	.2228	1.0000
	$\bar{C}_x = 1 \times 10^{-4}$			
100	0.9943	0.9950	0.8267	0.9950
300	.9845	.9852	.7820	.9852
1 000	.9512	.9524	.6652	.9523
3 000	.8690	.8696	.4597	.8695
10 000	.6664	.6667	.2078	.6666
	$\bar{C}_x = 1 \times 10^{-3}$			
100	0.9456	0.9524	0.7866	0.9519
300	.8641	.8696	.6909	.8691
1 000	.6638	.6667	.4891	.6663
3 000	.3993	.4000	.2663	.3998
10 000	.1668	.1667	.1024	.1666
	$\bar{C}_x = 1 \times 10^{-2}$			
100	0.6390	0.6667	0.5326	0.6635
300	.3935	.4000	.3222	.3980
1 000	.1678	.1667	.1356	.1658
3 000	.06357	.06250	.05112	.06218
10 000	.02003	.01961	.01607	.01951
	$\bar{C}_x = 1 \times 10^{-1}$			
100	0.1752	0.1667	0.1429	0.1587
300	.07252	.06250	.05876	.05942
1 000	.02374	.01961	.01919	.01863
3 000	.00812	.006622	.00656	.006292
10 000	.00246	.00200	.00199	.00190

$$\left.\begin{aligned} K_1(j) &= \bar{C}_x\left(2j^2 + \underline{s^2 \frac{a^2}{b^2} \frac{D_{xy}}{D_x}}\right) \\ K_2(j) &= \bar{C}_x\left[j^4 s^2 \frac{a^2}{b^2} \frac{D_{xy}}{D_x} + 2j^2 s^4 \frac{a^4}{b^4}\left(\frac{D_y}{D_x} - \frac{D_{xy}}{D_x} \mu_y\right) + \underline{\frac{s^6 a^6}{b^6} \frac{D_y}{D_x} \frac{D_{xy}}{D_x}}\right] + j^4 \end{aligned}\right\} \quad (28)$$

Now, since $\frac{D_y}{D_x}$ and $\frac{D_{xy}}{D_x}$ are both large quantities (for small values of D_x) of the same order of magnitude, only those terms in equations (28) that are underlined are retained. Therefore, λ_{cr} can be written (for any two modes and arbitrary stiffnesses) as

$$\lambda_{cr} = \frac{\pi^4\left(i^2 - r^2\right)^2}{8ir}\left(\frac{i^2 + r^2 - \bar{\bar{A}}}{1 + \bar{C}_x s^2 \frac{a^2}{b^2}\frac{D_{xy}}{D_x}}\right) \tag{29}$$

where $\bar{A}(i)$ has become

$$\bar{\bar{A}} = \left(1 + \bar{C}_x s^2 \frac{a^2}{b^2}\frac{D_{xy}}{D_x}\right)\bar{R}_x - 2s^2 \frac{a^2}{b^2}\left(\frac{D_{xy}}{D_x} + \mu_y\right)$$

This expression is extremely easy to evaluate in comparison with equation (26). Also, if $\bar{\bar{A}}$ is substituted into equation (29) directly, the effect of in-plane loading is seen to be linear. Thus, an increase in compressive loading lowers the critical dynamic-pressure parameter, and an increase in tensile loading raises it. This equation also shows that the influence of $\bar{R}_x$ decreases for the higher modes, and the largest effect is for i and r equal to 1 and 2. Finally, the influence of $\bar{R}_x$ is greater for panels with smaller values of $\bar{C}_x$ $\left(\text{or larger values of } D_{Q_x}\right)$. A plot of $\frac{\lambda_{cr}}{\lambda_{cr}(\infty)}$ with $\bar{A}$ for two representative values of $\bar{R}_x$ and a series of values of $\bar{C}_x$ is shown in figure 7. The values used for plotting figure 7 are also given in table II for easy reference.

The validity of the approximation used to obtain equation (29) may be seen if a comparison is made with the example given in figure 6. Thus, if $N_x = 0$, equation (29) can be rewritten in the following form (for any planform ratio and any two modes):

$$\frac{\lambda_{cr}}{\lambda_{cr}(\infty)} = \frac{1}{1 - \frac{\bar{A}\bar{C}_x}{2}} \tag{30}$$

The dashed curves drawn in figure 6(a) are the flutter curves specified by equation (30). These values are also presented in table II. The approximation is extremely good in all regions and indicates that for this case, expression (29) is accurate. It must be mentioned, however, that these conclusions are all based upon a two-mode Galerkin solution and it is known (ref. 1) that the Galerkin two-mode solution is conservative when compared with the exact solution, particularly for large negative $\bar{A}$ values.

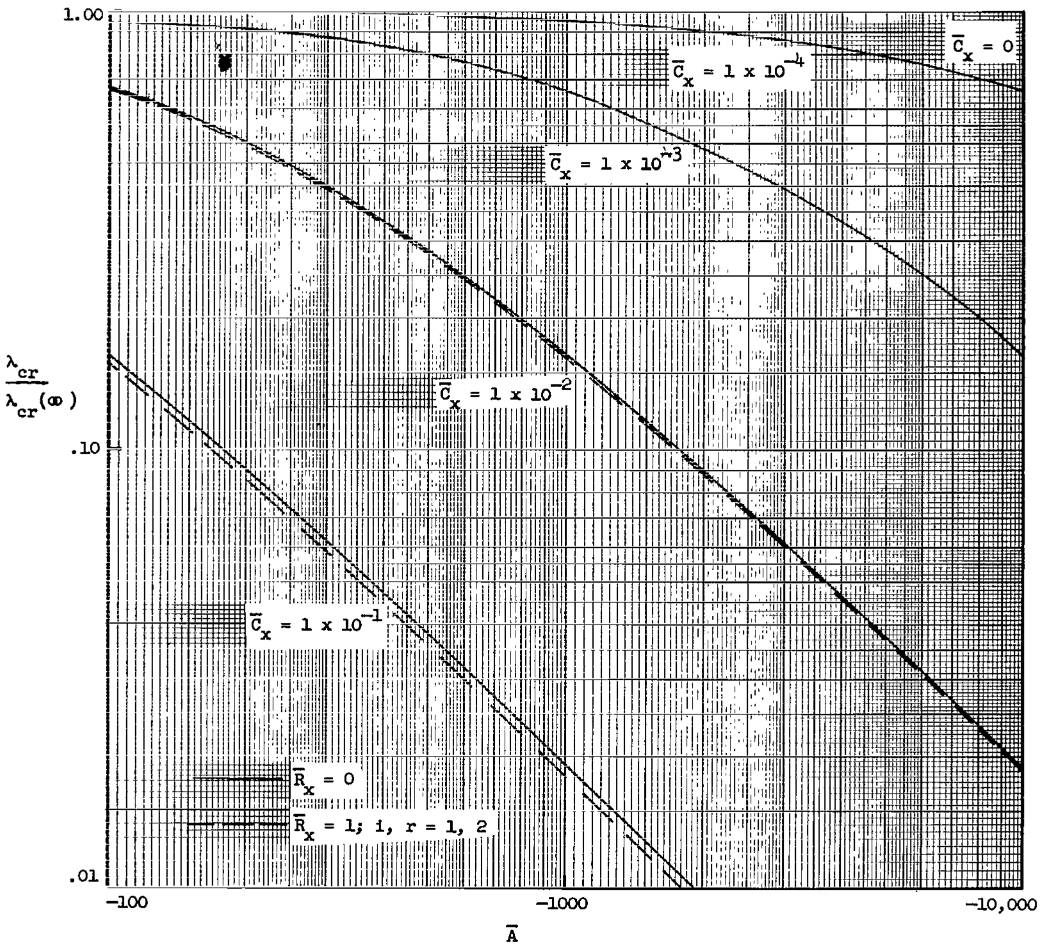


Figure 7.- The combined effect of D_{Q_X} and N_X on the flutter of a simply supported orthotropic panel (two-mode Galerkin solution, $\bar{C}_y = 0$) for relatively small values of D_x.

CONCLUDING REMARKS

Some important conclusions can be made on the effects of both side-edge boundary conditions and transverse shearing stiffnesses on panel flutter phenomena. Since these two effects were investigated independently, separate conclusions are presented. However, it must be emphasized that comparisons are made on the basis of two-term Galerkin solutions; exact solutions may not yield the same quantitative results, although the qualitative results are expected to remain the same.

Side-Edge Boundary Conditions

The influence of side-edge boundary conditions can be very significant. Changing boundary conditions from simply supported to clamped causes a relatively small increase in the critical flutter parameter λ_{cr}, but changing these boundary conditions to free edges causes a significant decrease in λ_{cr}. This decrease is primarily caused (for the examples analyzed) by the fact that the panels were not forced to bend between their side edges (their stiffest direction). Thus, if free boundary conditions exist along the side edges of a panel, flutter of that panel may occur at much lower dynamic pressures than expected for simply supported panels.

Transverse Shearing Stiffnesses

The two stiffnesses $\left(D_{Q_x} \text{ and } D_{Q_y}\right)$ can cause extremely large variation in the critical flutter parameter λ_{cr}, some reductions being as large as two orders of magnitude. These conclusions are based upon flutter calculations for an example series of panels that closely approximate typical corrugation-stiffened panels currently in use, and for such panels (with relatively small bending stiffness in the flow direction), it was noted that if there is a finite transverse shearing stiffness in the flow direction $\left(D_{Q_x}\right)$, the effect of D_{Q_y} is small. Therefore, a simple expression was derived for infinite D_{Q_y} that showed the influence of D_{Q_x}, in-plane load, and the mode numbers on the critical flutter parameter. This expression is extremely easy to use and agrees well with the more exact expression. Therefore, an estimate of the flutter parameter λ_{cr} may be easily calculated.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Station, Hampton, Va., September 17, 1965.

APPENDIX

GENERAL GALERKIN SOLUTION FOR THE FLUTTER OF ORTHOTROPIC PANELS IN SUPERSONIC FLOW USING STATIC STRIP THEORY

The orthotropic panels under consideration here are shown in figure 1, with arbitrary boundary conditions on all four edges. The differential equation for the lateral deflection w, both static strip theory and simple harmonic motion being assumed, is (eq. (1))

$$\frac{\partial^4 w}{\partial x^4} + 2\left(\frac{D_{xy}}{D_x} + \mu_y\right)\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{D_y}{D_x}\frac{\partial^4 w}{\partial y^4} + \frac{\pi^2 \bar{R}_x}{a^2}\frac{\partial^2 w}{\partial x^2} + \frac{\pi^2 \bar{R}_y}{a^2}\frac{\partial^2 w}{\partial y^2} + \frac{\lambda}{a^3}\frac{\partial w}{\partial x} - \frac{\bar{k}^2 \pi^4}{a^4} w = 0 \tag{A1}$$

where

$$\bar{R}_x = \frac{N_x a^2}{\pi^2 D_x}$$

$$\bar{R}_y = \frac{N_y a^2}{\pi^2 D_x}$$

$$\lambda = \frac{2qa^3}{\beta D_x}$$

$$\bar{k}^2 = \frac{m\omega^2 a^4}{\pi^4 D_x}$$

For the Galerkin solution, the deflection w was expanded in the series

$$w = \sum_n \sum_p a_{pn}\, G_{np}(x)\, F_{pn}(y) \tag{A2}$$

where each term $a_{pn}\, G_{np}(x)\, F_{pn}(y)$ must be chosen such that all the boundary conditions are satisfied. Substituting this function for w into equation (A1), multiplying by $F_{qm}(y)$, and integrating across the panel yields (for each value of q)

$$\sum_n \sum_p \left[bI_{qp}^{(0)}(m,n)\ G_{np}^{IV}(x) + \frac{2}{b}\left(\frac{D_{xy}}{D_x} + \mu_y\right) I_{qp}^{(2)}(m,n)\ G_{np}''(x) + \frac{D_y}{D_x}\frac{G_{np}(x)}{b^3}\ I_{qp}^{(4)}(m,n)\right.$$

$$+ \frac{b\bar{R}_x\pi^2}{a^2}\ I_{qp}^{(0)}(m,n)\ G_{np}''(x) + \frac{\bar{R}_y\pi^2}{ba^2}\ G_{np}(x)\ I_{qp}^{(2)}(m,n) - \frac{\bar{k}^2\pi^4 b}{a^4}\ I_{qp}^{(0)}(m,n)\ G_{np}(x)$$

$$\left. + \frac{b\lambda}{a^3}\ I_{qp}^{(0)}(m,n)\ G_{np}'(x)\right] a_{pn} = 0 \qquad \text{(A3)}$$

for q equal to all values taken by p and where primes denote differentiation with respect to x and

$$I_{qp}^{(r)}(m,n) = b^{r-1}\int_0^b F_{qm}(y)\ \frac{d^r}{dy^r}\ F_{pn}(y)\ dy$$

Multiplying equation (A3) by $G_{mq}(x)$ and integrating along the panel yields (for each combination of q and m)

$$\sum_p \sum_n \left[J_{mn}^{(4)}(q,p)\ I_{qp}^{(0)}(m,n) + \frac{2a^2}{b^2}\left(\frac{D_{xy}}{D_x} + \mu_y\right) J_{mn}^{(2)}(q,p)\ I_{qp}^{(2)}(m,n)\right.$$

$$+ \frac{D_y}{D_x}\frac{a^4}{b^4}\ J_{mn}^{(0)}(q,p)\ I_{qp}^{(4)}(m,n) + \bar{R}_x\pi^2 J_{mn}^{(2)}(q,p)\ I_{qp}^{(0)}(m,n)$$

$$+ \bar{R}_y\ \frac{\pi^2 a^2}{b^2}\ J_{mn}^{(0)}(q,p)\ I_{qp}^{(2)}(m,n) + \lambda J_{mn}^{(1)}(q,p)\ I_{qp}^{(0)}(m,n)$$

$$\left. - \bar{k}^2\pi^4 J_{mn}^{(0)}(q,p)\ I_{qp}^{(0)}(m,n)\right] a_{pn} = 0 \qquad \text{(A4)}$$

for q equal to all values taken by p and for m equal to all values taken by n and where

$$J_{mn}^{(r)}(q,p) = a^{r-1} \int_0^a G_{mq}(x) \frac{d^r}{dx^r} G_{np}(x)\, dx$$

For any given boundary conditions on all four edges of the panel, functions are selected that satisfy all the boundary conditions. These functions are then used to calculate the integral expressions $I_{qp}^{(r)}(m,n)$ and $J_{mn}^{(r)}(q,p)$ and the determinant of equation (A4) yields a relationship to solve for the flutter parameter λ. When generalized flutter parameters are introduced, equation (A4) becomes

$$\begin{aligned}
&\sum_p a_{pm}\left[J_{mm}^{(4)}(q,p) \frac{I_{qp}^{(0)}(m,m)}{I_{qq}^{(0)}(m,m)} + J_{mm}^{(2)}(q,p)\, \pi^2 \bar{A}_{qp}^{(m,m)} - J_{mm}^{(0)}(q,p)\, \pi^4 \bar{B}_{qp}^{(m,m)}\right. \\
&\left. + \lambda J_{mm}^{(1)}(q,p) \frac{I_{qp}^{(0)}(m,m)}{I_{qq}^{(0)}(m,m)}\right] + \sum_p \sum_{\substack{n \\ n\neq m}} a_{pn}\left[J_{mn}^{(4)}(q,p) \frac{I_{qp}^{(0)}(m,n)}{I_{qq}^{(0)}(m,m)}\right. \\
&\left. + J_{mn}^{(2)}(q,p)\, \pi^2 \bar{A}_{qp}^{(m,n)} - J_{mn}^{(0)}(q,p)\, \pi^4 \bar{B}_{qp}^{(m,n)} + \lambda J_{mn}^{(1)}(q,p) \frac{I_{qp}^{(0)}(m,n)}{I_{qq}^{(0)}(m,m)}\right] = 0 \qquad \text{(A5)}
\end{aligned}$$

for q equal to all values taken by p and for m equal to all values taken by n and where

$$\bar{A}_{qp}^{(m,n)} = \bar{R}_x \frac{I_{qp}^{(0)}(m,n)}{I_{qq}^{(0)}(m,m)} + \frac{2}{\pi^2} \frac{a^2}{b^2} \left(\frac{D_{xy}}{D_x} + \mu_y\right) \frac{I_{qp}^{(2)}(m,n)}{I_{qq}^{(0)}(m,m)}$$

and

$$\bar{B}_{qp}^{(m,n)} = \bar{k}^2 \frac{I_{qp}^{(0)}(m,n)}{I_{qq}^{(0)}(m,m)} - \bar{R}_y \frac{1}{\pi^2} \frac{a^2}{b^2} \frac{I_{qp}^{(2)}(m,n)}{I_{qq}^{(0)}(m,m)} - \frac{D_y}{D_x} \frac{1}{\pi^4} \frac{a^4}{b^4} \frac{I_{qp}^{(4)}(m,n)}{I_{qq}^{(0)}(m,m)}$$

If only one term is considered in the y-direction (as is often done), these equations become (for each value of m)

$$a_{qm}\left[J_{mm}^{(4)}(q,q) + J_{mm}^{(2)}(q,q)\ \pi^2\bar{A}_{qq}^{(m,m)} - J_{mm}^{(0)}(q,q)\ \bar{B}_{qq}^{(m,m)} + \lambda J_{mm}^{(1)}(q,q)\right]$$

$$+ \sum_{\substack{n \\ n\neq m}} a_{qn}\left[J_{mn}^{(4)}(q,q)\ \frac{I_{qq}^{(0)}(m,n)}{I_{qq}^{(0)}(m,m)} + J_{mn}^{(2)}(q,q)\ \pi^2\bar{A}_{qq}^{(m,n)} - J_{mn}^{(0)}(q,q)\ \pi^4\bar{B}_{qq}^{(m,n)}\right.$$

$$\left. + \lambda J_{mn}^{(1)}(q,q)\ \frac{I_{qq}^{(0)}(m,n)}{I_{qq}^{(0)}(m,m)}\right] = 0 \tag{A6}$$

for m equal to all values taken by n.

As an example of the use of equation (A6), the flutter parameter for a two-mode Galerkin solution can be written directly as a solution of the algebraic equation

$$\lambda^2\left[J_{mm}^{(1)}(q,q)\ J_{nn}^{(1)}(q,q) - J_{mn}^{(1)}(q,q)\ J_{nm}^{(1)}(q,q)\ \frac{I_{qq}^{(0)}(m,n)\ I_{qq}^{(0)}(n,m)}{I_{qq}^{(0)}(m,m)\ I_{qq}^{(0)}(n,n)}\right]$$

$$+ \lambda\left\{J_{mm}^{(1)}(q,q)\left[J_{nn}^{(4)}(q,q) + J_{nn}^{(2)}(q,q)\ \bar{A}_{qq}^{(n,n)} - J_{nn}^{(0)}(q,q)\ \bar{B}_{qq}^{(n,n)}\right]\right.$$

$$+ J_{nn}^{(1)}(q,q)\left[J_{mm}^{(4)}(q,q) + J_{mm}^{(2)}(q,q)\ \bar{A}_{qq}^{(m,m)} - J_{mm}^{(0)}(q,q)\ \bar{B}_{qq}^{(m,m)}\right]$$

$$- J_{nm}^{(1)}(q,q)\left[J_{mn}^{(4)}(q,q) + J_{mn}^{(2)}(q,q)\ \bar{A}_{qq}^{(m,n)} - J_{mn}^{(0)}(q,q)\ \bar{B}_{qq}^{(m,n)}\right]\frac{I_{qq}^{(0)}(m,n)\ I_{qq}^{(0)}(n,m)}{I_{qq}^{(0)}(m,m)\ I_{qq}^{(0)}(n,n)}$$

$$\left. - J_{mn}^{(1)}(q,q)\left[J_{nm}^{(4)}(q,q) + J_{nm}^{(2)}(q,q)\ \bar{A}_{qq}^{(n,m)} - J_{nm}^{(0)}(q,q)\ \bar{B}_{qq}^{(n,m)}\right]\frac{I_{qq}^{(0)}(n,m)\ I_{qq}^{(0)}(m,n)}{I_{qq}^{(0)}(n,n)\ I_{qq}^{(0)}(m,m)}\right\}$$

(Equation continued on next page)

$$+ \left\{ \left[J_{mm}^{(4)}(q,q) + J_{mm}^{(2)}(q,q)\ \bar{A}_{qq}^{(m,m)} - J_{mm}^{(0)}(q,q)\ \bar{B}_{qq}^{(m,m)} \right] \left[J_{nn}^{(4)}(q,q) \right. \right.$$

$$+ J_{nn}^{(2)}(q,q)\ \bar{A}_{qq}^{(n,n)} - J_{nn}^{(0)}(q,q)\ \bar{B}_{qq}^{(n,n)} \Big] - \Big[J_{mn}^{(4)}(q,q) + J_{mn}^{(2)}(q,q)\ \bar{A}_{qq}^{(m,n)}$$

$$- J_{mn}^{(0)}(q,q)\ \bar{B}_{qq}^{(m,n)} \Big] \Big[J_{nm}^{(4)}(q,q) + J_{nm}^{(2)}(q,q)\ \bar{A}_{qq}^{(n,m)}$$

$$\left. - J_{nm}^{(0)}(q,q)\ \bar{B}_{qq}^{(n,m)} \Big] \frac{I_{qq}^{(0)}(n,m)\ I_{qq}^{(0)}(m,n)}{I_{qq}^{(0)}(n,n)\ I_{qq}^{(0)}(m,m)} \right\} = 0 \qquad \text{(A7)}$$

These solutions are utilized in the text of this paper where applicable.

REFERENCES

1. Hedgepeth, John M.: Flutter of Rectangular Simply Supported Panels at High Supersonic Speeds. J. Aeron. Sci., vol. 24, no. 8, Aug. 1957, pp. 563-573, 586.

2. Fralich, Robert W.: Postbuckling Effects on the Flutter of Simply Supported Rectangular Panels at Supersonic Speeds. NASA TN D-1615, 1963.

3. Eisley, J. G.: The Flutter of Simply Supported Rectangular Plates in a Supersonic Flow. OSR-TN-55-236, GALCIT, July 1955.

4. Luke, Yudell L.; and St. John, Andrew D.: Supersonic Panel Flutter. WADC TR 57-252, U.S. Air Force, July 1957.

5. Voss, H. M.: The Effect of an External Supersonic Flow on the Vibration Characteristics of Thin Cylindrical Shells. Jour. Aerospace Sci., vol. 28, no. 12, Dec. 1961, pp. 945-956, 961.

6. McElman, John A.: Flutter of Curved and Flat Sandwich Panels Subjected to Supersonic Flow. NASA TN D-2192, 1964.

7. Cunningham, H. J.: Flutter Analysis of Flat Rectangular Panels Based on Three-Dimensional Supersonic Potential Flow. AIAA J., vol. 1, no. 8, Aug. 1963, pp. 1795-1801.

8. Bohon, Herman L.; and Dixon, Sidney C.: Some Recent Developments in Flutter of Flat Panels. J. Aircraft, vol. 1, no. 5, Sept.-Oct. 1964, pp. 280-288.

9. Weidman, Deene J.: Experimental Flutter Results for Corrugation-Stiffened and Unstiffened Panels. NASA TN D-3301, 1966.

10. Iguchi, S.: Eine Lösung für die Berechnung der biegsamen rechteckigen Platten. Julius Springer (Berlin), 1933.

11. Wittrick, W. H.: Buckling of Oblique Plates With Clamped Edges Under Uniform Compression. Aeron. Quart., vol. IV, pt. II, Feb. 1953, pp. 151-162.

12. Libove, Charles; and Batdorf, S. B.: A General Small-Deflection Theory for Flat Sandwich Plates. NACA Rept. 899, 1948.

13. Libove, Charles; and Hubka, Ralph E.: Elastic Constants for Corrugated-Core Sandwich Plates. NACA TN 2289, 1951.

NASA-Langley, 1966 L-3848

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