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by Robert L. Wright, John F. Newcomb, James L. Dillon, and Julie M. Willman

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SUMMARY

An analytical method has been developed for determining the flotation stability characteristics of a right-circular-conic frustum with the small face submerged. General nondimensional equations for the center-of-buoyancy location, metacentric height, righting moment arm, and the special case of the rearmost location of the center of gravity were derived in which the geometry, the mass, and the angle of inclination of the floating body with respect to the water line are the only variables. The equations are limited by the inclination angle for which any portion of the base becomes submerged. Numerical results were computed to provide the designer a convenient method for determining the desired flotation stability parameters. Several examples are presented as guidelines in the use of the method.

INTRODUCTION

Recovery of research payloads is playing an increasing role in free-flight rocket investigations. Certain biological and materials experiments and data capsules containing photographic or tape-recorder records require laboratory examination of the various components for more detailed analysis of the results; hence, recovery of the research package is necessary. After the recoverable package has landed safely on the water, location of the package becomes the principal problem of the recovery operation. It would be of little value to have a safe landing in the water, and then have the recoverable package oriented so that any location aids (flashing lights, radar reflecting surfaces, and radio antennas) are submerged. Therefore, knowledge of the flotation stability characteristics of the floating body is essential to effective employment of the location aids.

An analytical method for determining the flotation stability characteristics of a right-circular-conic frustum with the small face submerged has been developed. In the analysis, the frustum is always alined so that the small face (nose) is immersed in the fluid and the inclination (plane of the application of the force or moment) is in the positive direction. The equations of the analysis are limited for this configuration by the inclination angle for which any portion of the large face (base) becomes submerged. General nondimensional equations for the center-of-buoyancy location, metacentric height, and righting moment arm as functions of the body geometry and mass, and inclination angle with respect to the water line have been derived. The paper is divided into two parts, analysis and application. The analysis section serves to introduce a general development of the method and presents the governing equations. The appendix gives the complete development of the governing equations. Because of the explicit nature of the flotation stability equations of the method, it is not necessary to study their development to use the analysis. In the section on application, several examples are presented as guidelines in the use of the method. Numerical results were obtained from the governing equations by using the IBM 7094 electronic data processing system and are presented in tabular and graphic form for the following ranges of the variable parameters: frustum half-angles of 6° to 16° in 2° increments, inclination angles of 40° to 90° in 5° increments, and nondimensional volumes of 1 to 6 in 0.5 increments.

SYMBOLS

$A = 1 - \dfrac{\tan^2\theta}{\tan^2\phi}$

$B = \sin^2\phi \cos^2\theta - \cos^2\phi \sin^2\theta$ or $A \sin^2\phi \cos^2\theta$

$C = \dfrac{24}{\pi} \tan\theta \dfrac{V_f}{D^3} + 1$

D — diameter of submerged flat face of frustum

h_m — metacentric height (distance along center line from center of gravity to intersection of center line and a vertical line from center of buoyancy)

l — length of conic frustum, measured along center line

l_w — length of conic frustum to water line from submerged flat face of frustum, measured along center line

l_{tf} — length from tip to submerged flat face of frustum, measured along center line

l_{tw} — length from cone tip to water line, measured along center line

l_{wb} — length from water line to base of cone, measured along center line

x displacement in lateral direction

x_{cB} lateral displacement of center of buoyancy of frustum (perpendicular to center line)

x_M' righting moment arm (horizontal distance between center of gravity and center of buoyancy)

y displacement in longitudinal direction

y_{fB} longitudinal displacement of center of buoyancy of frustum (along center line)

y_{fcg} longitudinal displacement of center of gravity of frustum (along center line)

V submerged volume

γ angle between center line of right circular cone and line from apex to center of base of oblique cone

θ frustum half-angle

ϕ angle of inclination, measured from water line

Subscripts:

B buoyancy center

b base of cone

c cone

cg center of gravity

f flat-faced conic frustrum

M moment

m metacentric

lim limiting

s slant

t conical tip

w water line

ANALYSIS

After water impact of a recoverable payload, the motions of the buoyant body will be influenced by the state of the sea and by the inertia and stability characteristics of the body. Should the state of the sea be calm, the body will probably bob in the water with little or no rolling or tilting motions. However, under more severe sea conditions, wave action and body inertia will produce more violent tilting motions of the body. Since the exact state of the sea cannot be accurately analyzed, design considerations require a determination of the effects of varying tilt conditions on the flotation stability or righting capability of the body. In the present analysis, the body inertia is neglected, since this inertia does not affect the location of the center of buoyancy or metacentric height.

Figure 1 presents the general dimensions of the conic frustum of the analysis. The analysis requires that the flotation stability parameters be functions only of the body mass, body geometry, and inclination angle. The body geometry (body length, nose diameter, and cone half-angle) is fixed for a given body. The inclination angle may vary to some limiting angle (the angle for which any portion of the base becomes submerged). This angle is a function of the body geometry (freeboard of the body in the vertical position). The buoyant force of a floating body is equal to the weight of a volume of fluid which is equal to the volume of the submerged portion of the body (Archimedes' principle). The volume of the submerged portion of a frustum of a cone can be found by

$$V_f = V_c - V_t \tag{1}$$

By proper substitution of geometric relations into equation (1), as developed in the appendix, it is found that

$$\frac{V_f}{D^3} = \frac{\pi}{24} \cot \theta \left[\left(1 + \frac{2l_w}{D} \tan \theta\right)^3 \frac{1}{A^{3/2}} - 1\right] \tag{2}$$

Equation (2) can be rearranged to produce the nondimensionalized submerged length of the conic frustum measured along the center line.

$$\frac{l_w}{D} = \frac{1}{2 \tan \theta}\left[\left(\frac{24}{\pi} \tan \theta \frac{V_f}{D^3} + 1\right)^{1/3} A^{1/2} - 1\right] \tag{3}$$

Nondimensionalizing by the nose diameter D gives the submerged length as a function of θ, ϕ, and V_f/D^3. Numerical results for this parameter are given in table 1 for conic frustums for various θ, ϕ, and V_f/D^3. Figure 2 presents curves of the tabulated l_w/D data over a range of V_f/D^3 and ϕ for a typical conic frustum of 10° half-angle.

Actually, V_f could be computed more easily, since the submerged volume is equal to the mass of the body divided by the density of water. However, it was necessary to derive the l_w/D parameter for later use.

The longitudinal displacement of the center of buoyancy can be found by summing moments about the tip of the cone parallel to the center line of the right circular cone.

$$V_c y_c = V_f y_f + V_t y_t \tag{4}$$

Again, geometric substitution produces the nondimensional longitudinal displacement of the center of buoyancy of the frustum,

$$\frac{y_{fB}}{D} = \frac{3}{8} \cot \theta \left(\frac{C^{4/3} A^{-1/2} - 1}{C - 1} \right) - \frac{\cot \theta}{2} \tag{5}$$

Likewise, by summing moments perpendicular to the center line of the right circular cone, the lateral displacement of the center of buoyancy of the frustum is found to be

$$\frac{x_{cB}}{D} = \frac{3}{8} \frac{\tan \theta}{\tan \phi} \left(\frac{C^{4/3} A^{-1/2}}{C - 1} \right) \tag{6}$$

Numerical results from equations (5) and (6) are presented in tables 2 and 3, respectively. Figure 3 presents the data from table 2 for a range of V_f/D^3 and ϕ for a typical conic frustum of 10° half-angle. Figure 4 shows curves of the data from table 3 over the same range.

As indicated earlier, the inclination angle will vary to some limiting angle (angle for which any portion of the base becomes submerged) for which the equations of the present analysis are applicable. By setting up the relationship between the submerged center-line length of a cone and the slant height of a cone, the limiting inclination angle is obtained.

$$\phi_{lim} = \arc \tan \left[\frac{1 + 2\frac{l}{D} \tan \theta}{2\left(\frac{l}{D} - \frac{l_w}{D} \right)} \right] \tag{7}$$

The nondimensionalized submerged length of the conic frustum l_w/D defined earlier is noted to be a function of the inclination angle; therefore, an iteration is required to solve the equation for the limiting inclination angle for which the analysis is applicable. Only two iterations are required in most cases to provide sufficiently accurate results.

Thus, with the frustum half-angle and the displaced volume (body mass), values of the nondimensionalized submerged length along the center line l_w/D can be obtained from table 1 for the conic frustum in the vertical position ($\phi = 90^o$). Substitution of this value of l_w/D into equation (7) will produce an inclination angle ϕ. By reentering table 1, a new value of l_w/D is obtained and substituted into equation (7). This iteration process is continued until the difference between successive inclination angles ϕ is sufficiently small. The resultant inclination angle is the limiting inclination angle ϕ_{lim} for which the equations of the analysis are applicable. Tables 2 and 3 are entered at the computed value of ϕ_{lim} for the frustum half-angle and displaced volume, and the values of the lateral and longitudinal displacements of the center of buoyancy are determined.

Translating the axis and rotating through an angle $-(90 - \phi)$ produces the coordinate system developed in the appendix (designated by double primes) from which the righting moment arm is determined.

$$x'_M = x_{cB} \sin \phi + \left(y_{fB} - y_{fcg}\right)\cos \phi \tag{8}$$

The metacentric height is defined as the distance along the center line from the center of gravity to the intersection of the center line and a vertical line from the center of buoyancy.

$$h_m = x_{cB} \tan \phi + \left(y_{fB} - y_{fcg}\right) \tag{9}$$

The flotation stability parameters x'_M and h_m are shown in figure 5. Positive values of x'_M and h_m indicate that the conic-frustum configuration is stable, and the magnitude of these stability parameters provides the degree of stability.

For the designer, the rearmost location of the center of gravity without loss of flotation stability is of importance. Movement of the center of gravity rearward will provide the designer with more latitude in the location of internal instrumentation and will reduce the stringent requirement for small or compact instrumentation. The rearmost location of the center of gravity for the conic frustum inclined at any angle ϕ (up to and including ϕ_{lim}) is found by setting the metacentric height in equation (9) equal to zero and solving for y_{cg}.

$$y_{fcg} = x_{cB} \tan \phi + y_{fB} \tag{10}$$

APPLICATION OF THE METHOD

The following examples were calculated to illustrate the application of this analysis in determining the flotation stability characteristics of the conic-frustum configuration.

In all the examples, a frustum half-angle of 10° was used. Figures 2, 3, and 4 present the data given in tables 1, 2, and 3, respectively, for a 10° half-angle conic frustum for use with the examples. Figure 2 is used to determine the desired values of l_w/D for obtaining ϕ_{lim} from equation (7). Figures 3 and 4 provide the longitudinal and lateral displacements of the center of buoyancy.

The units used for the physical quantities are given both in the U.S. Customary Units and in the International System of Units (SI). Conversion factors are presented in reference 1.

Example 1

A recoverable body consists of a 10° half-angle conic frustum of 37.8 pounds (17.15 kg). The length of the spacecraft is 14 inches (35.6 cm) and the nose diameter is 8 inches (20.3 cm). The center of gravity y_{fcg} is located 4 inches (10.2 cm) aft of the flat-faced nose. Determine the limiting inclination angle within 20 minutes (for which the equations are applicable) and the magnitude of the stability parameters x_M' and h_m.

The volume displaced is the mass m divided by the density ρ of the fluid in which the body is submerged (64.0 lbm/ft^3 (1025.2 kg/m^3) for sea water):

$$V = \frac{m}{\rho} = \frac{37.8}{64.0} = 0.59 \text{ ft}^3 \left(0.017 \text{ m}^3\right)$$

Thus, $V = 1024 \text{ in}^3 \left(16\ 780 \text{ cm}^3\right)$. Nondimensionalizing by the diameter of the submerged flat face yields

$$\frac{V}{D^3} = \frac{1024}{8^3} = 2.0$$

From figure 2, for $\phi = 90^{\circ}$

$$\frac{l_w}{D} = 1.55$$

The limiting inclination angle for which the equations of the analysis are applicable can be determined by using equation (7)

$$\phi_{lim} = \arc\tan\left\{\frac{1 + 2\frac{l}{D}\tan\theta}{2\left[\frac{l}{D} - \left(\frac{l_w}{D}\right)_{\phi=90^\circ}\right]}\right\}$$

from which $\phi = 82^\circ\ 57'$. From figure 2 for the first iteration, $\frac{l_w}{D} = 1.547$ for $\phi = 82^\circ\ 57'$. Substituting into equation (7) yields

$$\phi_{lim} = \arc\tan\left\{\frac{1 + 2\frac{l}{D}\tan\theta}{2\left[\frac{l}{D} - \left(\frac{l_w}{D}\right)_{\phi=82^\circ\ 57'}\right]}\right\}$$

from which ϕ is determined to be $82^\circ\ 49'$. The difference between the calculated solution and the assumed solution is within the accuracy required; therefore, $82^\circ\ 49'$ is the limiting inclination angle for which the flotation characteristics of this configuration can be obtained from this analysis.

To determine the magnitude of the stability parameters, equations (8) and (9) are used

$$x'_M = x_{cB}\sin\phi + \left(y_{fB} - y_{fcg}\right)\cos\phi$$

$$h_m = x_{cB}\tan\phi + \left(y_{fB} - y_{fcg}\right)$$

From figures 3 and 4, $\frac{y_{fB}}{D} = 0.885$ and $\frac{x_{cB}}{D} = 0.018$, respectively, for $\phi = 82^\circ\ 49'$; $\frac{y_{fcg}}{D} = 0.50$. Therefore

$$x'_M = 0.018D\sin\phi + (0.885 - 0.50)D\cos\phi = 0.527\text{ inch (1.338 cm)}$$

$$h_m = 0.018D\tan\phi + (0.885 - 0.50)D = 4.22\text{ inches (10.72 cm)}$$

Since x'_M and $h_m > 0$, a positive righting moment of 19.9 inch-pounds (2.25 J) exists and the configuration is stable to the limits of the analysis.

Example 2

Example 2(a).- A recoverable body has a mass of 111 pounds (50.3 kg). The flat-faced nose is 10 inches (25.4 cm) in diameter and the 10° half-angle conic frustum is

30 inches (76.2 cm) long. Find the most rearward location of the center of gravity for which the body is stable (at the limits of the analysis).

The submerged volume is found to be 3000 in^3 $\left(49\ 161\ cm^3\right)$ and from figure 2 $l_w/D = 2.026$ for $\phi = 90^o$.

From equation (7),

$$\phi_{lim} = \text{arc tan}\left\{\frac{1 + 2\frac{l}{D}\tan\theta}{2\left[\frac{l}{D} - \left(\frac{l_w}{D}\right)_{\phi=90^o}\right]}\right\}$$

the limiting inclination angle ϕ is determined to be $46^o\ 35'$. For the first iteration, $\phi = 44^o\ 33'$. A second iteration produces $\phi = 44^o\ 17'$. The difference between the assumed and calculated ϕ is now sufficiently small; therefore, ϕ_{lim} is taken to be $44^o\ 17'$. From figures 3 and 4, $\frac{y_{fB}}{D} = 1.265$ and $\frac{x_{cB}}{D} = 0.1467$.

The most rearward location of the center of gravity is determined from equation (10)

$$y_{fcg} = x_{cB} \tan\phi + y_{fB}$$

$$y_{fcg} = 0.1467D \tan\phi + 1.265D = 14.08 \text{ inches } (35.76 \text{ cm})$$

Thus, for an inclination angle of $44^o\ 17'$ and a center of gravity located 14.08 inches (35.76 cm) from the flat face, the body is neutrally stable at the limits of the analysis.

Example 2(b).- The conditions are the same as for problem 2(a) except that y_{fcg} is 10 inches (25.4 cm). Determine the righting moment and plot the location of the center of buoyancy for several inclination angles including ϕ_{lim}.

The righting moment arm is found from equation (8):

$$x'_M = x_{cB} \sin\phi + \left(y_{fB} - y_{fcg}\right)\cos\phi$$

$$x'_M = 0.1467D \sin\phi + (1.265 - 1.0)D \cos\phi = 2.908 \text{ inches } (7.39 \text{ cm})$$

For the 111-pound (50.3-kg) body, the righting moment is 322.8 inch-pounds (36.48 J). The location of the center of buoyancy for this configuration is shown in figure 6 for several inclination angles including ϕ_{lim}.

CONCLUDING REMARKS

An analytical method has been developed for determining the flotation stability characteristics of a right-circular-conic frustum with the small face submerged in the fluid. General nondimensional equations for the center-of-buoyancy location, metacentric height, righting moment arm, and the special case of the rearmost location of the center of gravity have been derived. These equations are limited by the inclination angle for which any portion of the base becomes submerged. Numerical results have been tabulated for the following ranges of the variable parameters: frustum half-angles of 6° to 16° in 2° increments, inclination angles of 40° to 90° in 5° increments, and nondimensional volumes of 1 to 6 in 0.5 increments. Example problems are presented to serve as guidelines in the use of the method.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Station, Hampton, Va., September 14, 1965.

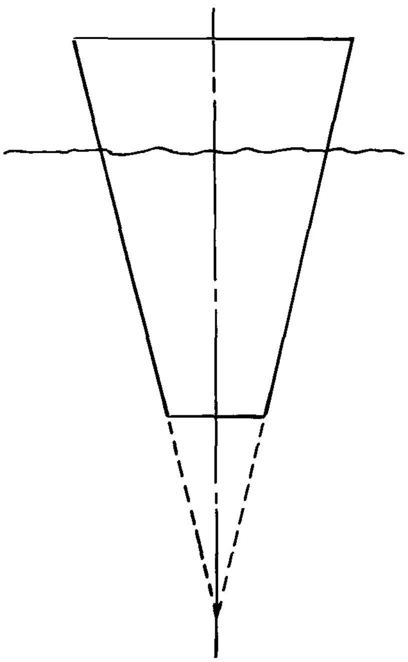
APPENDIX

DEVELOPMENT OF GOVERNING EQUATIONS

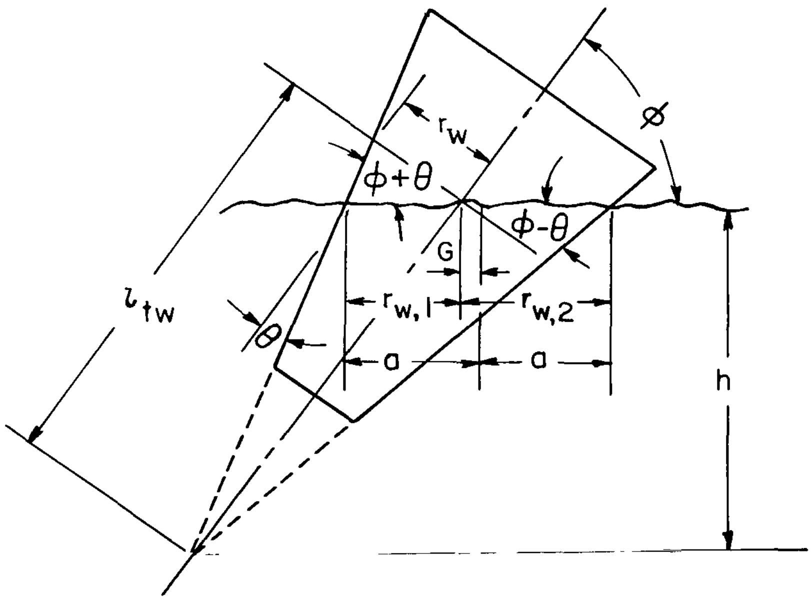
In developing the method for determining the flotation stability characteristics of a right-circular-conic frustum with the small face submerged, the general configuration of sketch (1a) was considered. By letting the cone be inclined at some angle ϕ less than 90° (measured from the water line to the geometric center line of the body), the plane of the water line cutting the inclined cone produces an ellipse whose semimajor axis a is

$$a = \frac{r_{w,1} + r_{w,2}}{2} \tag{A1}$$

Here, $r_{w,1}$ and $r_{w,2}$ are the distances at the water line along the major axis of the ellipse from the cone surface to the intersection of the water line and the geometric center line, as shown in sketch (1b). The radius of the conic frustum at the intersection of the water line with the center line is defined as r_w.



Sketch (1a)



Sketch (1b)

$$r_{w,1} = \frac{r_w \cos\theta}{\sin(\phi + \theta)} \tag{A2a}$$

$$r_{w,2} = \frac{r_w \cos\theta}{\sin(\phi - \theta)} \tag{A2b}$$

Since $r_w = l_{tw} \tan\theta$,

$$a = l_{tw}\left(\frac{\sin\phi \sin\theta \cos\theta}{\sin^2\phi \cos^2\theta - \cos^2\phi \sin^2\theta}\right) \tag{A3}$$

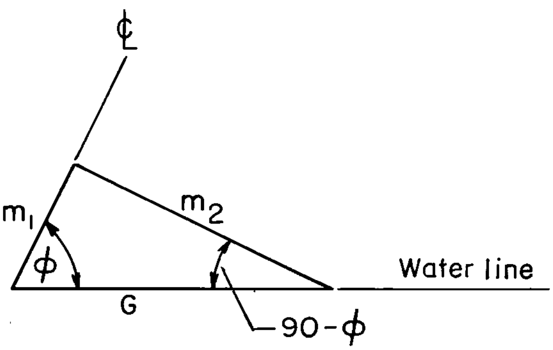
Let G be the distance $a - r_{w,1}$ along the major axis between the center line and the minor axis; then

$$G = l_{tw} \sin\theta\left(\frac{\cos\phi \sin\theta}{B}\right) \tag{A4}$$

where

$$B = \sin^2\phi \cos^2\theta - \cos^2\phi \sin^2\theta$$

From the right triangle between the geometric center line and the minor axis in sketch (2), the relations $m_1 = G\cos\phi$ and $m_2 = G\sin\phi$ are obtained.



Sketch (2)

Then, the use of equation (A4) yields

$$m_1 = l_{tw}\left(\frac{\cos^2\phi \sin^2\theta}{B}\right) \tag{A5}$$

$$m_2 = l_{tw} \tan\theta\left(\frac{\cos\phi \sin\phi \sin\theta \cos\theta}{B}\right) \tag{A6}$$

Define the distance from the apex of the cone to the plane of the minor axis as l_{tb}, such that

$$l_{tb} = l_{tw} + m_1$$

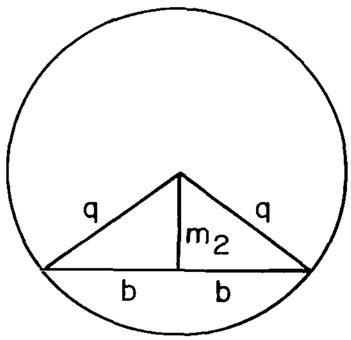
At this plane, the radius of the cone is

$$q = \left(l_{tw} + m_1\right)\tan\theta$$

Substituting for m_1 from equation (A5) yields

$$q = l_{tw}\tan\theta\left(\frac{\sin^2\phi\cos^2\theta}{B}\right) \tag{A7}$$

From the plane of the circle formed by cutting the cone perpendicular to the center line at distance l_{tb} (sketch (3)) $b = \left(q^2 - m_2{}^2\right)^{1/2}$



Sketch (3)

Substituting for q and m_2 yields the semiminor axis

$$b = l_{tw}\left(\frac{1}{B}\right)^{1/2}\sin\phi\sin\theta \tag{A8}$$

The volume of the submerged portion of the cone can now be determined from

$$V_c = \pi\frac{abh}{3}$$

where

$$a = \frac{l_{tw}\sin\phi\sin\theta\cos\theta}{B}$$

$$b = \frac{l_{tw} \sin\theta \sin\phi}{B^{1/2}}$$

where h is the length of the submerged portion of the cone. (See sketch (1b).) Solving for V_c gives

$$V_c = \frac{\pi}{3} l_{tw}{}^3 \left(\frac{\sin^3\phi \sin^2\theta \cos\theta}{B^{3/2}}\right) \tag{A9a}$$

Subtracting the volume of the tip

$$V_t = \frac{\pi}{3} r^3 \cos\theta \tag{A9b}$$

(where r is the radius of the flat face of the frustum) from the submerged portion of the cone produces the submerged volume of the conic frustum V_f

$$V_f = \frac{\pi}{3} l_{tw}{}^3 \left(\frac{\sin^3\phi \sin^2\theta \cos\theta}{B^{3/2}}\right) - \frac{\pi}{3} r^3 \cot\theta \tag{A10}$$

Since $l_{tw} = l_w + s$ and $l_{tf} = r \cot\theta$, then nondimensionalizing by D yields

$$\frac{l_{tw}{}^3}{D^3} = \left(\frac{l_w}{D} + \frac{\cot\theta}{2}\right)^3 \tag{A11}$$

Substituting into equation (A10) and simplifying yields

$$\frac{V_f}{D^3} = \frac{\pi}{24} \cot\theta \left[\left(1 + \frac{2l_w}{D} \tan\theta\right)^3 \frac{1}{A^{3/2}} - 1\right] \tag{A12}$$

where

$$A = 1 - \frac{\tan^2\theta}{\tan^2\phi}$$

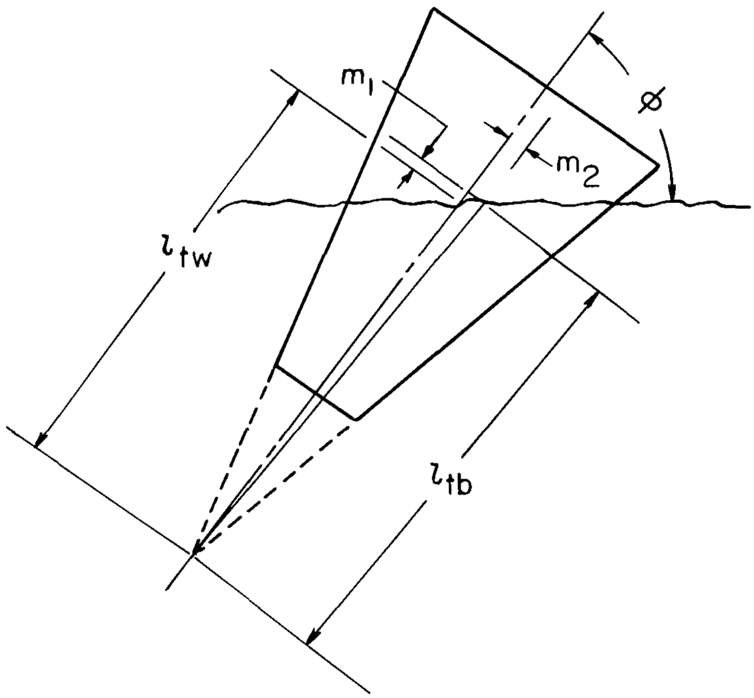
By rearranging equation (A12), the nondimensionalized equation for the submerged length along the center line of the conic frustum as a function of V_f/D^3, θ, and ϕ can be determined as

$$\frac{l_w}{D} = \frac{1}{2\tan\theta} \left[\left(\frac{24}{\pi} \frac{V_f}{D^3} \tan\theta + 1\right)^{1/3} A^{1/2} - 1\right] \tag{A13}$$

Numerical results from the solution of equation (A13) are given in table 1 for conic frustums with half-angles of 6°, 8°, 10°, 12°, 14°, and 16° for a V_f/D^3 range from 1 to 6 in increments of 0.5 and for inclination angles measured from the water line ranging from 90° to 40° in 5° increments. Figure 2 presents curves of l_w/D over the range of V_f/D^3 and ϕ for a frustum half-angle of 10°.

Center-of-Buoyancy Displacement

In order to determine the longitudinal and lateral displacements of the center of buoyancy, it is first necessary to obtain the angle γ between the geometric center line and the line from the tip to the center of the base of the submerged portion of the cone (fig. 1).



Sketch (4)

From the geometry of sketch (4),

$$l_{tb} = \left[\left(l_{tw} + m_1\right)^2 + \left(m_2\right)^2\right]^{1/2} \qquad \text{(A14)}$$

which simplifies to

$$l_{tb} = \frac{l_{tw} \sin \phi}{B}\left(\sin^2 \phi \cos^4 \theta + \cos^2 \phi \sin^4 \theta\right)^{1/2} \qquad \text{(A15)}$$

But,

$$\cos \gamma = \frac{l_{tw} + m_1}{l_{tb}}$$

Thus,

$$\cos \gamma = \frac{1}{\left(1 + \cot^2 \phi \tan^4 \theta\right)^{1/2}} \tag{A16}$$

It is now possible to determine the displacement of the center of buoyancy of the conic frustum by taking moments about the tip of the cone. First, summation of moments parallel to the center line of the right circular cone yields

$$V_c y_c = V_f y_f + V_t y_t \tag{A17}$$

where V_c and V_t are defined by equations (A9a) and (A9b), and

$$y_c = \frac{3}{4} l_{tb} \cos \gamma \tag{A18a}$$

$$y_t = \frac{3}{4} r \cot \theta \tag{A18b}$$

Substitution of these relations into equation (A17) yields

$$y_f = \frac{3}{4}\left[l_{tb} \cos \gamma - \frac{r^3 B^{3/2}\left(r \cot \theta - l_{tb} \cos \gamma\right)\cot \theta}{l_{tw}{}^3\left(\sin^3 \phi \sin^2 \theta \cos \theta\right) - r^3 B^{3/2} \cot \theta}\right] \tag{A19}$$

Upon substitution for l_{tb} and $\cos \gamma$, the following relation is obtained:

$$y_f = \frac{3}{4}\left[\frac{l_{tw} \sin^2 \phi \cos^2 \theta}{B} - \frac{r^3 B^{3/2}\left(r \cot \theta - l_{tw} B^{-1} \sin^2 \phi \cos^2 \theta\right)\cot \theta}{l_{tw}{}^3\left(\sin^3 \phi \sin^2 \theta \cos \theta\right) - r^3 B^{3/2} \cot \theta}\right] \tag{A20}$$

From equation (A11),

$$\frac{l_{tw}}{D} = \frac{l_w}{D} + \frac{\cot \theta}{2}$$

But, from equation (A13),

$$\frac{l_w}{D} = \frac{1}{2\tan\theta}\left(C^{1/3}A^{1/2} - 1\right)$$

where

$$C = \frac{24}{\pi}\frac{V_f}{D^3}\tan\theta + 1$$

therefore, the combination of these terms yields

$$l_{tw} = \frac{D}{2} C^{1/3}A^{1/2}\cot\theta \tag{A21}$$

Substitution of equation (A21) for l_{tw} in equation (A20) yields

$$y_f = \frac{3}{8}\cot\theta\left(\frac{DC^{1/3}A^{1/2}\sin^2\phi\cos^2\theta}{B} - \frac{D - DC^{1/3}A^{1/2}B^{-1}\sin^2\phi\cos^2\theta}{CA^{3/2}B^{-3/2}\sin^3\phi\cos^3\theta - 1}\right)$$

Since $B = A\sin^2\phi\cos^2\theta$, nondimensionalizing and reducing gives

$$\frac{y_f}{D} = \frac{3}{8}\cot\theta\left(\frac{C^{4/3}A^{-1/2} - 1}{C - 1}\right) \tag{A22}$$

From the submerged small face of the conic frustum, the location of the longidinal displacement of the center of buoyancy is found to be

$$y_{fB} = y_f - r\cot\theta$$

Therefore

$$\frac{y_{fB}}{D} = \frac{3}{8}\cot\theta\left(\frac{C^{4/3}A^{-1/2} - 1}{C - 1}\right) - \frac{\cot\theta}{2} \tag{A23}$$

Equation (A23) is the nondimensional equation for the longitudinal (along the center line) displacement of the center of buoyancy of a conic frustum as a function of V/D^3, θ, and ϕ. Table 2 presents numerical results of the variation of y_{fB}/D for conic frustums with half-angles of 6° to 16° at 2° intervals for a range of V_f/D^3 from 1 to 6 in increments of 0.5 and for inclination angles ranging from 90° to 40° at 5° intervals. Figure 3 presents plots of y_{fB}/D over the range of V_f/D^3 and ϕ for a frustum half-angle of 10°.

The lateral (perpendicular to the center line) displacement of the center of buoyancy of the conic frustum is found by taking moments about the tip of the right circular cone perpendicular to the center line

$$V_c x_c = V_f x_f + V_t x_t \tag{A24}$$

$$x_c = \frac{3}{4} m_2 \tag{A25a}$$

$$x_t = 0 \tag{A25b}$$

Substitution of equations (A25), (A9), and (A10) into equation (A24) yields

$$x_f = \frac{3}{4} \frac{l_{tw}^3 (\sin^3 \phi \sin^2 \theta \cos \theta) m_2}{l_{tw}^3 \sin^3 \phi \sin^2 \theta \cos \theta - r^3 B^{3/2} \cot \theta} \tag{A26}$$

From equation (A6)

$$m_2 = l_{tw} \tan \theta \left(\frac{\cos \phi \sin \phi \cos \theta \sin \theta}{B} \right)$$

therefore

$$x_f = \frac{3}{4} \left(\frac{l_{tw}^4 B^{-1} \tan \theta \sin^4 \phi \cos \phi \sin^3 \theta \cos^2 \theta}{l_{tw}^3 \sin^3 \phi \sin^2 \theta \cos \theta - r^3 B^{3/2} \cot \theta} \right)$$

Substitution for l_{tw} from equation (A21) yields

$$x_f = \frac{3}{8} D \frac{\tan \theta}{\tan \phi} \left(\frac{C^{4/3} A^2 B^{-5/2} \sin^5 \phi \cos^5 \theta}{C A^{3/2} B^{-3/2} \sin^3 \phi \cos^3 \theta - 1} \right)$$

Nondimensionalizing and reducing gives

$$\frac{x_c B}{D} = \frac{x_f}{D} = \frac{3}{8} \frac{\tan \theta}{\tan \phi} \left(\frac{C^{4/3} A^{-1/2}}{C - 1} \right) \tag{A27}$$

Equation (A27) is the general nondimensional equation for the lateral displacement of the center of buoyancy of a conic frustum as a function of V_f/D^3, ϕ, and θ. Table 3 presents numerical results for the variation of x_{cB}/D for the conic frustum for a range of θ from 6^{o} to 16^{o} at 2^{o} intervals and V_f/D^3 from 1 to 6 at 0.5 intervals and for ϕ ranging from 90^{o} to 40^{o} at 5^{o} intervals. Figure 4 presents curves of x_{cB}/D over the ϕ range and a V_f/D^3 range of 2 to 6 in increments of 1 for a frustum half-angle of 10^{o}.

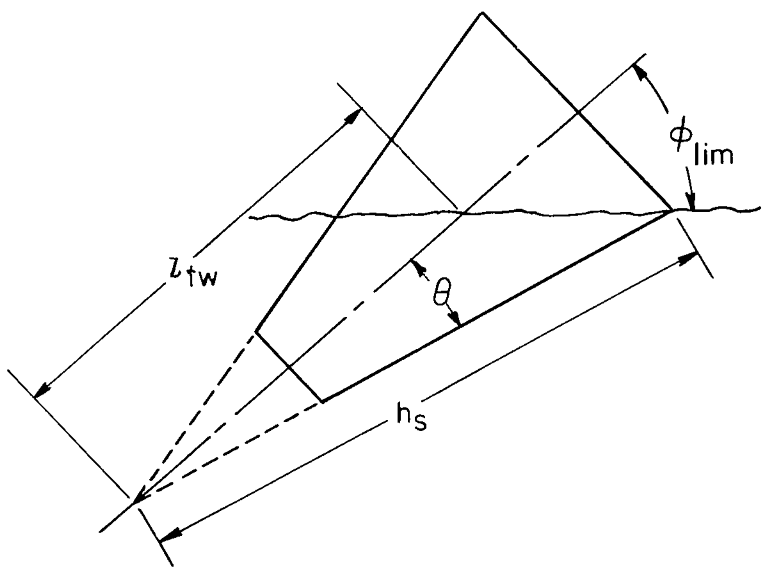
Limiting Inclination Angle

Equations (A13), (A23), and (A27) are limited for this configuration by the inclination angle for which any portion of the base becomes submerged. This is the limiting inclination angle ϕ_{lim} for which the present analysis is applicable.

The limiting inclination angle is found to occur when

$$l_{tw} \sin \phi = h_s \sin(\phi - \theta) \tag{A28}$$

where h_s is the slant height of the right circular cone. (See sketch (5).)



Sketch (5)

Substituting for l_{tw} and h_s and solving for ϕ gives

$$\phi = \text{arc tan}\left[\frac{1 + 2\frac{l}{D} \tan \theta}{2\left(\frac{l}{D} - \frac{l_w}{D}\right)}\right] \tag{A29}$$

APPENDIX

It is noted that l_w/D is a function of the inclination angle ϕ and therefore it is necessary to iterate to determine the limiting inclination angle.

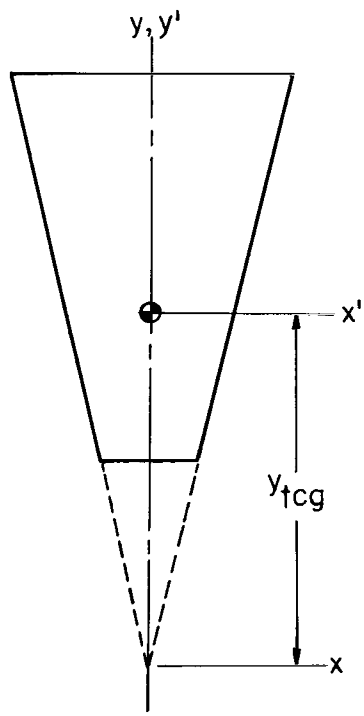
Determination of Stability

In order to determine whether the body is stable, it is necessary to define a new coordinate system which is obtained by translating the X,Y system such that

$$x' = x \tag{A30a}$$

$$y' = y - y_{tcg} \tag{A30b}$$

as shown in sketch (6).

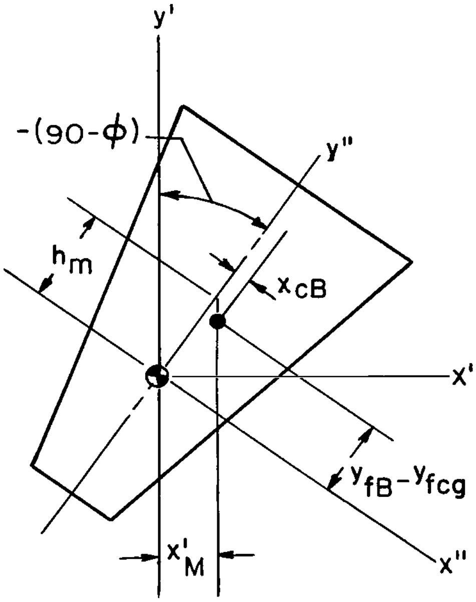


Sketch (6)

A rotation of the X',Y' system through the angle $-(90 - \phi)$ produces the coordinate system of sketch (7), where

$$x' = x'' \sin \phi + y'' \cos \phi \tag{A31a}$$

$$y' = -x'' \cos \phi + y'' \sin \phi \tag{A31b}$$



Sketch (7)

Substituting $x'' = x_{cB}$ and $y'' = y_{fB} - y_{fcg}$ into equations (A31a) and (A31b) gives the righting moment arm

$$x'_M = x_{cB} \sin \phi + \left(y_{fB} - y_{fcg}\right) \cos \phi \tag{A32}$$

and the metacentric height

$$h_m = x_{cB} \tan \phi + \left(y_{fB} - y_{fcg}\right) \tag{A33}$$

The righting moment arm is indicative of the stability. Whenever $x'_M > 0$, a positive righting moment exists and the flat-faced body is stable. The metacentric height is also a measure of the stability. The greater the metacentric height, the greater the stability.

Rearmost Location of Center of Gravity

The rearmost location of the center of gravity without loss of flotation stability at the limits of the present analysis is determined from equation (A33). By setting the metacentric height equal to zero and solving for y_{fcg}, the most rearward location of the center of gravity at ϕ_{lim} becomes

$$y_{fcg} = y_{fB} + x_{cB} \tan \phi \tag{A34}$$

Equation (A34) provides neutral or zero stability at ϕ_{lim}; that is, the metacentric height h_m and righting moment arm x'_M equals zero at ϕ_{lim}. To provide some degree of stability, a desired value of the metacentric height can be subtracted from equation (A33) and the equation can be solved for y_{fcg}:

$$y_{fcg} = y_{fB} + x_{cB} \tan \phi - h_m \tag{A35}$$

REFERENCE

1. Mechtly, E. A.: The International System of Units - Physical Constants and Conversion Factors. NASA SP-7012, 1964.

TABLE 1.- NONDIMENSIONAL CENTER-LINE DISTANCE FROM FLAT FACE TO WATER LINE

$\frac{V_f}{D^3}$	l_w/D for inclination angle ϕ of –										
	90	85	80	75	70	65	60	55	50	45	40
	$\theta = 6.0^{o}$										
1.0	1.0328	1.0325	1.0318	1.0305	1.0286	1.0258	1.0221	1.0171	1.0102	1.0007	0.9872
1.5	1.4341	1.4338	1.4330	1.4316	1.4296	1.4267	1.4227	1.4173	1.4100	1.3998	1.3853
2.0	1.7892	1.7889	1.7881	1.7866	1.7844	1.7813	1.7771	1.7714	1.7637	1.7529	1.7376
2.5	2.1095	2.1092	2.1083	2.1067	2.1044	2.1012	2.0968	2.0908	2.0827	2.0714	2.0554
3.0	2.4023	2.4020	2.4011	2.3995	2.3971	2.3937	2.3891	2.3829	2.3744	2.3627	2.3460
3.5	2.6730	2.6727	2.6718	2.6701	2.6676	2.6641	2.6594	2.6529	2.6441	2.6319	2.6145
4.0	2.9253	2.9250	2.9240	2.9223	2.9197	2.9161	2.9112	2.9045	2.8954	2.8828	2.8648
4.5	3.1621	3.1617	3.1607	3.1589	3.1563	3.1526	3.1475	3.1406	3.1312	3.1182	3.0997
5.0	3.3854	3.3851	3.3840	3.3822	3.3795	3.3757	3.3704	3.3634	3.3537	3.3403	3.3213
5.5	3.5972	3.5968	3.5957	3.5939	3.5911	3.5871	3.5818	3.5745	3.5646	3.5509	3.5314
6.0	3.7987	3.7983	3.7972	3.7953	3.7924	3.7884	3.7829	3.7755	3.7654	3.7513	3.7313
	$\theta = 8.0^{o}$										
1.0	0.9791	0.9787	0.9777	0.9759	0.9731	0.9693	0.9641	0.9571	0.9474	0.9341	0.9150
1.5	1.3409	1.3406	1.3394	1.3375	1.3345	1.3304	1.3248	1.3172	1.3068	1.2923	1.2717
2.0	1.6560	1.6556	1.6544	1.6523	1.6492	1.6448	1.6388	1.6307	1.6196	1.6043	1.5824
2.5	1.9370	1.9366	1.9354	1.9331	1.9298	1.9252	1.9189	1.9104	1.8987	1.8825	1.8594
3.0	2.1919	2.1915	2.1902	2.1878	2.1844	2.1796	2.1730	2.1640	2.1518	2.1349	2.1107
3.5	2.4260	2.4256	2.4242	2.4218	2.4182	2.4131	2.4063	2.3970	2.3843	2.3666	2.3415
4.0	2.6431	2.6426	2.6412	2.6387	2.6350	2.6298	2.6226	2.6130	2.5998	2.5815	2.5555
4.5	2.8459	2.8455	2.8440	2.8414	2.8376	2.8322	2.8248	2.8149	2.8013	2.7824	2.7555
5.0	3.0367	3.0362	3.0347	3.0320	3.0281	3.0225	3.0150	3.0047	2.9907	2.9713	2.9435
5.5	3.2170	3.2165	3.2149	3.2122	3.2082	3.2025	3.1947	3.1841	3.1698	3.1498	3.1213
6.0	3.3882	3.3877	3.3861	3.3833	3.3791	3.3733	3.3653	3.3545	3.3398	3.3193	3.2901
	$\theta = 10.0^{o}$										
1.0	0.9328	0.9323	0.9309	0.9286	0.9250	0.9200	0.9132	0.9039	0.8913	0.8737	0.8486
1.5	1.2634	1.2629	1.2614	1.2588	1.2549	1.2495	1.2421	1.2320	1.2183	1.1992	1.1719
2.0	1.5478	1.5473	1.5457	1.5430	1.5388	1.5330	1.5251	1.5143	1.4996	1.4792	1.4500
2.5	1.7995	1.7990	1.7973	1.7944	1.7900	1.7838	1.7754	1.7641	1.7485	1.7269	1.6960
3.0	2.0265	2.0259	2.0241	2.0211	2.0165	2.0100	2.0012	1.9893	1.9730	1.9503	1.9179
3.5	2.2340	2.2334	2.2316	2.2284	2.2236	2.2169	2.2077	2.1953	2.1782	2.1546	2.1208
4.0	2.4259	2.4252	2.4233	2.4200	2.4150	2.4080	2.3985	2.3856	2.3680	2.3434	2.3084
4.5	2.6046	2.6040	2.6020	2.5986	2.5934	2.5862	2.5764	2.5630	2.5448	2.5194	2.4832
5.0	2.7724	2.7717	2.7697	2.7661	2.7608	2.7534	2.7432	2.7295	2.7106	2.6845	2.6471
5.5	2.9306	2.9299	2.9278	2.9242	2.9187	2.9111	2.9006	2.8865	2.8672	2.8403	2.8019
6.0	3.0806	3.0799	3.0778	3.0740	3.0684	3.0606	3.0499	3.0354	3.0155	2.9879	2.9485

TABLE 1.- NONDIMENSIONAL CENTER-LINE DISTANCE FROM FLAT FACE TO WATER LINE – Concluded

$\frac{V_f}{D^3}$	l_w/D for inclination angle ϕ of –										
	90	85	80	75	70	65	60	55	50	45	40
	$\theta = 12.0^{\circ}$										
1.0	0.8921	0.8916	0.8898	0.887	0.8824	0.8761	0.8676	0.8560	0.8401	0.8180	0.7843
1.5	1.1972	1.1966	1.1947	1.1914	1.1866	1.1797	1.1704	1.1577	1.1403	1.1161	1.0814
2.0	1.4573	1.4566	1.4546	1.4511	1.4459	1.4385	1.4285	1.4148	1.3962	1.3702	1.3330
2.5	1.6860	1.6853	1.6832	1.6794	1.6739	1.6661	1.6555	1.6410	1.6212	1.5937	1.5543
3.0	1.8914	1.8906	1.8884	1.8845	1.8787	1.8705	1.8593	1.8441	1.8233	1.7944	1.7530
3.5	2.0786	2.0778	2.0755	2.0714	2.0653	2.0568	2.0451	2.0292	2.0076	1.9773	1.9341
4.0	2.2512	2.2504	2.2480	2.2437	2.2374	2.2285	2.2164	2.1999	2.1774	2.1460	2.1011
4.5	2.4118	2.4109	2.4084	2.4040	2.3975	2.3883	2.3758	2.3587	2.3354	2.3029	2.2564
5.0	2.5622	2.5613	2.5587	2.5542	2.5475	2.5380	2.5250	2.5074	2.4834	2.4499	2.4019
5.5	2.7039	2.7030	2.7004	2.6957	2.6888	2.6790	2.6657	2.6476	2.6228	2.5884	2.5390
6.0	2.8381	2.8372	2.8345	2.8297	2.8225	2.8125	2.7989	2.7803	2.7549	2.7195	2.6688
	$\theta = 14.0^{\circ}$										
1.0	0.8559	0.8553	0.8532	0.8495	0.8441	0.8365	0.8261	0.8120	0.7926	0.7656	0.7267
1.5	1.1396	1.1388	1.1366	1.1326	1.1266	1.1183	1.1068	1.0913	1.0700	1.0403	.9976
2.0	1.3796	1.3788	1.3764	1.3721	1.3657	1.3567	1.3444	1.3277	1.3047	1.2727	1.2268
2.5	1.5898	1.5889	1.5863	1.5817	1.5749	1.5654	1.5523	1.5346	1.5102	1.4762	1.4274
3.0	1.7778	1.7769	1.7742	1.7694	1.7622	1.7522	1.7384	1.7197	1.6941	1.6584	1.6070
3.5	1.9489	1.9479	1.9450	1.9400	1.9326	1.9221	1.9077	1.8881	1.8614	1.8240	1.7703
4.0	2.1063	2.1053	2.1023	2.0971	2.0893	2.0784	2.0634	2.0431	2.0153	1.9764	1.9206
4.5	2.2524	2.2514	2.2483	2.2429	2.2349	2.2236	2.2081	2.1871	2.1582	2.1180	2.0601
5.0	2.3892	2.3882	2.3850	2.3794	2.3711	2.3594	2.3435	2.3217	2.2920	2.2504	2.1907
5.5	2.5180	2.5169	2.5136	2.5079	2.4993	2.4873	2.4709	2.4485	2.4179	2.3751	2.3137
6.0	2.6398	2.6387	2.6353	2.6294	2.6206	2.6083	2.5914	2.5685	2.5370	2.4931	2.4300
	$\theta = 16.0^{\circ}$										
1.0	0.8233	0.8225	0.8200	0.8157	0.8093	0.8003	0.7879	0.7710	0.7479	0.7155	0.6688
1.5	1.0887	1.0878	1.0851	1.0803	1.0732	1.0632	1.0496	1.0310	1.0055	.9697	.9182
2.0	1.3119	1.3109	1.3080	1.3029	1.2953	1.2845	1.2697	1.2497	1.2221	1.1836	1.1280
2.5	1.5066	1.5055	1.5024	1.4970	1.4888	1.4774	1.4617	1.4404	1.4111	1.3701	1.3109
3.0	1.6803	1.6793	1.6759	1.6702	1.6616	1.6496	1.6331	1.6106	1.5797	1.5365	1.4742
3.5	1.8380	1.8369	1.8335	1.8275	1.8185	1.8059	1.7886	1.7651	1.7328	1.6876	1.6224
4.0	1.9830	1.9818	1.9782	1.9720	1.9626	1.9495	1.9316	1.9071	1.8735	1.8265	1.7586
4.5	2.1174	2.1162	2.1125	2.1060	2.0964	2.0828	2.0642	2.0388	2.0040	1.9553	1.8850
5.0	2.2432	2.2419	2.2381	2.2314	2.2214	2.2074	2.1881	2.1620	2.1260	2.0757	2.0031
5.5	2.3614	2.3601	2.3561	2.3493	2.3390	2.3245	2.3047	2.2778	2.2408	2.1890	2.1143
6.0	2.4732	2.4719	2.4678	2.4607	2.4502	2.4353	2.4150	2.3873	2.3493	2.2961	2.2193

TABLE 2.- NONDIMENSIONAL LONGITUDINAL DISPLACEMENT OF CENTER OF BUOYANCY

$\frac{V_f}{D^3}$	y_{fB}/D for inclination angle ϕ of –										
	90	85	80	75	70	65	60	55	50	45	40
θ = 6.0°											
1.0	0.5500	0.5504	0.5517	0.5539	0.5571	0.5617	0.5680	0.5765	0.5881	0.6043	0.6274
1.5	.7793	.7797	.7808	.7827	.7855	.7895	.7950	.8024	.8126	.8266	.8468
2.0	.9882	.9886	.9896	.9914	.9941	.9978	1.0029	1.0099	1.0194	1.0326	1.0515
2.5	1.1809	1.1813	1.1823	1.1840	1.1866	1.1902	1.1952	1.2019	1.2111	1.2239	1.2422
3.0	1.3604	1.3608	1.3618	1.3635	1.3660	1.3696	1.3745	1.3811	1.3902	1.4028	1.4208
3.5	1.5289	1.5292	1.5302	1.5319	1.5344	1.5380	1.5428	1.5494	1.5584	1.5709	1.5888
4.0	1.6878	1.6881	1.6891	1.6908	1.6934	1.6969	1.7018	1.7084	1.7174	1.7299	1.7478
4.5	1.8386	1.8389	1.8399	1.8416	1.8442	1.8477	1.8526	1.8592	1.8683	1.8809	1.8988
5.0	1.9822	1.9826	1.9835	1.9853	1.9878	1.9914	1.9963	2.0030	2.0121	2.0247	2.0428
5.5	2.1195	2.1198	2.1208	2.1226	2.1251	2.1288	2.1337	2.1404	2.1496	2.1623	2.1805
6.0	2.2511	2.2514	2.2524	2.2542	2.2568	2.2604	2.2654	2.2722	2.2814	2.2943	2.3126
θ = 8.0°											
1.0	0.5288	0.5293	0.5308	0.5335	0.5374	0.5430	0.5506	0.5609	0.5750	0.5947	0.6230
1.5	.7407	.7412	.7426	.7450	.7486	.7536	.7605	.7698	.7826	.8005	.8261
2.0	.9310	.9314	.9328	.9351	.9385	.9434	.9500	.9590	.9713	.9885	1.0131
2.5	1.1046	1.1050	1.1063	1.1086	1.1120	1.1168	1.1233	1.1322	1.1444	1.1613	1.1857
3.0	1.2649	1.2653	1.2666	1.2689	1.2723	1.2771	1.2836	1.2925	1.3046	1.3216	1.3459
3.5	1.4143	1.4147	1.4160	1.4183	1.4217	1.4265	1.4331	1.4420	1.4542	1.4712	1.4957
4.0	1.5545	1.5549	1.5562	1.5585	1.5620	1.5668	1.5734	1.5824	1.5948	1.6119	1.6366
4.5	1.6868	1.6873	1.6886	1.6909	1.6944	1.6993	1.7060	1.7151	1.7276	1.7449	1.7699
5.0	1.8124	1.8128	1.8142	1.8165	1.8201	1.8250	1.8318	1.8410	1.8536	1.8712	1.8964
5.5	1.9319	1.9324	1.9338	1.9362	1.9397	1.9447	1.9516	1.9609	1.9737	1.9915	2.0171
6.0	2.0462	2.0467	2.0481	2.0505	2.0541	2.0592	2.0661	2.0756	2.0885	2.1065	2.1325
θ = 10.0°											
1.0	0.5100	0.5106	0.5124	0.5155	0.5202	0.5267	0.5357	0.5480	0.5648	0.5884	0.6225
1.5	.7076	.7081	.7098	.7127	.7171	.7232	.7316	.7430	.7587	.7807	.8125
2.0	.8829	.8834	.8851	.8879	.8922	.8982	.9064	.9176	.9330	.9546	.9858
2.5	1.0415	1.0420	1.0437	1.0465	1.0508	1.0568	1.0651	1.0763	1.0917	1.1133	1.1445
3.0	1.1870	1.1876	1.1893	1.1921	1.1965	1.2025	1.2108	1.2221	1.2377	1.2595	1.2910
3.5	1.3220	1.3226	1.3242	1.3272	1.3315	1.3377	1.3461	1.3575	1.3733	1.3954	1.4273
4.0	1.4482	1.4487	1.4504	1.4534	1.4578	1.4640	1.4726	1.4842	1.5002	1.5226	1.5550
4.5	1.5668	1.5674	1.5691	1.5721	1.5766	1.5830	1.5917	1.6035	1.6197	1.6425	1.6754
5.0	1.6791	1.6797	1.6814	1.6845	1.6891	1.6955	1.7043	1.7163	1.7328	1.7560	1.7894
5.5	1.7857	1.7863	1.7881	1.7912	1.7959	1.8024	1.8114	1.8236	1.8404	1.8639	1.8978
6.0	1.8874	1.8880	1.8899	1.8930	1.8977	1.9044	1.9135	1.9259	1.9429	1.9668	2.0013

TABLE 2.- NONDIMENSIONAL LONGITUDINAL DISPLACEMENT

OF CENTER OF BUOYANCY – Concluded

$\frac{V_f}{D^3}$	y_{fB}/D for inclination angle ϕ of –										
	90	85	80	75	70	65	60	55	50	45	40
						θ = 12.0°					
1.0	0.4931	0.4937	0.4958	0.4995	0.5049	0.5125	0.5230	0.5374	0.5571	0.5850	0.6256
1.5	.6784	.6791	.6811	.6846	.6897	.6970	.7071	.7207	.7396	.7663	.8051
2.0	.8414	.8421	.8440	.8475	.8526	.8599	.8699	.8835	.9023	.9288	.9674
2.5	.9879	.9886	.9906	.9940	.9992	1.0066	1.0166	1.0304	1.0494	1.0762	1.1152
3.0	1.1216	1.1223	1.1243	1.1279	1.1332	1.1406	1.1509	1.1649	1.1842	1.2114	1.2510
3.5	1.2452	1.2458	1.2479	1.2515	1.2569	1.2645	1.2749	1.2892	1.3089	1.3366	1.3769
4.0	1.3605	1.3610	1.3631	1.3668	1.3723	1.3800	1.3906	1.4051	1.4252	1.4534	1.4946
4.5	1.4683	1.4690	1.4711	1.4749	1.4805	1.4884	1.4992	1.5140	1.5345	1.5633	1.6052
5.0	1.5702	1.5710	1.5731	1.5770	1.5827	1.5907	1.6018	1.6169	1.6377	1.6671	1.7098
5.5	1.6669	1.6676	1.6699	1.6738	1.6796	1.6878	1.6990	1.7144	1.7356	1.7655	1.8091
6.0	1.7590	1.7597	1.7620	1.7660	1.7719	1.7802	1.7917	1.8073	1.8289	1.8594	1.9037
						θ = 14.0°					
1.0	0.4776	0.4784	0.4808	0.4850	0.4912	0.5000	0.5121	0.5287	0.5517	0.5844	0.6324
1.5	.6525	.6533	.6556	.6596	.6657	.6742	.6860	.7022	.7246	.7563	.8031
2.0	.8050	.8058	.8081	.8122	.8183	.8269	.8387	.8550	.8776	.9095	.9566
2.5	.9414	.9422	.9445	.9487	.9549	.9637	.9757	.9923	1.0152	1.0478	1.0957
3.0	1.0654	1.0662	1.0686	1.0728	1.0792	1.0881	1.1005	1.1174	1.1408	1.1741	1.2231
3.5	1.1796	1.1804	1.1829	1.1872	1.1937	1.2028	1.2155	1.2328	1.2567	1.2908	1.3409
4.0	1.2857	1.2865	1.2891	1.2935	1.3001	1.3095	1.3224	1.3401	1.3646	1.3994	1.4507
4.5	1.3851	1.3859	1.3885	1.3931	1.3999	1.4094	1.4226	1.4407	1.4658	1.5014	1.5538
5.0	1.4787	1.4796	1.4822	1.4869	1.4938	1.5036	1.5171	1.5355	1.5611	1.5975	1.6510
5.5	1.5674	1.5683	1.5710	1.5757	1.5828	1.5928	1.6065	1.6254	1.6515	1.6886	1.7432
6.0	1.6517	1.6526	1.6554	1.6602	1.6674	1.6776	1.6916	1.7108	1.7375	1.7753	1.8310
						θ = 16.0°					
1.0	0.4634	0.4643	0.4670	0.4718	0.4788	0.4889	0.5027	0.5217	0.5483	0.5864	0.6431
1.5	.6291	.6299	.6326	.6373	.6443	.6542	.6678	.6867	.7129	.7505	.8066
2.0	.7726	.7735	.7762	.7809	.7880	.7981	.8120	.8311	.8578	.8960	.9529
2.5	.9003	.9012	.9040	.9089	.9161	.9264	.9407	.9602	.9876	1.0267	1.0850
3.0	1.0161	1.0170	1.0199	1.0249	1.0323	1.0429	1.0575	1.0776	1.1056	1.1458	1.2056
3.5	1.1224	1.1234	1.1263	1.1314	1.1391	1.1499	1.1649	1.1855	1.2143	1.2555	1.3169
4.0	1.2210	1.2220	1.2250	1.2303	1.2381	1.2493	1.2646	1.2858	1.3153	1.3575	1.4205
4.5	1.3133	1.3143	1.3174	1.3227	1.3308	1.3422	1.3579	1.3796	1.4098	1.4531	1.5176
5.0	1.4000	1.4011	1.4042	1.4097	1.4180	1.4296	1.4457	1.4679	1.4988	1.5431	1.6091
5.5	1.4821	1.4832	1.4864	1.4920	1.5004	1.5123	1.5288	1.5514	1.5830	1.6283	1.6957
6.0	1.5601	1.5612	1.5644	1.5702	1.5788	1.5909	1.6077	1.6308	1.6631	1.7093	1.7781

TABLE 3.- NONDIMENSIONAL LATERAL DISPLACEMENT

OF CENTER OF BUOYANCY

$\frac{V_f}{D^3}$	x_{cB}/D for inclination angle ϕ of –										
	90	85	80	75	70	65	60	55	50	45	40
						$\theta = 6.0^{\circ}$					
1.0	0.0000	0.00942	0.0190	0.0289	0.0392	0.0503	0.0623	0.0756	0.09074	0.1083	0.1294
1.5		.00821	.0166	.0252	.0342	.0438	.0543	.0659	.07908	.0944	.1128
2.0		.00770	.0155	.0236	.0321	.0411	.0509	.0618	.07414	.0885	.1057
2.5		.00746	.0150	.0228	.0310	.0398	.0493	.0598	.07180	.0857	.1024
3.0		.00734	.0148	.0225	.0306	.0392	.0486	.0589	.07071	.0844	.1008
3.5		.00730	.0147	.0224	.0304	.0390	.0483	.0586	.07031	.0839	.1003
4.0		.00730	.0147	.0224	.0304	.0390	.0483	.0586	.07031	.0839	.1003
4.5		.00733	.0148	.0225	.0305	.0391	.0485	.0588	.07057	.0842	.1006
5.0		.00737	.0149	.0226	.0307	.0393	.0487	.0592	.07099	.0847	.1012
5.5		.00743	.0150	.0228	.0309	.0396	.0491	.0596	.07151	.0854	.1020
6.0		.00749	.0151	.0229	.0312	.0400	.0495	.0601	.07211	.0861	.1028
						$\theta = 8.0^{\circ}$					
1.0	0.0000	0.01136	0.0229	0.0348	0.0473	0.0606	0.0752	0.0913	0.1097	0.1311	0.1569
1.5		.01029	.0207	.0315	.0429	.0550	.0681	.0828	.0994	.1188	.1422
2.0		.00990	.0200	.0304	.0413	.0529	.0656	.0797	.0957	.1143	.1368
2.5		.00978	.0197	.0300	.0407	.0522	.0647	.0786	.0944	.1128	.1351
3.0		.00977	.0197	.0299	.0407	.0522	.0647	.0785	.0943	.1127	.1349
3.5		.00982	.0198	.0301	.0409	.0524	.0650	.0790	.0948	.1134	.1357
4.0		.00991	.0200	.0304	.0413	.0529	.0656	.0797	.0956	.1144	.1369
4.5		.01002	.0202	.0307	.0417	.0535	.0663	.0806	.0967	.1156	.1384
5.0		.01014	.0204	.0311	.0422	.0542	.0671	.0815	.0979	.1170	.1401
5.5		.01027	.0207	.0315	.0428	.0548	.0680	.0826	.0992	.1185	.1419
6.0		.01040	0.0210	.0319	.0433	.0555	.0689	.0836	.1004	.1201	.1437
						$\theta = 10.0^{\circ}$					
1.0	0.0000	0.0134	0.0270	0.0411	0.0558	0.0716	0.0889	0.1080	0.1299	0.1555	0.1866
1.5		.0125	.0252	.0383	.0521	.0669	.0829	.1008	.1212	.1452	.1742
2.0		.0123	.0247	.0376	.0511	.0656	.0813	.0989	.1189	.1424	.1708
2.5		.0123	.0247	.0376	.0511	.0656	.0814	.0989	.1189	.1424	.1709
3.0		.0124	.0250	.0379	.0516	.0662	.0820	.0998	.1200	.1437	.1724
3.5		.0125	.0253	.0384	.0523	.0670	.0832	.1011	.1216	.1456	.1747
4.0		.0127	.0257	.0390	.0531	.0681	.0844	.1026	.1234	.1478	.1773
4.5		.0129	.0261	.0396	.0539	.0691	.0858	.1043	.1254	.1501	.1801
5.0		.0131	.0265	.0403	.0548	.0703	.0872	.1060	.1274	.1526	.1831
5.5		.0134	.0269	.0409	.0557	.0714	.0886	.1077	.1295	.1550	.1860
6.0		.0136	.0273	.0416	.0565	.0725	.0900	.1094	.1315	.1575	.1890

TABLE 3.- NONDIMENSIONAL LATERAL DISPLACEMENT

OF CENTER OF BUOYANCY – Concluded

$\frac{V_f}{D^3}$	x_{cB}/D for inclination angle ϕ of –										
	90	85	80	75	70	65	60	55	50	45	40
						θ = 12.0°					
1.0	0.0000	0.0145	0.0313	0.0477	0.0649	0.0832	0.1033	0.1258	0.1515	0.1818	0.2188
1.5		.0137	.0299	.0455	.0619	.0795	.0987	.1201	.1447	.1736	.2090
2.0		.0135	.0298	.0453	.0616	.0791	.0982	.1196	.1440	.1728	.2080
2.5		.0136	.0301	.0458	.0623	.0799	.0992	.1208	.1454	.1745	.2101
3.0		.0137	.0306	.0465	.0633	.0812	.1008	.1227	.1478	.1774	.2135
3.5		.0140	.0312	.0474	.0645	.0827	.1027	.1250	.1506	.1807	.2175
4.0		.0142	.0318	.0483	.0657	.0844	.1047	.1275	.1535	.1842	.2218
4.5		.0145	.0324	.0492	.0670	.0860	.1068	.1300	.1565	.1878	.2261
5.0		.0147	.0330	.0502	.0683	.0876	.1088	.1324	.1595	.1914	.2304
5.5		.0150	.0336	.0511	.0696	.0893	.1108	.1349	.1625	.1950	.2347
6.0		.0152	.0342	.0520	.0708	.0909	.1128	.1373	.1654	.1985	.2389
						θ = 14.0°					
1.0	0.0000	0.0178	0.0359	0.0546	0.0744	0.0955	0.1187	0.1447	0.1746	0.2101	0.2539
1.5		.0173	.0349	.0532	.0723	.0929	.1155	.1408	.1699	.2044	.2471
2.0		.0174	.0352	.0535	.0728	.0935	.1162	.1417	.1710	.2057	.2487
2.5		.0177	.0358	.0545	.0741	.0952	.1183	.1442	.1740	.2094	.2531
3.0		.0181	.0366	.0557	.0757	.0973	.1209	.1474	.1778	.2140	.2587
3.5		.0186	.0374	.0569	.0775	.0995	.1237	.1508	.1819	.2189	.2646
4.0		.0190	.0383	.0582	.0793	1.0181	.1265	.1542	.1861	.2239	.2707
4.5		.0194	.0391	.0595	.0810	1.0407	.1293	.1576	.1902	.2289	.2767
5.0		.0198	.0400	.0608	.0827	1.0630	.1321	.1610	.1943	.2338	.2826
5.5		.0202	.0408	.0620	.0844	1.0846	.1348	.1643	.1982	.2386	.2883
6.0		.0206	.0416	.0633	.0861	1.1058	.1374	.1675	.2021	.2432	.2940
						θ = 16.0°					
1.0	0.0000	0.0202	0.0407	0.0620	0.0844	0.1085	0.1350	0.1648	0.1993	0.2407	0.2924
1.5		.0199	.0402	.0612	.0834	.1072	.1334	.1628	.1970	.2378	.2889
2.0		.0203	.0409	.0622	.0847	.1089	.1355	.1654	.2001	.2416	.2935
2.5		.0207	.0418	.0637	.0867	.1115	.1388	.1694	.2049	.2474	.3006
3.0		.0213	.0430	.0654	.0890	.1145	.1424	.1739	.2103	.2539	.3085
3.5		.0219	.0441	.0671	.0914	.1175	.1462	.1785	.2159	.2606	.3166
4.0		.0224	.0452	.0688	.0937	.1205	.1499	.1830	.2213	.2673	.3247
4.5		.0230	.0463	.0705	.0960	.1234	.1535	.1875	.2267	.2738	.3326
5.0		.0235	.0474	.0721	.0982	.1262	.1571	.1918	.2319	.2801	.3402
5.5		.0240	.0484	.0737	.1003	.1290	.1605	.1960	.2370	.2862	.3477
6.0		.0245	.0494	.0752	.1024	.1317	.1638	.2000	.2419	.2921	.3548

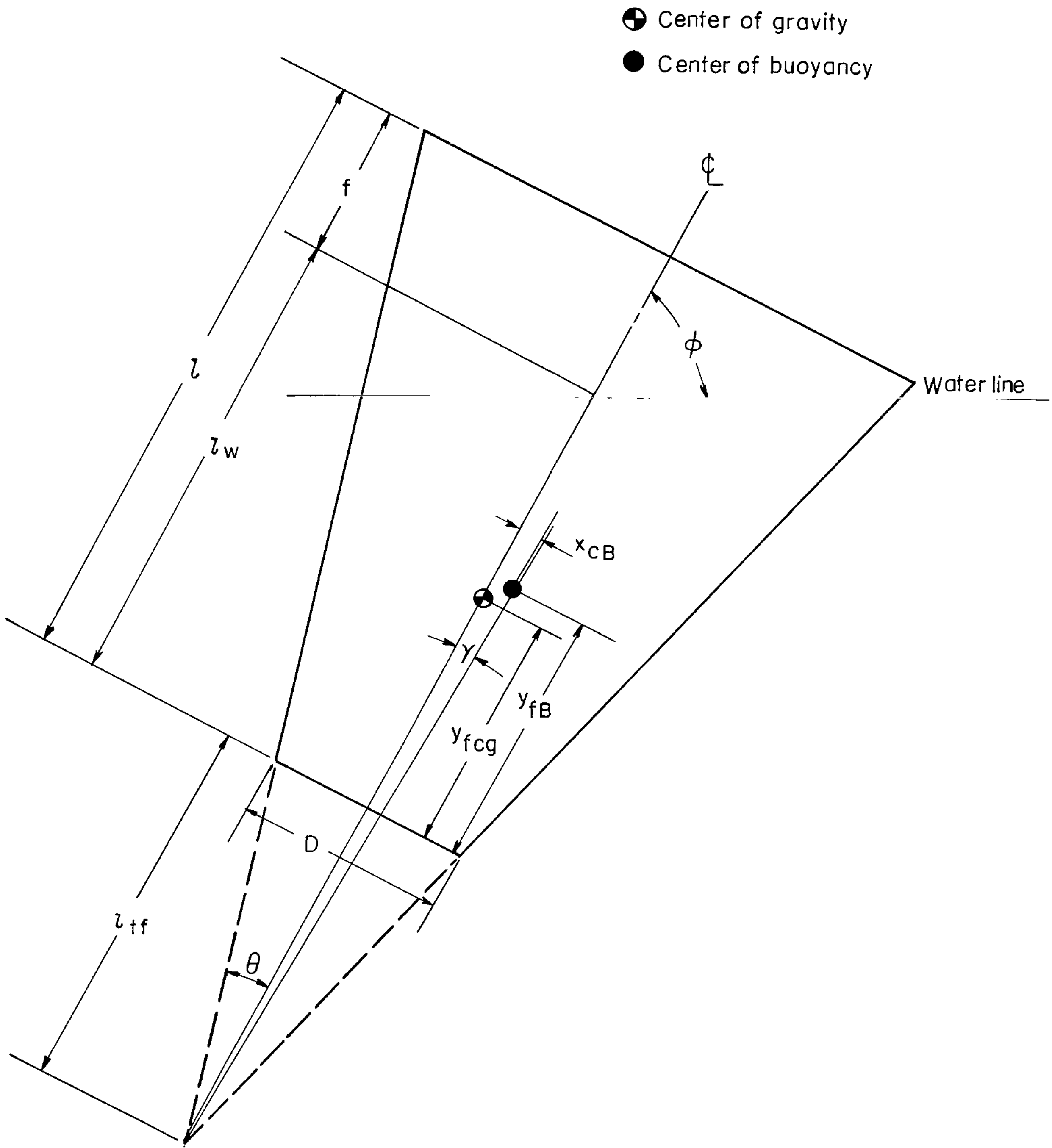


Figure 1.- General dimensions of the conic frustum.

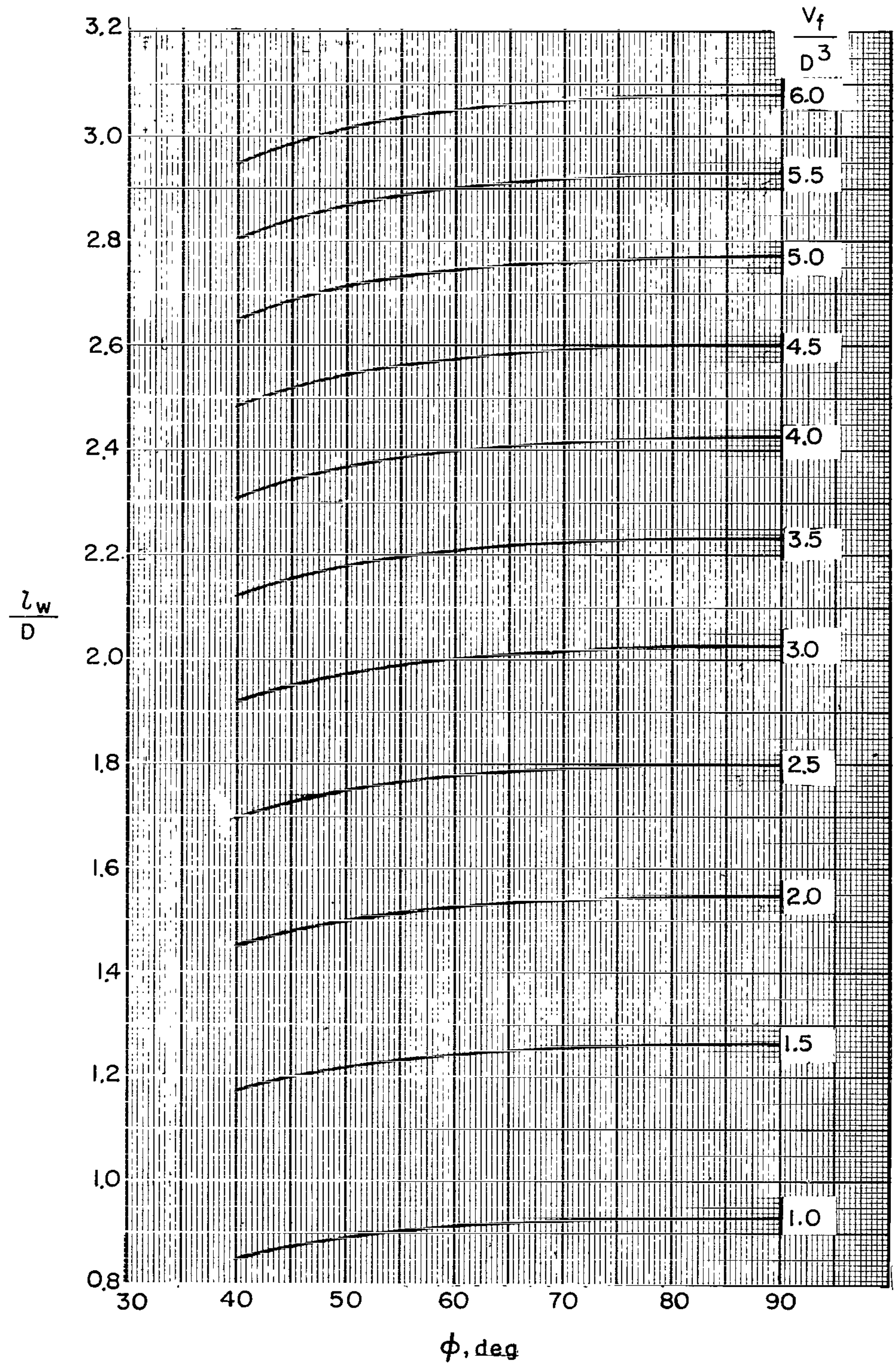


Figure 2.- Nondimensional center-line distance to water surface for $\theta = 10^{\circ}$.

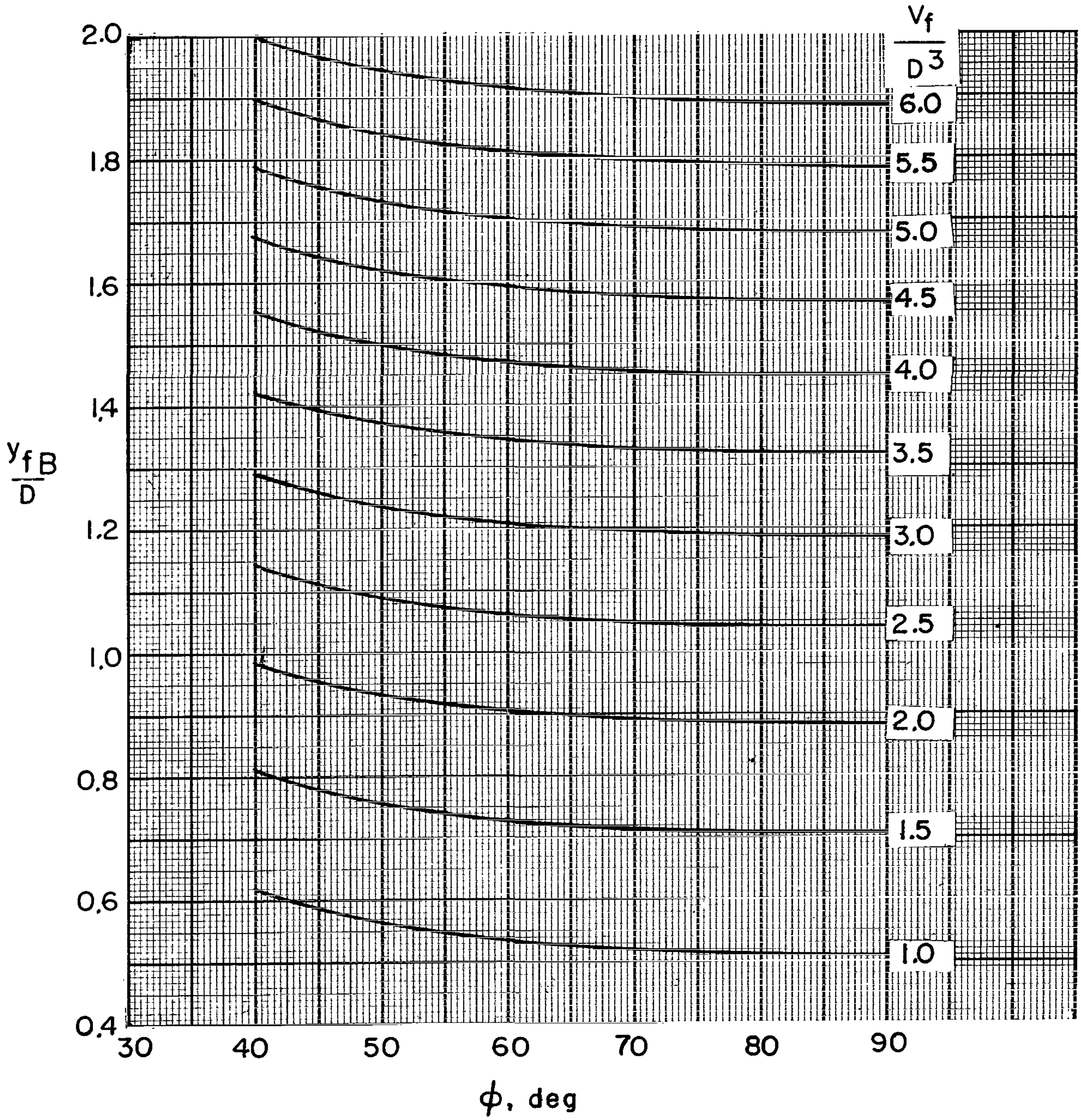


Figure 3.- Nondimensional longitudinal displacement of center of buoyancy for $\theta = 10^0$.

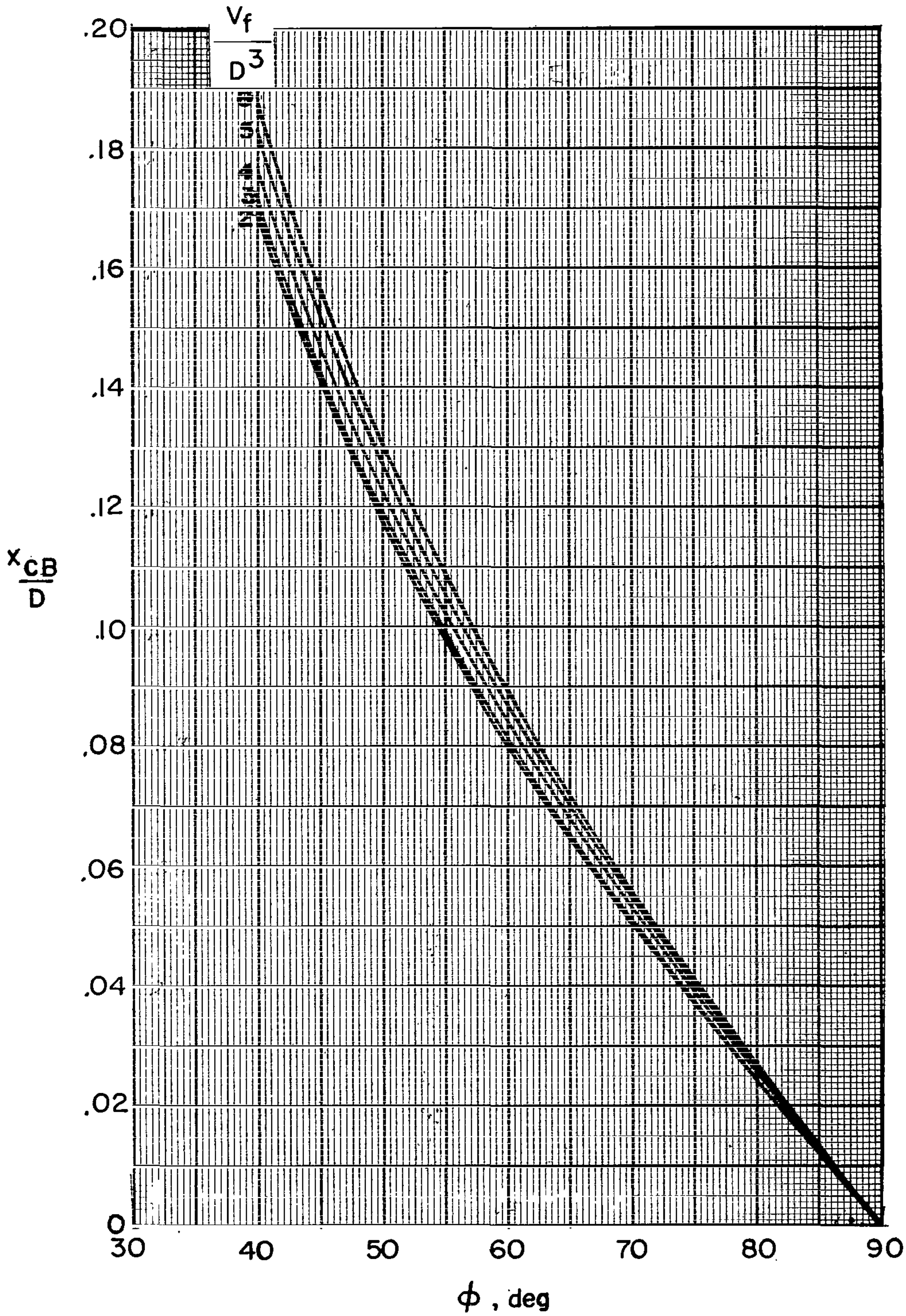


Figure 4.- Nondimensional lateral displacement of center of buoyancy for θ = 10°.

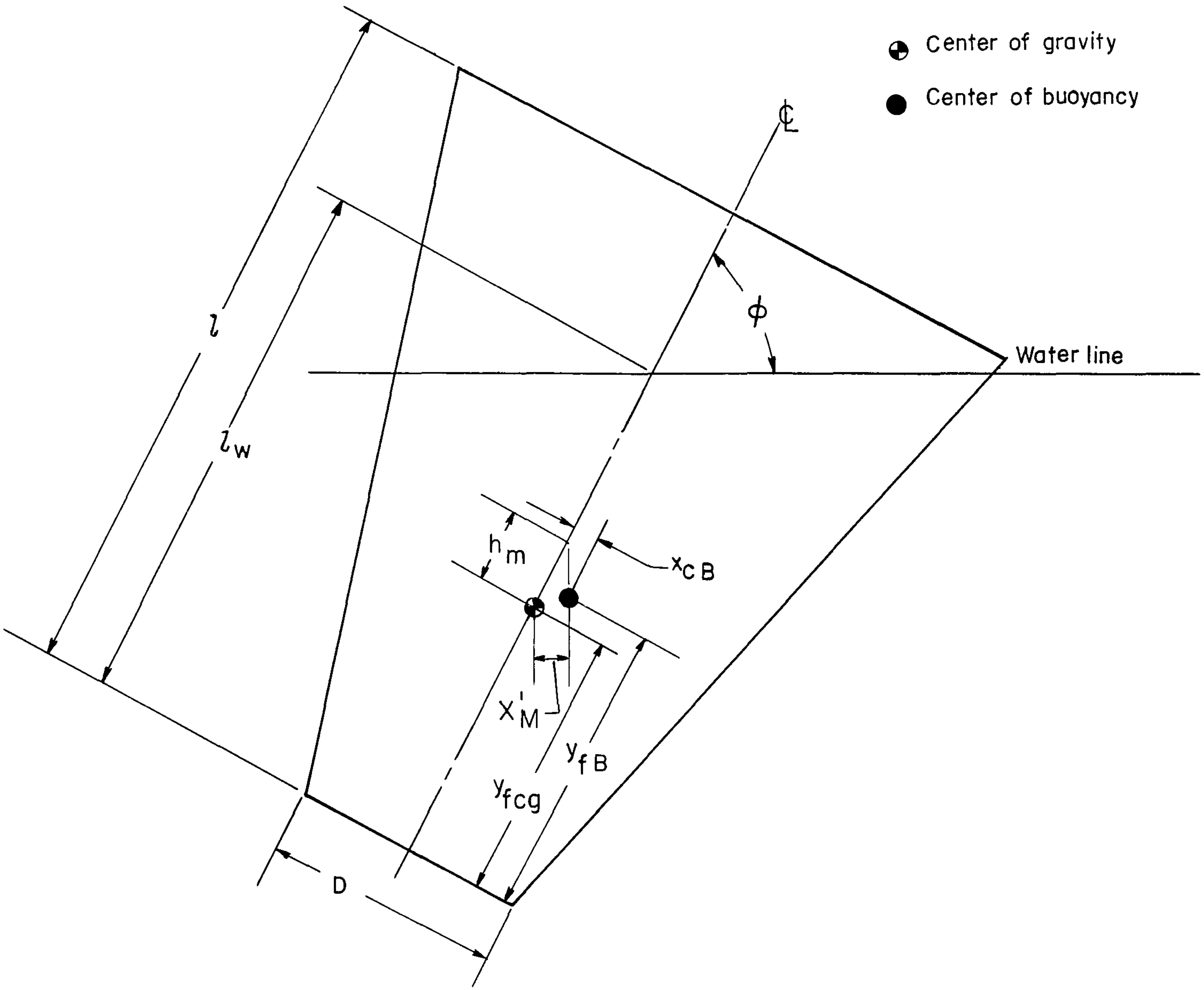


Figure 5.- General configuration of conic frustum showing flotation stability parameters.

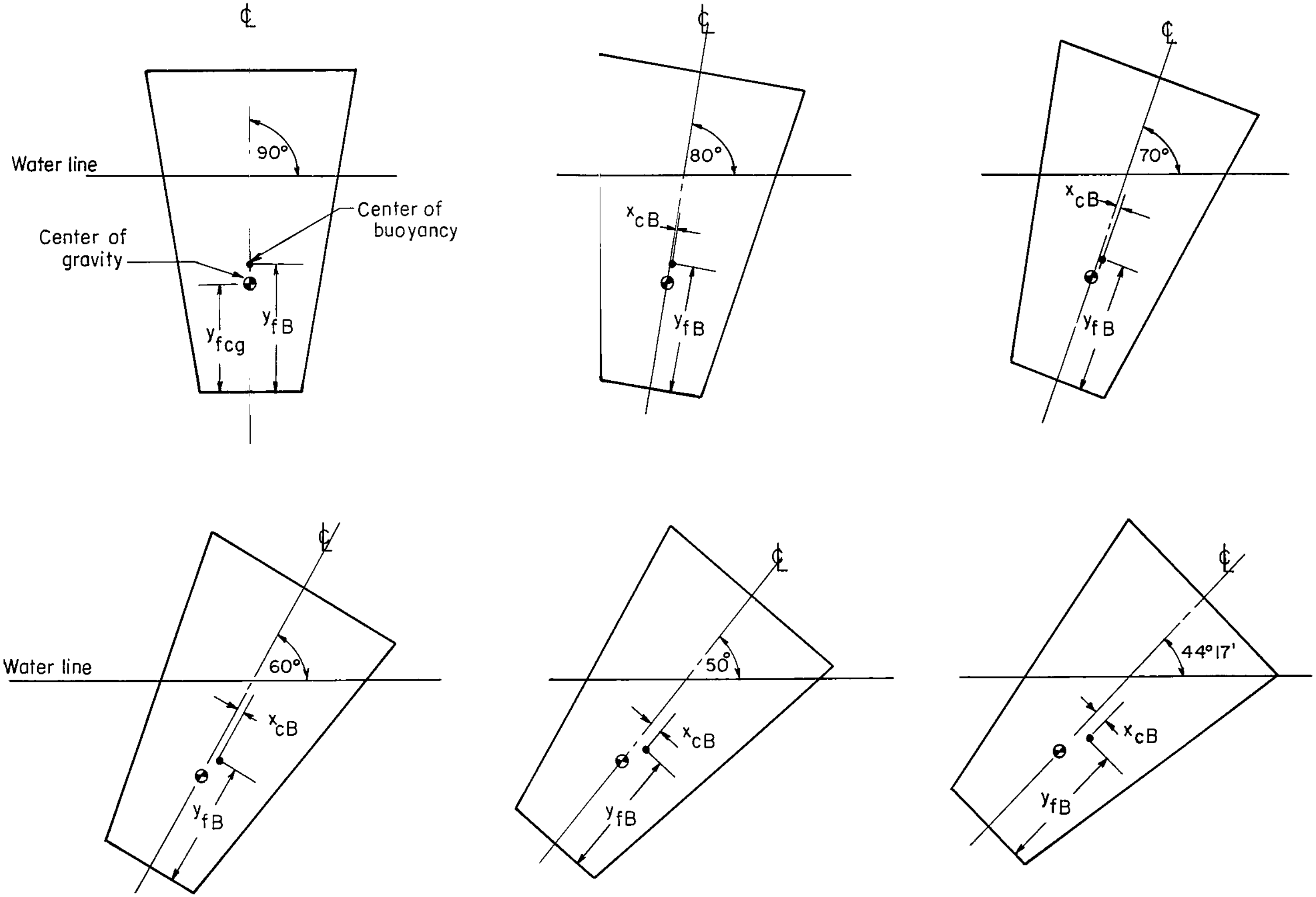


Figure 6.- Center-of-buoyancy locations for several inclination angles, including Φ_{lim}, used for example 2(b).

NASA-Langley, 1966 L-4574

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