Electron Acceleration in the Transition Region Behind the Earth's Bow Shock

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Abstract

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The highly disordered field found by Pioneer I in the transition suggests a possible explanation for the 'spikes' of high-energy electrons found by later satellites. Such a field must develop a great variety of local space-time patterns. Perhaps, occasionally, such a pattern may be roughly like that of a laboratory betatron, in which electron acceleration is much more effective than the 'betatron acceleration' in a spatially uniform field changing with time. If such an effective field pattern develops in the transition region, some of the electrons already in the high-energy tail of the velocity distribution may be accelerated to very much higher energies, and then they may escape from the pattern before reversed induction can take energy away from them. After escape, they may make further gains of energy by Fermi accelerations and related mechanisms. Thus they should spread over a wide part of the transition region, some of them even going out through the bow shock or into the magnetosphere; and if a spike found by any satellite is ascribed to such electrons, the field pattern in which they started to gain high energy was not necessarily very near that satellite. Author

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Introduction

Although many suggestions have been offered already for explaining the presence of supra-thermal electrons in the magnetosphere and the transition region between it and the bow shock, the present paper offers still another suggestion, relating the 'spikes' of such electrons in the transition region to the highly disordered magnetic fields found there.

While such spikes have been detected by several recent satellites, the most conclusive evidence that they are due to electrons seems to be that which was obtained from IMP 1 by Fan et al. [1964] and by Anderson et al. [1964], using electron detectors with lower energy limits of 30 and 45 keV, respectively. Also from IMP 1 came conclusive evidence of the reality of the bow shock, and of its location about as predicted by Spreiter and Jones [1963], through magnetic field measurements by Ness et al. [1964]. These measurements confirmed previous findings that the field in the transition region is very disordered, in comparison with the fields in the magnetosphere and the undisturbed solar wind. Being measurements of averages over intervals of 5.46 minutes, however, they could not show the extent of the disorder as fully as measurements with much higher resolution in time, which were obtained from Pioneer 1 by Sonett [1960, 1962], Sonett et al. [1960], and Sonett et al. [1962]. A typical sample of their data from the transition region is reproduced here in Figure 1. Fig. 1

Presumably any field that changes so fast and irregularly in time must also vary in some corresponding way in space. Thus it may be expected to have a great variety of local space-time patterns. Among these there may occasionally be one that roughly resembles a quarter-cycle of the field in the laboratory betatron invented by Kerst [1941] and treated mathematically by Kerst and Serber [1941]. This suggestion is obviously speculative. It is offered here because Kerst-Serber acceleration, as we may call it, should be very much more effective than most of what is now generally called betatron acceleration, and more widespread in its effects; also because the high-energy spikes, occurring unsystematically, seem to indicate some sort of acceleration that is both transient and fortuitous, as Kerst-Serber acceleration must be.

Kerst-Serber Acceleration

In Kerst's betatron, electrons gained energy by induction due to an axially symmetric field, of strength increasing with time, between the poles of an electromagnet. These electrons were in the fringing field of the magnet, called the 'guiding field,' which was only about half as strong, at any instant, as the 'inducing field' enclosed by the electrons' orbits. Electrons were injected into the guiding field steadily by a dc electron gun with a few hundred volts, while the magnet was excited by ac. Some of the electrons that came in during a short time interval in each cycle, while the field was very weak, were guided into orbits that approached a circle of fixed radius, coaxial with the field. Each of these electrons gained 2.3 MeV of energy, all within the quarter-cycle while the field was getting stronger. If these electrons had been injected into a spatially uniform field, changing with time like the guiding field, they would have gained only about 1 per cent of this energy.

In either field, if the orbit of an electron lies in a plane normal to the field, its kinetic energy W is related at any time to the guiding field B_g and the radius of curvature r_c of its orbit, through an equation used by Kerst and Serber, which may be written in the International System of Units (SI) as

$$W = eV_o\{[1 + (cr_cB_g/V_o)^2]^{1/2} - 1\} \tag{1}$$

with e and c as usual, and with

$$eV_o = m_oc^2 = 511 \text{ keV}. \tag{2}$$

Hence

$$V_o/c = 1.70 \times 10^{-3} \text{ V} \cdot \text{s/m}. \tag{3}$$

Since the SI unit of flux density is the tesla, T, defined as 1 V • s/m², the combination of units in (3) is the T • m. And since the gauss is 10^{-4} T, and the gamma is 10^{-9} T, or 1 nT, V_o/c may be called 1700 gauss cm or 1700 gamma km. In Kerst's betatron, for example, the 2.3 MeV value of W was attained with 1200 gauss for B_g and 7.5 cm for r_c. In space, 120 gamma and 75 km would do as well.

For nonrelativistic energies, writing (1) as

$$W = eV_o[(1/2)(cr_cB_g/V_o)^2 \; . \; . \; .], \tag{4}$$

we may see that acceleration as in Kerst's betatron makes W increase almost in proportion to $B_g{}^2$ if r_c stays constant.

Each electron that gained the 2.3 MeV in that betatron had to make about 200,000 revolutions in its orbit, all in a 'doughnut-shaped glass vessel' coaxial with the field. Stability of orbits was therefore essential. The electrons were injected at about 9.0 cm from the axis of the field and the doughnut, almost tangentially, and the circle toward which their orbits converged was 7.5 cm in radius.

Out in space, with no 'doughnut' and no need for so many revolutions, we still have need for a theorem, proved by Kerst and Serber [1941], about the radial stability of such orbits. In this theorem, the postulates about the field are as follows:

1. On the plane $z = 0$ in an $r\varphi z$ system,

$$\underset{\approx}{B} = \underset{\approx}{1}_z B_z(r, t), \tag{5}$$

with B_z increasing with time t at any r between zero and the maximum r in an orbit;

2. There is a circle $(r = r_o, z = 0)$, such that, at any time,

$$r_o{}^2 B_z(r_o, t) = \int_0^{r_o} B_z(r, t) r \, dr, \tag{6}$$

making the guiding field on this circle just half as strong as its inducing field; and

3. At any r near enough to r_0 for neglect of terms in $(r - r_0)^2$,

$$B_z(r, t) = B_z(r_0, t)(r_0/r)^n, \tag{7}$$

with n constant and algebraically less than 1.

Under these conditions, Kerst and Serber proved that a favorably injected electron would have its r approach r_0, either steadily or with oscillations decreasing in amplitude.

In space, of course, no field can be expected to satisfy these equations; and it seems impossible to say how far a quickly strengthening field can depart from them and yet give a favorably started electron an orbit with r neither approaching zero nor becoming infinite too soon to let the electron gain high energy. Not attempting any exact definition, therefore, we shall use the term Kerst-Serber field for any quickly strengthening field that can do this.

Axial stability of orbits was proved by Kerst and Serber to require the n of equation (7) to be positive. This requirement, however, is not included in the present definition of the term Kerst-Serber field, because their proof for it depended on a practically zero value for curl $\underset{\sim}{B}$ at the electron's orbit, and curl $\underset{\sim}{B}$ is not practically zero in a current-carrying plasma. Weak axial stability may occur in parts of a Kerst-Serber field in the transition region; but axial instability is likely to provide one of the ways for electrons to escape with whatever energy they have gained. Such a likelihood of axial escape was found by Swann [1933], in a theory of electron acceleration in sunspots, which anticipated a special case under Kerst-Serber theory, with circular orbits and $n = 1$.

The requirement for something like the ratio 1/2 of equation (6) obviously rules out any possibility of Kerst-Serber fields in the magnetosphere. In the interplanetary space outside the bow shock, also, there seems to be little reason to expect Kerst-Serber fields, even though Pioneer 1, with its high resolution in time, showed more variability than was shown by IMP 1. Thus the transition region is the only region where the possibility of Kerst-Serber fields should be considered, even on the speculative basis noted above.

In this region, the best chance for occurrence of such fields seems to be in locally convergent flows of plasma, which may be among turbulent phenomena suggested by Ness et al. [1964]. A locally convergent flow toward a line of force, with or without any divergent flow along the line, is well recognized as strengthening the field on this line. Among the reasons given for such strengthening, the one best in line with modern electromagnetic theory is that with high conductivity the electromotance of a closed circuit moving with the plasma is very weak. More specifically, it is very weak, at any instant, in comparison with the electromotance of a stationary circuit with which the moving circuit coincides at this instant. This comparison takes on a special significance in the theory of Kerst-Serber acceleration because an electron can have this acceleration only if its orbit is much more nearly stationary than any circuit that can be called 'moving with the plasma.'

'Moving with the plasma,' however, may be indefinable for a plasma of very low density, as Alfvén and Fälthammar [1963a] have pointed out, because the positive and negative particles making up the plasma may have different ways of moving. An additional obstacle to definition arises in the present case, because of the large and irregular deflections of the particles by the magnetic field; neither the protons nor the electrons are likely to have any clearly definable way of moving. Thus the concept of a circuit moving with this plasma seems so nonrigorous that any conclusions drawn from an assumed zero electromotance in it are merely qualitative.

For this reason, and also to see how the field transfers energy to electrons that acquire Kerst-Serber acceleration, we may consider an alternative approach, also qualitative, in terms of free paths and magnetic deflections. Assuming that the proton and electron temperatures have been brought together in the bow shock, say at a million ^{o}K, we may expect the thermal velocities of protons to be about 160 km/sec, and those of electrons to be about 6700 km/sec. Thus, if the flow velocity is estimated roughly as a few hundred km/sec, a large majority of the protons may be expected to have positive velocity components in the direction of the flow, while the electron velocities will be almost isotropic.

Considering the protons first, therefore, and of them only a few with velocities directed initially toward one line of force, in a plane normal to this line, and using Lorentz-Einstein electrodynamics (not ascribing any force on a particle to motion of the lines) the deflections of these protons are illustrated schematically in Figure 2. Fig. 2 As shown here, their velocities at their points of nearest approach to the 'central line' are all directed clockwise around this line. Of course, other protons, not initially moving directly toward the central line, must have different paths; but if a large majority of the protons have initial velocity components toward this line, the result of their deflections must be a temporary clockwise current around it. This is the sort of a current that is needed for a temporary increase in the field strength on this line and near it.

Since protons with different initial velocities must differ in their minimum distances from the central line, this temporary current must be distributed over a variety of distances from it. Thus the temporarily strengthened field must be somewhat like the field of a thick-walled solenoid, which is not so strong in the middle of the wall as at its inner surface. Such a field seems to have some chance of being a Kerst-Serber field.

Another result of the convergence of the protons is an increase of the proton density in the central region. If the electron velocities were initially isotropic, and the electron density initially uniform, the increased proton density would attract electrons so as to change their velocities toward convergence. Then their deflections would make them contribute to the clockwise current and thus increase the temporarily strong magnetic field.

This attraction of the electrons would, of course, increase their density in the central region, and thus cancel most of any radial component of $\underset{\sim}{E}$ that may have been built up by the protons. So long as the magnetic field is changing, however, there must be an azimuthal component of $\underset{\sim}{E}$. This component, being related to curl $\underset{\sim}{E}$, rather than to div $\underset{\sim}{E}$, is not subject to cancellation by any accumulation of electrons. Lenz's law, requiring this component of $\underset{\sim}{E}$ to oppose the clockwise current during the buildup of the magnetic field, makes it take energy away from all protons moving clockwise during this buildup and from all electrons moving counterclockwise.

Presumably most of this energy goes into the magnetic field, but some of it must be given to such electrons as are moving clockwise. Of these electrons, perhaps a few may be moving fast enough, and be directed well enough, to get into Kerst-Serber acceleration.

Any electrons that do this must receive relatively great quantities of energy from the azimuthal component of $\underset{\sim}{E}$. It is thus that some of the energy taken away from the converging plasma may be given to these relatively few electrons.

Moreover, they must be relatively few, not only because of the above considerations, but also because of the high speeds given to them. Each of them, at high speed, must offset the magnetic effect of many protons or other electrons. Thus it is essential to Kerst-Serber acceleration to high speeds that only relatively very few electrons attain it.

Starting and Stopping Kerst-Serber Acceleration

Assuming tentatively that the electrons in the transition region have been thermalized with the protons in the shock wave, and that they have thus acquired a Maxwell distribution of velocities, we may expect the very few of them that attain Kerst-Serber acceleration to come from the high-velocity tail of this distribution. They may, perhaps, be among the 3/4 of 1 per cent that have energies higher than four times the average. For four times the average, say at 10^6 °K, the Maxwellian function gives 517 eV.

For any such nonrelativistic initial energy W_1, and an initial guiding field B_{g1} and radius of curvature r_{c1}, equation (4) says that

$$r_{c1}B_{g1} = (V_0/c)(2W_1/eV_0)^{1/2}, \tag{8}$$

with V_0/c and eV_0 as in equations (3) and (2). If W_1 is 517 eV and B_{g1} is 1 gamma, as suggested by some of the minima of the field in Figure 1, r_{c1} is 76.5 km.

This value for the radius is consistent with a requirement that any region of convergent flow must be small in comparison with the thickness of the transition region. Thus this 76.5 km may serve for an illustrative example, in the absence of any more definite information on the sizes of convergent flows.

As the field gets stronger, r_c may either increase or decrease, so far as we can tell now, and so may any radius corresponding to the

r_o of equations (6) and (7). Assuming for this example, therefore, that r_c stays constant at 76.5 km, we may calculate the energy at any later time, using equation (1) when this energy is relativistic. Thus, if B_g reaches a value B_{g2} = 50 gamma, suggested by the peaks in Figure 1, equation (1) says that the energy is then W_2 = 747 keV.

This example, however, does not allow for the possibility that the electron may escape from the field because of orbital instability, either radial or axial, before B_g reaches 50 gamma. The Kerst-Serber theorem on radial stability, as stated above, showed it to be positive only if the exponent n in equation (7) was less than 1. Therefore radial stability may be lost if the field changes to a form such as equation (7) would describe with $n > 1$.

The theorem on axial stability made it positive only if n was positive, because, with curl $\underset{\sim}{B}$ zero at the electron's orbit, only a positive n would make the lines of force form a magnetic bottle. With a nonzero curl $\underset{\sim}{B}$, as in a current-carrying plasma, a magnetic bottle is not impossible, but it is not so easily guaranteed. If the flow converges toward one line of force, neighboring lines of force may be parallel to this one for some distance, making axial stability almost neutral. Perhaps the lines of force may form a weak magnetic bottle for a part of this distance; but they must curve away near the ends of the region of convergent flow, and thus cause axial instability there.

With such chances to escape, an electron starting as in the example considered above cannot be expected to gain all of the 747 keV calculated for it. But if B_g increases linearly with time at about 15 gamma/sec, as suggested by some of the peaks in Figure 1, and r_c stays constant, the electron gains more than half a keV in each revolution in its orbit, and 100 revolutions would get it into the ranges of high-energy electrons detected by Fan et al. [1964] and by Anderson et al. [1964], without any need for further gains.

Random Walks After Escapes

Even at the start in that example, the induced electric field exerts a force on the electron, well over a million times the dynamical friction exerted on it electrostatically by the few tens of protons and electrons per cm^3 that are around it; and such friction is well known to decrease with increasing speed of an electron that is already moving faster than the average. After a few revolutions, therefore, the Kerst-Serber electron is practically immune to dynamical friction. After a few more, perhaps, if it escapes, it may be going fast enough to be practically out of danger of being trapped by any other local field. Thus it may wander about, being deflected by local fields, for a long distance.

Of course such deflections are primarily magnetic; but in Lorentz-Einstein electrodynamics we must look to $\underset{\sim}{E}$ for any change in the kinetic energy of the wandering electron. Its wandering may be compared to the random walks postulated in theories of the origin of cosmic rays by Fermi [1949, 1954] and others for atomic nuclei in interstellar space. In Fermi's 'Type A acceleration,' for example, where the local magnetic field is idealized as an axially symmetric bottleneck of lines of force in moving plasma, Alfvén and Fälthammar [1963b] showed that the lines of $\underset{\sim}{E}$ are circles, coaxial with the bottleneck. Thus the energy given to the charged particle is given through the momentum component $p_\perp$, normal to $\underset{\sim}{B}$; all that the $\underset{\sim}{v} \times \underset{\sim}{B}$ forces do is to transfer energy from $p_\perp$ to $p_\parallel$, without changing the sum of the squares of these components.

This $\underset{\sim}{E}$ is calculable from its curl, div $\underset{\sim}{E}$ being zero in this case. In most other cases, however, div $\underset{\sim}{E}$ is not zero; it is not even negligible, as it is often supposed to be. Among such cases are encounters of the charged particles with the 'scattering centers' postulated by Parker [1956], which were described by Morrison [1961] as 'tight bundles of lines of force,' or 'knots or clouds of magnetic field,' with relatively weak fields between the clouds. When such clouds are moving, the parts of $\underset{\sim}{E}$ that are calculable from curl $\underset{\sim}{E}$ and from div $\underset{\sim}{E}$ are apt to be of the same order of magnitude, for reasons discussed by Webster [1961, 1963] with reference to circuits and magnets that are moving at laboratory speeds and are regarded by observers moving with them as uncharged. While the part calculable from curl $\underset{\sim}{E}$, when integrated around a stationary closed loop, would give the right electromotance for the loop, this part alone would not give nearly the right value for the energy transferred to an electron that encounters a magnetic cloud and leaves it without having made a closed loop in it.

Qualitatively, such an energy transfer may be regarded as a move toward equipartition of energy between this electron and the relatively great masses of protons and electrons that make up such clouds. An electron that has escaped from a Kerst-Serber field, after gaining a few keV of energy in it, has a speed higher than c/10, so much higher than the speed of the shocked solar wind that it must make many such moves toward equipartition before being carried far by the wind. Without any data on the sizes and speeds of the magnetic clouds, it seems impossible to estimate the number or effectiveness of these moves, but safe to guess that they may result in considerable further gains of kinetic energy. If they do, perhaps any Kerst-Serber acceleration that stops short of the range of energies found in the spikes may be regarded as a mechanism for injection of electrons into this generalized Fermi acceleration.

With such high speeds, many of the electrons that start to gain high energies in a Kerst-Serber field within the transition region must reach the boundaries of the region in a moderate number of seconds, despite the delays by reversals of their directions in large magnetic clouds or in clouds with strong fields. Any such electron that reaches the bow shock presumably escapes to interplanetary space. Other electrons, reaching the magnetopause, should usually be reflected by the stronger field behind it; but perhaps some of them may enter the magnetosphere through irregularities in its boundary, and become trapped in it. With these ways to escape from the transition region, and with the general drift downwind and the decrease in their number density as they spread, the electrons that start to gain high energy in any one Kerst-Serber field could not be expected to account for more than a 'spike' in the time graph of the number found by any one detector.

If such a spike does result in this way from one Kerst-Serber field, that field may have been fairly far from the detector. If, for example, the Kerst-Serber field is less than 100 km in radius, as in the example discussed above, though perhaps many hundred km in length along its lines of force, and its electrons are detectable at perhaps 10,000 km distance, the volume over which they are detectable is more than 10,000 times the volume of the Kerst-Serber field. Thus, if the spikes are ascribed to Kerst-Serber fields, either directly or through subsequent Fermi accelerations, the development of such a field covering any given point must be a very infrequent event - as we should expect - less than 10^{-4} as frequent as the detection of a spike by a detector at that point.

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Figure Legends

Fig. 1. A typical section of magnetometer data taken in the region of 12 to 13 R_e. The points which are connected with the line are corrected for electronic gain lag in the magnetometer amplifier, whereas the unconnected points are uncorrected.

Fig. 2. Paths of protons initially directed toward one line of $\underset{\sim}{B}$.

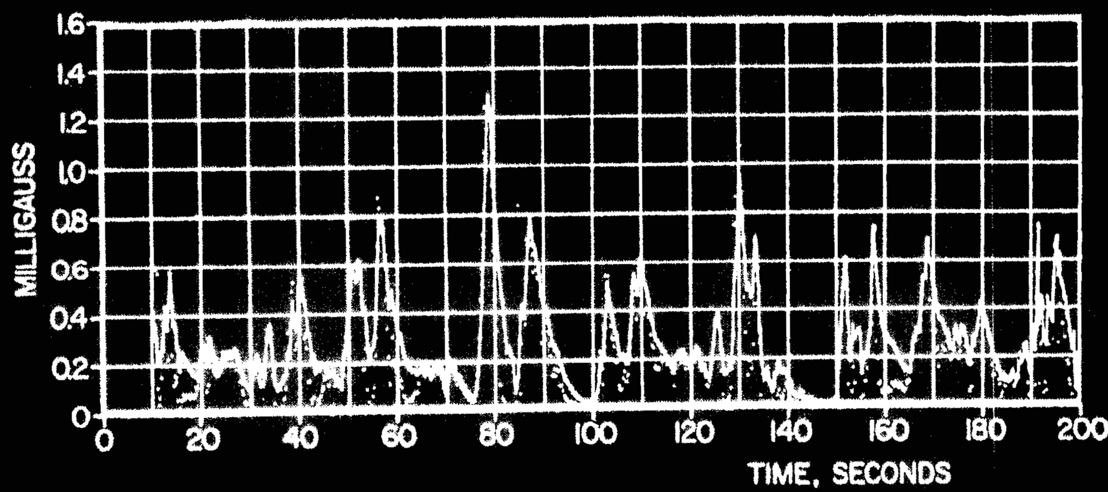
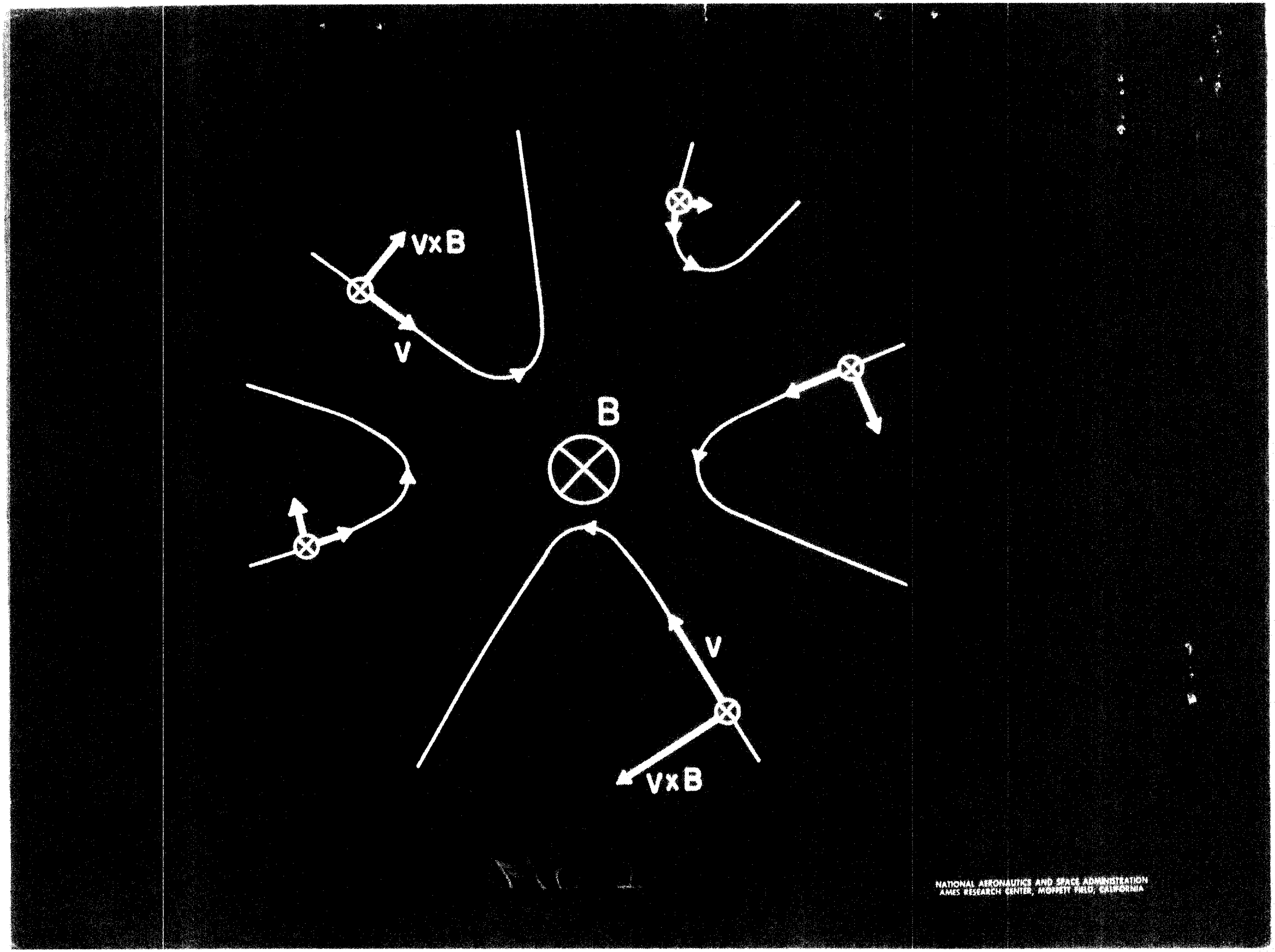

1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0
MILLIGAUSS
0
20
40
60
80
100
120
140
160
180
200
TIME, SECONDS

Fig. 1



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