A COMPARISON OF FRACTURE MECHANICS AND NOTCH ANALYSIS

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Presented at the American Society for Testing and Materials
Special Committee on Fracture Testing

Washington, D.C.
January 13, 1965

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INTRODUCTION

The "Fracture Mechanics" developed principally by Dr. George Irwin has been for a number of years an invaluable aid in the solution of grave problems of national importance. It has served to focus ideas and has provided concepts and experimental techniques for determining material properties relevant to the problem of crack strength. The American Society for Testing and Materials is naturally and vitally concerned with this subject, and the reports of the Fracture Committee of the ASTM are a much appreciated central source of information on the current state of the art in Fracture Mechanics (ref. 1).

While Fracture Mechanics was being developed, the Langley Research Center of the NASA was engaged in research aimed at improving fatigue design methods for aircraft. One result of this work was a method for predicting fatigue factors for notches (ref. 2). Somewhat later, it became necessary to consider the effect of fatigue cracks on the static strength. In view of the conditions characteristic of aircraft design, it was clear at the outset that it was highly desirable to develop a capability for structural analysis rather than a method for ranking materials. The problem as a whole thus fell about halfway between the conventional areas of materials science on the one hand and structural science on the other hand, an area that was a no-man's land for many years and still is, to a considerable extent.

In the initial NASA-Langley studies of the crack strength problem, transition temperature, Charpy test and other test methods were immediately found to be either inapplicable or inadequate. Fracture Mechanics was considered, but was judged to have insufficient accuracy as a design method. Finally, it was found that the previously developed method for predicting fatigue factors could be extended to handle the static-strength problem for notches, with cracks included as a limiting case. The resulting method of Notch Analysis (refs. 3, 4, and 5) thus has greater scope as well as greater accuracy than Fracture Mechanics with one major exception: in its present state of development, Notch Analysis cannot deal with static strength problems in thick parts - its application is confined to sheet-metal parts.

It is the purpose of the following presentation to compare Fracture Mechanics and Notch Analysis first in a general way with respect to fundamentals and to scope, and then in a more detailed way by comparisons with test results.

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SYMBOLS

F_{TU}	tensile strength, ksi
K_c	fracture-toughness parameter (based on critical crack length), ksi-in.$^{\frac{1}{2}}$
$\bar{K}_c$	"nominal" K_c (based on initial crack length)
K_F	fatigue notch factor, experimental
K_N	fatigue notch factor predicted by Notch Analysis
K_T	theoretical notch factor
S_N	stress on net section, ksi
S_Y	yield strength, ksi
a	one-half initial crack length, in.
w	width of specimen, in.
ρ'	size-effect constant used in Notch Analysis, in.

NOTCH ANALYSIS "MODES OF OPERATION"

Before proceeding with the comparisons, it should be pointed out that Notch Analysis has two main "modes of operation."

One mode may be called the "prediction mode" or "class mode." In this mode, the need for materials tests on cracked or notched specimens is eliminated because general relationships have been established for certain classes of materials. However, these relationships are based on empirical observations, not on precise laws; consequently, they are of limited accuracy. In many applications, it may therefore be advisable to determine the constants by specific tests rather than by use of the general relations. This may be called the "basic mode of operation."

When Notch Analysis is applied in the "basic mode of operation" to the problem of sheet specimens with cracks, it is similar to Fracture Mechanics. The similarity becomes very close when attention is confined to the region of validity of Fracture Mechanics as currently defined, that is, cases in which the net-section stress is less than 0.8 yield stress. Nevertheless, there is a crucial difference between the two methods, which will be discussed later.

COMPARISON OF FUNDAMENTALS

A comparison of the fundamentals of the two methods is shown in figure 1.

The first item of comparison is the basic approach. Notch Analysis is based on the engineering concepts that a notch causes a stress concentration, and that failure will take place when the peak stress at the notch reaches a limiting value. Fracture Mechanics is usually stated to be an energy approach, and is therefore often held to be fundamentally different. However, this question has been examined very carefully by J. L. Sanders (ref. 6), whose closing sentences are: "The path independence of the (contour) integral makes it clear that the amount of strain energy available in the cracked structure is irrelevant. The quantity that does matter is the strength of the square root singularity at the tip of the crack. This strongly suggests that the Griffith-Irwin approach to fracture mechanics via energy concepts is equivalent to an approach via stress concentration factors." It appears, then, that no fundamental difference really exists between the two methods in this respect.

The second item of comparison is the theory used. Fracture Mechanics uses linear elastic theory, which is unrealistic for a failure theory for ductile materials. Notch Analysis also begins with linear elastic theory. However, the application of a secant-modulus correction factor converts it, in effect, into an approximate nonlinear elastic-plastic theory. This results in a degree of realism which is adequate, first, to describe the peak stress at the tip of the crack, where failure begins, and second, to deal with any level of net-section stress.

The third item of comparison is the principles on which the methods are based.

Fracture Mechanics is based on the principle, or proposition, that the quantity $S_G\sqrt{a}$ may be regarded as a materials constant. This proposition was advanced by Griffith for brittle materials. It is derived from the stress-concentration concept on the basis of the assumption that the exact expression for the stress concentration factor (S.C.F.)

$$\text{S.C.F.} = 1 + \text{Const} \times \sqrt{a} \tag{1}$$

can be replaced by the approximate expression

$$\text{S.C.F.} = \text{Const} \times \sqrt{a} \tag{2}$$

In the problem investigated by Griffith (strength of glass), the S.C.F. involved was of the order to 100. Use of the approximate expression (2) therefore results in an error of only 1 percent, which is clearly tolerable.

In aero-space pressure vessel design, it is usually desired that the vessel be capable of carrying the yield stress in the proof test. Since the yield is about 2/3 of the ultimate or more, the S.C.F. of the flaws must be less than 3/2. Clearly, deleting 1.0 from 1.5 by using expression (2) instead of expression (1) introduces a very large error. If a limit of 10 percent is imposed on the error, the S.C.F. of the flaws must be greater than 10, that is, the application of the method must be restricted to stress levels less than 1/10 of the ultimate. This would be an extremely severe restriction for materials testing and would disqualify the method for practically all design analysis work.

The use of the "plastic zone correction" modifies the error considerations somewhat, in a manner that cannot be readily assessed in a general way. The numerical examples given later will show the actual errors incurred. Notch Analysis avoids all these errors by retaining the exact expression (1). <u>This is the crucial difference between Fracture Mechanics and Notch Analysis</u> as far as analysis of cracked sheet is concerned.

Notch Analysis utilizes a principle of size-effect correction. Use of this principle enables the method to deal with notches having finite radii, under either fatigue or static loading, resulting in wide scope. Use of the size-effect principle also results in a realistic stress computation for the point at the tip of a crack, permitting use of the tensile strength of the material as a failure criterion.

Fracture Mechanics uses linear elastic theory and conventional continuum mechanics, without invoking a principle of size effect. For a crack, this method of attack gives an infinite stress at the tip, a result which cannot be utilized to arrive at a failure criterion. In order to obtain a solution, Fracture Mechanics disregards the action not only at the tip of the crack, but in the entire plastic zone and uses the stress in the elastic region beyond the plastic zone as "index" for a failure criterion. From a physical point of view, this is hardly a very satisfying procedure.

COMPARISON OF SCOPE

A comparison of scope of the two methods is given in figure 2. The areas of notch effects under fatigue loading, notch effects under static loading, and crack effects under static loading are roughly of equal size and importance. Notch Analysis handles all three problem areas by one method. Fracture Mechanics handles only one of the three. The problem of cracks as stress raisers under fatigue loading can also be handled by Notch Analysis, but is not listed in figure 2 because it is only of academic interest.

The stress level must be less than 0.8 yield for Fracture Mechanics to be applicable, while Notch Analysis can be used at any stress level. As a consequence, Notch Analysis can always be used as a design analysis method, while Fracture Mechanics can be used as a design analysis method only for structures which fail at less than 0.8 times the yield stress.

The scope and usefulness of Notch Analysis is greatly extended in the "class mode of operation," which permits making estimates of crack strength when only the standard properties of a material are known (tensile strength, elongation, and Young's modulus). Fracture Mechanics has no equivalent mode of operation.

COMPARISONS WITH TESTS

Notches in fatigue.- Figure 3 shows fatigue data obtained on rotating beams of differing sizes with semicircular grooves. The two upper plots are for geometrically similar specimens; the two lower plots are for specimens in which the size of the groove was constant, while the diameter of the specimens increased. Predictions of the notch factors (dashed curves) are made by Notch Analysis in the "class mode of operation." The two upper plots, being for geometrically similar specimens, demonstrate directly the nature and importance of size effect.

Notches under static load.- Figure 4 shows static notch-strength data obtained on H-11 steel. Data from these tests were presented in the first report of the Fracture Committee (ref. 1), in the form of a plot of K_c versus $\sqrt{\rho}$. The present figure shows net-section stress divided by tensile strength rather than K_c; however, there is fairly close proportionality between S_N and K_c, and the two plots therefore appear to be almost identical as far as the test points are concerned.

Notch Analysis was used as follows to compute the curves shown. The average of the three tests with fatigue-cracked specimens ($\rho = 0$) was used to determine the size-effect constant ρ'. From handbook information, the elongation was estimated to be 9 percent. Since this estimate was approximate, being for a slightly different heat treatment, calculations were made assuming $e = 8$ percent and $e = 10$ percent, bracketing the estimated value and giving an indication of the sensitivity of the calculation.

By a comparison of the test points with the computed curves, Notch Analysis would arrive at the following conclusions:

(a) The tests on specimens with radii of 2, 3, and 4 mils, being in good agreement with the curves, indicate that these tests are consistent with those on the fatigue-cracked specimens by giving the same number for the size-effect constant ρ'.

(b) For the radius of 0.6 mil, the scatter between points in conjunction with an average far below the curve indicates that the machining of the radius was probably out of control, and that these results should be discarded.

(c) The results for the 1-mil radius are slightly low.

Thus, Notch Analysis makes full use at least of the specimens with 2-, 3-, and 4-mil radius. Fracture Mechanics, on the other hand, can only state that all these Vee-notch tests are useless for the purpose of obtaining K_c values, because all the radii are too large to simulate cracks. (The K_c value derived formally for the 4-mil radius is five times larger than the value derived for the cracked specimens.)

The application of Notch Analysis to Vee-notch tests, as exemplified here, can serve two very useful purposes:

(a) To extract information from the tremendous number of tests that have been made, information that would permit meaningful comparisons between different materials.

(b) To eventually stop the widespread practice of making direct comparisons between results obtained on specimens of different configurations - comparisons which generally are completely meaningless.

Cracks in sheet under static load.- Two sets of data for aluminum-alloy sheet will be examined, one for 2024-T3, one for 2219-T87 alloy.

The data for the 2024-T3 are shown in figure 5. Net-section stresses are plotted against crack length for specimen widths of 35, 12, and 2.25 inches. No coupon data for the material had been obtained; Notch Analysis was therefore used in the "prediction mode," using as input data typical material properties obtained from the materials manufacturers handbook. It may be seen that the predictions are in good agreement with the tests for all widths.

Next, K_c values were determined by fitting the test curves at a crack-length ratio of 0.3. Since only the initial crack length was known, the K_c values derived are "nominal" values and are denoted by $\overline{K}_c$. At this point, it should be remarked that this figure was originally prepared several years ago, when it was generally believed that computations of K retained useful comparative validity until $S_N = 1.1S_Y$. The $\overline{K}_c$ numbers obtained are 60, 90, and 105, which represents a rather large spread.

Under the current $0.8S_Y$ rule, the $\overline{K}_c$ numbers obtained from the 12-inch- and the 2.25-inch-wide specimens would be rejected as invalid, so no comparison of $\overline{K}_c$ numbers would be possible. However, a design analyst aware only of the results obtained on the 35-inch width might use the value of $\overline{K}_c = 105$ obtained on this width, which is definitely valid by all rules, to predict the results for a 12-inch and a 2.25-inch width (using standard K_c formulas). The results are shown by stars. The result for the 12-inch width would be declared invalid, being equal to the yield stress. The result for the 2.25-inch width would be declared valid; but it is evidently a rather poor prediction.

More recently, a set of tests on 2219-T87 aluminum-alloy sheet (0.1 inch thick) was performed by Boeing Aircraft and submitted to the MIL-HDBK-5 Committee for their consideration. This is an excellent set of data with very high internal consistency and was therefore used as basis for a detailed study.

The study is presented in terms of K_c. However, since calculations were made by Notch Analysis methods, the nominal $\overline{K}_c$ based on initial crack length was used as in the study of the 2024-T3 data. A comparison of K_c and $\overline{K}_c$ for all valid points showed that the ratio was 0.95 ± 0.04 for cracks 4 inches long or longer (14 points); for cracks less than 4 inches long, the ratio was up to 10 percent higher. Since the study is only concerned with trends rather than absolute values, the use of $\overline{K}_c$ instead of K_c should therefore have no significant effect.

Figure 6 shows plots of $\overline{K}_c$ (experimental values) against crack length for specimen widths of 24 and 48 inches. The 0.8 yield-stress rule eliminates two of the low points for short cracks as indicated, but does not eliminate any of the results for the long cracks. No rule appears to exist for eliminating long cracks; however, some rule should be used because it is well known that the $\tan(\pi a/w)$ formula used in Fracture Mechanics breaks down for long cracks. In this study, the arbitrary rule was adopted of disregarding all cracks longer than 50 percent of the width.

Even with the two rules in use, the test points in figure 6 still show quite a variation. Some additional rule must be used for selecting values if one wishes to make a study of width effect. As a first choice, it was decided to select points having a crack-length ratio of 0.33, very close to the ratio recommended by the ASTM Fracture Committee for specimen design.

Figure 7 shows $\overline{K}_c$ values for the chosen ratio $2a/w = 0.33$ plotted against specimen width. The lowest test point would be declared invalid by the 0.8 yield rule, but the other three are valid. It will be seen that there is a strong and systematic increase of $\overline{K}_c$ with width. Moreover, the test points are in very close agreement with the curve calculated by Notch Analysis. (Notch Analysis was used here in the "basic mode"; the appropriate constant was determined for all 24 tests in the series, and the average value was used for the calculations).

Fracture Mechanics contends - or assumes - that the curve becomes level for widths greater than about 18 inches. The Notch Analyses calculations, which are supported by close agreement with the test points, show that the curve still has a strong slope at a width of 48 inches. Considering only valid test points, the spread from the lowest to the highest K_c value is from 80 to 115.

Actual structures now are up to 300 inches long. However, extending the curve for a fixed crack length ratio of 0.33 to such widths would be unrealistic, because a crack 100 inches long would be an unrealistic case. Another set of calculations was therefore made for various fixed crack lengths; the results are shown in figure 8.

The figure shows curves calculated by Notch Analysis in full lines wherever the $\overline{K}_c$ values are valid. Test points are spotted in for the regions of greatest interest (lowest and highest). The curve for $2a/w = 0.33$ in figure 7 would form (approximately) an upper envelope for the curves.

Inspection of figure 8 shows that a test engineer, obeying both the 0.8 yield stress rule and the $2a/w < 50$ percent rule, could obtain valid values of $\overline{K}_c$ varying from 72 to 113, a spread of 56 percent. The spread increases to 79 percent if widths up to 300 inches are considered, which is within the size range of present-day structures to which these data might be applied.

Similar calculations have been made for more ductile alloys and for less ductile alloys. The former calculations simply confirm the well-known fact that the test engineer is forced to use larger and larger widths to obtain valid results; for the most ductile aluminum alloys used structurally, even the 48-inch width is not enough. As example for less ductile alloys, two materials were chosen. Both were assumed to have a tensile strength of 83 ksi, equal to the typical value for 7075-T6. The elongations assumed were 11 percent and 8 percent, respectively; the former is the typical value, the latter the minimum specification value for 7075-T6. For the material with 11-percent elongation, the ratio of maximum to minimum valid $\overline{K}_c$ was found to be about 2.2:1. For the material with 8-percent elongation, the ratio of maximum to minimum valid $\overline{K}_c$ was about 2.8:1. It should be emphasized that these variations in $\overline{K}_c$ are due entirely to the inherent weakness of the Griffith proposition, because the calculations assume in each case ideal material with zero scatter in properties.

CONCLUSIONS

The comparative study presented should afford food for considerable thought. The difference in scope of the two methods is worthy of notice. The variation of the notch-toughness constant K_c with specimen dimensions should be cause for grave concern, since this variation amounts to a factor of about three precisely for those aluminum alloys which are in greatest need of having their notch toughness well defined and controlled. Since the variation is due to the use of the Griffith proposition, a searching reappraisal of this proposition appears to be in order for any material which is expected - in actual use - to develop more than about one-tenth of its tensile strength.

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FUNDAMENTALS

FRACTURE MECHANICS	NOTCH ANALYSIS
BASIC APPROACH	
ENERGY (EQUIVALENT OF STRESS CONCENTRATION)	STRESS CONCENTRATION
THEORY	
LINEAR ELASTIC (UNREALISTIC)	APPROXIMATE NONLINEAR, ELASTIC-PLASTIC (REALISTIC TREATMENT OF PEAK STRESS AND $S_N > S_Y$)
PRINCIPLES	
(a) $S_G \sqrt{a}$ = CONSTANT (QUESTIONABLE IF $S_N > 0.1\, F_{TU}$)	SIZE EFFECT (PERMITS TREATMENT OF FINITE RADIUS)
(b) PLASTIC-ZONE CORRECTION	

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Figure 1.- Comparison of fundamentals.

	FRACTURE MECHANICS	NOTCH ANALYSIS
NOTCHES, FATIGUE	NO	YES
NOTCHES, STATIC	NO	YES
CRACKS, STATIC	YES	YES
STRESS LEVEL	$S_N < 0.8\ S_Y$	ANY
DESIGN ANALYSIS	ONLY IF $S_N < 0.8\ S_Y$	YES

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Figure 2.- Comparison of scope.

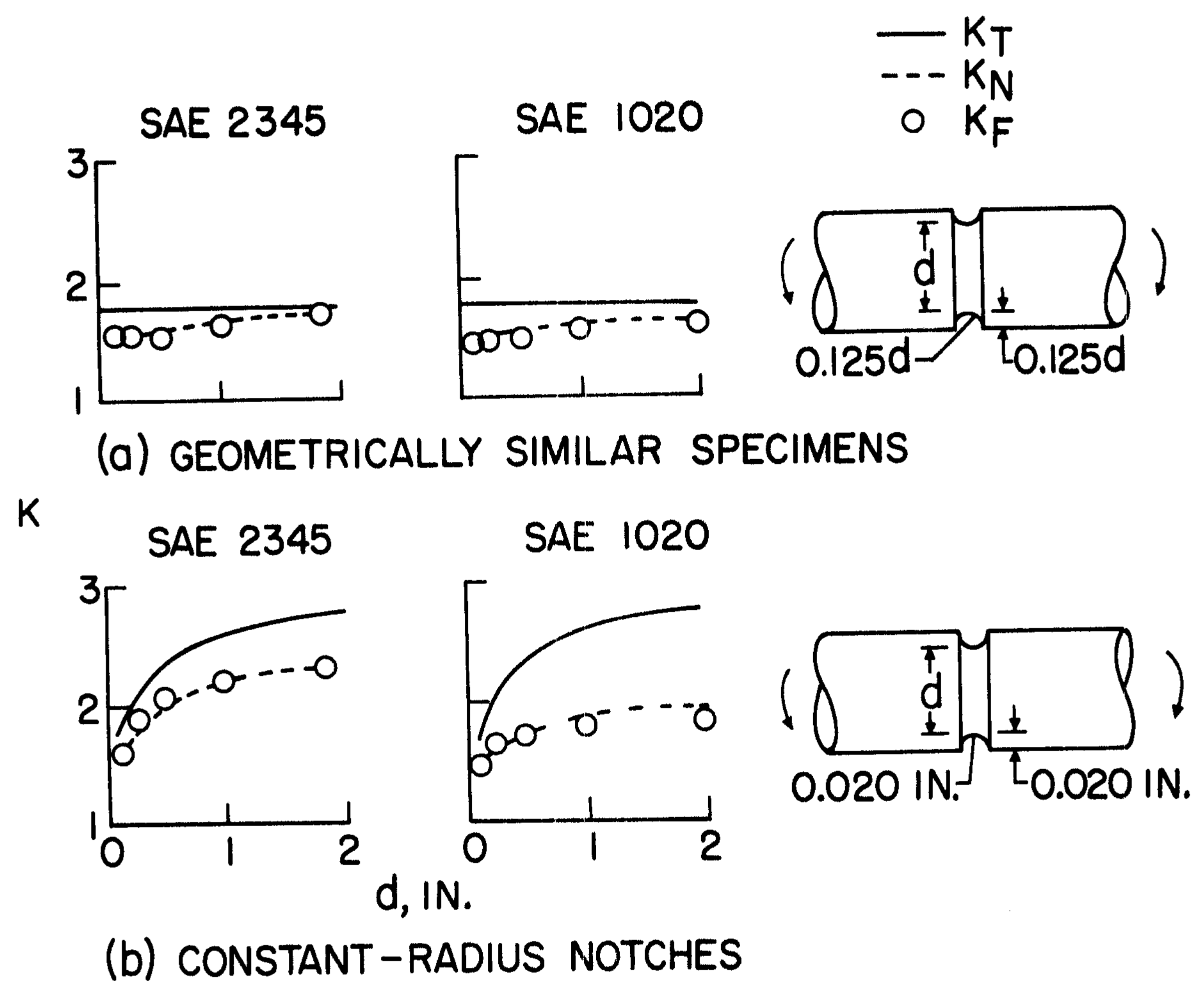

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Figure 3.- Notch-fatigue factors, low-alloy steel.

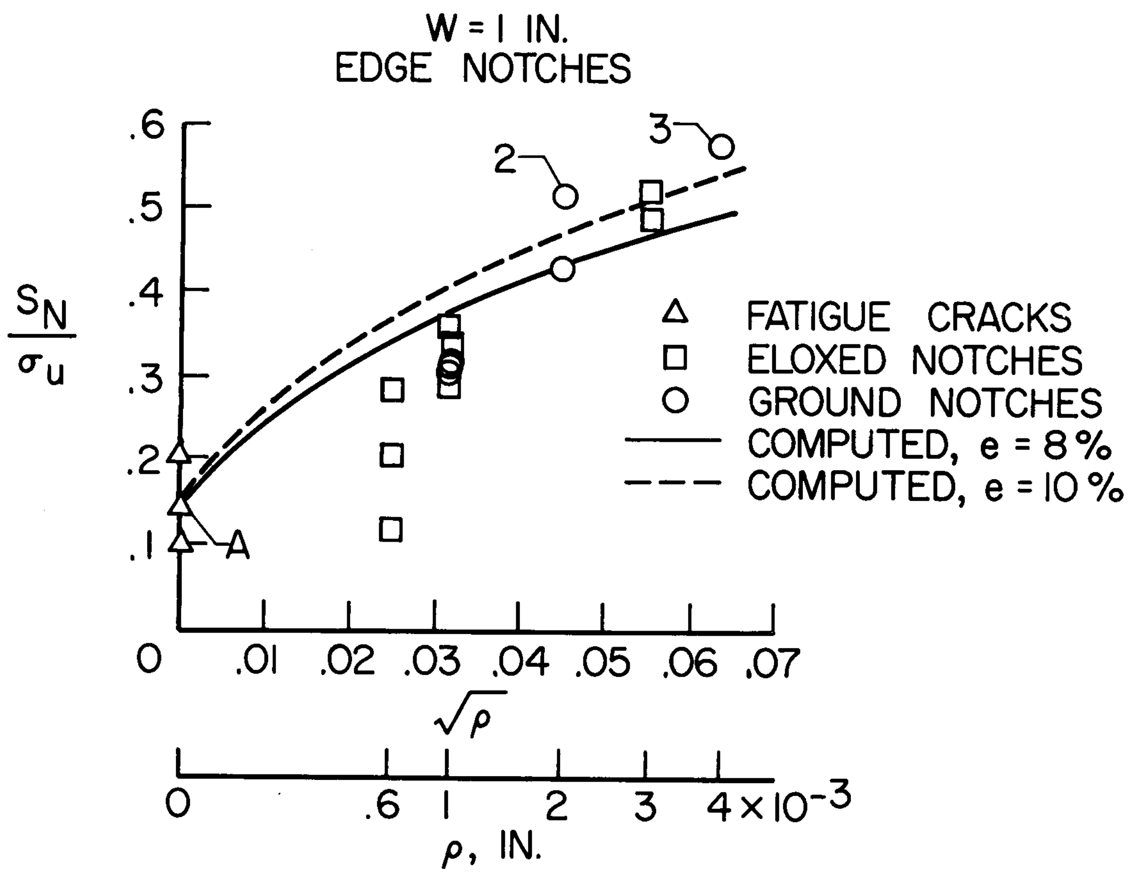

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Figure 4.- Notch strength ratios, H-11 steel.

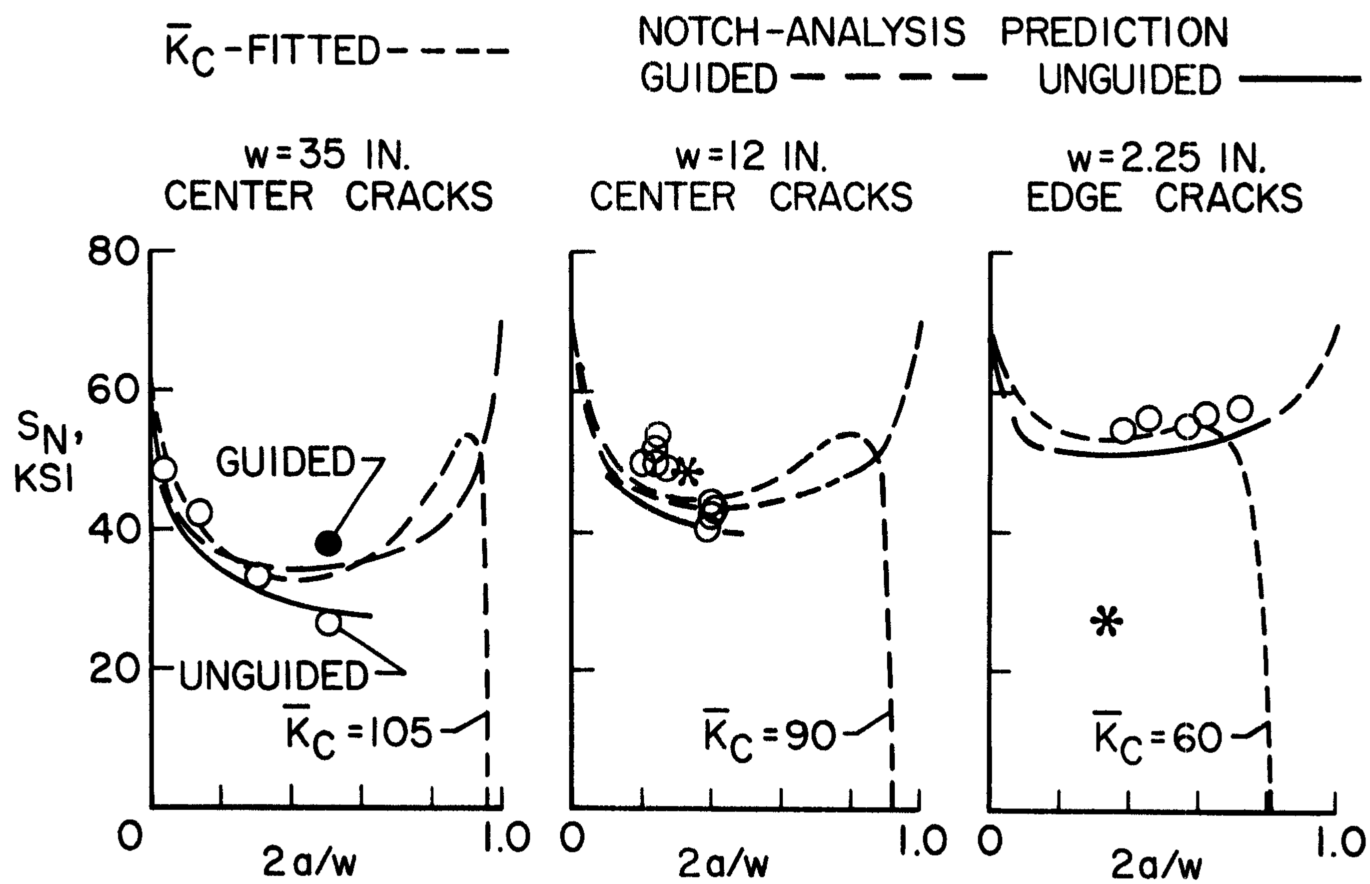

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Figure 5.- Crack strength of 2024-T3 sheet.

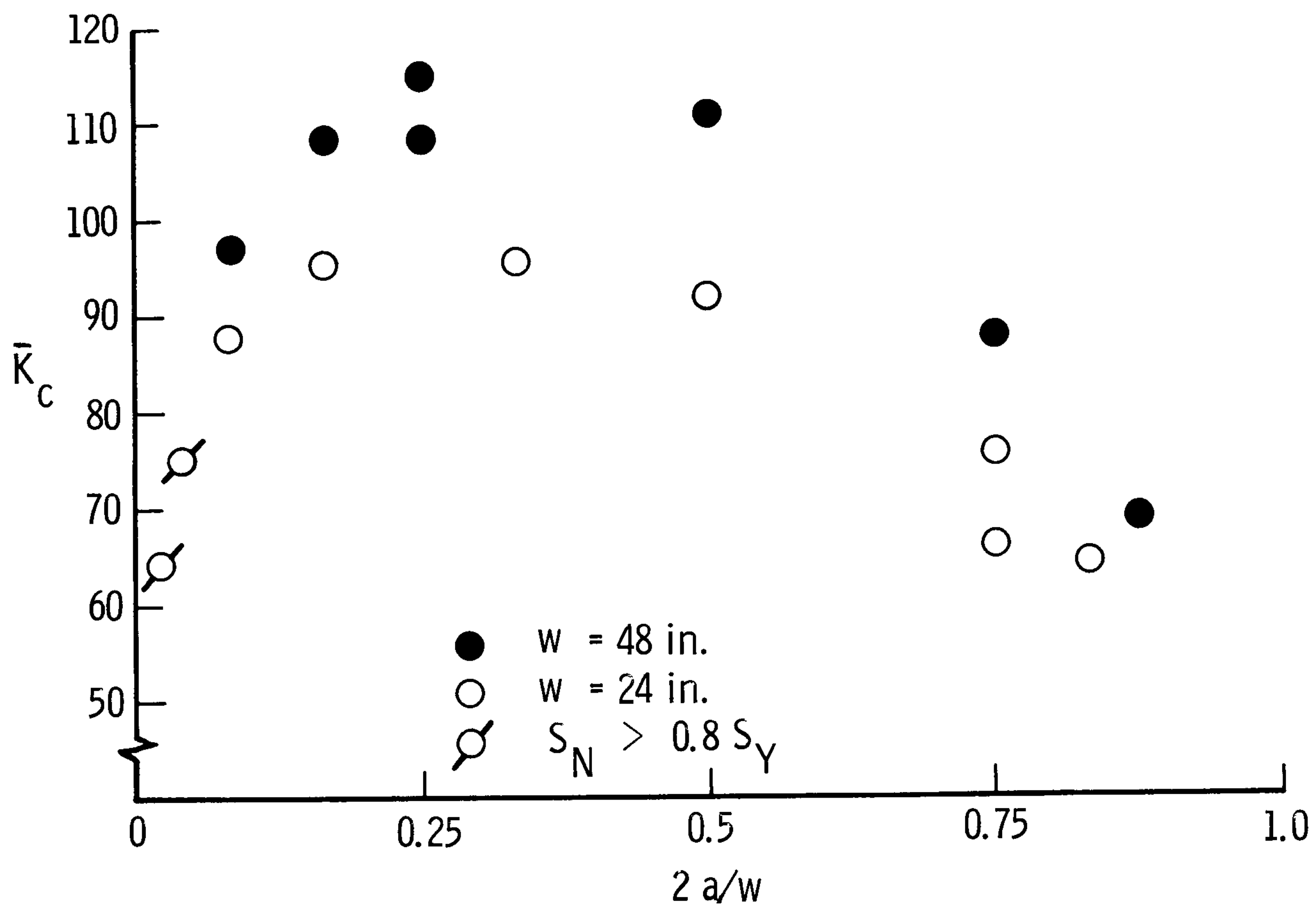

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Figure 6.- Nominal K_c for 2219-T87, w = 48 and w = 24 inches.

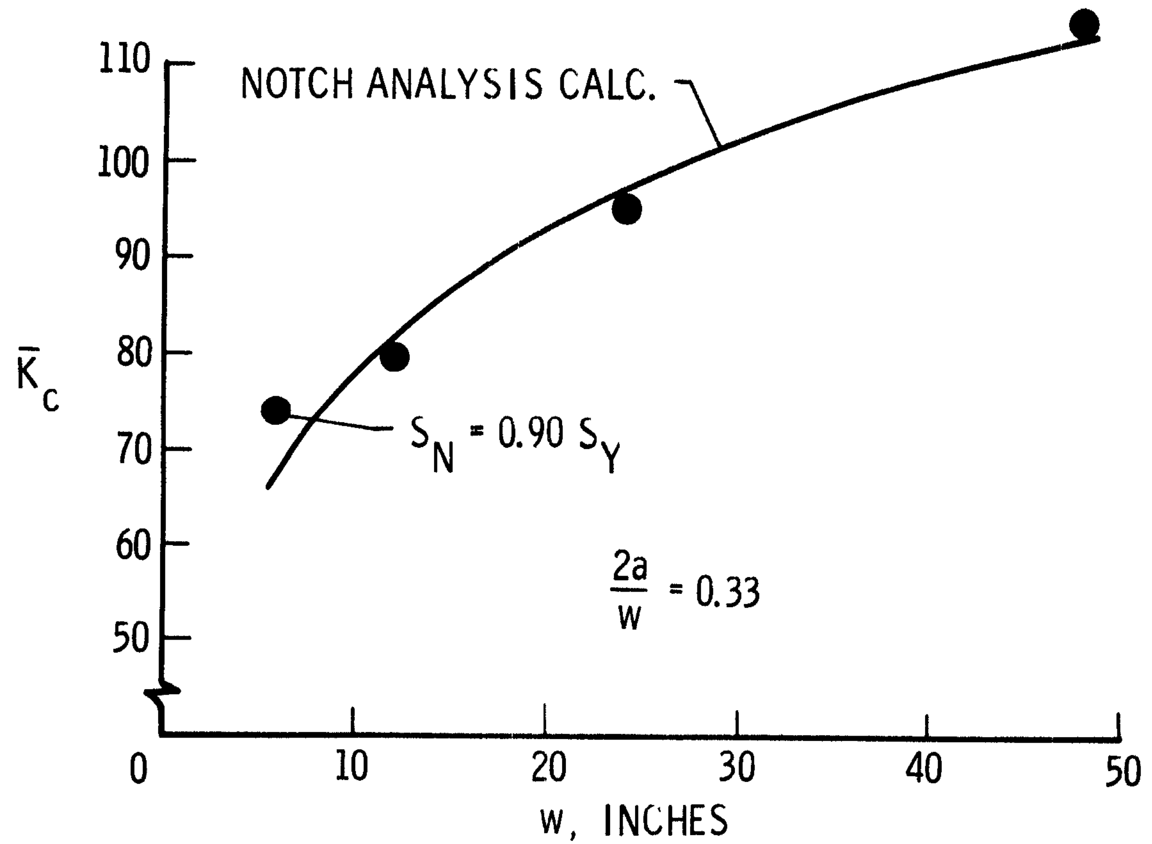

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Figure 7.- Nominal K_c for 2219-T87, $2a/w = 0.33$.

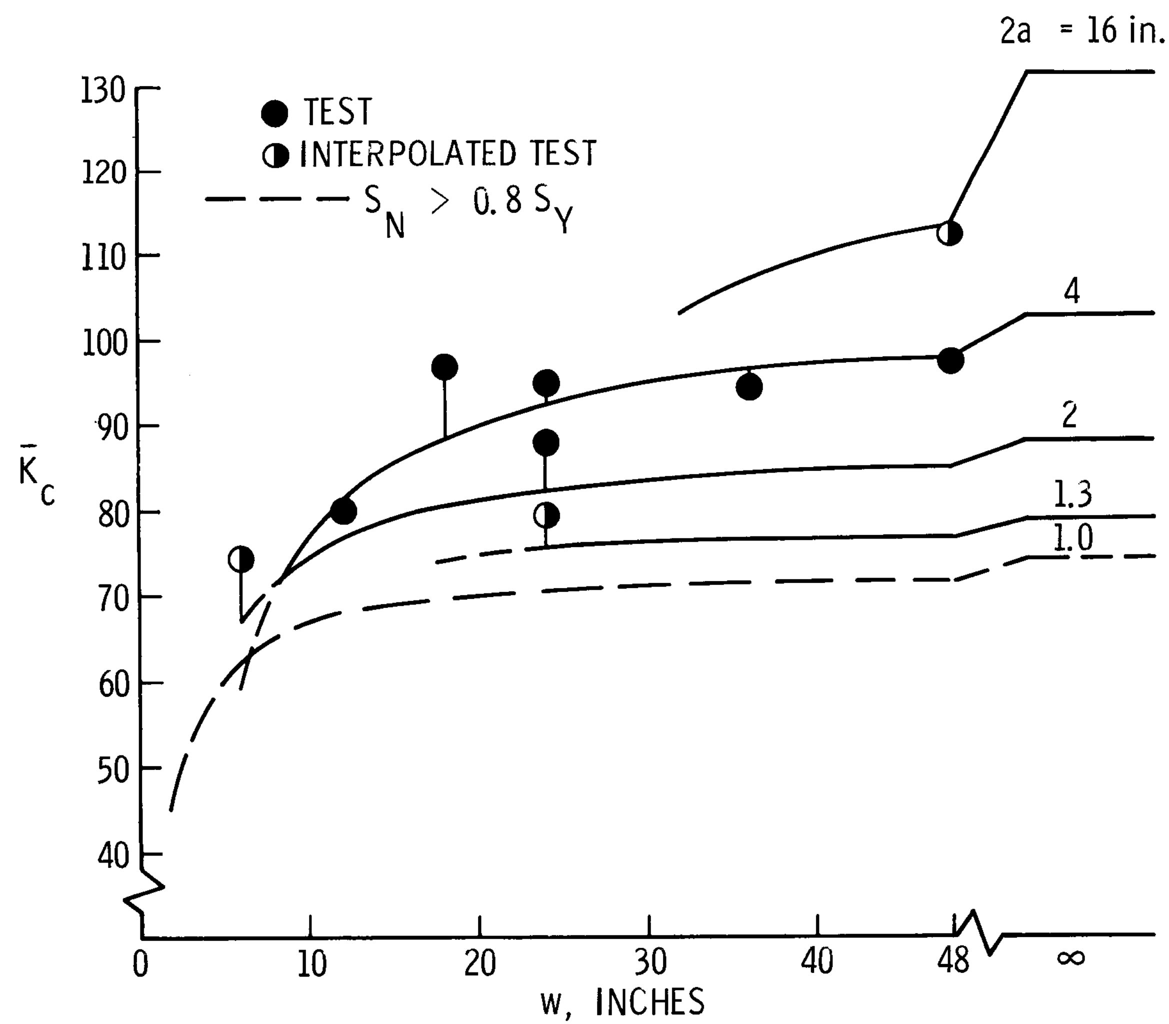

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Figure 8.- Nominal K_c for 2219-T87, 2a = 1.0 to 2a = 16 inches.

NASA-Langley, 1965