NASA TECHNICAL NOTE

NASA TN D-3404

NASA TN D-3404

A METHOD OF TRAJECTORY OPTIMIZATION BY FAST-TIME REPETITIVE COMPUTATIONS

by Rodney C. Wingrove, James S. Raby, and D. Francis Crane

Ames Research Center
Moffett Field, Calif.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • APRIL 1966

NASA TN D-3404

A METHOD OF TRAJECTORY OPTIMIZATION BY FAST-TIME REPETITIVE COMPUTATIONS

By Rodney C. Wingrove, James S. Raby,
and D. Francis Crane

Ames Research Center
Moffett Field, Calif.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

For sale by the Clearinghouse for Federal Scientific and Technical Information
Springfield, Virginia 22151 – Price $0.30

A METHOD OF TRAJECTORY OPTIMIZATION BY FAST-TIME REPETITIVE COMPUTATIONS

By Rodney C. Wingrove, James S. Raby,
and D. Francis Crane
Ames Research Center

SUMMARY

This report presents a perturbation method of computing optimum trajectories wherein control impulse response functions are determined by fast-time repetitive computations of the state equations. This method does not require the solution of the auxiliary set of adjoint equations used with other perturbation methods.

The mechanization of this computing method on a hybrid computer is discussed and an application to the steepest descent optimization of reentry trajectories is presented. In this example, the vehicle is to arrive at a desired range, the heat input to the vehicle is to be minimized, and the control is to remain within specified constraints. Optimum trajectories for this example could be obtained in about 8 minutes of computing time.

INTRODUCTION

It is important for space vehicle trajectories to be near optimum in the sense that some quantity is either maximized or minimized. For example, in reentry the trajectory to desired terminal conditions is near optimum when the total aerodynamic heating is a minimum. This report will consider a method for finding the time histories of nonlinear controls that correspond to optimum trajectories. Several perturbation methods, such as the calculus of variations, applications of the maximum principle, and direct steepest descent, have been considered for solving this control optimization problem. Reference 1 contains a good review of these various methods and reference 2 gives several analog and digital computing techniques for implementing them.

In principle, each of these techniques should give satisfactory results, but it has been found that for many trajectory problems the computer mechanizations are cumbersome and require programs that are difficult for engineers to formulate. The computing method to be reported herein was investigated in an attempt to alleviate these difficulties and to provide a more direct way of computing optimization solutions.

In previous optimization studies using perturbation techniques the computations have involved the dynamic solution of two sets of equations: (1) nonlinear state equations and (2) linear adjoint equations. The method to be reported herein differs in that only the solution of the nonlinear state equations is used. The response of given functions (e.g., terminal error or quantity to be optimized) to a control impulse is determined along the trajectory by fast-time repetitive computations rather than by a solution of the adjoint equations. Since auxiliary adjoint equations are not needed, the investigator should understand the optimization process more easily; also the computer program should be simpler. However, this new alternate computing method does require many solutions of the state equations. This task of computing a large number of dynamic solutions is ideally suited to high-speed repetitive hybrid computation as will be considered herein.

This report will present one application of this computing technique; that of trajectory optimization using the steepest descent method (refs. 3 and 4). The mechanization of this method on a hybrid computer will be discussed and results will be presented to illustrate the use of this procedure in the optimization of reentry trajectories. For the interested reader appendix A illustrates the relationship of the impulse response functions computed in this report to the solutions obtained with the adjoint equations and to the maximum principle of optimization. This appendix also provides a background for understanding the steepest descent optimization equations.

NOTATION

The following notation is used in the body of the text. Additional symbols are described as they are introduced in the appendixes.

$\frac{L}{D}$ control value of lift-drag ratio

n number of storage points in control time history

t time

t_f final time

t_o initial time

Δt time increment of control impulse

u control function

Δu control impulse

φ cost function at final time

$\Delta\varphi(t)$ change in cost function at final time due to control impulses at time t

ψ state value at final time

ψ_d desired state value at final time

$\Delta\psi(t)$ change in state value at final time due to control impulses at time t

METHOD

The method of steepest descent (refs. 3 and 4) is an iterative procedure that has been used for optimizing trajectories. The process commences with any nonoptimal trajectory from which a slightly improved one is derived. The improved trajectory is then used as a new nominal trajectory, and the procedure is repeated until the optimum or nearly optimum trajectory is found.

General Outline

The iteration is as follows: (1) Estimate a reasonable program that nearly satisfies the terminal conditions for specified initial conditions; (2) determine impulse response functions that describe the effects of small changes in the control on the terminal state and on the cost (the quantity to be minimized). These impulse response functions, combined with steepest descent computations, indicate the best possible way of making small changes to the control to decrease the cost and still arrive at the end point; (3) add this change in control to the previous nominal control program. The result is a new trajectory with a decreased cost; (4) repeat this process until there exists only a very small change in cost for each new trajectory, indicating that the control is very near a local optimum. A limit value of the control may be reached before the cost is completely minimized. In this case, the process is continued until the constraint (control limit) is reached, since no further optimization is possible.

The properties of steepest descent optimization have been documented in many previous studies (e.g., refs. 5-7). Although this method has been regarded as the most practical in many applications, there is no guarantee that it yields the absolute optimum. That is, for some initial choices of the nominal trajectory, the final optimized trajectory may represent only a local optimum path. Also, in some applications, where the cost function may be relatively insensitive to control variations, a large number of iterations may be necessary to approach the optimum solution.

Computation of Impulse Response Functions

To illustrate the computation of the impulse response functions let the quantity to be minimized be noted as φ, the cost evaluated at the final time. Let the state variable at the final time be noted as ψ and let the desired end-point value for this be denoted ψ_d.

Figure 1 illustrates the manner in which the influence of small control changes on φ and ψ are calculated in this report. The equations of motion are first solved with a control change, a positive control impulse at time t, superimposed upon the nominal control. During the next solution of the motion equations, a negative control impulse of the same magnitude is inserted at time t. The change in cost, $\Delta\varphi$, and the change in terminal state, $\Delta\psi$, are derived from these two solutions. In a similar manner the impulse response functions can be progressively determined at successive times along the trajectory, and the technique by which they are determined is the most important feature of this computing method. The computation of the full history of $\Delta\varphi(t)$ and $\Delta\psi(t)$ for the same control impulse at different times along the trajectory is termed one "iteration" since it corresponds to the previous optimization studies where one iteration with the adjoint equations was used to compute essentially these same functions along the trajectory. Appendix A shows the relationship of this experimental determination to the standard determination using adjoint equations. These experimental impulse response functions are shown to be directly related to the well-known "Green's functions."

Steepest Descent Optimization Equations

The impulse response functions are used in the steepest descent technique to modify the control toward the optimum in the following manner (see appendix A).

$$\begin{pmatrix}\text{New}\\ \text{nominal}\\ \text{control}\end{pmatrix} = \begin{pmatrix}\text{Previous}\\ \text{nominal}\\ \text{control}\end{pmatrix} + K_\varphi \, \Delta\varphi(t) + K_\psi \, \Delta\psi(t) \tag{1}$$

The gains K_φ and K_ψ are constants for each iteration. The gain K_φ weights the impulse response function for the cost; its sign is negative in order to decrease the cost. The magnitude of K_φ is determined experimentally for each problem. Too large a gain may cause instability in the convergence procedure, while too small a gain may extend the time of convergence.

The gain K_ψ must be calculated for each iteration so that the term $K_\psi \, \Delta\psi(t)$ will account for terminal displacement due to the optimizing term, $K_\varphi \, \Delta\varphi(t)$, and any terminal displacement error from the previous iteration will be corrected. The formulation for calculating K_ψ is as follows:

Small changes, $\delta\psi$, in the terminal state, ψ, due to small changes, $\delta u(t)$, in control can be approximated by:

$$\delta\psi = \frac{1}{2 \, \Delta u \, \Delta t} \int_{t_o}^{t_f} \delta u(t) \Delta\psi(t) \, dt \tag{2}$$

where Δu is the height of each control impulse and Δt is the time interval of each control impulse. Substituting $K_\varphi \, \Delta\varphi(t) + K_\psi \, \Delta\psi(t)$ from (1) for δu, we have:

$$\delta\psi = \frac{1}{2 \, \Delta u \, \Delta t} \int_{t_o}^{t_f} [K_\varphi \, \Delta\varphi(t)\Delta\psi(t) + K_\psi \, \Delta\psi^2(t)] \, dt \tag{3}$$

Solving for K_ψ and letting $-\delta\psi = \psi_d - \psi$ (previous terminal error) we obtain:

$$K_\psi = -K_\varphi \underbrace{\frac{\int_{t_o}^{t_f} \Delta\varphi(t)\Delta\psi(t)\,dt}{\int_{t_o}^{t_f} \Delta\psi^2(t)\,dt}}_{\text{Steepest descent optimization term}} + 2\,\Delta u\,\Delta t \underbrace{\frac{\psi_d - \psi}{\int_{t_o}^{t_f} \Delta\psi^2(t)\,dt}}_{\text{Terminal error correction term}} \tag{4}$$

This gives the general form of the steepest descent equations. The actual calculations are next considered in more detail.

HYBRID COMPUTER MECHANIZATION

The mechanization of the optimization procedure on a high-speed repetitive analog computer is presented in figure 2. Figure 2(a) is the flow diagram and figure 2(b) illustrates the logic used in automatically regulating the problem. The mechanization consists of an analog computer program for solving the trajectory equations; logic required to coordinate the procedure; and a serial memory storage unit for storing the nominal control program.

The serial memory unit is continuously driven by counter pulses (Logic no. 1). The output of the serial memory is the nominal control time history with n points that is used along with the appropriate control impulse, to solve the trajectory equations. These equations are started at the specified initial conditions with Logic no. 2 and stopped when the trajectory reaches the specified end point with Logic no. 3. The final values of the cost quantity, φ, and state quantity, ψ, are stored at the end of each run as indicated by Logic nos. 4 and 5. The positive or negative control impulse is added to the nominal control input with Logic nos. 6 and 7, respectively. Logic no. 8 inserts the modifying control $(K_\varphi\,\Delta\varphi + K_\psi\,\Delta\psi)$ into the serial memory. This procedure runs in essentially a continuous manner; that is, one point out of the n points in the nominal control history is updated after each two repetitive computations, and after $2n$ repetitive computations (one iteration), every point in storage has been modified and the process is repeated. For each iteration the gains K_φ and K_ψ are held as constants. As previously mentioned, the value of K_φ determines the relative speed and stability of the convergence onto the optimum. The corresponding value of K_ψ to be used with each new iteration is calculated by equation (4) as a function of the terminal error from each previous iteration $(\psi_d - \psi)$ and as a function of the following two integrated quantities from each previous iteration:

$$\int_{t_o}^{t_f} \Delta\varphi(t)\Delta\psi(t)\,dt \tag{5}$$

and

$$\int_{t_o}^{t_f} \Delta\psi^2(t)dt \tag{6}$$

The values for equations (5) and (6) were computed as integrals over the time period from $t = t_o$ to $t = t_f$. The time t_o was represented by a logic signal at the first repetitive computation in an iteration cycle and the time t_f was represented by a logic signal at the last computation in an iteration cycle. It should be noted that during those parts of the trajectory when the control was at a constraint limit, no further optimization was possible, and the integration of equations (5) and (6) was therefore not carried out during those times.

This type of computer mechanization will be illustrated in more detail for the following example problem.

APPLICATION TO REENTRY TRAJECTORY OPTIMIZATION

Statement of the Problem

The problem to be illustrated in this section is that of determining the time history of the variation of lift-drag ratio (control L/D) that must be flown for a vehicle returning into the earth's atmosphere so that:

The total heating load to the vehicle is minimized.

The vehicle arrives at a desired destination.

The control remains within specified constraints.

Mechanization

The equations of motion, presented in appendix B, were for a point mass in planar motion over a spherical nonrotating earth. The vehicle characteristics and flight conditions were those for a manned capsule returning from earth orbit.

Initial conditions were:

Altitude	76.3 km	(250,000 ft)
Velocity	7.63 km/s	(25,000 fps)
Flight-path angle	-1.8°	
Range to destination	1609 km	(1000 mi)

Final stopping conditions were:

Altitude	30.48 km	(100,000 ft)

Control limits were:

$$0 < \frac{L}{D} < 0.5$$

The main hardware elements used in the hybrid computer mechanization were:

Hardware elements	Program task
Analog computer	Solution of trajectory equations
Parallel digital logic units	Logic control of program
Track and store amplifiers	Storage of end-point values
Digital delay line memories[1] (with D/A and A/D converters)	Storage of control time history

The 64-word digital serial memory unit (13 bits per word) was accessed with the fastest allowable counter rate (0.002 sec). A complete 64-word cycle was then available every 0.128 second. To allow a complete solution of the trajectory equations within 0.128 second, the analog computer was time scaled at 3750 to 1.

Results

A series of computer runs for this problem is illustrated in figure 3. Figure 3(a) presents the details of each repetitive trajectory computation and figure 3(b) presents the details of the overall convergence onto the optimum nominal control. Figure 3(a) shows just a portion of iteration no. 0 as presented in figure 3(b).

In the upper trace of figure 3(a) the control impulses are superimposed upon the initial nominal control. Each control impulse had a magnitude of $L/D = \pm 0.25$ and a time increment of one clock pulse (0.002 sec). This control impulse was chosen because it gave variation in the final range and heat load on the order of ±5 percent. The range and integrated heat load along each of the repetitive trajectories are presented along with the final values as they are stored with track and store amplifiers. The difference between these stored quantities for each two pairs of subsequent runs is $\Delta\psi$ representing the range impulse response functions and $\Delta\varphi$ representing the heat load impulse response functions.

In figure 3(b) the first 10 iterations (each iteration consists of 128 repetitive computations) of the converging optimization procedure are illustrated along with the final iteration.

During the convergence procedure the range is seen to vary slightly about the desired value of 1609 km (1000 miles). The heat load is shown to be reduced about 10 percent during the first ten iterations and diminished to about 12 percent from the original with the final (optimum) control variation.

[1]A series of track and store amplifiers could also have been used for this storage.

The modifying control shown in the figure is the sum $K_\varphi \Delta\varphi + K_\psi \Delta\psi$. For this series of runs a constant value of $K_\varphi = -2.5\times10^{-7}$[units of $(L/D)/(J/m^2)$] was found to allow a fairly rapid convergence while maintaining program stability. The value of K_ψ was calculated by equation (4) to be that value for each iteration such as to allow convergence in the steepest descent manner.

In the lower trace of figure 3(b) the nominal control is recorded as it is read out of serial memory every 128 +1 counter pulses (with Logic no. 8). This gives a convenient time history to show the manner in which the control has been modified during each iteration. The control is seen to be limited within $0 < L/D < 0.5$. This was achieved simply by limiting the output of the serial memory to within these values.

As can be seen, the optimum control variation for this case is a bang-bang control. With the steepest descent method, it is usually found that near-optimum control can be achieved in the first few iterations, but that to "square up the corners" and achieve full optimum control a number of further iterations (on the order of 20 to 50) are required.

Convergence and Stability Considerations

One of the important aspects of any optimization scheme is the ability to converge, within a reasonable time, onto the optimum solution. For the particular method in this report it has been pointed out that this convergence primarily depends upon choosing the proper value of the gain K_φ. In the example problem, it was found that using any value of $|K_\varphi|$ less than 2.5×10^{-7} [units of $(L/D)/(J/m^2)$]resulted in smooth convergence; however, the convergence time (which was proportional to $1/K_\varphi$) became long. For initial values of K_φ greater than twice the aforementioned value the convergence became unstable, that is, the modifying δ control became so large as to change drastically the state variable from their nominal final values.

It was found that, as the optimum control was approached (after about 10 iterations), the value of K_φ could be increased and convergence stability maintained, because in these examples the control approached bang-bang and only small changes were possible near the saturation limits. The value of K_φ in these cases could be increased to about 10 times the aforementioned value, but increasing it much farther (without analog voltage scaling changes) would allow extraneous computer noise to be magnified to a point where it caused notable random fluctuations in the computations.

For a reasonable value of gain, such as that used for the example problem, the time to converge to a near optimum solution (11 iterations) was about 3 minutes, and to a full optimum solution (30 iterations), about 8 minutes. Further changes in these convergence times, of course, depend upon several factors. For instance, the convergence time in this computing setup was in proportion to n^2, where n is the number of points describing the control time history (64 points for the case cited). Also the allowable solution rates of the computer elements directly affect the convergence time. The continuing development and use of high-speed computing elements will certainly result in convergence times smaller than the time cited.

The results obtained by this computing method appear satisfactory for engineering purposes; however, the usual disadvantages of analog computation are inherent with this method. These disadvantages are primarily concerned with the extraneous noise in the computations and the absolute accuracy (only to within about 1 percent) of analog computer.

CONCLUDING REMARKS

This report has presented a perturbation method of computing optimum trajectories. The technique uses fast-time repetitive computations in determining control impulse response functions and requires only the dynamic solution of the state equations; whereas other perturbation computing techniques have required the solution of additional adjoint equations.

A hybrid computer was used in applying the method to the steepest descent optimization of reentry trajectories. Mechanizing the computer for this type of problem was relatively simple, and near optimum trajectories could be obtained in about 3 minutes of computing time and full optimum trajectories in about 8 minutes.

The advantage of the technique outlined here over alternative techniques is that the investigator need not be familiar with or use an auxiliary set of linear adjoint equations for the optimization. This technique does, however, require a large number of dynamic solutions of the state equations, but this computing task appears practical with the high-speed repetitive computation procedure presented in this report.

Ames Research Center
National Aeronautics and Space Administration
Moffett Field, Calif., Jan. 24, 1966

APPENDIX A

RELATIONSHIP OF EXPERIMENTAL IMPULSE RESPONSE FUNCTIONS TO ADJOINT SOLUTIONS AND THE MAXIMUM PRINCIPLE

Adjoint Solution of Impulse Response Functions

Let the state equations be noted as

$$\dot{x} = f[x(t), u(t)] \tag{A1}$$

where $x(t)$ is a vector of state variables, $u(t)$ is the control variable and f is a vector of known functions of $x(t)$ and $u(t)$.

The auxiliary adjoint equations can be noted as

$$\dot{\lambda} = -\left(\frac{\partial f}{\partial x}\right)^T \lambda \tag{A2}$$

where λ is a vector of influence functions and $(\partial f/\partial x)^T$ is the transpose of the matrix describing linear motions about the nominal path of $x(t)$ and $u(t)$.

It is well known that any small change in the control quantity $\delta u(t)$ along the nominal path will determine a change $\delta\varphi$ in any quantity φ at the final time as follows:

$$\delta\varphi = \int_{t_o}^{t_f} \left(\lambda_\varphi^T \frac{\partial f}{\partial u}\right) \delta u(t) dt \tag{A3}$$

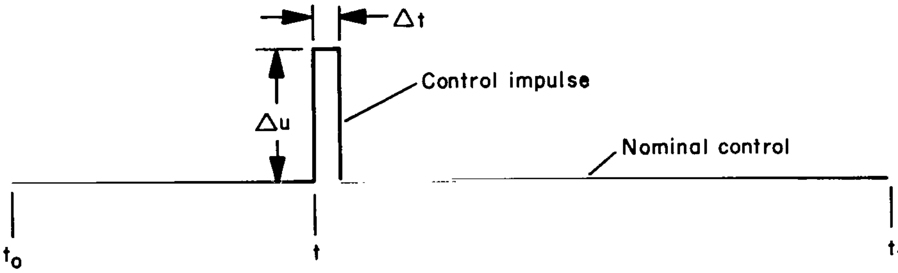
where λ_φ represents a solution of the adjoint equations with the boundary conditions at the final time of

$$\left(\lambda_\varphi^T\right)_{t=t_f} = \left(\frac{\partial\varphi}{\partial x}\right)_{t=t_f} \tag{A4}$$

The quantity $\lambda^T(\partial f/\partial u)$ within the integral is known as Green's function.

Experimental Determination of Impulse Response Functions

The method used in the test of this report for finding a change $\delta\varphi$ is to perturb the control experimentally in the following manner:



With two sequential dynamic solutions of the state equations using first a positive control impulse and then a negative control impulse, the following is available:

$$2\delta\varphi = \varphi(+\text{impulse}) - \varphi(-\text{impulse}) \tag{A5}$$

From equation (A3) we can write the change $2\delta\varphi = \Delta\varphi$, for a small control, $\delta u = \Delta u$, acting over a small time interval Δt, as follows:

$$2\delta\varphi = \Delta\varphi = \left(\lambda_\varphi^T \frac{\partial f}{\partial u}\right) 2\ \Delta u\ \Delta t \tag{A6}$$

This then represents the correspondence between the impulse response functions calculated in the text and those solved by the adjoint solution. Green's function evaluated at any time, t, along the trajectory can be noted as

$$\lambda_\varphi^T \frac{\partial f}{\partial u} = \frac{\Delta\varphi(t)}{2\ \Delta u\ \Delta t} \tag{A7}$$

and

$$\lambda_\psi^T \frac{\partial f}{\partial u} = \frac{\Delta\psi(t)}{2\ \Delta u\ \Delta t} \tag{A8}$$

Relationship to the Maximum Principle

The maximum principle (ref. 8) states that a necessary condition for a minimum (maximum) of the cost function is that the Hamiltonian be maximized (minimized) with respect to the control at all times. The Hamiltonian can be written as

$$H = \lambda^T f \tag{A9}$$

where the transversality condition must be satisfied at the final time,

$$(\lambda^T f)_{t=t_f} = -\left(\frac{\partial\varphi}{\partial t} + \eta \frac{\partial\psi}{\partial t}\right)_{t=t_f} \tag{A10}$$

and η is a Lagrange multiplier constant chosen so that the terminal constraint is satisfied. The boundary conditions on the adjoint equations at the final time are:

$$(\lambda^T)_{t=t_f} = \left(\frac{\partial\varphi}{\partial x} + \eta \frac{\partial\psi}{\partial x}\right)_{t=t_f} \tag{A11}$$

Now to determine if H is minimized with respect to the control we can take the partial derivative of H with respect to u:

$$\frac{\partial H}{\partial u} = \lambda^T \frac{\partial f}{\partial u} \tag{A12}$$

Or, noting the correspondence between equations (A4) and (A11), we can write

$$\frac{\partial H}{\partial u} = \lambda_\varphi^T \frac{\partial f}{\partial u} + \eta\lambda_\psi^T \frac{\partial f}{\partial u} \tag{A13}$$

where $\lambda_\varphi^T(\partial f/\partial u)$ is Green's function for the cost and $\lambda_\psi^T(\partial f/\partial u)$ is Green's function for the terminal constraint.

Recalling the correspondence between the adjoint solution for Green's function and that determined experimentally, we have the following:

$$\frac{\partial H}{\partial u} = \frac{\Delta\varphi(t)}{2\,\Delta u\,\Delta t} + \eta \frac{\Delta\psi(t)}{2\,\Delta u\,\Delta t} \tag{A14}$$

This, then, represents the relationship between the experimentally determined impulse response functions and the Hamiltonian. It is interesting that the maximum principle can be applied through this relationship without any need for solving the adjoint equations.

Steepest Descent Equations

The greatest change, $\delta\varphi$, in φ for a given value of $\int_{t_o}^{t_f} \delta u^2(t)dt$ is obtained (ref. 3) when

$$\delta u(t) = K_\varphi\,\Delta\varphi(t) + K_\psi\,\Delta\psi(t) \tag{A15}$$

where K_φ and K_ψ are constants. This is the steepest descent (or ascent) direction to the minimum (or maximum) φ.

When there are no state or control constraints, the steepest descent procedure converges toward the necessary conditions for an optimum solution as previously noted

$$\Delta\varphi(t) + \eta\,\Delta\psi(t) = 0 \tag{A16}$$

where in the steepest descent equations, $K_\psi/K_\varphi = \eta$, on the optimum solution with the terminal constraint satisfied.

APPENDIX B

REENTRY TRAJECTORY EQUATIONS

The following equations were programmed on the analog computer for the example considered in this report. These simplified equations were derived for flight within the atmosphere and the primary assumptions include: a spherical nonrotating earth, small flight-path angles, and a constant gravity term. The derivation of these simplified equations and their applicability have been considered in a number of reports. See for instance reference 9.

The equations are

$$\ddot{h} = -g + \frac{V^2}{r} + \left(\frac{C_D A}{m}\right) \frac{1}{2} \rho V^2 \left(\frac{L}{D} - \frac{\dot{h}}{V}\right)$$

$$\dot{V} = -\left(\frac{C_D A}{m}\right) \frac{1}{2} \rho V^2$$

$$\psi = \int_{t_o}^{t_f} V \, dt$$

$$\varphi = 3.75 \times 10^{-4} \int_{t_o}^{t_f} \sqrt{\rho} V^3 \, dt$$

where

$\frac{L}{D}$	control value of lift-drag ratio
h	altitude, m
V	horizontal velocity, m/s
ψ	final range, m
φ	total heat input, J/m^2
ρ	atmosphere density, $1.225 \, e^{-h/7160}$ kg/m^3
r	radius from earth center, 6.43×10^6 m
g	local gravitational acceleration, 9.8 m/s^2
$\left(\frac{C_D A}{m}\right)$	drag loading, 0.004 m^2/kg

REFERENCES

1. Leitmann, George, ed.: Optimization Techniques. Academic Press, 1962.

2. Balakrishnan, A. V.; and Neustadt, Lucien W., eds.: Computing Methods in Optimization Problems. Academic Press, 1964.

3. Bryson, Arthur E.; and Denham, Walter F.: A Steepest-Ascent Method for Solving Optimum Programming Problems. Raytheon Rep. BR 1303, 1961. Also J. Appl. Mech., vol. 29, no. 2, June 1962, pp. 247-257.

4. Kelley, H. J.: Gradient Theory of Optimal Flight Paths. ARS J., vol. 30, no. 10, Oct. 1960, pp. 947-954.

5. Bryson, A. E.; Denham, W. F.; Carroll, F. J.; and Mikami, K.: Determination of Lift or Drag Programs That Minimize Reentry Heating. J. Aerospace Sci., vol. 29, no. 4, April 1962, pp. 420-430.

6. Blanton, H. Elmore, ed.: Three-Dimensional Trajectory Optimization Study. Pt. 1 - Optimum Programming Formulation. NASA CR-57030, 1964. (Supersedes Aero. Sys. Div. Rep. ASD-TDR-62-295 and Raytheon Rep. Br-1759-1).

7. Hague, D. S.: Three-Degree-of-Freedom Problem Optimization Formulation. FDL-TDR-64-1, pt. 1, vol. 3, Oct. 1964.

8. Pontryagin, L. S., et al.: The Mathematical Theory of Optimal Processes. Wiley and Sons, 1962.

9. Chapman, Dean R.: An Approximate Analytical Method for Studying Entry Into Planetary Atmospheres. NASA TR R-11, 1959.

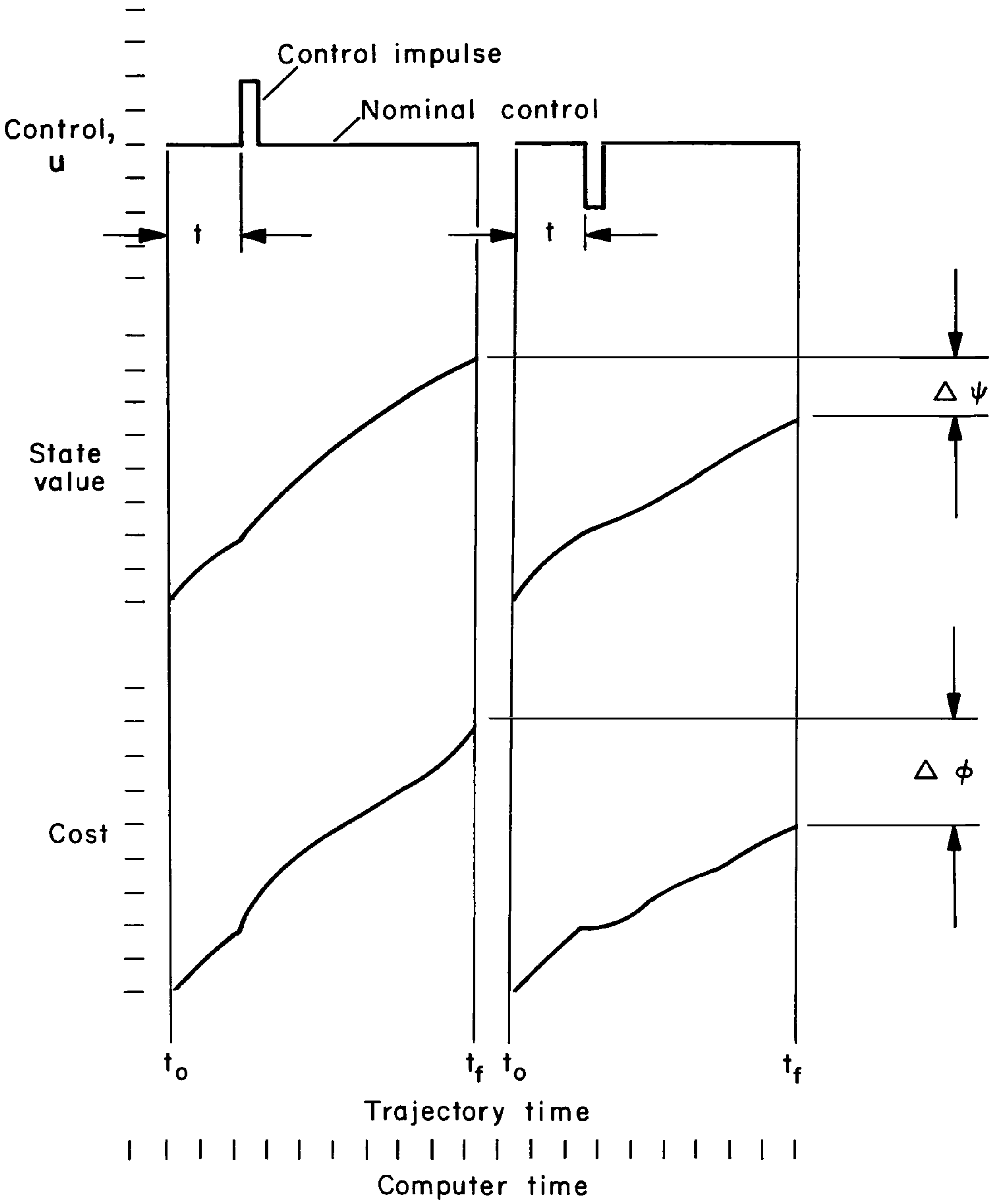
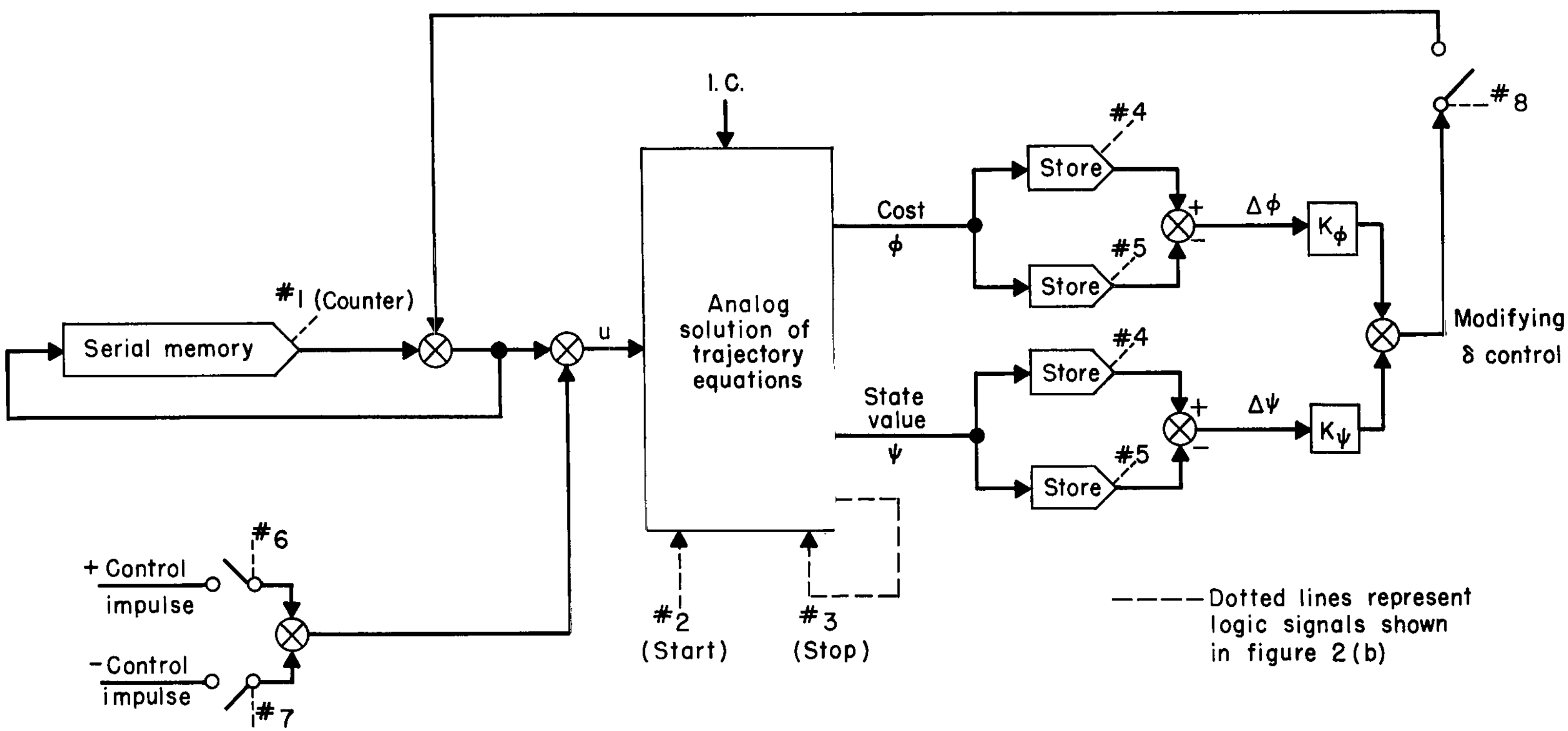
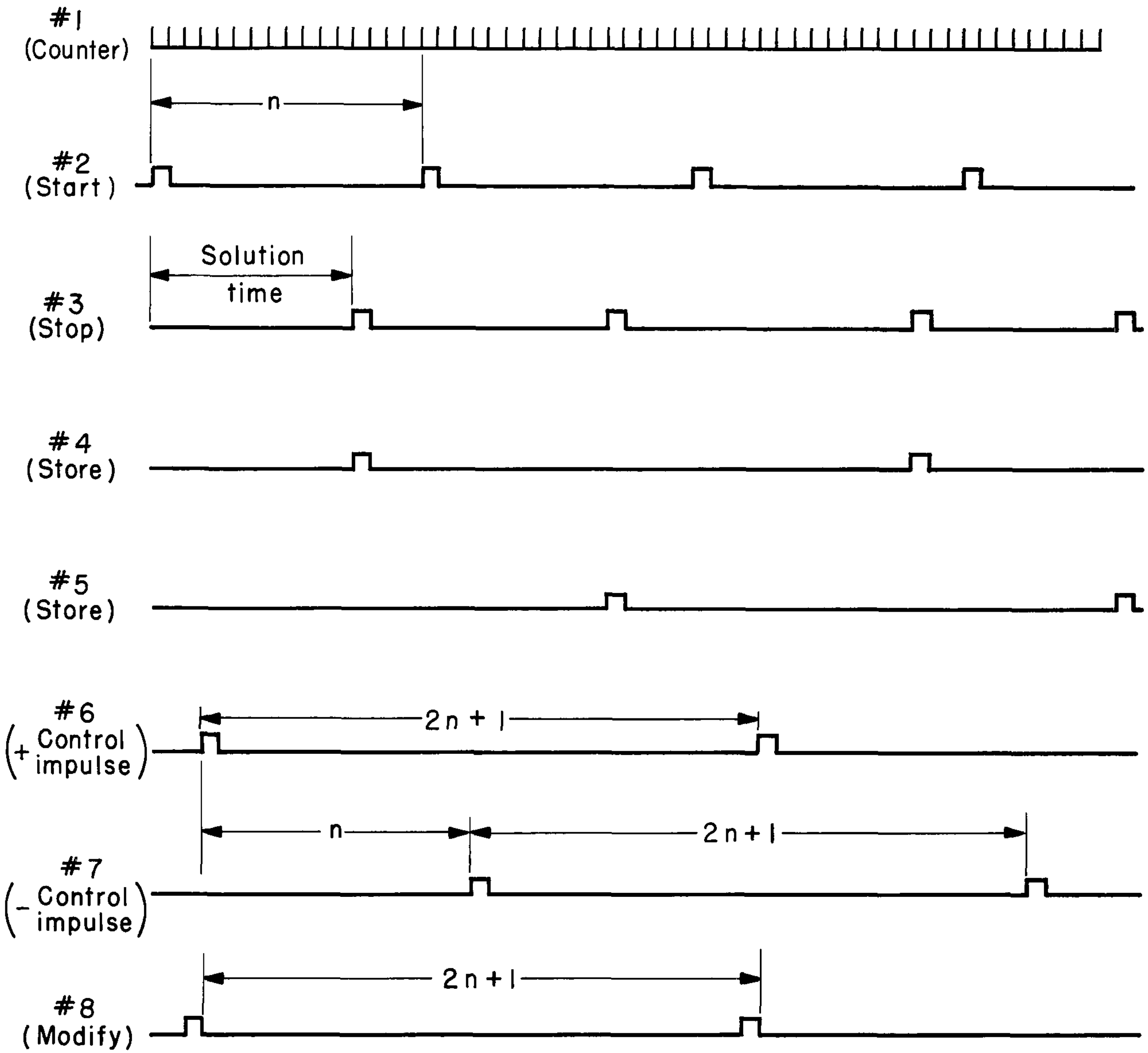


Figure 1.- Illustrative time history of repetitive analog computations.



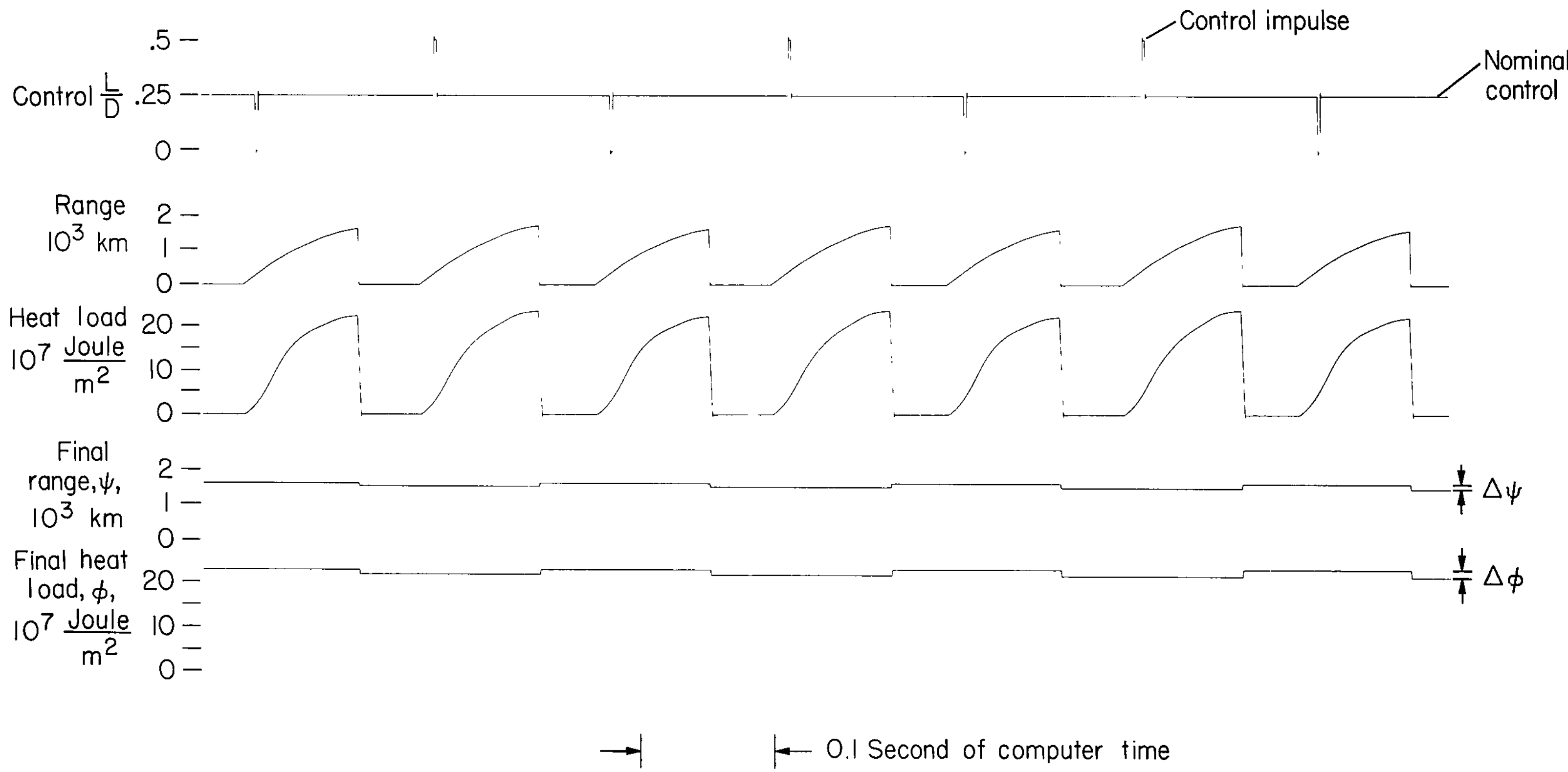
(a) Flow diagram.

Figure 2.- Mechanization of the optimization procedure.



(b) Problem logic.

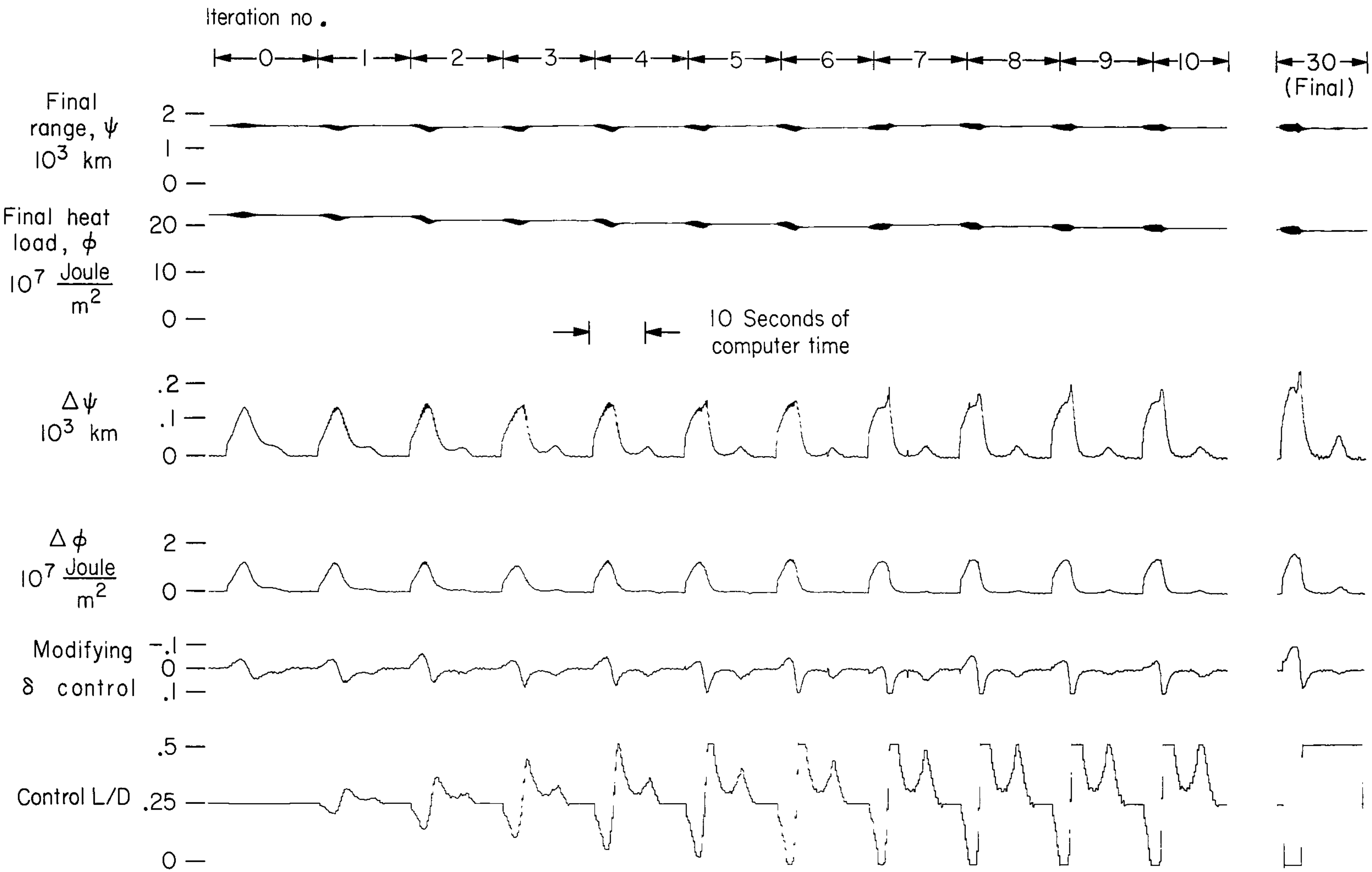
Figure 2.- Concluded.



(a) Details of repetitive trajectory computations.

Figure 3.- Recorded time histories for reentry trajectory optimization; minimum heat with terminal range constraint.

NASA-Langley, 1966 A-2201



(b) Details of overall convergence procedure.

Figure 3.- Concluded.

"The aeronautical and space activities of the United States shall be conducted so as to contribute . . . to the expansion of human knowledge of phenomena in the atmosphere and space. The Administration shall provide for the widest practicable and appropriate dissemination of information concerning its activities and the results thereof."

—NATIONAL AERONAUTICS AND SPACE ACT OF 1958

NASA SCIENTIFIC AND TECHNICAL PUBLICATIONS

TECHNICAL REPORTS: Scientific and technical information considered important, complete, and a lasting contribution to existing knowledge.

TECHNICAL NOTES: Information less broad in scope but nevertheless of importance as a contribution to existing knowledge.

TECHNICAL MEMORANDUMS: Information receiving limited distribution because of preliminary data, security classification, or other reasons.

CONTRACTOR REPORTS: Technical information generated in connection with a NASA contract or grant and released under NASA auspices.

TECHNICAL TRANSLATIONS: Information published in a foreign language considered to merit NASA distribution in English.

TECHNICAL REPRINTS: Information derived from NASA activities and initially published in the form of journal articles.

SPECIAL PUBLICATIONS: Information derived from or of value to NASA activities but not necessarily reporting the results of individual NASA-programmed scientific efforts. Publications include conference proceedings, monographs, data compilations, handbooks, sourcebooks, and special bibliographies.

Details on the availability of these publications may be obtained from:

SCIENTIFIC AND TECHNICAL INFORMATION DIVISION

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

Washington, D.C. 20546