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SIGNAL CONDITIONING TO IMPROVE HIGH RESOLUTION PROCESSING FOR PFM TELEMETRY

by Alan M. Demmerle and Thomas J. Karras

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ABSTRACT

High resolution processing of pulse frequency modulation telemetry can be improved with the use of a pre-conditioning filter bank. Analysis of the considerations which go into the design of this filter bank, including the transient response, frequency error, and signal-to-noise improvement for different filter characteristics are described. A block diagram of the filter bank is given, and the degree to which it has improved the data processing of pulsed frequency modulated data are discussed.

CONTENTS

Abstract	ii
INTRODUCTION.	1
ANALYSIS OF HIGH RESOLUTION PROCESSING (HRP) SYSTEM IMPROVEMENT.	1
ANALYSIS OF THE SETTling TIME OF A FILTER.	5
ANALYSIS OF THE EQUIVALENT NOISE BANDWIDTH (ENBW) OF A FILTER	5
DISCUSSION ON THE TRADE-OFF BETWEEN ENBW AND SETTling TIME FOR VARIOUS POLE CONFIGURATIONS.	6
LABORATORY MEASUREMENTS OF GAUSSIAN-SHAPED 1, 2, AND 3 POLE FILTERS.	6
DISCUSSION OF THE OPTIMUM FILTER CROSSOVER POINT	7
DESCRIPTION OF THE HRP SIGNAL D COMB FILTER BANK.	7
CONCLUSIONS.	8
References	8
Appendix A—The Frequency and Amplitude Response of a Single-Pole Filter to a Step Sinusoid	9
Appendix B—The Choice of the Point at Which the Gain Characteristics of the Contiguous Filters Overlap	13
Appendix C—The Filter Element Center Frequencies	17
Appendix D—The Signal D Comb Filter.	19

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INTRODUCTION

A high resolution processing (HRP) line is presently in use for the reduction of pulsed frequency modulation (PFM) telemetry data at the Goddard Space Flight Center. The performance of this line, in terms of its resolution of measurement, is described in Reference 1. The performance has been improved over what is described in this Reference by conditioning the PFM signal prior to its entry into the HRP system. This improvement was necessitated by a recent requirement to provide highly resolved data measurements, at yet lower signal-to-noise ratios. Herein is described the near-optimum preconditioning possible with the use of a filter bank, and the analysis used in selecting the filter transfer function, the bandwidth, and the number of contiguous filters for this optimum filter bank.

The use of such a preconditioning filter bank not only permits a lower allowable signal-to-noise ratio (S/N) for a specified resolution of measurement, but also provides a lower threshold S/N at which the HRP system can operate so that the system is usable at a lower S/N than without the preconditioning. This preconditioning filter bank is called the signal D comb filter and was first used on the F-7 PFM data reduction line of the Data Processing Branch (GSFC) in the reduction of data for the EPE-D satellite. It consists of 25 double-pole Gaussian-shaped filters in the 5 kc to 15 kc band. The contiguous filters in this bank are designed so that their gain characteristics overlap at 1.5db, and hence are separated in the center frequency by 400 cps.

ANALYSIS OF HIGH RESOLUTION PROCESSING (HRP) SYSTEM IMPROVEMENT

According to Reference 2, the S/N (lowest permissible), for a specified resolution of measurement using the HRP principle, is inversely proportional to the time available for measurement; i.e., the more time available for measurement the lower can be the S/N. A bank of preconditioning

contiguous gated filters provides S/N improvement by virtue of excluding the noise which is not in the immediate vicinity of the signal. This noise is excluded by gating on only the filter in which the signal occurs, and gating off all other filters. However, preconditioning filters also shorten the time available for making the measurement.

The lost time is caused by the fact that when energized with a frequency other than its exact resonant frequency, a filter has a transient response in the frequency-time domain, and it takes a settling time (T_s') before the filter is oscillating at the input frequency.

The system improvement for the HRP line is computed by the following relationship considering 5 kc to 15 kc PFM data bandwidth:

$$\begin{aligned} \text{S/N Improvement (db)} &= 10 \log \left(\frac{\text{Input equivalent noise bandwidth}}{\text{Equivalent noise bandwidth of a filter}} \right)^* \\ &- 20 \log \left(\frac{\text{Total duration of input PFM signal}}{\text{Time available out of the filter for making the frequency measurement}} \right) \\ \text{S/N Improvement (db)} &= 10 \log \left(\frac{10 \text{ kc}}{\text{ENBW}} \right) - 20 \log \left(\frac{T_B}{T_M} \right), \end{aligned}$$

where ENBW is the equivalent noise bandwidth, and

$$T_M = T_B - T_s'$$

$T_s' \triangleq$ the total time required for the filter to settle down to the input frequency.

*The signal-to-noise improvement of a filter is found by the following:

$$\begin{aligned} \text{S/N Improvement} &= 10 \log \frac{(S/N)_{out}}{(S/N)_{in}}, \\ &= 10 \log \left(\frac{N_{in}}{N_{out}} \right) - 20 \log \left(\frac{V_{in}}{V_{out}} \right) \text{ where } S_{in} = K V_{in}^2, \\ &\hspace{15em} S_{out} = K V_{out}^2, \text{ assuming the} \\ &\hspace{15em} \text{input and output impedances} \\ &\hspace{15em} \text{are the same.} \end{aligned}$$

and $N_{in} \triangleq$ Input equivalent noise bandwidth (assumed to be 10 kc band limited white noise)

$N_{out} \triangleq$ Equivalent noise bandwidth of a filter (See Analysis of ENBW Section).

Note: $20 \log \frac{V_{in}}{V_{out}} = 0\text{db}$ when $V_0 = V_{in}$ (The input frequency is at the center of the filter)
 $= 3\text{db}$ when $V_0 = .707 V_{in}$ (The input frequency is at the 3db point of the filter).

Taking the standard PFM case of $T_B = 9.1$ ms for a 55 cps PFM burst rate and allowing $\pm 1/2$ ms of uncertainty for the starting and ending of a measurement due to jitter in the word synchronizer,* the total duration of the burst available for making the measurement is $T_B = 8.1$ ms. Hence,

$$\text{S/N Improvement (db)} = 10 \log \left(\frac{10 \text{ kc}}{\text{ENBW}} \right) - 20 \log \left(\frac{8.1}{8.1 - T_s'} \right) . \quad (1)$$

To facilitate the analysis it is useful to establish the relationship between (a) the ENBW and the 3db bandwidth, and (b) the settling time (T_s') and the 3db bandwidth.

For this purpose let

$$\text{ENBW} = K_1 (f_{3db}) \text{ cps} , \quad (2)$$

and

$$T_s' = \frac{K_2}{(f_{3db})} \times 10^3 \text{ ms} , \quad (3)$$

where K_1 and K_2 are numbers which are determined by the transfer function of the filter. The steeper the sides of the filter response curve, the smaller the value of K_1 , to the point where with a rectangular filter, K_1 approaches one. Alternatively the steeper the filter response curve, the longer the time necessary for the filter to reach its steady state response, hence the greater is K_2 .

There is also a relationship between the number of filters in the filter bank, and the 3db bandwidth of each filter. If the filters overlap at the frequency where their response curve is attenuated by 3db and M is defined as the total number of filters needed to completely cover the 10 kc data band, then

$$M = \frac{10 \text{ kc}^\dagger}{f_{3db}} , \quad (4)$$

and Equations 2 and 3 become

$$\text{ENBW} = \frac{K_1 10 \text{ kc}}{M} \text{ cps} , \quad (5)$$

*The word synchronizer is the electronic device which is used to keep the data processing line in synchronism with the incoming telemetry signal. The output of the synchronizer is an estimate of the phase and frequency of the incoming data burst (word) rate.

†If the filters overlap at other than the 3db attenuation point, the definition of M would change accordingly.

and

$$T_s' = \frac{K_2 M}{10} \text{ ms} \quad (6)$$

Substituting (5) and (6) into Equation 1 results in

$$\text{S/N Improvement (db)} = 10 \log \frac{M}{K_1} - 20 \log \left(\frac{8.1}{8.1 - \frac{MK_2}{10}} \right) \quad (7)$$

For the purpose of illustration, Equation 7 is plotted in Figure 1 for the unrealizable case where $K_1 = K_2 = 1$. If the input frequency lies in the center of the filter, the improvement

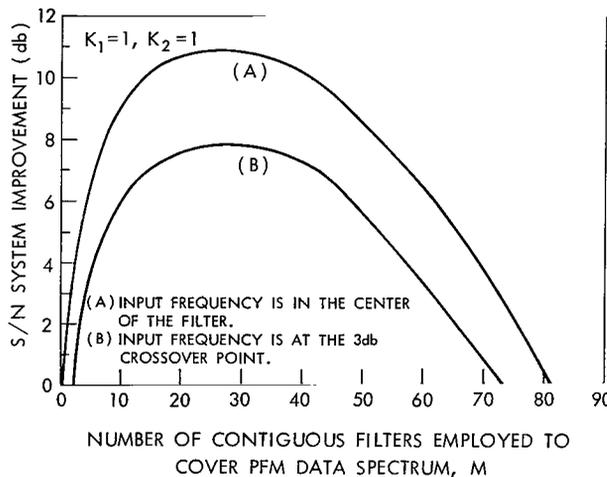


Figure 1—HRP system improvement versus number of filters in the 10kc data bandwidth (unrealizable case used only for illustration).

As the input frequency moves off the center frequency, the improvement will decrease since the output signal is attenuated while the noise power passing through the filter remains the same. The improvement when the input frequency is at the 3db crossover point is represented in Figure 1B. This assumes K_2 is independent of the offset frequency (Δf), where $\Delta f = | \text{resonant frequency} - \text{input frequency} |$. This assumption is not realistic (Appendix A) but introducing the functional relationship between K_2 and Δf unnecessarily complicates this analysis.

Twenty-seven filters which result in maximum system improvement for this unrealistic case is shown in Figure 1. Note that if more than 73 filters were used, no HRP system improvement results because of the excessive

time for the filter to settle down. With only a few filters the noise enhancement is such that the system improvement is low.

In general, the peak value of Equation 7 can be found by differentiating and setting the derivative equal to zero and solving for M peak. This results in

$$M_{\text{peak}} = \frac{27}{K_2} \quad (8)$$

Note that the peak value is a function of the settling time of the filter only.

ANALYSIS OF THE SETTLING TIME OF A FILTER

A simplification to the system improvement analysis which has an insignificant effect in the final results is a translation of the settling time variable T_s' . (The term settling time is chosen to be that time required for the output of the filter to be within 99.98 percent of its final value.) The translation of the variable is from T_s' to T_s , the settling time of the frequency-time response T_s' when the filter input is a step sinusoid, to the settling time of the amplitude-time response T_s when the filter input is a step sinusoid. An analysis to determine T_s and T_s' is shown in Appendix A. T_s and T_s' have been computed for various values of offset frequency Δf for the single-pole filter case. The average value of T_s and T_s' over the ensemble of Δf has been computed to be in both cases 2.61 ms ($K_2 = 1.31$). The rms values about this average for T_s' is 0.9 ms or 34.5 percent of the average value and for T_s is 0.311 ms or 11.9 percent of the average value. The maximum disagreement between these two functions is that when $\Delta f = 0$. The heuristic reason is that if $\Delta f = 0$, the filter will start and continue to respond with virtually no frequency errors, whereas it will always take a certain amount of time for the amplitude envelope to reach its final value. Similar computations of T_s and T_s' for the multipole filter are more complex. Measurement of T_s' is difficult, whereas measurement of T_s is easily made. Because of the similarity of these two functions, values of T_s determined by laboratory measurement of multipole filters are used as estimates of T_s' for these filters. From these estimates of T_s' , values of K_2 are calculated where $K_2 = T_s' f_{3db}/10^3$ and T_B , T_s , and T_m for a step sinusoidal input to a filter is illustrated in Figure 2.

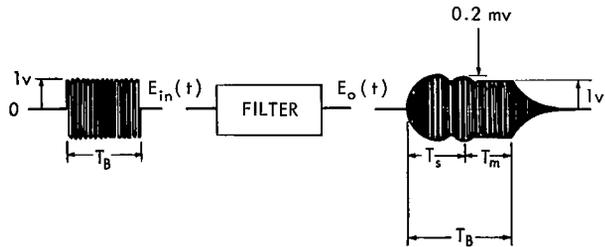


Figure 2—Illustrating T_B , T_s' , and T_m for a step sinusoidal input to a filter.

ANALYSIS OF THE EQUIVALENT NOISE BANDWIDTH (ENBW) OF A FILTER

The concept of ENBW is useful to express the power which would pass through a filter the input to which is white noise, regardless of the shape of that filter's response curve. The ENBW (in cps) is the bandwidth of the rectangular filter which would pass the same amount of white noise power as the filter to which the ENBW refers. Knowledge of the ENBW of a filter facilitates computation of the signal-to-noise ratio improvement offered by the filter to the system in which it is used. Methods for computation of ENBW, given the transfer function of a filter,

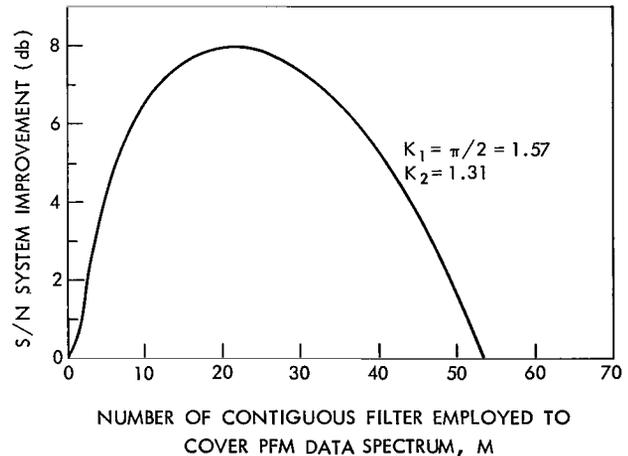


Figure 3—Theoretical HRP system improvement versus number of single-pole filters in the 10kc data bandwidth for 3db overlapping filter.

are known (Reference 3). The ENBW can also be determined by the measurement of the area under the filter's response curve. A plot of the maximum system improvement for a single-pole filter as a function of the number of filters in the filter bank is shown in Figure 3. It is based on the theoretical ENBW of $\frac{\pi}{2} f_{3db}$ ($K_1 = 1.57$) and the theoretical average settling time of 2.61 ms ($K_2 = 1.31$).

DISCUSSION ON THE TRADE-OFF BETWEEN ENBW AND SETTLING TIME FOR VARIOUS POLE CONFIGURATIONS

Generally speaking a trade-off exists between ENBW and settling time when choosing the pole configuration of a filter. This trade-off on a qualitative basis is illustrated in Figure 4.

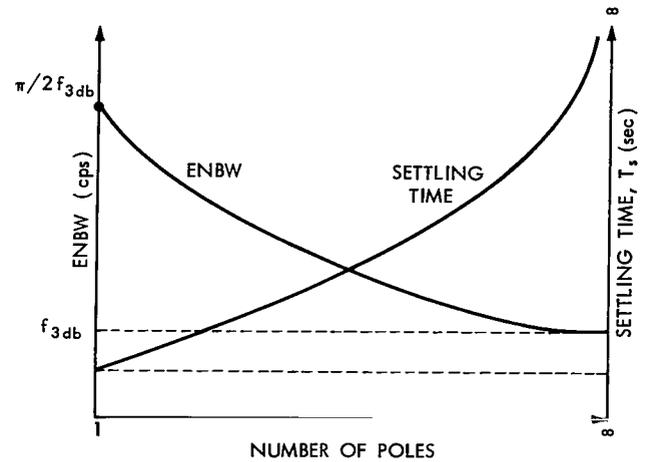


Figure 4—ENBW and settling time versus number of filter poles.

LABORATORY MEASUREMENTS OF GAUSSIAN-SHAPED 1, 2, AND 3 POLE FILTERS

The measured values of settling time T_s and ENBW are given in Table 1 for each of three filters. Each filter has the same 3db bandwidth (approximately 500 cps).

In theory, the ENBW for a 1-pole filter is $\frac{\pi}{2} f_{3db}$. The measured values differ from the theoretical values because the filter elements were not pure resistors, capacitors, and inductors; and because the physical circuit introduced stray, unknown inductance and capacitance making what was intended as a single-pole filter a more complex configuration.

The three above filters were designed to possess optimum transient settling time response (Gaussian type filter). Notice that as discussed previously the settling time increases with increased number of poles and the ENBW decreases with increased number of poles.

Table 1

Settling Time and ENBW for Three Filters.

Type of Filter	Settling Time, T_s	$T_s = K_2/f_{3db}$	ENBW = $K_1 f_{3db}$	M peak (filters)
1 pole	2.1 ms	$1.05/f_{3db}, \left(\frac{1.31}{f_{3db}}\right)^*$	$1.31 f_{3db}, \left(\frac{1.57}{f_{3db}}\right)^*$	25.7, (20.7)*
2 pole	2.3 ms	$1.15/f_{3db}$	$1.20 f_{3db}$	23.5
3 pole	3.0 ms	$1.50/f_{3db}$	$1.14 f_{3db}$	18.0

*Theoretical Values.

From the results in Table 1 and Equation 7, the system improvement for the three filter configurations were plotted in Figure 5. It can be seen that 17 double-pole filters offer the same improvement as 25 single-pole filters. Therefore the double-pole filter is used in this filter bank.

DISCUSSION OF THE OPTIMUM FILTER CROSSOVER POINT

From the results obtained from Figure 5 it can be seen that approximately 20 double-pole filters (500 cps 3db bandwidth) would be a good choice for the optimum number of filters for the HRP processing filter bank. One should remember that these curves assume each filter crosses at the 3db point. With 20 filters the system improvement will be 9.4db to 6.4db depending on the input frequency.

In order to narrow the range of system improvement over the passband of the filter while still maintaining the same ENBW and settling time, it would require adding more filters overlapping at some point other than the 3db attenuation point. The problem of the point on the filter gain curve at which the adjacent filter's gain curve should overlap is dealt with in Appendix B.

The analysis has led to the choice of 25 double-pole Gaussian shaped filters with a 3db bandwidth of 500 cps, but so arranged in a contiguous manner that their attenuation characteristics overlap at 1.5db. They are thus separated in center frequency by 400 cps.

A tabulation of the center frequency and the circuit configuration of these filters is shown in Appendix C.

DESCRIPTION OF THE HRP SIGNAL D COMB FILTER BANK

The filter elements which have been the main point of discussion thus far must be connected into a gated filter bank. The filter elements for the 5-15 kc PFM data band are called the D1 elements and those for the 20-60 kc ($4 \times$ speeded up PFM data band) are called the D4 elements. The D1 elements have a 500 cps 3db bandwidth and are separated in center frequency by 400 cps. The D4 elements have a 2000 cps 3db bandwidth and are separated in center frequency by 1600 cps. Both the D1 and D4 filters overlap at the 1.5db attenuation which for D1 is ± 200 cps from the center frequency and for D4 is ± 800 cps from the center frequency. A tabulation of the center frequencies and bandwidth of these filters is shown in Appendix C, Table C1.

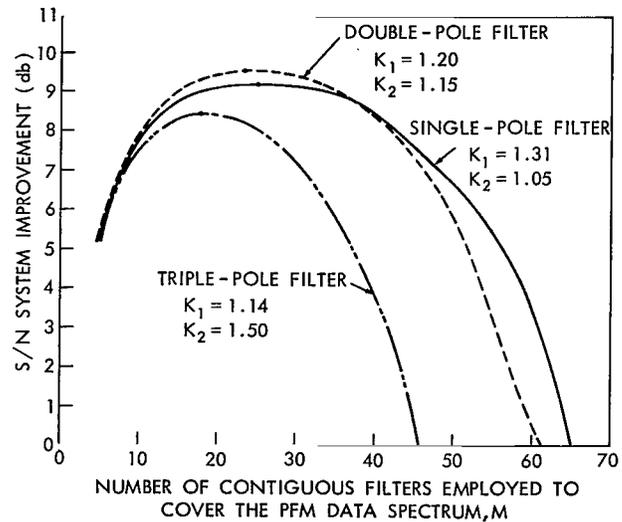


Figure 5—Experimental HRP system improvement versus number of filters for three different pole configurations in the 10 kc data bandwidth.

One D1 and one D4 filter element are placed on one printed circuit card along with a relay which selects which of the elements will be used. On that same card are two emitter followers, an amplifier, an envelope detector and an auction circuit.

The improvement offered by the filter bank is based on the assumption that only the filter in which the signal occurs has been gated on, and that all of the other filters are gated off.

The decision as to which filter is appropriate for the incoming signal, so that all other filters can be gated off and kept off for the remainder of the PFM burst, should be made as quickly as possible. The method for making this decision, and block diagrams and timing diagrams for the system are discussed in Appendix D.

CONCLUSIONS

There is a near optimum signal conditioning system for high resolution processing for pulsed frequency modulation. That system consists of 25 equally spaced filters across the 5 to 15 kc

PFM data band; the gain characteristics of each filter overlap at 1.5db attenuation points. This system results in an average computed improvement of 9.04db (for ENBW = 10 kc) for a 55cps PFM channel rate or 9.54db (an increase of 1/2db) for a 50 cps PFM channel rate. This results in lowering the present threshold of the HRP processing line from +10db to +0.46db (Figure 6). If the actual ENBW at the input to the HRP system is greater than 10 kc, then the S/N improvement is greater than that specified above. For example, if the ENBW is 20 kc, then the HRP threshold would occur at -2.54db for a 50 cps PFM channel rate.

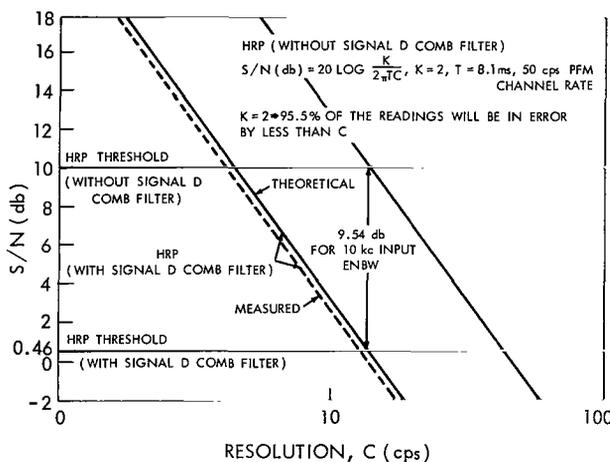


Figure 6—S/N versus resolution for the HRP line.

(Manuscript received August 31, 1965)

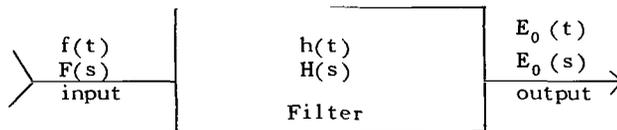
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2. Demmerle, A. M., "On Frequency Measurements and Resolution," NASA Technical Note D-2216, December 1964.
3. Karras, T. J., "Equivalent Noise Bandwidth Analysis From Transfer Functions," NASA Technical Note D-2842, November 1965.

Appendix A

The Frequency and Amplitude Response of a Single-Pole Filter to a Step Sinusoid

The following analysis* was used to compute the time required for the envelope of a single-pole filter's output to reach within 0.02 percent of its final value, and time required for the frequency of the filter's output to reach within 0.02 percent of its final value.



$$f(t) = A \sin W_{in} t \cdot U(t) \quad F(s) = \mathcal{L}^{-1} F(s)$$

$$F(s) = \frac{A W_{in}}{S^2 + W_{ir}^2}$$

$$H(s) = \frac{W_{3db} \cdot S}{S^2 + W_{3db} \cdot S + W_0^2}$$

W_0 = center frequency of the filter, W_{3db} = bandwidth of the filter between the 3db attenuation points.

$$E_0(s) = F(s)H(s), \quad E_0(t) = \mathcal{L}^{-1} E_0(s),$$

$$E_0(t) = \underbrace{W_{3db} A \sqrt{1 + e^{-W_{3db}t} - 2e^{-\frac{W_{3db}t}{2}} \cos \Delta \omega t}}_{(a)} \cdot \underbrace{\sin(W_{in} t + \phi_2 + \phi_3)}_{(b)}, \quad (1)$$

*Rochelle, R. W., "Pulse Frequency Modulation Telemetry," NASA Technical Report R-189, January 1964.

where

$$\Delta W = W_0 - W_{in} ,$$

$$\phi_2 = \tan^{-1} \left(\frac{2\Delta W}{W_{3db}} \right) ,$$

and

$$\phi_3 = \tan^{-1} \left\{ \frac{e^{-\frac{W_{3db}}{2} t} \cdot \sin \Delta W t}{e^{-\frac{W_{3db}}{2} t} \cdot \cos \Delta W t - 1} \right\} .$$

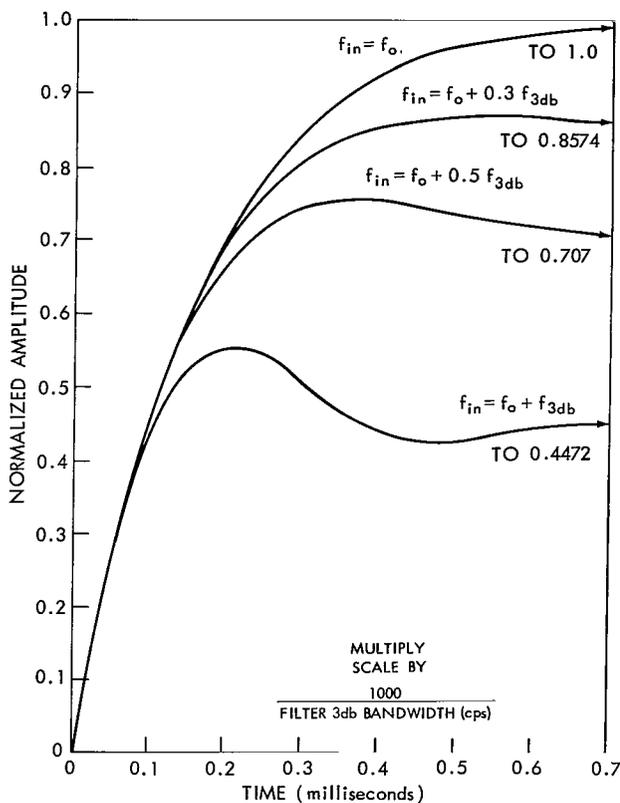


Figure A1—Amplitude response versus time for a step sinusoidal input applied to a single-pole filter for several offset frequencies.

Two digital computer programs were written to operate on Equation 1.

One program computed the magnitude of the envelope of Equation 1; i.e., factor (a), as a function of time t in families of offset frequency ΔW rad/sec. The results are plotted in Figure A1.

The other program, to compute the error in the frequency of the filter's output with relation to the filter's input, operated on factor (b) of Equation 1 in the following way:

The argument of factor (b) is the phase of the output, let

$$\theta = (W_{in} t + \phi_2 + \phi_3) .$$

To determine the output frequency, compute $d\theta/dt$

$$\frac{d\theta}{dt} = W_{in} + \frac{d\phi_3}{dt} .$$

The error in the output frequency is

$$\text{error} = \frac{\left(W_{in} + \frac{d\phi_3}{dt} \right) - W_{in}}{W_{in}} , \quad \text{error} = \frac{\frac{d\phi_3}{dt}}{W_{in}} .$$

This error was computed as a function of time in families of offset frequency and plotted in Figure A2.

From the data plotted in Figures A1 and A2 the time necessary for the filters to settle to within 0.02 percent of its final value (in one case amplitude, and in the other case final frequency) was determined and plotted in Figure B3 (Appendix B) as a modified function of offset frequency. From the data of eleven offset frequencies the average time for the filter output to settle to 0.02 percent of its final value was calculated and found to be for both the case of frequency error and amplitude error 2.61 ms for a filter of $F_{3db} = 500$ cps.

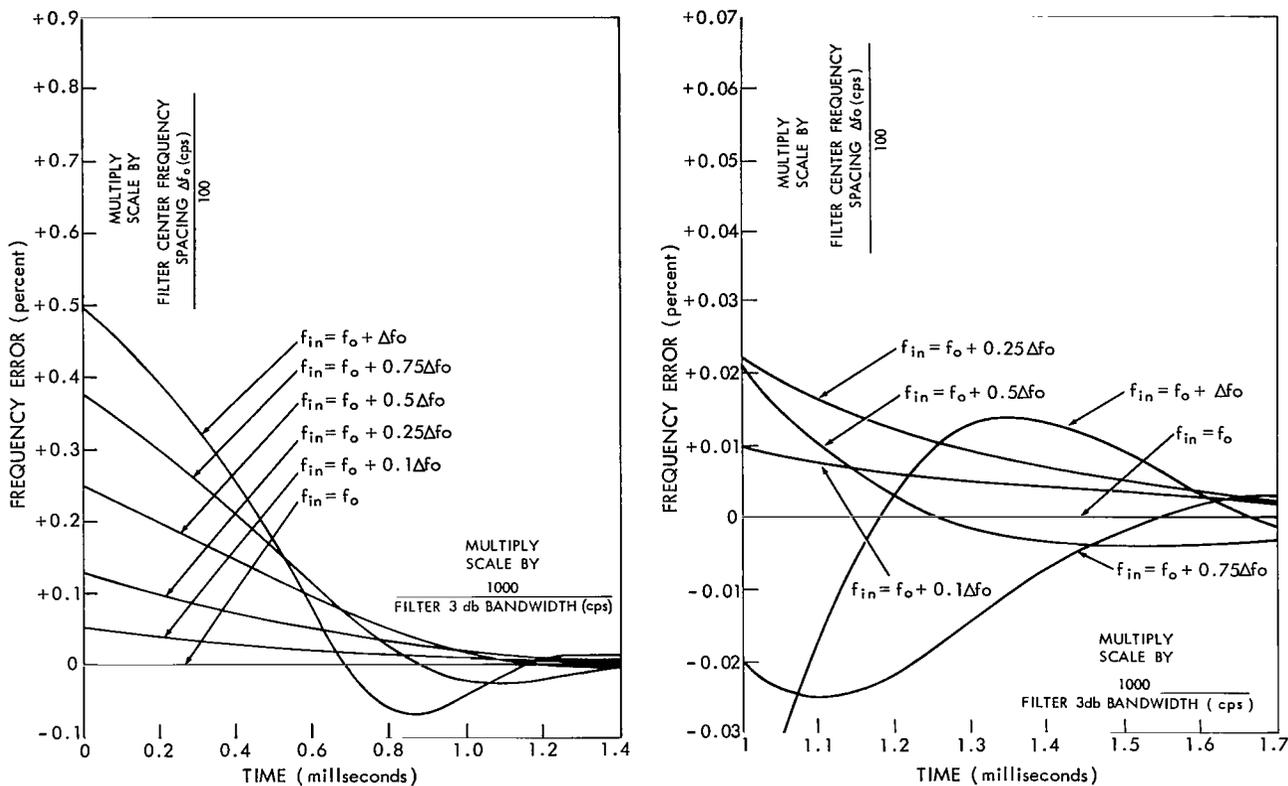
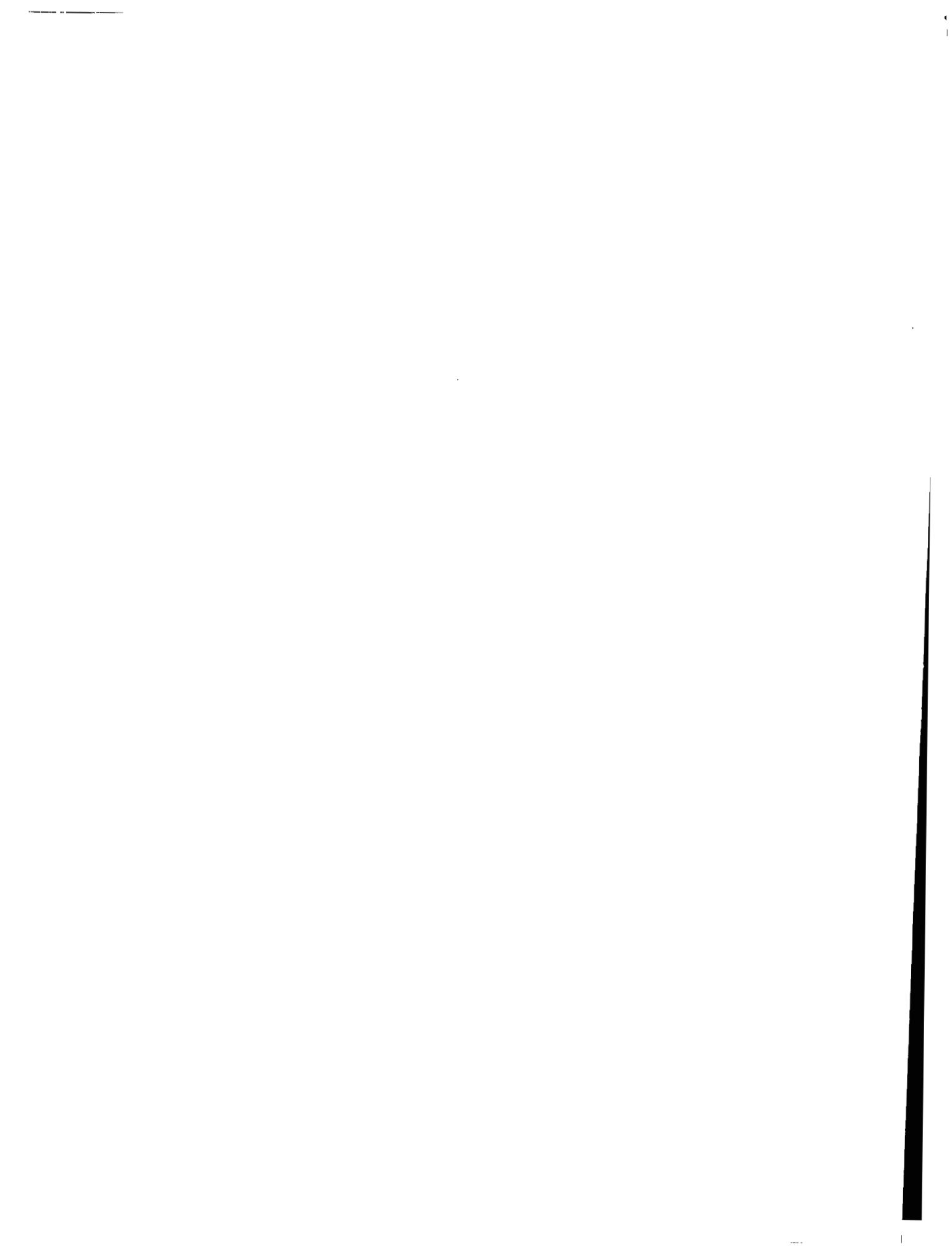


Figure A2—Two plots of frequency error versus time for a step sinusoidal input applied to a single-pole filter for several offset frequencies.



Appendix B

**The Choice of the Point at Which
the Gain Characteristics of the Contiguous Filters Overlap**

To determine the optimum crossover point for these filters, Figure B1 and Tables B1, B2 and B3 were constructed from experimental data. Figure B1 is useful for graphically computing the gain averaged across the frequency band of concern for the cases where the filter is active out to 1/2, 1, 1-1/2, 2, and 3db of its center frequency gain. The average gains are listed in Table B1.

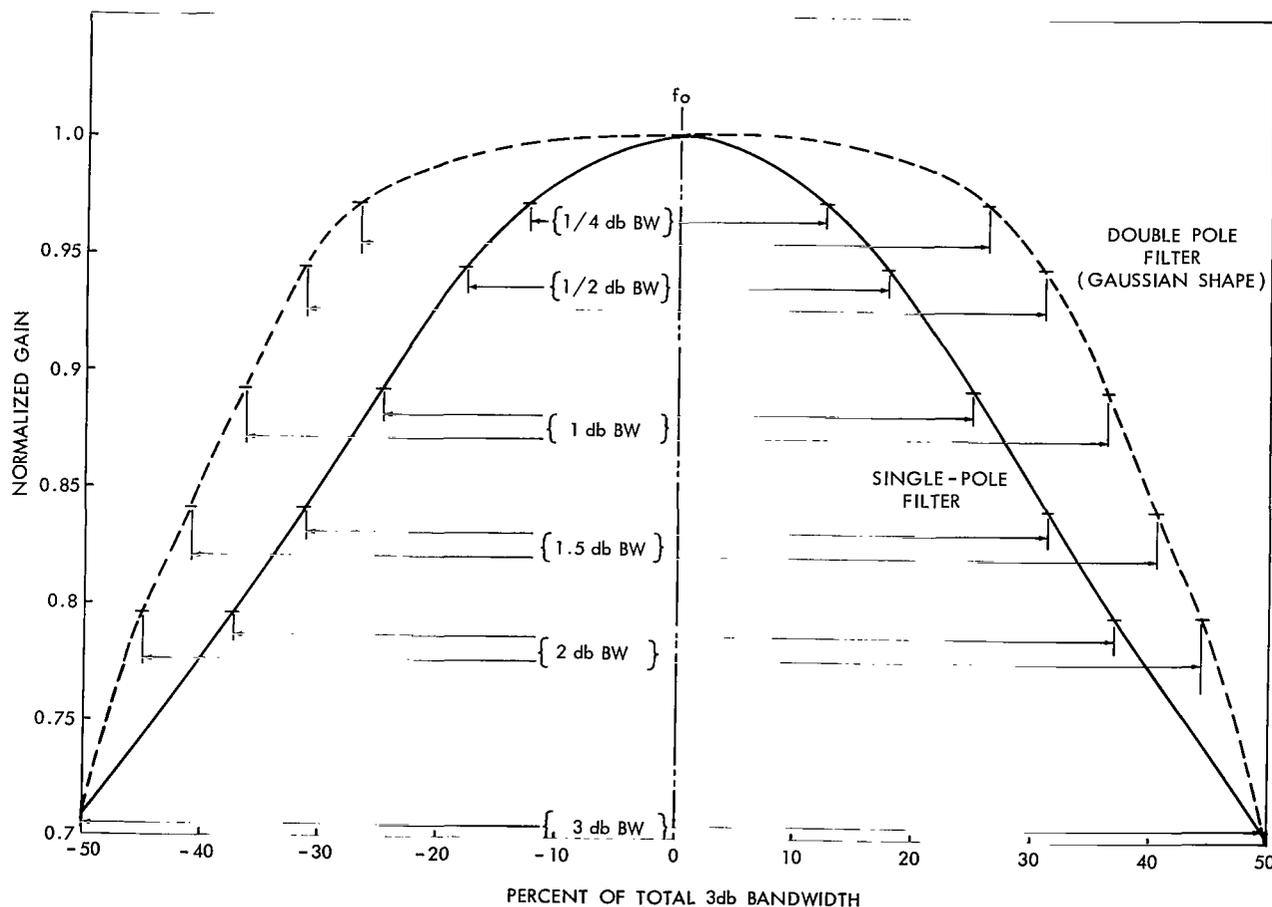


Figure B1—Amplitude response for a single-pole and Gaussian-shape double-pole filter versus frequency expressed in percentage of the 3db bandwidth.

Table B1

The Average Gain for a Single-Pole and Gaussian-Shaped Double-Pole Filters for Various Crossover Points.

Filter Attenuation Crossover Point	Single-pole Filter Average Gain	Double-pole Gaussian Shaped Filter Average Gain
3db	0.879 (-1.12db)	0.932 (-0.63db)
2db	0.924 (-0.70db)	0.954 (-0.41db)
1.5db	0.944 (-0.50db)	0.970 (-0.26db)
1.0db	0.961 (-0.35db)	0.979 (-0.20db)
0.5db	0.978 (-0.20db)	0.988 (-0.11db)

Table B2

The Relationship Between the Bandwidth at the 3db Point, and the Bandwidth at Other Attenuation Points.

Single-pole Filter BW at Other Attenuation Points = K_3 (3db BW)	Double-pole Gaussian Shaped Filter BW at Other Attenuation Points = K_4 (3db BW)
2.0db BW (cps) = 0.75 (3db BW) cps	2.0db BW (cps) = 0.90 (3db BW) cps
1.5db BW (cps) = 0.62 (3db BW) cps	1.5db BW (cps) = 0.81 (3db BW) cps
1.0db BW (cps) = 0.50 (3db BW) cps	1.0db BW (cps) = 0.73 (3db BW) cps
0.5db BW (cps) = 0.35 (3db BW) cps	0.5db BW (cps) = 0.63 (3db BW) cps

Table B3

Tabulation of the Relationship for the Number of Filters Required to Cover the Data Band for Various Crossover Points For the Single-Pole and Gaussian-Shaped Double-Pole Filters.

Crossover Point	Single-pole Filter		Double-pole Gaussian-Shaped Filter
	Experimental	Theoretical	
3.0db	25.7	20.7	23.5
2.0db	34.2	27.6	26.1
1.5db	41.5	33.4	29.0
1.0db	51.5	41.4	32.2

Table B2 shows the proportional relationship between the bandwidth at the 3db point, and the bandwidth at other attenuation points of interest.

Equation B1 shows that if a bank of filters is used which overlaps at the 3db point, the optimum number of filters (M peak) (that number which maximizes the system improvement), is

$$M_{\text{peak}} = \frac{27}{K_2} , \quad (\text{B1})$$

and Table B3 in the text shows M peak = 25.7 (for the single-pole filter) based on the experimentally determined values of K_1 and K_2 .

M peak = 23.5 for the double-pole filter based on the experimentally determined values of K_1 and K_2 .

If the crossover point of the filters is modified from 3db to 2, 1.5, and 1.0, these optimum numbers will increase in accordance with the K_3 and K_4 values of Table B2.

A tabulation of the optimum number of filters in the filter bank based on the experimentally determined values of K_1 and K_2 for the single-pole and Gaussian-shaped double-pole filter is shown in Table B3.

Figures B2 and B3 illustrate the narrowing of the improvement band as the number of filters is increased, while the characteristic of the filter is kept the same; i.e., the filters overlap at different attenuations.

Figure B2 illustrates that approximately 34 single-pole filters, each crossing at the 1.5db point offer an average system improvement of 7.7db. If the filters were to overlap at the 1db attenuation point, 6 more filters would be required and only 0.15db additional improvement would be gained.

Since the double-pole filter has been chosen from inspection of the experimental results shown in Figure 5 in the text, the same analysis of different crossover points was done for a double-pole filter based on the measured values of $K_2 = 1.15$ and $K_1 = 1.2$ (see Figure B3).

From Figure B3 one can see that 25 double-pole filters having a 1.5db crossover point offers an average system improvement of 9.04db; approximately 1.24db more than the single-pole filter bank with filters crossing at 1.5db points.*

*Laboratory measurement on the signal D comb filter indicate that a double-pole filter provided a 1.5db improvement over a single-pole filter.

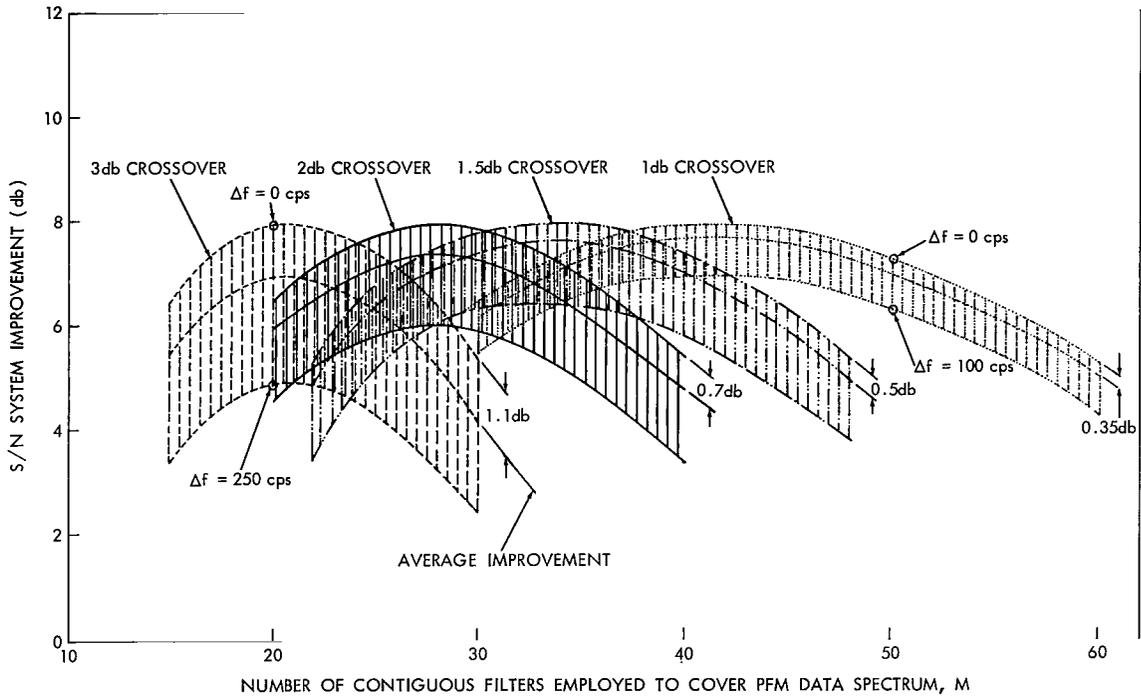


Figure B2—Theoretical HRP system improvement versus number of single-pole filters in the 10 kc data bandwidth for different crossover points (based on theoretical values; $K_1 = 1.57$ and $K_2 = 1.31$).

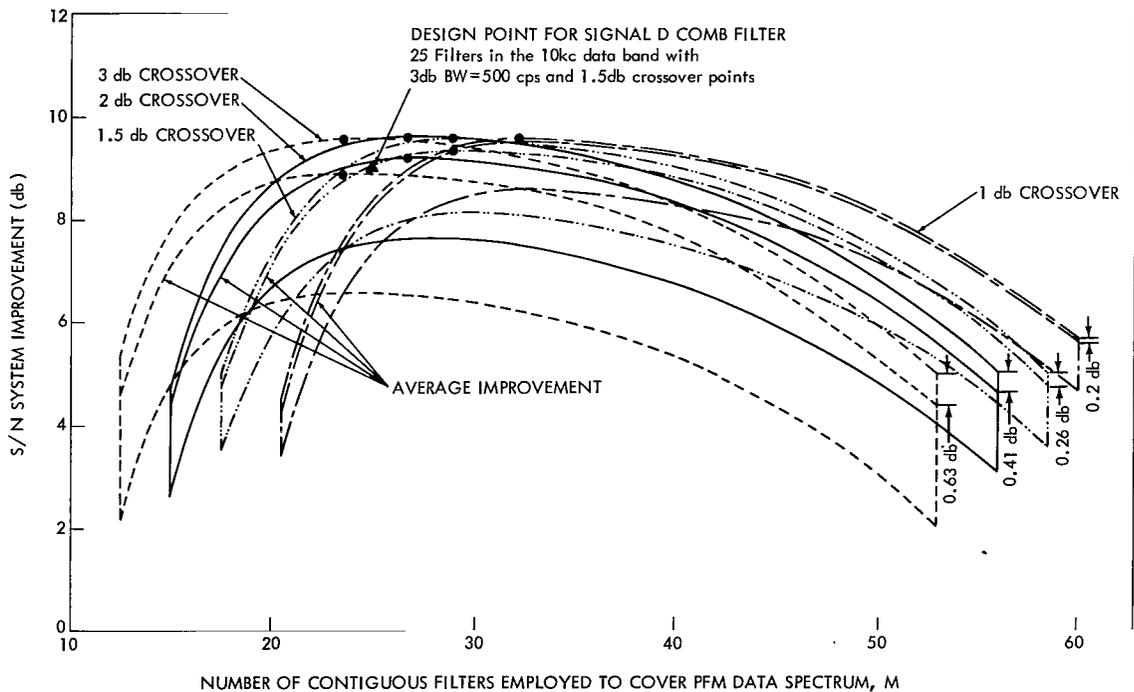


Figure B3—Theoretical HRP system improvement versus number of double-pole filters in the 10 kc data bandwidth for different crossover points (based on experimental values; $K_1 = 1.20$ and $K_2 = 1.15$).

Appendix C

The Filter Element Center Frequencies

A tabulation of the filter chosen for the HRP line is given in Table C1.

The system improvement characteristic (Figure B3 in Appendix B) has a broad peak so that one might have chosen 23 or 27 filters instead of 25. The number 25 has the advantage that if there

Table C1
2 Pole Filters for HRP Signal D Comb Filter.

Filter Number	D1 3db BW = 500 cps		D4 3db BW = 2 kc	
	Center Frequency	1.5db BW	Center Frequency	1.5db BW
Guard Band { 1	3.800 kc	±200 cps	15.200 kc	±800 cps
2	4.200		16.800	
Frame sync ← 3	4.600		18.400	
4	5.000		20.000	
5	5.400		21.600	
6	5.800		23.200	
7	6.200		24.800	
8	6.600		26.400	
9	7.000		28.000	
10	7.400		29.600	
11	7.800		31.200	
12	8.200		32.800	
13	8.600		34.400	
14	9.000		36.000	
15	9.400		37.600	
Data Band { 16	9.800		39.200	
26 filters { 17	10.200		40.800	
18	10.600		42.400	
19	11.000		44.000	
20	11.400		45.600	
21	11.800		47.200	
22	12.200		48.800	
23	12.600		50.400	
24	13.000		52.000	
25	13.400		53.600	
26	13.800		55.200	
27	14.200		56.800	
28	14.600		58.400	
29	15.000		60.000	
Bit Sync ← 30	15.400		61.600	
Guard Band { 31	15.800	63.200		
32	16.200	64.800		

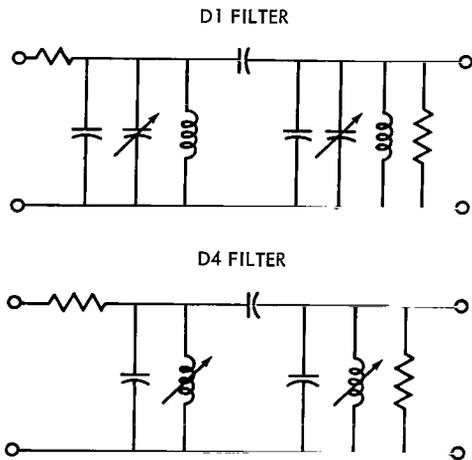


Figure C1—D1 and D4 tunable double-pole filters employed in the signal D comb filter.

are 25 filters divided evenly across the 10 kc data band, the center frequency of these filters, being 400 cps apart are easy to remember and hence desirable from the operational point of view.

Circuit diagram (Figure C1) shows the tunable double-pole filters used in the signal D comb filter.

Appendix D

The Signal D Comb Filter

The block diagram of the signal D comb filter is shown in Figure D1. The auction circuit allows only one output at one time to go to the flip flops labeled FA. The FA's are sequentially strobed to discover whether one has an input. When the sequential strobe reaches an FA with an input indicated by its corresponding auction circuit, that FA is set, the strobes are stopped and only the output from the filter corresponding to that FA is gated into the HRP input. Once the decision as to which filter should be gated out is made, it is irrevocable for that burst. Figure D2 is a timing diagram of the HRP signal D comb filter.

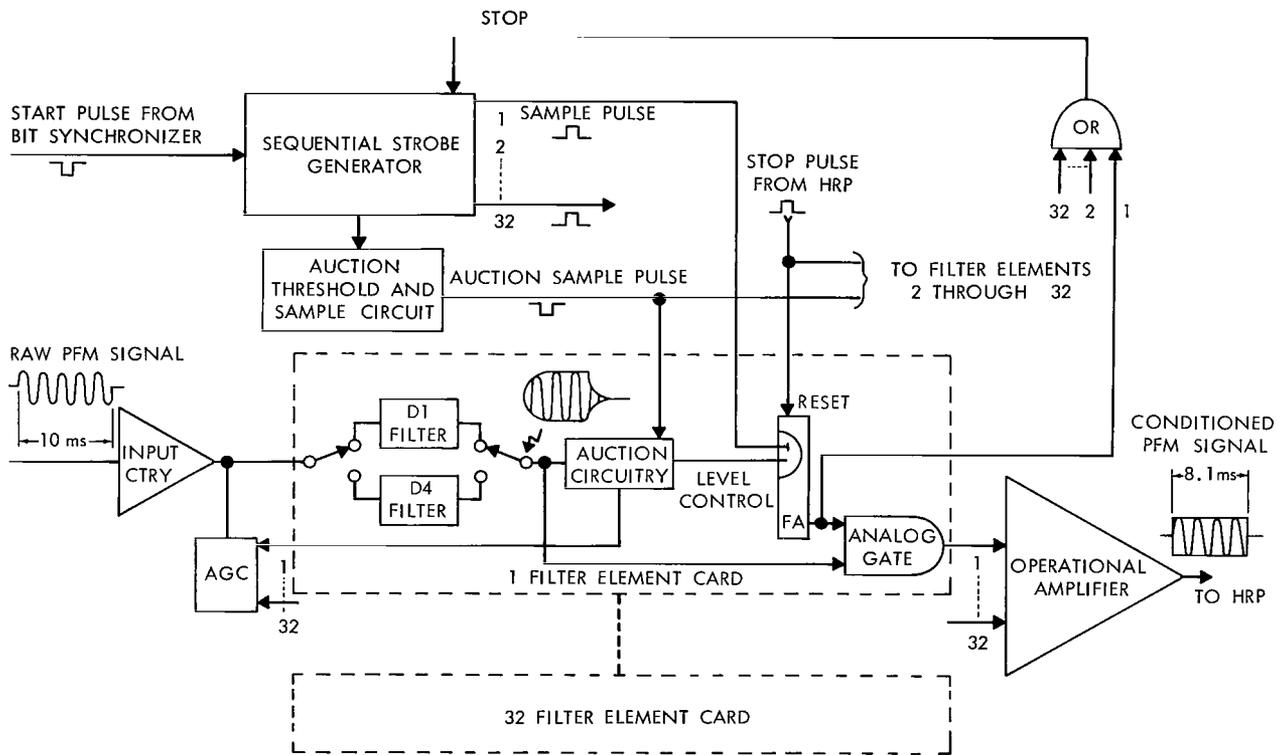


Figure D1—HRP signal D comb filter block diagram.

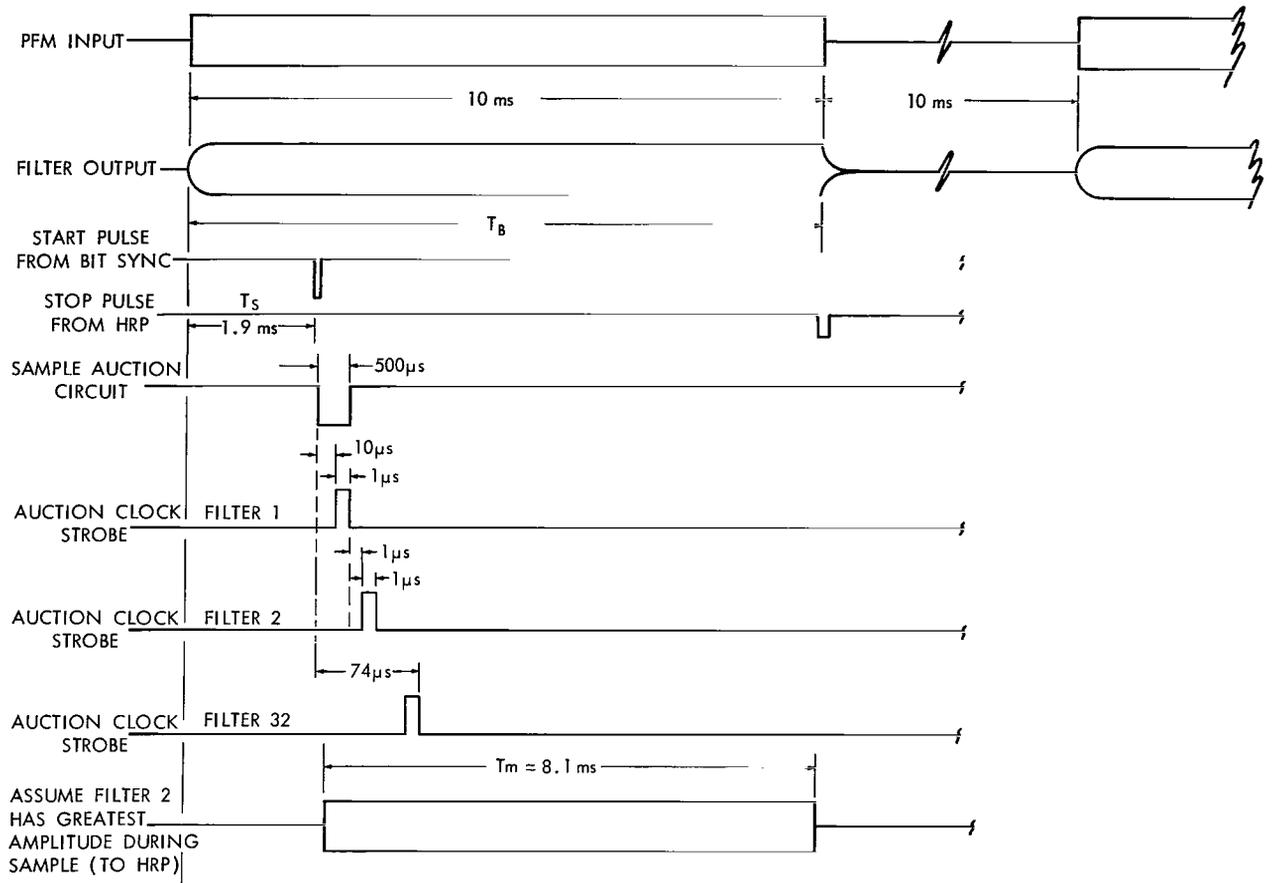


Figure D2—HRP signal D comb filter timing diagram.