EFFECTS OF PAYLOAD WEIGHT AND LENGTH ON THE MASS AND AERODYNAMIC CHARACTERISTICS OF THE APACHE AND CAJUN SOUNDING ROCKETS (CAPACHE)

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ABSTRACT

The effects of payload weight and length on the Capache mass and aerodynamic characteristics are compiled. The characteristics are based upon assumed payload properties and should be modified, if desired, for known payload properties.
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### SYMBOLS

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<tr>
<td>$b_0$</td>
<td>Burnout</td>
</tr>
<tr>
<td>$c_g$</td>
<td>Center of gravity</td>
</tr>
<tr>
<td>$C_{m_x}$</td>
<td>Static stability parameter, per radian</td>
</tr>
<tr>
<td>$C_{N_x}$</td>
<td>Normal force coefficient curve slope at $\alpha = 0$, per radian</td>
</tr>
<tr>
<td>$c_p$</td>
<td>Center of pressure</td>
</tr>
<tr>
<td>$i$</td>
<td>Ignition</td>
</tr>
<tr>
<td>$I_{xx}$</td>
<td>Roll moment of Inertia (slug ft$^2$)</td>
</tr>
<tr>
<td>$I_{yy}$</td>
<td>Pitch moment of inertia (slug ft$^2$)</td>
</tr>
<tr>
<td>$l$</td>
<td>Total vehicle length (inches)</td>
</tr>
<tr>
<td>$LA$</td>
<td>Launch angle (degrees)</td>
</tr>
<tr>
<td>$MEF$</td>
<td>Motor empty + fins</td>
</tr>
<tr>
<td>$MLF$</td>
<td>Motor loaded + fins</td>
</tr>
<tr>
<td>$PA$</td>
<td>Payload adapter junction</td>
</tr>
<tr>
<td>$p_l$</td>
<td>Propellant lost</td>
</tr>
<tr>
<td>$t$</td>
<td>Time (seconds)</td>
</tr>
<tr>
<td>$W$</td>
<td>Weight (lbs)</td>
</tr>
<tr>
<td>$x$</td>
<td>Longitudinal location measured from base (ft)</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Pitch natural frequency (cps)</td>
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<td>3</td>
</tr>
<tr>
<td>3</td>
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EFFECTS OF PAYLOAD WEIGHT AND LENGTH ON THE MASS AND AERODYNAMIC CHARACTERISTICS ON THE APACHE AND CAJUN SOUNDING ROCKETS

(CAPACHE)

INTRODUCTION

A brief study was conducted to define the effects of payload weight and length on the mass, static margin, static stability, and natural frequency characteristics of the Apache and Cajun sounding rockets. The characteristics have been available in memorandum form for some time and have been used primarily for preliminary estimates prior to payload finalization and for routine stability analysis.

METHOD

The computer programs shown in Appendixes A and B were used in this study. For the Apache vehicle, the program input characteristics were determined through the use of references 1, 2, 3, 4, and 5. For the Cajun vehicle, the program input characteristics were determined through the use of references 2, 3, 6, 7, 8, and 9.

The study was conducted for the standard 34-inch (11-degree) nose cone and a payload diameter of 6.75 inches. A nose cone weight of 14 pounds was assumed.

Computations were performed for payload weights and lengths of:
- 40 pounds: 40, 60, and 80 inches
- 80 pounds: 40, 60, and 80 inches
- 120 pounds: 40, 60, and 80 inches

The vehicle aerodynamic characteristics were determined for the extreme drag cases: drag I and drag IV.

The following assumptions were made in determining the mass characteristics:
- Propellant center-of-gravity location is constant.
- Nose cone is of uniform density.
- Payload cylindrical section is of uniform density.
- Pitch and roll moments of inertia vary linearly with burning time.
It must be noted that the vehicle characteristics presented in the following pages are only as valid as these assumptions.

RESULTS

Apache

Table 1 shows the Apache weight and inertia input characteristics. Figures 1 through 4 show the resulting weight-time, center-of-gravity location, pitch and yaw moment of inertia, and roll moment of inertia characteristics respectively.

Table 1

Apache Weight and Inertia Input Characteristics

| W_{MEF}  | 86 |
| W_{MLF}  | 217 |
| (x_{cg})_{MEF}  | 3.18 |
| (x_{cg})_{MLF}  | 4.1 |
| (x_{cg})_{pl}  | 4.71 |
| x_{PA}  | 8.92 |
| (I_{yy})_{MEF}  | 29.6 |
| (I_{yy})_{MLF}  | 58 |
| (I_{xx})_{MEF}  | 0.34 |
| (I_{xx})_{MLF}  | 0.52 |
| t_i  | 20.0 |
| t_{bo}  | 26.4 |

<table>
<thead>
<tr>
<th>Time (sec.)</th>
<th>Wt. Propellant Lost (lbs)</th>
<th>Time (sec.)</th>
<th>Wt. Propellant Lost (lbs)</th>
<th>Time (sec.)</th>
<th>Wt. Propellant Lost (lbs)</th>
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<td>20.0</td>
<td>0.0</td>
<td>22.45</td>
<td>51.1</td>
<td>25.57</td>
<td>124.2</td>
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<tr>
<td>20.07</td>
<td>0.7</td>
<td>22.69</td>
<td>56.3</td>
<td>25.76</td>
<td>127.4</td>
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<td>20.15</td>
<td>2.3</td>
<td>23.60</td>
<td>76.9</td>
<td>25.86</td>
<td>128.8</td>
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<td>20.22</td>
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<td>23.95</td>
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<td>25.98</td>
<td>129.9</td>
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<td>20.56</td>
<td>11.6</td>
<td>24.50</td>
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<td>130.5</td>
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<td>20.87</td>
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<td>24.69</td>
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<td>21.15</td>
<td>24.1</td>
<td>24.94</td>
<td>110.0</td>
<td>26.40</td>
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<tr>
<td>21.68</td>
<td>35.0</td>
<td>25.10</td>
<td>114.0</td>
<td>9999.0</td>
<td>131.0</td>
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</table>
Table 2 shows the Apache aerodynamic input characteristics. Figures 5, 6, and 7 show the resulting static margin, static stability, and natural frequency characteristics, respectively, for a zero-length launcher, 80-degree elevation launch from Wallops Island for extreme drag cases (cases I and IV).

### Table 2
Apache and Cajun (Capache) Aerodynamic Input Characteristics

<table>
<thead>
<tr>
<th>Mach No.</th>
<th>*C_{\infty}^X</th>
<th>x_{c_p}^{X} \ (l = 151)</th>
<th>x_{c_p}^{X} \ (l = 191)</th>
<th>Mach No.</th>
<th>*C_{\infty}^X</th>
<th>x_{c_p}^{X}</th>
<th>x_{c_p}^{X} \ (l = 191)</th>
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<tr>
<td>1.5</td>
<td>33.23</td>
<td>1.54</td>
<td>1.83</td>
<td>5.0</td>
<td>12.61</td>
<td>3.08</td>
<td>3.92</td>
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<tr>
<td>2.0</td>
<td>26.93</td>
<td>1.78</td>
<td>2.17</td>
<td>5.5</td>
<td>11.46</td>
<td>3.25</td>
<td>4.17</td>
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<tr>
<td>2.5</td>
<td>22.35</td>
<td>2.00</td>
<td>2.49</td>
<td>6.0</td>
<td>10.89</td>
<td>3.42</td>
<td>4.41</td>
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<tr>
<td>3.0</td>
<td>18.91</td>
<td>2.24</td>
<td>2.77</td>
<td>6.5</td>
<td>10.31</td>
<td>3.58</td>
<td>4.64</td>
</tr>
<tr>
<td>3.5</td>
<td>16.62</td>
<td>2.46</td>
<td>3.08</td>
<td>7.0</td>
<td>9.74</td>
<td>3.72</td>
<td>4.83</td>
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<tr>
<td>4.0</td>
<td>14.90</td>
<td>2.67</td>
<td>3.37</td>
<td>7.5</td>
<td>9.47</td>
<td>3.84</td>
<td>5.03</td>
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<tr>
<td>4.5</td>
<td>13.75</td>
<td>2.88</td>
<td>3.64</td>
<td>8.0</td>
<td>9.17</td>
<td>3.96</td>
<td>5.19</td>
</tr>
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</table>

*Aerodynamic reference length = \frac{6.5}{12} = 0.542 \text{ foot}

Aerodynamic reference area = \frac{\pi(6.5)^2}{4(144)} = 0.231 \text{ foot}^2

Figures 8 and 9 show the velocity and altitude histories for drag cases I and IV as obtained from references 4 and 5.

**Cajun**

Table 3 shows the Cajun weight and inertia input characteristics. Figures 10 through 13 show the resulting weight-time, center-of-gravity location, pitch and yaw moment of inertia, and roll moment of inertia characteristics respectively.

Table 2 shows the Cajun aerodynamic input characteristics. Figures 14, 15, and 16 show the resulting static margin, static stability, and natural frequency characteristics, respectively, for a zero-length launcher, 80-degree elevation launch from Wallops Island for extreme drag cases (cases I and IV).
Table 3

Cajun Weight and Inertia Input Characteristics

\[
\begin{align*}
W_{\text{MLF}} &= 83 \\
W'_{\text{MLF}} &= 202 \\
(x_{CG})_{\text{MEF}} &= 2.83 \\
(x_{CG})_{\text{MLF}} &= 4.04 \\
(x_{CG})_{p1} &= 4.78 \\
x_{PA} &= 8.92 \\
(I_{yy})_{\text{MEF}} &= 24 \\
(I_{yy})_{\text{MLF}} &= 53 \\
(I_{xx})_{\text{MEF}} &= 0.336 \\
(I_{xx})_{\text{MLF}} &= 0.50 \\
t_i &= 17.0 \\
t_{bo} &= 21.0
\end{align*}
\]

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<td>0.0</td>
<td>17.50</td>
<td>15.2</td>
<td>19.55</td>
<td>93.9</td>
<td>20.35</td>
<td>118.7</td>
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<tr>
<td>17.0</td>
<td>0.0</td>
<td>17.65</td>
<td>20.2</td>
<td>19.80</td>
<td>104.7</td>
<td>20.40</td>
<td>118.9</td>
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<td>17.05</td>
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<td>25.3</td>
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<td>20.50</td>
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<td>1.0</td>
<td>18.20</td>
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<td>21.00</td>
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<td>118.2</td>
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<tr>
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<td>10.2</td>
<td>19.35</td>
<td>85.6</td>
<td>20.25</td>
<td>118.5</td>
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Figures 17 and 18 show the velocity and altitude histories for drag cases I and IV as obtained from references 7 and 8.

CONCLUSIONS

As stated previously, the characteristics presented in this paper are only as valid as the aforementioned assumptions. Payloads exhibiting abnormal external configurations or weight distributions should be singularly investigated. The characteristics are presented without detailed discussion; however, four obvious conclusions are:
- The static margin and static stability histories are insensitive to payload length.

- The higher drag cases exhibit higher minimum static margins owing to decreased burnout Mach numbers.

- The undamped aerodynamic pitch frequency prior to resonance is insensitive to payload weight.

- The effects of varying payload weight and length on the undamped aerodynamic pitch frequency are small. The values of the pitch frequency at a given time are within 0.5 cps of the mean.
APPENDIX A

WEIGHT, CENTER-OF-GRAVITY LOCATION, PITCH AND ROLL MOMENT OF INERTIA DETERMINATION PROGRAM

INTRODUCTION

The enclosed equations were developed to facilitate determination of the weight, center-of-gravity location, and pitch and roll moment of inertia of the Capache sounding rocket as a function of time as required for input into the G.E. MASS Subprogram II. They are being programmed by Elva Glover of the Math and Computing Branch. An outline of the program follows:

ASSUMPTIONS

1. Propellant center-of-gravity location is constant.
2. Nose cone is of uniform density.
3. Payload cylindrical section is of uniform density.
4. Pitch and roll moments of inertia vary linearly with burning time.

SYMBOLS (See Figure A-1)

h = length of nose cone
I_{xx} = roll moment of inertia
I_{yy} = pitch moment of inertia
\ell = length of payload cylindrical section
r = radius of payload cylindrical section
t = time
W = weight
x = longitudinal location
x_{cg} = longitudinal center-of-gravity location
SUBSCRIPTS

bo  burnout

c  payload conical section

i  ignition

MF  motor + fins

MEF  motor empty + fins

MLF  motor loaded + fins

p  payload

PA  payload adapter juncture

p1  propellant lost

INPUT

Constants:  \( W_p \)

\( W_c \)

\( W_{MEF} \)

\( W_{MLF} \)

\( (x_{cg})_{MEF} \)

\( (x_{cg})_{MLF} \)

\( (x_{cg})_{p1} \)

\( x_{PA} \)

\( l \)

\( h \)

\( r \)
\[(I_{yy})_{MLF}\]
\[(I_{yy})_{MEF}\]
\[t_i\]
\[t_{bo}\]

**Variable:** \[W_{pl} = F(t)\]

**OUTPUT**

**Constants:** \[W_p\]
\[h\]
\[\ell\]
\[(x_{cg})_p\]
\[(I_{yy})_p\]
\[(I_{xx})_p\]

**Variables:** \[W = F(t)\]
\[x_{cg} = F(t)\]
\[I_{yy} = F(t)\]
\[I_{xx} = F(t)\]

**EQUATIONS**

\[(x_{cg})_p = x_{PA} + \frac{(W_p - W_c) \ell}{2W_p} + W_c \left(\ell + \frac{h}{4}\right)\]
\[(I_{yy})_p = \frac{(w_p - w_c)}{32.2} \left[ r^2 + \left( \frac{\ell^2}{3} \right) \right] / 4 \]

\[+ \frac{3w_c}{32.2} \left[ \left( \frac{r^2}{3} \right) + h^2 \right] / 5 \]

\[+ \left( \frac{w_p - w_c}{32.2} \right) \left[ \left( x_{PA} + \frac{\ell}{2} \right) - (x_{cg})_p \right]^2 \]

\[- \frac{w_c(0.75h)^2}{32.2} + \frac{w_c}{32.2} \left[ x_{PA} + 0.25h + \ell - (x_{cg})_p \right]^2 \]

\[(I_{xx})_p = 0.3 \left( \frac{w_c}{32.2} \right) r^2 + 0.5 \left( \frac{w_p - w_c}{32.2} \right) r^2 \]

\[x_{cg} = \frac{w_{MLF} (x_{cg})_{MLF} + w_p (x_{cg})_p - w_{p1} (x_{cg})_{p1}}{w_{MLF} + w_p - w_{p1}} \]

\[(I_{yy})_{t_i} = (I_{yy})_{MLF} + (I_{yy})_p \]

\[+ \frac{1}{32.2} \left\{ w_{MLF} \left[ (x_{cg})_{MLF} - (x_{cg})_{t_i} \right]^2 \right\} \]

\[+ w_p \left[ (x_{cg})_p - (x_{cg})_{t_i} \right]^2 \]  \quad \text{for} \quad t \leq t_i
\[(I_{yy})_{bo} = (I_{yy})_{MEF} + (I_{yy})_{p} + \frac{1}{32.2} \left\{ W_{MEF} \left[ (x_{cg})_{MEF} - (x_{cg})_{t_{bo}} \right]^2 \right. \\
\left. + W_{p} \left[ (x_{cg})_{p} - (x_{cg})_{t_{bo}} \right]^2 \right\} \quad t \geq t_{bo} \]

\[I_{yy} = (I_{yy})_{t_{i}} - \left[ (I_{yy})_{t_{i}} - (I_{yy})_{t_{bo}} \right] \frac{t - t_{i}}{t_{bo} - t_{i}} \quad t_{i} < t < t_{bo} \]

\[(I_{xx})_{t_{i}} = (I_{xx})_{p} + (I_{xx})_{MLF} \]

\[(I_{xx})_{t_{bo}} = (I_{xx})_{p} + (I_{xx})_{MEF} \]

\[I_{xx} = (I_{xx})_{t_{i}} - \left[ (I_{xx})_{t_{i}} - (I_{xx})_{t_{bo}} \right] \frac{t - t_{i}}{t_{bo} - t_{i}} \quad t_{i} \leq t \leq t_{bo} \]

\[W = W_{MLF} + W_{p} - W_{p1} \]
APPENDIX B

STATIC STABILITY AND NATURAL FREQUENCY PROGRAM

INTRODUCTION

In an effort to define the effects of payload weight and length on the Capache vehicle characteristics, several IBM 7094 computer programs have been developed within the Flight Performance Section. A program (herein designated WC&I) now in use (Appendix A) determines the weight, center-of-gravity location, and pitch and roll moment of inertia as functions of payload weight and length. This appendix describes a program (designated SS&W) now completed that determines the static stability and pitch natural frequency. The input format of the SS&W program was chosen to accept output cards from the WC&I program, thus minimizing program data transfer time. These programs have been verified by a comparison with hand calculations for flight 14.54. The programs were utilized to determine the natural frequency history for flight 14.37. They are currently being used in a parametric study to determine the effects of payload weight and length of the vehicle characteristics and natural frequency history of the Capache sounding rocket. The purpose of this appendix is to outline the SS&W program.

SYMBOLS

\[
\begin{align*}
a & \quad \text{Speed of sound} \\
\text{Alt.} & \quad \text{Altitude} \\
C_m & \quad \text{Pitching moment coefficient curve slope} \\
C_N & \quad \text{Normal force coefficient curve slope} \\
D & \quad \text{Reference diameter} \\
h & \quad \text{Length of nose cone} \\
I_{yy} & \quad \text{Pitch moment of inertia} \\
\ell & \quad \text{Length of payload cylindrical section} \\
M & \quad \text{Mach number} \\
q & \quad \text{Free-stream dynamic pressure}
\end{align*}
\]
S       Reference area \( \left( \frac{\pi D^2}{4} \right) \)

\( t \)       Time (sec)

\( V \)       Free-stream velocity

\( W_p \)     Payload weight (lbs)

\( x_{cg} \)  Center-of-gravity location

\( x_{cp} \)  Center-of-pressure location

\( \rho \)     Free-stream density

\( \omega \)   Pitch natural frequency (cps)

**STATIC STABILITY PARAMETER AND NATURAL FREQUENCY DETERMINATION PROGRAM**

**Inputs:** \( W_p, h, \ell, S, D \)

\[ x_{cg} = f(t) \]

\[ V \text{ and alt.} = f(t) \quad \text{Option 1*} \]

\[ C_{N_{\infty}} = f(M) \]

\[ x_{cp} = f(M, \ell_t) \]

\[ I_{yy} = f(t) \quad \text{Option 1*} \]

*The program was set up with two options as follows:

**Option 1:** Compute \( \omega = f(t) \) using velocity and altitude data

**Option 2:** Compute \( C_{m_{\infty}} = f(M, t) \) for input into the G.E. MASS Program

The 1959 ARDC atmosphere equations are used to determine the atmospheric ambient conditions.
Equations: 
\[ \ell_t = \ell + h \]
\[ q = \frac{1}{2} \rho V^2; \quad M = \frac{V}{a} \]
\[ C_{m_{\infty}} = C_{N_{\infty}} \left( \frac{x_{cp} - x_{cg}}{D} \right) \]
\[ \omega \text{ (rad/sec)} = \sqrt{-C_{m_{\infty}} q S D} \frac{1}{I_{yy}} \]
\[ \omega \text{ (CPS)} = \frac{1}{2 \pi} \omega \text{ (rad/sec)} \]

Output: Option 1*

- \( W_p, h, \ell, S, D \)
- \( C_{m_{\infty}}, \omega \text{ (rad/sec)}, \omega \text{ (CPS)}, q = f(t) \)

Option 2*

- \( W_p, h, \ell, S, D \)
- \( C_{m_{\infty}} = f(M, t) \)

*The program was set up with two options as follows:

Option 1: Compute \( \omega = f(t) \) using velocity and altitude data

Option 2: Compute \( C_{m_{\infty}} = f(M, t) \) for input into the G.E. MASS Program

The 1959 ARDC atmosphere equations are used to determine the atmospheric ambient conditions.
REFERENCES


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Figure 2. Effects of Payload Weight and Length on the Apache Center-of-Gravity Location
Figure 3. Effects of Payload Weight and Length on Apache Pitch and Yaw Moments of Inertia
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(b) PAYLOAD WEIGHT = 80 LBS.

Figure 8. (Continued)
Figure 8. (Concluded)

(c) PAYLOAD WEIGHT = 120 LBS.
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(a) PAYLOAD WEIGHT = 40 LBS.
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(a) PAYLOAD WEIGHT = 40 LBS.
Figure 18. (Continued)

(b) PAYLOAD WEIGHT = 80 LBS.
Figure 18. (Concluded)

(c) PAYLOAD WEIGHT = 120 LBS.