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SUMMARY

Working charts are presented giving values of additive drag coefficient and mass-flow ratio for inlets utilizing axisymmetric cones at zero angle of attack. Cone half-angles range from 50° to 350. The free-stream Mach number ranges from that which will yield sonic flow on the cone surface to 12. Perfect-gas relations are used throughout the calculations.

INTRODUCTION

The treatment of additive drag in reference 1 includes a study of axisymmetric inlets for a limited range of free-stream Mach numbers and cone half-angles which were of primary interest at the time the study was made. Utilizing axisymmetric inlets for hypersonic vehicles requires data over a larger range of Mach numbers and at smaller cone half-angles. Calculations for such data have been made and are presented herein as working charts for a range of Mach numbers from that which will yield sonic flow on the cone surface to 12, and for cone half-angles from 50° to 350°.

The computation of additive drag was accomplished by numerical integration of the pressures existing along the entering stream tube. The mass-flow ratios were determined from the radial coordinates of the streamline which corresponded to the limits of the pressure integration. The computations of the additive drag and the mass-flow ratio were made by adding a subroutine to a machine computer program which was used to calculate conical flow parameters. The computations of the present paper cover the ranges of variables presented in reference 1 in order that a complete set of data may be obtained from one source. It may be noted from a comparison of the two sets of results that some differences in shock angles exist. The shock angles computed herein agree with those presented in reference 2.
SYMBOLS AND NOTATIONS

A  cross-sectional area

\( C_{D,a} \)  additive drag coefficient, \( \frac{D_a}{qA} \)

\( D_a \)  additive drag

\( K \)  defined parameter, \( \frac{1}{\sin \theta_1} \left( \frac{1 - W_1^2}{W_{N,1}} \right)^{\gamma - 1} \)

\( L \)  integration index for cowl location

\( M \)  Mach number

\( w \)  mass flow

\( w/w_0 \)  ratio of mass flow entering inlet to that passing through area \( A \) under free-stream conditions

\( p \)  pressure

\( q \)  dynamic pressure, \( \frac{\gamma}{2} p_0 M_0^2 \)

\( r \)  radius

\( S \)  conical area generated by revolution of a ray

\( t \)  temperature

\( V \)  velocity of flow

\( V_{lim} \)  limiting velocity obtained by expanding flow to zero temperature

\( W \)  ratio of local velocity to limiting velocity, \( \frac{V}{V_{lim}} \)

\( z \)  distance along generatrix ray

\( \gamma \)  ratio of specific heats, 1.400 for air

\( 2 \)
METHOD OF SOLUTION

The solution of the flow field around cones at zero angle of attack has been published in references 3 and 4 and, therefore, will not be repeated herein. In the present investigation, the solution of the conical flow problem was derived by the use of an existing computer program in a manner similar to that of reference 3. Outputs were normal and resultant velocity ratio (W_N,W) for a given ray angle, cone half-angle, and free-stream Mach number (θ,η,M_0). The values of free-stream Mach number for sonic flow on the cone surface were taken from table 1 of reference 5.

For conical flow, the transition across the shock wave is governed by the oblique shock relations and followed by a continuous isentropic compression to surface conditions. Figure 1 shows a diagram of the flow field and additive drag parameters. The conditions just downstream of the shock and the coordinates of a point on the entering streamline must be defined in order to determine the mass-flow ratio.
The mass flow at any position downstream of the shock is given by

\[ w = \rho_l W_{N,1} S_l = \rho_l W_{N,1} S_1 \]

If the following relations are assumed:

\[ S_l = \pi r_l z_l \]

\[ r_l = z_l \sin \theta \]

\[ r_1 = 1 \]

\[ \left( \frac{\rho}{\rho_l} \right)_l = \left( 1 - W_l^2 \right)^{\gamma-1} \frac{1}{W_{N,l} \sin \theta_l} \]

the distance along a generatrix ray required to pass the mass flow \( \rho_l W_{N,1} S_1 \) is

\[ z_l = \sqrt{\frac{K}{\left( 1 - W_l^2 \right)^{\gamma-1} W_{N,l} \sin \theta_l}} \]
where \( K = \left( \frac{1 - W_1^2}{\sin \theta_1} \right)^{Y-1} W_{N,1} \), a convenient parameter defined only by conditions across the shock. The mass-flow ratio captured by a cowl lip at position \( l \) is defined as follows:

\[
\frac{w}{w_0} = \frac{\rho_0 V_0 A_0}{\rho_0 V_0 A_l}
\]

If \( A_l = \pi r_l^2 \) and \( r_1 = r_0 = 1 \), then

\[
\frac{w}{w_0} = \frac{1}{r_l^2}
\]

Additive drag is defined in reference 1 as

\[
D_a = \int_1^L (p - p_0) dA
\]

Equation (2) may be approximated by the method of numerical integration in the form:

\[
D_a = \sum_{J=2}^L \left( \frac{p_J + p_{J-1}}{2} - p_0 \right) (A_J - A_{J-1})
\]

The additive drag coefficient is given by

\[
C_{D,a} = \frac{D_a}{qA_l}
\]

Substituting equivalent expressions for the parameters in this equation and rearranging yield

\[
C_{D,a} = \frac{1}{\gamma M_0^2 r_l^2} \sum_{J=2}^L \left( \frac{p_J}{p_0} + \frac{p_{J-1}}{p_0} - 2 \right) (r_J^2 - r_{J-1}^2)
\]

where

\[
\frac{p_J}{p_0} = \left( \frac{p_t}{p_0} \right)^{J} \left( \frac{p_t}{p_0} \right)^{1/J}
\]
and, from reference 2, 

\[
\frac{p_1}{p_0} = \frac{2\gamma M_0^2 \sin^2 \theta_1 - (\gamma - 1)}{\gamma + 1}
\]

\[
\left( \frac{p}{p_0} \right)_J = \left( 1 - W_J^2 \right)^{\frac{\gamma}{1}}
\]

The drag coefficient on the cone surface is given by

\[
C_{D,c} = \frac{2}{\gamma M_0^2} \left( \frac{p_c}{p_0} - 1 \right)
\]

where

\[
\frac{p_c}{p_0} = \frac{p_1/p_0}{(p/p_t)_J} \left( 1 - W_c^2 \right)^{\frac{\gamma}{1}}
\]

Values of the cone drag coefficient are presented in the charts for which \( \theta_1 = \eta \).

In order to obtain sufficient accuracy, a 0.1° ray angle increment for numerical integration was chosen. Increments smaller than 0.1° showed no significant improvement in accuracy.

Perfect-gas relations (\( \gamma = 1.4 \)) are assumed for the calculations herein, but at the higher Mach numbers and cone half-angles this assumption is not correct. The curve in figure 2 indicates the limitations imposed by the assumption. This figure shows the relationship between the cone half-angle and Mach number for which the surface static temperature is 1000° R (556° K). At this temperature the value of \( \gamma \) is 98 percent of the ideal value. At \( M = 12 \), for example, the cone half-angle can be 11.5° without exceeding this temperature limitation. Since at hypersonic speeds the conical half-angles of interest are usually

![Figure 2. Variation of free-stream Mach number with cone half-angle for \( t_c = 1000° R \) (556° K).](image)
less than $10^\circ$, expansion of the computation to include effects of the specific-heat ratio was not considered necessary.

PRESENTATION OF CHARTS

Working charts are presented for cone half-angles between $5^\circ$ and $35^\circ$ in figures 3 to 14. For each cone half-angle, the additive drag coefficient is plotted as a function of the free-stream Mach number and the ray angle; the mass-flow ratio is plotted as a function of the free-stream Mach number and the ray angle; and the additive drag coefficient is plotted as a function of the mass-flow ratio for selected free-stream Mach numbers.

Figure 3.- Variation of additive drag and mass-flow ratio for $\eta = 5^\circ$.

(a) Variation of additive drag with Mach number.
(b) Variation of mass-flow ratio with Mach number.

Figure 3.- Continued.
(c) Variation of additive drag with mass-flow ratio.

Figure 3.- Concluded.
(a) Variation of additive drag with Mach number.

Figure 4.- Variation of additive drag and mass-flow ratio for $\eta = 60^\circ$. 

10
(b) Variation of mass-flow ratio with Mach number.

Figure 4.- Continued.
(c) Variation of additive drag with mass-flow ratio.

Figure 4 - Concluded.
Figure 5.- Variation of additive drag and mass-flow ratio for $\eta = 70$. 

(a) Variation of additive drag with Mach number.
(b) Variation of mass-flow ratio with Mach number.

Figure 5.- Continued.
Figure 5.- Concluded.

(c) Variation of additive drag with mass-flow ratio.
Figure 6. Variation of additive drag and mass-flow ratio for $\eta = 8^\circ$. 

(a) Variation of additive drag with Mach number.
(b) Variation of mass-flow ratio with Mach number.

Figure 6.- Continued.
(c) Variation of additive drag with mass-flow ratio.

Figure 6.- Concluded.
Figure 7. Variation of additive drag and mass-flow ratio for $\eta = 10^0$.
(b) Variation of mass-flow ratio with Mach number.

Figure 7.- Continued.
(c) Variation of additive drag with mass-flow ratio.

Figure 7.- Concluded.
(a) Variation of additive drag with Mach number.

Figure 8. Variation of additive drag and mass-flow ratio for $\eta = 12.5^\circ$. 

22
(b) Variation of mass-flow ratio with Mach number.

Figure 8.—Continued.
(c) Variation of additive drag with mass-flow ratio.

Figure 8.- Concluded.
Figure 9.- Variation of additive drag and mass-flow ratio for $\eta = 15^\circ$.  

(a) Variation of additive drag with Mach number.

25
(b) Variation of mass-flow ratio with Mach number.

Figure 9. Continued.
(c) Variation of additive drag with mass-flow ratio.

Figure 9. Concluded.
Figure 10. Variation of additive drag and mass-flow ratio for $\eta = 17.50$. 

(a) Variation of additive drag with Mach number.
Figure 10. (b) Variation of mass-flow ratio with Mach number.

Figure 10. Continued.
(c) Variation of additive drag with mass-flow ratio.

Figure 10. Concluded.
(a) Variation of additive drag with Mach number.

Figure 11.- Variation of additive drag and mass-flow ratio for $\eta = 20^\circ$. 
(b) Variation of mass-flow ratio with Mach number.

Figure 11.- Continued.
(c) Variation of additive drag with mass-flow ratio.

Figure 11.— Concluded.
(a) Variation of additive drag with Mach number.

Figure 12.- Variation of additive drag and mass-flow ratio for $\eta = 25^\circ$. 

0.6

0.5

0.4

0.3

0.2

0.1

0

0 1 2 3 4 5 6 7 8 9 10 11 12

Mach number

$C_{D,a}$

$\theta_i$
(b) Variation of mass-flow ratio with Mach number.

Figure 12. Continued.
(c) Variation of additive drag with mass-flow ratio.

Figure 12.- Concluded.
Figure 13.- Variation of additive drag and mass-flow ratio for $\eta = 30^\circ$. (a) Variation of additive drag with Mach number.
(b) Variation of mass-flow ratio with Mach number.

Figure 13.- Continued.
(c) Variation of additive drag with mass-flow ratio.

Figure 13.— Concluded.
(a) Variation of additive drag with Mach number.

Figure 14.- Variation of additive drag and mass-flow ratio for $\eta = 35^\circ$. 

40
(b) Variation of mass-flow ratio with Mach number.

Figure 14.- Continued.
Figure 14.- Concluded.
CONCLUDING REMARKS

Working charts are presented giving values of additive drag coefficient and mass-flow ratio for inlets utilizing axisymmetric cones at zero angle of attack. Cone half-angles range from $5^\circ$ to $35^\circ$. The free-stream Mach number ranges from that which will yield sonic flow on the cone surface to 12. Perfect-gas relations are used throughout the calculations.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Station, Hampton, Va., March 2, 1966.

REFERENCES


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