EFFECT OF TURBINE-COOLANT FLOW ON BRAYTON-CYCLE SPACE-POWER SYSTEM THERMODYNAMIC PERFORMANCE

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SUMMARY

The effects of turbine-coolant flow on the thermodynamic performance of Brayton-cycle space-power systems were studied. A reference system was established, and the interrelated effects of turbine-inlet temperature, coolant-flow rate, and turbine efficiency on radiator area were examined for turbine-inlet temperatures to 3500° R. Coolant-flow requirements as a function of temperature and turbine-efficiency change as a function of coolant flow were varied parametrically, with the maximum uncooled turbine-inlet temperature assumed to be 2500° R.

The most significant thermodynamic effect resulting from the use of turbine-coolant flow at any given turbine-inlet temperature was an increase in radiator area. The potential reduction in radiator area obtainable with increasing turbine-inlet temperature, therefore, depends strongly on the associated coolant-flow requirement as well as on the turbine-efficiency variation. Increasing turbine-inlet temperature through the use of turbine cooling can result in a radiator-area reduction, but this potential reduction can be nullified by a high-coolant-flow requirement or a severe degradation in turbine efficiency.

INTRODUCTION

Brayton-cycle systems have been studied mainly with respect to low- and intermediate-power (to about 300 kWe) applications (e.g., ref. 1), where low specific weight is not a prime requirement. Such has been the case because, as compared with Rankine-cycle systems, Brayton-cycle systems are relatively large and heavy primarily because of the nonisothermal nature of the heat-rejection process. In order for Brayton systems to be suitable for high-power-level electric-propulsion applications, turbine-inlet temperatures of 3000° R or higher appear to be necessary (ref. 2). At this high temperature level, cooling the turbine becomes a consideration, if not a necessity.
The turbine is commonly cooled by means of a small portion of gas extracted from a lower temperature region of the system. This gas is bypassed around the heat source, ducted through the turbine blading, and subsequently injected back into the main stream. As a result, the coolant flow does not produce work in the turbine to the same extent as does the main flow, and a penalty occurs in thermodynamic performance (i.e., lower cycle efficiency and a larger radiator). In addition, the injection of coolant flow into the main flow has an effect on turbine aerodynamic performance, which also manifests itself in terms of cycle thermodynamic performance. If the use of coolant flow is to prove advantageous, the benefits gained from the allowable increase in turbine-inlet temperature must be sufficient to offset the associated thermodynamic performance penalties.

In order to conduct realistic design studies for high-temperature Brayton-cycle systems, the pertinent factors associated with the use of turbine cooling must be understood. These factors include knowledge of coolant-flow requirements with increasing turbine-inlet temperature, turbine-efficiency variation with amount of coolant and method of cooling, as well as the basic thermodynamic effects of coolant bypass. The study presented herein was conducted primarily to explore the effects of coolant bypass flow on system thermodynamic performance. A reference system was established, and the interrelated effects of turbine-inlet temperature, coolant-flow rate, and turbine efficiency on radiator area were examined. Coolant-flow requirements as a function of temperature and turbine-efficiency change as a function of coolant flow were varied parametrically, with the maximum uncooled turbine-inlet temperature assumed to be 2500° R.

BASIC CYCLE CHARACTERISTICS

Brayton-cycle systems with either single- or dual-shaft turbomachinery have been studied (refs. 1 and 2). For the purpose of showing the thermodynamic effects of coolant bypass, a single-shaft system is used. Basic trends are the same for dual-shaft arrangements. In this section, the cycle is described, and the computational model is defined.

Cycle Description

A typical Brayton power cycle for space application is represented both schematically and thermodynamically in figure 1. Cold gas from the radiator is compressed and then heated. The heating takes place in two stages, first in the recuperator by means of the hotter turbine exhaust and then in the heat-source exchanger. As the gas expands next through the turbine, work is produced to drive the compressor and the alternator. The turbine exhaust gas is then cooled, first in the recuperator and finally in the radiator, to
The level for radiator heat-transfer coefficient reflects the relatively good heat-transfer properties that accompany the higher pressure levels associated with higher-power systems.

**Computational Model**

The computational model with respect to the coolant-bypass flow is indicated in figure 2. For purposes of computation, the coolant flow is assumed to bypass the turbine completely and then to mix with the turbine exhaust flow. The temperature after mixing...
is determined by a heat balance, and the higher pressure of the coolant stream is assumed to be lost as friction in the coolant ducts.

Turbine efficiency is defined basically as actual work divided by ideal work. When the turbine is cooled, there can be several ways to express this ratio, depending on which factors are considered in the determination of the actual and ideal works, as discussed in reference 3. The actual work, which always includes shaft work, may or may not include the work expended to pump the coolant out of the rotor blade. The ideal work, which always includes the work available from the main stream, may or may not include the work available from the coolant. For the coolant stream, the pumping work depends on the blade speed at the point of injection, and the available work depends on the fluid state at the point of injection and the position of injection. At the time that a preliminary thermodynamic analysis of a system is conducted, the factors required for the inclusion of the coolant pumping and available work terms are generally not yet established. For a cycle analysis of this type, consequently, it becomes most convenient to use the definition of turbine efficiency that excludes both of these terms. Turbine efficiency, therefore, is defined for this study as

\[ \eta_T = \frac{\text{Actual shaft work}}{\text{Main-stream ideal work}} \]

Coolant pumping and available work effects can be accounted for in terms of a work perturbation coefficient that acts on turbine efficiency in a manner to be discussed subsequently in the section TURBINE PERFORMANCE. In accordance with the definition of turbine efficiency assumed and the bypass flow model presented in figure 2, the computational definition of cooled-turbine efficiency becomes the same as that for uncooled-turbine efficiency.

The development of the basic equations required for this thermodynamic analysis is
similar to that presented in appendix B of reference 1 except that appropriate modifications must be made to account for the coolant bypass. Because of this similarity, the equations required to compute cycle efficiency and radiator area are stated rather than developed. All symbols used in this analysis are presented in appendix A. The thermodynamic equations for the single-shaft system assumed in this analysis are presented in appendix B.

EFFECT OF TURBINE-COOLANT BYPASS

The application of the thermodynamic-analysis equations for a constant turbine-inlet temperature ($3000^\circ R$ in this example) and the system parameters listed in table I yields radiator-area - cycle-efficiency characteristic curves as shown in figure 3 for several coolant-bypass ratios. These curves represent optimized variable values and were obtained in the manner described in reference 1. The dashed curve describes the locus of minimum-area points, which occur at a cycle temperature ratio of about 0.3. The basic thermodynamic effects associated with the use of coolant bypass can be seen from this figure. The minimum radiator area increases significantly with increasing bypass ratio. With respect to the uncooled case (zero bypass), a 4-percent bypass ratio results in a 17-percent increase in radiator area, and the required area is doubled with the use of a 16-percent bypass ratio. Increasing the bypass ratio also reduces the cycle efficiency. For the minimum-area operating points, the reduction in cycle efficiency is about 1 percent for each percent of bypass ratio. This reduction is much less significant than the...
increase in radiator area, and subsequent discussions of thermodynamic performance consider only the more significant radiator-area effect.

Minimum radiator areas, as obtained from plots such as figure 3, for several bypass ratios are plotted against turbine-inlet temperature in figure 4. With respect to radiator size, the detrimental effect of increasing bypass ratio and the beneficial effect of increasing turbine-inlet temperature are both evident and highly significant. Determining the net effect of these factors on radiator area requires that bypass ratio requirements be related to turbine-inlet temperature.

COOLANT-FLOW REQUIREMENTS

Any relation between coolant-flow requirements and turbine-inlet temperature depends on many factors. These factors include coolant inlet and exit temperatures, blade temperature, method of cooling, blade internal and external geometries, and turbine operating conditions. A general discussion of turbine cooling is presented in reference 4. With so many factors involved, there appears to be no simple general relation that can readily be established between coolant flow and temperature. In addition, all available information appears to be concerned with relatively large axial-flow turbines for jet-engine application. For Brayton-cycle space-power system applications, the turbines of interest are much smaller and include radial-inflow as well as axial-flow machines.

A comparison of relative coolant-flow requirements for several types of air-cooled blades presented in reference 4 showed that a linear form of relation between coolant flow and temperature appears to be a satisfactory estimate for a turbine-inlet-temperature range extending to about 1000° R above the maximum uncooled value. The same form of relation is assumed valid for the turbines of interest; consequently, for this study,

\[ y = k \times 10^{-4} (T_1 - T^*) \]

The temperature \( T^* \) is the maximum uncooled turbine-inlet temperature, and the constant \( k \), which is referred to as the "coolant-flow constant," can be interpreted as the percent coolant flow required per 100° R of allowable temperature rise.

Specific prime radiator area is plotted against turbine-inlet temperature in figure 5 for several values of \( k \) with \( T^* \) assumed equal to 2500° R.
The dashed curves, which are from figure 4, indicate the coolant-bypass ratio associated with each turbine-inlet temperature and coolant-flow constant. For each coolant-flow constant, there appears to be a turbine-inlet temperature at which a minimum area is obtained; this minimum-area turbine-inlet temperature decreases with increasing coolant rate until it becomes coincident with the maximum uncooled turbine-inlet temperature. With low coolant flows, such as 1 percent per 100° R (k = 1), the minimum area is not reached in the temperature range studied (to 3500° R where the coolant bypass ratio is 0.10). The use of higher turbine-inlet temperatures in this case appears definitely advantageous, since the required radiator area at 3500° R is about 45 percent of that at 2500° R and further reductions are still achievable. With a coolant flow of 2 percent per 100° R (k = 2), the minimum area is reached at approximately 3500° R, where the coolant-bypass ratio is 0.20. In this case, the maximum achievable reduction in radiator area is only about 30 percent. With a coolant flow greater than about 3 percent per 100° R (k > 3), the thermodynamic performance penalties more than offset the higher radiator temperatures, and the use of higher turbine-inlet temperatures result in larger radiator areas. Figure 5 shows that the level of required coolant flow strongly influences the potential benefits associated with turbine cooling. A coolant-flow schedule for a given machine or class of machines, therefore, must be established before the design of a system with turbine cooling can be finalized.

TURBINE PERFORMANCE

The curves in figure 5 and the results drawn therefrom are based on a constant value for turbine efficiency. As mentioned previously, turbine efficiency is defined herein as

\[ \eta_T = \frac{\Delta H}{w_T \Delta h_{id}} \]

which is the total shaft work divided by the ideal work available from the main-stream flow. In general, the introduction of coolant flow into the turbine affects the shaft work output. With efficiency so defined, this effect can be expressed in terms of a work perturbation coefficient that acts on the efficiency of an uncooled turbine to yield an efficiency differential attributable to the coolant flow. In this section, the efficiency for the cooled turbine is related to the work perturbation coefficient, and the effect of the work perturbation coefficient on radiator area is then examined parametrically.
Work Perturbation Coefficient

Both efficiency and shaft work can be expressed in terms of an uncooled-turbine component and a coolant-flow component as

\[ \eta_T = \eta_{T, \text{unc}} + \Delta \eta \]

and

\[ \Delta H = w_T \Delta h_{\text{unc}} + w_c \Delta h_c \]

Substituting these equations into the definition of efficiency yields

\[ \eta_{T, \text{unc}} + \Delta \eta = \frac{w_T \Delta h_{\text{unc}}}{w_T \Delta h_{\text{id}}} + \frac{w_c \Delta h_c}{w_T \Delta h_{\text{id}}} \]

Since

\[ \eta_{T, \text{unc}} = \frac{w_T \Delta h_{\text{unc}}}{w_T \Delta h_{\text{id}}} \]

then

\[ \Delta \eta = \frac{w_c \Delta h_c}{w_T \Delta h_{\text{id}}} \]

Further substitution of \( w_c/w_T = y/(1 - y) \) and \( \Delta h_{\text{id}} = \Delta h_{\text{unc}}/\eta_{T, \text{unc}} \) and rearrangement yield

\[ \frac{\Delta \eta}{\eta_{T, \text{unc}}} = \frac{y}{1 - y} \left( \frac{\Delta h_c}{\Delta h_{\text{unc}}} \right) \]

A work perturbation coefficient \( \xi \) is now defined as the ratio of shaft-work change attributable to the coolant flow to the shaft work produced in the uncooled turbine; that is,

\[ \xi = \frac{\Delta h_c}{\Delta h_{\text{unc}}} \]
Use of this definition yields

\[
\frac{\Delta \eta}{\eta_{T, \text{unc}}} = \frac{y}{1-y} \xi
\]

Finally, since

\[
\eta_T = \eta_{T, \text{unc}} + \Delta \eta
\]

then

\[
\frac{\eta_T}{\eta_{T, \text{unc}}} = 1 + \frac{y}{1-y} \xi
\]

which is the expression to be used for evaluating turbine efficiency.

**Effect of Turbine Performance Variation**

The value for work perturbation coefficient depends on the particular application and turbine configuration being studied. In general, this coefficient may be negative, zero, or positive. The effect of the work perturbation coefficient, consequently, is examined parametrically herein.

Radiator area is plotted against turbine-inlet temperature in figures 6(a) and (b) for coolant flows of 1 and 2 percent, respectively, per 100° R of temperature increase. A parametric variation from -1 to 1 is shown for the work perturbation coefficient \( \xi \). The curves for \( \xi = 1 \) approximate the uncooled case; the increase in specific work compensates for the deficit in mainstream flow. The effect of \( \xi \) is more pronounced at the higher coolant flow. With a relatively low coolant flow such as 1 percent per 100° R, even a value of \( \xi \) of -1 offers a potential reduction in radiator area of about 30 percent.
With a somewhat higher coolant flow of 2 percent per 100° R, any small negative value for $\xi$ makes potential radiator-area reductions relatively small, with a value of $\xi$ of about -0.5 being the value that eliminates the potential for any reduction at all. From these results, it can be generalized that the nature of the specific variation in turbine efficiency with coolant-bypass ratio does not affect the general nature of the results but does affect the level of achievable radiator-area reduction and associated minimum-area turbine-inlet temperature for any particular coolant-flow requirement. Also, the tolerable performance degradation becomes less as the coolant-flow requirement increases.

**SUMMARY OF RESULTS**

The study reported herein explores the effects of turbine-coolant flow on the thermodynamic performance of Brayton-cycle space-power systems. The use of turbine-inlet temperatures to 3500° R was examined, with the maximum uncooled value being 2500° R. Coolant-flow requirements were based on a parametric linear variation of coolant bypass ratio with increasing turbine-inlet temperature. Turbine efficiency was defined on the basis of main-stream ideal work, and the change in turbine efficiency with coolant flow was described in terms of coolant bypass ratio and a parametrically examined work-perturbation coefficient. The major results of this investigation are summarized as follows:

1. With constant values for turbine-inlet temperature and turbine efficiency, the basic thermodynamic effects of increasing coolant flow were a significant increase in minimum radiator area and a relatively minor reduction in associated cycle efficiency. At a turbine-inlet temperature of 3000° R, for example, a 16-percent coolant-bypass flow resulted in a 100-percent increase in radiator area and a 16-percent reduction in cycle efficiency.

2. Increasing turbine-inlet temperature through the use of turbine cooling resulted in a reduction in radiator area with a minimum area being achieved at some turbine-inlet temperature and associated coolant flow. The potential reduction in radiator area, however, is strongly dependent on the coolant-flow requirement. With no variation in turbine efficiency, for example, a coolant flow of 1 percent per 100° R of temperature increase yielded a radiator-area reduction of more than 55 percent, while any coolant flow above 3 percent per 100° R of temperature increase yielded no radiator-area reduction at all with respect to the 2500° R uncooled-turbine system.

3. The nature of the specific variation in turbine efficiency with coolant-bypass ratio did not affect the general nature of the results, but did affect the level of achievable radiator-area reduction and associated minimum-area turbine-inlet temperature. With a relatively low coolant flow such as 1 percent per 100° R, a work-perturbation coeffi-
cient as low as -1 still offered a potential reduction in radiator area of about 30 percent. With a somewhat higher coolant flow of 2 percent per $100^\circ$ R, a work perturbation coefficient of about -0.5 or less eliminated the potential for any reduction at all.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, March 9, 1966.
## APPENDIX A

### SYMBOLS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_R$</td>
<td>radiator prime surface area, sq ft</td>
</tr>
<tr>
<td>$c_p$</td>
<td>heat capacity at constant pressure, Btu/(lb) ($^0R$)</td>
</tr>
<tr>
<td>$E$</td>
<td>effectiveness</td>
</tr>
<tr>
<td>$\Delta H$</td>
<td>turbine shaft total work, Btu/sec</td>
</tr>
<tr>
<td>$\Delta h$</td>
<td>turbine shaft specific work, Btu/lb</td>
</tr>
<tr>
<td>$\Delta h_{id}$</td>
<td>turbine ideal specific work, Btu/lb</td>
</tr>
<tr>
<td>$h_R$</td>
<td>radiator gas heat-transfer coefficient, Btu/(hr) (sq ft prime area) ($^0R$)</td>
</tr>
<tr>
<td>$k$</td>
<td>coolant-flow constant, percent of compressor flow/100 $^0$ R increase in turbine-inlet temperature</td>
</tr>
<tr>
<td>$P$</td>
<td>net shaft power, kW</td>
</tr>
<tr>
<td>$r$</td>
<td>pressure ratio</td>
</tr>
<tr>
<td>$T$</td>
<td>absolute temperature, $^0$R</td>
</tr>
<tr>
<td>$T^*$</td>
<td>maximum uncooled turbine-inlet temperature, $^0$R</td>
</tr>
<tr>
<td>$T_{j1}$</td>
<td>ratio of $T_j/T_1$, $j=2,3,\ldots,6$</td>
</tr>
<tr>
<td>$w$</td>
<td>weight flow, lb/sec</td>
</tr>
<tr>
<td>$y$</td>
<td>coolant-bypass ratio $w_c/w_C$</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>heat-capacity ratio</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>emissivity</td>
</tr>
<tr>
<td>$\eta$</td>
<td>efficiency</td>
</tr>
<tr>
<td>$\xi$</td>
<td>work-perturbation coefficient</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Stefan-Boltzmann constant, $0.173\times10^{-8}$ Btu/(hr)(sq ft)($^0R^4$)</td>
</tr>
</tbody>
</table>

### Subscripts:

- $C$ compressor
- $c$ coolant
- $cy$ cycle
- $s$ sink
- $T$ turbine
- $unc$ uncooled
- $w$ wall
- $1,2,3,4,5,6$ state points defined in fig. 1
APPENDIX B

THERMODYNAMIC ANALYSIS EQUATIONS

The equations required for the computation of cycle efficiency and radiator area are presented herein for a system with single-shaft turbomachinery and turbine-coolant bypass flow. The development of these equations is similar to that for the more general analysis, presented in appendix B of reference 1, which was made for an uncooled-turbine system.

The modified expression for cycle efficiency is

\[ \eta_{cy} = \frac{(1 - y)(1 - T_{21}) - (T_{51} - T_{41})}{(1 - y)(1 - T_{61})} \]

As in reference 1, \( T_{j1} \) represents \( T_j / T_1 \). Specific prime radiator area is computed from

\[
\frac{A_R}{P} = 3600 \left( \frac{w_{cp}}{P} \right) \left\{ \frac{1}{h_R} \ln \frac{T_{w,4}^4 - T_s^4}{T_{w,3}^4 - T_s^4} + \frac{1}{4\sigma T_s^3} \ln \frac{(T_w,3 - T_s)(T_w,4 + T_s)}{(T_w,4 - T_s)(T_w,3 + T_s)} \right\} - 2 \left( \arctan \frac{T_{w,3}}{T_s} - \arctan \frac{T_{w,4}}{T_s} \right)
\]

where \( T_w \) is related to \( T \) by

\[ T = T_w + \frac{\sigma \varepsilon}{h_R} (T_w^4 - T_s^4) \]

With the variables \( T_{21}, T_{41}, \) and \( y \), as well as the system-design parameters of table I (p. 5) as input, only \( T_{51}, T_{61}, T_{31}, \) and \( w_{cp}/P \) remain to be evaluated.

From turbomachinery pressure ratio and system pressure loss considerations, \( T_{51} \) is obtained as

13
\[
T_{51} = T_{41} \left\{ 1 + \frac{1}{\eta_C} \left[ \frac{(r_T)}{(r_C)} \right]^{-(\gamma - 1)/\gamma} \right\} \left[ 1 - \frac{1}{\eta_T} (1 - T_{21}) \right] - 1
\]

The recuperator equations yield

\[
T_{61} = ET_{2'1} + (1 - E)T_{51}
\]

\[
T_{31} = (1 - y)ET_{51} + [1 - (1 - y)E]T_{2'1}
\]

where

\[
T_{2'1} = (1 - y)T_{21} + yT_{51}
\]

Specific capacity rate is

\[
\frac{w_c}{P} = \frac{0.9487}{T_1[(1 - y)(1 - T_{21}) - (T_{51} - T_{41})]}
\]

These relations comprise the thermodynamic analysis equations.
REFERENCES


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—NATIONAL AERONAUTICS AND SPACE ACT OF 1958

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