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THEORETICAL STUDIES OF SPACE PLASMAS

Summary Report

May 3, 1965 to May 1, 1966

by

D. B. Chang, N. A. Krall, L. D. Pearlstein and M. N. Rosenbluth

Prepared for
National Aeronautics and Space Administration
Office of Space Sciences, Geophysics and Astronomy Programs
under Contract NASw-1211

GENERAL ATOMIC

DIVISION OF

GENERAL DYNAMICS

JOHN JAY HOPKINS LABORATORY FOR PURE AND APPLIED SCIENCE

P O BOX 608 SAN DIEGO 12, CALIFORNIA

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ABSTRACT

Nine reports describing plasma phenomenon of relevance to the radiation belts are presented. The bulk of the reports are concerned with linear instability. Two papers consider non-linear effects and one is a compilation of tables of synchrotron radiation from trapped particles.

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I.

INTRODUCTION

The summary report under Contract NASw-1211 consists of nine reports, (GA-6591, GA-6822, GA-6708, GA-6967, GA-7113, GA-7114, GA-7087, GA-6696, Astrophysical Journal Supplement No. 111). The first three have been submitted for publication, the next four are expected to be submitted for publication in the near future, and the last two have been published. In this part we summarize the results arrived at in these reports. Part II, section 1 to 9 are the respective reports.

In Part II - sections 1 and 2, we determined the influence of finite β (ratio of particle to magnetic pressure) upon stability of the interchange mode. The parameters utilized pertain to the tail of the magnetosphere. It was found that inclusion of the influence of the finite β terms is, in general, stabilizing in that a marginally stable solution becomes stable when the plasma pressure is increased. In addition, we have shown that the $\beta = 0$ calculations are pessimistic. However, to determine the stability properties of the configuration, it is necessary to solve a differential equation using the known plasma parameters. The defining equations are given by Equations (2-15) and (2-16) of GA-6591.

In Part II - section 3, we examined a high frequency instability encountered in two component plasmas in mirror geometries. Here the basic ingredients are a tenuous high-temperature loss-cone limited plasma superimposed on a dense low-temperature background plasma--as is the case in the radiation belt. It was found the instabilities with characteristic

frequencies on the order of a harmonic of the species cyclotron frequency would result if the fractional density of hot plasma exceeded the inverse of the harmonic number. If the fractional density of hot plasma does not exceed the inverse of the harmonic number, instability would occur only if the harmonic frequency were equal to the appropriate hybrid frequency.

In Part II - section 4, it was demonstrated that the electrons from the Starfish debris although predicted unstable on purely energetic grounds are stabilized by the influence of ionospheric currents, vis. these currents effectively short out the lines of force.

In Part II - section 5, we describe a new approach toward handling non-linear interactions. This theory treats the non-linear interaction between plasma oscillation normal modes in a hydrodynamic approximation. A modified quasi-linear treatment is used, in which the normal modes are taken to affect each other through the slow non-linear temporal and spatial changes they induce in the (equilibrium) background quantities. Plasmons, i.e., wave packets of plasma oscillations, are found in this approximation to have an interaction similar in form to the electromagnetic and gravitational interactions. The plasmon interaction is describable in terms of scalar and vector potentials which satisfy wave equations of the form satisfied by their electromagnetic and gravitational counterparts. Here, quantities proportional to the slow changes induced in the background number and current densities play the role of the scalar and vector potentials, with the thermal velocity playing the role of the speed of light. The source terms in the potential wave equations are quadratic in the normal mode density perturbation. The normal mode density perturbation in turn satisfied a Klein Gordon-like equation, so that, normalized,

it would appear to play a role similar to the quantum mechanical probability wave function ψ. An interesting consequence of these equations is the presence of an instability arising out of the tendency of plasmons to condense.

Since many of the modes inherent to the radiation belt, especially those which occur in the stagnant layer surrounding the magnetosphere, give rise to strong turbulence, it may be useful to use such models to obtain information about the non-linear behavior.

In Part II - section 6, a study of the non-linear effects of an instability encountered by Krall and Rosenbluth[5] and later applied to auroral discharge by Chamberlain[6] is described. The references are listed in GA-7114. In the application by Chamberlain, it was conjectured that since the instability was characterized by the presence of an electric field parallel to the ambient magnetic field, there would be dumping of electrons out of the loss cone due to acceleration. Since only a density gradient is needed to drive this instability (although it is enhanced by temperature asymmetries), it might be expected that the mode strongly influences the distribution of particles in the belt.

Results using the quasi-linear approximation for the non-linear behavior indicate that the instability will be an effective mechanism for dumping particles out of the loss-cone. Specifically, the wave-spectrum interacts with the bulk of the protons in the belt. It is inferred that the electrons will be dragged along with the protons to maintain charge neutrality.

In Part II - section 7, finite-β resistive instabilities are studied. The decoupled-flow regions of resistive instabilities occurring in

magnetospheric tails are sufficiently narrow that magnetic and collisional viscosities may be of importance. To determine the effects of these viscosities, the resistive instabilities studied in the hydromagnetic approximation by Furth, Killeen and Rosenbluth (1963) and Johnson, Greene and Coppi (1963), are studied with plasma equations based on the Chapman and Cowling expansion. Treating the ratio β of the plasma pressure to magnetic pressure to be of order unity, three cases are considered, two leading to tearing instabilities and the third giving a gravitational instability. It is found in all three cases that the hydromagnetic equations are applicable when the ion collision frequency is larger than the ion gyro-frequency. When the ion gyro-frequency exceeds the ion collision frequency, the results are qualitatively the same except that ion-ion collisions replace ion inertia in determining the growth rates and the size of the decoupled-flow region. The conditions under which the Hall currents become important are defined.

As some representative numbers, we take a typical length of the shear field to be 10^8 cm, look at perturbation wavelengths of the same order and assume a density, temperature and magnetic field of 10^2 cm^{-3}, 0.1 eV and 10γ, respectively. In this case viscosity does determine the behavior. Here, the mean free path of $\sim 1/2 \times 10^9$ cm is one-thirtieth the effective parallel wavelength $k_{\parallel}^{-1}$ and the latter is comparable with the diameter of the geomagnetic tail. The ratio ϵ of the thickness of the decoupled-flow region to the thickness of the equilibrium current layer is 10^{-2}, and for the tearing mode the growth time is of the order of 10^{-2} times the resistive diffusion time. i.e. of the order of a day. Shorter growth times may be obtained by considering shorter wavelength perturbations.

A paper based on a talk given at the Advanced Study Institute, Radiation Trapped in the Earth's Magnetic Field, August 16 - September 3, 1965 at Bergen, Norway, and to be published in the proceedings comprises Part II - section 8. The paper was a summary of some results obtained during this and the preceding contract period.

In Part II - section 9, two-dimensional distributions of the intensity of the polarization of the synchrotron radiation from ultra-relativistic electrons trapped in a distant dipole field are calculated for comparison with recent radio astronomical observations. A derivation of the Stokes parameters is given and numerical results are presented in the form of fifty-two tables in which the radiation is described for various angles θ_o, between the dipole axis and the line of sight. Provision is made for shadowing by a centrally located object.

The results reflect the dependence of the intensity and polarization on the field alignment, radiation efficiency, electron density, and radiating angle. The brightness pattern tends to be more uniformly distributed for an isotropic distribution of particles approaching a thin vanishing ring as the limiting equatorial helix angle a_L, increases for $\theta_o = 0$ and a more slowly vanishing, thin, wedge-shaped equatorial strip as a_L increases for $\theta_o = 90^o$. In general, the pattern outline progresses smoothly from a circular to a flatter elliptical shape as θ_o increases, and becomes a butterfly pattern for $\theta_o = 90^o$ and large a_L. While the major axis of the distribution tends to lie in the equatorial plane, in cases of large sin a_L there exists an intermediate range of aspect angle θ_o, for which the distribution actually consists of two separated portions lying along the dipole axis.

Polarizations run the gamut as θ_o and a_L are varied, the over-all degree of polarization increasing as θ_o and a_L increased. The patterns depend much more strongly on the aspect angle and the helix angle than on the energy distribution, while the frequency behavior is dependent only on the power-law energy spectrum assumed and is independent of the aspect angle or the helix angle. The results for other angular and energy distributions may be approximated by combining the patterns of various figures. In this way situations may be constructed where both the polarization and the brightness distribution are frequency dependent.

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II.

Section 1

DRIFT INSTABILITIES AND ELECTRON CYCLOTRON OSCILLATIONS FOR ARBITRARY PLASMA PRESSURE

by

N. A. Krall and L. D. Pearlstein

General Atomic Report
GA-6591
1965

Drift Instabilities and Electron Cyclotron Oscillations
for Arbitrary Plasma Pressure*

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28667

Most of the familiar drift instabilities (flute, universal, etc.) were calculated in the limit β = (plasma pressure)/(magnetic pressure) = 0. In this limit the perturbed electric field is electrostatic, and the Vlasov equation is remarkably simplified. The β limit for validity of the electrostatic approximation depends on the particular mode considered and many present experiments fall outside the range. Two well known low frequency drift instabilities are examined at arbitrary β; the zero β regime is determined and stability criteria for larger values of β are derived. In addition, the influence of finite β on radiation at the electron cyclotron harmonics is examined. The finite Larmor radius (R) flute instability driven by mirror type curvature of the magnetic field is considered first. It is shown that the lowest order ominously large contribution, inversely proportional to R^2, cancels to all orders of β. The leading terms are then independent of R, and are included in the eigenvalue problem; stability criteria are obtained for various ranges of β. Next the universal instability $\vec{E}_{Perturbed} = \vec{E}\exp(ik_\perp r_\perp)\exp(ik_\parallel r_\parallel)$ in a uniform magnetic field $\vec{B} = B_0\hat{i}_\parallel$ is considered, simulating field curvature by a fictitious gravity. Here the zero β limit is dependent upon wavelengths. If β < (electron mass)/(ion mass), or if $\beta < (k_\parallel r_p)^2$ where r_p = plasma radius and $(1/k_\parallel) \geq$ plasma length, the zero β limit obtains. Since the instability only exists for $(k_\parallel r_p)^2 < (6.5)^{-2}$ a modest plasma pressure violates the zero β limit. The eigenvalue for $\beta > (k_\parallel r_p)^2$ is calculated as are conditions for cusp stabilization. The dominant high frequency non-relativistic modes of the stable zero β plasma are longitudinal waves at electron gyrofrequency harmonics. If excited these modes can radiate only if coupled to transverse waves; in zero β this coupling occurs at a surface or at a local inhomogeneity. Since a prominent effect of finite β is to couple longitudinal and transverse waves the spectrum and coupling of electron cyclotron oscillations is calculated for arbitrary β.

Author

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1. INTRODUCTION

Most of the familiar drift instabilities (interchange, universal, etc.) [1-5] were calculated in the limit β = plasma pressure/magnetic pressure = 0. In this limit the unstable modes are electrostatic, and the Vlasov equation is remarkably simplified. The β limit for validity of the electrostatic approximation depends on the particular mode considered, and many present experiments fall outside the range. We examine two well known low frequency instabilities, with β finite, and determine both the range of the zero β approximation and the stability criteria for general values of β. In addition we calculate the influence of finite β on electron cyclotron harmonic waves.

In Section 2 we consider the finite Larmor radius (R_i) flute, or interchange, instability [1-3]. It is shown that the ominously large lowest order contribution, inversely proportional to R_i^2, cancels to all orders of β. The remaining terms prove to be stabilizing. We treat this problem by an equation involving only macroscopic quantities, density, pressure, β, etc.; this equation evolves from the Vlasov equation when we introduce the ordering scheme appropriate to the flute mode ($\omega/\Omega \sim k^2R_i^2 \ll 1$, $\beta \sim 1$), and allows a variational treatment of the stability problem. It also allows us to treat large $\beta < 1/k^2R_i^2$.

In Section 3 we examine the universal instability [4,5] $\vec{E}_1 = \vec{E}\exp(iky)\exp(iKz)$ in a linear magnetic field $B_z(x)$. This mode can be stabilized by several mechanisms in zero β [5]. One significant effect, stabilization by a strong magnetic cusp, is shown to be unaffected by finite β considerations. Another effect, finite length stabilization, is shown to be enhanced by finite β, allowing longer stable machines.

In Section 4 we study the high frequency mode structure of an infinite uniform plasma in a magnetic field. We show that in addition to the familiar longitudinal Bernstein mode [6], transverse and mixed waves exist at electron cyclotron harmonic frequencies. We calculate the absorption coefficients and show that the transverse and mixed waves can be excited to the black body level if β is finite. Like the Bernstein modes, these waves require a coupling mechanism in order to radiate out of the plasma. They also require frequencies below the plasma frequency and could be of interest in connection with experiments which observe intense radiation at cyclotron harmonics [7,8].

2. THE FINITE β FLUTE INSTABILITY

In the earliest investigation of the interchange mode the Vlasov equation was solved by direct integration over the unperturbed orbits [1]; in the later calculations the solution was obtained using an expansion technique in which the ion Larmor radius (R_i) was assumed small compared to a characteristic length (L) in the plasma [2,3]. In the method of Rosenbluth and Simon [3] it is necessary to order, from the beginning, all quantities in terms of a smallness parameter; the ordering presupposes knowledge of the solution [1]. However, in contrast to the method of direct integration it is then practical to solve problems of considerable complexity.

We use this ordering technique [3] to write down a closed set of equations to first order in the perturbation which enables us to incorporate β to all orders. Curvature is represented by an effective gravity (g). The geometry is one-dimensional, i.e., no quantities vary in the direction of a one-dimentional magnetic field B and all unperturbed quantities vary only in the direction of gravity

perpendicular to B. The appropriate ordering [1-3] in terms of the smallness parameter $\epsilon = R_i/L = 1/\Omega_i L\sqrt{2\alpha_i}$ is

$$\frac{\omega}{\Omega_i} \sim \frac{cE}{LB\Omega_i} \sim \frac{R_i^2}{L}\frac{\nabla B}{B} \sim \sqrt{\frac{g}{L\Omega_i^2}} \sim \epsilon^2; \qquad \beta \sim \epsilon^0$$

with Ω the cyclotron frequency, B the magnetic field, E the electric field, ω the eigenfrequency, and L the system size.

The first effect of finite β is to produce an apparent charge separation in lowest order due to the ∇B drift. However, this large effect cancels to all orders in β, leading to a theory which depends only upon the macroscopic plasma properties. We examine herein the cylindrical case and demonstrate by a variational technique that in general finite β is stabilizing.

Since the method employed follows that of Ref. [3] we shall, wherever convenient, use their notation. Our starting point is the Vlasov equation written in a frame moving with the drift velocity, with equilibrium fields and gravity varying only radially.

$$\vec{V} = \frac{\vec{G} \times \vec{\Omega}}{\Omega^2}, \qquad \vec{v} = \vec{v}' + \vec{V}, \qquad \vec{B}_0 = B_0(r)\hat{i}_z \tag{2.1}$$

$$D_t f + \vec{v}'\cdot\nabla f + [\vec{v}' \times \vec{\Omega} - D_t\vec{V} - (\vec{V}\cdot\nabla)\vec{V}]\cdot\nabla_v f = 0 \tag{2.2}$$

where

$$D_t = \frac{\partial}{\partial t} + \vec{V}\cdot\nabla \tag{2.3}$$

and $\vec{G}$ represents the acceleration arising from all forces save the Lorentz force. Note that $\vec{V}$ has no component in the direction of $\vec{B}$. If we now convert to polar coordinates in velocity space and define

$$\partial^{\pm} = \frac{\partial}{\partial x} \pm i\frac{\partial}{\partial y} = e^{\pm i\theta}\left(\frac{\partial}{\partial r} \pm i\frac{1}{r}\frac{\partial}{\partial\theta}\right) \tag{2.4}$$

with similar definitions for all other vectors we can, by substituting

$$f = \sum_{n=0}^{\infty}\sum_{q=-n}^{n} \epsilon^n e^{iq\phi} f_{n,q} \tag{2.5}$$

into the Vlasov equation obtain a formal solution by equating coefficients. We then have to lowest order

$$\epsilon f_{1,1} = -\frac{i}{2\Omega} v\partial^- f_0$$

$$2D_t f_0 + \epsilon v(\partial^+ f_{1,1} + \partial^- f_{1,-1}) - v \cdot \vec{V} \frac{\partial f_0}{\partial v} . \tag{2.6}$$

This equation along with the next order equation coupled with Maxwell's equations

$$\nabla \cdot \vec{E} = 4\pi \sum e \int v dv [f_0 + \epsilon^2 f_{2,0}] \tag{2.7}$$

$$\left[c\nabla \times \vec{B} - \frac{\partial \vec{E}}{\partial t}\right]^{\pm} = 4\pi \sum e \int v dv \{V^{\pm}[f_0 + \epsilon^2 f_{2,0}] + v[\epsilon f_{1,\mp 1} + \epsilon^2 f_{3,\mp 1}]\} \tag{2.8}$$

$$c \frac{\partial \vec{B}}{\partial t} = - \nabla \times \vec{E} \tag{2.9}$$

completely determine the solutions. If we now substitute Eq. (2.6) into Eqs. (2.7) and (2.8) we obtain, to lowest order, the equilibrium condition ($p = vdvv^2Mf/2$)

$$B \frac{\partial B}{\partial r} + 4\pi \sum \frac{\partial p}{\partial r} = 0, \tag{2.10}$$

and with the perturbations $f', E', B' \sim e^{i\omega t} e^{im\phi} f(r)$ we obtain, for the perturbed distribution function with $\eta = (\partial B/\partial r)/B$,

$$f' = -\frac{i}{\bar{\omega} + mV_D/r} \frac{\nabla \cdot \vec{V}'}{\eta} \left(\frac{\partial f}{\partial r} + \frac{\eta v}{2} \frac{\partial f}{\partial v}\right) + \frac{i}{\omega\eta} \frac{c\nabla \times \vec{E}}{B} \frac{\partial f}{\partial r} , \tag{2.11}$$

and for both the perturbed Maxwell's equations

$$0 = \nabla \cdot \vec{V} \sum e \int \frac{vdv}{\omega + mV_D/r} \left(\frac{\partial f}{\partial r} + \frac{\eta v}{2} \frac{\partial f}{\partial v}\right) . \tag{2.12}$$

In the above equations primed terms refer to perturbed quantities and the sum is over species. To arrive at these results we have assumed that the Debye length is of the same order in ϵ as is the Larmor radius; hence terms like $\nabla \cdot \vec{E}'$ are of higher order. Also note that $\vec{V}'$ does not contain the gravitational drift to this

order. In addition we have used the following definitions

$$\bar{\omega} = \omega - mcE/rB, \qquad V_D = (\partial B/\partial r)/2\alpha\Omega B. \tag{2.13}$$

We immediately see that the r.h.s. of Eq. (2.12) is the perturbed charge separation due to the ∇B drift which vanishes to lowest order in ϵ. Consequently, we must conclude that either the integral must vanish (which gives rise to a universal instability generated by a temperature gradient [9]) or $\nabla\cdot\vec{V}'$ must vanish producing the flute mode with the polarization

$$\frac{c}{B}(\nabla \times \vec{E}')_z = \frac{1}{B}\frac{\partial B}{\partial r}\frac{\omega}{\bar{\omega}}\frac{cE'_\theta}{B} + \Phi'(\epsilon^2). \tag{2.14}$$

Now, when this polarization relation is substituted into the next contributing order and $\Phi'(\epsilon^2)$ is eliminated from the lowest order equations (it of course does not occur in the higher order equations) by taking a suitable linear combination of Eqs. (2.7) and (2.8) we obtain

$$\frac{1}{r}\frac{\partial}{\partial r} r^3 T\rho \frac{\partial\psi}{\partial r} + \left[(1-m^2)\, T\rho + m^2 g \frac{\partial\rho}{\partial r} + r\omega^2 \frac{\partial}{\partial r}\rho(1+b^2)\right]\psi = 0 \tag{2.15}$$

where

$$b^2 = B^2/4\pi\rho c^2, \qquad \eta = (\partial B/\partial r)/B, \qquad \psi = cE'_\theta/B\bar{\omega}, \qquad \delta R = \int Mv^4 f\, vdv$$

$$T = \bar{\omega}^2(1+b^2) + m\bar{\omega}[\eta p + \partial p/\partial r]/\Omega r + m^2\eta(\partial R/\partial r)/r^2\Omega^2. \tag{2.16}$$

Again, all quantities refer to ion parameters. Finally, if we assume the plasma is isothermal with equal ion and electron temperatures we have

$$T = [\bar{\omega} - mV_c(1-\beta)/r]^2 - [mV_c(1+\beta)/r]^2$$

$$V_c = -(\partial p/\partial r)/2\alpha\Omega \tag{2.17}$$

where for convenience we have dropped the b^2 term, a valid approximation for all but very low density plasmas. In determining stability behavior we shall work solely with this last form for T, Eq. (2.17).

As in Ref. [3] Eq. (2.15) can be obtained from a variational principle with the property that the eigenvalues are in general complex. This variational principle is expressed for general equilibrium in Eqs. (2.25) - (2.27). In general we can only extract from this a sufficient condition for stability. However, as in Ref. [3], there is a special equilibrium $E(\rho)$ in which there is a minimal expression for $\bar{\omega}^2$. It can be shown, as in Ref. [3], that varying $E(\rho)$ away from the special equilibrium is destabilizing.

The variational principle allows us to make stability statements when β, ρ, B, and E are general functions of positions, and can be used to analyze quite realistic physical situations. In this work we are concerned only with the stability behavior arising from a variation of β. There are two useful approaches. We can find corrections to calculations [1-3] which assume a ρ, B, E and ignore the β contributions implied by ρ and B, or we can calculate the stability consistently with finite $\beta(r)$, and ask whether an increase in β (which changes ρ, B, etc.) is stabilizing. To distinguish β effects uniquely in the latter case is difficult; we attempt this by varying β with ρ'/ρ fixed, and taking first $E(\rho)$ given by the special equilibrium and later E = 0 independent of β.

We consider first the special equilibrium

$$cE/B = - V_c(1-\beta) \tag{2.18}$$

from which we obtain after multiplying by $r\psi$ and integrating

$$\frac{\omega^2}{m^2} = \frac{\int drr\rho[V_c(1+\beta)]^2 \Lambda^2 + \int drrg \frac{\partial\rho}{\partial r} \psi^2}{\int drr^3\rho\left[\left(\frac{\partial\psi}{\partial r} + \frac{\psi}{r}\right)^2 + \frac{m^2\psi^2}{r^2}\right]} \tag{2.19}$$

where

$$\Lambda^2 = \left(\frac{\partial\psi}{\partial r}\right)^2 + \frac{\psi^2(m^2-1)}{r^2} . \tag{2.20}$$

As in Ref. [3], with g = 0, $\omega^2 = 0$ is a solution for m = 1. This is seen by choosing as a trial function ψ constant everywhere except at the boundary; thus the numerator can be as small as desired. Consequently there is no stabilization for the m = 1 mode even for general β.

To obtain the behavior of a variation of β which maintains the special equilibrium we write

$$\rho = K\, \rho_0(x). \tag{2.21}$$

This form allows us to change β without altering the plasma profile ρ'/ρ, by changing K, since

$$\delta\beta = \beta(1+2\beta)\frac{\delta K}{K} . \tag{2.22}$$

To obtain Eq. (2.22) we have used the pressure balance equation which leads to

$$(B/B_0)^2 = (1+2\beta)^{-1} \tag{2.23}$$

with B_0 the value of B at the boundary. If we now assume that we have marginal stability for $\delta K = 0$, we may solve for $\delta\omega^2$ to order δK by varying the equilibrium only, since $\delta\omega^2/\delta\psi = 0$, and obtain

$$\delta\omega^2 \sim \delta K \int dr r\rho\Lambda^2 V_c{}^2\beta(6\beta^2 + 10\beta + 4). \tag{2.24}$$

Hence increasing β stabilizes.

For a general equilibrium the variational principle is

$$\frac{\delta\omega}{\delta\psi} = \frac{\delta\omega}{\delta\psi^*} = 0 \tag{2.25}$$

with

$$\omega = (- B \pm \sqrt{B^2-4AC})/2A \tag{2.26}$$

and

$$A = -\int dr\rho r^3\Lambda^2 + \int dr r^2(\partial\rho/\partial r)|\psi|^2$$

$$B = 2\int dr\rho r\Lambda^2[cE/B + \frac{1}{2} V_c(1-\beta)]$$

$$C = -\int dr\rho r\Lambda^2[(cE/B + V_c)(cE/B - V_c\beta)] \tag{2.27}$$

where Λ^2 is defined as in Eq. (2.20) except now absolute values are implied.

We next consider the influence of finite β on the stability for the case in which ρ is gaussian, $g = 0$, and (i) $E \sim r$, (ii) $u = -cE/B - V_c \sim r$ (uniform rotation). Note that for B = constant these two conditions are equivalent and the problem reduces to a well known calculation [1]. To proceed, we have as a

sufficient condition for stability $B^2-4AC \geq 0$. We assume that for $\beta = 0$ the system is at marginal stability and compute the correction to first order in β. We thus have for case (i) with

$$\bar{\alpha} = -\int dr\rho' r^2 \psi^2 / \int dr\rho r^3 \Lambda^3 > 0$$

$$\bar{\beta} = \int dr\rho r^3 \beta \Lambda^2 / \int dr\rho r^3 \Lambda^2 > 0 \tag{2.28}$$

the condition for stability

$$[(V_c/2r)^2 + \bar{\alpha} ucE/r^2 B] + \bar{\beta}[(V_c/r)^2 - \bar{\alpha}(3uV_c+u^2)/r^2] > 0 \tag{2.29}$$

All parameters occurring are to be taken in the $\beta = 0$ limit. The first square bracket vanishes by the assumption of marginal stability in the $\beta = 0$ limit. If we now substitute this result into the remaining term we obtain from Eq. (2.29)

$$(V_c)^2/8\,\bar{\beta}\left[\frac{1}{\bar{\alpha}} + 1 \pm \sqrt{\frac{1}{\bar{\alpha}} + 1}\right] > 0 \tag{2.30}$$

for stability, which is automatically satisfied. Thus we see β stabilizes; the $\beta = 0$ stability criterion is pessimistic [1]. For case (ii) we obtain twice the result of Eq. (2.30).

If we perform the analogous calculation for linear geometry where we assume all quantities depart for uniformity by a slow linear term we obtain the finite β correction to another well known result [1]. Our β modified stability criteria is

$$(1+2\beta)(kV_c)^2 \geq -kg(\partial\rho/\partial x)/\rho \tag{2.31}$$

and again β is stabilizing.

Finally we examine the effect of varying ρ subject to Eq. (2.21) with $E = 0$ and obtain after some algebra the condition for stability

$$\frac{\delta K}{K}\left\{\frac{3}{2}\,(\overline{\beta^2 V_c})(\overline{\beta V_c}) + \frac{3}{2}\,[\overline{\beta^2 V_c^2} - (\overline{\beta^2 V_c})\overline{V_c}] + \left(\frac{5}{2}\,\overline{\beta^2+\beta}\right)\overline{V_c^2} + \bar{\alpha}\,\overline{\beta(1+4\beta)V_c^2}\right\} \geq 0 \tag{2.32}$$

In Eq. (2.32) we have used the definition

$$\bar{f} = \int dr\rho r^3 \Lambda^2 f / \int dr\rho r^3 \Lambda^2 \tag{2.33}$$

and in addition we have used Eq. (2.28) for $\bar{\alpha}$. We immediately see that if ρ' is negative the first term and the last term in Eq. (2.32) are positive and by the Schwartz inequality the contents of the square bracket are positive. Hence, if δK and thus $\delta\beta$ are positive, we have stability.

Our results can be summarized as follows:

1) The special equilibrium is the most stable for a given plasma profile provided $\rho' < 0$;

2) Increasing β and holding E fixed within the special equilibrium or setting $E = 0$ for a general equilibrium is stabilizing provided the density ρ is varied with a fixed ρ'/ρ;

3) The $\beta = 0$ stability criteria are pessimistic since the lowest order β corrections are stabilizing.

3. UNIVERSAL INSTABILITY

One low frequency instability of an inhomogeneous collisionless plasma in a magnetic field $B_z(x)$ is a nearly electrostatic wave ($E_1 \sim e^{iky} e^{iKz} e^{i\omega t}$, $k \gg K$) with group velocity parallel to particle drifts and phase velocity parallel to B_z. This instability was calculated for a simple equilibrium (density and temperature gradients only) [4] with arbitrary β; in a zero β calculation [5] it was shown that the mode could be stabilized by more complex equilibria, where the effects of finite length, magnetic cusp curvature, shear, cold plasma, etc. were included. Here we examine the influence of finite β on finite length and magnetic cusp stabilization.

We simplify the calculation by assuming a linear plasma $f_0 = n\left(\frac{\alpha}{\pi}\right)^{3/2} e^{-\alpha v^2 + 2\alpha g x} \times [1-\epsilon'(x+v_y/\Omega)]$, $\vec{B}_0 = B_0 \hat{i}_z(1+\epsilon x)$, simulate curvature by gravity $g = -v_{thermal}^2/R_c$, and use a local approximation to evaluate the final equations at $x = 0$ [4, 5]. The perturbed electric field is $\vec{E} = -\nabla\phi + \vec{E}_z$, where E_z vanishes in zero β, and E_x is a correction smaller than E_z for this mode.

Previous work demonstrates that the maximum growth mode occurs for $1/K_M \sim R_p/\sqrt{\beta}$, or $1/K_M \sim R_p\sqrt{M/m}$ the former when $\beta > m/M$, and R_p is the plasma radius. This worst mode can be eliminated by making the system short enough, $L < 1/K_M$. In fact all modes were shown in zero β [5] to require $1/K > 1/K_0 \sim 6R_p$, the finite length stabilization effect. We expect the same effect in finite β. Since we also expect to find stabilization for cusp curvature, we solve the case of marginal stability, $\mathrm{Re}\,\omega \gg \mathrm{Im}\,\omega$, calculate as in Ref. [5], using the Vlasov equation, solved by the method of characteristics, assume $\omega < Kv_e$, and arrive at the approximate dispersion relation including curvature and finite β

$$1 + \frac{T_i}{T_e} - I_0 e^{-b}(1+W)(1-\omega^x/\omega') = -\frac{\left\{I_0 e^{-b}(\omega'-\omega^x)W + \frac{T_i}{T_e}\left[\omega'+\omega^x\frac{T_e}{T_i} - \frac{kv^2}{\Omega R_c}\left(1+\frac{T_e}{T_i}\right)\right]\right\}^2}{K^2 V_A^2 k^2 R^2} \tag{3.1}$$

$$\mathrm{Im}\,\omega = -\frac{i\sqrt{\pi}}{Kv_e}\left[\omega'+\omega^x\frac{T_e}{T_i} - k^2v^2\left(1+\frac{T_e}{T_i}\right)/\Omega R_c\right]\left[A+B/K^2V_A^2k^2R^2\right] + i\,(\text{Ion Terms}) \tag{3.2}$$

where all parameters except T_e, v_e refer to ions, I_0 is the modified Bessel function, $\omega' = \omega - kv^2/\Omega R_c$, $V_A = B/\sqrt{4\pi nM}$, V = thermal velocity, $\omega^x = kR_i v_i \, d(\ln n/dx)$, $b = k^2R^2$, R = Larmor radius, $W = -(1/\sqrt{\pi}) \int_{-\infty}^{\infty} xe^{-x^2} dx/(x + \omega'/Kv_i - i\epsilon)$. The quantities A and B are positive real functions of the parameters, and the ion terms are stabilizing. Reference [5] solved Eq. (3.1, 3.2) with $K^2V_A^2 = K^2V_i^2/\beta \to \infty$. To see the finite length effect we solve Eq. (3.1) (with no curvature) for $\omega^x/\omega = (Kv/\omega)kR(d\ln n/dx)/K$, write $Kv/\omega \equiv 1/\lambda$,

$$kRI_0e^{-b}\frac{1}{K}\frac{d\ln n}{dx} = -\left\{1 + \frac{T_i}{T_e} - I_0e^{-b}(1+W) + \frac{\left[I_0e^{-b}(\omega-\omega^x)W(\lambda) + \frac{T_i}{T_e}\left(\omega'+\omega^x\frac{T_e}{T_i}\right)\right]^2}{K^2V_A^2b}\right\}\frac{\lambda}{1+W(\lambda)} \tag{3.3}$$

Observing that $\lambda/(1+W(\lambda))$ has a minimum at $\lambda_0 \sim 1$ we conclude that there is a minimum value of $(d\ln n/dx)/K$ for this mode to exist. With $K^2V_A^2 \Rightarrow \infty$ the minimum is that obtained in Ref. [5], $\pi/K > (20.6)(d\ln n/dx)^{-1}$, occurring for modes with $k^2R^2 \gtrsim 1$, $W(\lambda) \sim 0$. Including the finite β effects the minimum is displaced and

$$L_{stability} < \pi/K_0 = 20.6\left\{\frac{1 + T_i/T_e}{2} + \frac{1}{2}(6.5)^2\beta_i\right\}R_p, \qquad R_p \equiv (d\ln n/dx)^{-1} \tag{3.4}$$

We see that finite β permits longer stable machines and that the effect is enhanced by the factor $(d\ln n/dx)/K \sim 6.5$ for the shortest parallel wavelength mode.

The cusp stability criterion is apparent from Eq. (3.2). We note that ω' and k^2v^2/R_c are the stabilizing terms. From Eq. (3.1) it can be seen that ω'/ω^x may be made small by choosing $kR \gg 1$; in addition the finite β corrections to (ω'/ω^x) are small. Then stability is achieved by taking $kv^2/\Omega R_c > \omega^x = (kv^2/2\Omega)(d\ln n/dx)$. From the form of Eq. (3.2) we see that this stabilization is unaltered by finite β corrections, since $(\omega'+\omega^x - kv^2/\Omega R_c)$ is a common factor of all destabilizing contributions.

We conclude that finite β increases the allowed length of a stable machine but does not alter the criteria for magnetic cusp stabilization.

4. ELECTRON CYCLOTRON HARMONIC OSCILLATIONS

Explanations of the features of observed radiation [7,8] at electron cyclotron harmonic frequencies (intensities flat over several harmonics, comparable to black body radiation at the plasma temperature) usually involve the Bernstein electrostatic mode [6]. In this paper we show that two other modes can exist at harmonic frequencies. One is purely transverse, the other half longitudinal, half transverse. We calculate, for the two new modes, the absorption length, indicative of the size of a plasma needed to excite these modes to black body level. This size is infinite for a $\beta = 0$ plasma, but is not unreasonably large for the values of β found in experiments. We simplify the problem by considering waves with k-vector nearly perpendicular to B.

Consider an infinite Maxwellian plasma in equilibrium with a uniform magnetic field $\vec{B}_0 = B_0\hat{i}_z$, and perturb it with a wave $\vec{E}e^{iky}e^{iKz}e^{i\omega t}$, $K \ll k$ $\omega \sim -\ell\Omega = -\ell eB/mc$. Straightforward integration of the Vlasov equatipn gives the dispersion relations (for K = 0)

$$a_{11}E_x + a_{12}E_y = 0, \qquad a_{21}E_x + a_{22}E_y = 0, \qquad a_{33}E_z = 0 \tag{4.1}$$

$$a_{11} = (k^2c^2-\omega^2)/\omega_p^2 + (\ell^2+2b^2-2b^2 I_\ell'/I_\ell)\chi_\ell/b; \qquad \chi_\ell \equiv \frac{I_\ell e^{-b}}{1+\ell\Omega/\omega}$$

$$a_{22} = (\omega_p^2+\Omega^2-\omega^2)/\omega_p^2(1-\Omega^2/\omega^2) + (\omega^2/b\Omega^2)\chi_\ell \tag{4.2}$$

$$a_{12} = a_{21}^* = -i(\omega/\Omega)\left[1/\left(\frac{\omega^2}{\Omega^2} - 1\right) + \left(I_\ell'/I_\ell - 1\right)\chi_\ell\right]$$

$$a_{33} = (k^2c^2-\omega^2)/\omega_p^2 + \chi_\ell \tag{4.3}$$

where $b = k^2R_e^2/2$, I_ℓ is the Bessel function of imaginary argument, $I_\ell' = dI_\ell/db$, Ω and R_e are the electron gyration frequency and radius. The dispersion relation $a_{11}a_{22} = |a_{12}|^2$ can be solved in the limit $k^2c^2 \gg \omega_p^2$, $\chi_\ell \ll b$, yielding the hybrid mode and the $\omega = kc$ mode. The limit $\chi_\ell \sim b$ yields the Bernstein mode, $\omega \sim -\ell\Omega$, $E_y = i(k^2c^2/\omega_p^2)E_x$, while the limit $\chi_\ell \sim k^2c^2/\omega_p^2 \gg 1$ gives the new modes

$$\omega_1 = -\ell\Omega - bI_\ell e^{-b}\frac{\ell\Omega\omega_p^2}{k^2c^2}\left[1 + W\left(\frac{\omega+\ell\Omega}{Kv}\right)\right]; \qquad E_y = -iE_x \tag{4.4}$$

$$\omega_2 = -\ell\Omega - \omega I_\ell e^{-b}(\omega_p^2/k^2c^2)\left[2\left(\frac{\omega+\ell\Omega}{Kv}\right)^2 W\left(\frac{\omega+\ell\Omega}{Kv}\right)\right]; \qquad E = E_z \tag{4.5}$$

Equation (4.4) is the solution to $a_{11}a_{22} = |a_{12}|^2$, and is mixed in polarization independent of the small parameter ω_p/kc. Equation (4.5) solves $a_{33} = 0$ and is a pure transverse harmonic mode. The corrections for finite K, $W(\lambda) = -\frac{1}{\sqrt{\pi}}\int_{-\infty}^{\infty} xe^{-x^2}dx/(x+\lambda-i\epsilon)$ are included, and are necessary for calculation of the absorption coefficient $\alpha = \mathrm{Im}\,\omega/|d\omega/d\vec{k}|$. We average over a cone of small values of K to obtain the absorption length for the two modes

$$L_1 \sim \frac{R_e}{\beta}(I_\ell e^{-b})^{-1}; \qquad L_2 \sim \frac{R_e}{\beta}(b/I_\ell e^{-b}) \sim R_e/\beta \tag{4.6}$$

These modes, flat over harmonics with $b \sim \ell$ could be excited to black body level in a plasma of size $R \sim R_e/\beta$. These new modes share with the longitudinal wave a difficulty in propagating out of the plasma ($\omega_p^2 \to 0$ requires complex k, giving reflections), but could be coupled at a sharp boundary, since they are transverse waves. The results above occur only when $\Omega = eB/m_0c\sqrt{1-v^2c^2}$ is replaced by eB/m_0c. The relativistic correction places a limit on the possible size of the harmonic term, allowing solutions Eq. (4.4, 4.5) only when $\omega/\omega_p < 1/\sqrt{\ell}$. Since the Bernstein modes exhibit no such behavior this offers another means of experimentally distinguishing among the various contributions to the electron harmonic spectrum.

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Section 2

FINITE β EFFECTS IN A WEAKLY UNSTABLE PLASMA

by

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Finite β Effects in a Weakly Unstable Plasma*

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The effect of finite plasma pressure (β) on the interchange mode is calculated in the finite Larmor radius limit. The method follows earlier zero β calculations of Rosenbluth and Simon, expanding the Vlasov equation with the ion Larmor radius a_i considered small compared with the plasma radius, L. The plasma pressure is taken to be zero order in a_i/L. A differential equation is obtained for the stability properties of systems with arbitrary radial dependence of density, electric field, magnetic field and β. Several simple examples show that increasing β moves marginally flute unstable systems to stability, if the change in β does not include violent changes in the density and field profiles.

author

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I. Introduction

The purpose of this paper is to calculate the effect of finite plasma pressure on the interchange mode in the finite Larmor radius, "weakly" unstable, regime. The previous low plasma pressure calculations were done in the electrostatic approximation.[1,2,3] In the earliest calculation the solution was obtained from direct integration of the Vlasov equation over the particle orbits.[1] In later calculations the solution to the Vlasov equation proceeded from an expansion technique in which the ion Larmor radius (a_i) was assumed small compared to a characteristic length (L) of the plasma, i.e. the radius of the plasma.[2,3] Although the direct integration of the Vlasov equation leads in principle to a solution of the most complex problems, in practice few problems are amenable to solution with this method. On the other hand the expansion technique, which presupposes knowledge of the solution to obtain the proper ordering scheme, enables one to solve problems of considerable complexity.

We use this latter technique, specifically the method of Rosenbluth and Simon,[3] to obtain a closed set of equations to first order in the perturbation. The contribution of β, the ratio of plasma to magnetic pressure, a quantity which we treat as order unity is included to all orders.

The model examined is one-dimensional in the equilibrium state with a uniform magnetic field. Curvature is represented by a fictitious gravity which points in the direction (orthogonal to the magnetic field) of variation of the equilibrium state (particle distribution and electrostatic potential). We consider flute type perturbations i.e., there is no motion or variation in the direction of the uniform magnetic field;

thus the perturbed magnetic field has no component orthogonal to the equilibrium magnetic field.

The appropriate ("finite Larmor radius") ordering scheme[1] in terms of the smallness parameter $\epsilon = a_i/L$ is

$$\frac{\omega}{\Omega_i} \sim \epsilon^2$$

$$\frac{cE}{LB\Omega_i} \sim \epsilon^2$$

$$\frac{1}{2\alpha}\,\Omega\,\frac{\nabla B}{B} \sim \epsilon^2$$

$$\frac{g}{L\Omega^2} \sim \epsilon^4$$

$$\frac{\lambda_D}{a_i} \sim \epsilon^0$$

$$\beta \sim \epsilon^0$$

where ω is the eigenfrequency, E the electric field, B the magnetic field, Ω the cyclotron frequency, $M/2\alpha$ the temperature and λ_D the Debye length.

The immediate effect of the inclusion of finite β terms is to produce to lowest order in ϵ an apparent charge separation due to the ∇B drift, which however cancels to all orders in β leading to a theory which depends only upon macroscopic plasma parameters. In a previous paper[4] the definitive differential equation was written down without proof and several cases were examined to determine the influence of finite β on stability. In the present treatment we derive this equation for rectangular geometry and add to the previous stability analysis. For completeness we will comment on some of the previous results as well.

In Section II we derive, for a linear perturbation, an eigenvalue equation for stability in rectangular geometry; we also include the analogous equation for cylindrical geometry. In both cases we include an equlibrium electric field. In Section III we determine the influence of finite β on stability through the use of a variational technique.

II. Linear Perturbed Equation

Since the method employed will parallel that of Rosenbluth and Simon[3] we use, wherever convenient, their notation. We start with the Vlasov equation written in the frame moving with the drift velocity

$$\underset{\sim}{V} = \frac{(\frac{e}{M}\underset{\sim}{E} + \underset{\sim}{g}) \times \underset{\sim}{\Omega}}{\Omega^2} \tag{1}$$

which is given by

$$\left\{D + \underset{\sim}{v}' \cdot \underset{\sim}{\nabla} + [\underset{\sim}{v}' \times \underset{\sim}{\Omega} - D\underset{\sim}{V} - \underset{\sim}{v}' \cdot \underset{\sim}{\nabla}\,\underset{\sim}{V}] \cdot \underset{\sim}{\nabla}_{v'}\right\} f = 0 \tag{2}$$

with

$$D = \frac{\partial}{\partial t} + \underset{\sim}{V} \cdot \underset{\sim}{\nabla} \tag{3}$$

and

$$\underset{\sim}{v} = \underset{\sim}{v}' + \underset{\sim}{V} \tag{4}$$

If we next convert to polar coordinates in velocity space ($\underset{\sim}{B}$ is chosen in the z-direction, $\underset{\sim}{\Omega} = \Omega\hat{z}$

$$v'_x = v\cos\phi$$

$$v'_y = v\sin\phi \tag{5}$$

The Vlasov equation can be written in the form*

$$Df - \Omega\frac{\partial f}{\partial \phi} + \Big[\tfrac{1}{2}ve^{-i\phi}\partial^+ f - \tfrac{1}{2}e^{-i\phi}(DV^+)\,\frac{1}{v}\frac{\partial vf}{\partial v} + \frac{i}{2v}\frac{\partial}{\partial\phi}\,e^{-i\phi}(DV^+)f - i\,\frac{v}{4}(\partial^+V^-)$$

$$\times\,\frac{\partial f}{\partial\phi} - \frac{1}{4v}(\partial^+V^+)e^{-2i\phi}\,\frac{\partial v^2 f}{\partial v} - \frac{v}{4}(\partial^+V^-)\frac{\partial f}{\partial v} + \frac{i}{4}\,\frac{\partial}{\partial\phi}(\partial^+V^+)e^{-2i\phi}f + cc\Big] = 0 \tag{6}$$

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*As a definition by "cc" we mean $\partial^{\pm}(V^{\pm}) \to \partial^{\mp}(V^{\mp})$, $f_{n,m} \to f_{n,-m}$ and everywhere i appears explicitely it is replaced by -i.

In Eq. (6) we have defined

$$\partial^{\pm} = \frac{\partial}{\partial x} \pm i \frac{\partial}{\partial y}$$

$$V^{\pm} = V_x \pm i V_y \tag{7}$$

Also note that derivatives operate on all quantities to the right unless a bracket is interposed. A formal solution to the above equation can be obtained by substituting

$$f = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} \epsilon^n e^{im\phi} f_{n,m} \tag{8}$$

and equating coefficients. The truncation of the sum over m is a property of the finite Larmor radius ordering. Recall that all terms in Eq. (6) are now uniquely ordered by the aforementioned ordering scheme. Thus, we have to lowest order

$$Df_o + \frac{\epsilon v}{2} (\partial^+ f_{1,1} + cc) - \frac{v}{2} (\underset{\sim}{\nabla} \cdot \underset{\sim}{V}) \frac{\partial f_o}{\partial v} = 0 \tag{9}$$

with

$$\epsilon f_{1,1} = - \frac{i}{2\Omega} v\partial^- f_o \tag{10}$$

Recall that since the gravitational drift is included in $\underset{\sim}{V}$ there are terms of higher order also present. To the next order we obtain

$$\epsilon^2 Df_{2,0} + \epsilon^3 \frac{v}{2} [\partial^+ f_{3,1} + cc] - \epsilon^2 \left[\frac{1}{4v}(\partial^+ V^+) \frac{\partial v^2 f_{2,2}}{\partial v} + \frac{v}{4}(\underset{\sim}{\nabla} \cdot \underset{\sim}{V}) \frac{\partial f_{2,0}}{\partial v} + cc\right]$$

$$- \frac{\epsilon}{2v} \left[\frac{\partial v f_{1,1}}{\partial v} (DV^+) + cc\right] = 0 \tag{11}$$

with

$$\epsilon^3 i\Omega f_{3,1} = \frac{\epsilon^2 v}{2} \left[\partial^+ f_{2,2} + \partial^- f_{2,0}\right] + \epsilon \left[Df_{1,1} - \frac{1}{4}(\partial^- V^-) \frac{\partial v f_{1,1}}{\partial v}\right.$$

$$\left. - \frac{v}{2}(\underset{\sim}{\nabla} \cdot \underset{\sim}{V}) \frac{\partial f_{1,1}}{\partial v} + \frac{1}{4} (\partial^+ V^- - \partial^- V^+) f_{1,1}\right] - \frac{1}{2} \frac{\partial f_o}{\partial v} (DV^-) \tag{12}$$

and

$$2i\Omega f_{2,2} = \frac{v}{2}\,\epsilon\,\partial^- f_{1,1} - \frac{v}{4}\,(\partial^- V^-)\,\frac{\partial f_o}{\partial v} \tag{13}$$

The terms odd in m + n form a separate equivalent chain as in reference 3. If the terms of Eqs. (9)-(13) are combined we obtain for the lowest order contribution

$$Df_o - \frac{v}{2}\,(\underset{\sim}{\nabla}\cdot\underset{\sim}{V})\,\frac{\partial f_o}{\partial v} + \left[\frac{v^2}{4i}\,\partial^+\,\frac{1}{\Omega}\,\partial^- f_o + cc\right] = 0 \tag{9'}$$

and to the next order

$$\epsilon^2\left(Df_{2,0} - \frac{v}{2}\,(\nabla\cdot V)\,\frac{\partial f_{2,0}}{\partial v}\right) + \frac{v}{2}\,(\partial^+\,\frac{1}{\Omega}\,X^- + cc) + \epsilon^2\left(\frac{v^2}{4i}\,\partial^+\,\frac{1}{\Omega}\,\partial^- f_{2,0} + cc\right)$$

$$- \epsilon^2\left[\frac{1}{4v}\,(\partial^+ V^+)\,\frac{\partial v^2 f_{2,2}}{\partial v} + cc\right] - \epsilon\left[\frac{1}{2v}(DV^+)\,\frac{\partial v f_{1,1}}{\partial v}\right] = 0 \tag{11'}$$

where

$$X^- = -\frac{v^3}{16i}\,\partial^+\,\frac{1}{\Omega}\,\partial^-\,\frac{1}{\Omega}\,\partial^- f_o + \frac{v^2}{16}\,\partial^+\,\frac{1}{\Omega}\,(\partial^- V^-)\,\frac{\partial f_o}{\partial v} - \frac{1}{8\Omega}\,(\partial^- V^-)\partial^+\,\frac{\partial v^2 f_o}{\partial v}$$

$$- \frac{v}{8\Omega}\,(\partial^+ V^- - \partial^- V^+)\partial^- f_o + \frac{v}{4\Omega}(\underset{\sim}{\nabla}\cdot\underset{\sim}{V})\,\partial^-\frac{\partial v f_o}{\partial v} + \frac{i}{2}(DV^-)\,\frac{\partial f_o}{\partial v} - \frac{v}{2}\,D\,\frac{1}{\Omega}\,\partial^- f_o$$

$$= \epsilon^3\Omega f_{3,1} + \frac{i\epsilon^2 v}{2}\,\partial^- f_{2,0} \tag{14}$$

These equations, Eqs. (9'), (11') and (14) coupled with Maxwell's equations

$$\underset{\sim}{\nabla}\cdot\underset{\sim}{E} = 4\pi\,\Sigma\,e\int v dv\,(f_o + \epsilon^2 f_{2,0}), \tag{15a}$$

$$(\underset{\sim}{\nabla}\times\underset{\sim}{B})^{\pm} - \frac{1}{c}\,\frac{\partial \underset{\sim}{E}^{\pm}}{\partial t} = \frac{4\pi}{c}\,\Sigma\,e\int v dv\left[V^{\pm}(f_o + \epsilon^2 f_{2,0})\right.$$

$$\left. + (\epsilon\,f_{1,\pm 1} + \epsilon^3 f_{3,\pm 1})\,v\right] \tag{15b}$$

and

$$\frac{\partial B}{\partial t} = -\frac{1}{c}\,\underset{\sim}{\nabla}\times\underset{\sim}{E} \tag{15c}$$

completely determine our solution. The x and y components of Eq. (15b) can be obtained by taking suitable linear combinations e.g. $E_y = \frac{i}{2}(E^- - E^+)$.

To proceed, we choose all equilibrium quantities to be functions of x; then g and the equilibrium electric field point in the x-direction. Since we are concerned with the linearized perturbed equation we Fourier analyze with respect to time (ω) and the y-variation (k).

First, we obtain from the y-component of Eq. (15b) the equilibrium pressure balance equation

$$0 = \frac{1}{4\pi}\left(B\frac{\partial B}{\partial x} - E\frac{\partial E}{\partial x}\right) + \Sigma\frac{\partial p}{\partial x} + \Sigma g\rho - \frac{1}{2}\frac{\partial}{\partial x}\frac{1}{\Omega}\frac{\partial}{\partial x}\frac{1}{\Omega}\frac{\partial R}{\partial x}$$

$$+ \frac{1}{2}\frac{\partial}{\partial x}\frac{1}{\Omega}\left(\frac{\partial}{\partial x}\frac{cE}{B}\right)p \qquad (16)$$

where when indicated we sum over species; otherwise all quantities refer to ion parameters. Also in Eq. (16) we have utilized the following definitions

$$\rho = M\int v dv\,(f_o + \epsilon^2 f_{2,0}) \qquad \text{(Mass density)}$$

$$p = \frac{M}{2}\int v^3 dv\,(f_o + \epsilon^2 f_{2,0}) \qquad \text{(Perpendicular pressure)}$$

$$R = \frac{M}{8}\int v^5 dv\,(f_o + \epsilon^2 f_{2,0}) \qquad (17)$$

Note that for an isothermal Maxwellian velocity distribution

$$p = \rho/2\alpha$$

$$R = p/2\alpha = \rho/4\alpha^2 \qquad (18)$$

where $M/2\alpha$ is the perpendicular temperature.

Next, we compute the lowest order perturbed contributions. From Eq. (9') we have

$$i\left(\bar{\omega} + kV_D - \frac{kg}{\Omega}\right) f_o' - \frac{v}{2}\left(\underset{\sim}{\nabla}\cdot\underset{\sim}{V}'\right)\frac{\partial f_o}{\partial v} + \frac{kv^2\xi}{2\omega\Omega}\frac{\partial f_o}{\partial x} + \frac{cE_y'}{B}\frac{\partial f_o}{\partial x} = 0 \qquad (19)$$

with

$$V_D = \frac{v^2\eta}{2\Omega}$$

$$\eta = \frac{1}{B}\frac{\partial B}{\partial x}$$

$$\bar{\omega} = \omega - \frac{kcE}{B}$$

$$\xi = \hat{i}_z \cdot (\underset{\sim}{\nabla} \times \underset{\sim}{E}')\, c/B$$

Primed quantities refer to perturbed variables. For convenience we have retained the contribution from the gravitational drift velocity, a term of relative order ϵ^2. Also, note that Eq. (19) holds for both ions and electrons and except where confusing we shall supress indices. Now,

$$\underset{\sim}{\nabla}\cdot\underset{\sim}{V}' = \xi\frac{\bar{\omega}}{\omega} - \eta\frac{cE_y'}{B} - k\,\xi\frac{g}{\Omega\omega} \qquad (21)$$

which can be written

$$\underset{\sim}{\nabla}\cdot\underset{\sim}{V}' = -\frac{1}{B}\,DB \qquad (22)$$

Hence, to this order the perturbed distribution is given by

$$f_o' = \frac{i}{\omega\eta}\,\xi\,\frac{\partial f_o}{\partial x} - \frac{i}{\bar{\omega} + kV_D - \frac{kg}{\Omega}}\,\frac{1}{\eta}\left(\underset{\sim}{\nabla}\cdot\underset{\sim}{V}'\right)\left(\frac{\partial f_o}{\partial x} + \frac{\eta v}{2}\frac{\partial f_o}{\partial v}\right) \qquad (23)$$

Inserting Eq. (21) and (23) into Eq. (15) we then have

$$\underset{\sim}{\nabla}\cdot\underset{\sim}{E}' = \frac{i}{\omega\eta}\,\xi\,\frac{\partial^2 E}{\partial x^2} - \left(\xi\frac{\bar{\omega}}{\omega} - \eta\frac{cE_y'}{B}\right)\frac{4\pi i}{\eta}\sum e\int v\,dv\; I$$
$$+ \frac{k}{\omega}\frac{4\pi i}{\eta}\,\xi\sum\frac{eg}{\Omega}\int v\,dv\; I \qquad (24)$$

and from the $\hat{x}$-component of Eq. (15b) using Eq. (10) we have

$$-\frac{kB\xi}{\omega}\left(1 + \frac{4\pi}{B^2\eta}\sum\frac{\partial p}{\partial x}\right) - \frac{i\omega E_x'}{c} - \frac{E_y'}{B}\frac{\partial E}{\partial x}$$
$$= \left(\xi\frac{\bar{\omega}}{\omega} - \eta\frac{cE_y'}{B}\right)\frac{4\pi}{\eta c}\sum e\int\frac{kv^2}{2\Omega}\,I\,v\,dv - \frac{4\pi}{\eta c}\,\xi\sum e\,\frac{kg}{\omega\Omega}\int v\,dv\,\frac{kv^2}{2\Omega}\,I\,. \qquad (25)$$

In Eqs. (24 and (25) we have defined

$$I = \frac{\frac{\partial f_0}{\partial x} + \frac{v\eta}{2}\frac{\partial f_0}{\partial v}}{\bar{\omega} + kV_D - \frac{kg}{\Omega}} \tag{26}$$

Using

$$\sum 4\pi e \int vdv \frac{kv^2\eta}{2\Omega} I = \frac{\partial^2 E}{\partial x^2} - \eta\frac{\partial E}{\partial x} - \sum 4\pi e(\bar{\omega} - \frac{kg}{\Omega}) \int vdv\ I \tag{27}$$

we have to lowest order in ϵ (recall $\frac{\lambda_D}{a_i} \sim \epsilon^0$, $\frac{kg}{\Omega^2} \sim \epsilon^4$ and thus the first **two terms of Eq. (24) are the same order in ϵ as the term on the r.h.s proportional to gravity)**

$$0 = (\xi\frac{\bar{\omega}}{\omega} - \eta\frac{cE_y'}{B}) \sum e \int vdv\ I \tag{28}$$

for both Eqs. (24) and (25). The first term on the l.h.s of Eq. (25) vanishes to lowest order due to the equilibrium condition Eq. (16). This equation can be satisfied by either the first bracket vanishing which, as will be subsequently seen, determines the polarization for the interchange mode; or by the sum vanishing which gives rise to a universal instability driven by a temperature gradient.[5] Note the r.h.s of Eq. (28) is just the lowest order charge separation due to the ∇B drift [see Eq. (24)].

Thus, since we are interested in the interchange mode we must have

$$\frac{\bar{\omega}}{\omega}\xi - \eta c\frac{E_y'}{B} = \epsilon^2\Phi, \tag{29}$$

viz, $\underset{\sim}{\nabla} \cdot \underset{\sim}{V}'$ vanishes to lowest order in ϵ. Consequently, we may set $\Phi = 0$ in all terms of higher order, such as the contribution of $f_{2,0}$ to the charge density, the Debye length terms etc. With these restrictions in mind we have

$$\underset{\sim}{\nabla} \cdot \underset{\sim}{E}' - i \frac{cE_y'}{B\bar{\omega}} \frac{\partial^2 E}{\partial x^2} - 4\pi i \frac{cE_y'}{\bar{\omega}B} \sum e \int vdv\, I \frac{kg}{\Omega}$$

$$= -\epsilon^2 \frac{4\pi i}{\eta} \Phi \sum e \int Ivdv + \epsilon^2 4\pi e \int vdv\, f_{20}' \tag{30}$$

and

$$- \frac{kcE_y'\eta}{\bar{\omega}} \left[1 + \frac{4\pi}{B^2\eta} \frac{\partial p}{\partial x}\right] - \frac{i\omega}{c} E_x' - \frac{E_y'}{B} \frac{\partial E}{\partial x} + \frac{4\pi i}{\eta\bar{\omega}} \frac{E_y'}{B} \sum e \int vdv \frac{kg}{\Omega} \frac{kv^2\eta}{2} I$$

$$= \epsilon^2 \Phi \frac{4\pi\bar{\omega}}{\eta c} \sum e \int vdv\, I + \epsilon^3 \frac{4\pi e}{c} \int vdv \frac{v}{2} (f_{3,1}' + f_{3,-1}'), \tag{31}$$

where the gravitational drift has been dropped from the denominator of I [see Eq. (26)]. Note that in the terms arising from $f_{2,0}'$ and $f_{3,1}'$ only the ions contribute; also as a reminder, all quantities refer to ion parameters unless a sum is explicitly indicated. We may now eliminate Φ from Eqs. (30) and (31) by multiplying Eq. (31) by $\frac{i\eta c}{\bar{\omega}}$ and adding it to Eq. (30) to obtain

$$\underset{\sim}{\nabla} \cdot \underset{\sim}{E}' - \frac{icE_y'}{B\bar{\omega}} \frac{\partial^2 E}{\partial x^2} + \frac{\eta\omega}{\bar{\omega}} E_x' - \frac{icE_y'}{B\bar{\omega}} \eta \frac{\partial E}{\partial x} - \frac{ik}{\bar{\omega}^2} c^2 E_y' \left[\frac{4\pi}{B^2} \sum \left(g\frac{\partial \rho}{\partial x} - g\eta\rho\right)\right.$$

$$\left. + \eta^2 \left(1 + \frac{4\pi}{B^2\eta} \sum \frac{\partial p}{\partial x}\right)\right] = 4\pi e \int vdv\, \epsilon^2 f_{2,0}' \left(1 + \frac{kV_D}{\bar{\omega}}\right) + \frac{4\pi ie}{\bar{\omega}} \int vdv$$

$$\times \left[\frac{v\eta}{2\Omega} (X^- + cc)\right]' \tag{32}$$

where X^- is defined by Eq. (14). We are of course only interested in the linear perturbed part of the last integral of Eq. (32). Finally we can rewrite the linear perturbed part of Eq. (11') in the form

$$\epsilon^2 i (\bar{\omega} + kV_D) f_{2,0}' + \frac{v}{2} \left(\partial^+ \frac{1}{\Omega} X^- + cc\right)' = 0 \tag{33}$$

where we have dropped higher order terms in ϵ and also we have not included the last two terms of Eq. (11') since these terms vanish after an integration over velocity when substituted into Eq. (32). We then have for the r.h.s of Eq. (32)

$$\frac{4\pi e i}{\bar{\omega}} \int \frac{v^2 dv}{2} \left[(\partial^+ + \eta) \frac{1}{\Omega} X^- + cc \right]. \tag{34}$$

At this point a few remarks are in order. First we note [see Eq. (14)] that there are no denominators of the form $\bar{\omega} + kV_p$ to be integrated over; hence there are no resonant particle contributions to this order in ϵ. Second, we see that only the macroscopic plasma parameters listed in Eq. (17) contribute. Hence, we see that the cancellation of the apparent charge separation to lowest order which forced the polarization restraint, Eq. (29) affords extensive simplification to the finite β interchange mode in the "weak instability" limit.

We next evaluate Eq. (34). If Ω is expanded to first order in the perturbation we have

$$\left[(\partial^+ + \eta) \frac{X^-}{\Omega} + cc \right]' = \left[\frac{1}{\Omega} \partial^+ X'^- + cc \right] + \left[i \frac{\eta k}{\bar{\omega}\Omega} \frac{cE_y'}{B} X^- + cc \right]. \tag{35}$$

wherein we have utilized the polarization relation, Eq. (29), to lowest order, viz.

$$\frac{\bar{\omega}}{\omega} \xi - \eta \frac{cE_y'}{B} = \frac{1}{B} DB = \underset{\sim}{\nabla} \cdot \underset{\sim}{V}' = 0 \tag{29'}$$

We now express the last bracket of Eq.(35) in terms of macroscopic quantities, using the definition of X^- Eq.(14) along with a linear combination of Eqs.(15b). This contribution to Eq.(32) combines with $[\eta^2(1+\frac{4\pi}{B^2\eta}\sum \partial p/\partial x) - \sum \frac{4\pi}{B^2} g\eta p]$ to produce $\frac{\eta}{B^2} E \frac{\partial E}{\partial x}$, see Eq.(16). Thus Eq.(32) can be rewritten as

$$\underset{\sim}{\nabla} \cdot \underset{\sim}{E}' \; \frac{-icE_y'}{B\bar{\omega}} \frac{\partial^2 E}{\partial x^2} + \eta \frac{\omega}{\bar{\omega}} E_x' - \frac{icE_y'}{B\bar{\omega}} \eta \frac{\partial E}{\partial X} - \frac{ikc^2\eta}{\bar{\omega}^2 B^2} E_y' E \frac{\partial E}{\partial X} - \frac{4\pi i}{\bar{\omega}^2} \frac{kc^2 E_y'}{B^2} \sum g \frac{\partial p}{\partial x}$$

$$+ \frac{4\pi i e}{\bar{\omega}} \int \frac{v dv}{\Omega} \left\{ \partial^+ \overset{(1)}{\frac{1}{2i} (DV^-) f_o} - \frac{v^2}{4} \partial^+ D \overset{(2)}{\frac{1}{\Omega} \partial^- f_o} + \frac{v^2}{8} \left[\partial^+ (\partial^- V^-) \frac{1}{\Omega} \partial^+ f_o \right.\right.$$

$$\left.\left. \overset{(3)}{-(\partial^+)^2 (\partial^- V^-) \frac{f_o}{\Omega}} \overset{(4)}{- \partial^+ (\partial^+ V^-) \frac{1}{\Omega} \partial^- f_o} \right] + \frac{iv^4}{32} (\partial^+)^2 \frac{1}{\Omega} \partial^- \frac{1}{\Omega} \partial^- f_o + cc \right\} \tag{36}$$

where in all terms proportional to $\frac{\partial f_o}{\partial v}$ an integration over v has been performed. The bracketed numbers which appear in Eq. (36) serve to label the respective terms. Recall that f_o is the lowest order distribution function including both the equilibrium and perturbed contributions.

To continue with the evaluation of the integral it is convenient to record the following relations

$$f_o' = i\Psi \frac{\partial f_o}{\partial x}$$

$$\frac{B'}{B} = i\eta\Psi \tag{37}$$

where we have utilized Eqs. (15c), (22'), (23) and (29') and

$$\Psi = \frac{cE_y'}{B\bar{\omega}} \tag{38}$$

We now have for the various terms of the integral

$$(1) \quad I_1 = \frac{1}{2i}\,\partial^+ (DV^-)\, f_o + cc = -k\Psi\bar{\omega}^2 + i\,\frac{\partial}{\partial x}\,\bar{\omega}\,\frac{cE_x'}{B}$$

$$+ \frac{\partial}{\partial x}\,\frac{c^2E_y'}{B^2}\,\frac{\partial E}{\partial x}$$

where we have utilized Eq. (22'). If we now incorporate the polarization condition Eq. (29')

$$I_1 = \frac{1}{k}\left[\frac{\partial}{\partial x}\,\bar{\omega}^2\,\left(\frac{\partial\Psi}{\partial x}\right)f_o - k^2\,\bar{\omega}^2\Psi\, f_o\right]. \tag{39}$$

$$(2) \quad I_2 = \frac{v^2}{4}\,\partial^+ D\,\frac{1}{\Omega}\,\partial^- f_o = -\frac{v^2}{2}\left[i\left(\frac{\partial}{\partial x}\,\frac{\bar{\omega}}{\omega}\,\frac{\partial f_o'}{\partial x} - \frac{k^2\bar{\omega}}{\Omega}\, f_o'\right)\right.$$

$$\left. + \frac{\partial}{\partial x}\,\frac{\bar{\omega}}{\Omega}\,\Psi\,\frac{\partial^2 f_o}{\partial x^2}\right]$$

and substituting Eq. (37) we obtain

$$I_2 = \frac{v^2}{2}\left[\frac{\partial}{\partial x}\,\frac{\bar{\omega}}{\Omega}\,\frac{\partial\Psi}{\partial x}\,\frac{\partial f_o}{\partial x} - \frac{k^2}{\Omega}\,\bar{\omega}\,\Psi\,\frac{\partial f_o}{\partial x}\right]. \tag{40}$$

$$(3)\quad I_3 = \frac{v^2}{8}\left[-\partial^+(\partial^+ \frac{1}{\Omega}\partial^- V^-)f_o + \partial^+ \frac{1}{\Omega}(\partial^- V^-)\,\partial^- f_o + cc\right]$$

$$= \frac{v^2}{4}\left\{\left[-2(\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2})\frac{1}{\Omega}\frac{\partial}{\partial x}\bar{\omega}\Psi - 2\frac{\partial}{\partial x}(-\frac{1}{\Omega}\frac{\partial^2}{\partial x^2}\bar{\omega}\Psi - \frac{k^2\bar{\omega}\Psi}{\Omega}\right.\right.$$

$$\left. + \frac{\eta\Psi}{\Omega}\frac{\partial\bar{\omega}}{\partial x})\right] f_o + 2\left[-\frac{\partial}{\partial x}\frac{1}{\Omega}\frac{\partial}{\partial x}\bar{\omega}\Psi + \frac{1}{\Omega}\frac{\partial^2}{\partial x^2}\bar{\omega}\Psi\right.$$

$$\left.\left. - \frac{\eta\Psi}{\Omega}\frac{\partial\bar{\omega}}{\partial x}\right]\frac{\partial f_o}{\partial x}\right\}$$

wherein Eqs. (22') and (37) have been utilized. After further algebra we obtain

$$I_3 = \frac{v^2}{2}\left[\frac{\partial}{\partial x}\frac{\bar{\omega}\eta}{\Omega}\frac{\partial\Psi}{\partial x} f_o - \frac{k^2\eta}{\Omega}\bar{\omega}\Psi\, f_o\right]. \tag{41}$$

$$(4)\quad I_4 = \frac{iv^4}{32}(\partial^+)^2\frac{1}{\Omega}\partial^-\frac{1}{\Omega}\partial^- f_o + cc$$

$$= \frac{v^2}{16}\left[(\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2})\frac{1}{\Omega}(\frac{\partial}{\partial x}\frac{1}{\Omega}\frac{\partial f_o}{\partial y} + \frac{\partial}{\partial y}\frac{1}{\Omega}\frac{\partial f_o}{\partial x})\right.$$

$$\left. - 2\frac{\partial}{\partial x}\frac{\partial}{\partial y}\frac{1}{\Omega}(\frac{\partial}{\partial x}\frac{1}{\Omega}\frac{\partial f_o}{\partial x} + \frac{\partial}{\partial y}\frac{1}{\Omega}\frac{\partial f_o}{\partial y})\right]$$

which can be written with Eq. (37)

$$I_4 = \frac{v^4}{16}\left[(\frac{\partial^2}{\partial x^2} + k^2)\frac{ik}{\Omega}(\frac{\partial}{\partial x}\frac{1}{\Omega}f_o' + \frac{1}{\Omega}\frac{\partial f_o'}{\partial x}) - 2\frac{\partial}{\partial x}\frac{ik}{\Omega}(\frac{\partial}{\partial x}\frac{1}{\Omega}\frac{\partial f_o'}{\partial x} + \frac{k^2}{\Omega}f_o')\right.$$

$$\left. + (\frac{\partial^2}{\partial x^2} + k^2)\frac{1}{\Omega}\frac{k\eta\Psi}{\Omega}\frac{\partial f_o}{\partial x} - 2\frac{\partial}{\partial x}(\frac{1}{\Omega}\frac{\partial}{\partial x}\frac{k\eta\Psi}{\Omega}\frac{\partial f_o}{\partial x} + \frac{k\eta\Psi}{\Omega}\frac{\partial}{\partial x}\frac{1}{\Omega}\frac{\partial f_o}{\partial x})\right]$$

Now with Eq. (37) and some minor algebra

$$I_4 = \frac{v^4}{8}\left[\frac{\partial}{\partial x}\frac{k\eta}{\Omega^2}\frac{\partial\Psi}{\partial x}\frac{\partial f_o}{\partial x} - \frac{k^3\eta}{\Omega^2}\Psi\frac{\partial f_o}{\partial x}\right] \tag{42}$$

Collecting all terms we have for the integral in Eq. (36) with Eq. (17)

$$\frac{c4\pi i}{kB\bar{\omega}}\left[\frac{\partial}{\partial x}\frac{\partial\Psi}{\partial x}\left(\bar{\omega}^2\rho + \frac{k\bar{\omega}}{\Omega}(\frac{\partial p}{\partial x} + \eta p) + \frac{k^2\eta}{\Omega^2}\frac{\partial R}{\partial x}\right) - k^2\Psi\left(\bar{\omega}^2\rho + \frac{k\bar{\omega}}{\Omega}(\frac{\partial p}{\partial x} + \eta p)\right.\right.$$

$$\left.\left. + \frac{k^2\eta}{\Omega^2}\frac{\partial R}{\partial x}\right)\right] \tag{43}$$

We next turn to the **evaluation** of the first five terms on the l.h.s of Eq. (36), the Debye length contributions. If the polarization relation Eq. (29') is employed we have for these terms

$$\frac{1}{ik}\left[-k^2E_y' + \frac{\partial^2 E_y'}{\partial x^2} - E_y'\left[\frac{\partial}{\partial x}\eta\frac{\omega}{\bar\omega} + \eta^2\left(\frac{\omega}{\bar\omega}\right)^2\right] + \Psi\frac{\partial^2}{\partial x^2}(\omega-\bar\omega)\frac{B}{c} + k\Psi\eta\frac{\partial E}{\partial x} + k\Psi\eta\frac{kcE}{B\bar\omega}\frac{\partial E}{\partial x}\right]$$

or

$$\frac{1}{ik}\left[-k^2E_y' + \frac{\partial^2 E_y'}{\partial x^2} - \Psi\frac{\partial^2}{\partial x^2}\frac{B\bar\omega}{c} + \frac{B}{c}\Psi\left(+\frac{\eta\omega}{\bar\omega}\frac{\partial\bar\omega}{\partial x} + \eta^2\omega + \frac{k\eta c}{B}\frac{\partial E}{\partial x} - \frac{\eta^2\omega^2}{\bar\omega} + \frac{k\eta c}{\bar\omega B}\frac{kc}{B}\frac{\partial E}{\partial x}\right)\right]$$

Inserting the definition for $\bar\omega$ [See Eq. (20)] we see that the round bracket vanishes and finally, the Debye length terms contribute

$$\frac{c}{B\bar\omega}\frac{1}{ik}\left[\left(\frac{B}{c}\bar\omega\right)^2 k^2\Psi - \frac{\partial}{\partial x}\left(\frac{B\bar\omega}{c}\right)^2\frac{\partial\Psi}{\partial x}\right] \tag{44}$$

Consequently, we have for our final equation [from Eqs. (36), (43) and (44)]

$$\frac{\partial}{\partial x}\rho T\frac{\partial\Psi}{\partial x} - k^2T\rho\Psi + k^2\sum g\frac{\partial\rho}{\partial x}\Psi = 0 \tag{45}$$

with

$$T = (1+b^2)\bar\omega^2 + \frac{k\bar\omega}{\rho\Omega}\left(\frac{\partial p}{\partial x} + \eta p\right) + \frac{k^2\eta}{\Omega^2}\frac{1}{\rho}\frac{\partial R}{\partial x} \tag{46}$$

$$b^2 = \frac{B^2}{4\pi\rho c^2}$$

To be complete we now write down the analogous equation for cylindrical geometry, namely

$$\frac{1}{r}\frac{\partial}{\partial r}\rho Tr^3\frac{\partial\Psi}{\partial r} + \left[(1-m^2)\rho T + m^2\sum g\frac{\partial\rho}{\partial r} + r\omega^2\frac{\partial}{\partial r}\rho(1+b^2)\right]\Psi = 0 \tag{47}$$

with

$$\bar\omega = \omega - \frac{mcE_\theta}{rB}$$

$$\Psi = \frac{cE_\theta'}{B\bar\omega} \tag{48}$$

and

$$T = (1 + b^2)\,\bar{\omega}^2 + \frac{m\bar{\omega}}{r\Omega\rho}\left(\frac{\partial p}{\partial r} + \eta p\right) + \frac{m^2\eta}{\Omega^2 r^2}\,\frac{1}{\rho}\,\frac{\partial R}{\partial r}\ ,$$

where $\quad \eta = \frac{1}{B}\frac{\partial B}{\partial r}$. (49)

The lowest order equilibrium is again just

$$\frac{\partial}{\partial r}\left(\frac{B^2}{8\pi} + \sum p\right) = 0; \tag{50}$$

the higher order terms are not required since all terms appearing in Eq. (47) are already of order ϵ^2.

Note that Y is the displacement in the frame drifting with the ∇B drift.

If we assume that the plasma can be represented by an isothermal Maxwellian velocity distribution with ions and electrons having equal temperatures, T can be considerably simplified and is given by

$$T = (1 + b^2)\left[\bar{\omega} + \frac{k}{2\Omega\rho}\,\frac{\partial p}{\partial x}\,(1-\beta)/(1+b^2)\right]^2$$
$$- \left[\frac{k}{2\Omega\rho}\,\frac{\partial p}{\partial x}\,(1+\beta)\right]^2 (1+b^2)^{-1} - \left(\frac{k}{\Omega\rho}\,\frac{\partial p}{\partial x}\right)^2 \frac{4\pi}{\alpha c^2(1+b^2)} \tag{51}$$

for the linear case with $\frac{k\partial p}{\partial x}$ replaced by $\frac{m}{r}\frac{\partial p}{\partial r}$ for the cylindrical case. Recall that all parameters in the above expression refer to ions only, thus $\beta = \frac{8\pi p_{ion}}{B^2}$.

III. ANALYSIS AND RESULTS

The differential equation Eq. (51) has the same structure as the zero β equations derived by Rosenbluth and Simon[3], and can be treated by the same computational or variational techniques with little added complication. They can be integrated directly to make stability predictions about plasma configurations with general $E_0(r)$, $B(r)$, $\rho(r)$, $P(r)$. Clearly changes in β will affect stability, but the effect will also depend on what changes are made in $E_0(r)$, $B(r)$, $\rho(r)$ etc. Indeed the theory written down here includes the variation $\beta(r)$, with $\beta = 0$ outside the boundary of the plasma increasing to some β_{max} inside the plasma. Thus the stability will be sensitive to gross changes in the profile $\rho(r)$, $\beta(r)$ as well as the value of β_{max}. We will try to isolate the effect of increasing β by choosing an equilibrium which is neutrally stable, increasing β keeping the gross spatial features of the plasma fixed, and calculating the shift from neutral stability. We will also calculate the sizes of the β corrections to a zero β theory and will show that the zero-β limit is a pessimistic estimate of flute stability.

Following Ref. (3) we put Eq. (51) in the form

$$\omega^2 O_1 \Psi + \omega O_2 \Psi + O_3 \Psi = 0. \tag{52}$$

Multiplying by Ψ^* on the left and integrating, we obtain

$$\omega^2 A + \omega B + C = 0 \tag{53}$$

where $$A = -\int dr\ \rho(1 + b^2)r^3\Lambda^2 + \int dr(\partial\rho/\partial r)r^2|\Psi|^2$$

$$B = -\int dr\ \rho(1 + b^2)\left[V_c\frac{(1-\beta)}{1+b^2} - 2\frac{cE_o}{B}\right] mr^2\Lambda^2$$

$$C = -\int dr\ \rho(1 + b^2)\left[\frac{cE_o}{B} + \frac{\beta}{1+b^2}V_c\right]\left[\frac{cE_o}{B} - \frac{1}{1+b^2}V_c\right]$$

$$m^2r\Lambda^2 + \int dr\ m^2r\Sigma g\frac{\partial\rho}{\partial r}|\Psi|^2$$

and

$$\Lambda^2 \equiv \left|\frac{\partial\Psi}{\partial r}\right|^2 + \frac{(m^2-1)}{r^2}|\Psi|^2, \quad V_c = \frac{1}{\Omega\rho}\frac{\partial p}{\partial r}$$

For our first example we choose an equilibrium with $2cE_o/B = V_c(1-\beta)(1+b^2)^{-1}$ This choice makes the constant B in Eq. (53) vanish and gives the simple results $\omega^2 = -C/A$. Rosenbluth and Simon[3] refer to this as the special equilibrium, and show that varying $E(\rho)$ away from this equilibrium is destabilizing. Now choosing a trial function Ψ constant everywhere except at the boundary we note that the only contribution to C for $m = 1$ modes is the destabilizing contribution from g; indeed $\omega^2 = 0$ is a solution for $m = 1$ and $g = 0$. Thus even for general β there is no stabilization for the $m = 1$ mode from the finite Larmor radius equation.

For higher m values we can adjust the strength of E_o/B and g to give marginal stability, $\omega^2 = 0$ and see what $\delta\omega$ is for a change $\delta\beta$.

To obtain the behavior of a variation of ρ which maintains the special equilibrium we vary the uniform temperature keeping the density fixed. From our equilibrium conditions

$$2p + \frac{B^2}{8\pi} = \frac{B_o^2}{8\pi}$$

where B_o is the magnetic field outside the plasma; we have with $\beta = \frac{8\pi p}{B^2}$ $= \frac{8\pi\rho}{2\alpha B^2}$

$$B = \frac{B_0}{\sqrt{1 + 2\beta}} \, . \tag{54}$$

By varying α we then have

$$\delta\beta = -\beta(1+2\beta)\,\frac{\delta\alpha}{\alpha} \tag{55}$$

which when inserted into the "special equilibrium" variational expression leads to

$$\frac{\delta\omega^2}{m^2} \sim -\frac{\delta\alpha}{\alpha}\int dr\rho r\left(\frac{1}{\rho}\frac{\partial\rho}{\partial r}\;\frac{1}{4\alpha\Omega}\right)^2 (1-2\beta)(1+\beta)(1+3\beta)\,\Lambda^2 \tag{56}$$

hence increasing β, i.e., $\delta\alpha/\alpha<0$, stabilizes.

For our next examples we choose either $E \sim r$ or $u = cE/B + \frac{1}{2}V_c \sim r$. This is the case of uniform rotation; if B = constant the two conditions are equivalent and the problem reduces to that of Rosenbluth, Krall and Rostoker.[1] We assume that for $\beta = 0$ the system is marginally stable ($B^2 = 4AC$) and calculate the corrections to first order in β, giving $\frac{d(B^2 - 4AC)}{d\beta}\,\delta\beta>0$ for stability.

Dropping β^2 contributions, using $B^2 = 4AC$ in the β contributions, and defining

$$\bar{\alpha} = -\frac{\int dr\, r^2\psi^2(\partial\rho/\partial r)}{\int dr\rho r^3\Lambda^2} > 0$$

we obtain

$$\left[\frac{1}{\bar{\alpha}} + 1 \pm \sqrt{1 + \frac{1}{\bar{\alpha}}}\right] > 0 \quad \text{for stability}$$

which is automatically satisfied. Thus β stabilizes in this case; the $\beta=0$ stability criterion[1] is pessimistic in that it ignores the stabilizing finite β contributions implied by the choice of p and B. The modified RKR results is (in linear geometry)

$$(1 + 2\beta)(kV_c)^2 > -4k\,g\,\frac{1}{\rho}\frac{d\rho}{dx} \quad \text{for stability.} \tag{57}$$

The superficial result of RKR that increasing the temperature with fixed magnetic field increased the finite Larmor radius stabilization is not contradicted here. Indeed it is enhanced by the increasing size of the finite β contribution.

A similar calculation with $E = 0$ shows that for most reasonable plasma profiles $\delta\beta > 0$ is stabilizing but a pathological geometry could lead to the opposite result.

We conclude that the effect of finite β on a finite Larmor radius flute instability is to generally drive a marginally stable system towards stability. The situation is complicated by the influence of the actual geometry on the instability. The best procedure is to compute the stability directly from the differential equation (5) with parameters and profiles appropriate to the plasmas being studied. The primary virtue of Eq. (51) is that it has the scope to handle rather straightforwardly realistic values of E, B, ρ and β, all functions of radius, for any value of $\beta < (k^2 a_i^2)^{-1} \gg 1$.

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Section 3

HIGH-FREQUENCY "LOSS-CONE" FLUTE INSTABILITIES INHERENT TO TWO-COMPONENT PLASMAS

by

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High-Frequency "Loss-Cone" Flute Instabilities Inherent to Two-Component Plasmas+

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It is shown that when a low temperature homogeneous plasma is added to a high temperature "loss-cone" limited homogeneous plasma in the presence of a uniform magnetic field, high frequency "flute" type unstable oscillations occur. For a fractional density of hot plasma (Δ) on the order of unity the frequency and growth rates are on the order of the ion (electron) cyclotron harmonic frequency where the harmonic number (N) is determined by the ratio of plasma to cyclotron frequency for ions (electrons). If $\Delta N \sim 1$ the instability occurs at the lower (upper) hybrid frequency. For $\Delta N \ll 1$ the instability only occurs when the ion (electron) cyclotron harmonic is equal to the lower (upper) hybrid frequency. Author

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Previously it has been shown[1] that a high frequency, electrostatic, convective instability ($\underset{\sim}{k} \cdot \underset{\sim}{B} \neq 0$) develops for the case in which the particles are described by a loss-cone distribution. It was also pointed out in this work that flute type ($\underset{\sim}{k} \cdot \underset{\sim}{B} = 0$) unstable modes would also occur in the presence of a density gradient.[2] We demonstrate here that an unstable flute mode develops when a low energy plasma component is incorporated. Specifically, we consider the case of a two-component (with reference to temperature), loss cone limited homogeneous plasma in the presence of a uniform magnetic field which we treat in the electrostatic limit.

We solve the perturbed Vlasov and Poisson equations to obtain

$$\nabla^2\varphi = -4\pi e \Sigma_j n_j^1 \tag{1}$$

and

$$n_j^1 = \frac{e_j}{m_j}\int dv_\perp^2 \frac{\partial f}{\partial v_\perp^2}\left[1 - \sum_{n=-\infty}^{+\infty} \frac{\omega}{\omega+n\Omega} J_n^2\left(\frac{kv_\perp}{\Omega}\right)\right]\varphi \tag{2}$$

where we have performed the standard orbit integrals to obtain the latter equation. Here Ω is the cyclotron frequency, k-ω is the wavenumber-frequency of the perturbation, $v_\perp$ is the perpendicular velocity and f is the equilibrium distribution function integrated over the parallel velocity. In the above we have set $k_\parallel = 0$. Note also that for a loss cone distribution the first term in the integral vanishes. In what follows we treat $kv_\perp/\Omega \gg 1$ (asymptotic) for the hot species and $kv_\perp/\Omega \ll 1$ (convergent) for the cold species insofar as the expansion of the Bessel function is concerned. It

should be remarked that it is only in the limit $kv_\perp/\Omega \gg 1$ that the presence of a loss cone distribution plays a role. In the convergent limit the contribution to the perturbed density is unaffected by the absence of particles with zero perpendicular energy; hence the low temperature component need not be loss cone limited. With the latter restrictions in mind we obtain, after an integration by parts, with[3]

$$R \equiv \frac{1}{\Omega}\left(\frac{\int dv_\perp^{\,2}\, f v_\perp^{\,-3}}{\int dv_\perp^{\,2}\, f}\right)^{-1/3} \equiv \frac{\bar{v}}{\Omega} \tag{3}$$

the dispersion relation

$$F \equiv -\left(\frac{K}{1-\Delta} + \frac{1}{1-y^2}\right)\frac{\tan \pi y}{y} = \frac{\Delta}{1-\Delta}\,\frac{1}{|kR|^3} \equiv \beta \tag{4}$$

where we have utilized $\Sigma\, 1/(y+n) = \pi \operatorname{ctn} \pi y$. We have assumed that f is slowly varying over the range in which $\cos^2\left[\frac{kv_\perp}{\Omega} - (n + \frac{1}{2})\frac{\pi}{2}\right]$, from the asymptotic expansion of the Bessel function is rapidly oscillating, in which case this term is replaced by its average value of 1/2. Also in Equation (4) we have set

$$K = \frac{\Omega_e^2}{\omega_{p_e}^2}$$

$$y = \omega/\Omega_e \tag{5}$$

$$R = R_e$$

for $\omega \sim \Omega_e$ and

$$K = \frac{m}{M} + \frac{\Omega^2_i}{\omega^2_{p_i}}$$

$$y = \frac{\omega}{\Omega_i} \qquad (6)$$

$$R = R_i$$

for $\omega \sim \Omega_i$. Here Δ is the fractional density of hot particles $\left(n_{hot}/(n_{hot}+n_{cold})\right)$ with ω_p the plasma frequency for the total density. Note that the effect of the loss cone distribution is to reverse the sign of the integral in Equation (2) for the case in which $kv_\perp/\Omega >> 1$.

To determine the stability properties we examine the dispersion relation Equation (4) for y real and look for values of parameters at which solutions disappear. Such behavior implies an instability since roots must occur in conjugate pairs. We sketch in Figure (1) the behavior of F as a function of y real. First note that the zeros of F occur for integer values of y save y = 1. In addition there is one other zero associated with the vanishing of the bracket. Now if the bracket vanishes for $y > 3/2$ we have two extrema other than that of y = 0 as depicted in Figure 1; in which case if β (see Eq. (4)) lies between these two extrema a pair of roots disappear and instability is achieved. On the other hand, if the bracket vanishes for $y \leq 3/2$ there are no additional extrema and all the roots are accounted for since all branches (except the first) now intersect the axis. Thus, in order for instability to occur we must have

$$\frac{1-\Delta}{K} < \left(\frac{3}{2}\right)^2 - 1 \quad . \qquad (7)$$

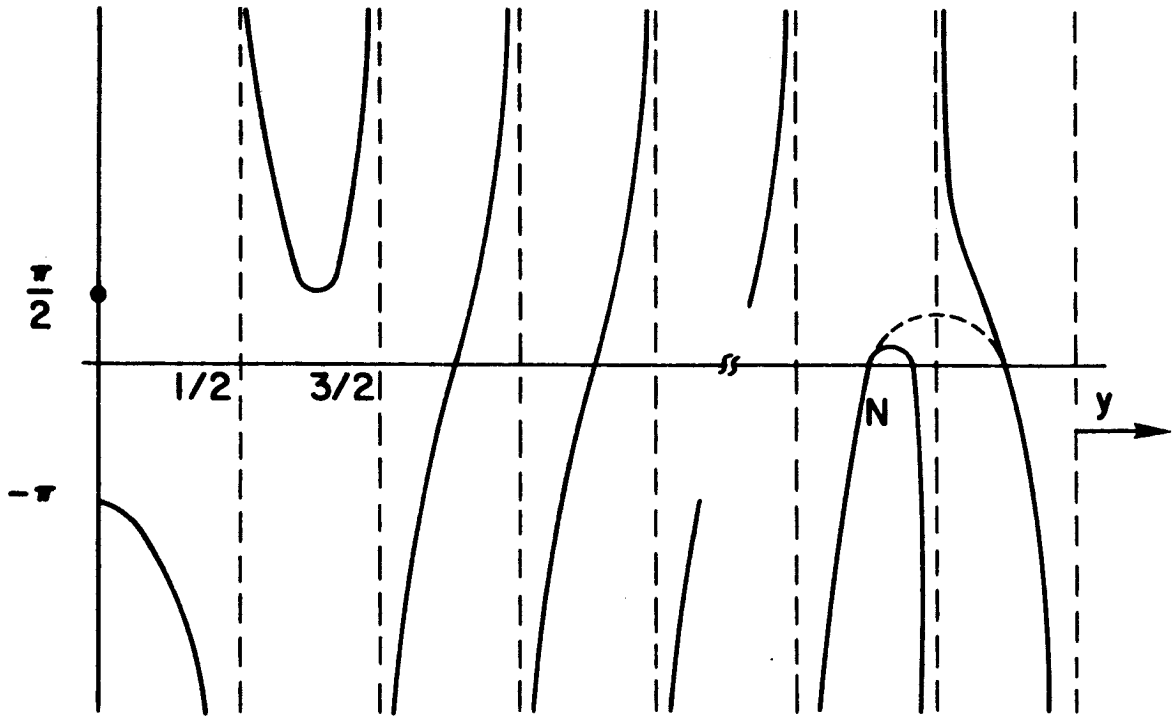

Fig. 1--F as a function of y real

Previously, we have seen that if β (see Eq. (4)) lies between the two extrema (see Fig. 1) instability is achieved. We now evaluate the magnitudes of these extrema to determine stability criteria. In what follows we have

$$N^2 - 1 \leq \frac{1-\Delta}{K} \leq (N+1)^2 - 1 \qquad N = 1,2\ldots\ldots \tag{8}$$

which determines the branch on which the bracket of Equation (4) vanishes. Now, the magnitude of F at the minimum is a monotonically increasing function of $(1-\Delta)/K$ with the minimum value occurring at $(1-\Delta)/K = 5/4$ with $y = 3/2$ and the maximum value occurring at $(1-\Delta)/K \to 0$ with $y \sim 1$. We then have

$$\frac{2}{9\pi} < F_{min} < \frac{\pi}{2} \quad . \tag{9}$$

The magnitude at the maximum on the other hand varies both within a branch and from branch to branch. In fact within a branch the value of F at the maximum varies from $F_{max} = 0$ at

$$\frac{1-\Delta}{K} = N^2 - 1 \tag{10}$$

to

$$F_{max} = \frac{2}{\pi}\left[\left(N + \frac{1}{2}\right)^2 - 1\right]^{-2} \tag{11}$$

with

$$\frac{1-\Delta}{K} = \left(N + \frac{1}{2}\right)^2 - 1 \quad . \tag{12}$$

It is immediately obvious that the F_{min} and F_{max} never overlap.

Thus we have as a condition for instability

$$\frac{2}{\pi}\left[\left(N+\frac{1}{2}\right)^2-1\right]^{-2} < \frac{\Delta}{1-\Delta}\frac{1}{(kR)^3} < F_{min} \tag{13}$$

with the range of F_{min} given by Equation (9). If the left hand side of the above inequality is violated, instability would still persist provided Equation (10) is maintained, leading to a sharply resonant mode. Actually the unique relation is necessary only for the strict $\Delta = 0$ limit; there is of course a narrow range of K dependent upon Δ for which instability still persists. Since kR is unbounded[3] (except for the restriction imposed by the convergent expansion of the cold plasma) the right hand side of the above inequality is not a constraint. Hence we have as a final stability criterion

$$\frac{\Delta}{1-\Delta} < \frac{2N^3}{\pi}\left[\left(N+\frac{1}{2}\right)^2-1\right]^{-2} \quad . \tag{14}$$

We now solve for the roots to the dispersion relation Equation (4) for the two limits $\Delta \ll 1$ and $(1-\Delta)/\Delta \lesssim 1$. Provided the parameters are well within the region of instability (see Eqs. (8) and (14)) we obtain for the root ($\tan y = -i$ for $-\mathrm{Im}\,y > 1$)

$$y = kR\left(\frac{1-\Delta}{\Delta}\right)^{1/3}\left[\pm\frac{\sqrt{2}}{2}-\frac{i}{2}\right] \quad , \quad \frac{1-\Delta}{\Delta} \lesssim 1 \tag{15}$$

and

$$y = \pm\left(\frac{1+K}{K}\right)^{\frac{1}{2}} - i\,\frac{\Delta}{2K^2}\frac{1}{(kR)^3} \quad , \qquad \Delta \ll 1 \tag{16}$$

for $K \ll 1$. Equation (16) can be written with $kR \sim N$

$$y = \pm\left(\frac{1+K}{K}\right)^{\frac{1}{2}} - i\,\frac{\Delta N}{2} \quad . \tag{17}$$

Recall that the dispersion relation was a symmetric function of y real which accounts for the two unstable roots.

For Δ small enough the inequality Equation (14) may be satisfied in which case for instability to occur Equation (10) must be maintained. We then have the root

$$y = \pm N - i\,\xi \qquad , \qquad \xi \ll N \tag{18}$$

where ξ satisfies the transcendental equation

$$\xi \tanh \pi\xi = \frac{\Delta(N^2-1)^2}{2(kR)^3} \tag{19}$$

which for $(1-\Delta)/K \ll 1$ and thus $N \gg 1$ has the approximate solution

$$\xi = \left(\frac{\Delta N}{2\pi}\right)^{1/2} < \frac{1}{\pi} \tag{20}$$

wherein we have utilized Equation (14) in this limit.

As an example we apply these results to two situations. In I there is a small density of hot electrons and we examine the behavior at $\omega \sim \Omega_e$. In II we assume a small density of hot ions and look for effects at $\omega \sim \Omega_i$. For convenience we assume $K \ll 1$. Both these regimes apply to the parameters in the Van Allen Belt.[4] In addition, there have been several laboratory experiments which operate in the regime described by I and II.[5]

For I with $\omega_{p_e}/\Omega_e = K^{-1/2} \sim N$ we have instability so long as

$$\Delta > \frac{2}{\pi N} \sim \frac{2}{\pi}\frac{\Omega_e}{\omega_{p_e}} \qquad \text{(see Eq. (14))} \tag{21}$$

with the characteristic frequency[5]

$$\begin{aligned} \omega &= \pm\, \omega_{UH} + i\,\frac{\Delta}{2}\,\frac{\omega_{p_e}^{\;2}}{\Omega_e}\,\frac{1}{(kR)^3} \\ &\approx \pm\, \omega_{UH} + i\,\Omega_e\,\frac{\Delta N}{2} \end{aligned} \tag{22}$$

On the other hand, if the inequality is not satisfied we have instability if

$$\frac{\omega_{UH}}{|\Omega_e|} = N \qquad N = 2,3 \ldots \tag{23}$$

with the frequency

$$\omega = \pm N \Omega_e - i|\Omega_e|\left(\frac{\Delta N}{2\pi}\right)^{1/2} \quad . \tag{24}$$

Thus for this more restrictive range of parameters, sharp electron cyclotron harmonic radiation would occur every time condition (23) occurred, at which

$$\mathrm{Re}\ \omega = \pm \omega_{UH} = \pm N \ \Omega_e \quad , \tag{25}$$

assuming that the instability would build up an electrostatic wave to high enough amplitude so that coupling with a transverse wave would be efficient.

For case II we have as a condition for instability

$$\Delta > \frac{2}{\pi} N \sim \frac{2}{\pi}\left(\frac{m}{M} + \frac{\Omega_i^2}{\omega_{p_i}^2}\right)^{1/2} \tag{26}$$

with the characteristic frequency[6]

$$\omega = \pm \omega_{LH} - i \frac{\Delta}{2} \Omega_i \ (kR)^{-3}\left(\frac{m}{M} + \frac{\Omega_i^2}{\omega_{p_i}^2}\right)^{\frac{1}{2}}$$

$$\approx \pm \omega_{LH} - i \ \Omega_i \frac{\Delta N}{2} \quad . \tag{27}$$

If the inequality is not satisfied we have instability if

$$\frac{\omega_{LH}}{\Omega_i} = N \qquad N = 2,3 \ldots \tag{28}$$

with the frequency

$$\omega = \pm N \Omega_i - i \Omega_i \left(\frac{\Delta N}{2\pi}\right)^{1/2} \quad . \tag{29}$$

For the parameters of the Van Allen belt ($n_i \approx 10^4$, $\Omega_i \approx 4 \times 10^2$ at 2 earth radii ($L = 2$)) we have as conditions for instability (excluding the sharp modes)

$$\Delta > \frac{5}{\pi} \times 10^{-2} \quad \text{for ion frequencies} \tag{30a}$$

$$\Delta > .16 \frac{1}{L^{3/2}} \quad \text{for electron frequencies} \quad . \tag{30b}$$

Hence, except for very high lines, the belt can be grossly unstable at only ion frequencies. If we assume the bulk of the hot plasma is at 10 keV possessing on the order of one tenth the magnetic energy density at 2 earth radii ($\Delta \sim 10^{-2}$) we have the condition Equation (30a) roughly satisfied. Hence this instability might play a role in determining the energy content of the belt.

In conclusion we see that provided the inequality given by Equation (14) is violated the plasma is grossly unstable to high frequency waves with growth rates comparable to the appropriate cyclotron harmonic. If these conditions are not maintained we have instabilities only at sharp frequencies for which

$$\mathrm{Re}\,\omega = \pm N \Omega_e = \pm \omega_{UH}$$

or

$$\mathrm{Re}\,\omega = \pm N \Omega_i = \pm \omega_{LH} \quad .$$

For the sharp instabilities the growth rates are small by $\sqrt{\Delta/N}$ compared to the frequency in which case we might expect a significant non-linear limitation of the growth. We conjecture that non-linear effects might put sufficient structure on the distribution function (still hardly discernible from a smooth function) which could change the sign of the integral for the high energy component in Equation (2) and completely cut off the instability. It should be remarked that the occurrence of the present instability relies upon the smooth variation of the distribution function; whereas in a prior calculation[7] sufficiently peaked distribution functions could produce instabilities for a one component plasma.*

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*After this paper had been prepared, R. F. Post kindly brought to our attention similar results obtained by L. S. Hall and W. Heckrotte [UCRL-12447 (1965)] in which the non-resonant case was treated using a delta function for the high energy perpendicular velocity distribution.

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Section 4

IONOSPHERIC STABILIZATION OF THE INTERCHANGE MODE

by

L. D. Pearlstein

General Atomic Report
GA-6967
1966

INTRODUCTION

Recent analysis of the proton data in the earth's radiation belt demonstrates that the energetic protons (>100 kev) are stable, to interchanges of equal tubes of flux, on purely energetic grounds.[1,2,3] However, the electrons from the beta decay of the fission products deposited during the starfish event do not satisfy the energy principle stability criteria for the interchange mode.[1] The presence of ionospheric currents which has not been included in the energy principle analysis tends to short out the growing electric field which drives the plasma across the magnetic lines of force. Thus, to properly pursue a stability analysis of particles trapped in the radiation belt the stabilizing influence of the ionospheric currents should be included.

A calculation by D. B. Chang et al[4] treats just this problem and we shall apply the results to the electron debris of the starfish event where the distribution is represented by a fission fragment energy spectrum with a phenomonologic fit to the spatial dependence.[2] The results of this calculation show that with daytime ionospheric parameters line tying, the effect of the ionospheric currents, stabilizes the interchange mode. For nighttime ionospheric parameters, where values are more sporadic, and with the initial density of trapped electrons, the stability criterion is violated by roughly an order of magnitude for the more extreme data.

The condition for stability is essentially that, in a tube of flux, the divergence of the ionospheric currents be greater than the divergence of the currents driving the flute in the radiation belt. Now, the ionosphere behaves as a resistive medium for fast characteristic frequencies and as a reactive medium if the characteristic

frequency is slow enough for the neutral particles to take part in the motion. For growth rates on the order or greater than the drift frequencies of the electrons, a condition necessary to violate the "flux" invariant and thus affect the distribution of trapped electrons, the ionosphere behaves resistively. Consequently, stability would occur if the ratio of energetic electrons to the number of ionospheric ions on a given tube of flux were small enough. An estimate of this condition is

$$m\, N_{ion.} > \frac{L}{\Delta L} N_e \quad \text{[see Eq. (14)]}$$

where $N_{ion.}$ is the effective number of ionospheric ions on a line [see Eq. (13)] L is the shell parameter with ΔL the width of the distribution function in the belt, m is the azimuthal mode number and N_e is the total number of trapped electrons on the line. It should be remarked that the above relation breaks down where $\Delta L \sim \frac{1}{7} L$, since the stabilizing effects associated with the increasing volume of a tube of flux with L which have not been included now compete. Finally, if we reintroduce the dependence of $N_{ion.}$ upon the shell parameter we obtain as a final stability criterion

$$N_e < 3 \times 10^{28} \frac{(\Delta L)^2}{L^3}$$

where the numerical factor comes from the detailed calculation.

To apply this result to a general spatial distribution of particles one should associate ΔL with the distance over which the number of particles changes and not with the total width of the distribution. In addition, N_e is now taken to be the total number of particles in the layer of width ΔL in which the density is decreasing. Thus, this stability criterion can then be applied to particles distributed smoothly over distances greater than the earth's radius provided the density drops off rapidly enough at its outer boundary.

CALCULATIONS AND RESULTS

Since the basic formulae were derived in reference 4 we shall extract the appropriate formulae and label them by the number which appears in that work; we also use the same notation. As a starting point we write down the differential equation determining the perturbed electrostatic potential (Φ_m):

$$\oint ds\, B \frac{\partial}{\partial\alpha}\left[r^2 \left(1 + \frac{4\pi\sigma_1}{i\omega}\right) \frac{\partial\Phi_m}{\partial\alpha}\right] - \left[m^2 \oint \frac{ds}{Br^2}\left(1 + \frac{4\pi\sigma_1}{i\omega}\right)\right.$$

$$\left. + \oint ds\, B \frac{\partial}{\partial\alpha}\left(\frac{4\pi\sigma_2 m}{B\omega}\right)\right]\Phi_m = -8\pi\Phi_m \Sigma \iint d\mu dJ$$

$$\times \frac{ecm \dfrac{\partial F_o}{\partial\alpha}}{\omega + \dfrac{cm}{e}\left(\oint \dfrac{ds}{V_\parallel}\right)^{-1} \oint \dfrac{ds}{V_\parallel}\left(\dfrac{1}{B}\dfrac{\partial B}{\partial\alpha}\right) m_o\gamma \left(V_\parallel^2 + \dfrac{V_\perp^2}{2}\right)}$$

$$= -\mathcal{M}(r_E)\Phi_m \qquad (4\text{-}46)$$

where the denominator on the r.h.s of the above equation is nothing but

$$\omega + \frac{cm}{e}\frac{\partial K_o}{\partial\alpha} = \omega - \frac{3mcr_E}{eM}\,\frac{K_o^2 - m_o c^2}{K_o}\,(0.35 + .15 \sin\alpha_E). \qquad (1)$$

Also in Eq. (4-46) the components of the conductivity tensor ($\sigma_{1,2}$) are given by

$$\sigma_0 \approx ne^2 \Sigma \frac{1}{m_j \nu_j}$$

$$\sigma_1 \approx ne^2 \Sigma \frac{\nu_j}{m_j\left[\nu_j^2 + \Omega_j^2\right]}$$

$$\sigma_2 \approx ne^2 \Sigma \frac{\Omega_j}{m_j\left[\nu_j^2 + \Omega_j^2\right]} \qquad (4\text{-}44a)$$

since $\omega \gg \rho_i \nu_i / \rho_o$ and the ionosphere behaves as a resistive medium. Before proceeding further it will be useful to define the various symbols which occur: μ is the magnetic moment, J is the longitudinal invariant, $\underset{\sim}{B} = \underset{\sim}{\nabla}\alpha \times \underset{\sim}{\nabla}\beta$ is the magnetic field with $\beta = \varphi$ for a cylindrically symmetric field, $F(\mu,J,\alpha,\beta)$ is the number of particles at (μ,J,α,β) in the interval $d\mu dJ d\alpha d\beta$, K_o is the total particle energy defined in terms of J by

$$J = \oint \frac{ds}{c} \left[(K_o - e\Phi)^2 - m_o^2 c^4 - 2\mu B m_o c^2 \right]^{\frac{1}{2}} \qquad (4\text{-}34)$$

r is the cylindrical radius, s is the distance along a line of force, r_E is the equatorial radius of a line $(r_E = -\frac{M}{\alpha})$, $\Phi = \Sigma_m \Phi_m(\alpha) e^{im\varphi}$ is the perturbed potential, Ω_j, ν_j, n_j, m_j are the species gyrofrequency, collision frequency with neutrals, density, mass respectively, and V is the particle velocity.

Performing a Nyquist analysis on Eq. (4-46) determines our stability criterion

$$m^2 \oint \frac{4\pi\sigma_1}{r^2 B} \, ds > \mathrm{Im}(-\omega \mathcal{M}) \ , \qquad \omega \text{ real} \qquad (4\text{-}65)$$

where σ_1 is given by

$$\sigma_1 = \frac{\omega_{p_i}^2}{2\Omega_i} \Delta\rho \ \delta(\rho - \rho_1) \qquad (4\text{-}59)$$

with ρ the spherical radius, i.e. $r = \rho \sin\theta$.

Before proceeding it is interesting to derive from Eq. (4-46) the energy principle stability criteria.[1] If $\omega \gg \frac{cm}{e} \frac{\partial K_o}{\partial \alpha}$ [see Eq. (1)] we have after multiplying by Φ_m^* and integrating over α and expanding the denominator

$$d\alpha \oint B \, ds \left(1 + \frac{4\pi m_i n_i}{B^2}\right) \left\{ \left|\frac{\partial \Phi_m}{\partial \alpha}\right|^2 r^2 + \frac{m^2}{r^2 B^2} \left|\Phi_m\right|^2 \right\} - 8\pi \int d\alpha \left|\Phi_m\right|^2$$

$$\times \sum \iint d\mu dJ \, \frac{m^2 c^2}{\omega^2} \frac{\partial K_o}{\partial \alpha} \frac{\partial F_o}{\partial \alpha} < 0 \qquad (2)$$

wherein we have neglected the ionospheric contributions and have employed the collisionless form of the conductivity tensor (4-44b) suitable for the radiation belt. We now obtain as a sufficient condition for stability

$$8\pi \sum \int d\mu dJ \, \frac{\partial K_o}{\partial \alpha} \frac{\partial F_o}{\partial \alpha} < 0 \, . \tag{3}$$

If we now assume that the pitch angle distribution of particles is peaked at $\alpha_E = \frac{\pi}{2}$ we have with $\sin\alpha_E \approx \frac{\pi}{2}$

$$E = \sqrt{p^2c^2 + m_o^2c^4} \approx m_oc^2 + \frac{p^2}{2m} = m_oc^2 + \frac{\mu B}{\sin^2\alpha_E} \approx m_oc^2 + \mu B \, , \tag{4}$$

(the non-relativistic limit),

$$- \frac{24\pi}{r_E^2} \frac{\partial}{\partial \alpha} \int d\mu dJ \, \mu \, F_o < 0 \tag{5}$$

for Eq. (3). Now, with

$$f = \frac{2BV_{\parallel}}{T} F_o \tag{4-14}$$

where T is the bounce time we have

$$- \frac{3}{2r_E^2} \frac{\partial}{\partial \alpha} \oint ds \, \beta < 0 \, , \tag{6}$$

with β the ratio of particle to magnetic pressure, for stability.

Next, we turn to an evaluation of the various terms of the stability condition Eq. (4-65). First for the l.h.s we have [see Eq. (4-59)]

$$m^2 \oint \frac{4\pi\sigma_1}{r^2B} ds = \frac{\Delta\rho m^2 r_E}{\sqrt{1-\rho_1/r_E}} \times \left. \frac{2\omega p_i^2}{M\Omega_i} \right|_{\rho = \rho_1} \tag{7}$$

where we have assumed a dipole field, i.e. $B = \frac{M}{\rho^3}\sqrt{4 - 3\,\rho/r_E}$. For

the right hand side we first obtain an estimate true for a general distribution as a function of r_E. The remainder of the paper will then be concerned with the evaluation of the r.h.s for the fission fragment distribution[2]

$$F(E,\eta,L) = 1.9\times10^3 \frac{(1 - \frac{\eta}{\eta_c})^4}{\eta_c} \exp\left[-\left(\frac{L-L_o}{\Delta L}\right)^2\right] \exp\left[-0.575E - 0.055E^2\right]$$

$$\times \frac{\sqrt{E(E+4)}}{E+2} \tag{8}$$

where E is the kinetic energy in Mev, $\eta = \cos\theta$, L is the shell parameter equal to the ratio of r_E to the radius of the earth (R_E) for a dipole field and $L_o = 1.15$, $\Delta L = 0.1$. To obtain an estimate of the r.h.s we utilize the non-relativistic limit and set $\alpha_E = \pi/2$ since we are generally concerned with flat helix distributions. We then obtain in the limit $\mathrm{Im}\,\omega \to 0^-$

$$\mathrm{Im}(-\omega\eta) = 8\pi^{3/2} ecm \int dJ\, \mu \frac{\partial F_o}{\partial \alpha}\Big|_{\mu = \frac{er_E^2\omega}{3mc}} \tag{9}$$

where e is the charge of an electron; only electrons contribute. Now, we are interested in the maximum value of Eq. (9) as a function of ω since if the stability condition is then satisfied it will be satisfied for all ω. Thus as the integrand of Eq. (9) will be peaked about some μ with width $\Delta\mu$ we may write as an estimate

$$\mathrm{Im}(-\omega\eta) = 8\pi^{3/2} ecm \frac{1}{\Delta\mu}\int \mu \frac{\partial F_o}{\partial\alpha}\, d_\mu dJ$$

$$= \frac{\pi^{\frac{1}{2}}}{2}\, ecm \frac{1}{\Delta\mu}\frac{\partial}{\partial\alpha}\oint \beta ds\ . \tag{10}$$

We now have for stability

$$mn_{ion.}\ \Delta\rho > \sqrt{1-R_E/r_E}\ \frac{M}{16\Delta\mu\pi^{\frac{1}{2}}}\ \frac{r_E}{R_E^3}\ \frac{\partial}{\partial r_E}\oint \beta ds$$

or approximately

$$mn_{ion.}\ \Delta\rho > \sqrt{1-R_E/r_E}\ \frac{r_E^{\ 4}}{R_E^{\ 3}\Delta r_E}\ \frac{1}{16\pi^{\frac{1}{2}}}\oint n_e\ ds \tag{12}$$

where $n_{ion.}$ is the density of ions in the ionosphere at $\rho = \rho_1 \approx R_E$ and Δr_E is the width of the distribution function in the radiation belt. Also, n_e is the density of trapped electrons. Recall that it is the conductivity which is peaked at ρ_1 with width $\Delta\rho$ thus the total number of ions in the ionosphere affecting stabilization is equal to the value of the density of the peak (n_{ion}) times the width ($\Delta\rho$) integrated over **the cross-sectional area of the tube of flux containing the trapped electrons or**

$$N_{ion.} = \int n_{ion.}\ d^3V = \int n_{ion.}\frac{r_E^{\ 4}}{M}\ d\alpha d\beta\ \sin^7\theta\ d\theta$$

$$= 2\pi\ n_{ion.}\Delta\rho\int_{\Delta r_E} r_E\ dr_E \left(\frac{\rho_1}{r_E}\right)^3 (1 - \rho_1/r_E)^{-1/2}$$

$$N_{ion.} = 2\pi\ n_{ion.}\ \rho_1^{\ 2}\Delta\rho\ \frac{\rho_1\Delta r_E}{r_E^{\ 2}}\ (1 - \rho_1/r_E)^{-1/2} \tag{13}$$

wherein we have utilized Eq. (4-50) for the volume of a tube of flux. Hence we now obtain as our sufficient condition for stability

$$mN_{ion.} > \frac{\pi^{\frac{1}{2}}}{4}\ r_E^{\ 2}\oint n_e\ ds \approx \frac{r_E}{\Delta r_E}\ N_e \tag{14}$$

To obtain the last form of the inequality we have represented n_e by a Gaussian of width Δr_E peaked at r_E in addition have assumed a flat helix distribution of particles in the belt.

We now calculate the stability criteria for the fission fragment energy distribution. The average energy of the trapped electrons is

roughly 1.7 Mev thus the non-relativistic limit is not strictly valid. However, in the extreme relativistic limit, $E = K_o \gg m_o c^2$, the demoninator (Eq. 1) can be written $\omega - \frac{3mcr_E}{2eM} E$ which differs only by a factor of 2 from the non-relativistic limit. Moreover, as will be seen when maximizing $\mathrm{Im}(-\omega\mathcal{M})$ with respect to ω the amplitude is unaltered from one approximation to the other; only the frequency changes, which is not significant. Consequently, we assume the drift frequency is a linear function of E. To obtain the expression for $\mathrm{Im}(-\omega\mathcal{M})$ we use Eq. (29) of the addenda[5] to reference 4, except we no longer take the non-relativistic limit, with the result

$$\mathrm{Im}(-\omega\mathcal{M}) = 12\pi^2 \frac{ecmr_E^5}{M^2} \int d(\sin\alpha_E)\, E \left\{ r_E \frac{\partial f}{\partial r_E} + 1/4 \frac{(E-m_oc^2)(E+2m_oc^2)}{(E+m_oc^2)^2} f - \frac{3}{2} \frac{E + 2m_oc^2}{E + m_oc^2} E^{3/2} \frac{\partial}{\partial E} \frac{f}{E^{1/2}} \right\} \Bigg|_{E = \frac{3M\omega\lambda}{3mcr_E}}, \tag{15}$$

where $\lambda = 1(2)$ for the non-relativistic (extreme relativistic) limit. We now must maximize with respect to ω or as we have already indicated with respect to E, it is easily seen that the value at the maximum is unaffected by the value of λ.

Evaluating the above expression numerically and inserting the result into Eq. (4-65) leads to the stability criterion

$$mn_{ion.} > 3 \times 10^4 \sqrt{1 - \frac{1}{L}}\; L^4 \left(1 - \frac{1}{L^3} \frac{1}{\sqrt{4-3/L}}\right) G(L) \tag{16}$$

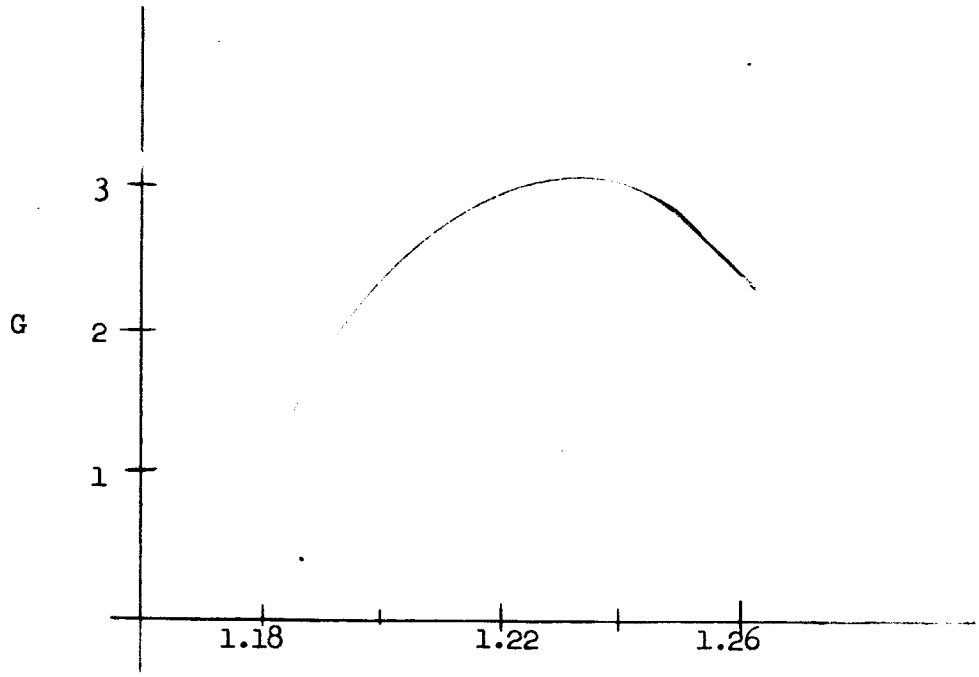

Fig. 1

with G(L) plotted in Fig. 1. If the maximum value of G(L) is employed we obtain

$$mn_{ion.} > 2.5 \times 10^4 . \tag{17}$$

To arrive at these results we have utilized $\Delta\rho = 5 \times 10^6$ and $\rho_1 \approx R_E$. Now at the peak of $\sigma_1, n_{ion} = 2 \times 10^5 (>3 \times 10^3)$ during the day (night). Thus without an average over night and day parameters (the drift time is about 1/30 of a day) this calculation shows that the stability criteria, with night-time parameters and with the initial number of trapped electrons, is violated for the more extreme data. With the daytime parameters stability is predicted

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Section 5

ON THE INTERACTION OF PLASMONS

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ON THE INTERACTION OF PLASMONS

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The nonlinear interaction between plasma oscillation normal modes is studied in a hydrodynamic approximation. A modified quasilinear treatment is used, in which the normal modes are taken to affect each other through the slow nonlinear temporal and spatial changes they induce in the (equilibrium) background quantities. Plasmons, i.e. wave packets of plasma oscillations, are found in this approximation to have an interaction similar in form to the electromagnetic interaction. The plasmon interaction is describable in terms of scalar and vector potentials which satisfy wave equations of the form satisfied by their electromagnetic counterparts. Here, quantities proportional to the slow changes induced in the background number and current densities play the roles of the scalar and vector potentials, with the thermal velocity playing the role of the speed of light. The source terms in the potential wave equations are quadratic in the normal mode density perturbation. The normal mode density perturbation in turn satisfies a Klein Gordon-like equation, so that, normalized, it would appear to play a role similar to the quantum mechanical probability wave function Ψ. The nonlinear interaction leads to an instability with a growth rate proportional to the square root of the ratio of the plasma oscillation energy to the background thermal energy.

Author

I. INTRODUCTION

Wave-particle duality has been widely discussed for a variety of physical phenomena. The Hamilton-Jacobi method of replacing the partial differential equations of wave motion by the ordinary differential equations of particle motion in the eikonal (or WKB) approximation[1,2], and the use of invariant quadratic forms to introduce appropriate distribution functions for the corresponding particles[3,4,5,6], have found application in problems ranging from the dual wave-particle aspects of light and matter to the behavior of quasi-particles in solid, liquid and gaseous many-body systems.[7,8,9] A linearized treatment of the collective motion of a many-body system gives the normal modes of the wave motion, corresponding in the dual particle description to the motion of noninteracting particles: A nonlinear perturbation analysis then describes the interaction of normal modes and the scattering of the dual particles.

In the following we consider the wave-particle duality of plasma oscillations in a hydrodynamic approximation to see if nonlinear interactions between plasma oscillations normal modes, i.e. between plasmons, may be described in terms of simple potentials. Treating these interactions in a modified quasilinear approximation[10,11], it is found that the plasmon interaction bears a resemblance to the electromagnetic interaction. A Klein Gordon-like equation is obtained for the plasmons in which the interactions between plasmons are described by scalar and vector potentials satisfying wave equations of the form satisfied by their electromagnetic counterparts.

II. WAVE EQUATIONS

1. Linearized wave equation

Treating the ions as providing a uniform background of positive charge, the normal mode equations for electron plasma oscillations in a uniform stationary plasma are obtained by inserting in Poisson's equation

$$\nabla^2\Phi^{(1)} = 4\pi e n^{(1)} , \tag{1}$$

the perturbed electron density $n^{(1)}\left(\Phi^{(1)}\right)$ given by Vlasov's equation

$$\frac{\partial f}{\partial t} + v\cdot\nabla f + \frac{e}{m}\nabla\Phi^{(1)}\cdot\nabla_v f = 0 . \tag{2}$$

Here $\Phi^{(1)}$ is the electrostatic potential, -e and m are the electronic charge and mass, and $f(\vec{x},\vec{v},t)\, d^3x\, d^3v$ denotes the number of electrons in the position interval d^3x at $\vec{x}$ and velocity interval d^3v at $\vec{v}$ at time t. We shall find it convenient to deal with the first and second velocity moments of eq. (2):

$$\frac{\partial n}{\partial t} + \nabla\cdot(n\vec{V}) = 0 , \tag{3}$$

$$\frac{\partial}{\partial t}(mn\vec{V}) + \nabla\cdot\overleftrightarrow{\Pi} - en\nabla\Phi^{(1)} = 0 ; \tag{4}$$

where

$$n(\vec{x},t) = \int d^3v f(\vec{x},\vec{v},t) , \tag{5}$$

and

$$nm\vec{V}(\vec{x},t) = \int d^3v m\vec{v} f(\vec{x},\vec{v},t) , \tag{6}$$

are the number and momentum density, respectively, and where

$$\overleftrightarrow{\pi}(\vec{x},t) = m\int d^3v\, \vec{v}\, \vec{v}\, f(\vec{x},\vec{v},t) \tag{7}$$

is related to the pressure tensor

$$\overleftrightarrow{P}(\vec{x},t) = m\int d^3v(\vec{v}-\vec{V})(\vec{v}-\vec{V})f(\vec{x},\vec{v},t) \ , \tag{8}$$

by

$$\overleftrightarrow{\pi} = nm\vec{V}\vec{V} + \overleftrightarrow{P} \ . \tag{9}$$

On differentiating eq. (3) with respect to time and combining this with the divergence of eq. (4), there results

$$\frac{\partial^2 n}{\partial t^2} - \frac{1}{m}\nabla\cdot\nabla\cdot\overleftrightarrow{\pi} + \frac{e}{m}\nabla\cdot(n\nabla\Phi) = 0 \ . \tag{10}$$

On linearizing, eq. (10) becomes

$$\frac{\partial^2}{\partial t^2} n^{(1)}\left(\Phi^{(1)}\right) - \frac{1}{m}\nabla\cdot\nabla\cdot\overleftrightarrow{\pi}^{(1)}\left(\Phi^{(1)}\right) + \frac{n_o e}{m}\nabla^2\Phi^{(1)} = 0 \ , \tag{11}$$

where the perturbations on the initially uniform n_o and $\overleftrightarrow{\pi}$ are denoted by $n^{(1)}$ and $\overleftrightarrow{\pi}^{(1)}$. For our present purposes, we need only consider eq. (10) in the hydrodynamic approximation, ignoring any Landau damping contributions to n and $\overleftrightarrow{\pi}$. In fact, the essential features of the development may be seen with the simple fluid approximation

$$\overleftrightarrow{\pi}^{(1)} = \overleftrightarrow{I}mn^{(1)}v_\theta^2 \ , \tag{12}$$

where v_θ is a thermal velocity and $\overleftrightarrow{I}$ is the unit dyadic. With this approximation, the linearized normal mode equation is simply

$$\frac{\partial^2}{\partial t^2} n^{(1)} - v_\theta^2\nabla^2 n^{(1)} + \omega_p^2 n^{(1)} = 0 \ , \tag{13}$$

giving the usual plasma oscillation of time scale $O(\omega_p^{-1})$, where $\omega_p = [4\pi n_o e^2/m]^{\frac{1}{2}}$ is the plasma frequency.

2. Nonlinear wave equation

The problem now is to find how eq. (13) is modified if nonlinear effects are included. We shall include nonlinear effects approximately by adopting the spirit of the quasilinear theory developed by Drummond and Pines and by Vedenov, Velikhov and Sagdeev. In their quasilinear theory, attention is directed to the Fourier-transformed Vlasov equation:

$$\frac{\partial f_{\vec{k}}}{\partial t} + i\vec{k}\cdot\vec{v}f_{\vec{k}} + \frac{ie}{m}\Phi_{\vec{k}}\vec{k}\cdot\nabla_v f_o + \frac{ie}{m}\sum_{\vec{q}\neq 0}(\vec{k}-\vec{q})\Phi_{\vec{k}-\vec{q}}\cdot\nabla_v f_{\vec{q}} = 0 \ , \tag{14}$$

where

$$\{f_{\vec{k}}, \Phi_{\vec{k}}\} = \int d^3x \ e^{i\vec{k}\cdot\vec{x}} \{f(\vec{x},\vec{v}),\Phi(\vec{x})\}. \tag{15}$$

Their quasilinear approximation consists of retaining only $\vec{q} = 0$ in the $\vec{k} \neq 0$ equations:

$$\frac{\partial f_{\vec{k}}}{\partial t} + i\vec{k}\cdot\vec{v}f_{\vec{k}} + \frac{ie}{m}\Phi_{\vec{k}}\ \vec{k}\cdot\nabla_v f_o \approx 0 \tag{16a}$$

$$\frac{\partial f_o}{\partial t} - \frac{ie}{m}\sum_{\vec{q}\neq 0}\Phi_{\vec{q}}\ \vec{q}\cdot\nabla_v f_{\vec{q}} = 0, \tag{16b}$$

i.e. the nonlinearities are assumed to act on any $f_{\vec{k}}$ only through their effect on the $\vec{k} = 0$ Fourier component. The $f_{\vec{k}}$ then have two time scales, the usual fast time scale of $O(1/\omega_p)$ and a slow time scale associated with the changes in $f_{\vec{k}=0}$.

A modification to the quasilinear approximation which retains slightly more phase information than the treatments using eq. (16) may be had by relaxing the condition that the nonlinearities act directly only on the $\vec{k} = 0$ component. This may be done by applying the spirit of the quasilinear

approximation to quantities in $\vec{x}$-space directly rather than to their Fourier counterparts. More explicitly, the effects of nonlinearities may be assumed to act on any normal mode only through the slow changes [in space as well as in time] they cause in the background equilibrium quantities. Apparently, the normal modes will now have two time scales, the usual fast time scale of $O(1/\omega_p)$ and a much slower time scale associated with the modulation introduced by the slow changes in the background quantities. A reasonable nonlinear approximation would then consist of dividing out this slow modulation, leaving the remaining fast time-scale portions of the normal mode variables to satisfy the regular linearized normal mode equations, but with the background equilibrium quantities in these equations possessing their modified values.

Thus, consider the Vlasov equations for the electrons and ions, including the ion dynamics now since a slow time scale phenomenon is involved:

$$\frac{\partial f_j}{\partial t} + \vec{v}\cdot\nabla f_j - \frac{e_j}{m_j}\nabla\Phi\cdot\nabla_v f_j = 0 \ . \tag{17}$$

Here, the subscript $j = e,i$ denotes the electron or ion species. In a linear treatment, eq. (17) is written

$$\frac{\partial f_j^{(1)}}{\partial t} + \vec{v}\cdot\nabla f_j^{(1)} - \frac{e_j}{m_j}\nabla\Phi^{(1)}\cdot\nabla_v f_j^{(0)} = 0 \ , \tag{18}$$

where $f_j^{(0)}(\vec{v})$ is the uniform equilibrium distribution of the jth species. In a usual perturbation procedure, the equation for the nonlinear $f_j^{(2)}$ correction to f_j is

$$\frac{\partial f_j^{(2)}}{\partial t} + \vec{v}\cdot\nabla f_j^{(2)} - \frac{e_j}{m_j}\nabla\Phi^{(2)}\cdot\nabla_v f_j^{(0)} - \frac{e_j}{m_j}\nabla\Phi^{(1)}\cdot\nabla_v f_j^{(1)} = 0 \ , \tag{19}$$

where the potentials in eqs. (18) and (19) satisfy

$$\nabla^2\Phi^{(\alpha)} = -\sum_j 4\pi e_j \int d^3v f_j^{(\alpha)} \ . \tag{20}$$

The electron plasma oscillation solution of eq. (18) has a time scale of $O(\omega_p^{-1})$. Thus, the

$$\frac{e_j}{m_j} \nabla\Phi^{(1)} \cdot \nabla_v f_j^{(1)}$$

term in Eq. (19) will consist of terms of two time scales: almost constant and $O(1/2\omega_p)$. The distribution $f_j^{(2)}$ will then modify the background distribution $f_j^{(0)}$ and in addition have a rapdily oscillating component of frequency $\sim 2\omega_p$. The approximation we now wish to make is to ignore the $2\omega_p$ component and to say that the normal mode solutions of eq. (20) are affected by nonlinearities only through the modification of the background.

In this approximation, then, the distribution function $f_j(\vec{x},\vec{v},t)$ is written as

$$f_j(\vec{x},\vec{v},t) = f_j^{(0)}(\vec{v}) + f_j^{(s)}(\vec{x},\vec{v},t) + \delta f_j(\vec{x},\vec{v},t) \ , \tag{21}$$

and eqs. (18)-(20) are replaced by

$$\frac{\partial \delta f_j}{\partial t} + \vec{v}\cdot\nabla\delta f_j - \frac{e_j}{m_j}\nabla\Phi^{(s)}\cdot\nabla_v\delta f_j - \frac{e_j}{m_j}\nabla\delta\Phi\cdot\nabla_v\left[f_j^{(0)} + f_j^{(s)}\right] = 0 \ , \tag{22}$$

$$\frac{\partial f_j^{(s)}}{\partial t} + \vec{v}\cdot\nabla f_j^{(s)} - \frac{e_j}{m_j}\nabla\Phi^{(s)}\cdot\nabla_v f_j^{(0)} - \frac{e_j}{m_j}\left[\nabla\delta\Phi\cdot\nabla_v\delta f_j\right]^{(s)} = 0 \ , \tag{23}$$

$$\nabla^2\Phi^{(s)} = -\sum_j 4\pi e_j \int d^3v f_j^{(s)} \ , \tag{24}$$

and

$$\nabla^2\delta\Phi = -\sum_j 4\pi e_j \int d^3v\ \delta f_j \ . \tag{25}$$

Here $f_j^{(0)}(\vec{v})$ is again the original uniform equilibrium distribution, and the superscript "s" stands for "slowly time-varying portion." Equation (22) for δf_j differs from eq. (18) for $f_j^{(1)}$ in the presence of the $\nabla\Phi^{(s)}$ and $f_j^{(s)}$ terms describing the background modification. The latter are seen in eq. (23) to arise from the $\omega \sim 0$ portion of $\nabla\delta\Phi\cdot\nabla_v\delta f_j$.

The solution to eq. (23) may be written

$$f_j^{(s)}(\vec{x},\vec{v};t) = \int_{-\infty}^{t} dt'\ \frac{e_j}{m_j}\ \nabla\Phi^{(s)}[\vec{x}(t'),t']\cdot\nabla_v f_j^{(0)}(\vec{v}) + F_j^{(s)}(\vec{x},\vec{v};t)\ , \tag{26}$$

where

$$F_j^{(s)}(\vec{x},\vec{v},t) = \int_{-\infty}^{t} dt'\ \frac{e_j}{m_j}\ \{\nabla\delta\Phi[\vec{x}(t'),t']\cdot\nabla_v\delta f_j[\vec{x}(t'),\vec{v},t']\}^{(s)} \tag{27}$$

and

$$\vec{x}(t') = \vec{x} + \vec{v}(t'-t)\ . \tag{28}$$

The quantity $F_j^{(s)}(\vec{x},\vec{v};t)$ satisfies the equation

$$\frac{\partial F_j^{(s)}}{\partial t} + \vec{v}\cdot\nabla F_j^{(s)} = \frac{e_j}{m_j}\ [\nabla\delta\Phi\cdot\nabla_v\delta f_j]^{(s)}\ , \tag{29}$$

and is what $f_j^{(s)}\ (\vec{x},\vec{v};t)$ would be if $\nabla\Phi^s$ could be ignored. The distributions $F_e^{(s)}$ and $F_i^{(s)}$ obtained from eq. (29) may be regarded as electron and ion test particle distribution functions, in which case the

$$\int_{-\infty}^{t} dt' \frac{e_j}{m_j} \nabla' \Phi^{(s)}[\vec{x}(t'),t'] \cdot \nabla_v f_j^{(0)}(\vec{v})$$

term in eq. (26) describes the polarization response of the jth-species to the $\Phi^{(s)}$ induced by the test particles.

The essential features of the present development may be seen without evaluating $f_j^{(s)}(\vec{x},\vec{v},t)$ precisely, and in the following a number of approximations will be made. For frequencies of the order of the electron plasma frequency, $(e_i/m_i)\delta f_i$ may be ignored compared to $(-e/m_e)\delta f_e$; so from eq. (29) we shall ignore $F_i^{(s)}$ compared with $F_e^{(s)}$. The distribution $F_e^{(s)}$ will be regarded as changing sufficiently slowly to allow both the ion and electron polarization charges to respond adiabatically, i.e. fractional changes for the jth species will be taken proportional to $e_j\Phi_j/KT_j$, where K is Boltzmann's constant and T_j is the temperature of the jth species. For equal electron and ion temperatures, and assuming singly charged ions, this has the consequence that the polarization cloud responding to $N_e^{(s)}$ electron test particles will be composed of $-N_e^{(s)}/2$ electrons and $+N_e^{(s)}/2$ ions, giving a net involvement of $N_e^{(s)}/2$ electrons and $N_e^{(s)}/s$ ions, and a net zero charge. Thus, we shall make the approximation of replacing quantities evaluated from $f_e^{(s)}(\vec{x},\vec{v},t)$ by simply one-half the corresponding quantities evaluated from $F_e^{(s)}(\vec{x},\vec{v},t)$, and shall use the quasi-equality of electron and ion densities evaluated from $f_e^{(s)}$ to ignore the $\nabla\Phi^{(s)}$ term in eq. (22).

2a. Equation of motion

In the crudest approximation, then, nonlinear effects enter the linearized hydrodynamic normal mode equations through n_s and $(n\vec{v})_s$:

$$\left(\frac{\partial}{\partial t} + \frac{(n\vec{v})_s \cdot \nabla}{n_o}\right) \delta n_e + (n_o + n_s)\, \nabla \cdot \delta\vec{v}_e = 0 \ , \tag{30}$$

$$m_e(n_o + n_s)\left(\frac{\partial}{\partial t} + \frac{(n\vec{v})_s \cdot \nabla}{n_o}\right) \delta\vec{v}_e = - m v_\theta^2 \nabla \delta n_e + e(n_o + n_s)\nabla\delta\Phi \ , \tag{31}$$

$$\nabla^2 \delta\Phi = 4\pi\delta n_e \ . \tag{32}$$

Here, space and time derivatives of $F_e^{(s)}$ have been ignored compared to derivatives of δf_e, δn_e and $\delta\vec{v}_e$ are the density and velocity calculated from δf_e, the perturbed pressure has been set equal to $m v_\theta^2 \delta n_e$ as in eq. (12), n_o and n_s are the densities obtained from $f_e^{(o)}$ and $F_e^{(s)}/2$, and

$$\frac{(n\vec{v})_s}{n_o} = \frac{\int d^3v\ \vec{v}\left(f_e^{(o)} + (F_e^{(s)}/2)\right)}{\int\left(f_e^{(o)} + (F_e^{(s)}/2)\right) d^3v} \approx \frac{\int d^3v\ \vec{v}(F_e^{(s)}/2)}{\int f_e^{(o)}\ d^3v} \ , \tag{33}$$

the last equality holding when terms of $O\left([F_e^{(s)}]^2\right)$ are ignored. Operating on eq. (30) with

$$\left(\frac{\partial}{\partial t} + \frac{(n\vec{v})_s \cdot \nabla}{n_o}\right)$$

and taking the divergence of eq. (31), we find

$$\frac{\partial^2 \delta n_e}{\partial t^2} - v_\theta^2 \nabla^2 \delta n_e + \omega_p^2 \left(1 + \frac{n_s}{n_o}\right)\delta n_e + 2\frac{(n\vec{v})_s}{n_o} \cdot \nabla \frac{\partial \delta n_e}{\partial t} = 0 \tag{34}$$

where again derivatives of $F_e^{(s)}$ have been ignored compared to derivatives of δf_e, terms of $O\left\{[F_e^{(s)}]^2\right\}$ have been ignored, and $\omega_p = [4\pi n_o e^2/m_e]^{\frac{1}{2}}$.

It is of interest to consider the eikonal equations corresponding to eq. (34). Writing $\delta n_e = \eta(\vec{x},t)e^{i(\int \vec{k}\cdot d\vec{x} - \int \omega dt)}$, where the variations in $\eta(\vec{x},t)$ result from slow variations in n_s and $(n\vec{v})_s$ and are assumed to be much slower than the variations described by $\vec{k}$ and ω, the first order WKB solution[1,2] of eq. (34) gives the dispersion relation

$$\omega^2 - \omega_p^2\left(1 + \frac{n_s}{n_o}\right) - k^2 v_\theta^2 - \frac{2\omega}{n_o}(n\vec{v})_s\cdot\vec{k} = 0 \ . \tag{35}$$

Since the n_s and $(n\vec{v})_s$ terms are regarded as small, we may consider the first iterate solution of eq. (25):

$$\omega^2 = \omega_p^2 + k^2 v_\theta^2 + \omega_p^2\frac{n_s}{n_o} + \frac{2\omega_p}{n_o}(n\vec{v})_s\cdot\vec{k} \ . \tag{36}$$

[Equation (36) may also be obtained directly from the first iterate solution of the Vlasov equation, eq. (22), with $\nabla\Phi^{(s)} = 0$ on ignoring terms of $O[(f_e^{(s)})^2, k^3]$ and ignoring Landau damping.] The eikonal equations corresponding to eq. (36) are

$$\vec{V} = \frac{\partial\omega}{\partial\vec{k}} = \frac{\vec{k}v_\theta^2}{\omega} + \frac{\omega_p}{\omega n_o}(n\vec{v})_s \tag{37}$$

and

$$\frac{d\vec{k}}{dt} = -\frac{\partial\omega}{\partial\vec{x}} = -\frac{\omega_p^2}{2\omega n_o}\nabla n_s - \frac{(\vec{k}\cdot\nabla)}{\omega}\left(\frac{\omega_p}{n_o}(n\vec{v})_s\right) - \frac{\vec{k}}{\omega}\times\left\{\nabla\times\left[\frac{2\omega_p}{n_o}(n\vec{v})_s\right]\right\} , \tag{38}$$

where

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \vec{V}\cdot\nabla \ . \tag{39}$$

On combining eqs. (36)-(38), we find on ignoring terms of $O(n_s^2)$:

$$\frac{d}{dt}(\omega\vec{V}) = -\frac{v_\theta^2\omega_p^2}{2\omega n_o}\nabla n_s + \frac{\omega_p}{n_o}\frac{\partial}{\partial t}(n\vec{v})_s - \frac{\omega_p\vec{V}}{n_o}\times\left(\nabla\times(n\vec{v})_s\right) , \qquad (40a)$$

which when $\omega \approx \omega_p$ [the only case in which a hydrodynamic treatment is valid] becomes

$$\frac{d\vec{V}}{dt} \approx -\frac{v_\theta^2}{2n_o}\nabla n_s + \frac{1}{n_o}\frac{\partial}{\partial t}(n\vec{v})_s - \frac{1}{n_o}\vec{V}\times\left(\nabla\times(n\vec{v})_s\right) \qquad [\omega \approx \omega_p] . \qquad (40b)$$

This equation, which describes the acceleration of a plasmon with $V << v_\theta$ in the presence of variations in n_s and $(n\vec{v})_s$, resembles the $F = ma$ equation describing the acceleration of a nonrelativistic charged particle in the presence of an electric and magnetic field. Here n_s and $(n\vec{v})_s$ play the roles of scalar and vector potentials.

2b. Equations for interaction potentials

In the foregoing, the nonlinear quantities n_s and $(n\vec{v})_s$ are to be evaluated from $F_e^{(s)}/2$. Wave equations for n_s and $(n\vec{v})_s$ may be obtained from eq. (29). Differentiation of eq. (29) with respect to time gives

$$\frac{\partial^2 F_e^{(s)}}{\partial t^2} - (\vec{v}\cdot\nabla)^2 F_e^{(s)} = \frac{e}{m_e}\left\{(\vec{v}\cdot\nabla)(\nabla\delta\Phi\cdot\nabla_v\delta f_e)^{(s)} - \frac{\partial}{\partial t}(\nabla\delta\Phi\cdot\nabla_v\delta f_e)^{(s)}\right\}, \quad (41)$$

so that

$$\frac{\partial^2 n_s}{\partial t^2} - \int d^3v\ (\vec{v}\cdot\nabla)^2(F_e^{(s)}/2) = \frac{e}{2m_e}\int d^3v(\vec{v}\cdot\nabla)(\nabla\delta\Phi\cdot\nabla_v\delta f_e)^{(s)}$$

$$= -\frac{e}{2m_e}\nabla\cdot[\delta n_e\nabla\delta\Phi]^{(s)}, \quad (42)$$

and

$$\frac{\partial^2 (n\vec{v})_s}{\partial t^2} - \int d^3v(\vec{v}\cdot\nabla)^2\ \vec{v}\ (F_e^{(s)}/2) = \frac{e}{2m_e}\left[\int d^3v\ \vec{v}\ (\vec{v}\cdot\nabla)(\nabla\delta\Phi\cdot\nabla_v\delta f_e)^{(s)}\right.$$

$$\left. + \frac{\partial}{\partial t}[\delta n_e\nabla\delta\Phi]^{(s)}\right] = \frac{e}{2m_e}\left[-\left\{[\nabla\cdot\delta(n\vec{v})_e + \delta(n\vec{v})_e\cdot\nabla]\ \nabla\delta\Phi\right.\right.$$

$$\left.\left. + [\nabla^2\delta\Phi + (\nabla\delta\Phi)\cdot\nabla]\ \delta(n\vec{v})_e\right\}^{(s)} + \frac{\partial}{\partial t}[\delta n_e\nabla\delta\Phi]^{(s)}\right], \quad (43)$$

where

$$\delta(n\vec{v}) = \int d^3v\ \vec{v}\ \delta f_e\ . \quad (44)$$

Since δf_e is the source of $F_e^{(s)}$, and since for plasma oscillations of wavelengths much larger then the Debye length δf_e essentially describes a velocity oscillation of amplitude $\simeq \frac{e|\nabla\delta\Phi|}{m_e\omega_p}$ imposed on the equilibrium

Maxwellian distribution, we shall [for a moderately uniform spatial distribution of $[\nabla\delta\Phi\cdot\nabla_v\delta f_e]^{(s)}$] regard $F_e^{(s)}$ at any $\vec{x}$ as describing some small drift imposed on the equilibrium Maxwellian. In that case,

$$\int d^3v(\vec{v}.\nabla)^2\ (F_e^{(s)}/2) \approx v_\theta^2\nabla^2 n_s \tag{45}$$

and

$$\int d^3v\ (\vec{v}.\nabla)^2\ \vec{v}\ (F_e^{(s)}/2) \approx v_\theta^2\nabla^2(n\vec{v})_s\ , \tag{46}$$

so that eqs. (42) and (43) are wave equations describing the generation of n_s and $(n\vec{v})_s$ by nonlinear sources and the propagation of n_s and $(n\vec{v})_s$ at the thermal speed v_θ. In passing, we note that similar equations would arise if the time scale of $F_e^{(s)}$ were such that only the electrons responded adiabatically: if the electron temperature were much larger than the ion temperature, the background perturbations would be due to ion acoustic waves propagating at the thermal velocity of ions with the electron temperature.

It is convenient to express the source terms in eqs. (42) and (43) in somewhat simpler form. If we ignore nonlinear terms in evaluating δn_e and $\delta(n\vec{v})_e$ in the source expressions, we may use eq. (13) and Poisson's equation to write

$$\delta n_e = \frac{1}{4\pi e}\nabla^2\delta\Phi \approx \frac{1}{4\pi e v_\theta^2}\left(\frac{\partial^2\delta\Phi}{\partial t^2} + \omega_p^2\delta\Phi\right). \tag{47}$$

This then gives

$$[\delta n_e \nabla\delta\Phi]^{(s)} = \frac{1}{T}\int_t^{t+T} dt' \, (\delta n_e \nabla\delta\Phi)$$

$$= \frac{1}{4\pi e v_\theta^2 T}\left\{ (\nabla\delta\Phi)\,\frac{\partial\delta\Phi}{\partial t}\Big]_t^{t+T} + \int_t^{t+T} dt \right.$$

$$\left. \times\ \nabla \left(-\frac{1}{2}\left(\frac{\partial\delta\Phi}{\partial t}\right)^2 + \frac{1}{2}\,\omega_p^2(\delta\Phi)^2 \right)\right\} . \tag{48}$$

For large T and bounded $\nabla\delta\Phi$ and $\frac{\partial\delta\Phi}{\partial t}$, the end point term is small compared to the integral, leading to

$$[\delta n_e \nabla\delta\Phi]^{(s)} \approx \frac{1}{8\pi e v_\theta^2}\left\langle \nabla\left\{ -\left(\frac{\partial\delta\Phi}{\partial t}\right)^2 + \omega_p^2\,(\delta\Phi)^2 \right\}\right\rangle , \tag{49}$$

where the brackets $\langle\ \rangle$ denote a time average over times longer than ω_p^{-1}. Similarly, the $\delta(n\vec{v})_e$ in the source expression for $(n\vec{v})_s$ may be expressed in terms of $\delta\Phi$. For the linearized electrostatic plasma oscillation, the perturbed magnetic field $\delta\vec{B}$ is zero:

$$0 = c\nabla \times \delta\vec{B} \approx -\,4\pi e\ \delta(n\vec{v})_e - \frac{\partial}{\partial t}\nabla\delta\Phi\ . \tag{50}$$

Thus,

$$-\,\{[\nabla.\delta(n\vec{v})_e + \delta(n\vec{v})_e.\nabla]\ \nabla\delta\Phi + [\nabla^2\delta\Phi + (\nabla\delta\Phi).\nabla]\delta(n\vec{v})_e\}^{(s)}$$

$$= \frac{1}{4\pi e}\left\{\frac{\partial}{\partial t}\,[(\nabla^2\delta\Phi + \nabla\delta\Phi.\nabla)\nabla\delta\Phi]\right\}^{(s)}$$

$$= \frac{1}{4\pi e}\,\frac{\partial}{\partial t}\left\{(\nabla^2\delta\Phi + \nabla\delta\Phi.\nabla)\nabla\delta\Phi\right\}^{(s)}$$

$$= \frac{\partial}{\partial t}\,[\delta n_e\nabla\delta\Phi]^{(s)} + \frac{1}{8\pi e}\,\{\nabla(\nabla\delta\Phi)^2\}^{(s)}\ . \tag{51}$$

Ignoring nonlinear terms in δn_e and $\delta(n\vec{v})_e$ in evaluating the source terms for n_s and $(n\vec{v})_s$, then, we find

$$\frac{\partial^2 n_s}{\partial t^2} - v_\theta^2 \nabla^2 n_s = - \frac{1}{16\pi m_e v_\theta^2} \nabla \cdot \left\langle \nabla \left\{ - \left(\frac{\partial \delta\Phi}{\partial t}\right)^2 + \omega_p^2 \, (\delta\Phi)^2 \right\} \right\rangle \tag{52}$$

$$\frac{\partial^2 (n\vec{v})_s}{\partial t^2} - v_\theta^2 \nabla^2 (n\vec{v})_s = \frac{1}{16\pi m_e v_\theta^2} \frac{\partial}{\partial t} \left\langle \nabla \left\{ -2 \left(\frac{\partial \delta\Phi}{\partial t}\right)^2 + 2\omega_p^2 (\delta\Phi)^2 + v_\theta^2 (\nabla\delta\Phi)^2 \right\} \right\rangle . \tag{53}$$

A further approximation may be made. Thus, if an average over both space and time--which we shall indicate by $\langle\langle \; \rangle\rangle$ --is taken of $\delta\Phi$ times the linearized equation

$$\nabla^2 \delta\Phi - \frac{1}{v_\theta^2} \frac{\partial^2}{\partial t^2} \delta\Phi - \frac{\omega_p^2}{v_\theta^2} \delta\Phi \approx 0 \; , \tag{54}$$

there results

$$\left\langle\left\langle v_\theta^2 (\nabla\delta\Phi)^2 - \left(\frac{\partial \delta\Phi}{\partial t}\right)^2 + \omega_p^2 \, (\delta\Phi)^2 \right\rangle\right\rangle = 0 \; . \tag{55}$$

This suggests replacing

$$\left(\frac{\partial \delta\Phi}{\partial t}\right)^2 - \omega_p^2 \, (\delta\Phi)^2$$

in eqs.(52) and (53) by $v_\theta^2 (\nabla\delta\Phi)^2$, and the right-hand sides of eqs. (46) and (47) become

$$- \frac{1}{16\pi m_e v_\theta^2} \nabla \cdot \left\langle \nabla \left\{ - \left(\frac{\partial \delta\Phi}{\partial t}\right)^2 + \omega_p^2 \, (\delta\Phi)^2 \right\} \right\rangle \approx \frac{1}{2m_e} \nabla \cdot \left\langle \nabla \frac{(\nabla\delta\Phi)^2}{8\pi} \right\rangle \tag{56}$$

and

$$\frac{1}{16\pi m_e v_\theta^2} \frac{\partial}{\partial t} \left\langle \nabla \left\{ - 2\left(\frac{\partial \delta\Phi}{\partial t}\right)^2 + 2\omega_p^2 (\delta\Phi)^2 + v_\theta^2 (\nabla\delta\Phi)^2 \right\} \right\rangle$$

$$\approx - \frac{1}{2m_e} \frac{\partial}{\partial t} \left\langle \nabla \frac{(\nabla\delta\Phi)^2}{8\pi} \right\rangle \tag{57}$$

the latter being the expression which would have been obtained from the $\frac{\partial}{\partial t} [\delta n_e \nabla\delta\Phi]^{(s)}$ term in eq. (43) alone. The source terms are proportional to the gradient of the electrostatic energy density. With the total perturbed energy density given by

$$\frac{n_o m_e}{2} (\delta\vec{v}_e)^2 + \frac{m_e v_\theta^2}{2n_o} (\delta n_e)^2 + \frac{(\nabla\delta\Phi)^2}{8\pi}$$

when nonlinear effects are ignored in evaluating $\delta\vec{v}$ and δn_e --as may be seen by multiplying the linearized momentum equation

$$n_o m_e \frac{\partial \delta\vec{v}_e}{\partial t} \approx \frac{e}{m_e} \nabla\delta\Phi - \frac{v_\theta^2}{n_o} \nabla\delta n_e$$

by $n_o m_e \, \delta\vec{v}$ and adding this to $\frac{\nabla\delta\Phi}{4\pi}$ times eq. (44), we find with equipartition of the time averaged energy between particles and the electric field that in eqs. (56) and (57),

$$\left\langle \frac{\nabla(\nabla\delta\Phi)^2}{8\pi} \right\rangle$$

may be written

$$\left\langle \nabla \left[\frac{n_o m_e}{2} (\delta\vec{v}_e)^2 + \frac{m_e v_\theta^2}{2n_o} (\delta n_e)^2 \right] \right\rangle .$$

Denoting a representative phase velocity of a plasma oscillation mode by ω/k, the linearized continuity equation gives $\langle(\delta v_e)^2\rangle = (\omega/k)^2(\delta n_e/n_o)^2$, so that finally

$$\frac{1}{2m_e}\left\langle \nabla \frac{(\nabla\delta\Phi)^2}{8\pi}\right\rangle \approx n_o v_\theta^2 \left\langle \nabla \left(\frac{\delta n_e}{n_o}\right)^2 c_1\right\rangle \tag{58}$$

where

$$c_1 = \frac{1}{4}\left[1 + (\omega/kv_\theta)^2\right] \tag{59}$$

is a quantity--which we shall treat as a proportionality constant--of O (1).

III. DISCUSSION

The nonlinear relations of eqs. (22)-(25) have led in the present approximation to the wave equations

$$\frac{\partial^2}{\partial t^2}\,\delta n_e - v_\theta^2\nabla^2\delta n_e + \omega_p^2\left(1 + \frac{n_s}{n_o}\right)\delta n_e + \frac{2(n\vec{v})_s}{n_o}\cdot\nabla\frac{\partial\delta n_e}{\partial t} = 0\ , \qquad (34)$$

$$\frac{\partial^2 n_s}{\partial t^2} - v_\theta^2\nabla^2 n_s = c_1 n_o v_\theta^2\,\nabla\cdot\left\langle\nabla\left(\frac{\delta n_e}{n_o}\right)^2\right\rangle, \qquad (60)$$

$$\frac{\partial^2}{\partial t^2}\,(n\vec{v})_s - v_\theta^2\nabla^2(n\vec{v})_s = -\,c_1 n_o v_\theta^2\,\frac{\partial}{\partial t}\left\langle\nabla\left(\frac{\delta n_e}{n_o}\right)^2\right\rangle, \qquad (61)$$

and in the eikonal solution of eq. (34) when $\omega \approx \omega_p$ to the plasmon equation of motion

$$\frac{d\vec{V}}{dt} = -\,\frac{v_\theta^2}{2n_o}\,\nabla n_s + \frac{1}{n_o}\frac{\partial}{\partial t}\,(n\vec{v})_s - \frac{1}{n_o}\,\vec{V}\times\left(\nabla\times(n\vec{v})_s\right) \qquad \left[\frac{V}{v_o} \ll 1\right]. \qquad (62)$$

It is of interest to compare these equations which describe the generation and propagation of "interaction potentials" n_s and $(n\vec{v})_s$ and the motion of a plasmon in those potentials, with the electromagnetic field equations and the Klein-Gordon equation describing the motion of a spinless particle in that field:

$$\left(\frac{\hbar}{i}\frac{\partial}{\partial t} - e\varphi\right)^2 \Psi_\alpha - c^2\left(\frac{\hbar}{i}\nabla - \frac{e}{c}\vec{A}\right)^2 \Psi_\alpha - m_o^2 c^4\,\Psi_\alpha = 0\ , \qquad (63)$$

$$\left(\frac{\partial^2}{\partial t^2} - c^2\nabla^2\right)\varphi = 4\pi c^2\rho = 4\pi c^2\sum_{\alpha=1}^{N} e_\alpha\left[\frac{i\hbar}{2m_\alpha c^2}\left(\Psi_\alpha^*\frac{\partial\Psi_\alpha}{\partial t} - \frac{\partial\Psi_\alpha^*}{\partial t}\,\Psi_\alpha\right) - \frac{e_\alpha}{m_\alpha c^2}\,\varphi\,\Psi_\alpha^*\Psi_\alpha\right] \qquad (64)$$

$$\left(\frac{\partial^2}{\partial t^2} - c^2\nabla^2\right)\vec{A} = 4\pi c\vec{j} = 4\pi c \sum_{\alpha=1}^{N} e_\alpha\left[\frac{\hbar}{2m_\alpha i}\left(\Psi_\alpha^*\nabla\Psi_\alpha - (\nabla\Psi_\alpha^*)\Psi_\alpha\right) + \frac{e_\alpha}{m_\alpha c^2}\vec{A}\Psi_\alpha^*\Psi_\alpha\right] . \tag{65}$$

The eikonal solution of eq. (63) in the nonrelativistic limit is

$$\frac{d\vec{v}}{dt} = -\frac{e}{m_o}\nabla\varphi - \frac{e}{m_o c}\frac{\partial\vec{A}}{\partial t} + \frac{e\vec{v}\times}{m_o c}(\nabla\times\vec{A}) \quad . \tag{66}$$

Here $\hbar$ is Planck's constant, c is the speed of light, e, m_o and $\vec{v}$ are the charge, mass and velocity of the particle described by the wave function Ψ, φ and $\vec{A}$ are the scalar and vector electromagnetic potentials, while ρ and $\vec{j}$ are the charge and current densities. In eqs. (64) and (65) we have assumed N particles to be present and have evaluated the charge and current densities in terms of the wave functions of the N particles in the Hartree-Fock approximation.

There exist some similarities between these two sets of equations. It would appear that a normalized δn_e plays the role of a wave function, just as the perturbed background density and current, n_s and $(n\vec{v})_s$, play the roles of interaction scalar and vector potentials. The plasma frequency ω_p corresponds in eqs. (34), (60)-(62) to $(m_o c^2/\hbar)$ in eqs. (63)-(66). The thermal velocity v_θ corresponds to the speed of light in eqs. (63)-(66), so that the $\{V/v_\theta \ll 1,\ \omega \approx \omega_p\}$ condition under which the hydrodynamic treatment applies corresponds to the nonrelativistic treatment of eqs. (63)-(66). Ignoring φ and $\vec{A}$ in the right hand sides of eqs. (64) and (65), the source terms in this nonrelativistic limit reduce to

$$\frac{i\hbar}{2m_\alpha c^2}\left(\Psi_\alpha^*\frac{\partial\Psi_\alpha}{\partial t} - \frac{\partial\Psi_\alpha^*}{\partial t}\Psi_\alpha\right) \to \Psi_\alpha^*\Psi_\alpha \to \Psi^*\Psi \quad , \tag{67}$$

and

$$\frac{\hbar}{2m_\alpha i}\left(\Psi_\alpha^*\nabla\Psi_\alpha - (\nabla\Psi_\alpha^*)\Psi_\alpha\right) \to \vec{v}\Psi_\alpha^*\Psi_\alpha \to \vec{v}\Psi^*\Psi \quad , \tag{68}$$

where the subscript α may be dropped if nonoverlapping wave functions are assumed and $\Psi(\vec{x})$ is taken to represent the value of the wave function for whatever particle is present at $\vec{x}$ [so that $\int \Psi^*\Psi d^3x = N$]. The right-hand sides of eqs. (60) and (61) also depend on the square of the "wavefunction" δn_e, and may be regarded as being related through the group (particle) velocity $\vec{V}$, since if $(\delta n_e)^2$ has an extent L, then

$$\nabla \cdot \left\langle \nabla\left(\frac{\delta n_e}{n_o}\right)^2 \right\rangle \sim -\frac{1}{L^2}\left(\frac{\delta n_e}{n_o}\right)^2 \tag{69}$$

$$\frac{\partial}{\partial t}\left\langle \nabla\left(\frac{\delta n_e}{n_o}\right)^2 \right\rangle \sim -\vec{V}\frac{1}{L^2}\left(\frac{\delta n_e}{n_o}\right)^2 \quad , \tag{70}$$

the minus sign in eq. (69) arising if $(\delta n_e)^2$ is assumed to have a continuous derivative so that $\nabla^2[(\delta n_e)^2]$ is negative where $(\delta n_e)^2$ is largest, and the minus sign in eq. (70) arising on setting $\partial/\partial t \sim -\vec{V}\cdot\nabla$.

On the other hand, there is a marked difference between the behavior of the plasmon interaction potentials of eqs. (60) and (61) and the inverse-distance electromagnetic potentials. Thus, a localized disturbance $[\delta n_e/n_o]^2$ moving at a group velocity $\vec{u} = \vec{u}i_x$ creates potentials at a distant point $\vec{r} = x\vec{i}_x + y\vec{i}_y + z\vec{i}_z$ given approximately by

$$n_s \approx \left\{\int dx^3 \frac{c_1 n_o}{4\pi}\left(\frac{\delta n}{n_o}\right)^2\right\} \nabla^2\left(\frac{1}{R}\right) \quad , \tag{71}$$

$$(n\vec{v})_s \approx \left\{ \int dx^3 \frac{c_1 n_o}{4\pi} \left(\frac{\delta n}{n_o}\right)^2 \right\} (\vec{u}.\nabla)\ \nabla\left(\frac{1}{R}\right) , \tag{72}$$

$$R = \left[x^2 + \left(1 - \frac{u^2}{v_\theta^2}\right) (y^2+z^2) \right]^{\frac{1}{2}} , \tag{73}$$

where the disturbance is assumed localized about the point $x = y = z = 0$, where the time derivatives in eqs. (60) and (61) have been expressed in terms of $\vec{u}$ by $\partial/\partial t = - \vec{u}.\nabla$, and where the monopole and dipole contributions to the potentials have integrated to zero. We note that in this approximation a stationary plasmon does not give rise to a long range potential. The long range interaction comes from a moving plasmon, and in that case is of a quadrupole nature.

Equations (34), (60) and (61) are somewhat interesting even for the case of stationary plasmons, where no long range potential exists. For instance, in the case where $\partial/\partial t \langle \nabla (\delta n_e/n_o)^2 \rangle = 0$, a potential-well phenomenon still exists. In this case, eqs. (34) and (60) become

$$\frac{\partial^2 \delta n_e}{\partial t^2} - v_\theta^2 \nabla^2 \delta n_e + \omega_p^2 \left(1 + \frac{n_s}{n_o}\right) \delta n_e = 0 \tag{74}$$

$$\frac{n_s}{n_o} = - \left\langle c_1 \left(\frac{\delta n_e}{n_o}\right)^2 \right\rangle , \tag{75}$$

eq. (75) expressing the fact that the sum of the change in the background pressure and the pressure due to the electrostatic oscillations is practically zero. If a solution of the form $\delta n_e = \xi(\vec{x}) e^{i\omega t}$ is inserted in these equations, there results:

$$\nabla^2\xi + \left[\frac{\omega^2-\omega_p^2}{v_\theta^2} + \left(\frac{c_1\omega_p^2}{2n_o^2v_\theta^2}\right)\xi^2\right]\xi = 0 \quad . \tag{76}$$

In the one-dimensional case where $\nabla^2\xi$ is replaced by $\partial^2\xi/\partial x^2$ and where the frequency is less than the plasma frequency, $\omega < \omega_p$, eq. (76) has the solution:

$$|\,\xi(x)\,| = \frac{[4(\omega_p^2-\omega^2)n_o^2/c_1\omega_p^2]^{\frac{1}{2}}}{\cosh\left[c_o + \left[\frac{\omega_p^2-\omega^2}{v_\theta^2}\right]^{\frac{1}{2}}|\,x\,|\right]} \tag{77}$$

where c_o is given by the value ξ has at $x = 0$. Thus, when $\omega < \omega_p$ the nonlinear term in eq. (76) strengthens the possibility of localized (non-singular) plasma oscillations, i.e. even in this stationary case the non-linear term can behave like a potential well.

In fact, it would appear that there is a tendency for plasma oscillations to "condense". Again considering a one-dimensional example, if we insert a solution of the form

$$\delta n_e(x,t) = \sum_\omega \eta_\omega(x,t)\cos\left[\omega t - \int k_\omega(x,t)\,dx\right] \tag{78}$$

into eq. (74) and set the coefficients of the sine and cosine terms equal to zero separately, we find

$$2\omega\frac{\partial\eta_\omega}{\partial t} + 2v_\theta^2\frac{\partial\eta_\omega}{\partial x}k_\omega + v_\theta^2\eta_\omega\frac{\partial k_\omega}{\partial x} - \left\{\eta_\omega\frac{\partial^2}{\partial t^2}\int k_\omega(x,t)\,dx\right\} = 0 \tag{79}$$

and

$$\eta_\omega \left[-\omega^2 + \omega_p^2 \left(1 + \frac{n_s}{n_o}\right) + k_\omega^2 v_\theta^2\right] + \left\{\frac{\partial^2 \eta_\omega}{\partial t^2} - v_\theta^2 \nabla^2 \eta_\omega\right\} = 0 \quad . \tag{80}$$

Ignoring the terms in the curly brackets as being much smaller than the ω and $\vec{k}$ terms, eq. (79) may be written

$$\frac{\partial}{\partial t}\left(\eta_\omega^2\right) + \frac{\partial}{\partial x}\left[\left(\eta_\omega^2\right) \boldsymbol{v}_{g\omega}\right] = 0 \tag{81}$$

where

$$\boldsymbol{v}_{g\omega} = \frac{k_\omega v_\theta^2}{\omega} \; ; \tag{82}$$

and eq. (80) yields the dispersion relation

$$\omega^2 = \omega_p^2 \left(1 + \frac{n_s}{n_o}\right) + k_\omega^2 v_\theta^2 \quad . \tag{83}$$

When $\omega \approx \omega_p$, application of $dk/dt = - \partial\omega/\partial x$ to eq. (83) gives

$$\frac{d\boldsymbol{v}_{g\omega}}{dt} = \frac{d}{dt}\left(\frac{k_\omega v_\theta^2}{\omega}\right) \approx - \frac{v_\theta^2}{2n_o} \frac{\partial n_s}{\partial x} \quad . \tag{84}$$

Equation (81) may be regarded as a continuity equation for the density η_ω^2, and eq. (84) as the momentum equation. The quantity n_s may be expressed in terms of η_ω^2 by combining eqs. (75) and (78):

$$n_s \approx - \frac{c_1}{2n_o} \sum_\omega \eta_\omega^2 \quad . \tag{85}$$

Let us introduce an amplitude modulation of the plasma oscillations, i.e. write

$$\eta_\omega^2 = (\eta_\omega^2)_o + (\eta_\omega^2)_1 \tag{86}$$

$$\mathbf{v}_{g\omega} = (\mathbf{v}_{g\omega})_o + (\mathbf{v}_{g\omega})_1$$

where the zero subscript denotes quantities independent of time and space, and the quantities with the subscript 1 denote perturbations. Taking $(\eta_\omega^2)_1$ and $(\mathbf{v}_{g\omega})_1$, to have the form

$$(\eta_\omega^2)_1 = (n_\omega^2)_1 \, e^{i[\Omega t - Kx]} \tag{87}$$

$$(\mathbf{v}_{g\omega})_1 = (v_{g\omega})_1 \, e^{i[\Omega t - Kx]} \quad ,$$

where $\Omega \ll \omega$ and $K \ll k$; eqs. (81), (84) and (85) yield

$$\Omega(n_\omega^2)_1 - (\mathbf{v}_{g\omega})_o \, K \, (n_\omega^2)_1 - (\eta_\omega^2)_o \, K \, (v_{g\omega})_1 = 0 \tag{88}$$

$$\Omega(v_{g\omega})_1 = - \frac{c_1 v_\theta^2}{(2\eta_o)^2} K \sum_\omega (n_\omega^2)_1 \quad . \tag{89}$$

On combining these two equations, a dispersion relation is obtained for $\Omega(K)$:

$$1 = - \sum_\omega \left\{ \frac{(\eta_\omega^2)_o \, K^2 c_1 v_\theta^2}{\Omega \, (2n_o)^2 \left[\Omega - K \, (\mathbf{v}_{g\omega})_o \right]} \right\} \quad . \tag{90}$$

If $K(\mathbf{v}_{g\omega})_o$ is ignored compared with Ω, this gives

$$\Omega^2 = - K^2 v_\theta^2 \left(\frac{|n_s|_o}{2n_o} \right) \quad . \tag{91}$$

Since eq. (58) shows $|n_s|_o/2n_o$ to be of the order of the ratio of the electrostatic energy density $\mathcal{E}^2/8\pi$ to the thermal energy density $n_o m_e v_\theta^2$, eq. (91) indicates that the modulation increases in amplitude with a growth rate

$$i\Omega \sim Kv_\theta \left(\frac{\mathcal{E}^2}{8\pi n_o m_e v_\theta^2} \right)^{\frac{1}{2}} \quad . \tag{92}$$

This "condensation" tendency apparently results from the fact that $m_e n_s v_\theta^2$ decreases in the presence of an electrostatic pressure $\mathcal{E}^2/8\pi$, giving by eq. (84) a plasmon accelerating force directed toward regions where $\mathcal{E}^2/8\pi$ is already the largest. Nonlinear instabilities of this growth rate involving ion acoustic waves have also been found by Vedenov and Rudakov[12] and Harris.[13]

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Section 6

EFFECT OF A UNIVERSAL INSTABILITY ON AURORAL BOMBARDMENT

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I. INTRODUCTION

Plasma instabilities have frequently been mentioned as a possible mechanism for the dumping of charged particles from the Van Allen radiation belt[1] into the aurora. The low frequency instabilities most familiar in the study of laboratory plasmas[2,3,4] are unsatisfactory as an explanation for auroral bombardment; these familiar instabilities are electrostatic waves propagating perpendicular to the background magnetic field $\vec{B}_o$ and to the direction of the density gradient and should cause the plasma to move across the field lines rather than along B_o. However, there is one low frequency universal instability which is pure transverse;[5] this mode also propagates perpendicular to $\vec{B}_o$ but has an electric vector and current parallel to $\vec{B}_o$, and Chamberlain[6] has recently suggested that it might account for auroral dumping and for a number of other auroral characteristics, including[6] ray structure, north-south thinness of auroral formations, simultaneous electron-proton bombardment, angular and velocity distribution of auroral primaries.

Since the original theory was based on infinitesimal linear perturbations we now attempt to calculate the time evolution of this instability and to estimate the associated particle diffusion to see if the notion of diffusion along $\vec{B}_o$, suggested by linear theory is borne out by a non-linear calculation.

The approach will use the so-called "quasi-linear" theory[7,8], which takes account of non-linear effects in only one sense, i.e. the distortion

of the plasma distribution function due to the interaction with the waves. The justification of this approach and of the approximations generally made in employing it is an independent research program of major proportions,[9] but it is widely held that this is a correct way of deriving the original diffusion and distortion properties associated with the instability. In this approximation the particle velocity distribution function is written as a sum of two parts: a slowly varying $f_o(v,t)$ and a rapidly oscillating $f_k(v,t)$. The slow change in f_o is calculated from the averaged quadratic effects of the low-amplitude rapid oscillations. The non-linear coupling between different modes is not taken into account. In Section II we derive the linear instability, in Section III we perform the quasi-linear ritual and discuss our conclusions.

II. LINEAR STABILITY ANALYSIS

The equilibrium consists of a magnetic field

$$\left[\vec{B} = B_o \hat{n}_z \ (1 + \epsilon x)\right] , \tag{1}$$

which confines a plasma whose distribution function is

$$f_{oj} = n_o[1 - \epsilon'(x + V_y/\Omega)]_j \ \exp \ (- \alpha_j v^2)\left[\frac{\alpha_j}{\pi}\right]^{3/2} \tag{2}$$

where $\Omega_j = e_j B/m_j c$ is the cyclotron frequency and j refers to particle species. This equilibrium is chosen for simplicity, but reflects the essential features of the radiation belt, i.e. density gradient, magnetic field gradient, particle drifts ($\sim \nabla B/B$) and particle currents ($\sim \nabla n/n$). An earlier calculation[5] demonstrated that the linear stability results did not depend on the choice $\vec{B} = B_o \hat{n}_z \ (1 + \epsilon x)$, but were valid for general $\vec{B}_o = B_o(x)\hat{n}_z$ as long as B_o does not change by much over a wavelength or an ion Larmor radius. The equilibrium also satisfies the Vlasov equation

$$\frac{\partial f}{\partial t} + \vec{v}.\nabla f + \frac{e}{m}\left(\vec{E} + \frac{\vec{v}\times\vec{B}}{c}\right).\nabla_v f = 0 \tag{3}$$

$$\nabla\times\vec{B} = \frac{1}{c}\frac{\partial\vec{E}}{\partial t} + \frac{4\pi}{c}\Sigma \ e_j \int \vec{v} f_{oj} d^3v \tag{4}$$

$$\nabla.\vec{E} = \Sigma \ e_j \int f_{oj} d^3v \tag{5}$$

with $E_o = \partial f/\partial t = 0$.

We perturb the equilibrium with $\vec{E}_1 = E(y,t)\hat{n}_z$, $\vec{B} = \vec{B}_o + b(y,t)\hat{n}_x$, $f = f_o + f_1(y,v,t)$, where from Maxwell's eqs.

$$\frac{dE_z}{dy} = -\frac{1}{c}\frac{\partial b}{\partial t} \quad .$$

In linear theory we write $f_1 = f_k e^{i\omega_k t} e^{iky}$. Eq. (3) thus becomes

$$\frac{\partial f_k}{\partial t} + i\vec{k}.\vec{v}f_k + \frac{e}{m}\left(\frac{\vec{v}\times\vec{B}_o}{c}\right).\nabla_v f_k = -\frac{e}{m}\left[\vec{E} + \frac{\vec{v}\times\vec{b}}{c}\right]_k.\nabla_v f_o \tag{6}$$

which can be solved by the method of characteristics,[5] giving for each species

$$f_k = -\frac{eE_k}{m}\left[\frac{\partial}{\partial V_z} - \frac{kV_z}{\omega\Omega}\frac{\partial}{\partial x}\right] f_o\, J_o^2\left(kv_\perp/\Omega\right)\Big/ i(\omega + k\epsilon v_\perp^2/2\Omega) \tag{7}$$

where we have assumed that $\omega \ll \Omega = eB/mc$ and dropped terms

$$\sum_{\ell\neq 0} J_\ell^2/(\omega + \ell\Omega + k\epsilon v_\perp^2/2\Omega) \quad .$$

Substituting eq. (7) into the z-component of eq. (4) we obtain the dispersion relation for ω_k

$$k^2c^2 = 4\pi\sum_j \frac{e^2}{m}\int \frac{2V_z^2\left(\frac{\partial}{\partial V_z^2} - \frac{k}{2\omega\Omega}\frac{\partial}{\partial x}\right) f_o J_o^2\,(kv_\perp/\Omega)d^3v}{1 + k\epsilon v_\perp^2/2\Omega\omega} \quad . \tag{8}$$

The factor $\frac{e^2}{m}$ enhances the electron term, ignoring for the moment the ion contributions and setting $k\epsilon v_\perp^2/2\Omega\omega \ll 1$ Eq. (8) becomes $k^2c^2 = -\omega_p^2(1 + k\epsilon/2\alpha\Omega\omega)$, so that

$$\omega_k \sim -k\left(\frac{1}{n}\frac{dn}{dx}\Big/ 2\alpha\Omega\right)\Big/\left(1 + k^2c^2/\omega_p^2\right) \quad . \tag{9}$$

In solving eq. (8) for eq. (9) we have assumed wavelength larger than the electron gyroradius, to set

$$J_o^2\left(\frac{kv_{\perp e}}{\Omega}\right) = 1 \ .$$

We observe that since $\omega/k < 0$, the electron integral is not resonant. However the ion integral does have a resonance, at $v_\perp^2 = 2\Omega_i\omega/k\epsilon$, which contributes an imaginary part to the complete dispersion relation

$$\omega_k + i\gamma_k = \frac{-k|\epsilon'/2\alpha\Omega|_e}{1 + k^2c^2/\omega_{pe}^2}\left\{1 + \frac{|k|}{k}\frac{m}{M}\frac{\epsilon'}{\epsilon}\frac{J_o^2\left[\sqrt{k^2\frac{\epsilon'}{\epsilon}\frac{T_e}{T_i}\frac{1}{\alpha_i\Omega_i^2(1 + k^2c^2/\omega_p^2)}}\right]}{1 + k^2c^2/\omega_{pe}^2}\right.$$

$$\left.\exp^{-\frac{\epsilon'}{\epsilon}\frac{T_e}{T_i}\Big/\frac{1 + k^2c^2}{\omega_{pe}^2}}\right\} \ . \qquad (10)$$

Equations (8)-(10) contain the features which make a quasi-linear approach plausible. The instability comes from a resonance of particles, with waves, it grows most rapidly for a limited range of wave numbers k, and it leads to a "weak" turbulence i.e. $\gamma_k \ll \omega_k$.

III. QUASI-LINEAR EVOLUTION OF THE INSTABILITY

A Fourier analysis of Eq. (3), with $f = F_o(r,v) + f_1(r,v,t)$, $B = B_o(x) + b(r,t)$, $E = E(r,t)$, retaining non-linear terms, gives

$$\frac{\partial f_{1k}}{\partial t} + i\vec{k}\cdot\vec{v}f_{1k} + \frac{e}{m}\sum_q \left(\vec{E} + \frac{\vec{v}\times\vec{b}}{c}\right)_{k-q}\cdot\nabla_v f_q + \frac{e}{m}(\vec{v}\times\vec{B}_o)\cdot\nabla_v f_k = -\frac{e}{m}\left(\vec{E} + \frac{\vec{v}\times\vec{b}}{c}\right)_k\cdot\nabla_v F_o \quad . \tag{11}$$

In the quasi-linear approach we separate the distribution function into a slowly varying part $g = F_o + f_o(v,t)$ and a rapidly varying part $f_k, k \neq 0$. The slowly varying part of Eq. (11) is

$$\frac{\partial f_o}{\partial t} + \frac{e}{m}\sum_q \left(\vec{E} + \frac{\vec{v}\times\vec{b}}{c}\right)_{-q}\cdot\nabla_v f_q + \frac{e}{m}(\vec{v}\times\vec{B}_o)\cdot\nabla_v f_o = 0 \tag{12}$$

while the equation for the rapidly oscillating distribution f_k is

$$\frac{\partial f_k}{\partial t} + \frac{e}{m}(\vec{v}\times\vec{B}_o)\cdot\nabla_v f_k + i\vec{k}\cdot\vec{v}f_k = -\frac{e}{m}\left(\vec{E} + \frac{\vec{v}\times\vec{b}}{c}\right)_k\cdot\nabla_v g \tag{13}$$

where in Eq. (13) we have dropped the "mode coupling" term

$$\sum_{q\neq o}\left(\vec{E} + \frac{\vec{v}\times\vec{b}}{c}\right)_{k-q}\cdot\nabla_v f_q \quad ,$$

which in quasi-linear theory can be demonstrated[8] to contribute on a time scale longer than the initial growth of the perturbation. Eq. (13) is identical with the linear Eq. (6), and reflects the basis of the quasi-linear approach, i.e. we use linear theory to derive the rapidly varying part of the perturbation f_k and E_k, along with ω_k and γ_k, from the background distribution which exists at time t, and then use these values

of f_k etc. through Eq. (12) to find their effect on the slowly varying background distribution $g = F(v) + f_o(v,t)$.

Solving Eq. 13 as in linear theory we obtain (separating $\omega \to \omega - i\gamma$) for each species

$$f_k = -\frac{eE_k}{mM}\left(\frac{\partial}{\partial V_z} - \frac{kV_z}{\omega\Omega}\frac{\partial}{\partial x}\right)\frac{g(x,v,t)\ J_o^2\ (kv_\perp/\Omega)}{i(\omega + k\epsilon v_\perp^2/2\Omega - i\gamma)} \quad . \tag{14}$$

Substituting this f_k into Eq. (12) gives

$$\frac{\partial g}{\partial t} = \frac{e^2}{m^2}\sum_q{}' E_{-q}\left(\frac{\partial}{\partial V_z} - \frac{qV_z}{\omega\Omega}\frac{\partial}{\partial x}\right)\frac{J_o^2\ (kv_\perp/\Omega)}{i\left[\omega + q\epsilon v_\perp^2/2\Omega - i\gamma\right]} E_q\left(\frac{\partial}{\partial V_z} - \frac{qV_z}{\omega\Omega}\frac{\partial}{\partial x}\right)g \ . \tag{15}$$

The sum in Eq. (15) is from $q = -\infty$ to $q = \infty$, excluding $q = 0$, but since $\omega_q = -\omega_{-q}$ and $\gamma_q = \gamma_{-q}$, we can write it as

$$\frac{\partial g}{\partial t} = 8\frac{e^2}{m^2}\pi\sum_{q=o}^{\infty}{}'\left(\frac{\partial}{\partial V_z} - \frac{qV_z}{\omega\Omega}\frac{\partial}{\partial x}\right)\frac{2J_o^2(kv_\perp/\Omega)\ \gamma_q\mathcal{E}_q\ \frac{\omega^2}{q^2c^2}}{(\omega + q\epsilon v_\perp^2/2\Omega)^2 + \gamma^2}\left(\frac{\partial}{\partial V_z} - \frac{qV_z}{\omega\Omega}\frac{\partial}{\partial x}\right)g \tag{16}$$

with $\mathcal{E}_q = \frac{1}{8\pi}\left[|E_q|^2 + |B_q|^2\right] = \frac{E_q^2}{8\pi}\left(1 + \frac{k^2c^2}{\omega^2}\right)$ and $\frac{\partial\mathcal{E}_q}{\partial t} = 2\gamma_q\mathcal{E}_q$.

Now ω_q and q are related by Eq. (9), and Eq. (16) can be written

$$\frac{\partial g}{\partial t} = 8\pi\frac{e^2}{m^2}\sum_{q=0}^{\infty}{}'\left[\frac{\partial}{\partial V_z} - \frac{qV_z}{\omega\Omega}\frac{\partial}{\partial x}\right]2J_o^2\mathcal{E}_q\left[\frac{2\alpha\Omega}{q\epsilon}\right]\frac{\left(\gamma 2\frac{\alpha\Omega}{q\epsilon}\right)\frac{\omega^2}{q^2c^2}}{\left(\frac{\gamma 2\alpha\Omega}{q\epsilon}\right)^2 + \left(\alpha v_\perp^2 - \frac{\epsilon'/\epsilon}{1+q^2c^2/\omega_p^2}\right)^2}$$

$$\left(\frac{\partial}{\partial V_z} - \frac{qV_z}{\omega\Omega}\frac{\partial}{\partial x}\right)g \tag{17}$$

For all values of q, $2\alpha\Omega\gamma/q\epsilon < m/M \ll 1$ where m/M is the electron-ion mass ratio, so that Eq. (17) has a strong resonance behavior as a function of $v_\perp^2$ and q. From this linear theory we know that the growth rate γ is maximum for $\Gamma = (\epsilon'/\epsilon)/(1+q^2c^2/\omega_p^2) = 2$, decreasing exponentially for larger values and quadratically for smaller values of the parameter Γ. The perturbed electric energy $\mathcal{E}_q$ will be maximum for that value of $q = \hat{q} = \omega_p/c\ (\epsilon'/2\epsilon - 1)^{1/2}$, at least at times early enough so that mode coupling can be ignored. If we select the fastest growing mode $q = \hat{q}$ from the sum, and write

$$\frac{(2\alpha\Omega\gamma/q\epsilon)}{(2\alpha\Omega\gamma/q\epsilon)^2 + \left(\alpha v_\perp^2 - \dfrac{\epsilon'/\epsilon}{1 + q^2c^2/\omega_p^2}\right)^2} = \pi\delta\ (\alpha v_\perp^2 - \Gamma) \quad .$$

We use the dispersion relation to relate ω_q and q in Eq. (17), and limit the sum $\sum_q$ to values $q > \omega_p/c$. For smaller values of q the growth rate γ_q and the field energy $\mathcal{E}_q$ are exponentially small. With these restrictions Eq. (17) becomes (ω_p = the electron plasma frequency)

$$\frac{\partial g_i}{\partial t} = -\frac{8\pi e^2}{M^2} \sum_{q > \omega_p/c} \frac{J_o^2\ \epsilon'}{\alpha\Omega\epsilon c\omega_p} \left(\frac{\omega_p}{qc}\right)^5 \left(\frac{\partial}{\partial V_z} + \frac{2\alpha V_z q^2c^2}{\epsilon'\omega_p^2}\frac{\partial}{\partial x}\right)$$

$$\frac{\dfrac{2\alpha\Omega c}{\omega_p}\sqrt{\dfrac{2}{\epsilon'\epsilon}}\ \gamma_q\ \mathcal{E}_q\ \dfrac{1}{n}\dfrac{dn}{dx}}{\left[\dfrac{2\alpha\Omega c}{\omega_p}\sqrt{\dfrac{2}{\epsilon'\epsilon}}\ \gamma_q\right]^2 + \left(\alpha v_\perp^2 - \dfrac{\epsilon'}{\epsilon}\dfrac{\omega_p^2}{q^2c^2}\right)^2} \left(\frac{\partial}{\partial V_z} + \frac{2\alpha V_z q^2c^2}{\epsilon'\omega_p^2}\frac{\partial}{\partial x}\right) g \qquad (18)$$

$$\gamma_q = \frac{2\pi^2e^2\left(\frac{1}{n}\frac{dn}{dx}\right)}{M\alpha\Omega c\omega_p}\left(\frac{\epsilon'}{\epsilon}\right)^2\left(\frac{\omega_p}{qc}\right)^5 \int v_z \left(\frac{\partial}{\partial v_z} - \frac{2\alpha v_z q^2c^2}{\epsilon'\omega_p^2}\frac{\partial}{\partial x}\right) g\ J_o^2\ \delta\left(\alpha v_\perp^2 - \frac{\epsilon'}{\epsilon}\frac{\omega_p^2}{q^2c^2}\right) d^3v \qquad (19)$$

$$\frac{\partial \mathcal{E}_q}{\partial t} = 2\gamma_q \mathcal{E}_q \tag{20}$$

This is the set of equations to be solved for the early time development of the instability. The sum $\sum_q$ in Eq. (18) includes the mode energy $\mathcal{E}_q$, peaked at the q which gives maximum growth, and the resonant function $\Gamma/\Gamma^2 + (\alpha v_\perp^2 - \xi)^2$. The half width of the latter can be shown (using Eq. (19) for γ) to be $\Delta(\epsilon'/\epsilon\ \omega_p^2/q^2c^2) \sim m/M$, while the mode energy $\mathcal{E}_q$ is peaked with half width $\Delta(\epsilon'/\epsilon\ \omega_p^2/q^2c^2) \sim 1$. This smooth behavior of $\mathcal{E}_q$ allows the replacement of $\Gamma/\Gamma^2 + (\alpha v_\perp^2 - \xi)^2$ by $\pi\delta(\alpha v_\perp{}^2 - \xi)$, and also, along with the condition $\gamma \ll \omega$, appears to justify neglect of the mode coupling terms.[10] The energy transfer processes which must be considered in the set of Eqs. (18-20) include resonant and non-resonant particle energy change computed from $\int d^3v\ Mv^2\ \partial g/\partial t$, mode energy change, and changes in the energy associated with the diamagnetic currents and drifts; these drift energies drive the instability. The velocity variation $\partial/\partial v_z$ is smaller than the space variations

$$\frac{2\alpha v_z q^2 c^2}{\epsilon' \omega_p^2} \frac{\partial}{\partial x} \sim \frac{1}{\beta} \frac{\partial}{\partial v_z} \quad ;$$

thus quasi-linear "flattening" of the distribution is seen to be a flattening in coordinate space, and should be traced by a numerical integration of Eq. (18-20). Eq. (18) is resonant in $v_\perp^2$, and the location of the resonance is fixed by the original temperature $1/\alpha$. There is no resonance in v_z^2, and the energy transfer can allow a slow increase in T_z. This is the basis for our tentative expectation that the energy released by reducing the density gradient will appear as mode energy $\vec{E}_z$, and as an increase in the

energy of parallel oscillations; this upward drift in T_z has no effect on diffusion in closed or infinite systems, but can cause escape through the loss cone of a magnetic mirror system, as in the radiation belt. We hope that these cursory conclusions will be borne out by detailed calculations now underway.

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Section 7

FINITE-β RESISTIVE INSTABILITIES OF MAGNETOSPHERIC TAILS

by

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Finite-β Resistive Instabilities of Magnetospheric Tails†

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The decoupled-flow regions of resistive instabilities occurring in magnetospheric tails are sufficiently narrow that magnetic and collisional viscosities may be of importance. To determine the effects of these viscosities, the resistive instabilities studied in the hydromagnetic approximation by Furth, Killeen and Rosenbluth (1963) and Johnson, Greene and Coppi (1963), are studied with plasma equations based on the Chapman and Cowling expansion. Treating the ratio β of the plasma pressure to magnetic pressure to be of order unity, three cases are considered, two leading to tearing instabilities and the third giving a gravitational instability. It is found in all three cases that the hydromagnetic equations are applicable when the ion collision frequency is larger than the ion gyro-frequency. When the ion gyro-frequency exceeds the ion collision frequency, the results are qualitatively the same except that ion-ion collisions replace ion inertia in determining the growth rates and the size of the decoupled flow region. The conditions under which the Hall currents become important are defined.

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I.

INTRODUCTION

There has been some discussion recently on the effects that the annihilation and recombination of magnetic field lines have on the structure of magnetospheric tails [Piddington (1960, 1965a,b); Dungey (1961, 1962, 1963, 1965); Axford and Hines (1961, 1964); Alfvén (1963); Dessler (1964); Levy, Petschek and Siscoe (1964); Axford, Petschek and Siscoe (1965); Sonnerup (1965); Ness (1965); Dessler and Michel (1966a)]. The rate at which magnetic field lines merge could be of importance both in determining the stability of twisted filamentary features and neutral sheets in magnetospheric tails, and in determining whether or not planetary field lines join onto interplanetary lines to form open or closed magnetospheres. The discussions of the magnetic field annihilation resulting from finite resistivity are related to Dungey's demonstration that a current concentration can develop at a neutral point of a magnetic field in a resistive plasma [Dungey (1958)]. Sweet (1958) and Parker (1963) have described a steady state two-dimensional situation in which a plasma of resistivity η in the region of a neutral sheet flows along the sheet at the Alfvén speed V_A while flowing perpendicular to the sheet at the resistive diffusion speed. This leads to a neutral sheet thickness, a, which is equal to the breadth of the sheet, L, divided by the square root of a magnetic Reynold's number $\mathcal{R} = 4\pi L\eta^{-1}V_A^{-1}$, and to equilibrium fluid velocities perpendicular to the neutral sheet equal to the Alfvén speed divided by $\mathcal{R}^{\frac{1}{2}}$. Petschek (1964) has argued that in this two-dimensional situation, the fluid flow perpendicular to the neutral sheet does not have to be restricted to resistive diffusion over the entire

breadth L, but rather that over part of the breadth hydromagnetic flow normal to the sheet can be sustained by the normal magnetic field component which joins the regions of oppositely directed magnetic fields. Petschek's analysis gives equilibria in which fluid velocities along the sheet are of the order of the Alfvén speed, while flow velocities perpendicular to the sheet can have values between that given by Parker and a maximum u_m given by $u_m \approx V_A \, \ell n^{-1} [2\mathcal{R} u_m^2 \bar{V}_A^2]$.

Furth, Killeen and Rosenbluth (1963) and Johnson, Greene and Coppi (1963) have analyzed the stability of a neutral sheet in which the equilibrium fluid velocities perpendicular to the sheet may be ignored. They find instabilities with growth rates proportional to fractional powers of the resistive diffusion time $4\pi \, a^2/\eta$. These instabilities may therefore have some relevance to the equilibrium discussed by Sweet and Parker, and to those equilibria discussed by Petschek in which the perpendicular flow velocities are sufficiently limited by boundary conditions. Ness (1965) has suggested that these resistive instabilities may occur in the earth's magnetospheric tail.

In the high-conductivity limit, these resistive instability modes are characterized by two regions: a narrow central region where finite conductivity permits relative motions of field and fluid, and an outer region where field and fluid are coupled as in the infinite-conductivity case. On the one hand, the confinement of resistive effects to a narrow central region leads to simplification in the analysis, in that equilibrium quantities in this region may be approximated by linear expansions, and in that this region may be treated as planar--approximating the effect of geometric curvature here by a gravitational force. On the other hand, as

pointed out by Furth, Killeen and Rosenbluth [hereinafter referred to as FKR] and by Coppi (1964) the large gradients of the narrow central region suggest that effects such as "finite Larmor radius" not taken into account in their hydromagnetic analysis may be of importance. More explicitly, it is known that in a finite-temperature plasma, ion-ion collisions and finite Larmor radius result in an Ohm's law which does not relate the current to the Lorentz force through a scalar resistivity, and in a pressure tensor which does not give just a scalar pressure [Chapman and Cowling (1939)]. The effect of ion-ion collisions has been treated qualitatively in Appendix H of FKR, where it was shown that a scalar viscosity does not qualitatively change the resistive modes, but that it does change the growth rate and scales involved.

With the long mean free paths in magnetospheric tails, it might be expected that decoupled flow regions there would be very narrow, so that both Larmor radius and viscosity effects would be of importance. Accordingly, in the following we examine the resistive instabilities using plasma equations based on the Chapman and Cowling expansion. As the ratio β of plasma pressure to magnetic pressure might not be too small in the tails, we treat β to be of order unity. In section II, the problem is delineated and the basic equations presented. The approximation of an isotropic resistivity is introduced there, the effect of an anisotropic resistivity being discussed in Appendix A. In section III, the structure of the equations is discussed in relation to the two-region solution of FKR, and three cases of interest are considered. It is shown there that the effect of viscosity is important even at low temperatures, but that the finite Larmor radius terms in the pressure tensor and the additional Ohm's law

terms do not affect the growth rates appreciably until the validity of the Chapman and Cowling expansion itself becomes somewhat questionable. These correction terms are considered in Appendix B. In Section IV, the results are summarized and the validity of the analysis discussed.

II.

ASSUMPTIONS AND BASIC EQUATIONS

We consider an infinite plane current layer, choosing a coordinate system so that equilibrium quantities change in the y-direction only. The equilibrium magnetic field is taken to lie in the x-z plane,

$$\vec{B}^{o} = B_x^{o}(y)\,\vec{i}_x + B_z^{o}(y)\,\vec{i}_z \ , \tag{1}$$

and the effect of curvature or acceleration is represented by a constant gravitational field of magnitude g directed along the y-axis. Following the discussion in FKR's Appendix F of the stabilizing effect of thermal conductivity on instabilities driven by temperature gradients, we shall ignore all temperature gradients, taking any variation in the y-direction of the equilibrium plasma transport coefficients to result from variations in plasma density. Perturbed quantities are denoted by the superscript 1,

$$f^{1}(\vec{r},t) = f^{1}(y)e^{ikx + \omega t} \ , \tag{2}$$

where without loss of generality the x-axis has been chosen to lie in the direction of the wave vector $\vec{k}$.

The plasma equations based on the Chapman and Cowling expansion of the Boltzmann equation have been much discussed.* As calculated by Marshall (and corrected by Haas and Vaughn-Williams), the momentum, current density and heat flux equations have the forms [Thompson (1962)]:

$$\rho\frac{d\vec{v}}{dt} = \vec{j}\times\vec{B} - \nabla\cdot\overleftrightarrow{P} + \rho\vec{g} \ , \tag{3}$$

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* For a list of references, see the historical survey in B. B. Robinson and I. B. Bernstein (1962).

$$\vec{j} = \sigma_{\parallel}\vec{D}_{\parallel} + \sigma_{\perp}\vec{D}_{\perp} + \sigma_H\, \vec{b}\times\vec{D} + \Phi_{\parallel}\nabla_{\parallel}T + \Phi_{\perp}\nabla_{\perp}T + \Phi_H\, \vec{b}\times\nabla T\ , \tag{4}$$

and

$$\vec{q} = -\lambda_{\parallel}\nabla_{\parallel}T - \lambda_{\perp}\nabla_{\perp}T - \lambda_H\, \vec{b}\times\nabla T + \Psi_{\parallel}\vec{j}_{\parallel} + \Psi_{\perp}\vec{j}_{\perp} + \Psi_H\, \vec{b}\times\vec{j}\ . \tag{5}$$

Here, ρ, T, $\vec{v}$, $\vec{j}$, $\vec{q}$, $\vec{B}$ and $\vec{g}$ are the mass density, temperature, velocity, current density, heat flux, magnetic field and gravitational field, respectively. The vector $\vec{b}$ is the unit vector along the magnetic field, and the subscripts $\parallel$ and $\perp$ denote the vector components parallel and perpendicular to the magnetic field. The vector $\vec{D}$ is an effective electric field,

$$\vec{D} = \vec{E} + \vec{v}\times\vec{B} + \frac{1}{ne}\nabla p\ , \tag{6}$$

where $\vec{E}$ is the actual electric field, n is the total number density, e is the magnitude of the electron charge, and p is one-third the trace of the pressure tensor $\overleftrightarrow{P}$. In a right-handed coordinate system $\vec{a}$, $\vec{b}$, $\vec{c}$ (with $\vec{b}$ along the magnetic field), the pressure tensor has the form

$$\begin{aligned}
P_{aa} &= p - \frac{2\mu}{1 + r_+^2}\{S_{aa} + \tfrac{1}{2}r_+^2(S_{aa} + S_{cc}) - r_+S_{ac}\} \\
P_{bb} &= p - 2\mu S_{bb} \\
P_{cc} &= p - \frac{2\mu}{1 + r_+^2}\{S_{cc} + \tfrac{1}{2}r_+^2(S_{aa} + S_{cc}) + r_+S_{ac}\} \\
P_{ab} &= P_{ba} = -\frac{2\mu}{1 + \frac{1}{4}r_+^2}\{S_{ab} - \tfrac{1}{2}r_+S_{bc}\} \\
P_{bc} &= P_{cb} = -\frac{2\mu}{1 + \frac{1}{4}r_+^2}\{S_{bc} + \tfrac{1}{2}r_+S_{ab}\} \\
P_{ac} &= P_{ca} = \frac{2\mu}{1 + \frac{1}{4}r_+^2}\{S_{bc} + \tfrac{1}{2}r_+(S_{aa} - S_{cc})\}\ ,
\end{aligned} \tag{7}$$

with

$$S_{ij} = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} - \frac{1}{3}\delta_{ij}\,\mathrm{div}\,\vec{v}\right). \tag{8}$$

The coefficients in eqs. (4), (5) and (7) involve the collision frequencies

$$\nu_- = \frac{2}{3}(2\pi/m_-)^{\frac{1}{2}} ne^4 \log \Lambda/(KT)^{3/2} \quad \text{with} \quad \Lambda = 2(KT)^{3/2}/e^3(\pi n)^{\frac{1}{2}}$$

$$\nu_+ = \frac{2\sqrt{2}}{5}(m_-/m_+)^{\frac{1}{2}}\,\nu_- , \tag{9}$$

the gyro-frequencies

$$\Omega_- = eB/m_- \qquad \Omega_+ = eB/m_+ , \tag{10}$$

the ratios

$$r = \Omega_-/\nu_- \qquad r_+ = \Omega_+/\nu_+$$

$$M_- = m_-/(m_- + m_+) , \tag{11}$$

the unmodified transport coefficients

$$\sigma_o = ne^2/m_-\nu_- , \qquad \lambda_o = \sigma_o K^2 T/e^2$$

$$\Phi_o = \sigma_o/e , \qquad \Psi_o = KT/e \tag{12}$$

and the r-dependent factors

$$\bar{\Delta}_1^{-1} = r^4 + 6.28r^2 + 0.93$$

$$\bar{\Delta}_2^{-1} = r^4 + 16.20r^2 + 44.3$$

$$\bar{\Delta}_3^{-1} = r^2M_- + 9M_- + 3.39(M_-)^{\frac{1}{2}} + 0.32 \tag{13}$$

$$\bar{\Delta}_4^{-1} = r^2 + 3.48$$

$$\bar{\Delta}_5^{-1} = r^2 + 12.72 .$$

Here, m_- and m_+ are the electron and positive ion masses, and K is Boltzmann's constant. Explicitly, the transport coefficients are

$$\sigma_{\parallel} = \sigma_o(1.93/2) \qquad \Phi_{\parallel} = 0.777\ \Phi_o$$

$$\sigma_{\perp} = \sigma_o\tfrac{1}{2}(r^2 + 1.8)\Delta_1 \qquad \Phi_{\perp} = -\ \Phi_o\ 0.75(r^2 - 0.966)\Delta_1$$

$$\sigma_H = \sigma_o\tfrac{1}{2}r(r^2 + 4.4)\Delta_1\ , \qquad \Phi_H = -\ \Phi_o\ 2.15r\Delta_1\ ,$$

$$\lambda_{\parallel} = 1.02\ \lambda_o$$

$$\lambda_{\perp} = 1.25\ \lambda_o\ [(3M_- + 0.56M_-^{\frac{1}{2}})\Delta_3 + (5.43r^2 + 36.1)\Delta_2 - 3.56\Delta_5]$$

$$\lambda_H = 1.25\ \lambda_o\ [rM_-\Delta_3 - 9.28r\Delta_2 - r\Delta_5]\ ,$$

$$\Psi_{\parallel} = 3.30\ \Psi_o \tag{14}$$

$$\Psi_{\perp} = \Psi_o[2.5r^2 + 11.5]\Delta_4$$

$$\Psi_H = \Psi_o 1.5r\Delta_4\ ,$$

and

$$\mu = \frac{1}{4}\ nKT/\nu_+\ .$$

Equation (4) for the current density takes on a more familiar form when temperature gradients are ignored and the $r \to \infty$ limit is considered. For with $\Delta T = 0$, eq. (4) gives

$$\vec{D} = \frac{1}{\sigma_{\parallel}}\ \vec{j}_{\parallel} + \left(\frac{\sigma_{\perp}}{\sigma_{\perp}^2 + \sigma_H^2}\right)\vec{j}_{\perp} + \left(\frac{\sigma_H}{\sigma_{\perp}^2 + \sigma_H^2}\right)\vec{j}\times\vec{b}\ ; \tag{15}$$

and since as $r \to \infty$, the conductivities become

$$\sigma_{\parallel} \to \sigma_o\ \frac{1.93}{2}\ , \qquad \frac{\sigma_{\perp}}{\sigma_{\perp}^2 + \sigma_H^2} \to \frac{2}{\sigma_o}\ , \qquad \frac{\sigma_H}{\sigma_{\perp}^2 + \sigma_H^2} \to \frac{2r}{\sigma_o}\ , \tag{16}$$

equation (15) becomes

$$E + \vec{v}\times\vec{B} = \overleftrightarrow{\eta}\cdot\vec{j} - \frac{1}{ne}\left[\nabla p - 2\vec{j}\times\vec{B}\right] \tag{17}$$

with

$$\overleftrightarrow{\eta}\cdot\vec{j} = \left(\frac{2}{1.93\sigma_o}\right)\vec{j}_{\parallel} + \left(\frac{2}{\sigma_o}\right)\vec{j}_{\perp} \ . \tag{18}$$

As we shall see, for the conditions of interest the ratio of the temperature relaxation time to the resistive instability growth time is of the order of $(m_-/m_+)^{\frac{1}{2}}$ or less. Denoting by a the thickness of the equilibrium current layer and by ϵa the much smaller thickness of the decoupled flow, the ratio of times is of order ϵ for the tearing mode. Accordingly, in the present calculation ∇T is set equal to zero. In passing, we note that if $\vec{q}_{\parallel} \approx 0$, equation (5) states that $\nabla_{\parallel}T \approx (\Psi_{\parallel}/\lambda_{\parallel})\vec{j}_{\parallel}$: the calculations of the appendix and the heuristic considerations of FKR both indicate that the most important current in the resistive instability is $\vec{j}_{\parallel}$, and thus the effect of including $\nabla_{\parallel}T$ here would be to modify the value of the resistivity along the field lines. With $\nabla T = 0$, the scalar portion of the pressure tensor in equation (7) is proportional to the density

$$\frac{p^1}{p^o} = \frac{\rho^1}{\rho^o} \ . \tag{19}$$

Another simplification may be introduced in the analysis by setting

$$\overleftrightarrow{\eta} = \eta^o\,\overleftrightarrow{I} \tag{20}$$

in eq. (17), where $\overleftrightarrow{I}$ is the unit dyadic. In Appendix A, it is shown that the anisotropic resistivity of eq. (18) complicates the algebra but only slightly modifies the scalar resistivity results, the reason being that the important current in the instability is just $\vec{j}_{\parallel}$.

To summarize, then, the pressure tensor of eq. (7) is used in the momentum equation, eq. (3). The Ohm's law of eq. (17) is used with the scalar resistivity of eq. (20). In addition, we need the continuity equation

$$\frac{\partial\rho}{\partial t} + \nabla\cdot(\rho\vec{v}) = 0 \ , \tag{21}$$

eq. (19) relating the density ρ to the scalar portion of the pressure tensor p, and the three Maxwell equations

$$\begin{aligned} \nabla\times\vec{E} &= -\frac{\partial\vec{B}}{\partial t} \\ \nabla\times\vec{B} &= 4\pi\vec{j} \\ \nabla\cdot\vec{B} &= 0 \ , \end{aligned} \tag{22}$$

(ignoring the displacement current for the slow growth rates under study). Following FKR, we regard $\vec{B}$ as the basic electromagnetic variable, eliminating $\vec{j}$ by eq. (22) and eliminating $\vec{E}$ by taking the curl of Ohm's law and using eq. (22). Eliminating one component of $\vec{B}$ with $\nabla\cdot\vec{B} = 0$, the curl of Ohm's law then gives two independent equations for two components of $\vec{B}$. These two equations involve three components of the macroscopic velocity $\vec{v}$, and these may be obtained from the momentum equation. On linearizing about a stationary state, this leads to the following five coupled equations.

$$-\left[\omega\rho^{o} + \frac{d}{dy}\left(\frac{g\rho^{o}}{\omega}\right)\right]v_y - \frac{d}{dy}\left[\frac{1}{4\pi}\left(B_z^{o}B_z^{1} - \frac{B_x^{o}}{ik}\frac{dB_y^{1}}{dy}\right) + P_\perp^{1}\right] + \frac{ik}{4\pi}B_x^{o}B_y^{1} - \frac{\rho^{o}g}{\omega}\nabla\cdot\vec{v}$$
$$- (\nabla\cdot\delta\overleftrightarrow{P})_y = 0 \tag{23}$$

$$-\omega\rho^{o}v_z + \frac{B_y^{1}}{4\pi}\frac{dB_z^{o}}{dy} + \frac{ik}{4\pi}B_x^{o}B_z^{1} - ik\sin\Phi\cos\Phi\,(P_\parallel^{1}-P_\perp^{1}) - (\nabla\cdot\delta\overleftrightarrow{P})_z = 0 \tag{24}$$

$$- \omega\rho^o\left(ikv_y + \frac{1}{ik}\frac{d^2v_y}{dy^2}\right) + \frac{\omega\rho^o}{ik}\frac{d}{dy}(\nabla\cdot\vec{v}) - \frac{d}{dy}\left(\frac{igk\rho^o}{\omega}\right)v_y - \frac{d}{dy}\left[\frac{B_y^1}{4\pi}\frac{dB_x^o}{dy} -\right.$$

$$\left. - \frac{B_x^o}{4\pi}\frac{dB_y^1}{dy} - iksin^2\Phi(P_\parallel^1 - P_\perp^1)\right] - \frac{k^2}{4\pi}B_x^oB_y^1 - \frac{ik}{\omega}\rho^o g\ \nabla\cdot\vec{v} - \{\nabla\times(\nabla\cdot\delta\overleftrightarrow{P})\}_z = 0 \qquad (25)$$

$$\omega B_y^1 - ikB_x^ov_y - \frac{\eta^o}{4\pi}\left(\frac{d^2}{dy^2}B_y^1 - k^2B_y^1\right) - \frac{ik}{4\pi n_e^o e}\left(ikB_x^oB_z^1 + B_y^1\frac{dB_z^o}{dy}\right) = 0 \qquad (26)$$

$$\left(\frac{\eta^o k^2}{4\pi} + \omega\right)B_z^1 - ikB_x^ov_z + v_y\frac{dB_z^o}{dy} + B_z^o\ \nabla\cdot\vec{v} - \frac{d}{dy}\left(\frac{\eta^o}{4\pi}\frac{dB_z^1}{dy}\right) + \frac{1}{4\pi en_o^e\rho^o}$$

$$\times\left[\frac{d\rho^o}{dy}\left(-ikB_z^oB_z^1 + B_y^1\frac{dB_x^o}{dy}\right) + \rho^o\left\{-k^2B_x^oB_y^1 + \frac{d}{dy}\left(B_x^o\frac{dB_y^1}{dy} - B_y^1\frac{dB_x^o}{dy}\right)\right\} - \frac{ikB^o}{\omega}\right.$$

$$\left.\times\frac{dB^o}{dy}\left(v_y\frac{d\rho^o}{dy} + \rho^o\ \nabla\cdot\vec{v}\right)\right] = 0\ . \qquad (27)$$

Here,

$$B_x^o = B^o\sin\Phi \qquad\qquad B_z^o = B^o\cos\Phi\ , \qquad (28)$$

and $n_e^o = (n^o/2)$ is the equilibrium electron number density. In these equations the variable v_x has been replaced by

$$\nabla\cdot\vec{v} = ikv_x + \frac{dv_y}{dy} \qquad (29)$$

to display the effect of compressibility explicitly. The modifications in the pressure tensor to the scalar pressure are of three types, behaving in the $r \to \infty$ limit of interest as μ, (μ/r) and (μ/r^2); the remaining terms are represented by $P_\perp^1$ and $P_\parallel^1$, which in the $r \to \infty$ limit are given by

$$P_\perp^1 = -\mu\sin^2\Phi\frac{dv_y}{dy} - \frac{P^o}{\omega\rho^o}\frac{d\rho^o}{dy}v_y + \mu ik\sin\Phi\cos\Phi v_z - \left[\frac{P^o}{\omega} + \mu(\cos^2\Phi - \frac{2}{3})\right]\nabla\cdot\vec{v} \qquad (30)$$

and

$$P_{\parallel}^{1} - P_{\perp}^{1} = 3\mu\sin^2\Phi \frac{dv_y}{dy} - 3\mu ik\sin\Phi\cos\Phi\, v_z + \mu(\cos^2\Phi - 2\sin^2\Phi)\, \nabla.\vec{v} \,. \tag{31}$$

If only the largest terms of each type are retained in $\delta\overleftrightarrow{P}$, the following approximate expressions are obtained:

$$(\nabla.\delta\overleftrightarrow{P})_y = -\frac{3\mu\cos\Phi}{4r_+ik}\frac{d^3v_y}{dy^3} - \frac{3\mu\sin\Phi}{4r_+}\frac{d^2v_z}{dy^2} - \frac{3\mu\cos\Phi}{4r_+}\frac{d\Phi}{dy}\frac{dv_z}{dy}$$

$$+ \frac{3\mu\cos\Phi}{4r_+ik}\frac{d^2}{dy^2}(\nabla.\vec{v}) \tag{32}$$

$$(\nabla.\delta\overleftrightarrow{P})_z = \frac{27\mu\sin\Phi\cos\Phi}{16r_+^2ik}\frac{d^3v_y}{dy^3} + \frac{27\mu\cos^2\Phi}{16r_+^2ik}\frac{d\Phi}{dy}\frac{d^2v_y}{dy^2} - \frac{3\mu\sin\Phi}{4r_+}(2\cos^2\Phi-1)$$

$$\times \frac{d^2v_y}{dy^2} - \frac{3\mu\cos\Phi}{4r_+}(3\cos^2\Phi-1)\frac{d\Phi}{dy}\frac{dv_y}{dy} - \frac{9\mu\cos^2\Phi}{4r_+^2}\frac{d^2v_z}{dy^2}$$

$$+ \left[-6ik\frac{\mu}{r_+}\cos^2\Phi\sin\Phi\frac{d\Phi}{dy} + \frac{3}{2}ik\cos^3\Phi\frac{d}{dy}\left(\frac{\mu}{r_+}\right)\right]v_z$$

$$- \frac{27\mu\sin\Phi\cos\Phi}{16r_+^2ik}\frac{d^2}{dy^2}(\nabla.\vec{v}) - \frac{27\mu\cos^2\Phi}{16r_+^2ik}\frac{d\Phi}{dy}\frac{d}{dy}(\nabla.\vec{v})$$

$$+ \frac{3\mu\sin\Phi\cos^2\Phi}{2r_+}\frac{d}{dy}(\nabla.\vec{v}) + \frac{9\mu\cos^3\Phi}{4r_+}\frac{d\Phi}{dy}(\nabla.\vec{v}) \tag{33}$$

and

$$\{\nabla \times (\nabla.\delta\overleftrightarrow{P})\}_z = -\frac{9\mu\cos^2\Phi}{16r_+^2 ik}\frac{d^4v_y}{dy^4} + \left[\frac{3\mu}{4r_+}\sin\Phi(5\cos^2\Phi-1)\frac{d\Phi}{dy} + \frac{3}{4}\cos\Phi\right.$$

$$\times\left.(1+\cos^2\Phi)\frac{d}{dy}\left(\frac{\mu}{r_+}\right)\right]\frac{d^2v_y}{dy^2} + \frac{3\mu\cos\Phi}{4r_+}(5\cos^2\Phi-1)$$

$$\times\left(\frac{d\Phi}{dy}\right)^2\frac{dv_y}{dy} + \frac{27\mu\sin\Phi\cos\Phi}{16r_+^2}\frac{d^3v_z}{dy^3} + \frac{27\mu\cos^2\Phi}{8r_+^2}\frac{d\Phi}{dy}\frac{d^2v_z}{dy^2}$$

$$+\frac{3\mu ik\sin\Phi}{4r_+}(2\cos^2\Phi-1)\frac{d^2v_z}{dy^2} + \frac{9\mu\cos^2\Phi}{16r_+^2 ik}\frac{d^3}{dy^3}(\nabla.\vec{v})$$

$$+\frac{3\mu\cos\Phi}{4r_+}\frac{d^2}{dy^2}(\nabla.\vec{v}). \qquad (34)$$

In arriving at these approximate $\delta\overleftrightarrow{P}$ expressions, we have referred to the FKR result of a narrow decoupled-flow region in which the perturbation approximately follows along a field line, and thus have regarded $\frac{d}{dy}$ acting on a velocity as introducing a factor $1/\epsilon$, and have taken $\sin\Phi$ to be of order ϵ.

By way of interpretation, eqs. (34) and (24) are the y and z components of the momentum equation, respectively, and eqs. (26) and (27) are the y and z components of the curl of Ohm's law. Equation (25) is the z component of the curl of the momentum equation; using this equation instead of the x component of the momentum equation removes the scalar portion of the pressure and puts things in a form resembling the FKR equation. In the incompressible hydromagnetic treatment of FKR, eqs. (25) and (26) were the only equations needed as they involved only B_y^1 and v_y. As will become

clearer later, eq. (23) says that the sum of the magnetic and perpendicular particle pressure remains approximately constant in a perturbation; it may be regarded as an equation for $\nabla.\vec{v}$. Equation (24) states that for a given driving current associated with the shear field B_x^o, the velocity along the unsheared field component B_z^o is limited by viscosity. Equation (25) defines the size of the decoupled flow region, again through the viscosity; and eq. (26) in conjunction with (25) gives the growth rate. Finally, eq. (27) states that for a given current and ion velocity, the electron velocity must adjust itself so that the current is related through the resistivity to the electric field in the electron reference frame.

In dimensionless form, eqs. (23)-(34) become

$$\left(\frac{p\tilde{\rho}}{S^2} - \frac{G\tilde{\rho}}{p}\right) W - (H\chi)' + (F\Psi')' - \alpha^2 F\Psi - \frac{G\tilde{\rho}^2}{p\rho} \operatorname{div} \vec{v}$$

$$- i\alpha\beta \left[\frac{p^o}{3i\alpha p} \left\{ \frac{W\tilde{\rho}'}{\rho} + \operatorname{div} \vec{v} \right\} + \frac{A\tilde{p}^o}{\rho\eta} \left\{ \frac{F^2 W'}{i\alpha I^2} - \frac{FHZ}{I^2} + \frac{1}{i\alpha}\left(- \frac{2}{3} + \frac{H^2}{I^2} \right) \operatorname{div} \vec{v} \right\} \right]'$$

$$+ \frac{3i\beta\tilde{p}^o A}{4r_+ I\alpha\rho\eta} \left[HW''' + i\alpha FZ'' + i\alpha I\left(\frac{F}{I}\right)' Z' - H(\operatorname{div} \vec{v})'' \right] = 0 \qquad (35)$$

$$\frac{p\tilde{\rho}Z}{S^2 i\alpha} + \Psi H' + F\chi - i\alpha\beta\,\frac{FH\rho^{o}A}{I^4\tilde{\rho}\tilde{\eta}}\left[-\frac{3F^2}{i\alpha}W' + 3FHZ - \frac{H^2}{i\alpha}\,\mathrm{div}\,\vec{v}\right]$$

$$-\frac{27H\beta\rho^{o}A}{16I^2\alpha^2 r_+^2\tilde{\rho}\tilde{\eta}}\left[FW''' + I\left(\frac{F}{I}\right)'W'' + \frac{4FIr_+\alpha}{9Hi}\left(\frac{2H^2}{I^2} - 1\right)W'' + \frac{4I^2\alpha r_+}{9iH}\right.$$

$$\times\left(\frac{F}{I}\right)'\left(\frac{3H^2}{I^2} - 1\right)W' + \frac{4H\alpha}{3i}Z'' + \frac{32}{9}Fr_+\alpha^2\left(\frac{F}{I}\right)'Z - \frac{8H^2\alpha^2 r_+^2}{9I\mu}\left(\frac{\mu}{r_+}\right)'Z$$

$$\left. - F(\mathrm{div}\,\vec{v})'' - I\left(\frac{F}{I}\right)'(\mathrm{div}\,\vec{v})' - \frac{4Hr_+\alpha}{3i}\left(\frac{F}{I}\right)'\mathrm{div}\,\vec{v} - \frac{8FHr_+\alpha}{9Ii}(\mathrm{div}\,\vec{v})'\right] = 0 \qquad (36)$$

$$\left(\frac{p\tilde{\rho}}{S^2} - \frac{G\tilde{\rho}}{p}\right)W - \frac{p\tilde{\rho}W''}{\alpha^2 S^2} + \frac{p\tilde{\rho}}{\alpha^2 S^2}(\mathrm{div}\,\vec{v})' - (\Psi F')' + (F\Psi')' - \alpha^2 F\Psi$$

$$-\frac{G\tilde{\rho}^2}{p\tilde{\rho}'}\,\mathrm{div}\,\vec{v} + \left[\frac{i\alpha\beta F^2\rho^{o}A}{I^4\tilde{\rho}\tilde{\eta}}\left(-\frac{3F^2}{i\alpha}W' + 3FHZ - \frac{H^2}{i\alpha}\,\mathrm{div}\,\vec{v}\right)\right]'$$

$$-\frac{3H^2\beta\rho^{o}A}{4I^2\alpha^2 r_+^2\tilde{\rho}\tilde{\eta}}\left[-\frac{3}{4}W'''' + \frac{I^2\alpha r_+}{iH^2}\left(\frac{5H^2}{I^2} - 1\right)\left(\frac{H}{I}\right)'W'' - \frac{I\alpha r_+^2}{Hi\mu}\left(\frac{H^2}{I^2} + 1\right)\right.$$

$$\times\left(\frac{\mu}{r_+}\right)'W'' + iIH\alpha r_+(\Phi')^2\left(\frac{5H^2}{I^2} - 1\right)W' - \frac{9F\alpha}{4Hi}Z''' - \frac{9I\alpha}{2Hi}\left(\frac{F}{I}\right)'Z''$$

$$\left. - \frac{FI\alpha^2 r_+}{H^2}\left(\frac{2H^2}{I^2} - 1\right)Z'' + \frac{3}{4}(\mathrm{div}\,\vec{v})''' - \frac{I\alpha r_+}{Hi}(\mathrm{div}\,\vec{v})''\right] = 0 \qquad (37)$$

$$\Psi(p + \tilde{\eta}\alpha^2) - \tilde{\eta}\Psi'' + FW - \frac{i\alpha r}{\rho}(\Psi H' + F\chi) = 0 \qquad (38)$$

and

$$\left(-i\tilde{\eta}\alpha + \frac{p}{i\alpha}\right)\chi + FZ - \frac{WH'}{i\alpha} - \frac{H}{i\alpha}\,\mathrm{div}\,\vec{v} - \frac{1}{i\alpha}(\tilde{\eta}\chi')' - \frac{\tilde{\rho}'r}{(\tilde{\rho})^2}(\chi H - \Psi F')$$

$$+ \frac{r}{\rho}\left[-\alpha^2 F\Psi + (F\Psi' - \Psi F')'\right] + \frac{rII'}{(\tilde{\rho})^2 p}(W\tilde{\rho}' + \tilde{\rho}\,\mathrm{div}\,\vec{v}) = 0\,, \qquad (39)$$

where

$$\Psi = B_y'/B$$

$$\chi = i\alpha B_z'/B$$

$$W = -ik\tau_R v_y$$

$$Z = -\, ik\tau_R v_z$$

$$\mathrm{div}\,\vec{v} = -i\alpha\tau_R\ \nabla\cdot\vec{v}$$

$$F = B_x^o/B$$

$$H = B_z^o/B$$

$$I = B^o/B$$

$$\tilde{\rho} = \rho^o/\langle\rho\rangle$$

$$\tilde{\eta} = \eta^o/\langle\eta\rangle$$

$$p^o = 3p^o/\langle P\rangle$$

$$G = -g\tau_H^2\,\frac{1}{\rho^o}\,\frac{\partial\rho^o}{\partial y}$$

$$\beta = 4\pi\langle P\rangle/B^2$$

$$A = \frac{5}{18\sqrt{2}}\left(\frac{m_+}{m_-}\right)^{\frac{1}{2}}\frac{c^2}{a^2\langle\omega_p^2\rangle}$$

$$\alpha = ka$$

$$\tau_R = 4\pi a^2/\langle\eta\rangle$$

$$\tau_H = a(4\pi\langle\rho\rangle)^{\frac{1}{2}}/B$$

$$S = \tau_R/\tau_H$$

$$p = \omega\tau_R$$

with c being the speed of light, and μ, $\mathbf{r}$, r_+, m_+ and m_- as defined earlier. The primes denote differentiation with respect to a dimensionless variable $u = y/a$, where a is a measure of the thickness of the current layer. The quantities B, $\langle\eta\rangle$, $\langle\rho\rangle$, $\langle P\rangle$ and $\langle\omega_p^2\rangle$ are measures of the field strength, resistivity, mass density, pressure and plasma frequency, respectively.

III.

THE REGION OF DECOUPLED FLOW

We wish now to discuss the structure of eqs. (35)-(39) in relation to the hydromagnetic results. FKR demonstrated that in the $S \to \infty$ limit the hydromagnetic equations gave two regions, an outer region where the fluid was frozen to the field and a narrow central region near $F = 0$ where the fluid and field were decoupled: their heuristic derivation of this result indicates that the situation is not changed using the plasma equations. Thus, we shall follow the same general procedure as in FKR; i.e., the solution in the outer region will be used only to give a value for

$$\Delta' = \frac{\Psi_2'}{\Psi_2} - \frac{\Psi_1'}{\Psi_1} \quad , \tag{40}$$

the difference between the logarithmic derivative of Ψ at the right and left boundaries of the inner region. From FKR's eqs. (23) and (26), the outer solution gives

$$\Delta' = -2\alpha + 0\left(\frac{1}{\alpha}\right) \qquad \text{as} \qquad \alpha \to \infty \qquad \text{(FKR 23)}$$

$$\Delta' = \frac{1}{\alpha}(F')^2\left(\frac{1}{F_{-\infty}^2} + \frac{1}{F_{\infty}^2}\right) \qquad \text{as} \qquad \alpha \to 0 \quad , \qquad \text{(FKR 26)}$$

where the subscripts $\pm\infty$ on F indicate the values of F as $y \to \pm\infty$, and the F' refers to the value of F' at $F = 0$. The problem, then, consists of solving eqs. (35)-(39) in a small region about the point $F = 0$, requiring that this solution gives the Δ' determined from the outer region and gives quantities which will be well-behaved in the outer region.

In the small inner region the equilibrium quantities may be approximated by linear expansions, e.g. $F = F'\mu$, where for convenience $\mu = 0$ is chosen at $F = 0$. Defining a new independent variable

$$\theta = \mu/\epsilon \quad , \tag{41}$$

eqs. (35)-(39) then take on the form

$$\frac{F'\epsilon}{\tilde{\eta}}\left(p - \frac{H' i\alpha r}{\tilde{\rho}}\right)\Psi\theta + \frac{F'}{\epsilon}\frac{d\Psi}{d\theta} - \frac{i\alpha r\epsilon^2}{\tilde{\eta}\tilde{\rho}}(F')^2\chi\theta^2 - \frac{H}{\epsilon}\frac{d\chi}{d\theta} + \left[\frac{p\tilde{\rho}}{S^2} - \frac{G\tilde{\rho}}{p} + \frac{(F'\epsilon)^2}{\tilde{\eta}}\theta^2\right] W$$

$$- \left[\frac{\beta\rho^o\tilde{\rho}'}{3p\epsilon\tilde{\rho}} + \frac{32\alpha\epsilon^6}{9\tilde{\eta}H^2} r_+^2(F')^4\,\theta\right]\frac{dW}{d\theta} - \frac{16\alpha\epsilon^6}{9\tilde{\eta}H^2} r_+^2(F')^4\,\theta^2\frac{d^2W}{d\theta^2}$$

$$+ \frac{4i\epsilon^3 I\alpha}{3\tilde{\eta}H} r_+(F')^2\frac{d^3W}{d\theta^3} - \frac{G\tilde{\rho}^2}{\tilde{\rho}'p}\,\mathrm{div}\,\vec{v} - \left[\frac{\beta\rho^o}{3p\epsilon} + \left(\frac{H^2}{I^2} - \frac{2}{3}\right)\frac{16\alpha\epsilon^5 I^2}{9\tilde{\eta}H^2} r_+^2(F')^2\right]$$

$$\times\frac{d}{d\theta}(\mathrm{div}\,\vec{v}) - \frac{4i\epsilon^4 I\alpha}{3\tilde{\eta}H} r_+(F')^2\frac{d^2}{d\theta^2}(\mathrm{div}\,\vec{v}) + \frac{i16\alpha^2\epsilon^6 r_+^2(F')^3}{9\tilde{\eta}H} Z$$

$$+ \left[-\frac{4\alpha^2\epsilon^5 I^2}{3\tilde{\eta}H^2} r_+(F')^2\left(\frac{F}{I}\right)' + \frac{i16\alpha^2\epsilon^6}{9\tilde{\eta}H} r_+^2(F')^3\,\theta\right]\frac{dz}{d\theta}$$

$$- \frac{4\alpha^2\epsilon^5 I}{3\tilde{\eta}H^2} r_+(F')^3\,\theta\frac{d^2Z}{d\theta^2} = 0 \tag{42}$$

$$H'\Psi + F'\epsilon\theta\chi + \left[\frac{16\alpha^2(F')^5 r_+^2\epsilon^8\theta^3}{3\tilde{\eta}HI^2} - \frac{4\epsilon^5(F')^2 I^2\alpha}{3H^2\tilde{\eta}i} r_+ \left(\frac{F}{I}\right)' \left(\frac{3H^2}{I^2} - 1\right)\right] \frac{dW}{d\theta}$$

$$+ \left[-\frac{3\epsilon^4 I(F')^2}{H\tilde{\eta}} \left(\frac{F}{I}\right)' - \frac{4\epsilon^5(F')^3}{3H^2\tilde{\eta}i} Ir_+\alpha \left(\frac{2H^2}{I^2} - 1\right)\theta\right] \frac{d^2W}{d\theta^2} - \frac{3\epsilon^4(F')^3}{H\tilde{\eta}}\theta\frac{d^3W}{d\theta^3}$$

$$+ \left[\frac{4\epsilon^6(F')^2 r_+\alpha}{i\tilde{\eta}} \left(\frac{F}{I}\right)' + \frac{16\alpha^2(F')^3}{9\tilde{\eta}I^2} r_+^2\epsilon^7 H\theta\right] \operatorname{div}\vec{v}$$

$$+ \left[\frac{3\epsilon^5 I(F')^2}{H\tilde{\eta}} \left(\frac{F}{I}\right)' + \frac{8\epsilon^6(F')^3}{3\tilde{\eta}Ii} r_+\alpha\theta\right] \frac{d}{d\theta}(\operatorname{div}\vec{v}) + \frac{3\epsilon^5(F')^3}{H\tilde{\eta}}\theta\frac{d^2}{d\theta^2}(\operatorname{div}\vec{v})$$

$$+ \left[\frac{p\tilde{\rho}}{S^2 i\alpha} - \frac{16i\alpha^3(F')^4\epsilon^8}{3\tilde{\eta}I^2} r_+^2\theta^2 - \frac{32(F')^3\epsilon^7}{3H\tilde{\eta}} r_+\alpha^2 \left(\frac{F}{I}\right)'\theta\right.$$

$$\left. + \frac{8\epsilon^6(F')^2 H\alpha^2}{3\tilde{\eta}I\mu} r_+^2 \left(\frac{\mu}{r_+}\right)'\right] Z - \frac{4\epsilon^4(F')^2\alpha}{i\tilde{\eta}} \frac{d^2Z}{d\theta^2} = 0 \qquad (43)$$

$$\left[-\frac{\tilde{\eta}F''}{(F'\epsilon)^2} + \frac{\theta}{F'\epsilon}\left(p - \frac{i\alpha H'}{\tilde{\rho}} r\right)\right]\Psi - \frac{i\alpha r}{\tilde{\rho}}\theta^2\chi + \left[\frac{\tilde{\eta}\tilde{\rho}}{(F'\epsilon)^2}\left(\frac{p}{S^2} - \frac{G}{p}\right) + \theta^2\right] W$$

$$+ \left[\frac{16\alpha}{3i} r_+(\Phi')^2\epsilon^3 I^2 - \frac{64(F')^4\alpha^2}{3I^2H^2} r_+^2\epsilon^6\theta^3\right]\frac{dW}{d\theta} + \left[-\frac{p\tilde{\rho}\tilde{\eta}}{\alpha^2 S^2(F')^2\epsilon^4} - \frac{16(F')^4\alpha^2 r_+^2\epsilon^6}{3I^2H^2}\theta^4\right.$$

$$\left. + \frac{4I^2\alpha r_+ F'\Phi'\epsilon^3}{3iH^2}\left(\frac{5H^2}{I^2} - 1\right)\theta + \frac{\alpha r_+^2\epsilon^2}{3i\mu}\left(1 + \frac{H^2}{I^2}\right)\left(\frac{5H^2}{I^2} - 1\right)\left(\frac{\mu}{r_+}\right)'\right]\frac{d^2W}{d\theta^2}$$

$$+ \frac{d^4W}{d\theta^4} + \left[-\frac{G\tilde{\rho}^2\tilde{\eta}}{p\tilde{\rho}(F'\epsilon)^2} - \frac{32\epsilon^5(F')^2\alpha^2 r_+^2}{9I^2}\theta\right]\operatorname{div}\vec{v} + \left[\frac{p\tilde{\rho}\tilde{\eta}}{\alpha^2 S^2(F')^2\epsilon^3} - \frac{16\epsilon^5(F')^2\alpha^2 r_+^2}{9I^2}\theta^2\right]$$

$$\times\frac{d}{d\theta}(\operatorname{div}\vec{v}) + \frac{4\epsilon^2 I\alpha r_+}{3iH}\frac{d^2}{d\theta^2}(\operatorname{div}\vec{v}) - \epsilon\frac{d^3}{d\theta^3}(\operatorname{div}\vec{v}) + \frac{i\alpha^3 r_+^2(F')^3 16\epsilon^6}{I^2H}\theta^2 Z \qquad (44)$$

$$+ \frac{i\alpha^3(F')^3\epsilon^6 16}{3I^2H} r_+^2\theta^3\frac{dz}{d\theta} + \left[\frac{6I\alpha\epsilon^2}{Hi}\left(\frac{F}{I}\right)' + \frac{4F'I\alpha^2 r_+\epsilon^3}{3H^2}\left(\frac{2H^2}{I^2} - 1\right)\theta\right]\frac{dZ^2}{d\theta^2} + \frac{3F'\alpha\epsilon^2}{Hi}\theta\frac{d^3Z}{d\theta^3} = 0$$

$$- \frac{\tilde{\eta}}{\epsilon^2} \frac{d^2\Psi}{d\theta^2} + \left[p + \tilde{\eta}\alpha^2 - \frac{i\alpha H' r}{\tilde{\rho}}\right]\Psi + F'\epsilon\theta W - \frac{i\alpha\epsilon F'}{\tilde{\rho}} r\theta\chi = 0 \tag{45}$$

$$\left[\frac{r}{\tilde{\rho}}\left(\frac{\tilde{\rho}'F'}{\tilde{\rho}} - F''\right) + \frac{rF'\epsilon}{\tilde{\rho}\tilde{\eta}}\left(p - \frac{H' i\alpha}{\tilde{\rho}} r\right)\theta\right]\Psi + \left[- i\tilde{\eta}\alpha + \frac{p}{i\alpha} - \frac{\tilde{\rho}' rH}{\tilde{\rho}^2} - \frac{i\alpha\epsilon^2 (F')^2 r^2\theta^2}{\tilde{\rho}^2\tilde{\eta}}\right]\chi$$

$$- \frac{\tilde{\eta}}{i\alpha\epsilon^2} \frac{d^2\chi}{d\theta^2} + \left[- \frac{H'}{i\alpha} + \frac{\tilde{\rho}' rII'}{\tilde{\rho}^2 p} + \frac{(F')^2\epsilon^2 r\theta^2}{\tilde{\rho}\tilde{\eta}}\right] W + \left[- \frac{H'}{i\alpha} + \frac{II' r}{p\tilde{\rho}}\right] \operatorname{div} \vec{v} + F'\epsilon\theta Z = 0 \tag{46}$$

where

$$\epsilon^6 = \frac{5}{32\sqrt{2}} \left(\frac{m_+}{m_-}\right)^{\frac{1}{2}} \frac{H^2\beta\rho^o}{I^2\alpha^2 (F')^2\, \tilde{\rho}\, S^2} \quad . \tag{47}$$

The equations are rather formidable as they stand. To make further progress, the nonhydromagnetic terms will be regarded as perturbations (except for the viscosity terms which, when $r_+ \gg 1$, must be used to define the scale of the region). Specifically, we regard the $r\epsilon^\gamma$ and $r_+\epsilon^\gamma = (m_-/m_+)^{\frac{1}{2}} r\epsilon^\gamma$ appearing in eqs. (42)-(46) as small quantities, and consider the effect of an iterative solution on the Δ' calculated with these $r\epsilon^\gamma$ equal to zero. Increasing the size of the perturbation terms corresponds to increasing the temperature. Apparently, such a procedure should give results dependent on the $r\epsilon^\gamma = 0$ ordering of quantities. We consider three cases:

(1) $H' = O(1)$, $G = 0$

(2) $H' = O(1)$, $G = O(1)$

(3) $H' = 0$, $G = 0$

the $r\epsilon^\gamma = 0$ relations between Ψ, χ, Z, W and $\operatorname{div} \vec{v}$ being different in each of these three. The first and third cases describe the tearing modes, occurring where the pressure is maximum in the third case (since the

equilibrium equation is $- 1/3\,\rho_0' - II'/\beta - (\tilde{\rho})^2\, G/\tilde{\rho}'\beta = 0)$; and the second case describes the gravitational mode.

A rough estimate of some relative magnitudes in the small $r\epsilon^{\gamma}$ limit may be had by examining the frozen-field condition

$$-\omega\vec{B}^1 + \nabla\times(\vec{v}\times\vec{B}^0) = 0 \quad , \tag{48}$$

as this must be approached toward the boundaries of the inner region. In component form, this is

$$p\Psi + WF = 0 \tag{49}$$

$$- p\chi - i\alpha ZF + WH' + H \operatorname{div} \vec{v} = 0 \quad .$$

When $r\epsilon^{\gamma} \to 0$, then,

$$\begin{aligned} W &= O\left(\frac{\Psi p}{\epsilon}\right) \\ \chi &= O\left(\frac{\Psi H'}{\epsilon}\right) \quad , \end{aligned} \tag{50}$$

where slippage with respect to the shear component has been allowed in the second equation. The last expression suggests that when $H' = 0$, χ may be expected to be small.

Since the desired result of these calculations is the real integral Δ', the perturbation terms should be analyzed as to their effect on this quantity. Thus, three features of each term should be examined: (1) the phase - i.e. the complex nature, (2) the parity - i.e. the eveness or oddness, and of course (3) the magnitude. Following FKR, we consider even-Ψ solutions.

Case (1): $H' = O(1)$, $G = 0$

If only the dominant terms are retained in the $r\epsilon^{\gamma} \to 0$ limit, eqs. (42)-(46) reduce to

$$\frac{H}{\epsilon}\frac{d\chi}{d\theta} + \frac{\beta\mathcal{P}^{o}}{3p\epsilon}\left[\frac{\tilde{\rho}'}{\tilde{\rho}}\frac{dW}{d\theta} + (\mathrm{div}\,\vec{v})\right] = 0 \tag{51}$$

$$\chi F'\epsilon\theta - \frac{4\epsilon^{4}(F')^{2}\alpha}{i\tilde{\eta}}\frac{d^{2}Z}{d\theta^{2}} + \Psi H' = 0 \tag{52}$$

$$\frac{d^{4}W}{d\theta^{4}} + W\theta^{2} + \Psi\left[-\frac{\tilde{\eta}\,F''}{(F'\epsilon)^{2}} + \frac{p\theta}{F'\epsilon}\right] + \frac{6I\alpha\epsilon^{2}}{Hi}\left(\frac{F}{I}\right)'\frac{d^{2}Z}{d\theta^{2}} + \frac{3F'\alpha\epsilon^{2}}{Hi}\,\theta\,\frac{d^{3}Z}{d\theta^{3}} = 0 \tag{53}$$

$$\frac{\tilde{\eta}}{\epsilon^{2}}\frac{d^{2}\Psi}{d\theta^{2}} = \Psi p + WF'\epsilon\theta \tag{54}$$

$$-\frac{\tilde{\eta}}{i\alpha\epsilon^{2}}\frac{d^{2}\chi}{d\theta^{2}} + F'\epsilon\theta Z = 0 \quad . \tag{55}$$

For this case, eqs. (53) and (54) state that $p\epsilon = O(1)$: Eq. (54) may be regarded as giving $\Psi p + F''\epsilon\theta W = O(\Psi/\epsilon)$ in order to give $\Delta' = O(1)$. So eq. (53) consists of terms of $O(W, \epsilon^{2}Z, \Psi/\epsilon)$. Eliminating Z by using eqs. (52) and (55), and using eq. (50) to estimate W, these orders become $O(\psi p/\epsilon, \Psi/\epsilon^{2}, \Psi/\epsilon)$. Thus, we must have $p\epsilon = O(1)$. In the $r\epsilon^{\gamma} \to 0$ limit, then, $W = O(\Psi/\epsilon^{2})$, $\chi = O(\Psi/\epsilon)$, $Z = O(\Psi/\epsilon^{4})$ and $\mathrm{div}\,\vec{v} = O(\Psi/\epsilon^{2})$, with Z the only imaginary quantity.

With this ordering, the largest correction terms are the χ and Ψ terms in eqs. (44) and (45), these being of $O(r\epsilon)$ compared to those in eqs. (51)-(55). They, however, have the wrong phase to give a real Δ' contribution directly in the first interation. They would be expected simply to introduce overstability. Similarly, examination shows that there

are a host of terms which are of $O\left[\epsilon,\ r_+\epsilon,\ r_+\epsilon^2,\ r\epsilon^2,\ (r_+\epsilon^2)^2,\ (r\epsilon^2)^2\right]$ compared to the dominant terms. Successive iterations in which the $O(r\epsilon)$ terms comingle with these terms do lead to real contributions to Δ'. With $(r/r_+) = (m_+/m_-)^{\frac{1}{2}} \gg 1$, we then expect that with increasing temperature the first corrections to the low temperature results are appreciable when $(r\epsilon)\ (r\epsilon^2) = r^2\epsilon^3 = O(1)$. Since the terms containing r arise from the Hall current in Ohm's law, the first temperature corrections arise from the Ohm's law and not from the pressure tensor in the momentum equation. Incidentally, we note that since terms of $O(r^2\epsilon^3)$ can arise from the product of quantities of order $(r\epsilon)$, $(r\epsilon)$ and ϵ, and since $O(r\epsilon)$ terms arise in the equation for $d^2\Psi/d\theta^2$, it is necessary in calculating the complete $O(r^2\epsilon^3)$ correction to abandon the constant-Ψ approximation of FKR.

Case (2): $H' = O(1)$, $G = O(1)$

In this case, if only the dominant terms are retained in the $r\epsilon^\gamma \to 0$ limit, eqs. (42), (43) and (45) again reduce to eqs. (51), (52) and (54), respectively. However, eqs. (44) and (46) become

$$\frac{d^4W}{d\theta^4} + W\left[-\frac{G\tilde{\Pi}\tilde{\rho}}{p\epsilon^2(F')^2} + \theta^2\right] - \frac{G\tilde{\rho}^2\tilde{\Pi}}{p\tilde{\rho}'(F'\epsilon)^2}\,\mathrm{div}\,\vec{v} + \Psi\,\frac{\theta p}{F'\epsilon} = 0 \qquad (56)$$

and

$$-\frac{\tilde{\Pi}}{i\alpha\epsilon^2}\frac{d^2\chi}{d\theta^2} + \frac{p}{i\alpha}\chi - \frac{H'W}{i\alpha} - \frac{H'}{i\alpha}\,\mathrm{div}\,\vec{v} + F'\epsilon\theta Z = 0 \quad . \qquad (57)$$

For this case, eqs. (56) and (54) give $p\epsilon^2 = O(1)$: eq. (56) consists of terms of $O(W,\ GW/p\epsilon^2,\ G\,\mathrm{div}\,\vec{v}/p\epsilon^2,\ \Psi/\epsilon^2)$, where use has been made of the $\Delta' = O(1)$ constraint. Eliminating div $\vec{v}$ by eqs. (51), (52) and (57) and using the frozen field constraint to eliminate W, these orders become $O(\Psi p/\epsilon,\ G\Psi/\epsilon^3,\ G\Psi/\epsilon^3,\ \Psi/\epsilon^2)$. Thus, we must have $p\epsilon^2 = O(1)$ when $G = O(1)$. So

in the low temperature limit, $W = O(\Psi/\epsilon^3)$, $\chi = O(\Psi/\epsilon)$, $Z = O(\Psi/\epsilon^4)$, $\operatorname{div} \vec{v} = O(\Psi/\epsilon^3)$, with W and χ odd, Z even, and only Z imaginary.*

With this ordering, we note that there are no correction terms of $O(r\epsilon,\ r_+\epsilon)$. Again there are a host of terms of $O\left[\epsilon, r\epsilon^2, r_+\epsilon^2, (r\epsilon^2)^2, (r_+\epsilon^2)^2\right]$, the terms involving r arising from the Hall current in Ohm's law and those involving r_+ arising from the pressure tensor in the momentum equation. These terms all have the wrong phase to contribute to a real Δ' in a first iteration. However, in a successive iteration in which these terms are evaluated by insertion of the first iterative solutions, contributions to a real Δ' do result. Thus, with $(r/r_+) = (m_+/m_-)^{\frac{1}{2}} \gg 1$, we expect that for Case (2), the Ohm's law is responsible for the first corrections as the temperature is raised, and these become appreciable when $r\epsilon^2 = O(1)$.

<u>Case (3)</u>: $H' = 0$, $G = 0$

When $H' = 0$, χ and Z are much smaller than in the two previous cases. Retaining only the dominant terms in the $r\epsilon^\gamma \to 0$ limit, eqs. (42)-(46) reduce to

$$-\frac{\beta\rho^0}{3p\epsilon}\left[\frac{\tilde{\rho}'}{\tilde{\rho}}\frac{dW}{d\theta} + \frac{d}{d\theta}(\operatorname{div}\vec{v})\right] + \frac{F'}{\epsilon}\frac{d\Psi}{d\theta} = 0 \tag{58}$$

$$\chi F'\epsilon\theta - \frac{4\epsilon^4(F')^2\alpha}{i\tilde{\eta}}\frac{d^2Z}{d\theta^2} - \frac{3\epsilon^4 I(F')^2}{H\tilde{\eta}}\left(\frac{F}{I}\right)'\frac{d^2W}{d\theta^2} - \frac{3\epsilon^4(F')^3}{H\tilde{\eta}}\theta\frac{d^3W}{d\theta^3} = 0 \tag{59}$$

$$\frac{d^4W}{d\theta^4} + \theta^2 W + \Psi\left(-\frac{\tilde{\eta}F''}{(F'\epsilon)^2} + \frac{p\theta}{F'\epsilon}\right) = 0 \tag{60}$$

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*Johnson (1964) has pointed out that the presence of $\operatorname{div}\vec{v}$ in Case (2), may lead to the stabilization of the gravitational mode under some circumstances.

$$\frac{\tilde{\eta}}{\epsilon^2}\frac{d^2\Psi}{d\theta^2} = \Psi p + F'\epsilon\theta W \tag{61}$$

$$-\frac{\tilde{\eta}}{i\alpha\epsilon^2}\frac{d^2\chi}{d\theta^2} + F'\epsilon\theta Z = 0 \quad . \tag{62}$$

As in Case (1), $p\epsilon = O(1)$. The low temperature magnitudes are $W = O(\Psi/\epsilon^2)$, $\chi = O(\epsilon\chi)$, $Z = O(\Psi/\epsilon^2)$, and $\text{div}\,\vec{v} = O(\Psi/\epsilon)$. Here eqs. (60) and (61) are decoupled from the remaining three equations, as in the incompressible hydromagnetic calculation.

With this ordering, the largest corrections are of $O(r\epsilon, r_+\epsilon)$ and are in the χ and Z equations. The $O(r\epsilon)$ corrections occur in eq. (46) and the $O(r_+\epsilon)$ terms are in eq. (43). These do not contribute to a real Δ' in this order, however, since they do not contribute directly to quantities in the equation for $d^2\Psi/d\theta^2$. The first correction terms to occur in the equations for W and Ψ are of $O(\epsilon, r\epsilon^3, r_+\epsilon^3)$. They enter with the wrong phase to give a direct contribution to real Δ' (except for the $O(\epsilon)$ term); however, since these terms involve χ and Z, an iteration in which these terms comingle with the $O(r\epsilon, r_+\epsilon)$ terms in the χ and Z equations will give contributions to Δ' of $O(r^2\epsilon^4,\ r\,r_+\epsilon^4,\ r_+^2\epsilon^4)$. The phases and parities are such that these latter contributions are real. Again, with $r \gg r_+$, the Hall current in the Ohm's law is responsible for the first temperature corrections, and these become appreciable when $r\epsilon^2 = O(1)$.

To summarize, it is found that with $\Omega_+ \gg \nu_+$, at low temperature the plasma equations give the same results as the hydromagnetic equations, except that the ion-ion collision contribution to the pressure tensor determines the size of the region in which the plasma and magnetic field are decoupled. The first temperature corrections to the growth rates arise

from the Hall-current terms in Ohm's law. These are appreciable when $r\epsilon^{\gamma} = O(1)$, with $\gamma = 3/2$ in Case (1) and $\gamma = 2$ in Cases (2) and (3).

As the temperature continues to increase, the pressure tensor corrections in the momentum equation become appreciable. However, this occurs when the validity of the Chapman and Cowling plasma equations become quite suspect. Thus, with ϵ determined by viscosity,

$$r\epsilon^2 \approx 3\ (r_I/\epsilon a) \quad , \tag{63}$$

where r_I is the ion Larmor radius. This quantity has been assumed to be small in the Chapman and Cowling expansion. Thus, even the Hall current corrections to the growth rate do not enter until the validity of the expansion becomes somewhat questionable. The condition $r_+\epsilon^2 = O(1)$ corresponds then to $r_I/\epsilon a = O\left[(m_+/m_-)^{\frac{1}{2}}\right]$, and this is quite outside the range where the Chapman and Cowling expansion is known to be valid. It appears that another set of equations is needed for an accurate treatment of this regime. It is of interest, nevertheless, to speculate on the type of effects which might be expected here on the basis of the present type of equations and this is the subject of Appendix B.

IV.

DISCUSSION

With the various assumptions involved in the Chapman and Cowling expansion and in our treatment of the resulting equations, the range of conditions to which the foregoing analysis may be applied is somewhat limited. Thus, where viscosity dominates, all of the following quantities have been assumed to be less than unity:

$$r_I/\epsilon a \approx 10^7 \, [\beta^{1/6}\alpha^{-2/3}a^{-2/3}(\log \Lambda)^{-1/3}] \, T^{2/3}n^{-1/2} \tag{65}$$

$$k_{\parallel}v_{\theta}/\nu_+ \approx 6 \times 10^{13} \, [\beta^{1/3}\alpha^{2/3}a^{-4/3} \, (\log \Lambda)^{-2/3}] \, T^{4/3}n^{-1} \tag{66}$$

$$\omega/\nu_+ \approx 5 \times 10^{13} \, [\beta^{-2/3}\alpha^{2/3}a^{-4/3}(\log \Lambda)^{-2/3}] \, p\epsilon^2 T^{4/3}n^{-1} \tag{67}$$

$$\omega/\Omega_+ \approx 10^7 \, [\beta^{-1/6}\alpha^{2/3}a^{-4/3}(\log \Lambda)^{1/3}] \, p\epsilon^2 T^{-2/3}n^{-1/2} \tag{68}$$

$$\nu_+/\Omega_+ \approx 2 \times 10^{-6} \, [\beta^{1/2} \log \Lambda] \, T^{-2}n^{1/2} \tag{69}$$

$$\omega\tau_T = 0\left(p\epsilon^2\beta^{-1}(m_-/m_+)^{1/2}\right) \tag{70}$$

$$\epsilon \approx \beta^{1/3} \, (a\alpha)^{-1/3} \, (\log \Lambda)^{1/3} \, T^{-2/3} \tag{71}$$

$$S^{-1} \approx \frac{1}{6} \, [\beta^{1/2}a^{-1} \log \Lambda] \, T^{-2} \quad . \tag{72}$$

Here, the number density n is expressed in cm^{-3}, the temperature T is expressed in eV, and the thickness of the equilibrium current layer a is in cm. The first four quantities are assumed to be small in the Chapman and Cowling expansion when $r_+ = \Omega_+/\nu_+$ is large, and we have used the form of the pressure tensor which is correct only when r_+ is large. Equation (70)

gives the ratio of the temperature relaxation time τ_T estimated from the ∇T terms of eq. (5) to the growth time, and its significance has been discussed following eq. (18). In this connection, the energy conservation equation in its linearized form is

$$\frac{3}{2}\,\omega p^1 + \frac{3}{2}\,\vec{v}\cdot\nabla p^o + \frac{5}{2}\,p^o\,\nabla\cdot\vec{v} + \nabla\cdot\vec{q} = \vec{j}^o\cdot[\vec{E} + \vec{v}\times\vec{B}^o] \quad . \tag{64}$$

With the orderings discussed in Cases (1)-(3), the first three terms dominate this equation. Invoking the continuity equation, we then find

$$T^1 = O\left(\frac{T^o}{\omega}\,\nabla\cdot\vec{v}\right) \quad . \tag{65}$$

In Appendix A, and in the heuristic arguments of FKR, it is shown that the important current in the instabilities is j_z, since it acts on the sheared component of the magnetic field to give a dynamic force in the y-direction. When $r \gg 1$, the ratio of the j_z evaluated ignoring ∇T to the temperature-driven j_z evaluated from eq. (65) is of order $r^2/\beta \gg 1$ in Cases (1) and (2) and of order $r^2/\epsilon\beta \gg 1$ in Case (3). Finally, the division of the problem into two regions, a very narrow decoupled-flow region surrounded by a frozen-field region, assumes $\epsilon \sim S^{-1/3}$ small. For β of order unity, the most restrictive of these conditions seems to be eq. (65).

In the foregoing, the ion gyrofrequency has been regarded as being much larger than the ion collision frequency. In that case, the ratio of the viscosity term to the inertial term in eq. (37) is:

$$\frac{\text{viscosity}}{\text{inertia}} = O\left[\beta\left(\frac{m_+}{m_-}\right)^{1/2}\left(\frac{T_-}{T_+}\right)^{3/2}\frac{1}{p\epsilon^2}\right] \quad . \tag{73}$$

Since

$$p\epsilon = O(1) \qquad \text{(tearing mode)}$$

$$p\epsilon^2 = O(1) \qquad \text{(gravitational mode)}, \tag{74}$$

and since the factor $(m_+/m_-)^{\frac{1}{2}}$ appears in the expression, unless β is quite small the viscosity term dominates the inertia term. On the other hand, if we had regarded the opposite limit where $r_+ = \Omega_+/\nu_+$ was very small, then eqs. (7)-(13) show that the right hand side of eq. (73) would be multiplied by r_+^2. In the $r_+ \ll 1$ limit, then, inertia dominates the viscosity, and the hydromagnetic results of Furth, Killeen and Rosenbluth and Johnson, Greene and Coppi are recovered.

The results are summarized in Figure 1. The ordinate is the density (in cm^{-3}) and the abscissa is the temperature (in eV), with normalizing factors to allow for the size of the current layer a (in cm), for $\alpha = ka$ and for the ratio of the pressures β. The lower boundary is where the ion gyroradius is as large as the width of the decoupled-flow region. The equations may be expected to apply only above this line, with the qualification that the minimum density must be multiplied by the largest of

$$\frac{\alpha^2}{\pi}, \frac{p\epsilon^2\alpha^2}{4\beta}, \frac{(p\epsilon)^2(\log\Lambda)^{4/3}\alpha^{8/3}}{a^{4/3}\,\beta^{2/3}\,T^{8/3}} \quad ,$$

if any of the three is greater than 1. (This corresponds to the most restrictive condition being $\{k_\parallel v_\theta/\nu_+ < 1,\ \omega/\nu_+ < 1,\ \omega/\Omega_+ < 1\}$, respectively.) The dashed line indicates where the ion gyrofrequency and collision frequency become equal. In the region of low temperatures and high densities above and to the left of the dashed line, the finite-resistivity hydromagnetic results are valid. In the region of higher temperatures to the right

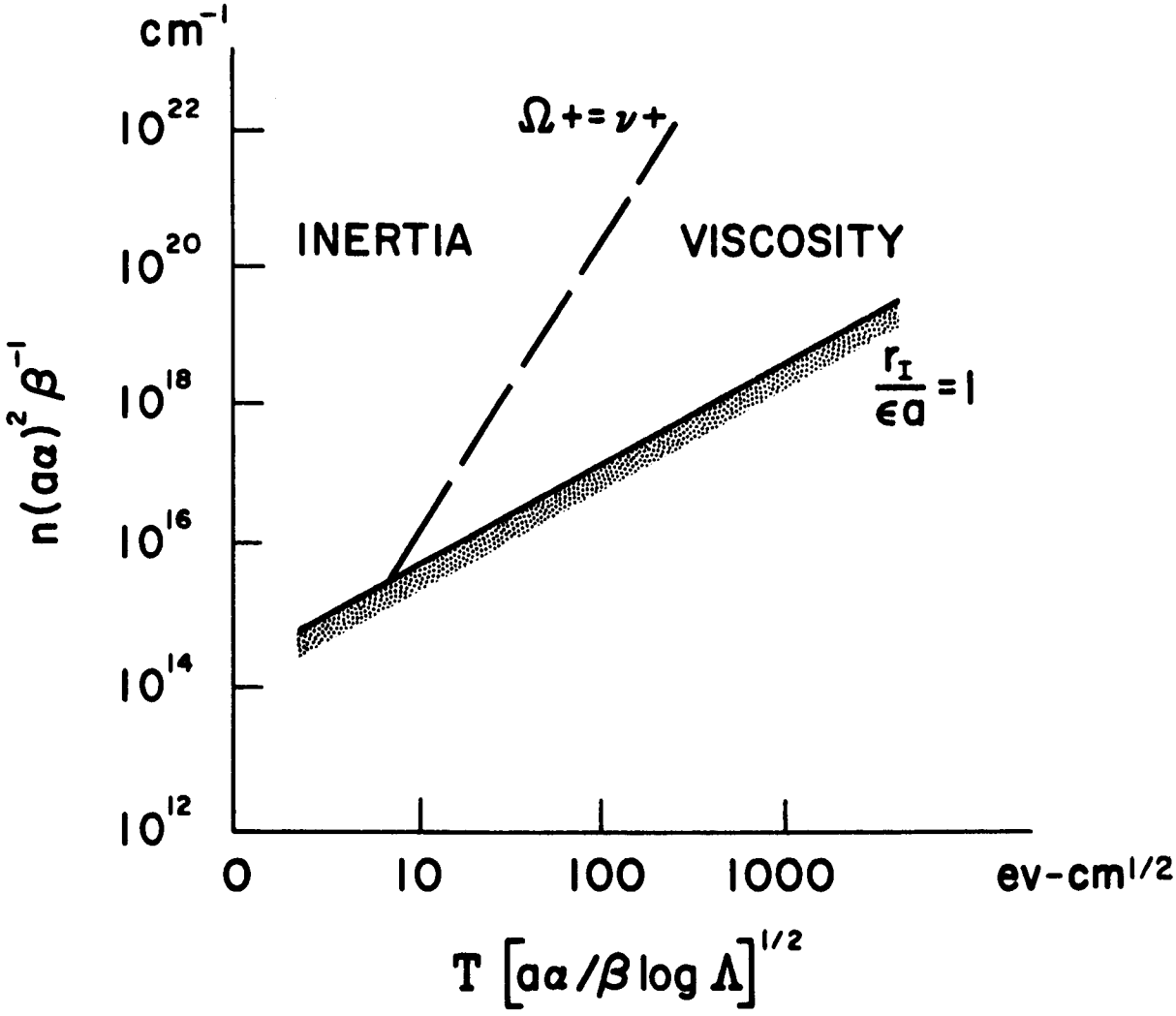

Fig. 1--Regions of validity for Chapman and Cowling analysis of resistive instabilities. n_{min} must be multiplied by the largest of

$$\frac{\alpha^2}{\pi},\ \frac{p\epsilon^2\alpha^2}{4\beta},\ \frac{(p\epsilon^2)^2\ (\ell og\ \Lambda)^{4/3}\ \alpha^{8/3}}{a^{4/3}\beta^{2/3}w^{8/3}}$$

if any of the three is greater than 1. This corresponds to the most restrictive condition being $\{k_{\parallel}v_{\theta}^{+}/\nu_{+} < 1,\ \omega/\nu_{+} < 1,\ \omega/\Omega_{+} < 1\}$ respectively. Here, a is expressed in cm, n is in cm^{-3}, T is in ev, and $\beta = O(\epsilon^{o})$.

of the dashed line and above the lower boundary, the instabilities are qualitatively the same as the hydromagnetic results, except that viscosity replaces inertia in determining the size of the decoupled flow region and the growth rate. In the viscosity-dominated region, overstability is introduced when $r\epsilon = O(1)$ for the tearing mode occurring where the pressure is not a maximum (Case 1), when $r\epsilon^2 = O(1)$ for the gravitational mode (Case 2), and does not occur for the tearing mode occurring at pressure maximum (Case 3). Corrections to the growth rate arising from the Hall-current terms in Ohm's law do not become important until $r\epsilon^\gamma = O(1)$, with $\gamma = 3/2$ in Case (1) and $\gamma = 2$ in Cases (2) and (3). In the region of higher temperatures and lower densities below the $r_I/\epsilon a = 1$ curve, another set of equations is needed: the considerations of Appendix B suggest that the instabilities might persist under some conditions there also, but a much more careful treatment of this region is required.

Magnetic fields in the earth's magnetospheric tail have been measured by the Explorer 10 [Heppner, Ness, Scearce and Skillman (1963)], Explorer 14 [Cahill (1964)] and Imp 1 [Ness (1965)] satellites. Fields in the range of 10-30 γ have been found; the existence of a neutral sheet of thickness $\sim$ 600 km has been established; and the diameter of the tail at a distance of thirty earth radii has been determined to be about forty earth radii. Howard, Eshleman, Barry and Fenwick (1964) and Yoh, Howard, Lusignan and Eshleman (1966) have concluded on the basis of lunar radar echoes that the average electron density in the tail is about 200 cm^{-3} greater than the solar wind electron density (which is of the order of 10 cm^{-3}). If the plasma in the tail comes primarily from the solar wind, temperatures in the range of 10^{70} K might be expected, whereas if it comes primarily from

evaporation from the polar cap ionospheres, temperature in the range of 10^3 °K might be expected [Dessler and Michel (1966b)].

As some representative numbers, then, we take a typical length of the shear field to be 10^8 cm, look at perturbation wavelengths of the same order and assume a density, temperature and magnetic field of 10^2 cm^{-3}, 0.1 eV and 10γ, respectively. Equations (65)-(72) show that in this case viscosity does determine the behavior. Here, the mean free path of $\sim 1/2 \times 10^9$ cm is one-thirtieth the effective parallel wavelength $k_{\parallel}^{-1}$, and the latter is comparable with the diameter of the geomagnetic tail. The ratio ϵ of the thickness of the decoupled flow region to the thickness of the equilibrium current layer is 10^{-2}, and for the tearing mode the growth time is of the order of 10^{-2} times the resistive diffusion time, i.e. of the order of a day. Equations (71) and (74) show that shorter growth times may be obtained by considering shorter wavelength perturbations.

APPENDIX A

The effect of an anisotropic resistivity is examined here by replacing the scalar resistivity term in FKR's incompressible hydromagnetic treatement by

$$\overleftrightarrow{\eta}\,\vec{j} = \eta_{\parallel}\vec{j}_{\parallel} + \eta_{\perp}\,\vec{j}_{\perp} \quad . \tag{A1}$$

Linearizing about a stationary state, the basic equations are

$$\omega\vec{B}^{1} = \nabla\times(\vec{v}\times\vec{B}^{o}) - \frac{1}{4\pi}\nabla\times[\overleftrightarrow{\eta}^{o}\,\nabla\times\vec{B}^{1} + \eta^{1}\,\nabla\times\vec{B}^{o}] \tag{A2}$$

$$\omega\nabla\times(\rho^{o}\vec{v}) = \nabla\times\left[\frac{1}{4\pi}\{(\vec{B}^{o}\cdot\nabla)\,\vec{B}^{1} + (\vec{B}^{1}\cdot\nabla)\,\vec{B}^{o}\} + \rho^{1}\vec{g}\right] \tag{A3}$$

$$\nabla\cdot\vec{v} = 0 \tag{A4}$$

$$\nabla\cdot\vec{B}^{1} = 0 \tag{A5}$$

$$\omega\overleftrightarrow{\eta}^{1} + (\vec{v}\cdot\nabla)\,\overleftrightarrow{\eta}^{o} = 0 \tag{A6}$$

$$\omega\rho^{1} + \vec{v}\cdot\nabla\,\rho^{o} = 0 \quad . \tag{A7}$$

Following the procedure described in Section II, the following three coupled dimensionless equations are obtained:

$$\frac{\Psi''}{\alpha^2} = \Psi\left[1 + \frac{p}{\tilde{\eta}\alpha^2}\right] + \frac{W}{\eta\alpha^2}\left[F + \frac{1}{p}\left\{\tilde{\eta}'F' + \frac{FI'}{I}(\delta\tilde{\eta})'\right\}\right] + \frac{FH}{I^2\alpha^2}\frac{\delta\tilde{\eta}}{\tilde{\eta}}\chi' \tag{A8}$$

$$\frac{1}{\alpha^2}[\tilde{\rho}W']' = W\left[\tilde{\rho} - \frac{S^2G}{p^2} + \frac{S^2F}{p\tilde{\eta}}\left\{F + \frac{1}{p}\left(\tilde{\eta}'F' + \frac{FI'}{I}(\delta\tilde{\eta})'\right)\right\}\right]$$

$$+ S^2\Psi\left[\frac{F}{\tilde{\eta}} - \frac{F''}{p}\right] + \frac{S^2F^2H\delta\tilde{\eta}}{pI^2\tilde{\eta}}\chi' \tag{A9}$$

$$\left[\left(\tilde{\eta} + \frac{H^2}{I^2}\,\delta\tilde{\eta}\right)\chi'\right]' = \chi\left[p + \frac{\alpha^2 S^2 F^2}{p\tilde{\rho}} + \alpha^2\,(\tilde{\eta} + \delta\tilde{\eta})\right] - WH' + \frac{\alpha^2 S^2}{\tilde{\rho}p}\,FH'\Psi$$

$$- \frac{1}{p}\left[\left\{H'\tilde{\eta}' + \frac{HI'}{I}\,(\delta\tilde{\eta})'\right\} W\right]' + \left[\frac{FH}{I^2}\,\delta\tilde{\eta}\,(\Psi'' - \alpha^2\Psi)\right]' , \tag{A10}$$

where quantities are as defined in Section II, with $\tilde{\eta}_\perp$ and $\delta\tilde{\eta}$ given by

$$\delta\tilde{\eta} = (\eta_\perp - \eta_\parallel)/\langle\eta\rangle \tag{A11}$$

$$\tilde{\eta}_\perp = \tilde{\eta}\left(1 + \frac{F^2\delta\tilde{\eta}}{I^2\tilde{\eta}}\right) . \tag{A12}$$

With $\chi = O\,(H'\Psi/F'\epsilon)$ and $W = O\,(\Psi p/F'\epsilon)$, if only the dominant terms in the region of decoupled flow are retained, eqs. (A8)-(A10) reduce to

$$\frac{d^2\Psi}{d\theta_1^2} - \epsilon_1^2\alpha^2\Psi = \epsilon_1\Omega\left[4\Psi + U(\theta_1 + 2\delta_1) + 4\delta_4\theta_1\frac{d\chi}{d\theta_1}\right] \tag{A13}$$

$$\frac{d^2U}{d\theta_1^2} + U\left(\Lambda_o - \frac{\theta_1^2}{4} - \frac{\theta_1\delta_1}{2}\right) = \Psi(\theta_1 - \delta_5) + \delta_4\theta_1^2\frac{d\chi}{d\theta_1} \tag{A14}$$

$$\left(\frac{\tilde{\eta} + \delta\tilde{\eta}}{\tilde{\eta}}\right)\frac{d^2\chi}{d\theta_1^2} - \frac{\theta_1^2\chi}{4} = \frac{H'}{4\epsilon_1 F'}\,\theta_1 \tag{A15}$$

where

$$U = \frac{4\epsilon_1 F'}{p}\,W$$

$$\theta_1 = \frac{\mu}{\epsilon_1}$$

$$\epsilon_1 = \left[\frac{p\tilde{\eta}\tilde{\rho}}{4\alpha^2 S^2 (F')^2}\right]^{\frac{1}{4}}$$

$$\Lambda_o = \frac{S^2\alpha^2\epsilon_1^2\, G}{p^2\tilde{\rho}} - \alpha^2\epsilon_1^2$$

$$\delta_1 = \frac{\tilde{\eta}'}{2p\epsilon_1}$$

$$\delta_4 = \frac{F'H\tilde{\eta}\,\delta\tilde{\eta}}{I^2 p\,\tilde{\eta}}$$

$$\delta_5 = \frac{F''}{4\Omega F'}$$

$$\Omega = \frac{p\epsilon_1}{4\tilde{\eta}} \quad .$$

In eqs. (A13) and (A14), the arisotropic resistivity enters only through the $\delta_4\, d\chi/d\theta_1$ terms. Equation (A15) states that χ is an odd function of θ_1, since Ψ may be regarded as constant in eqs. (A14) and (A15). Thus, the $\delta_4\, d\chi/d\theta_1$ term in (A13) is odd, and on integration to form Δ' will contribute nothing. Similarly, in eq. (A14) the $\delta_4\, d\chi/d\theta_1$ term contributes to an even U: this has the effect of simply changing the value of δ_1 in eq. (A13). As may be seen from FKR's eq. (47), variations in δ_1 do not change the growth rates appreciably, the important quantity in (FKR 47) being the argument of the gamma-function. Thus, we conclude that an anisotropic resistivity does not effect the resistive instabilities in an essential way. This is the same conclusion as would be drawn from FKR's heuristic considerations, since there it was argued that the important current in the instability is just that along the unsheared component of the field.

APPENDIX B

Even though the Hall current corrections in Ohm's law and the finite Larmor radius corrections in the momentum equation do not contribute appreciably to the growth rates until the validity of the Chapman and Cowling equations becomes somewhat questionable, it is of interest to see what these types of equations predict for their effect. In the first part of this appendix, the Chapman and Cowling equations are applied beyond their range of validity in the case with the simplest equations (case (3)) to see what they predict for the effect of the Hall current. In the second part of the appendix, a crude estimate of the effect of finite Larmor radius is made by constructing approximate equations for the extreme limit where the ion gyroradius is much larger than the size of the decoupled flow region.

Part 1

When $r\epsilon^2 = O(1)$ in Case (3), the dominant terms in eqs. (42)-(46) lead to the following four coupled equations:

$$\bar{X}\theta = C_1 \frac{d^2\bar{z}}{d\theta^2} \tag{B1}$$

$$\frac{d^4W}{d\theta^4} + \theta^2 W + \Psi\theta - \bar{X}\theta^2 = 0 \tag{B2}$$

$$\frac{1}{C_2}\frac{d^2\bar{\Psi}}{d\theta^2} = \bar{\Psi} + \theta W - \theta\bar{X} \tag{B3}$$

$$\frac{1}{C_1}\frac{d^2\bar{X}}{d\theta^2} + \theta\left[\frac{\bar{z}}{4} + \bar{\Psi} + \theta W - \theta\bar{X}\right] = 0 \quad , \tag{B4}$$

where

$$\bar{\Psi} = \frac{\Psi p}{F'\epsilon}$$

$$\bar{\chi} = \frac{i\alpha r\chi}{\tilde{\rho}}$$

$$\bar{z} = \frac{4\tilde{\rho}\tilde{\eta}Z}{F'r\epsilon}$$

$$C_1 = \frac{r^2\epsilon^4\alpha^2(F')^2}{(\tilde{\rho}\tilde{\eta})^2}$$

$$C_2 = \frac{p\epsilon^2}{\tilde{\eta}}$$

and where we consider only the case $F'' = 0$. Eliminating $\bar{\chi}$ from the equations by using eq. (B1), we may construct from eqs. (B2) and (B4) a quadratic form for Δ' involving W and $\bar{z}$. On setting the first variation of this expression equal to zero, the equations are recovered; using the stationary property, it is estimated that for large C_1,

$$p\epsilon \sim C_1^{\frac{1}{4}} \quad . \tag{B5}$$

With $\beta = O(1)$, $\epsilon \propto T^{-2/3}$, $r \propto T^2$, $\tau_R \propto T^{3/2}$, this gives a growth rate which varies inversely as the square root of the temperature.

Part 2

We consider a situation where the ion gyroradius is much larger than the width of the decoupled flow region. Then, for the perturbed ion motion, we may ignore the magnetic field in the decoupled-flow region, regarding the dominant force on the ions to arise from the electric field resulting from the difference between the forces acting on the ions and electrons. On

regarding the limit $r_+ \gg 1$, ion-ion collisions may be ignored compared to the inertia. We relate the electric field in the frame of reference drifting with the electrons to the current through electron-ion collisions. Then, the basic equations are

$$m_+ \frac{d\tilde{v}_\pm}{dt} = e\vec{\mathcal{E}} + m_+\vec{g} \tag{B6}$$

$$\vec{\mathcal{E}} + \tilde{v}_- \times \vec{B} = \eta\vec{j} \tag{B7}$$

where $\tilde{v}_+$ and $\tilde{v}_-$ are the perturbed drift velocities of the ions and electrons. Writing eqs. (B6) and (B7) in terms of macroscopic quantities (allowing for the difference between the electron drift and macroscopic velocities resulting from the magnetization current), we obtain

$$\rho \frac{d\vec{v}}{dt} = \vec{j}\times\vec{B} - \nabla p + \rho\vec{g} + \frac{e\rho}{m_+}\left[\eta\vec{j} - \vec{v}\times\vec{B}\right] \tag{B8}$$

$$\vec{\mathcal{E}} + \vec{v}\times\vec{B} = \eta\vec{j} - \frac{1}{n_e e}\left[\nabla p_e - \vec{j}\times\vec{B}\right] \tag{B9}$$

with the subscript e denoting electron quantities. Equation (B8) states that the driving current is furnished by the electrons, with the inertia provided by the ions.

Writing eqs. (B8) and (B9) for the planar geometry,

$$-\frac{e}{m_+} B_y' = -ikv_z \tag{B10}$$

$$-\frac{e}{m_+} B_z' = ikv_y + \frac{1}{ik}\frac{d^2 v_y}{dy^2} - \frac{1}{ik}\frac{d}{dy}(\nabla\cdot\vec{v}) + \frac{g}{\omega^2}\left(ik\nabla\cdot\vec{v} + \frac{ik}{\rho^o}\frac{d\rho^o}{dy}\right) \tag{B11}$$

$$\frac{\eta^o}{4\pi}\frac{d^2 B_y'}{dy^2} = \omega B_y' - ikB_x^o v_y + \frac{\eta^o k^2}{4\pi} B_y' - \frac{ik}{4\pi n_e^o e}\left(ikB_x^o B_z' + B_y'\frac{dB_z^o}{dy}\right) \tag{B12}$$

$$\left(\omega + \frac{\eta^o k^2}{4\pi}\right) B_z' - \frac{d}{dy}\left(\frac{\eta^o}{4\pi}\frac{dB_z'}{dy}\right) - ikB_x^o v_z + v_y \frac{dB_z^o}{dy} + B_z^o\,(\nabla\cdot\vec{v})$$

$$= \frac{1}{4\pi e n_e^o \rho^o}\left[\frac{d\rho^o}{dy}\left(-\,ikB_z^o B_z^1 + B_y^1 \frac{dB_x^o}{dy}\right) + \rho^o\left\{-k^2 B_x^o B_y^1 + B_x^o \frac{d^2 B_y^1}{dy^2} - B_y^1 \frac{d^2 B_x^o}{dy^2}\right\}\right.$$

$$\left. - \frac{ikB^o}{\omega}\frac{dB^o}{dy}\left(v_y \frac{d\rho^o}{dy} + \rho^o\,\nabla\cdot\vec{v}\right)\right], \qquad \text{(B13)}$$

we note that now v_z varies slowly.

Consider Case (3) again: $g = 0$, $dB_z^o/dy = 0$. Setting $\nabla\cdot\vec{v} = 0$, the dominant terms give

$$-\frac{e}{m_+} B_z' = \frac{1}{ik}\frac{d^2 v_y}{dy^2} \qquad \text{(B14)}$$

$$\frac{\eta^o}{4\pi}\frac{d^2 B_y'}{dy^2} = \omega B_y' - ikB_x^o v_y + \frac{k^2 B_x^o B_z'}{4\pi n_e^o e} \qquad \text{(B15)}$$

$$-\frac{d}{dy}\left(\frac{\eta^o}{4\pi}\frac{dB_z'}{dy}\right) = \frac{B_x^o}{4\pi e n_e^o}\frac{d^2 B_y'}{dy^2} \quad . \qquad \text{(B16)}$$

With the frozen field constraint at the boundaries where $\vec{j}^1 \to 0$, these equations are consistent with

$$p\epsilon = O(1) \qquad \text{(B17)}$$

and

$$\epsilon^6 = \frac{p\epsilon}{s^2\alpha^2}$$

(the B_z' term in (B15) making a negligible contribution to Δ' because of (B14)), and are a valid approximation to equations (B10)-(B13) when

$$r_I/\epsilon a \gtrsim O\,(1/\epsilon^{\frac{1}{2}}) \quad . \tag{B18}$$

The assumptions leading to eqs. (B6) and (B7) are similar to those made in treating magnetic sheath problems (i.e. the magnetic field acts only on the electrons, but these in turn act on the ions through the resulting electric field). Presumably, this is valid in the "decoupled flow" region when $r_I/\epsilon a \gg 1$. However, the use of eq. (B7) here severely restricts the range of validity, since now eq. (66) requiring that $k_{\parallel} v_{\theta}/\nu \ll 1$ sets the lower limit on allowable densities. Violation of the condition raises the possibility of Landau damping along the field lines. To conclude, the considerations of this appendix suggest that the resistive instabilities might persist under some conditions even when $r_I/\epsilon a \gtrsim 1$: however, a much more careful treatment of this region is necessary.

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Section 8

SOME PLASMA INSTABILITIES OF THE MAGNETOSPHERE

by

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Some Plasma Instabilities of the Magnetosphere*

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Four plasma instabilities which may operate in planetary magnetospheres are described: (1) the low frequency Rayleigh-Taylor-type flute instability as affected by a resistive ionosphere, a dipole field geometry and by resonant particle-wave interactions, (2) a whistler instability, with the concomitant particle diffusion, which occurs in a plasma containing a small component of flat-helix energetic electrons, (3) a nonconvective high frequency two energy component electrostatic loss-cone instability, and (4) the resistive instabilities occurring in sheared magnetic fields, as affected by the Chapman and Cowling collisional and magnetic viscosities and Hall currents.

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I. INTRODUCTION

A frightening number of plasma instabilities has been uncovered in the last few years in connection with the work on controlled fusion, and it seems likely that some of these could operate in planetary magnetospheres. I'd like to describe four candidates for magnetospheric instability that M. N. Rosenbluth, L. D. Pearlstein and I have been studying.

(1) The first is the low frequency Rayleigh-Taylor-type flute instability which may occur when a plasma is supported by a magnetic field against some destabilizing force. At first glance it would seem that a planetary radiation belt should fall victim to such an instability since a particle confined in a dipole-like field experiences an outwardly directed inertial force associated with its motion in the curved field. On the other hand, the presence of a conducting ionosphere tends to short out the electric fields which drive the instability. In addition, the geometry of the field itself can play a stabilizing role. We were interested in studying the competition between these various effects to see if a flute instability could ever occur fast enough to cause diffusion of the energetic belt particles.

(2) The second instability concerns disturbances of a much higher frequency, in fact frequencies high enough to resonantly violate the magnetic moments of the energetic trapped particles. Study of this instability was prompted by observations of the sporadic circularly polarized decameter radiation which comes from the planet Jupiter. The sporadic nature of the radiation suggests that it might be associated with some

instability, and the circular polarization suggests that it might be an instability of electromagnetic waves propagating along a magnetic field. Other radio observations of Jupiter in the shorter decimeter wavelengths indicate that the plasma in Jupiter's belts has an energetic electron component comprised mostly of particles with very flat helices. What we have done, then, is to see if a plasma, the bulk of which is thermalized and of low temperature but which contains a small number of quite flat helix energetic particles, can exhibit maser action. It's interesting also to look at what effect this energy exchange between the waves and particles has on the particle distribution, and a quasilinear diffusion equation has been constructed to look at this.

(3) Recently, Rosenbluth and Post[1] have pointed out that there are some high frequency electrostatic plasma instabilities which are peculiar to "loss-cone" particle distributions such as exist in mirror confinement geometries. They treated the case in which there is only one energy component of each species present, and found that in a uniform plasma the instability was convected ($k_{\parallel} \neq 0$) along the magnetic field. Rosenbluth and Post have pointed out that gradients of the density could lead to nonconvective ($k_{\parallel} = 0$) instabilities. With the Van Allen belt in mind, we have looked at a loss-cone instability in a plasma containing both high and low energy components, and it turns out that the presence of two components also leads to nonconvective instabilities of rather large growth rates.

(4) The fourth instability studied is a low frequency resistive type which occurs in sheared magnetic fields. This type may have some chance

of existing in long twisted magnetospheric tails, and if it does it could play some role in the annihilation and recombination of field lines. The instability owes its existence to the local relaxation of the constraint that the plasma must remain attached to a magnetic field. In the instability, finite conductivity permits the system to go to a lower potential energy by allowing lines of force which are initially distinct to link up. Furth, Killeen and Rosenbluth[2], and Johnson, Greene and Coppi[3] have studied these instabilities with hydromagnetic equations which contain a scalar resistivity. For these resistive instabilities, the hydromagnetic equations are only directly applicable when the ion collision frequency is larger than the ion gyro-frequency (when the plasma and magnetic pressures are comparable). Since the mean free paths in magnetospheres are quite long and the plasma and magnetic pressures in the tails may be comparable, such a description would not be directly applicable. We have tried to analyze the instabilities with the Chapman and Cowling plasma equations to see how they behave under a broader range of conditions.

II. LOW FREQUENCY FLUTE

Gold[4] and Sonnerup and Laird[5] have looked at the energetics of interchanges in a dipole field, and treating the plasma as a fluid have defined circumstances under which the energy change would permit an interchange instability to occur. By looking at the dynamics of the instability, a dispersion relation can be constructed to see if an interchange can occur fast enough to affect the energetic trapped particles. It appears that the ionosphere could play an important role in this instability, as it would strongly affect any driving electric fields. The ionosphere would behave as a resistive medium for rapidly occurring instability and as a reactive medium for interchanges occurring slowly enough for the neutral ionospheric particles to take part in the motion. (Here we do not consider those situations in which the change in ionospheric gravitational energy is important.[5]) The character of the instability would be different in the two cases: Thus, hydromagnetically a reactive ionosphere might be expected to give the usual Rayleigh-Taylor-type dispersion relation in which growth rates are proportional to the square-root of the product of the destabilizing force and the density gradient of the energetic particles which drive the instability. The growth rate in this case would be smaller the smaller the ratio of the number of energetic driving particles to the number of driven ionospheric particles, and stability of the energetic particles might be expected if this ratio were small enough. A resistive ionosphere, on the other hand, would give a dispersion relation with hydromagnetic growth rates proportional to the effective collision

frequency of the resistivity and to the density gradient of the driving energetic particles. Again, stability of the energetic particles might be expected if the ratio of the number of energetic particles to the number of ionospheric particles were small enough.

We have studied the resistive situation in the electrostatic approximation to see if diffusion of the energetic particles could ever result.[6] The driving currents of the energetic particles have been obtained from the description of Northrup and Teller[7] of particle motion in a mirror system when the magnetic moment and mirror actions are invariant. The polarization and resistive currents have been described by a dielectric tensor of a zero temperature three fluid (ions, electrons, neutrals) plasma model. In this situation, currents due to higher order drifts are not important: even though the frequencies of interest are of the order of the drift frequencies of the energetic particles - so that their finite Larmor radius and polarization drifts are of the same order, their polarization currents are already negligible compared with that of the low energy belt component, and especially compared with the ionospheric currents.

The resulting dispersion relation shows geometry itself to play a stabilizing role, associated with the fact that the flux tubes in a dipole field increase in volume with increasing radial distance. The lowest order solution to the dispersion relation (i.e., the solution obtained by treating the growth rate as being much larger than the azimuthal drift frequencies - the latter being obtained from the time required for a particle to drift through a wavelength of the disturbance) yields the familiar

interchange energy $\delta^2 W > 0$ sufficiency - condition[8] for stability*. If this condition is violated, or if one is interested in instabilities in which the growth rates are not treated as being much larger than the azimuthal drift frequencies - e.g., those in which resonant interactions between the disturbances and the azimuthal drift velocities play a role, then a Nyquist analysis of the dispersion relation gives another sufficiency condition for stability of the energetic particles which takes into account the effect of a resistive ionosphere. This more general condition is essentially the requirement that the growth rates be less than the azimuthal drift frequencies of the energetic particles.

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*Dungey, Hess and Nakada (see discussion following Van Allen's Aug. 23 talk) have pointed out that this condition seems to be satisfied over a large portion of the earth's magnetosphere. G. Davidson and M. Walt have reached similar conclusions (private communication).

III. WHISTLER MASER

To date, three different kinds of radio emission have been detected from Jupiter.[9] Below wavelengths of 3 cm, the radiation is thermal, following closely the λ^{-2} dependence of the Rayleigh-Jeans law. In the decimeter region beyond 3 cm, the flux behavior differs considerably from a λ^{-2} law, remaining quite constant between 3 cm and 70 cm. Observations of the intensity, polarization and spatial extent of the decimeter radiation strongly suggest that the decimeter radiation might be synchrotron radiation from energetic electrons in Jovian Van Allen belts. To explain the high degree of linear polarization that is observed, it's necessary to postulate electrons with very flat helices. The third kind of radiation occurs in the decameter band, and is quite different from the other two in that it is quite sporadic. It is observed in the 8 to 80 meter range, occurs in bursts, has features which are strongly correlated with longitude on Jupiter, and 90% of the time is very strongly RH circularly polarized. A variety of mechanisms has been suggested over the years for the origin of this sporadic radiation. The storms have been attributed to thunderstorm activity, atmospheric chemical explosions, plasma oscillations, spin-flip, the focusing by Jupiter's magnetosphere of radio waves from distant sources, magnetic bremsstrahlung from electrons dumped into the Jovian magnetic polar region, and coherent emission by relativistic electrons in Jupiter's Van Allen belts. An instability of whistlers may have some relevance to the last two explanations.[10]

The whistler instability calculation consists first of evaluating an absorption coefficient from a Vlasov equation, and then obtaining the

corresponding emissivity by Einstein coefficient arguments. For a plasma the bulk of which is thermalized but which also contains a small number of energetic electrons with very flat helices, it is found that efficient amplification is possible under two quite different conditions, when the flat-helix relativistic electrons have a net drift along the field. Amplification occurs for waves traveling in the same direction as the drift. The two conditions are: (1) $\omega_p^2 \gg \omega_c\omega \gg \omega^2$, and (2) $\omega \approx \omega_c$, $\omega_p^2 \gg \omega_c\omega - \omega^2 > 0$. Here, ω, ω_c and ω_p are the whistler frequency, (non-relativistic) electron gyrofrequency, and plasma frequency, respectively. It appears that case (1) may exist in Jupiter's ionosphere. Zhelezniakov[11] has estimated from photoionization considerations that the electron density there might be 10^7 cm^{-3}, and it appears that higher densities may be obtained on taking into account the ionizing particle flux from the sun and particles dumped from the belts. For some numbers, suppose the field in the ionosphere is 16 gauss, the whistler frequency desired is 10 Mc/s and the electron density is 10^8 cm^{-3}, say: then $\omega_p^2/\omega\omega_c \approx 13$, and case (1) is satisfied. Beams of particles traveling down the field might also be important in this case. Case (2), on the other hand, might easily exist in the belts themselves. The decameter radiation has been observed between frequencies of 5 and 39 Mc/s, and these correspond to gyrofrequencies for fields between 1.8 and 14 gauss. Fields of this order have already been required to provide the decimeter radiation.

If amplified whistlers are the source of the decameter radiation, then it appears reasonable to associate the sporadic nature of the radiation with the only occasional occurrence of net drift motions along the

magnetic field of the relativistic electrons (and possibly in case (1) with the only occasional occurrence of the required plasma densities). A net drift motion might exist, for instance, when increased solar wind activity causes dumping of radiation belt electrons, as might be suggested by the correlation between the Jovian decameter activity and solar events. The predominant presence of right-handed polarization may be attributed to a combination of the preference in drift velocity directions - for instance, dumping would correspond to drifts from the belts toward the planet - and to geometry. The geometrical aspects of the problem have been discussed in detail by Warwick,[12] Field[13] and Carr.[14] Insertion of reasonable Jovian parameters in the expressions give acceptable emissivities and growth rates, so it seems possible that some such process might supply the decameter radiation.

It is interesting to ask for the effect of these circularly polarized waves on the particles with which they resonantly interact. A Fokker-Planck equation may be constructed to describe the behavior of a group of particles in the presence of damped or growing waves.[15] In this way it can be seen that the behavior of particles which resonate with a wave differs considerably from that of particles which are far from resonance. The nonresonant particles have wave-induced motions which increase or decrease as the amplitude of a wave increases or decreases: they are described by a "diffusion coefficient" which can be either positive or negative. The resonant particles, on the other hand, are exposed to an almost constant electric field which carries them rapidly away from resonance: a real (positive) diffusion coefficient results.

Alternatively, the diffusive behavior of the particles may be studied directly from the Vlasov equation by constructing a quasilinear diffusion equation following the prescription of Drummond and Pines,[16] directing attention to resonant interactions in which some doppler-shifted wave frequency is equal to a particle's gyrofrequency.[17]

If we assume that the dispersion relation of the waves has the form $\omega \sim k^b$ - e.g., for whistlers b = 2 and for Alfven waves, b = 1, and if we assume that the energy spectrum of the waves goes like $k^{-\alpha}$ - for whistlers resulting from lightning discharges, Dungey has suggested an α close to unity, whereas Alfven waves might have this form if they result from some turbulence spectrum, and if we make the approximations that the particles can be treated nonrelativistically, that the phase velocities are much less than the particle velocities, and that only particles in flat helices are to be considered; then the diffusion equation takes a very simple form in which energy changes may be ignored compared to changes in the helix angles. If particles are inserted initially at one helix angle, then this equation shows the peak of the distribution to move to flatter helices while at the same time the particles leak off to steeper helices where they disappear into a loss cone. Eventually the simple equation doesn't apply since the derivatives with respect to energy cannot be ignored compared to the derivatives with respect to helix angle when the latter become small. The steady state diffusion equation keeping both the energy and angle derivatives has as possible solutions some very flat helix distributions. These may have interesting properties, e.g., it's possible to have distributions in which energy and helix angle are strongly correlated.

IV. TWO-COMPONENT ELECTROSTATIC LOSS-CONE

The dispersion relation for the high frequency flute-like two energy component loss-cone instability in a uniform magnetic field is shown below in dimensionless variables.[18]

$$-\left(\frac{K}{1-\Delta}+\frac{1}{1-y^2}\right)\frac{\tan \pi y}{y}=\frac{\Delta}{1-\Delta}\frac{1}{|kR|^3} \tag{1}$$

For a situation in which there are $n_H + n_L$ low energy electrons, n_L low energy ions and n_H energetic ions in a loss cone distribution,

$$y=\frac{\omega}{\Omega_i} \qquad K=\frac{m_e}{m_i}+\frac{\Omega_i^2}{\omega_{pi}^2}$$
$$R=\frac{\overline{V}_i}{\Omega_i} \qquad \Delta=\frac{n_H}{n_H+n_L} \tag{2}$$

Here, ω is the frequency, k is the wave number, Ω_i is the ion cyclotron frequency, ω_{pi} is the ion plasma frequency, m_e and m_i are the electron and ion masses, and $\overline{V}_i^{-3}$ is obtained from the perpendicular velocity $V_\perp$ as the average value of $V_\perp^{-3}$ for the energetic ions. For a situation in which there are n_L low energy electrons and n_H high energy electrons in a loss cone distribution with the ions treated as unperturbed, the quantities are

$$y=\frac{\omega}{\Omega_e} \qquad K=\frac{\Omega_e^2}{\omega_{p_e}^2}$$
$$R=\frac{\overline{V}_e}{\Omega_e} \qquad \Delta=\frac{n_H}{n_H+n_L} \tag{3}$$

Here the subscript e denotes electron quantities.

To determine stability properties, we examine the dispersion relation for real ω and look for values of the parameters at which real solutions

disappear. Except for a special resonant case, the condition for instability found in this way is

$$\frac{2}{\pi}\left(\frac{K}{1-\Delta}\right)^2 \approx \frac{2}{\pi[1-(N+\frac{1}{2})^2]^2} \lesssim \left(\frac{\Delta}{1-\Delta}\right)\frac{1}{(kR)^3} \lesssim \left(\frac{\Delta}{1-\Delta}\right)\frac{1}{N^3} \lesssim \frac{\pi}{2} \tag{4}$$

where $N = 2, 3, \ldots$. For the $\Delta \ll 1$ case, we have as the instability condition

$$\Delta \gtrsim \frac{2}{\pi}\sqrt{K} \tag{5}$$

and as a solution

$$y \approx \pm\frac{1}{\sqrt{K}} - i\,\frac{\Delta}{2K^2(kR)^3} \tag{6}$$

For electrons, Eqs. (5) and (6) become

$$(n_H/n_L) \gtrsim (2/\pi)\left(\Omega_e/\omega_{p_e}\right) \tag{7}$$

$$\omega \approx \omega_{p_e} - i\Omega_e \tag{8}$$

whereas for ions, they become

$$(n_H/n_L) \gtrsim (2/\pi)\left[(m_e/m_i) + \left(\Omega_i^2/\omega_{p_i}^2\right)\right]^{1/2} \tag{9}$$

$$\omega \approx \Omega_i\left[(m_e/m_i) + \left(\Omega_i^2/\omega_{p_i}^2\right)\right]^{-1/2} - i\Omega_i \tag{10}$$

For a magnetic field of $\frac{1}{24}$ gauss and $n = 10^4$ cm^{-3}, say, $\frac{\Omega_e}{\omega_{p_e}} \approx 1$; so that the electron case wouldn't be satisfied except at rather large L in the earth's field. In the ion case, the requirement for these numbers is that $\frac{n_H}{n_L} \gtrsim \frac{1}{65}$. If the instabilities do occur, the growth rates are quite large (on the order of the cyclotron frequencies): On the other hand,

the wavelengths are quite small so that turbulence, enhanced resistivity and diffusion rather than mass motion would be expected.

Finally, a special resonance case occurs when

$$\frac{1-\Delta}{K} = N^2 - 1 \quad , \Delta < \frac{2}{\pi}\sqrt{K} \tag{11}$$

In that case, we have for the electrons,

$$\left(\omega_{p_e}/\Omega_e\right)^2 = N^2 - 1 \tag{12}$$

$$\omega \approx N\Omega_e - i\Omega_e \left[(n_H/n_L)(N/2\pi)\right]^{1/2} \tag{13}$$

and for the ions

$$\left[(m_e/m_i) + \left(\Omega_i/\omega_{p_i}\right)^2\right]^{-1} = N^2 - 1 \tag{14}$$

$$\omega \approx N\Omega_i - i\Omega_i \left[(n_H/n_L)(N/2\pi)\right]^{1/2} \tag{15}$$

We have not yet looked into the effect of nonlinearities on these instabilities, but these might be expected to be important in quenching the instabilities by putting enough structure on the distribution function to reduce the contribution of the energetic portion.

V. RESISTIVE INSTABILITIES

Finally, we come to the resistive instabilities which, it has been suggested,[19] might occur in magnetospheric tails. One of the interesting features of these instabilities is that the purely resistive part of the instabilities is very strongly localized in the high conductivity limit. The finite conductivity actually only allows the plasma to decouple from the magnetic field in a very small region, about where the perturbation practically lies along a field line. The longer the mean free path, the smaller is the size of this decoupled region. The very large gradients associated with this narrow decoupled-flow region then give rise to the possibility that viscosity and finite Larmor radius effects might be important. With the long mean free paths in magnetospheric tails, this is likely to be the case there. To see how these effects modify the finite resistivity hydromagnetic results, we have been looking at the instabilities with the Chapman and Cowling equations.[20] As the ratio β of plasma pressure to magnetic pressure might not be too small in the tails, we have treated β to be of order unity.

When temperature gradients may be ignored, the Chapman and Cowling equations differ from the hydromagnetic equations principally at two points. In the momentum equation, the hydromagnetic approximation has a scalar pressure. In the Chapman and Cowling momentum equation the scalar pressure is accompanied by a pressure tensor which accounts for the viscosity which arises from ion-ion collisions and for the current which arises when the ions because of their larger gyroradii sample a different environment than the electrons. The second place where the equations

differ is in the Ohm's law. In the hydromagnetic approximation, the Ohm's law relates the electric field in the fluid frame of reference to the current through a scalar resistivity. The Chapman and Cowling Ohm's law states that the electron-ion collisions should relate the current to the electric field in the frame of reference drifting with the electrons. In addition, the scalar resistivity of the hydromagnetic approximation is replaced by an anisotropic resistivity which results from the presence of a magnetic field.

At low temperatures, when the ion gyrofrequency is less than the ion collision frequency these equations give the same results as the finite-resistivity hydromagnetic equations: there are again tearing and gravitational modes. As the temperature is increased, and the ion gyrofrequency becomes larger than the collision frequency, the first correction to the growth rates comes from the ion-ion collision contribution to the pressure tensor. In the curl of the momentum equation, the ratio of the viscosity term to the inertial term is of the order of β, the ratio of the plasma and magnetic pressures, times the square root of the ratio of the ion to electron mass, times the quantity $1/p\epsilon^2$. "p" is the growth rate expressed in terms of the resistive diffusion time, and ϵ is the ratio of the width of the decoupled-flow region to the width of the original current layer. For the tearing mode, $p\epsilon^2$ is of order ϵ. At any rate, with the square root of the mass ratio occurring here too, unless β is very small, the viscosity term will dominate the inertial term. This then gives an ϵ that is inversely proportional to the cube root of the magnetic Reynold's number. But otherwise, the results are qualitatively the same as the

hydromagnetic results: the only difference is that viscosity replaces inertia here, confirming the conclusions of Furth, Killeen and Rosenbluth.[2]

As the temperature is increased further, the phases and magnitudes of the quantities are such that the next correction to the growth rates arises from the Hall current term in Ohm's law. The Hall current corrections do not become appreciable, though, until $r\epsilon^{\delta} = O(1)$, where r is the ratio of the electron gyrofrequency to the electron collision frequency; and δ is equal to 2 for the gravitational mode and for the tearing mode which occurs where the pressure is maximum. With ϵ determined by viscosity, the quantity $r\epsilon^2$ is just equal to three times the ratio of the ion Larmor radius to the size of the decoupled flow region. This ratio has been assumed to be small in the Chapman and Cowling expansion, so these Hall current corrections to the growth rate do not become important until the validity of the Chapman and Cowling plasma equations itself becomes somewhat questionable. The finite Larmor radius terms in the momentum equations pressure tensor do not contribute sizeable corrections to the growth rate until even higher temperatures, when $r\epsilon^{\delta}$ is of the order of the square root of the ratio of the ion mass to the electron mass. This is quite outside the range where the Chapman and Cowling equations are known to be valid.

As some representative numbers, if we take a typical length of the shear field to be 10^9 cm, look at perturbation wavelengths of the same order, assume a density of 10 cm^{-3}, set $\beta = \frac{1}{10}$ and assume a temperature of 1 ev (so that the magnetic field is on the order of 10 gamma), the Chapman and Cowling equations show that viscosity dominates the behavior. In this

case, ϵ is on the order of 10^{-3}, and for the tearing mode the growth time is on the order of 10^{-3} times the resistive diffusion time, i.e., on the order of a year. Smaller lengths would give faster growth times, but then these equations would become suspect.

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APPENDIX

Summary of Instability Equations

(1) Two stability sufficiency criteria are given for the electrostatic low frequency flute, the first (1a) applying only to gross hydromagnetic interchanges, and the second (1b) including those in which resonant interactions between waves and particles are important.

(1a) Specifically, the sufficiency condition for stability against gross hydromagnetic interchanges is:

$$\iint d\mu dJ \frac{\partial F_o}{\partial \alpha} \frac{\partial K_o}{\partial \alpha} < 0 \tag{A1}$$

where $F_o(\mu,J,\alpha)\, d\mu dJ d\alpha$ is the number of particles with magnetic moment and bounce actions in the intervals $(\mu, \mu + d\mu)$ and $(J, J + dJ)$, located in the magnetic flux interval $(\alpha, \alpha + d\alpha)$.*

$K_o\ (\mu,J,\alpha)$ is the relativistic energy of a particle.

(1b) The second sufficiency condition takes a simple approximate form for disturbances with large azimuthal (longitudinal) variation when the ionosphere behaves resistively:

$$m^2 \oint \frac{4\pi\sigma_1}{R^2 B}\, ds > \omega\, \mathrm{Im}(-\mathcal{M}) \tag{A2}$$

where

$$\mathcal{M} = 8\pi\, \Sigma \iint d\mu dJ \frac{ecm \dfrac{\partial F_o}{\partial \alpha}}{\omega + \dfrac{cm}{e} \dfrac{\partial F_o}{\partial \alpha}} \tag{A3}$$

---

*For a dipole field of moment M, a line of force with an equatorial distance of r_E has $\alpha = -M/r_E$.

Here ω and m are the frequency and azimuthal wave number (disturbances of the form $e^{i(\omega t + m\Theta)}$ are assumed, where Θ is the longitude), σ_1 is the Pederson conductivity, B is the magnetic field intensity, R is the distance from the dipole axis, the integral $\oint ds$ is over the line of force whose stability is in question, c and e are the velocity of light and electron charge, and the sum is over the different particle species.

(2) The whistler instability condition may be expressed in terms of the sign of an integral. Specifically, in a plasma which is comprised mostly of low energy, thermal particles but which also contains a small number of relativistic electrons, the condition for the instability of a whistler of frequency ω propagating along the magnetic field is:

$$I(p_1,p_2) > 0 \tag{A4}$$

where, approximately,

$$I(p_1,p_2) = -2\int_{p_1}^{p_2} dp\, pF_o(p,\Theta)\left[1 - \left(1 - \frac{\xi}{(1+p^2)^{1/2}}\right)\left(\frac{\xi-1}{\xi+\chi^2-1}\right)\right]$$

$$\pm \frac{\chi^2}{\left[(\xi+\chi^2-1)(\xi-1)\right]^{1/2}} \int_{p_1}^{p_2} dp\, \frac{p^2 \sin\Theta}{(1+p^2)^{1/2}} \frac{\partial F_o}{\partial\Theta} \tag{A5}$$

(and where here the mass of the electron and the velocity of light have been set equal to unity). The plus sign is used when $\xi > 1$ and the minus sign is used when $\xi < 1$. Here, $\chi = \omega_p/\omega$ and $\xi = \omega_c/\omega$, with ω_p and ω_c the plasma frequency and (nonrelativistic) cyclotron frequency. $F_o(p,\Theta)\, dpd\Theta$ is the number of electrons with relativistic momentum p in the interval

$(p, p + dp)$ and helix angle in the interval $(\Theta, \Theta + d\Theta)$. In the integrands, Θ is determined from

$$\cos \Theta = - \frac{\omega}{kp} \left[(1 + p^2)^{1/2} - \frac{\omega_c}{\omega} \right] \tag{A6}$$

$$\left(\frac{k}{\omega}\right)^2 = 1 - \frac{\chi^2}{1-\xi} \tag{A7}$$

and p_1 and p_2 are the smallest and largest momenta for which $|\cos \Theta| = 1$. When the relativistic electrons are in flat helices $(\Theta \approx \frac{\pi}{2})$, the condition $I\ (p_1, p_2) > 0$ can be easily satisfied for slight drift motions of the relativistic electrons along the field under two quite different conditions of low phase velocity:

$$(1) \qquad \omega_p^2 \gg \omega_c \omega \gg \omega^2 \tag{A8}$$

$$(2) \qquad \omega \approx \omega_c \text{ and } \omega_p^2 \gg \omega_c \omega - \omega^2 > 0. \tag{A9}$$

(3) The two component electrostatic loss-cone instability condition is given in the general nonresonant case for electron and ion frequencies by Eqs. (7) and (9) of the text, respectively. For the resonant case, the instability condition is given by Eqs. (12) and (14) for electron and ion frequencies, respectively.

(4) For the resistive instabilities occurring for comparable plasma and magnetic pressures and governed by viscosity in the absence of temperature gradients, the Chapman and Cowling equations apply when all of the following quantities are less than unity:

$$r_I/\varrho a \approx 10^7 \left[\beta^{1/6}\ \alpha^{-2/3}\ a^{-2/3} (\log \Lambda)^{-1/3} \right] T^{2/3}\ n^{-1/2} \tag{A10}$$

$$k_{\parallel} v_{\Theta}/\nu_{+} \approx 6 \times 10^{13} \left[\beta^{1/3} \alpha^{2/3} a^{-4/3} (\log \Lambda)^{-2/3}\right] T^{4/3} n^{-1} \quad (A11)$$

$$\omega/\nu_{+} \approx 5 \times 10^{13} \left[\beta^{-2/3} \alpha^{2/3} a^{-4/3} (\log \Lambda)^{-2/3}\right] p\epsilon^{2} T^{4/3} n^{-1} \quad (A12)$$

$$\omega/\Omega_{+} \approx 10^{7} \left[\beta^{-1/6} \alpha^{2/3} a^{-4/3} (\log \Lambda)^{1/3}\right] p\epsilon^{2} T^{-2/3} n^{-1/2} \quad (A13)$$

$$\nu_{+}/\Omega_{+} \approx 2 \times 10^{-6} \left[\beta^{1/2} \log \Lambda\right] T^{-2} n^{1/2} \quad (A14)$$

$$\omega\tau_{T} = 0 \left[p\epsilon^{2} \beta^{-1} (m_{-}/m_{+})^{1/2}\right] \quad (A15)$$

$$\epsilon \approx \beta^{1/3} (a\alpha)^{-1/3} (\log \Lambda)^{1/3} T^{-2/3} \quad (A16)$$

$$S^{-1} \approx (1/6) \left[\beta^{1/2} a^{-1} \log \Lambda\right] T^{-2} \quad (A17)$$

Here β is the ratio of the plasma to the magnetic pressure, a is the width of the equilibrium current layer expressed in cm, the dimensionless quantity α is the product of a and the perturbation wave number k, T is the temperature in ev, n is the density in cm^{-3}, r_I is the ion gyroradius, ϵ is the ratio of the width of the decoupled flow region to a, $k_{\parallel}$ is a measure of the projection of k onto the magnetic field in the decoupled flow region, v_{Θ} is the ion thermal velocity, ω is the growth rate, ν_{+} is the ion-ion collision frequency, Ω_{+} is the ion gyrofrequency, τ_T is the temperature relaxation time, S is the magnetic Reynold's number, p is the product of the growth rate and the resistive diffusion time, m_{-} and m_{+} are the electron and ion masses, and Λ is the ratio of the maximum to minimum impact parameter for coulomb collisions.

When these quantities are less than unity, then the growth rate may

be cbtained from

$$p\epsilon = O(1) \tag{A18}$$

for the tearing mode (the mode which is driven by magnetic pressure) and from

$$p\epsilon^2 = O(1) \tag{A19}$$

for the gravitational mode (the mode which is driven by a destabilizing force, such as the inertial force of acceleration).

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Section 9

SYNCHROTRON RADIATION FROM A DIPOLE FIELD

by

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SYNCHROTRON RADIATION FROM A DIPOLE FIELD

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ABSTRACT

Tables of the two-dimensional distribution of Stokes parameters for synchrotron radiation from ultrarelativistic electrons in a dipole field are compiled for various energy and spatial distributions, and for varying angles between the dipole axis and the line of sight.

Author

I. INTRODUCTION

Recent radio astronomical observations of Jupiter (Morris and Berge 1962; Miller and Gary 1962; Morris and Bartlett 1963; Rose, Bologna, and Sloanaker 1963; Gary 1963; Bash, Drake, Gundermann, and Heiles 1964; Roberts and Komesaroff 1964, 1965; Berge and Morris 1964) call for the calculation of two-dimensional distributions of the intensity and polarization of the synchrotron radiation from ultrarelativistic electrons trapped in a distant dipole field. Past treatments (Chang and Davis 1962; Field 1962; Roberts and Huguenin 1962; Korchak 1963; Thorne 1963) have been restricted to calculations of either one-dimensional or integrated distributions. The purpose of this paper is to extend these treatments to calculate the two-dimensional distribution of the synchrotron radiation Stokes parameters. The results are presented in the form of fifty-two tables in which the radiation is described for various angles between the dipole axis and the line of sight, and for various energy and spatial distributions of the radiating electrons. The expressions for the Stokes parameters are derived in § II. In § III, these expressions are summarized and the results discussed. The tabulated results are presented in the Appendix.

II. SYNCHROTRON RADIATION FROM A DIPOLE FIELD

The Stokes parameters for synchrotron radiation from the power-law distribution,

$$\rho(r,E,\alpha) = n(r,\alpha)C_2E^{-(\gamma+1)}\,, \tag{1}$$

are given by Chang (1962):

$$I(f)dV = D_1 f^{-\gamma/2}B^{(\gamma+2)/2}(\sin\alpha^*)^{\gamma/2}n(\alpha^*)dV\,, \tag{2}$$

$$Q(f)dV = D_2\cos 2\chi I(f)dV\,, \tag{3}$$

$$U(f)dV = D_2\sin 2\chi I(f)dV\,, \tag{4}$$

$$V'(f)dV = 0\,, \tag{5}$$

where D_1 and D_2 are defined as

$$D_1 = R^{-2}CC_2L^{\gamma/2}2^{(\gamma-4)/2}\left(\frac{3\gamma+10}{3\gamma+6}\right)\Gamma\left(\frac{3\gamma+2}{12}\right)\Gamma\left(\frac{3\gamma+10}{12}\right), \tag{6}$$

$$D_2 = -\left(\frac{3\gamma+6}{3\gamma+10}\right). \tag{7}$$

C and L are proportionality constants with the values:

$$C = 3.73 \times 10^{-23} \text{ erg sec}^{-1} \text{ gauss}^{-1}$$

$$L = 1.608 \times 10^{13} \text{ c/s gauss}^{-1} \text{ BeV}^{-2} .$$

R is the distance to the observer, f is the frequency of the radiation, x is the angle measured clockwise by the observer from the (arbitrary) vector i_l involved in Chandrasekhar's (1960) definition of the Stokes parameters to the projection of the magnetic field onto a plane at right angles to the line of sight, and $\rho(\boldsymbol{r},E,\boldsymbol{\alpha})dVdEd\boldsymbol{\alpha}$ is the number of particles in an element of volume dV at $\boldsymbol{r}$, energy interval dE at energy E, and helix angle interval $d\boldsymbol{\alpha}$ at $\boldsymbol{\alpha}$.

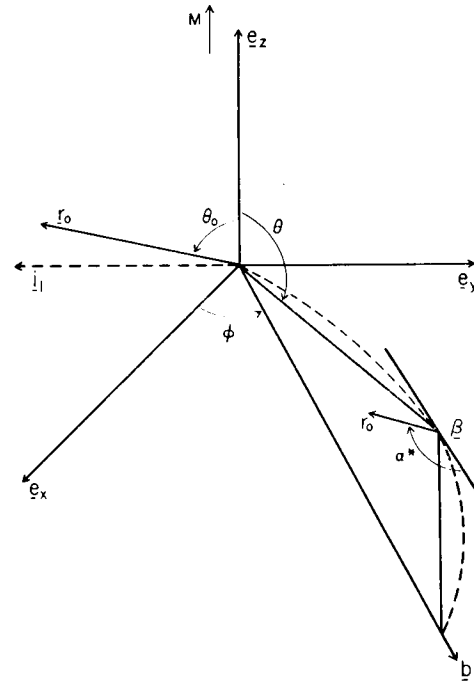

FIG. 1.—The coordinate systems

To evaluate these expressions for electrons in a dipole field, we adopt the coordinate systems indicated in Figure 1. The origin is at the center of the dipole. The unit vector $\boldsymbol{e}_z$ is taken along the dipole axis. The unit vector $\boldsymbol{e}_y$ is the positive (l) direction and is the intersection of the (l,w)-plane (which is the plane perpendicular to the line of sight to the observer and containing the center of the dipole axis) with the equatorial plane of the dipole. The third unit vector $\boldsymbol{e}_x$ also lies in the equatorial plane. The unit vector in the w direction in the (l,w)-plane is $\boldsymbol{i}_r$, while that in the direction of the observer (and thus perpendicular to the (l,w)-plane is $\boldsymbol{r}_0$. The angle between $\boldsymbol{r}_0$ and $\boldsymbol{e}_z$ is θ_0, the aspect angle, and is one of the parameters of the calculation.

Chang and Davis (1962) calculated the one-dimensional distribution of Stokes parameters in the case where θ_0 was $\pi/2$ (the (l,w)-plane perpendicular to the dipole equatorial plane) with $\boldsymbol{e}_x \equiv \boldsymbol{r}_0$. Their unit vectors, ($\boldsymbol{i}_l$, $\boldsymbol{i}$, $\boldsymbol{i}_r$), are also shown and have the following relationships:

$$\boldsymbol{i}_l = -\boldsymbol{e}_y , \qquad \boldsymbol{i} = \boldsymbol{r}_0 , \qquad \boldsymbol{i}_r = \boldsymbol{i}_r , \qquad \boldsymbol{i}_r \times \boldsymbol{r}_0 = \boldsymbol{e}_z \times \boldsymbol{e}_x = \boldsymbol{e}_y . \tag{8}$$

A polar coordinate system (r,θ,ϕ) is also shown in Figure 1. The projection of $\boldsymbol{r}$ onto the $\boldsymbol{e}_y$ direction is l, while that onto the $\boldsymbol{i}_r$ direction is w. Thus,

$$l = r \sin\theta \sin\phi \,, \tag{9}$$

$$w = r\,(\cos\theta \sin\theta_0 - \sin\theta \cos\theta \cos\phi)\,. \tag{10}$$

The $(x$-$z)$-plane is shown in Figure 2. The distance (d) of a radiating volume dV from the (l,w)-plane is

$$d = x \sin\theta_0 + z \cos\theta_0 \,, \tag{11}$$

where

$$x = r \sin\theta \cos\phi \,, \quad z = r \cos\theta \,, \quad d = r\,(\sin\theta_0 \sin\theta \cos\phi + \cos\theta_0 \cos\theta)\,. \tag{12}$$

We shall only need the relative position of the radiating volume to the (l,w)-plane, that is, only the sign of d. Figure 3 is a section through the dipole field (the $(y$-$z)$-plane of

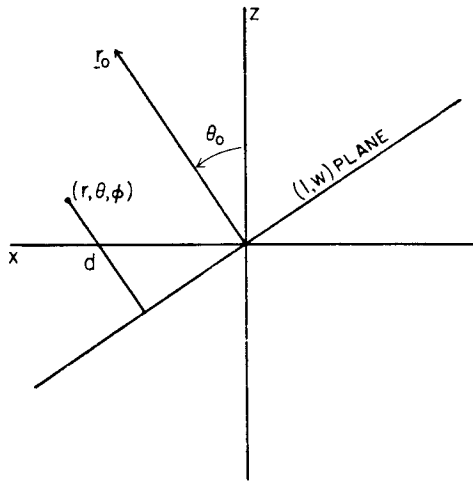

FIG. 2.—The (x,z) plane of Fig. 1

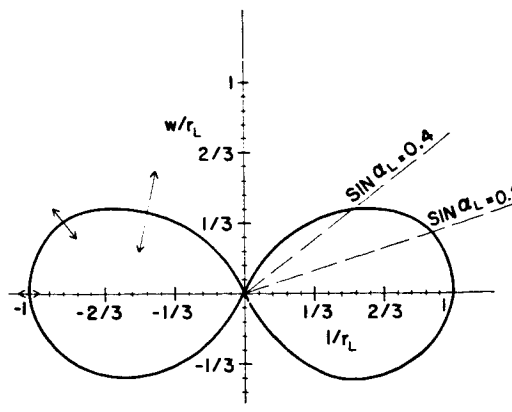

FIG. 3.—The (y,z) plane of Fig. 1 illustrating sample planes of polarization and the mirror points for limiting helix angles.

Fig. 1) illustrating the planes of polarization of the synchrotron radiation and the mirror points with limiting helix angles α_L, at the equatorial plane.

For the Stokes parameter expressions of equations (2)–(4), it is necessary to express the following quantities—(i) B; (ii) $\sin \alpha^*$; (iii) dV; (iv) $\cos 2\chi$, $\sin 2\chi$; (v) $n(\alpha^*)$—in terms of the polar coordinates (r,θ,ϕ) and the parameters θ_0 and α_L.

i) The magnetic field, B: The plane containing the point (r,θ,ϕ) and the magnetic-field line passing through it is shown in Figure 4. The field line is defined by

$$r = r_E \sin^2 \theta \,, \tag{13}$$

where r_E is the value of r at the intersection with the dipole equatorial plane ($\theta = \pi/2$). The field may be written

$$\boldsymbol{B}(r,\theta,\phi) = B(r_E,\pi/2)h^{1/2} \sin^{-6} \theta \,, \tag{14}$$

where we define

$$h \equiv 3 \cos^2 \theta + 1 \,. \tag{15}$$

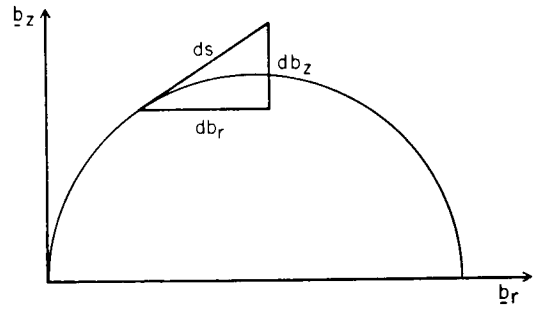

FIG. 4.—The plane containing a magnetic field line such as that in Fig. 1

ii) The angle between $\boldsymbol{B}$ and r_0, α^*: The angle α^* is the angle between the field and the observer (i.e., between $\boldsymbol{B}$ and r_0) and is given by

$$\cos \alpha^* = (r_0 \cdot \boldsymbol{B})/B \,, \tag{16}$$

$$\cos \alpha^* = \cos \theta^* \cos \theta_0 + \sin \theta^* \cos \theta_0 \cos \phi \,, \tag{17}$$

where θ^* is shown in Figure 4. The other quantities in Figure 4 are given by

$$b_r = r \sin \theta = r_E \sin^3 \theta \,, \tag{18}$$

$$b_z = r \cos \theta = r_E (\cos \theta - \cos^3 \theta) \,; \tag{19}$$

differentiation gives

$$\frac{d b_r}{d b_z} = \tan \theta^* = 3 \cos \theta \sin \theta (h - 2)^{-1} \,. \tag{20}$$

The differential arc length, ds,

$$ds^2 = db_r^2 + db_z^2 \,, \tag{21}$$

may be written

$$ds = r_E \sin\theta\, h^{1/2} d\theta \tag{22}$$

and

$$\sin\theta^* = \frac{db_r}{ds} = 3\sin\theta\cos\theta\, h^{-1/2}, \qquad \cos\theta^* = \frac{db_z}{ds} = (h-2)\, h^{-1/2}. \tag{23}$$

Combining equation (17) with equations (23) yields

$$\cos\alpha^* = h^{-1/2}[(h-2)\cos\theta_0 + 3\sin\theta_0 \sin\theta\cos\theta\cos\phi], \tag{24}$$

$$\sin^2\alpha^* = h^{-1}\sigma, \tag{25}$$

where σ is defined as

$$\sigma \equiv h - [(h-2)\cos\theta_0 + 3\sin\theta_0\sin\theta\cos\theta\cos\phi]^2.$$

iii) The incremental volume, dV: The radiating electrons are assumed to pass through the equatorial plane in a thin shell with thickness dr_E. The incremental volume dV along a corresponding tube of flux is given in polar coordinates by Chang and Davis (1962):

$$dV = r_E^2 \sin^7\theta\, d\theta\, d\phi\, dr_E. \tag{26}$$

iv) The angle between $\boldsymbol{i}_t$ and $\boldsymbol{B}, \chi$: The quantities $\cos 2\chi$ and $\sin 2\chi$ may be found from

$$\cos 2\chi = [(\boldsymbol{B}\cdot\boldsymbol{e}_y)^2 - (\boldsymbol{B}\cdot\boldsymbol{i}_r)^2](B^2\sin^2\alpha^*)^{-1}, \tag{27}$$

$$\sin 2\chi = -2[(\boldsymbol{B}\cdot\boldsymbol{i}_r)(\boldsymbol{B}\cdot\boldsymbol{e}_y)](B^2\sin^2\alpha^*)^{-1}, \tag{28}$$

where

$$\boldsymbol{B}\cdot\boldsymbol{e}_y = B\frac{dy}{ds} = 3Bh^{-1/2}\sin\theta\cos\theta\sin\phi, \tag{29}$$

$$\boldsymbol{B}\cdot\boldsymbol{i}_r = B\left[\frac{dx}{ds}(-\cos\theta_0) + \frac{dz}{ds}\sin\theta_0\right] \tag{30}$$

$$= Bh^{-1/2}[\sin\theta_0(h-2) - 3\cos\theta_0\sin\theta\cos\theta\sin\phi].$$

v) The angular distribution, $n(\alpha^*)$: Finally $\rho(\alpha^*)$ is required where $\rho(\alpha)dVdEd\alpha$ is the number of particles in an element of volume dV, energy interval dE, and helix angle interval $d\alpha$. It is convenient to express $\rho(\alpha)$ in terms of the distribution function in the equatorial plane ρ_0. This may be done using Liouvilles' theorem. Thus, let $\rho_0\, dVdEd\Omega$ be the number of particles in the volume dV in the solid angle $d\Omega$ at pitch angle $\pi/2$. Further, let $\rho_1\, dVdEd\Omega$ be the number at any other location and any other helix angle, α. Then, with ρ_0 isotropic for helix angles between limiting angles α_L and $\pi - \alpha_L$,

$$\rho_1 = \begin{cases} \rho_0 \sin\alpha > \beta^{-1}\sin\alpha_L \\ 0 \text{ otherwise}, \end{cases} \tag{31}$$

where

$$\beta = \sin^3\theta h^{-1/4} = B^{1/2}(r_E, \pi/2)B^{-1/2}(r_E\sin^2\theta, \theta). \tag{32}$$

The distributions $\rho(\alpha)$ and $\rho_1(\alpha)$ are related by

$$\rho = \rho_1(2\pi\sin\alpha). \tag{33}$$

Taking

$$\rho_0 = \begin{cases} \dfrac{\eta(E)}{4\pi \cos \alpha_L R_E \delta R_E} & \begin{cases} \sin \alpha_E > \sin \alpha_L \\ R_E < r_E < R_E + \delta R_E \end{cases} \\ 0 & \text{otherwise}, \end{cases} \tag{34}$$

we then have

$$\rho = \begin{cases} \dfrac{\eta(E) \sin \alpha}{2 \cos \alpha_L R_E \delta R_E} & \begin{cases} \sin \alpha > \beta^{-1} \sin \alpha_L \\ R_E < r_E < R_E + \delta R_E \end{cases} \\ 0 & \text{otherwise}. \end{cases} \tag{35}$$

Now, by comparison of the assumed form of the distribution in equation (7) with

$$\eta(E) = C_2 E^{-(\gamma+1)}, \tag{36}$$

we have for $n(r,\alpha)$

$$n(r,\alpha) = \begin{cases} \dfrac{\sin \alpha}{2 \cos \alpha_L R_E \delta R_E} & \begin{cases} \sin \alpha > \beta^{-1} \sin \alpha_L \\ R_E < r_E < R_E + \delta R_E \end{cases} \\ 0 & \text{otherwise}, \end{cases} \tag{37}$$

and for $n(\alpha^*)$

$$n(\alpha^*) = \begin{cases} \dfrac{\sin \alpha^*}{2 \cos \alpha_L R_E \delta R_E} & \begin{cases} \sin \alpha > \beta^{-1} \sin \alpha_L \\ R_E < r_E < R_E + \delta R_E \end{cases} \\ 0 & \text{otherwise}. \end{cases} \tag{38}$$

vi) Total number of electrons in a shell, $N_E\, dE$: We now have the required functions to obtain the Stokes parameters in terms of the variables (θ,ϕ) and the parameters γ, θ_0, and α_L. It is of interest, however, to digress and relate the equatorial distribution function, ρ_0, of equation (34) to the total number of electrons, $N_E dE$, in the shells, δR_E, with energies in the range dE. From equation (35), the number of particles per unit volume at colatitude θ with energy in dE and with any helix angle is

$$\rho_V(\theta) = \frac{\eta(E) \cos \alpha_m}{R_E \delta R_E \cos \alpha_L}, \tag{39}$$

where $\sin \alpha_m = \beta^{-1} \sin \alpha_L$. Integration gives

$$N_E = \int_{R_E}^{R_E+\delta R_E} \int_0^{2\pi} \int_{\theta_m}^{\pi-\theta_m} \rho_V(\theta) r_E^2 \sin^7 \theta\, d\theta\, d\phi\, dr_E = \frac{4\pi \eta(E) R_E}{\cos \alpha_L} J(\theta_m), \tag{40}$$

where

$$J(\theta_m) = \int_{\theta_m}^{\pi/2} \sin^7 \theta (1 - \beta^{-2}(\theta) \sin^2 \alpha_L)^{1/2}\, d\theta \tag{41}$$

and $\theta = \theta_m$ is the colatitude at which the limiting particles mirror, i.e., $\beta(\theta_m) = \sin \alpha_L$.

To obtain an approximate expression for $J(\theta_m)$ we note that for an isotropic distribution, i.e., $\alpha_L = 0$;

$$J(\theta_m) = \int_0^{\pi/2} \sin^7 \theta\, d\theta = 0.457. \tag{42}$$

At the opposite extreme the electrons are all in very flat orbits, i.e., $(\pi/2 - \alpha) \ll 1$. Setting $\cos \theta = x \ll 1$, we have

$$J(\theta_m) = \int_0^{x_m} (1 - 3x^2 + 3x^4 + \ldots) \left[1 - \frac{(1 - \frac{7}{2}x_m^2 + \frac{87}{8}x_m^4 + \ldots)}{(1 - \frac{7}{2}x^2 + \frac{87}{8}x^4)} \right]^{1/2} dx, \tag{43}$$

where

$$\sin^2 \alpha_L = \beta^2(\theta_m) = 1 - \tfrac{9}{2}x_m^2 + \tfrac{87}{8}x_m^4 + \dots . \tag{44}$$

This gives

$$J(\theta_m) = \frac{3}{4\sqrt{2}}\pi x_m^2[1 - \tfrac{109}{96}x_m^2 + \dots] \tag{45}$$

when $(\pi/2 - \alpha_L) \ll 1$. Comparison of the extreme cases of equations (42) and (45) suggests that a reasonable approximation to $J(\theta_m)$ over the entire range of θ_m is

$$J(\theta_m) \doteq \frac{3\pi}{4\sqrt{2}} x_m^2(1 - 1.15x_m^2 + 0.43x_m^4). \tag{46}$$

This yields

$$\begin{aligned} N_E &= (4\pi\eta(E)R_E)\frac{(3\pi/4\sqrt{2})x_m^2(1 - 1.15x_m^2 + 0.43x_m^4)}{(3/\sqrt{2})x_m(1 - 1.21x_m^2 + \dots)} \\ &\doteq \pi^2\eta(E)R_E x_m(1 + \tfrac{7}{12}x_m^2) \end{aligned} \tag{47}$$

with x_m determined from equation (44).

III. SUMMARY AND RESULTS

a) *The Equations*

If we now combine the expressions for the various functions derived in the previous section the first Stokes parameter, I, may be written in the following form

$$IdV = D_I d\theta d\phi \sin^{1-3\gamma}\theta\sigma^{(\gamma+2)/4} \tag{48}$$

or

$$I_0 \equiv \frac{IdV}{D_I d\theta d\phi} = \sin^{1-3\gamma}\theta\,\sigma^{(\gamma+2)/4}, \tag{49}$$

where

$$D_I \equiv \tfrac{1}{2}f^{-\gamma/2}B^{(\gamma+4)/2}(r_E,\pi/2)R_E\cos^{-1}\alpha_L, \tag{50}$$

$$\sigma = (3\cos^2\theta + 1) - [(3\cos^2\theta - 1)\cos\theta_0 + 3\sin\theta_0\sin\theta\cos\theta\cos\phi]^2. \tag{51}$$

Similarly the second and third Stokes parameters may be written

$$Q_0 = \frac{QdV}{D_Q d\theta d\phi} = I_0\frac{a^2 - b^2}{a^2 + b^2}, \tag{52}$$

$$U_0 = \frac{UdV}{D_U d\theta d\phi} = I_0 a b\sigma^{-1}, \tag{53}$$

where

$$a \equiv 3\sin\theta\cos\theta\sin\phi, \tag{54}$$

$$b \equiv (3\cos^2\theta - 1)\sin\theta_0 - 3\cos\theta_0\sin\theta\cos\theta\cos\phi, \tag{55}$$

$$D_Q \equiv D_I D_2 \equiv -\tfrac{1}{2}D_U. \tag{56}$$

Finally, from equations (2), (3), and (13)

$$\frac{l}{r_E} = \sin^3\theta\sin\phi, \tag{57}$$

$$\frac{w}{r_E} = \sin^2\theta\cos\theta\sin\theta_0 - \sin^3\theta\cos\theta\cos\phi. \tag{58}$$

b) The Calculations

The functions I_0, Q_0, and U_0 were calculated and are presented as a function of l/r_E and w/r_E. The final equations and definition of constants are summarized in the Appendix. The distributions of I_0, Q_0, and U_0 were obtained for

$$0 \leq \theta_i \leq \pi/2, \quad 0 \leq \phi_i \leq \pi \tag{59}$$

in two degree increments (i.e., $d\theta = d\phi = 2°$). The method of calculation used in the computer consisted of (i) setting ϕ, (ii) calculating I_0, Q_0, and U_0, (iii) calculating l/r_E and w/r_E, (iv) finding the proper square in the (l,w)-plane, (v) determining the relative position of the radiating volume with respect to the (l,w)-plane (the sign of eq. [6]), and finally (vi) combining it with the other contributions coming from that square.

Because of symmetry about the dipole axis, calculation of the distribution in one quadrant is sufficient. However, allowance for blocking by a planet requires separation of the contributions to the parameters from each side of the (l,w)-plane. Therefore, the distributions of the total contribution from both sides of the plane are given in the first quadrant for the *first* quadrant and that of the difference between the front and back contributions are given in the fourth quadrant for the *fourth* quadrant.

It was convenient for computing purposes to compute the differences in the fourth quadrant in terms of the functions in the first quadrant. Let the subscript designate the quadrant and the asterisk designate contribution from the back side of the (l,w)-plane, $(d < 0)$. By examination of symmetry in equations (8)–(10),

$$\begin{gathered} I_1 = I_4^*, \quad I_1^* = I_4; \quad Q_1 = Q_4^*, \quad Q_1^* = Q_4\,; \\ U_1 = -U_4^*, \quad U_1^* = -U_4\,. \end{gathered} \tag{60}$$

Thus the components in the fourth quadrant may be expressed in terms of the first-quadrant components

$$I_4 - I_4^* = I_1^* - I_1\,, \quad Q_4 - Q_4^* = Q_1^* - Q_1\,, \quad U_4 - U_4^* = U_1 - U_1^*\,. \tag{61}$$

The right-hand sides of equations (61) were calculated. The results are presented in the form of a sign (if negative), decimal point, two digits, and sign and magnitude of the power of the tens multiplier (e.g., $-0.31 \times 10^4 = -0.31 + 04$). The (l,w)-plane is divided into 225 squares, that is:

$$\Delta(l/r_E) = \Delta(w/r_E) = \tfrac{1}{15}\,. \tag{62}$$

The total contribution to the three Stokes parameters from each square are listed in the order I_0, Q_0, U_0, the numbers themselves forming the squares. Where no contribution to any of the Stokes parameters occurs, the resulting zeros were masked off. Occasionally zero times ten to different powers occurs (e.g., 00-31 and 00-04). There is no significance to be attached to this difference in zeros. There is just no contribution to that particular parameter for that square.

The distributions were calculated for all but five (the cases for $\gamma = 1.5$, $\theta_0 = 10°$) of the possible combinations of the values of the three parameters γ, θ_0, and α_L listed in Table 1 of the Appendix. The calculated distributions are presented in the Appendix with the exception of those cases for which no contributions were obtained. The values of the parameters in these cases are indicated in Table 2 of the Appendix.

Tables 3–30 are for the $\gamma = 0$ energy distributions while Tables 31–54 are for the $\gamma = 1.5$ energy distributions. They are grouped within each set by aspect angle, θ_0: increasing from 0° to 90°. Therefore each group of five (four when there are no contributions) has the same aspect angle with successive distributions corresponding to increasing values of the sine of the cutoff helix angle, α_L.

c) *Accuracy of Calculations*

The calculations were done on a Raytheon 250 computer with subroutine error less than 0.01 per cent. Several sample distributions were obtained using values of $d\theta$ and $d\phi$ of 1°, 2°, and 5°.

A considerable improvement in the smoothness of the distributions was evident between the 5° and 2° increment distributions. Little seemed to be gained using the 1° increment while the computation time was quadrupled. The final accuracy of the distributions was estimated less than 0.2 per cent. However, because of difficulty in the computer's interpreter program used, the final results were truncated instead of the normal rounding off. Thus the calculated value lies between the reported value and the next lower value, i.e., for a reported value of $I = 0.73 + 01$,

$$0.72 + 01 < I_0 \leq 0.73 + 01 \,, \tag{63}$$

instead of $(0.73 \pm 0.005) + 01$.

By summing the total contributions in three columns (i.e., all the contributions in the strip $0 < l/r_E \leq 0.2$) a comparison may be made with the sum of the equivalent two strips in Chang and Davis (1962). Agreement was within 4 per cent for increments of 5°.

d) *Discussion*

The two-dimensional results reflect the dependence of the intensity and polarization on the field alignment, radiation efficiency, electron density, and radiating angle. Thus the brightness pattern tends to be more uniformly distributed for an isotropic distribution of particles approaching a thin vanishing ring as α_L increases for $\theta_0 = 0$ and a more slowly vanishing thin wedge-shaped equatorial strip as α_L increases for $\theta_0 = 90°$. In general, the pattern outline progresses smoothly from a circular to a flatter elliptical shape as θ_0 increases, and becomes a butterfly pattern for $\theta_0 = 90°$ and large α_L. Shadowing by a small centrally located object would appear to be least important for large α_L. It is interesting to note that for the case $\theta_0 = 30°$ and $\sin \alpha_L = 0.8$ the distribution actually consists of two separated portions lying along the dipole axis.

Polarizations run the gamut as θ_0 and α_L are varied, the over-all degree of polarization increasing as θ_0 and α_L are increased. The patterns depend much more strongly on the aspect angle and the helix angle than on the energy distribution, i.e., γ, while the frequency behavior is dependent only on the energy spectrum assumed and is independent of the aspect angle or the helix angle.

The results for other angular and energy distributions may be approximated by combining the patterns of various figures. In this way situations may be constructed where both the polarization and the brightness distribution are frequency dependent. It must be remembered, however, that the calculations do not provide for either differential absorption or differential Faraday rotation. It is straightforward to include these effects if they occur primarily external to the source, but in the present figures, these effects occurring within the source itself can only be approximated to the extent different correction factors may be applied to the radiation originating in front of and behind the plane which intersects the dipole and is normal to the line of sight.

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APPENDIX

The Stokes parameters as evaluated for a power-law distribution of the form

$$\rho(r,E,\alpha) = n(r,\alpha)C_2E^{-(\gamma+1)}$$

were

$$I_0 = \frac{I\,dV}{D_I d\theta d\phi} = \sin^{1-3\gamma}\theta\{(3\cos^2\theta+1)$$

$$-[(3\cos^2\theta-1)\cos\theta_0+3\sin\theta_0\sin\theta\cos\theta\sin\phi]^2\}^{(\gamma+2)/4},$$

$$Q_0 = \frac{Q\,dV}{D_Q d\theta d\phi}$$

$$= I_0\frac{(3\sin\theta\cos\theta\sin\phi)^2-[(3\cos^2\theta-1)\sin\theta_0-3\cos\theta_0\sin\theta\cos\theta\cos\phi]^2}{(3\sin\theta\cos\theta\sin\phi)^2+[(3\cos^2\theta-1)\sin\theta_0-3\cos\theta_0\sin\theta\cos\theta\cos\phi]^2},$$

$$U_0 = \frac{U\,dV}{D_U d\theta d\phi}$$

$$= I_0\frac{(3\sin\theta\cos\theta\sin\phi)[(3\cos^2\theta-1)\sin\theta_0-3\cos\theta_0\sin\theta\cos\theta\cos\phi]}{(3\cos^2\theta+1)-[(3\cos^2\theta-1)\cos\theta_0+3\sin\theta_0\sin\theta\cos\theta\cos\phi]^2},$$

when

$$\sin\alpha > \frac{\sin\alpha_L B^{1/2}(r_E\sin^2\theta,\theta)}{B^{1/2}(r_E,\pi/2)}$$

and were zero otherwise. The three parameters of the distributions are the power-law exponent γ, the aspect angle θ_0, and the cutoff helix angle α_L. The variables are the polar coordinate angles (θ,ϕ) as defined in Figure 1 of the text. They were varied in 2° increments in the range from 0 to π radians.

The constants are given by

$$D_I = \tfrac{1}{2}D_1 f^{-\gamma/2}B^{(\gamma+4)/2}(r_E,\pi/2)\,R_E\cos^{-1}\alpha_L\,;$$

$$D_Q = \tfrac{1}{2}D_U \equiv -D_I\left(\frac{3\gamma+6}{3\gamma+10}\right);$$

$$D_1 \equiv R^{-2}CC_2L^{\gamma/2}2^{(\gamma-4)/2}\left(\frac{3\gamma+10}{3\gamma+6}\right)\Gamma\left(\frac{3\gamma+2}{12}\right)\Gamma\left(\frac{3\gamma+10}{12}\right);$$

$$C \doteq 3.73\times10^{-23}\ \text{erg sec}^{-1}\ \text{gauss}^{-1}\,;$$

$$L \doteq 1.608\times10^{13}\ \text{c/s gauss}^{-1}\ \text{BeV}^{-2}\,.$$

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TABLE 1

THE VALUES OF THE PARAMETERS USED IN THE DISTRIBUTIONS

Electron Energy Distribution, E^{γ} γ	Aspect Angle, θ_0 (degrees) θ_0	Cutoff Helix Angle, α_L sin α_L
0	0	0.0
1.5	10*	0.2
	30	0.4
	60	0.6
	80	0.8
	90	

* All combinations of γ, θ_0, and α_L were used except for those five cases with $\gamma=1.5$ and $\theta_0=10°$.

TABLE 2

THE VALUES OF THE PARAMETERS FOR WHICH THERE ARE NO CONTRIBUTIONS TO THE DISTRIBUTION

γ	θ_0	sin α_L
0	0	0.8
0	10	0.8
1.5	0	0.8

TABLE 3

DISTRIBUTION FOR $\gamma = 0$, $\theta_0 = 0°$, AND $\sin \alpha_L = 0$

.73+01	.73+01	.62+01	.42+01	.25+01										
-.73+01	-.71+01	-.58+01	-.37+01	-.21+01										
.85-01	.25+00	.34+00	.32+00	.25+00										
.86+01	.86+01	.65+01	.85+01	.73+01	.73+01	.42+01	.41+00							
-.86+01	-.84+01	-.61+01	-.75+01	-.59+01	-.52+01	-.26+01	-.22+00							
.10+00	.30+00	.37+00	.67+00	.72+00	.85+00	.55+00	.59-01							
.72+01	.10+02	.83+01	.81+01	.73+01	.81+01	.71+01	.74+01	.25+01						
-.72+01	-.10+02	-.76+01	-.68+01	-.56+01	-.54+01	-.40+01	-.35+01	-.94+00						
.84-01	.43+00	.53+00	.72+00	.79+00	.10+01	.97+00	.10+01	.40+00						
.10+02	.10+02	.77+01	.11+02	.89+01	.85+01	.78+01	.94+01	.79+01	.43+01					
-.10+02	-.97+01	-.70+01	-.95+01	-.65+01	-.53+01	-.39+01	-.37+01	-.22+01	-.79+00					
.15+00	.43+00	.52+00	.10+01	.10+01	.11+01	.11+01	.14+01	.12+01	.70+00					
.10+02	.43+01	.12+02	.84+01	.12+02	.77+01	.92+01	.67+01	.72+01	.66+01	.69+01				
-.10+02	-.41+01	-.11+02	-.68+01	-.84+01	-.43+01	-.41+01	-.20+01	-.15+01	-.68+00	-.10-07				
.16+00	.19+00	.92+00	.83+00	.14+01	.10+01	.13+01	.10+01	.11+01	.10+01	.11+01				
.90+01	.13+02	.13+02	.88+01	.87+01	.10+02	.81+01	.11+02	.85+01	.72+01	.66+01	.43+01			
-.89+01	-.12+02	-.11+02	-.65+01	-.55+01	-.54+01	-.30+01	-.28+01	-.96+00	-.10-07	.68+00	.79+00			
.18+00	.69+00	.11+01	.98+00	.11+01	.15+01	.12+01	.18+01	.14+01	.12+01	.10+01	.70+00			
.13+02	.18+02	.68+01	.15+02	.11+02	.88+01	.12+02	.10+02	.93+01	.85+01	.72+01	.79+01	.25+01		
-.13+02	-.17+02	-.57+01	-.11+02	-.60+01	-.35+01	-.32+01	-.12+01	-.14-07	.96+00	.15+01	.22+01	.94+00		
.24+00	.10+01	.64+00	.18+01	.15+01	.13+01	.20+01	.16+01	.15+01	.14+01	.11+01	.12+01	.40+00		
.16+02	.92+01	.18+02	.92+01	.11+02	.13+02	.89+01	.10+02	.10+02	.11+02	.67+01	.94+01	.74+01	.41+00	
-.15+02	-.83+01	-.14+02	-.56+01	-.56+01	-.42+01	-.12+01	-.16-07	.12+01	.28+01	.20+01	.37+01	.35+01	.22+00	
.33+00	.62+00	.18+01	.12+01	.16+01	.21+01	.14+01	.17+01	.16+01	.18+01	.10+01	.14+01	.10+01	.59-01	
.11+02	.20+02	.15+02	.16+02	.13+02	.11+02	.13+02	.89+01	.12+02	.81+01	.92+01	.78+01	.71+01	.42+01	
-.11+02	-.18+02	-.11+02	-.87+01	-.47+01	-.17+01	-.20-07	.12+01	.32+01	.30+01	.41+01	.39+01	.40+01	.26+01	
.29+00	.14+01	.17+01	.22+01	.21+01	.18+01	.22+01	.14+01	.20+01	.12+01	.13+01	.11+01	.97+00	.55+00	
.21+02	.21+02	.17+02	.15+02	.15+02	.13+02	.11+02	.13+02	.88+01	.10+02	.77+01	.85+01	.81+01	.73+01	
-.21+02	-.18+02	-.11+02	-.64+01	-.33+01	-.20-07	.17+01	.42+01	.35+01	.54+01	.43+01	.53+01	.54+01	.52+01	
.62+00	.17+01	.22+01	.23+01	.25+01	.22+01	.18+01	.21+01	.13+01	.15+01	.10+01	.11+01	.10+01	.85+00	
.25+02	.20+02	.20+02	.19+02	.20+02	.15+02	.13+02	.11+02	.11+02	.87+01	.12+02	.89+01	.73+01	.73+01	.25+01
-.25+02	-.16+02	-.11+02	-.50+01	-.30-07	.33+01	.47+01	.56+01	.60+01	.55+01	.84+01	.65+01	.56+01	.59+01	.21+01
.96+00	.19+01	.28+01	.31+01	.33+01	.25+01	.21+01	.16+01	.15+01	.11+01	.14+01	.10+01	.79+00	.72+00	.25+00
.30+02	.27+02	.27+02	.16+02	.19+02	.15+02	.16+02	.92+01	.15+02	.88+01	.84+01	.11+02	.81+01	.85+01	.42+01
-.28+02	-.18+02	-.86+01	-.24-07	.50+01	.64+01	.87+01	.56+01	.11+02	.65+01	.68+01	.95+01	.68+01	.75+01	.37+01
.14+01	.33+01	.43+01	.27+01	.31+01	.23+01	.22+01	.12+01	.18+01	.98+00	.83+00	.10+01	.72+00	.67+00	.32+00
.42+02	.39+02	.32+02	.27+02	.20+02	.17+02	.15+02	.18+02	.68+01	.13+02	.12+02	.77+01	.83+01	.65+01	.62+01
-.38+02	-.17+02	-.47-07	.86+01	.11+02	.11+02	.11+02	.14+02	.57+01	.11+02	.11+02	.70+01	.76+01	.61+01	.58+01
.23+01	.56+01	.53+01	.43+01	.28+01	.22+01	.17+01	.18+01	.64+00	.11+01	.92+00	.52+00	.53+00	.37+00	.34+00
.72+02	.47+02	.39+02	.27+02	.20+02	.21+02	.20+02	.92+01	.18+02	.13+02	.43+01	.10+02	.10+02	.86+01	.73+01
-.53+02	-.67-07	.17+02	.18+02	.16+02	.18+02	.18+02	.83+01	.17+02	.12+02	.41+01	.97+01	.10+02	.84+01	.71+01
.69+01	.77+01	.56+01	.33+01	.19+01	.17+01	.14+01	.62+00	.10+01	.69+00	.19+00	.43+00	.43+00	.30+00	.25+00
.19+03	.72+02	.42+02	.30+02	.25+02	.21+02	.11+02	.16+02	.13+02	.90+01	.10+02	.10+02	.72+01	.86+01	.73+01
-.88-06	.53+02	.38+02	.28+02	.25+02	.21+02	.11+02	.15+02	.13+02	.89+01	.10+02	.10+02	.72+01	.86+01	.73+01
.21+02	.69+01	.23+01	.14+01	.96+00	.62+00	.29+00	.33+00	.24+00	.18+00	.16+00	.15+00	.84-01	.10+00	.85-01

TABLE 4

DISTRIBUTION FOR $\gamma = 0$, $\theta_0 = 0°$, AND $\sin \alpha_L = 0.2$

.65+01	.65+01	.54+01	.33+01	.17+01										
-.64+01	-.63+01	-.50+01	-.30+01	-.14+01										
.75-01	.22+00	.30+00	.26+00	.16+00										
.86+01	.86+01	.65+01	.85+01	.73+01	.65+01	.33+01								
-.86+01	-.84+01	-.61+01	-.75+01	-.59+01	-.46+01	-.21+01								
.10+00	.30+00	.37+00	.67+00	.72+00	.75+00	.44+00								
.72+01	.10+02	.83+01	.81+01	.73+01	.81+01	.71+01	.70+01	.17+01						
-.72+01	-.10+02	-.76+01	-.68+01	-.56+01	-.54+01	-.40+01	-.33+01	-.63+00						
.84-01	.43+00	.53+00	.72+00	.79+00	.10+01	.97+00	.10+01	.27+00						
.10+02	.10+02	.77+01	.11+02	.89+01	.85+01	.78+01	.94+01	.76+01	.33+01					
-.10+02	-.97+01	-.70+01	-.95+01	-.65+01	-.53+01	-.39+01	-.37+01	-.21+01	-.62+00					
.15+00	.43+00	.52+00	.10+01	.10+01	.11+01	.11+01	.14+01	.12+01	.55+00					
.10+02	.43+01	.12+02	.84+01	.12+02	.77+01	.92+01	.67+01	.72+01	.66+01	.57+01				
-.10+02	-.41+01	-.11+02	-.68+01	-.84+01	-.43+01	-.41+01	-.20+01	-.15+01	-.68+00	-.85-08				
.16+00	.19+00	.92+00	.83+00	.14+01	.10+01	.13+01	.10+01	.11+01	.10+01	.95+00				
.90+01	.13+02	.13+02	.88+01	.87+01	.10+02	.81+01	.11+02	.85+01	.72+01	.66+01	.33+01			
-.89+01	-.12+02	-.11+02	-.65+01	-.55+01	-.54+01	-.30+01	-.28+01	-.96+00	-.10-07	.68+00	.62+00			
.18+00	.69+00	.11+01	.98+00	.11+01	.15+01	.12+01	.18+01	.14+01	.12+01	.10+01	.55+00			
.13+02	.18+02	.68+01	.15+02	.11+02	.88+01	.12+02	.10+02	.93+01	.85+01	.72+01	.76+01	.17+01		
-.13+02	-.17+02	-.57+01	-.11+02	-.60+01	-.35+01	-.32+01	-.12+01	-.14-07	.96+00	.15+01	.21+01	.63+00		
.24+00	.10+01	.64+00	.18+01	.15+01	.13+01	.20+01	.16+01	.15+01	.14+01	.11+01	.12+01	.27+00		
.16+02	.92+01	.18+02	.92+01	.11+02	.13+02	.89+01	.10+02	.10+02	.11+02	.67+01	.94+01	.70+01		
-.15+02	-.83+01	-.14+02	-.56+01	-.56+01	-.42+01	-.12+01	-.16-07	.12+01	.28+01	.20+01	.37+01	.33+01		
.33+00	.62+00	.18+01	.12+01	.16+01	.21+01	.14+01	.17+01	.16+01	.18+01	.10+01	.14+01	.10+01		
.11+02	.20+02	.15+02	.16+02	.13+02	.11+02	.13+02	.89+01	.12+02	.81+01	.92+01	.78+01	.71+01	.33+01	
-.11+02	-.18+02	-.11+02	-.87+01	-.47+01	-.17+01	-.20-07	.12+01	.32+01	.30+01	.41+01	.39+01	.40+01	.21+01	
.29+00	.14+01	.17+01	.22+01	.21+01	.18+01	.22+01	.14+01	.20+01	.12+01	.13+01	.11+01	.97+00	.44+00	
.21+02	.21+02	.17+02	.15+02	.15+02	.13+02	.11+02	.13+02	.88+01	.10+02	.77+01	.85+01	.81+01	.65+01	
-.21+02	-.18+02	-.11+02	-.64+01	-.33+01	-.20-07	.17+01	.42+01	.35+01	.54+01	.43+01	.53+01	.54+01	.46+01	
.62+00	.17+01	.22+01	.23+01	.25+01	.22+01	.18+01	.21+01	.13+01	.15+01	.10+01	.11+01	.10+01	.75+00	
.12+02	.14+02	.20+02	.19+02	.20+02	.15+02	.13+02	.11+02	.11+02	.87+01	.12+02	.89+01	.73+01	.73+01	.17+01
-.11+02	-.11+02	-.11+02	-.50+01	-.30-07	.33+01	.47+01	.56+01	.60+01	.55+01	.84+01	.65+01	.56+01	.59+01	.14+01
.42+00	.14+01	.28+01	.31+01	.33+01	.25+01	.21+01	.16+01	.15+01	.11+01	.14+01	.10+01	.79+00	.72+00	.16+00
		.81+01	.16+02	.19+02	.15+02	.16+02	.92+01	.15+02	.88+01	.84+01	.11+02	.81+01	.85+01	.33+01
		-.25+01	-.24-07	.50+01	.64+01	.87+01	.56+01	.11+02	.65+01	.68+01	.95+01	.68+01	.75+01	.30+01
		.12+01	.27+01	.31+01	.23+01	.22+01	.12+01	.18+01	.98+00	.83+00	.10+01	.72+00	.67+00	.26+00
			.81+01	.20+02	.17+02	.15+02	.18+02	.68+01	.13+02	.12+02	.77+01	.83+01	.65+01	.54+01
			.25+01	.11+02	.11+02	.11+02	.14+02	.57+01	.11+02	.11+02	.70+01	.76+01	.61+01	.50+01
			.12+01	.28+01	.22+01	.17+01	.18+01	.64+00	.11+01	.92+00	.52+00	.53+00	.37+00	.30+00
				.14+02	.21+02	.20+02	.92+01	.18+02	.13+02	.43+01	.10+02	.10+02	.86+01	.65+01
				.11+02	.18+02	.18+02	.83+01	.17+02	.12+02	.41+01	.97+01	.10+02	.84+01	.63+01
				.14+01	.17+01	.14+01	.62+00	.10+01	.69+00	.19+00	.43+00	.43+00	.30+00	.22+00
				.12+02	.21+02	.11+02	.16+02	.13+02	.90+01	.10+02	.10+02	.72+01	.86+01	.65+01
				.11+02	.21+02	.11+02	.15+02	.13+02	.89+01	.10+02	.10+02	.72+01	.86+01	.64+01
				.42+00	.62+00	.29+00	.33+00	.24+00	.18+00	.16+00	.15+00	.84-01	.10+00	.75-01

TABLE 5

DISTRIBUTION FOR $\gamma = 0$, $\theta_0 = 0°$, AND $\sin \alpha_L = 0.4$

.40+01	.40+01	.22+04	.91+00										
-.40+01	-.39+01	-.27+01	-.82+00										
.46-01	.13+00	.16+00	.66-01										
.86+01	.86+01	.65+01	.85+01	.66+01	.40+01	.91+00							
-.86+01	-.84+01	-.61+01	-.75+01	-.53+01	-.29+01	-.58+00							
.10+00	.30+00	.37+00	.67+00	.64+00	.46+00	.11+00							
.72+01	.10+02	.83+01	.81+01	.73+01	.81+01	.71+01	.38+01						
-.72+01	-.10+02	-.76+01	-.68+01	-.56+01	-.54+01	-.40+01	-.18+01						
.84-01	.43+00	.53+00	.72+00	.79+00	.10+01	.97+00	.56+00						
.10+02	.10+02	.77+01	.11+02	.89+01	.85+01	.78+01	.94+01	.64+01	.91+00				
-.10+02	-.97+01	-.70+01	-.95+01	-.65+01	-.53+01	-.39+01	-.37+01	-.18+01	-.19+00				
.15+00	.43+00	.52+00	.10+01	.10+01	.11+01	.11+01	.14+01	.10+01	.14+00				
.10+02	.43+01	.12+02	.84+01	.12+02	.77+01	.92+01	.67+01	.72+01	.66+01	.20+01			
-.10+02	-.41+01	-.11+02	-.68+01	-.84+01	-.43+01	-.41+01	-.20+01	-.15+01	-.68+00	-.30-08			
.16+00	.19+00	.92+00	.83+00	.14+01	.10+01	.13+01	.10+01	.11+01	.10+01	.33+00			
.90+01	.13+02	.13+02	.88+01	.87+01	.10+02	.81+01	.11+02	.85+01	.72+01	.66+01	.91+00		
-.89+01	-.12+02	-.11+02	-.65+01	-.55+01	-.54+01	-.30+01	-.28+01	-.96+00	-.10-07	.68+00	.19+00		
.18+00	.69+00	.11+01	.98+00	.11+01	.15+01	.12+01	.18+01	.14+01	.12+01	.10+01	.14+00		
.13+02	.18+02	.68+01	.15+02	.11+02	.88+01	.12+02	.10+02	.93+01	.85+01	.72+01	.64+01		
-.13+02	-.17+02	-.57+01	-.11+02	-.60+01	-.35+01	-.32+01	-.12+01	-.14-07	.96+00	.15+01	.18+01		
.24+00	.10+01	.64+00	.18+01	.15+01	.13+01	.20+01	.16+01	.15+01	.14+01	.11+01	.10+01		
.92+01	.92+01	.18+02	.92+01	.11+02	.13+02	.89+01	.10+02	.10+02	.11+02	.67+01	.94+01	.38+01	
-.90+01	-.83+01	-.14+02	-.56+01	-.56+01	-.42+01	-.12+01	-.16-07	.12+01	.28+01	.20+01	.37+01	.18+01	
.21+00	.62+00	.18+01	.12+01	.16+01	.21+01	.14+01	.17+01	.16+01	.18+01	.10+01	.14+01	.56+00	
			.92+01	.13+02	.11+02	.13+02	.89+01	.12+02	.81+01	.92+01	.78+01	.71+01	.91+00
			-.51+01	-.47+01	-.17+01	-.20-07	.12+01	.32+01	.30+01	.41+01	.39+01	.40+01	.58+00
			.12+01	.21+01	.18+01	.22+01	.14+01	.20+01	.12+01	.13+01	.11+01	.97+00	.11+00
				.23+01	.13+02	.11+02	.13+02	.88+01	.10+02	.77+01	.85+01	.81+01	.40+01
				-.47+00	-.20-07	.17+01	.42+01	.35+01	.54+01	.43+01	.53+01	.54+01	.29+01
				.37+00	.22+01	.18+01	.21+01	.13+01	.15+01	.10+01	.11+01	.10+01	.46+00
					.23+01	.13+02	.11+02	.11+02	.87+01	.12+02	.89+01	.73+01	.66+01
					.47+00	.47+01	.56+01	.60+01	.55+01	.84+01	.65+01	.56+01	.53+01
					.37+00	.21+01	.16+01	.15+01	.11+01	.14+01	.10+01	.79+00	.64+00
						.92+01	.92+01	.15+02	.88+01	.84+01	.11+02	.81+01	.85+01	.91+00
						.51+01	.56+01	.11+02	.65+01	.68+01	.95+01	.68+01	.75+01	.82+00
						.12+01	.12+01	.18+01	.98+00	.83+00	.10+01	.72+00	.67+00	.66-01
							.18+02	.68+01	.13+02	.12+02	.77+01	.83+01	.65+01	.29+01
							.14+02	.57+01	.11+02	.11+02	.70+01	.76+01	.61+01	.27+01
							.18+01	.64+00	.11+01	.92+00	.52+00	.53+00	.37+00	.16+00
							.92+01	.18+02	.13+02	.43+01	.10+02	.10+02	.86+01	.40+01
							.83+01	.17+02	.12+02	.41+01	.97+01	.10+02	.84+01	.39+01
							.62+00	.10+01	.69+00	.19+00	.43+00	.43+00	.30+00	.13+00
							.92+01	.13+02	.90+01	.10+02	.10+02	.72+01	.86+01	.40+0
							.90+01	.13+02	.89+01	.10+02	.10+02	.72+01	.86+01	.40+0
							.21+00	.24+00	.18+00	.16+00	.15+00	.84-01	.10+00	.46-0

TABLE 6

DISTRIBUTION FOR $\gamma = 0$, $\theta_0 = 0°$, AND $\sin \alpha_L = 0.6$

.32+01	.32+01													
-.31+01	-.31+01													
.37-01	.11+00													
.72+01	.10+02	.83+01	.67+01	.32+01	.16+01									
-.72+01	-.10+02	-.76+01	-.56+01	-.24+01	-.11+01									
.84-01	.43+00	.53+00	.59+00	.34+00	.19+00									
.10+02	.10+02	.77+01	.11+02	.89+01	.85+01	.49+01	.16+01							
-.10+02	-.97+01	-.70+01	-.95+01	-.65+01	-.53+01	-.25+01	-.65+00							
.15+00	.43+00	.52+00	.10+01	.10+01	.11+01	.70+00	.24+00							
		.62+01	.41+01	.12+02	.77+01	.92+01	.67+01	.16+01						
		-.56+01	-.33+01	-.84+01	-.43+01	-.41+01	-.20+01	-.33+00						
		.45+00	.40+00	.14+01	.10+01	.13+01	.10+01	.26+00						
					.62+01	.81+01	.11+02	.85+01	.16+01					
					-.33+01	-.30+01	-.28+01	-.96+00	-.23-08					
					.88+00	.12+01	.18+01	.14+01	.26+00					
						.20+01	.10+02	.93+01	.85+01	.16+01				
						-.57+00	-.12+01	-.14-07	.96+00	.33+00				
						.33+00	.16+01	.15+01	.14+01	.26+00				
							.20+01	.10+02	.11+02	.67+01	.16+01			
							-.31-08	.12+01	.28+01	.20+01	.65+00			
							.34+00	.16+01	.18+01	.10+01	.24+00			
								.20+01	.81+01	.92+01	.49+01			
								.57+00	.30+01	.41+01	.25+01			
								.33+00	.12+01	.13+01	.70+00			
									.62+01	.77+01	.85+01	.16+01		
									.33+01	.43+01	.53+01	.11+01		
									.88+00	.10+01	.11+01	.19+00		
										.12+02	.89+01	.32+01		
										.84+01	.65+01	.24+01		
										.14+01	.10+01	.34+00		
										.41+01	.11+02	.67+01		
										.33+01	.95+01	.56+01		
										.40+00	.10+01	.59+00		
										.62+01	.77+01	.83+01		
										.56+01	.70+01	.76+01		
										.45+00	.52+00	.53+00		
											.10+02	.10+02	.32+01	
											.97+01	.10+02	.31+01	
											.43+00	.43+00	.11+00	
											.10+02	.72+01	.32+01	
											.10+02	.72+01	.31+01	
											.15+00	.84-01	.37-01	

TABLE 7

DISTRIBUTION FOR $\gamma = 0$, $\theta_0 = 10°$, AND $\sin \alpha_L = 0$

.61+01	.61+01	.47+01	.34+01	.13+01										
-.61+01	-.59+01	-.42+01	-.27+01	-.66+00										
.67-01	.20+00	.24+00	.22+00	.85-01										
.81+01	.88+01	.80+01	.85+01	.79+01	.58+01	.29+01	.84-01							
-.81+01	-.86+01	-.74+01	-.73+01	-.61+01	-.34+01	-.12+01	.51-01							
.94-01	.32+00	.48+00	.69+00	.77+00	.58+00	.24+00	-.11-01							
.69+01	.86+01	.83+01	.80+01	.81+01	.90+01	.66+01	.76+01	.25+01						
-.69+01	-.84+01	-.76+01	-.68+01	-.61+01	-.59+01	-.34+01	-.27+01	-.68+00						
.80-01	.33+00	.52+00	.69+00	.86+00	.10+01	.86+00	.92+00	.16+00						
.98+01	.99+01	.96+01	.83+01	.99+01	.84+01	.90+01	.76+01	.68+01	.39+01					
-.98+01	-.95+01	-.86+01	-.69+01	-.71+01	-.52+01	-.46+01	-.28+01	-.13+01	-.27+00					
.15+00	.41+00	.69+00	.77+00	.11+01	.10+01	.12+01	.11+01	.93+00	.30+00					
.11+02	.10+02	.84+01	.12+02	.79+01	.86+01	.81+01	.98+01	.90+01	.77+01	.63+01				
-.11+02	-.10+02	-.75+01	-.98+01	-.54+01	-.48+01	-.35+01	-.28+01	-.19+01	-.92+00	.18+00				
.19+00	.50+00	.62+00	.12+01	.96+00	.11+01	.12+01	.15+01	.14+01	.11+01	.50+00				
.10+02	.10+02	.12+02	.99+01	.11+02	.93+01	.10+02	.94+01	.93+01	.73+01	.79+01	.47+01			
-.10+02	-.95+01	-.10+02	-.73+01	-.75+01	-.44+01	-.36+01	-.21+01	-.28+00	.20-01	.44+00	.72+00			
.17+00	.52+00	.98+00	.10+01	.14+01	.13+01	.15+01	.14+01	.15+01	.11+01	.11+01	.30+00			
.11+02	.13+02	.13+02	.11+02	.89+01	.11+02	.10+02	.10+02	.85+01	.85+01	.79+01	.83+01	.29+01		
-.11+02	-.12+02	-.11+02	-.77+01	-.48+01	-.45+01	-.24+01	-.12+01	-.15+00	.14+01	.12+01	.16+01	.72+00		
.22+00	.81+00	.12+01	.13+01	.12+01	.16+01	.17+01	.16+01	.13+01	.13+01	.12+01	.10+01	.68-01		
.16+02	.17+02	.11+02	.13+02	.13+02	.10+02	.10+02	.10+02	.91+01	.94+01	.75+01	.93+01	.74+01	.52+00	
-.15+02	-.15+02	-.88+01	-.88+01	-.61+01	-.29+01	-.12+01	-.11+00	.12+01	.21+01	.26+01	.33+01	.18+01	-.25+00	
.32+00	.10+01	.11+01	.17+01	.20+01	.16+01	.16+01	.17+01	.14+01	.14+01	.11+01	.13+01	.66+00	-.26-01	
.15+02	.14+02	.19+02	.13+02	.12+02	.12+02	.13+02	.11+02	.11+02	.10+02	.83+01	.86+01	.84+01	.42+01	
-.15+02	-.13+02	-.14+02	-.69+01	-.43+01	-.20+01	-.21+00	.16+01	.30+01	.36+01	.37+01	.43+01	.38+01	.46+00	
.41+00	.10+01	.21+01	.19+01	.19+01	.20+01	.22+01	.17+01	.18+01	.15+01	.11+01	.11+01	.10+01	.16+00	
.18+02	.20+02	.16+02	.15+02	.18+02	.12+02	.12+02	.10+02	.11+02	.94+01	.88+01	.86+01	.75+01	.96+01	
-.17+02	-.17+02	-.10+02	-.66+01	-.33+01	.13-01	.22+01	.31+01	.42+01	.49+01	.46+01	.50+01	.47+01	.41+01	
.47+00	.17+01	.20+01	.23+01	.29+01	.20+01	.20+01	.15+01	.16+01	.12+01	.11+01	.10+01	.81+00	.50+00	
.22+02	.24+02	.22+02	.18+02	.16+02	.15+02	.14+02	.12+02	.13+02	.97+01	.91+01	.92+01	.91+01	.76+01	.38+01
-.21+02	-.19+02	-.11+02	-.44+01	.57+00	.30+01	.52+01	.55+01	.77+01	.58+01	.62+01	.63+01	.64+01	.50+01	.50+00
.86+00	.24+01	.31+01	.29+01	.27+01	.25+01	.22+01	.18+01	.17+01	.12+01	.10+01	.99+00	.89+00	.61+00	.36-02
.33+02	.26+02	.28+02	.20+02	.19+02	.17+02	.14+02	.13+02	.91+01	.11+02	.11+02	.10+02	.83+01	.74+01	.53+01
-.31+02	-.18+02	-.83+01	.10+00	.44+01	.75+01	.82+01	.93+01	.57+01	.85+01	.90+01	.83+01	.64+01	.55+01	.14+01
.14+01	.31+01	.43+01	.33+01	.30+01	.26+01	.19+01	.16+01	.11+01	.10+01	.10+01	.86+00	.65+00	.50+00	.95-01
.46+02	.38+02	.31+02	.24+02	.20+02	.18+02	.17+02	.10+02	.16+02	.11+02	.75+01	.90+01	.87+01	.83+01	.75+01
-.41+02	-.16+02	.51+00	.86+01	.10+02	.12+02	.12+02	.77+01	.13+02	.96+01	.63+01	.76+01	.75+01	.66+01	.31+01
.30+01	.55+01	.50+01	.37+01	.28+01	.22+01	.18+01	.10+01	.14+01	.80+00	.59+00	.64+00	.52+00	.53+00	.13+00
.69+02	.51+02	.36+02	.28+02	.23+02	.20+02	.14+02	.17+02	.11+02	.11+02	.11+02	.10+02	.89+01	.72+01	.84+01
-.49+02	.15+01	.16+02	.18+02	.18+02	.17+02	.12+02	.15+02	.10+02	.10+02	.10+02	.93+01	.81+01	.64+01	.38+01
.66+01	.81+01	.51+01	.33+01	.21+01	.15+01	.10+01	.11+01	.47+00	.66+00	.56+00	.31+00	.32+00	.13+00	.21+00

.54+01	-.15+01	-.10+01	-.11+01	-.84+00	-.83+00	-.70+00	-.91+00	-.67+00	-.54+00	-.50+00	-.54+00	-.47+00	-.75+00	-.10+01	
-.20+02	-.10+02	-.25+01	-.74+00	-.29+01	-.32+01	.10+01	.11+01	-.46+01	.22+01	.99+00	-.21+01	-.13+00	-.14+01	.95+00	
.10+02	-.10+02	-.71+01	-.49+01	-.51+01	-.52+01	-.37+00	-.59+00	-.51+01	.11+01	-.13-01	-.28+01	-.86+00	-.20+01	-.94+00	
.24+01	.16+01	-.20+00	-.59+00	-.32+00	-.44+00	-.52+00	-.71+00	-.18+00	-.57+00	-.58+00	-.42+00	-.49+00	-.47+00	-.97+00	
-.98+01	-.89+01	-.26+01	-.68+01	-.13-01	-.10+01	-.11+01	.11+01	-.89-02	-.22+01	.11+01	.95+00	.22-01	.12+01	-.10-01	
.73+01	-.43+00	-.51+01	-.62+01	-.36+01	-.38+01	-.33+01	-.64+00	-.20+01	-.33+01	.55-01	-.34+00	-.11+01	-.35+00	-.26+01	
.11+01	.16+01	.41+00	.80+00	-.36+00	-.32+00	-.34+00	-.43+00	-.52+00	-.26+00	-.37+00	-.48+00	-.48+00	-.67+00	-.80+00	
-.99+01	-.64+01	-.69+01	-.21+01	.38-01	-.22+01	-.11+01	-.46+01	.22+01	-.22+01	-.10+01	-.93+00	-.40-01	-.92-01	.19+00	
.93+01	.26+01	-.18+01	-.34+01	-.37+01	-.40+01	-.32+01	-.51+01	-.31-01	-.34+01	-.26+01	-.26+01	-.16+01	-.19+01	-.23+01	
.47+00	.10+01	.12+01	.36+00	-.16+00	.99-01	-.14+00	.26+00	-.51+00	-.11+00	-.32+00	-.38+00	-.44+00	-.55+00	-.57+00	
-.20+01	-.41+01	-.31+01	-.11+01	-.33+01	-.22+01	-.11+01	.11+01	-.22+01	.10+01	-.10+01	.77-02	.59-01	-.65-01	.44+00	
.17+01	.21+01	-.35+00	-.20+01	-.28+01	-.31+01	-.33+01	-.15+01	-.40+01	-.11+01	-.24+01	-.20+01	-.23+01	-.26+01	-.20+01	
.24+00	.64+00	.64+00	.27+00	.53+00	.28+00	-.46-02	-.35+00	-.68-02	-.38+00	-.17+00	-.36+00	-.47+00	-.58+00	-.33+00	
-.92+00	-.11+01	-.10+01	-.22+01	-.22+01	-.11+01	-.11+01	-.11+01	.36-03	-.10+01	.98+00	-.56-01	-.62+00	-.26+00		
.85+00	.24+00	-.26+00	-.49+00	-.20+01	-.22+01	-.25+01	-.22+01	-.23+01	-.26+01	-.14+01	-.23+01	-.27+01	-.54+01		
.52-01	.30+00	.27+00	.44+00	.45+00	.19+00	.11+00	.79-01	-.16+00	-.67-01	-.36+00	-.29+00	-.29+00	-.54+00		
-.66+01	-.10+01	-.34+01	-.22+01	-.11+01	-.11+01	-.18-01	-.13-01	-.10+01	-.35-01	-.95+00	-.81+00	.52+00	.11+01		
.65+01	.78+00	.14+01	-.49-01	-.13+01	-.16+01	-.24+01	-.23+01	-.27+01	-.23+01	-.26+01	-.30+01	-.32+01	-.25+01		
.20+00	.13+00	.56+00	.44+00	.28+00	.24+00	.10-01	-.57-01	.60-01	-.15+00	-.46-01	-.15+00	-.46+00	-.23+00		
-.23+01	-.34+01	-.22+01	.29-03	-.22+01	-.11+01	-.11+01	-.46-01	-.10+01	.11-01	-.83+00	.14+00	.10+01	.52+00		
.22+01	.28+01	.12+01	-.11+01	-.57+00	-.10+01	-.16+01	-.21+01	-.21+01	-.24+01	-.25+01	-.34+01	-.40+01	-.25+00		
.88-01	.34+00	.34+00	.15+00	.48+00	.25+00	.22+00	.10-01	.13+00	-.10+00	-.91-02	-.27+00	-.33+00	.26-01		
.22+01	.79-02	-.22+01	-.12-01	-.22+01	-.20-02	-.21+01	.14-01	.90+00	-.17+01	.60+00	.31+00	.93+00			
-.22+01	-.29+00	.13+01	-.88+00	.35+00	-.15+01	-.12+01	-.21+01	-.20+01	-.25+01	-.26+01	-.43+01	-.11+01			
-.10-01	.12+00	.34+00	.14+00	.40+00	.12+00	.41+00	.42-01	-.14+00	.23+00	-.18+00	-.13+00	.32-01			
-.33+01	-.33+01	.11+01	-.32+01	.10+01	-.10+01	-.18-01	-.27-02	-.25+01	.13+00	.35+00	.13+01				
.33+01	.31+01	-.14+01	.18+01	-.18+01	-.69+00	-.16+01	-.19+01	-.20+01	-.23+01	-.36+01	-.20+01				
.63-01	.21+00	.54-01	.47+00	.15-01	.26+00	.11+00	.77-01	.44+00	-.25-01	-.10+00	-.76-03				
-.99+00	-.21+01	-.81-01	.44-01	-.39-01	-.97+00	-.95+00	-.17+01	.78+00	.42+00	.25+01					
.96+00	.18+01	-.20+00	-.76+00	-.67+00	-.45+00	-.83+00	-.15+01	-.28+01	-.33+01	-.21+01					
.39-01	.18+00	.96-01	.15+00	.11+00	.24+00	.24+00	.38+00	-.29-01	-.16-02	-.11-01					
-.18+01	.20+01	-.21+01	-.84+00	-.99+00	.69-01	.30-01	-.31-01	.20+00	.17+01						
.18+01	-.20+01	.16+01	.30+00	-.17+00	-.11+01	-.16+01	-.20+01	-.21+01	-.11+01						
.68-01	-.24-01	.23+00	.17+00	.27+00	.13+00	.16+00	.15+00	.15+00	.42-01						
.91-01	-.16+01	-.42-01	.79-01	-.77+00	.31-01	-.72+00	.19+01	.18+01							
-.96-01	.14+01	-.26+00	-.55+00	-.20+00	-.14+01	-.12+01	-.24+01	-.83+00							
.93-02	.12+00	.12+00	.12+00	.24+00	.21+00	.29+00	.30-01	-.53-01							
.16+00	-.58+00	-.77+00	-.44+00	.97+00	.23+01	.22+01	.84-01								
-.17+00	.46+00	.44+00	-.24+00	-.16+01	-.18+01	-.11+01	.51-01								
.17-01	.95-01	.16+00	.21+00	.11+00	-.72-01	-.14+00	.11-01								
.26+01	.26+01	.21+01	.26+01	.13+01											
-.26+01	-.25+01	-.19+01	-.20+01	-.66+00											
-.11-01	-.36-01	-.59-01	-.14+00	-.85-01											

TABLE 8

DISTRIBUTION FOR $\gamma = 0$, $\theta_0 = 10°$, AND $\sin \alpha_L = 0.2$

.53+01	.53+01	.38+01	.25+01	.52+00									
-.53+01	-.52+01	-.35+01	-.22+01	-.40+00									
.57-01	.17+00	.19+00	.16+00	.45-01									
.81+01	.88+01	.80+01	.85+01	.77+01	.52+01	.22+01							
-.81+01	-.86+01	-.74+01	-.73+01	-.60+01	-.33+01	-.11+01							
.94-01	.32+00	.48+00	.69+00	.73+00	.57+00	.24+00							
.69+01	.86+01	.83+01	.80+01	.81+01	.90+01	.66+01	.70+01	.17+01					
-.69+01	-.84+01	-.76+01	-.68+01	-.61+01	-.59+01	-.34+01	-.27+01	-.81+00					
.80-01	.33+00	.52+00	.69+00	.86+00	.10+01	.86+00	.91+00	.18+00					
.98+01	.99+01	.96+01	.83+01	.99+01	.84+01	.90+01	.76+01	.65+01	.30+01				
-.98+01	-.95+01	-.86+01	-.69+01	-.71+01	-.52+01	-.46+01	-.28+01	-.15+01	-.36+00				
.15+00	.41+00	.69+00	.77+00	.11+01	.10+01	.12+01	.11+01	.94+00	.35+00				
.11+02	.10+02	.84+01	.12+02	.79+01	.86+01	.81+01	.98+01	.90+01	.77+01	.49+01			
-.11+02	-.10+02	-.75+01	-.98+01	-.54+01	-.48+01	-.35+01	-.28+01	-.19+01	-.92+00	-.18+00			
.19+00	.50+00	.62+00	.12+01	.96+00	.11+01	.12+01	.15+01	.14+01	.11+01	.58+00			
.10+02	.10+02	.12+02	.99+01	.11+02	.93+01	.10+02	.94+01	.93+01	.73+01	.79+01	.37+01		
-.10+02	-.95+01	-.10+02	-.73+01	-.75+01	-.44+01	-.36+01	-.21+01	-.28+00	.20-01	.44+00	.44+00		
.17+00	.52+00	.98+00	.10+01	.14+01	.13+01	.15+01	.14+01	.15+01	.11+01	.11+01	.37+00		
.11+02	.13+02	.13+02	.11+02	.89+01	.11+02	.10+02	.10+02	.85+01	.85+01	.79+01	.80+01	.23+01	
-.11+02	-.12+02	-.11+02	-.77+01	-.48+01	-.45+01	-.24+01	-.12+01	-.15+00	.14+01	.12+01	.16+01	.85+00	
.22+00	.81+00	.12+01	.13+01	.12+01	.16+01	.17+01	.16+01	.13+01	.13+01	.12+01	.10+01	.14+00	
.16+02	.17+02	.11+02	.13+02	.13+02	.10+02	.10+02	.10+02	.91+01	.94+01	.75+01	.93+01	.71+01	.20+00
-.15+02	-.15+02	-.88+01	-.88+01	-.61+01	-.29+01	-.12+01	-.11+00	.12+01	.21+01	.26+01	.33+01	.19+01	-.18+00
.32+00	.10+01	.11+01	.17+01	.20+01	.16+01	.16+01	.17+01	.14+01	.14+01	.11+01	.13+01	.70+00	.14-01
.15+02	.14+02	.19+02	.13+02	.12+02	.12+02	.13+02	.11+02	.11+02	.10+02	.83+01	.86+01	.84+01	.36+01
-.15+02	-.13+02	-.14+02	-.69+01	-.43+01	-.20+01	-.21+00	.16+01	.30+01	.36+01	.37+01	.43+01	.38+01	.62+00
.41+00	.10+01	.21+01	.19+01	.19+01	.20+01	.22+01	.17+01	.18+01	.15+01	.11+01	.11+01	.10+01	.25+00
.18+02	.20+02	.16+02	.15+02	.18+02	.12+02	.12+02	.10+02	.11+02	.94+01	.88+01	.86+01	.75+01	.85+01
-.17+02	-.17+02	-.10+02	-.66+01	-.33+01	.13-01	.22+01	.31+01	.42+01	.49+01	.46+01	.50+01	.47+01	.47+01
.47+00	.17+01	.20+01	.23+01	.29+01	.20+01	.20+01	.15+01	.16+01	.12+01	.11+01	.10+01	.81+00	.56+00
.14+02	.17+02	.22+02	.18+02	.16+02	.15+02	.14+02	.12+02	.13+02	.97+01	.91+01	.92+01	.91+01	.76+01	.27+01
-.13+02	-.14+02	-.11+02	-.44+01	.57+00	.30+01	.52+01	.55+01	.77+01	.58+01	.62+01	.63+01	.64+01	.50+01	.10+01
.53+00	.16+01	.31+01	.29+01	.27+01	.25+01	.22+01	.18+01	.17+01	.12+01	.10+01	.99+00	.89+00	.61+00	.66-01
.56+01	.66+01	.13+02	.20+02	.19+02	.17+02	.14+02	.13+02	.91+01	.11+02	.11+02	.10+02	.82+01	.74+01	.44+01
-.54+01	-.50+01	-.55+01	.10+00	.44+01	.75+01	.82+01	.93+01	.57+01	.85+01	.90+01	.83+01	.64+01	.55+01	.20+01
.21+00	.70+00	.20+01	.33+01	.30+01	.26+01	.19+01	.16+01	.11+01	.10+01	.10+01	.86+00	.65+00	.50+00	.12+00
		.28+01	.14+02	.20+02	.18+02	.17+02	.10+02	.16+02	.11+02	.75+01	.90+01	.87+01	.83+01	.68+01
		-.49+00	.42+01	.10+02	.12+02	.12+02	.77+01	.13+02	.96+01	.63+01	.76+01	.75+01	.66+01	.37+01
		.46+00	.23+01	.28+01	.22+01	.18+01	.10+01	.14+01	.80+00	.59+00	.64+00	.52+00	.53+00	.13+00
			.29+01	.23+02	.20+02	.14+02	.17+02	.11+02	.11+02	.11+02	.10+02	.89+01	.72+01	.77+01
			.13+01	.18+02	.17+02	.12+02	.15+02	.10+02	.10+02	.10+02	.93+01	.81+01	.64+01	.44+01
			.43+00	.21+01	.15+01	.10+01	.11+01	.47+00	.66+00	.56+00	.31+00	.32+00	.13+00	.21+00

.29+01	-.29+01	-.32+01	.10+01	.11+01	-.46+01	.22+01	.99+00	-.21+01	-.13+00	-.14+01	.57+00		
.13+01	-.51+01	-.52+01	-.37+00	-.59+00	-.51+01	.11+01	-.13-01	-.28+01	-.86+00	-.20+01	-.63+00		
-.43+00	-.32+00	-.44+00	-.52+00	-.71+00	-.18+00	-.57+00	-.58+00	-.42+00	-.49+00	-.47+00	-.93+00		
.28+01	.27+01	-.13-01	-.10+01	-.11+01	.11+01	-.89-02	-.22+01	.11+01	.95+00	.22-01	.12+01	-.73+00	
-.49+00	-.18+01	-.36+01	-.38+01	-.33+01	-.64+00	-.20+01	-.33+01	.55-01	-.34+00	-.11+01	-.35+00	-.20+01	
-.46+00	-.58+00	-.36+00	-.32+00	-.34+00	-.43+00	-.52+00	-.26+00	-.37+00	-.48+00	-.48+00	-.67+00	-.80+00	
.56+01	.66+01	.76+01	-.21+01	.38-01	-.22+01	-.11+01	-.46+01	.22+01	-.22+01	-.10+01	-.93+00	-.40-01	-.92-01	-.32+00
-.54+01	-.50+01	-.45+01	-.34+01	-.37+01	-.40+01	-.32+01	-.51+01	-.31-01	-.34+01	-.26+01	-.26+01	-.16+01	-.19+01	-.17+01
-.21+00	-.70+00	-.10+01	.36+00	-.16+00	.99-01	-.14+00	.26+00	-.51+00	-.11+00	-.32+00	-.38+00	-.44+00	-.55+00	-.54+00
.58+01	.28+01	-.31+01	-.11+01	-.33+01	-.22+01	-.11+01	.11+01	-.22+01	.10+01	-.10+01	.77-02	.59-01	-.65-01	-.27+00
-.58+01	-.29+01	-.35+00	-.20+01	-.28+01	-.31+01	-.33+01	-.15+01	-.40+01	-.11+01	-.24+01	-.20+01	-.23+01	-.26+01	-.14+01
-.87-01	-.12+00	.64+00	.27+00	.53+00	.28+00	-.46-02	-.35+00	-.68-02	-.38+00	-.17+00	-.36+00	-.47+00	-.58+00	-.33+00
-.92+00	-.11+01	-.10+01	-.22+01	-.22+01	-.11+01	-.11+01	-.11+01	.36-03	-.10+01	.98+00	-.56-01	-.62+00	-.13+01	
.85+00	.24+00	-.26+00	-.49+00	-.20+01	-.22+01	-.25+01	-.22+01	-.23+01	-.26+01	-.14+01	-.23+01	-.27+01	-.48+01	
.52-01	.30+00	.27+00	.44+00	.45+00	.19+00	.11+00	.79-01	-.16+00	-.67-01	-.36+00	-.29+00	-.29+00	-.60+00	
-.66+01	-.10+01	-.34+01	-.22+01	-.11+01	-.11+01	-.18-01	-.13-01	-.10+01	-.35-01	-.95+00	-.81+00	.52+00	.52+00	
.65+01	.78+00	.14+01	-.49-01	-.13+01	-.16+01	-.24+01	-.23+01	-.27+01	-.23+01	-.26+01	-.30+01	-.32+01	-.23+01	
.20+00	.13+00	.56+00	.44+00	.28+00	.24+00	.10-01	-.57-01	.60-01	-.15+00	-.46-01	-.15+00	-.46+00	-.31+00	
-.23+01	-.34+01	-.22+01	.29-03	-.22+01	-.11+01	-.11+01	-.46-01	-.10+01	.11-01	-.83+00	.14+00	.69+00	.20+00	
.22+01	.28+01	.12+01	-.11+01	-.57+00	-.10+01	-.16+01	-.21+01	-.21+01	-.24+01	-.25+01	-.34+01	-.39+01	-.18+00	
.88-01	.34+00	.34+00	.15+00	.48+00	.25+00	.22+00	.10-01	.13+00	-.10+00	-.91-02	-.27+00	-.37+00	-.14-01	
.22+01	.79-02	-.22+01	-.12-01	-.22+01	-.20-02	-.21+01	.14-01	.90+00	-.17+01	.60+00	.17-01	.31+00		
-.22+01	-.29+00	.13+01	-.88+00	.35+00	-.15+01	-.12+01	-.21+01	-.20+01	-.25+01	-.26+01	-.42+01	-.10+01		
-.10-01	.12+00	.34+00	.14+00	.40+00	.12+00	.41+00	.42-01	-.14+00	.23+00	-.18+00	-.17+00	-.43-01		
-.33+01	-.33+01	.11+01	-.32+01	.10+01	-.10+01	-.18-01	-.27-02	-.25+01	.13+00	.35+00	.38+00			
.33+01	.31+01	-.14+01	.18+01	-.18+01	-.69+00	-.16+01	-.19+01	-.20+01	-.23+01	-.36+01	-.23+01			
.63-01	.21+00	.54-01	.47+00	.15-01	.26+00	.11+00	.77-01	.44+00	-.25-01	-.10+00	-.71-01			
-.99+00	-.21+01	-.81-01	.44-01	-.39-01	-.97+00	-.95+00	-.17+01	.78+00	.42+00	.11+01				
.96+00	.18+01	-.20+00	-.76+00	-.67+00	-.45+00	-.83+00	-.15+01	-.28+01	-.33+01	-.25+01				
.39-01	.18+00	.96-01	.15+00	.11+00	.24+00	.24+00	.38+00	-.29-01	-.16-02	-.90-01				
-.18+01	.20+01	-.21+01	-.84+00	-.99+00	.69-01	.30-01	-.31-01	-.57-01	.72+00					
.18+01	-.20+01	.16+01	.30+00	-.17+00	-.11+01	-.16+01	-.20+01	-.23+01	-.12+01					
.68-01	-.24-01	.23+00	.17+00	.27+00	.13+00	.16+00	.15+00	.15+00	-.31-02					
.91-01	-.16+01	-.42-01	.79-01	-.77+00	.31-01	-.72+00	.13+01	.10+01						
-.96-01	.14+01	-.26+00	-.55+00	-.20+00	-.14+01	-.12+01	-.24+01	-.96+00						
.93-02	.12+00	.12+00	.12+00	.24+00	.21+00	.29+00	.33-01	-.71-01						
.16+00	-.58+00	-.77+00	-.44+00	.76+00	.17+01	.16+01								
-.17+00	.46+00	.44+00	-.24+00	-.15+01	-.17+01	-.10+01								
.17-01	.95-01	.16+00	.21+00	.14+00	-.65-01	-.14+00								
.18+01	.18+01	.12+01	.17+01	.52+00										
-.18+01	-.18+01	-.12+01	-.15+01	-.40+00										
-.22-02	-.68-02	-.10-01	-.82-01	-.45-01										

TABLE 9

DISTRIBUTION FOR $\gamma = 0$, $\theta_0 = 10°$, AND $\sin \alpha_L = 0.4$

.28+01	.28+01	.13+01	.42+00										
-.28+01	-.28+01	-.12+01	-.40+00										
.29-01	.87-01	.62-01	.19-01										
.81+01	.88+01	.80+01	.81+01	.57+01	.18+01	.41+00							
-.81+01	-.86+01	-.74+01	-.70+01	-.45+01	-.14+01	-.35+00							
.94-01	.32+00	.48+00	.65+00	.55+00	.17+00	.35-01							
.69+01	.86+01	.83+01	.80+01	.81+01	.90+01	.54+01	.34+01						
-.69+01	-.84+01	-.76+01	-.68+01	-.61+01	-.59+01	-.29+01	-.19+01						
.80-01	.33+00	.52+00	.69+00	.86+00	.10+01	.71+00	.43+00						
.98+01	.99+01	.96+01	.83+01	.99+01	.84+01	.90+01	.76+01	.32+01					
-.98+01	-.95+01	-.86+01	-.69+01	-.71+01	-.52+01	-.46+01	-.28+01	-.87+00					
.15+00	.41+00	.69+00	.77+00	.11+01	.10+01	.12+01	.11+01	.48+00					
.11+02	.10+02	.84+01	.12+02	.79+01	.86+01	.81+01	.98+01	.90+01	.67+01	.14+01			
-.11+02	-.10+02	-.75+01	-.98+01	-.54+01	-.48+01	-.35+01	-.28+01	-.19+01	-.62+00	.22+00			
.19+00	.50+00	.62+00	.12+01	.96+00	.11+01	.12+01	.15+01	.14+01	.10+01	.20+00			
.10+02	.10+02	.12+02	.99+01	.11+02	.93+01	.10+02	.94+01	.93+01	.73+01	.68+01	.95+00		
-.10+02	-.95+01	-.10+02	-.73+01	-.75+01	-.44+01	-.36+01	-.21+01	-.28+00	.20-01	.59+00	.14+00		
.17+00	.52+00	.98+00	.10+01	.14+01	.13+01	.15+01	.14+01	.15+01	.11+01	.10+01	.13+00		
.11+02	.13+02	.13+02	.11+02	.89+01	.11+02	.10+02	.10+02	.85+01	.85+01	.79+01	.59+01		
-.11+02	-.12+02	-.11+02	-.77+01	-.48+01	-.45+01	-.24+01	-.12+01	-.15+00	.14+01	.12+01	.17+01		
.22+00	.81+00	.12+01	.13+01	.12+01	.16+01	.17+01	.16+01	.13+01	.13+01	.12+01	.81+00		
.68+01	.68+01	.57+01	.11+02	.13+02	.10+02	.10+02	.10+02	.91+01	.94+01	.75+01	.93+01	.35+01	
-.68+01	-.64+01	-.46+01	-.75+01	-.61+01	-.29+01	-.12+01	-.11+00	.12+01	.21+01	.26+01	.33+01	.10+01	
.12+00	.36+00	.56+00	.14+01	.20+01	.16+01	.16+01	.17+01	.14+01	.14+01	.11+01	.13+01	.45+00	
.44+01	.67+01	.79+01	.57+01	.68+01	.12+02	.13+02	.11+02	.11+02	.10+02	.83+01	.86+01	.76+01	.45+00
-.44+01	-.61+01	-.62+01	-.35+01	-.30+01	-.20+01	-.21+00	.16+01	.30+01	.36+01	.37+01	.43+01	.35+01	-.67-02
.10+00	.46+00	.80+00	.74+00	.10+01	.20+01	.22+01	.17+01	.18+01	.15+01	.11+01	.11+01	.93+00	.76-01
		.11+01	.45+01	.79+01	.80+01	.12+02	.10+02	.11+02	.94+01	.88+01	.86+01	.75+01	.45+01
		-.76+00	-.24+01	-.27+01	-.69+00	.22+01	.31+01	.42+01	.49+01	.46+01	.50+01	.47+01	.28+01
		.13+00	.63+00	.12+01	.13+01	.20+01	.15+01	.16+01	.12+01	.11+01	.10+01	.81+00	.38+00
				.11+01	.68+01	.11+02	.12+02	.13+02	.97+01	.91+01	.92+01	.91+01	.76+01
				-.20+00	-.71-01	.35+01	.55+01	.77+01	.58+01	.62+01	.63+01	.64+01	.50+01
				.18+00	.11+01	.17+01	.18+01	.17+01	.12+01	.10+01	.99+00	.83+00	.31+00
					.22+01	.68+01	.13+02	.91+01	.11+02	.11+02	.10+02	.82+01	.74+01	.17+01
					.46+00	.24+01	.93+01	.57+01	.85+01	.90+01	.63+01	.64+01	.55+01	.11+01
					.37+00	.10+01	.16+01	.11+01	.10+01	.10+01	.86+00	.65+00	.50+00	.56-01
						.45+01	.10+02	.16+02	.11+02	.75+01	.90+01	.87+01	.83+01	.39+01
						.26+01	.77+01	.13+02	.96+01	.63+01	.76+01	.75+01	.66+01	.27+01
						.61+00	.10+01	.14+01	.80+00	.59+00	.64+00	.52+00	.53+00	.16+00
							.13+02	.11+02	.11+02	.11+02	.10+02	.89+01	.72+01	.42+01
							.11+02	.10+02	.10+02	.10+02	.93+01	.81+01	.64+01	.32+01
							.10+01	.47+00	.66+00	.56+00	.31+00	.32+00	.13+00	.11+00

							.45+01	-.46+01	.22+01	.99+00	-.21+01	-.13+00	-.14+01	.12+00
							.27+01	-.51+01	.11+01	-.13-01	-.28+01	-.86+00	-.20+01	-.38+00
							-.79+00	-.18+00	-.57+00	-.58+00	-.42+00	-.49+00	-.47+00	-.42+00
						.45+01	.11+01	-.89-02	-.22+01	.11+01	.95+00	.22-01	.12+01	-.15+00
						.26+01	-.64+00	-.20+01	-.33+01	.55-01	-.34+00	-.11+01	-.35+00	-.10+01
						-.61+00	-.43+00	-.52+00	-.26+00	-.37+00	-.48+00	-.48+00	-.67+00	-.39+00
					.22+01	.68+01	-.46+01	.22+01	-.22+01	-.10+01	-.93+00	-.40-01	-.92-01	-.36-01
					.46+00	.24+01	-.51+01	-.31-01	-.34+01	-.26+01	-.26+01	-.16+01	-.19+01	-.63+00
					-.37+00	-.10+01	.26+00	-.51+00	-.11+00	-.32+00	-.38+00	-.44+00	-.55+00	-.18+00
				.11+01	.68+01	.22+01	.11+01	-.22+01	.10+01	-.10+01	.77-02	.59-01	-.65-01	
				-.20+00	-.71-01	-.15+01	-.15+01	-.40+01	-.11+01	-.24+01	-.20+01	-.23+01	-.26+01	
				-.18+00	-.11+01	-.49+00	-.35+00	-.68-02	-.38+00	-.17+00	-.36+00	-.47+00	-.58+00	
		.11+01	.45+01	.79+01	.34+01	-.11+01	-.11+01	.36-03	-.10+01	.98+00	-.56-01	-.62+00	-.60+00	
		-.76+00	-.24+01	-.27+01	-.14+01	-.25+01	-.22+01	-.23+01	-.26+01	-.14+01	-.23+01	-.27+01	-.22+01	
		-.13+00	-.63+00	-.12+01	-.56+00	.11+00	.79-01	-.16+00	-.67-01	-.36+00	-.29+00	-.29+00	-.26+00	
.44+01	.67+01	.79+01	.57+01	.45+01	-.11+01	-.18-01	-.13-01	-.10+01	-.35-01	-.95+00	-.81+00	.56+00	.45+00	
-.44+01	-.61+01	-.62+01	-.35+01	-.26+01	-.16+01	-.24+01	-.23+01	-.27+01	-.23+01	-.26+01	-.30+01	-.27+01	-.67-02	
-.10+00	-.46+00	-.80+00	-.74+00	-.64+00	.24+00	.10-01	-.57-01	.60-01	-.15+00	-.46-01	-.15+00	-.42+00	-.76-01	
.68+01	.68+01	.34+01	.22+01	-.22+01	-.11+01	-.11+01	-.46-01	-.10+01	.11-01	-.83+00	.14+00	.39+00		
-.68+01	-.64+01	-.30+01	-.24+01	-.57+00	-.10+01	-.16+01	-.21+01	-.21+01	-.24+01	-.25+01	-.34+01	-.16+01		
-.12+00	-.36+00	-.28+00	-.15+00	.48+00	.25+00	.22+00	.10-01	.13+00	-.10+00	-.91-02	-.27+00	-.19+00		
.22+01	.79-02	-.22+01	-.12-01	-.22+01	-.20-02	-.21+01	.14-01	.90+00	-.17+01	.60+00	-.48+00			
-.22+01	-.29+00	.13+01	-.88+00	.35+00	-.15+01	-.12+01	-.21+01	-.20+01	-.25+01	-.26+01	-.28+01			
-.10-01	.12+00	.34+00	.14+00	.40+00	.12+00	.41+00	.42-01	-.14+00	.23+00	-.18+00	-.72-01			
-.33+01	-.33+01	.11+01	-.32+01	.10+01	-.10+01	-.18-01	-.27-02	-.25+01	.13+00	.33-01	.12-01			
.33+01	.31+01	-.14+01	.18+01	-.18+01	-.69+00	-.16+01	-.19+01	-.20+01	-.23+01	-.29+01	-.48+00			
.63-01	.21+00	.54-01	.47+00	.15-01	.26+00	.11+00	.77-01	.44+00	-.25-01	-.54-01	-.16-01			
-.99+00	-.21+01	-.81-01	.44-01	-.39-01	-.97+00	-.95+00	-.17+01	.78+00	.73-01	-.42+00				
.96+00	.18+01	-.20+00	-.76+00	-.67+00	-.45+00	-.83+00	-.15+01	-.28+01	-.26+01	-.71+00				
.39-01	.18+00	.96-01	.15+00	.11+00	.24+00	.24+00	.38+00	-.29-01	.32-01	.59-01				
-.18+01	.20+01	-.21+01	-.84+00	-.39+00	.69-01	.30-01	-.31-01	-.88-01						
.18+01	-.20+01	.16+01	.30+00	-.17+00	-.11+01	-.16+01	-.20+01	-.11+01						
.68-01	-.24-01	.23+00	.17+00	.27+00	.13+00	.16+00	.15+00	.72-01						
.91-01	-.16+01	-.42-01	.79-01	-.77+00	.31-01	-.20+00	.13+01							
-.96-01	.14+01	-.26+00	-.55+00	-.20+00	-.14+01	-.12+01	-.16+01							
.93-02	.12+00	.12+00	.12+00	.24+00	.21+00	.18+00	-.91-01							
.16+00	-.58+00	-.77+00	-.41-01	.39+00	.84+00	.41+00								
-.17+00	.46+00	.44+00	-.54+00	-.10+01	-.93+00	-.35+00								
.17-01	.95-01	.16+00	.17+00	.12+00	-.32-01	-.35-01								
.91+00	.90+00	.35+00	.42+00											
-.91+00	-.92+00	-.37+00	-.40+00											
.46-03	.11-02	.28-02	-.19-01											

TABLE 10

DISTRIBUTION FOR $\gamma = 0$, $\theta_0 = 10°$, AND $\sin \alpha_L = 0.6$

.29+01	.29+01	.13+01	.13+01									
-.29+01	-.28+01	-.13+01	-.12+01									
.27-01	.82-01	.63-01	.87-01									
.35+01	.35+01	.31+01	.40+01	.36+01	.28+01							
-.35+01	-.34+01	-.39+01	-.36+01	-.31+01	-.22+01							
.35-01	.10+00	.19+00	.28+00	.31+00	.28+00							
.39+01	.59+01	.37+01	.37+01	.44+01	.42+01	.45+01	.21+01					
-.39+01	-.58+01	-.35+01	-.33+01	-.36+01	-.31+01	-.31+01	-.13+01					
.42-01	.21+00	.22+00	.29+00	.42+00	.47+00	.55+00	.27+00					
.54+01	.42+01	.41+01	.62+01	.39+01	.38+01	.35+01	.40+01	.36+01				
-.53+01	-.41+01	-.38+01	-.53+01	-.30+01	-.26+01	-.21+01	-.21+01	-.17+01				
.77-01	.16+00	.26+00	.54+00	.42+00	.46+00	.47+00	.57+00	.53+00				
.33+01	.33+01	.66+01	.33+01	.64+01	.41+01	.50+01	.47+01	.34+01	.30+01			
-.33+01	-.32+01	-.60+01	-.27+01	-.47+01	-.25+01	-.26+01	-.20+01	-.11+01	-.92+00			
.56-01	.15+00	.46+00	.30+00	.73+00	.54+00	.70+00	.70+00	.53+00	.48+00			
			.22+01	.33+01	.55+01	.42+01	.50+01	.47+01	.34+01	.30+01		
			-.17+01	-.22+01	-.30+01	-.18+01	-.17+01	-.10+01	-.57+00	-.39+00		
			.23+00	.41+00	.76+00	.64+00	.79+00	.76+00	.55+00	.49+00		
					.11+01	.44+01	.53+01	.40+01	.74+01	.51+01	.22+01	
					-.49+00	-.14+01	-.11+01	-.48+00	.13+01	.11+01	.82-01	
					.16+00	.69+00	.87+00	.67+00	.11+01	.79+00	.37+00	
						.11+01	.44+01	.53+01	.50+01	.83+01	.55+01	.71+00
						-.21+00	-.28+00	.16+00	.65+00	.37+01	.19+01	.15+00
						.18+00	.73+00	.88+00	.83+00	.11+01	.81+00	.11+00
								.55+01	.42+01	.88+01	.86+01	.22+01
								.97+00	.11+01	.46+01	.50+01	.69+00
								.90+00	.67+00	.11+01	.10+01	.35+00
								.11+01	.54+01	.80+01	.92+01	.62+01
								.37+00	.23+01	.53+01	.63+01	.36+01
								.17+00	.81+00	.93+00	.99+00	.76+00
									.33+01	.83+01	.10+02	.82+01
									.19+01	.61+01	.83+01	.64+01
									.45+00	.85+00	.86+00	.65+00
									.11+01	.43+01	.90+01	.87+01	.23+01
									.74+00	.32+01	.76+01	.75+01	.18+01
									.13+00	.48+00	.64+00	.52+00	.17+00
										.64+01	.10+02	.89+01	.31+01
										.54+01	.93+01	.81+01	.27+01
										.57+00	.31+00	.32+00	.11+00

.64+01	-.21+01	-.13+00	-.88-01	
.54+01	-.28+01	-.86+00	-.41+00	
-.57+00	-.42+00	-.49+00	-.20+00	

.11+01	.43+01	.95+00	.22-01	.73+00
.74+00	.32+01	-.34+00	-.11+01	.24+00
-.13+00	-.48+00	-.48+00	-.48+00	-.19+00

.33+01	.21+01	-.93+00	-.40-01
.19+01	.26+00	-.26+01	-.16+01
-.45+00	-.53+00	-.38+00	-.44+00

.11+01	.54+01	.58-01	.77-02	.29+01
.37+00	.23+01	-.15+01	-.20+01	.49+00
-.17+00	-.81+00	-.26+00	-.36+00	-.61+00

.55+01	.42+01	.98+00	-.56-01	.22+01
.97+00	.11+01	-.14+01	-.23+01	.69+00
-.90+00	-.67+00	-.36+00	-.29+00	-.35+00

.11+01	.44+01	.53+01	.50+01	-.95+00	.22+01	.71+00
-.21+00	-.28+00	.16+00	.65+00	-.26+01	-.67+00	.15+00
-.18+00	-.73+00	-.88+00	-.83+00	-.46-01	-.47+00	-.11+00

.11+01	.44+01	.53+01	.40+01	.19+01	.16+01	.22+01
-.49+00	-.14+01	-.11+01	-.48+00	-.15+01	-.10+01	.82-01
-.16+00	-.69+00	-.87+00	-.67+00	-.39+00	-.34+00	-.37+00

.22+01	.33+01	.55+01	.42+01	.50+01	.47+01	.34+01	.30+01
-.17+01	-.22+01	-.30+01	-.18+01	-.17+01	-.10+01	-.57+00	-.39+00
-.23+00	-.41+00	-.76+00	-.64+00	-.79+00	-.76+00	-.55+00	-.49+00

.33+01	.33+01	.66+01	.33+01	.64+01	.41+01	.50+01	.47+01	.34+01	.30+01
-.33+01	-.32+01	-.60+01	-.27+01	-.47+01	-.25+01	-.26+01	-.20+01	-.11+01	-.92+00
-.56-01	-.15+00	-.46+00	-.30+00	-.73+00	-.54+00	-.70+00	-.70+00	-.53+00	-.48+00

.54+01	.42+01	.41+01	.62+01	.39+01	.38+01	.35+01	.40+01	.36+01
-.53+01	-.41+01	-.38+01	-.53+01	-.30+01	-.26+01	-.21+01	-.21+01	-.17+01
-.77-01	-.16+00	-.26+00	-.54+00	-.42+00	-.46+00	-.47+00	-.57+00	-.53+00

.39+01	.59+01	.37+01	.37+01	.44+01	.42+01	.45+01	.21+01
-.39+01	-.58+01	-.35+01	-.33+01	-.36+01	-.31+01	-.31+01	-.13+01
-.42-01	-.21+00	-.22+00	-.29+00	-.42+00	-.47+00	-.55+00	-.27+00

.35+01	.35+01	.41+01	.40+01	.36+01	.28+01
-.35+01	-.34+01	-.39+01	-.36+01	-.31+01	-.22+01
-.35-01	-.10+00	-.19+00	-.28+00	-.31+00	-.28+00

.29+01	.29+01	.13+01	.13+01
-.29+01	-.28+01	-.13+01	-.12+01
-.27-01	-.82-01	-.63-01	-.87-01

TABLE 11

DISTRIBUTION FOR $\gamma = 0$, $\theta_0 = 30°$, AND $\sin \alpha_L = 0$

.50+01	.56+01	.39+01	.27+01	.74+00									
-.49+01	-.48+01	-.23+01	-.41+00	.23+00									
.51-01	.18+00	.16+00	.90-01	-.13-01									
.81+01	.86+01	.86+01	.93+01	.84+01	.68+01	.38+01	.51+00						
-.80+01	-.81+01	-.75+01	-.66+01	-.32+01	-.35+00	.88+00	.11+00						
.10+00	.35+00	.53+00	.75+00	.59+00	.22+00	-.94-01	-.77-01						
.98+01	.73+01	.95+01	.76+01	.10+02	.90+01	.86+01	.92+01	.42+01					
-.98+01	-.69+01	-.82+01	-.60+01	-.63+01	-.40+01	-.20+01	.14+01	.49+00					
.15+00	.32+00	.69+00	.64+00	.10+01	.98+00	.79+00	.19+00	-.29+00					
.95+01	.10+02	.93+01	.11+02	.79+01	.92+01	.10+02	.95+01	.10+02	.83+01	.11+01			
-.94+01	-.96+01	-.80+01	-.86+01	-.48+01	-.44+01	-.35+01	-.12+01	.43+00	.14+01	.15+00			
.14+00	.51+00	.72+00	.12+01	.89+00	.11+01	.12+01	.10+01	.54+00	-.39+00	-.14+00			
.92+01	.12+02	.12+02	.90+01	.11+02	.95+01	.92+01	.10+02	.96+01	.86+01	.10+02	.16+01		
-.91+01	-.11+02	-.10+02	-.59+01	-.66+01	-.36+01	-.11+01	-.19+01	-.16+00	.15+00	.46+00	.37+00		
.17+00	.67+00	.10+01	.10+01	.14+01	.12+01	.13+01	.12+01	.10+01	.51+00	-.55+00	-.22+00		
.12+02	.16+02	.11+02	.11+02	.11+02	.11+02	.10+02	.10+02	.99+01	.10+02	.10+02	.11+02	.83+00	
-.12+02	-.15+02	-.91+01	-.74+01	-.51+01	-.41+01	-.15+01	-.86+00	.15+01	.24+01	-.29+00	.87+00	-.19+00	
.25+00	.93+00	.11+01	.13+01	.15+01	.15+01	.14+01	.14+01	.12+01	.10+01	.59+00	-.71+00	-.13+00	
.16+02	.13+02	.12+02	.14+02	.12+02	.13+02	.11+02	.11+02	.10+02	.83+01	.10+02	.11+02	.98+01	
-.16+02	-.11+02	-.87+01	-.88+01	-.37+01	-.20+01	-.13+01	.14+01	.13+01	.17+01	.24+01	.65+00	-.56+00	
.38+00	.90+00	.13+01	.17+01	.17+01	.19+01	.16+01	.15+01	.13+01	.90+00	.91+00	.35+00	-.91+00	
.17+02	.13+02	.18+02	.14+02	.15+02	.13+02	.12+02	.12+02	.11+02	.10+02	.83+01	.10+02	.12+02	.58+01
-.16+02	-.11+02	-.13+02	-.60+01	-.49+01	-.34+00	.12+01	.16+01	.40+01	.32+01	.41+01	.30+01	.51+00	-.19+01
.54+00	.10+01	.20+01	.20+01	.22+01	.20+01	.17+01	.16+01	.13+01	.11+01	.61+00	.64+00	-.18+00	-.62+00
.15+02	.22+02	.19+02	.15+02	.15+02	.14+02	.13+02	.12+02	.12+02	.10+02	.10+02	.98+01	.95+01	.11+02
-.15+02	-.18+02	-.10+02	-.49+01	-.71+00	.12+01	.36+01	.45+01	.40+01	.49+01	.45+01	.50+01	.22+01	-.37+01
.45+00	.20+01	.25+01	.22+01	.23+01	.21+01	.18+01	.15+01	.13+01	.10+01	.91+00	.46+00	.19+00	-.79+00
.24+02	.26+02	.21+02	.21+02	.17+02	.13+02	.17+02	.13+02	.12+02	.12+02	.93+01	.11+02	.90+01	.13+02	.39+01
-.23+02	-.19+02	-.86+01	-.34+01	.25+01	.40+01	.74+01	.46+01	.69+01	.71+01	.47+01	.46+01	.21+01	-.97+00	-.23+01
.96+00	.26+01	.31+01	.32+01	.25+01	.18+01	.20+01	.16+01	.11+01	.91+00	.66+00	.73+00	.46+00	-.19+00	-.40+00
.36+02	.32+02	.26+02	.22+02	.20+02	.20+02	.13+02	.11+02	.15+02	.11+02	.85+01	.11+02	.11+02	.10+02	.10+02
-.34+02	-.18+02	-.47+01	.34+01	.68+01	.10+02	.66+01	.65+01	.93+01	.58+01	.45+01	.59+01	.51+01	.13+01	-.53+01
.17+01	.41+01	.40+01	.33+01	.26+01	.23+01	.14+01	.10+01	.12+01	.89+00	.58+00	.51+00	.38+00	.10+00	-.65+00
.49+02	.42+02	.33+02	.25+02	.22+02	.18+02	.13+02	.15+02	.13+02	.11+02	.12+02	.10+02	.10+02	.11+02	.13+02
-.42+02	-.11+02	.41+01	.11+02	.11+02	.11+02	.73+01	.11+02	.92+01	.69+01	.76+01	.57+01	.45+01	.33+01	-.56+01
.35+01	.62+01	.48+01	.31+01	.23+01	.15+01	.11+01	.94+00	.72+00	.66+00	.54+00	.44+00	.46+00	-.79-01	-.55+00
.87+02	.55+02	.38+02	.29+02	.23+02	.19+02	.19+02	.12+02	.14+02	.11+02	.88+01	.11+02	.96+01	.10+02	.14+02
-.55+02	.13+02	.20+02	.17+02	.16+02	.14+02	.13+02	.81+01	.10+02	.80+01	.52+01	.70+01	.53+01	.35+01	-.68+01
.89+01	.75+01	.40+01	.21+01	.14+01	.91+00	.93+00	.71+00	.53+00	.17+00	.54+00	.18+00	-.64-01	-.62-02	-.13+00

-.62+02	-.34+02	-.17+02	-.75+01	-.56+01	-.51+01	-.22+01	.10+01	-.12+01	-.23+01	.21+01	-.12+01	-.20+01	-.93+00	.26+01
.32+02	-.26+02	-.26+02	-.17+02	-.12+02	-.10+02	-.75+01	-.25+01	-.44+01	-.41+01	-.50+00	-.32+01	-.31+01	-.27+01	-.41+01
.76+01	.48+01	.74+00	-.13+01	-.14+01	-.14+01	-.17+01	-.13+01	-.14+01	-.11+01	-.10+01	-.14+01	-.12+01	-.16+01	-.16+01
-.34+02	-.25+02	-.13+02	-.12+02	-.60+01	-.54+01	-.59-01	-.46+01	-.23+01	-.80-01	-.12+01	-.15+00	.75+00	-.18+01	.21+01
.27+02	-.17+01	-.15+02	-.16+02	-.13+02	-.11+02	-.60+01	-.92+01	-.66+01	-.40+01	-.50+01	-.36+01	-.33+01	-.48+01	-.59+01
.30+01	.45+01	.21+01	.98+00	-.36+00	-.62+00	-.10+01	-.90+00	-.11+01	-.11+01	-.12+01	-.12+01	-.13+01	-.15+01	-.11+01
-.21+02	-.17+02	-.12+02	-.10+02	-.73+01	-.78+01	-.23+01	-.23+01	-.24+01	-.12+00	-.14+00	-.12+01	-.10+01	-.13+00	.48+01
.19+02	.54+01	-.50+01	-.97+01	-.11+02	-.13+02	-.80+01	-.68+01	-.89+01	-.53+01	-.40+01	-.57+01	-.61+01	-.51+01	-.62+01
.13+01	.29+01	.23+01	.15+01	.68+00	.23+00	-.41+00	-.47+00	-.10+01	-.93+00	-.81+00	-.11+01	-.13+01	-.14+01	-.30+00
-.11+02	-.98+01	-.11+02	-.63+01	-.66+01	-.45+01	-.58+01	-.15+00	-.35+01	-.34+01	-.11+01	-.19+00	.71+00	.12+01	.39+01
.11+02	.48+01	.18+00	-.66+01	-.82+01	-.76+01	-.10+02	-.75+01	-.82+01	-.80+01	-.57+01	-.67+01	-.53+01	-.83+01	-.23+01
.72+00	.16+01	.21+01	.13+01	.10+01	.46+00	.24+00	-.56+00	-.33+00	-.50+00	-.68+00	-.11+01	-.10+01	-.13+01	.40+00
-.29+01	-.94+01	-.86+01	-.43+01	-.67+01	-.34+01	-.46+01	-.35+01	-.12+01	-.22+01	-.13+01	-.28+01	-.10+01	.79+01	
.26+01	.60+01	.10+01	-.34+01	-.51+01	-.71+01	-.76+01	-.78+01	-.75+01	-.71+01	-.75+01	-.76+01	-.73+01	-.62+01	
.22+00	.13+01	.16+01	.10+01	.12+01	.54+00	.52+00	.15+00	-.30+00	-.32+00	-.60+00	-.61+00	-.90+00	.33+00	
-.10+02	-.43+01	-.54+01	-.78+01	-.23+01	-.46+01	-.34+01	-.12+01	-.33+01	-.11+01	-.29+01	-.94+00	.38+01	.58+01	
.10+02	.28+01	.11+01	.45+00	-.50+01	-.54+01	-.62+01	-.70+01	-.78+01	-.75+01	-.67+01	-.83+01	-.71+01	-.19+01	
.42+00	.63+00	.11+01	.14+01	.75+00	.87+00	.53+00	.58-01	.14+00	-.27+00	-.16+00	-.67+00	-.34+00	.62+00	
-.53+01	-.65+01	-.54+01	-.11+01	-.56+01	-.44+01	-.11+01	-.33+01	-.11+01	-.11+01	-.10+01	.38+01	.85+01		
.52+01	.53+01	.25+01	-.26+01	-.14+01	-.43+01	-.56+01	-.65+01	-.71+01	-.61+01	-.87+01	-.66+01	-.18+01		
.23+00	.66+00	.91+00	.63+00	.11+01	.95+00	.36+00	.49+00	.68-01	-.43-01	-.32+00	-.35+00	.84+00		
-.42+01	-.31+01	-.53+01	-.32+01	-.32+01	-.11+01	-.22+01	-.11+01	-.29+01	-.26+01	.60+01	.11+02	.83+00		
.41+01	.22+01	.30+01	-.22+00	-.20+01	-.41+01	-.44+01	-.59+01	-.66+01	-.84+01	-.45+01	.87+00	-.19+00		
.16+00	.49+00	.82+00	.79+00	.84+00	.53+00	.56+00	.33+00	.44+00	.18+00	-.36+00	.71+00	.13+00		
-.10+01	-.40+01	-.20+01	-.30+01	-.20+01	-.21+01	-.37+01	-.19+00	.31+01	.71+01	.10+02	.16+01			
.92+00	.32+01	.38+00	.35+00	-.21+01	-.27+01	-.34+01	-.63+01	-.42+01	-.10+01	.46+00	.37+00			
.10+00	.46+00	.55+00	.67+00	.72+00	.65+00	.80+00	.32+00	-.16+00	-.36+00	.55+00	.22+00			
-.19+01	-.27+01	-.18+01	-.26+01	.73+00	.57+00	.51+01	.79+01	.10+02	.83+01	.11+01				
.19+01	.22+01	.68+00	-.12+00	-.22+01	-.32+01	-.41+01	-.17+01	.43+00	.14+01	.15+00				
.91-01	.34+00	.45+00	.75+00	.22+00	.32+00	-.38+00	-.78+00	-.54+00	.39+00	.14+00				
.62+01	.73+01	.78+01	.76+01	.10+02	.90+01	.86+01	.92+01	.42+01						
-.62+01	-.69+01	-.70+01	-.60+01	-.63+01	-.40+01	-.20+01	.14+01	.49+00						
-.10+00	-.32+00	-.49+00	-.64+00	-.10+01	-.98+00	-.79+00	-.19+00	.29+00						
.81+01	.86+01	.86+01	.93+01	.84+01	.68+01	.38+01	.51+00							
-.80+01	-.81+01	-.75+01	-.66+01	-.32+01	-.35+00	.88+00	.11+00							
-.10+00	-.35+00	-.53+00	-.75+00	-.59+00	-.22+00	.94-01	.77-01							
.50+01	.56+01	.39+01	.27+01	.74+00										
-.49+01	-.48+01	-.23+01	-.41+00	.23+00										
-.51-01	-.18+00	-.16+00	-.90-01	.13-01										

TABLE 12

DISTRIBUTION FOR $\gamma = 0$, $\theta_0 = 30°$, AND $\sin \alpha_L = 0.2$

.37+01	.46+01	.28+01	.14+01										
-.36+01	-.43+01	-.21+01	-.63+00										
.27-01	.17+00	.16+00	.10+00										
.81+01	.86+01	.86+01	.93+01	.81+01	.60+01	.38+01	.51+00						
-.80+01	-.81+01	-.75+01	-.66+01	-.35+01	-.98+00	.88+00	.11+00						
.10+00	.35+00	.53+00	.75+00	.61+00	.27+00	-.94-01	-.77-01						
.98+01	.73+01	.95+01	.76+01	.10+02	.90+01	.86+01	.92+01	.42+01					
-.98+01	-.69+01	-.82+01	-.60+01	-.63+01	-.40+01	-.20+01	.14+01	.49+00					
.15+00	.32+00	.69+00	.64+00	.10+01	.98+00	.79+00	.19+00	-.29+00					
.95+01	.10+02	.93+01	.11+02	.79+01	.92+01	.10+02	.95+01	.10+02	.83+01	.11+01			
-.94+01	-.96+01	-.80+01	-.86+01	-.48+01	-.44+01	-.35+01	-.12+01	.43+00	.14+01	.15+00			
.14+00	.51+00	.72+00	.12+01	.89+00	.11+01	.12+01	.10+01	.54+00	-.39+00	-.14+00			
.92+01	.12+02	.12+02	.90+01	.11+02	.95+01	.92+01	.10+02	.96+01	.86+01	.10+02	.16+01		
-.91+01	-.11+02	-.10+02	-.59+01	-.66+01	-.36+01	-.11+01	-.19+01	-.16+00	.15+00	.46+00	.37+00		
.17+00	.67+00	.10+01	.10+01	.14+01	.12+01	.13+01	.12+01	.10+01	.51+00	-.55+00	-.22+00		
.12+02	.16+02	.11+02	.11+02	.11+02	.11+02	.10+02	.10+02	.99+01	.10+02	.10+02	.11+02	.83+00	
-.12+02	-.15+02	-.91+01	-.74+01	-.51+01	-.41+01	-.15+01	-.86+00	.15+01	.24+01	-.29+00	.87+00	-.19+00	
.25+00	.93+00	.11+01	.13+01	.15+01	.15+01	.14+01	.14+01	.12+01	.10+01	.59+00	-.71+00	-.13+00	
.16+02	.13+02	.12+02	.14+02	.12+02	.13+02	.11+02	.11+02	.10+02	.83+01	.10+02	.11+02	.98+01	
-.16+02	-.11+02	-.87+01	-.88+01	-.37+01	-.20+01	-.13+01	.14+01	.13+01	.17+01	.24+01	.65+00	-.56+00	
.38+00	.90+00	.13+01	.17+01	.17+01	.19+01	.16+01	.15+01	.13+01	.90+00	.91+00	.35+00	-.91+00	
.17+02	.13+02	.18+02	.14+02	.15+02	.13+02	.12+02	.12+02	.11+02	.10+02	.83+01	.10+02	.12+02	.58+01
-.16+02	-.11+02	-.13+02	-.60+01	-.49+01	-.34+00	.12+01	.16+01	.40+01	.32+01	.41+01	.30+01	.51+00	-.19+01
.54+00	.10+01	.20+01	.20+01	.22+01	.20+01	.17+01	.16+01	.13+01	.11+01	.61+00	.64+00	-.18+00	-.62+00
.64+01	.14+02	.18+02	.15+02	.15+02	.14+02	.13+02	.12+02	.12+02	.10+02	.10+02	.98+01	.95+01	.11+02
-.64+01	-.12+02	-.98+01	-.49+01	-.71+00	.12+01	.36+01	.45+01	.40+01	.49+01	.45+01	.50+01	.22+01	-.37+01
.11+00	.11+01	.24+01	.22+01	.23+01	.21+01	.18+01	.15+01	.13+01	.10+01	.91+00	.46+00	.19+00	-.79+00
.61+01	.82+01	.52+01	.19+02	.17+02	.13+02	.17+02	.13+02	.12+02	.12+02	.93+01	.11+02	.90+01	.13+02	.39+01
-.60+01	-.75+01	-.42+01	-.36+01	.25+01	.40+01	.74+01	.46+01	.69+01	.71+01	.47+01	.46+01	.21+01	-.97+00	-.23+01
.12+00	.52+00	.51+00	.28+01	.25+01	.18+01	.20+01	.16+01	.11+01	.91+00	.66+00	.73+00	.46+00	-.19+00	-.40+00
.75+01	.75+01	.70+01	.93+01	.20+02	.20+02	.13+02	.11+02	.15+02	.11+02	.85+01	.11+02	.11+02	.10+02	.10+02
-.73+01	-.65+01	-.49+01	-.17+01	.68+01	.10+02	.66+01	.65+01	.93+01	.58+01	.45+01	.59+01	.51+01	.13+01	-.53+01
.21+00	.61+00	.82+00	.13+01	.26+01	.23+01	.14+01	.10+01	.12+01	.89+00	.58+00	.51+00	.38+00	.10+00	-.65+00
.74+01	.84+01	.10+02	.68+01	.22+02	.18+02	.13+02	.15+02	.13+02	.11+02	.12+02	.10+02	.10+02	.11+02	.13+02
-.72+01	-.67+01	-.57+01	-.21+01	.11+02	.11+02	.73+01	.11+02	.92+01	.69+01	.76+01	.57+01	.45+01	.33+01	-.56+01
.25+00	.83+00	.13+01	.10+01	.23+01	.15+01	.11+01	.94+00	.72+00	.66+00	.54+00	.44+00	.46+00	-.79-01	-.55+00
		.66+01	.10+02	.16+02	.19+02	.19+02	.12+02	.14+02	.11+02	.88+01	.11+02	.96+01	.10+02	.14+02
		-.17+01	-.41-01	.95+01	.14+02	.13+02	.81+01	.10+02	.80+01	.52+01	.70+01	.53+01	.35+01	-.68+01
		.10+01	.17+01	.13+01	.91+00	.93+00	.71+00	.53+00	.17+00	.54+00	.18+00	-.64-01	-.62-02	-.13+00

		.66+01	.10+02	.99+00	-.51+01	-.22+01	.10+01	-.12+01	-.23+01	.21+01	-.12+01	-.20+01	-.93+00	.26+01
		-.17+01	-.41-01	-.60+01	-.10+02	-.75+01	-.25+01	-.44+01	-.41+01	-.50+00	-.32+01	-.31+01	-.27+01	-.41+01
		-.10+01	-.17+01	-.15+01	-.14+01	-.17+01	-.13+01	-.14+01	-.11+01	-.10+01	-.14+01	-.12+01	-.16+01	-.16+01
.74+01	.84+01	.10+02	.68+01	-.60+01	-.54+01	-.59-01	-.46+01	-.23+01	-.80-01	-.12+01	-.15+00	.75+00	-.18+01	.21+01
-.72+01	-.67+01	-.57+01	-.21+01	-.13+02	-.11+02	-.60+01	-.92+01	-.66+01	-.40+01	-.50+01	-.36+01	-.33+01	-.48+01	-.59+01
-.25+00	-.83+00	-.13+01	-.10+01	-.36+00	-.62+00	-.10+01	-.90+00	-.11+01	-.11+01	-.12+01	-.12+01	-.13+01	-.15+01	-.11+01
.75+01	.75+01	.70+01	.29+01	-.73+01	-.78+01	-.23+01	-.23+01	-.24+01	-.12+00	-.14+00	-.12+01	-.10+01	-.13+00	.48+01
-.73+01	-.65+01	-.49+01	-.45+01	-.11+02	-.13+02	-.80+01	-.68+01	-.89+01	-.53+01	-.40+01	-.57+01	-.61+01	-.51+01	-.62+01
-.21+00	-.61+00	-.82+00	-.40+00	.68+00	.23+00	-.41+00	-.47+00	-.10+01	-.93+00	-.81+00	-.11+01	-.13+01	-.14+01	-.30+00
.61+01	.82+01	.52+01	-.43+01	-.66+01	-.45+01	-.58+01	-.15+00	-.35+01	-.34+01	-.11+01	-.19+00	.71+00	.12+01	.39+01
-.60+01	-.75+01	-.42+01	-.64+01	-.82+01	-.76+01	-.10+02	-.75+01	-.82+01	-.80+01	-.57+01	-.67+01	-.53+01	-.83+01	-.23+01
-.12+00	-.52+00	-.51+00	.10+01	.10+01	.46+00	.24+00	-.56+00	-.33+00	-.50+00	-.68+00	-.11+01	-.10+01	-.13+01	.40+00
.64+01	-.10+01	-.75+01	-.43+01	-.67+01	-.34+01	-.46+01	-.35+01	-.12+01	-.22+01	-.13+01	-.28+01	-.10+01	.79+01	
-.64+01	-.22+00	.47+00	-.34+01	-.51+01	-.71+01	-.76+01	-.78+01	-.75+01	-.71+01	-.75+01	-.76+01	-.73+01	-.62+01	
-.11+00	.41+00	.15+01	.10+01	.12+01	.54+00	.52+00	.15+00	-.30+00	-.32+00	-.60+00	-.61+00	-.90+00	.33+00	
-.10+02	-.43+01	-.54+01	-.78+01	-.23+01	-.46+01	-.34+01	-.12+01	-.33+01	-.11+01	-.29+01	-.94+00	.38+01	.58+01	
.10+02	.28+01	.11+01	.45+00	-.50+01	-.54+01	-.62+01	-.70+01	-.78+01	-.75+01	-.67+01	-.83+01	-.71+01	-.19+01	
.42+00	.63+00	.11+01	.14+01	.75+00	.87+00	.53+00	.58-01	.14+00	-.27+00	-.16+00	-.67+00	-.34+00	.62+00	
-.53+01	-.65+01	-.54+01	-.11+01	-.56+01	-.44+01	-.11+01	-.33+01	-.11+01	-.11+01	-.10+01	.38+01	.85+01		
.52+01	.53+01	.25+01	-.26+01	-.14+01	-.43+01	-.56+01	-.65+01	-.71+01	-.61+01	-.87+01	-.66+01	-.18+01		
.23+00	.66+00	.91+00	.63+00	.11+01	.95+00	.36+00	.49+00	.68-01	-.43-01	-.32+00	-.35+00	.84+00		
-.42+01	-.31+01	-.53+01	-.32+01	-.32+01	-.11+01	-.22+01	-.11+01	-.29+01	-.26+01	.60+01	.11+02	.83+00		
.41+01	.22+01	.30+01	-.22+00	-.20+01	-.41+01	-.44+01	-.59+01	-.66+01	-.84+01	-.45+01	.87+00	-.19+00		
.16+00	.49+00	.82+00	.79+00	.84+00	.53+00	.56+00	.33+00	.44+00	.18+00	-.36+00	.71+00	.13+00		
-.10+01	-.40+01	-.20+01	-.30+01	-.20+01	-.21+01	-.37+01	-.19+00	.31+01	.71+01	.10+02	.16+01			
.92+00	.32+01	.38+00	.35+00	-.21+01	-.27+01	-.34+01	-.63+01	-.42+01	-.10+01	.46+00	.37+00			
.10+00	.46+00	.55+00	.67+00	.72+00	.65+00	.80+00	.32+00	-.16+00	-.36+00	.55+00	.22+00			
-.19+01	-.27+01	-.18+01	-.26+01	.73+00	.57+00	.51+01	.79+01	.10+02	.83+01	.11+01				
.19+01	.22+01	.68+00	-.12+00	-.22+01	-.32+01	-.41+01	-.17+01	.43+00	.14+01	.15+00				
.91-01	.34+00	.45+00	.75+00	.22+00	.32+00	-.38+00	-.78+00	-.54+00	.39+00	.14+00				
.62+01	.73+01	.78+01	.76+01	.10+02	.90+01	.86+01	.92+01	.42+01						
-.62+01	-.69+01	-.70+01	-.60+01	-.63+01	-.40+01	-.20+01	.14+01	.49+00						
-.10+00	-.32+00	-.49+00	-.64+00	-.10+01	-.98+00	-.79+00	-.19+00	.29+00						
.81+01	.86+01	.86+01	.93+01	.81+01	.60+01	.38+01	.51+00							
-.80+01	-.81+01	-.75+01	-.66+01	-.35+01	-.98+00	.88+00	.11+00							
-.10+00	-.35+00	-.53+00	-.75+00	-.61+00	-.27+00	.94-01	.77-01							
.37+01	.46+01	.28+01	.14+01											
-.36+01	-.43+01	-.21+01	-.63+00											
-.27-01	-.17+00	-.16+00	-.10+00											

TABLE 13

DISTRIBUTION FOR $\gamma = 0$, $\theta_0 = 30°$, AND $\sin \alpha_L = 0.4$

.90+00	.90+00												
-.90+00	-.90+00												
-.12-02	-.36-02												
.58+01	.51+01	.60+01	.51+01	.25+01	.91+00								
-.58+01	-.50+01	-.55+01	-.43+01	-.21+01	-.91+00								
.65-01	.12+00	.23+00	.28+00	.95-01	-.13-01								
.98+01	.73+01	.95+01	.76+01	.10+02	.84+01	.63+01	.32+01	.46+00					
-.98+01	-.69+01	-.82+01	-.60+01	-.63+01	-.40+01	-.25+01	-.58+00	-.45+00					
.15+00	.32+00	.69+00	.64+00	.10+01	.89+00	.54+00	.24+00	-.94-02					
.75+01	.10+02	.93+01	.11+02	.79+01	.92+01	.10+02	.95+01	.77+01	.34+01				
-.74+01	-.96+01	-.80+01	-.86+01	-.48+01	-.44+01	-.35+01	-.12+01	-.25+00	-.30+00				
.12+00	.51+00	.72+00	.12+01	.89+00	.11+01	.12+01	.10+01	.60+00	.43-01				
.41+01	.71+01	.92+01	.90+01	.11+02	.95+01	.92+01	.10+02	.96+01	.86+01	.70+01	.44+00		
-.41+01	-.67+01	-.79+01	-.59+01	-.66+01	-.36+01	-.11+01	-.19+01	-.16+00	.15+00	-.47+00	.36+00		
.35-01	.32+00	.67+00	.10+01	.14+01	.12+01	.13+01	.12+01	.10+01	.51+00	-.82-01	-.43-01		
.43+01	.65+01	.32+01	.42+01	.73+01	.11+02	.10+02	.10+02	.99+01	.10+02	.10+02	.92+01	.41+00	
-.43+01	-.63+01	-.30+01	-.38+01	-.42+01	-.41+01	-.15+01	-.86+00	.15+01	.24+01	-.29+00	.10+01	-.19+00	
.42-01	.21+00	.17+00	.30+00	.88+00	.15+01	.14+01	.14+01	.12+01	.10+01	.59+00	-.40+00	-.60-01	
.55+01	.33+01	.33+01	.66+01	.32+01	.12+02	.11+02	.11+02	.10+02	.83+01	.10+02	.11+02	.98+01	
-.55+01	-.32+01	-.30+01	-.57+01	-.26+01	-.21+01	-.13+01	.14+01	.13+01	.17+01	.24+01	.65+00	-.56+00	
.77-01	.12+00	.20+00	.54+00	.32+00	.17+01	.16+01	.15+01	.13+01	.90+00	.91+00	.35+00	-.91+00	
.33+01	.44+01	.66+01	.33+01	.66+01	.44+01	.12+02	.12+02	.11+02	.10+02	.83+01	.10+02	.12+02	.58+01
-.32+01	-.42+01	-.60+01	-.27+01	-.49+01	-.28+01	.12+01	.16+01	.40+01	.32+01	.41+01	.30+01	.51+00	-.19+01
.56-01	.19+00	.46+00	.30+00	.74+00	.56+00	.17+01	.16+01	.13+01	.11+01	.61+00	.64+00	-.18+00	-.62+00
.64+01	.65+01	.54+01	.55+01	.44+01	.56+01	.67+01	.12+02	.12+02	.10+02	.10+02	.98+01	.95+01	.11+02
-.64+01	-.61+01	-.47+01	-.41+01	-.29+01	-.29+01	-.48+00	.45+01	.40+01	.49+01	.45+01	.50+01	.22+01	-.37+01
.11+00	.34+00	.45+00	.60+00	.55+00	.78+00	.95+00	.15+01	.13+01	.10+01	.91+00	.46+00	.19+00	-.79+00
.61+01	.82+01	.52+01	.75+01	.55+01	.44+01	.56+01	.13+02	.12+02	.12+02	.93+01	.11+02	.90+01	.13+02	.39+01
-.60+01	-.75+01	-.42+01	-.50+01	-.28+01	-.17+01	-.16+01	.46+01	.69+01	.71+01	.47+01	.46+01	.21+01	-.97+00	-.23+01
.12+00	.52+00	.51+00	.92+00	.78+00	.67+00	.89+00	.16+01	.11+01	.91+00	.66+00	.73+00	.46+00	-.19+00	-.40+00
		.30+01	.41+01	.63+01	.65+01	.56+01	.91+01	.15+02	.11+02	.85+01	.11+02	.11+02	.10+02	.10+02
		-.22+01	-.21+01	-.25+01	-.14+01	-.69+00	.43+01	.93+01	.58+01	.45+01	.59+01	.51+01	.13+01	-.53+01
		.34+00	.58+00	.97+00	.10+01	.92+00	.93+00	.12+01	.89+00	.58+00	.51+00	.38+00	.10+00	-.65+00
				.21+01	.54+01	.66+01	.11+02	.13+02	.11+02	.12+02	.10+02	.10+02	.11+02	.13+02
				-.38+00	.74-01	.63+00	.67+01	.92+01	.69+01	.76+01	.57+01	.45+01	.33+01	-.56+01
				.34+00	.89+00	.11+01	.89+00	.72+00	.66+00	.54+00	.44+00	.46+00	-.79-01	-.55+00
						.77+01	.12+02	.14+02	.11+02	.88+01	.11+02	.96+01	.10+02	.14+02
						.27+01	.81+01	.10+02	.80+01	.52+01	.70+01	.53+01	.35+01	-.68+01
						.11+01	.71+00	.53+00	.17+00	.54+00	.18+00	-.64-01	-.62-02	-.13+00

							.77+01	.10+01	-.12+01	-.23+01	.21+01	-.12+01	-.20+01	-.93+00	.26+01
							.27+01	-.25+01	-.44+01	-.41+01	-.50+00	-.32+01	-.31+01	-.27+01	-.41+01
							-.11+01	-.13+01	-.14+01	-.11+01	-.10+01	-.14+01	-.12+01	-.16+01	-.16+01
						.21+01	.54+01	.66+01	-.90-01	-.23+01	-.80-01	-.12+01	-.15+00	.75+00	-.18+01
						-.38+00	.74-01	.63+00	-.46+01	-.66+01	-.40+01	-.50+01	-.36+01	-.33+01	-.48+01
						-.34+00	-.89+00	-.11+01	-.95+00	-.11+01	-.11+01	-.12+01	-.12+01	-.13+01	-.15+01
			.30+01	.41+01	.63+01	.65+01	.56+01	-.91-01	-.24+01	-.12+00	-.14+00	-.12+01	-.10+01	-.13+00	.48+01
			-.22+01	-.21+01	-.25+01	-.14+01	-.69+00	-.45+01	-.89+01	-.53+01	-.40+01	-.57+01	-.61+01	-.51+01	-.62+01
			-.34+00	-.58+00	-.97+00	-.10+01	-.92+00	-.56+00	-.10+01	-.93+00	-.81+00	-.11+01	-.13+01	-.14+01	-.30+00
.61+01	.82+01	.52+01	.75+01	.55+01	.44+01	.56+01	-.15+00	-.35+01	-.34+01	-.11+01	-.19+00	.71+00	.12+01	.39+01	
-.60+01	-.75+01	-.42+01	-.50+01	-.28+01	-.17+01	-.16+01	-.75+01	-.82+01	-.80+01	-.57+01	-.67+01	-.53+01	-.83+01	-.23+01	
-.12+00	-.52+00	-.51+00	-.92+00	-.78+00	-.67+00	-.89+00	-.56+00	-.33+00	-.50+00	-.68+00	-.11+01	-.10+01	-.13+01	.40+00	
.64+01	.65+01	.54+01	.55+01	.44+01	.56+01	.22+01	-.35+01	-.12+01	-.22+01	-.13+01	-.28+01	-.10+01	.79+01		
-.64+01	-.61+01	-.47+01	-.41+01	-.29+01	-.29+01	-.34+01	-.78+01	-.75+01	-.71+01	-.75+01	-.76+01	-.73+01	-.62+01		
-.11+00	-.34+00	-.45+00	-.60+00	-.55+00	-.78+00	-.38+00	.15+00	-.30+00	-.32+00	-.60+00	-.61+00	-.90+00	.33+00		
.33+01	.44+01	.66+01	.33+01	.66+01	.44+01	-.34+01	-.12+01	-.33+01	-.11+01	-.29+01	-.94+00	.38+01	.58+01		
-.32+01	-.42+01	-.60+01	-.27+01	-.49+01	-.28+01	-.62+01	-.70+01	-.78+01	-.75+01	-.67+01	-.83+01	-.71+01	-.19+01		
-.56-01	-.19+00	-.46+00	-.30+00	-.74+00	-.56+00	.53+00	.58-01	.14+00	-.27+00	-.16+00	-.67+00	-.34+00	.62+00		
.55+01	.33+01	.33+01	.66+01	.32+01	-.33+01	-.11+01	-.33+01	-.11+01	-.11+01	-.10+01	.38+01	.85+01			
-.55+01	-.32+01	-.30+01	-.57+01	-.26+01	-.41+01	-.56+01	-.65+01	-.71+01	-.61+01	-.87+01	-.66+01	-.18+01			
-.77-01	-.12+00	-.20+00	-.54+00	-.32+00	.77+00	.36+00	.49+00	.68-01	-.43-01	-.32+00	-.35+00	.84+00			
.43+01	.65+01	.32+01	.42+01	.10+01	-.11+01	-.22+01	-.11+01	-.29+01	-.26+01	.60+01	.92+01	.41+00			
-.43+01	-.63+01	-.30+01	-.38+01	-.29+01	-.41+01	-.44+01	-.59+01	-.66+01	-.84+01	-.45+01	.10+01	-.19+00			
-.42-01	-.21+00	-.17+00	-.30+00	.14+00	.53+00	.56+00	.33+00	.44+00	.18+00	-.36+00	.40+00	.60-01			
.41+01	.11+01	.11+01	-.30+01	-.20+01	-.21+01	-.37+01	-.19+00	.31+01	.71+01	.70+01	.44+00				
-.41+01	-.13+01	-.19+01	.35+00	-.21+01	-.27+01	-.34+01	-.63+01	-.42+01	-.10+01	-.47+00	.36+00				
-.35-01	.10+00	.20+00	.67+00	.72+00	.65+00	.80+00	.32+00	-.16+00	-.36+00	.82-01	.43-01				
.18-01	-.27+01	-.18+01	-.26+01	.73+00	.57+00	.51+01	.79+01	.77+01	.34+01						
-.65-01	.22+01	.68+00	-.12+00	-.22+01	-.32+01	-.41+01	-.17+01	-.25+00	-.30+00						
.64-01	.34+00	.45+00	.75+00	.22+00	.32+00	-.38+00	-.78+00	-.60+00	-.43-01						
.62+01	.73+01	.78+01	.76+01	.10+02	.84+01	.63+01	.32+01	.46+00							
-.62+01	-.69+01	-.70+01	-.60+01	-.63+01	-.40+01	-.25+01	-.58+00	-.45+00							
-.10+00	-.32+00	-.49+00	-.64+00	-.10+01	-.89+00	-.54+00	-.24+00	.94-02							
.58+01	.51+01	.60+01	.51+01	.25+01	.91+00										
-.58+01	-.50+01	-.55+01	-.43+01	-.21+01	-.91+00										
-.65-01	-.12+00	-.23+00	-.28+00	-.95-01	.13-01										
.90+00	.90+00														
-.90+00	-.90+00														
.12-02	.36-02														

TABLE 14

DISTRIBUTION FOR $\gamma = 0$, $\theta_0 = 30°$, AND $\sin \alpha_L = 0.6$

.26+01	.26+01	.19+01	.12+01										
-.26+01	-.26+01	-.19+01	-.12+01										
.96-02	.28-01	.31-01	.25-01										
.32+01	.32+01	.39+01	.38+01	.42+01	.26+01	.12+01							
-.32+01	-.32+01	-.39+01	-.37+01	-.41+01	-.25+01	-.12+01							
.19-01	.56-01	.11+00	.13+00	.15+00	.10+00	.45-01							
.37+01	.37+01	.37+01	.45+01	.35+01	.41+01	.46+01	.33+01	.19+01					
-.37+01	-.37+01	-.36+01	-.43+01	-.32+01	-.38+01	-.42+01	-.31+01	-.18+01					
.27-01	.82-01	.13+00	.22+00	.20+00	.26+00	.30+00	.19+00	.10+00					
.41+01	.41+01	.51+01	.30+01	.48+01	.37+01	.27+01	.49+01	.43+01	.25+01				
-.41+01	-.40+01	-.49+01	-.28+01	-.43+01	-.32+01	-.22+01	-.41+01	-.37+01	-.23+01				
.35-01	.10+00	.23+00	.18+00	.36+00	.31+00	.25+00	.43+00	.34+00	.17+00				
.43+01	.65+01	.32+01	.42+01	.42+01	.51+01	.39+01	.47+01	.34+01	.39+01	.34+01			
-.43+01	-.63+01	-.30+01	-.38+01	-.36+01	-.41+01	-.30+01	-.34+01	-.25+01	-.29+01	-.28+01			
.42-01	.21+00	.17+00	.30+00	.36+00	.50+00	.42+00	.53+00	.39+00	.42+00	.33+00			
.55+01	.33+01	.33+01	.66+01	.32+01	.43+01	.52+01	.40+01	.48+01	.36+01	.48+01	.34+01		
-.55+01	-.32+01	-.30+01	-.57+01	-.26+01	-.31+01	-.35+01	-.25+01	-.28+01	-.21+01	-.31+01	-.26+01		
.77-01	.12+00	.20+00	.54+00	.32+00	.49+00	.65+00	.53+00	.64+00	.47+00	.62+00	.36+00		
.33+01	.44+01	.66+01	.33+01	.66+01	.44+01	.44+01	.53+01	.41+01	.48+01	.26+01	.47+01	.27+01	
-.32+01	-.42+01	-.60+01	-.27+01	-.49+01	-.28+01	-.25+01	-.27+01	-.18+01	-.21+01	-.13+01	-.26+01	-.19+01	
.56-01	.19+00	.46+00	.30+00	.74+00	.56+00	.60+00	.77+00	.61+00	.72+00	.39+00	.66+00	.31+00	
			.22+01	.33+01	.56+01	.44+01	.44+01	.53+01	.41+01	.48+01	.98+01	.67+01	.63+00
			-.17+01	-.22+01	-.29+01	-.19+01	-.16+01	-.17+01	-.11+01	-.15+01	.50+01	-.22+00	-.46+00
			.23+00	.41+00	.78+00	.66+00	.68+00	.84+00	.66+00	.75+00	.46+00	.42+00	.71-01
						.45+01	.67+01	.44+01	.42+01	.61+01	.11+02	.90+01	.55+01
						-.13+01	-.14+01	-.68+00	-.46+00	.23-04	.46+01	.21+01	-.11+01
						.71+00	.10+01	.72+00	.70+00	.66+00	.73+00	.46+00	.38+00
							.22+01	.67+01	.54+01	.52+01	.11+02	.11+02	.88+01
							-.12+00	.20+00	.29+00	.13+01	.59+01	.51+01	.67+00
							.37+00	.11+01	.91+00	.65+00	.51+00	.38+00	.29+00
								.33+01	.56+01	.87+01	.10+02	.10+02	.11+02	.26+01
								.75+00	.14+01	.43+01	.57+01	.45+01	.33+01	-.18+00
								.55+00	.90+00	.69+00	.44+00	.46+00	-.79-01	-.20-01
									.45+01	.88+01	.11+02	.96+01	.10+02	.47+01
									.19+01	.52+01	.70+01	.53+01	.35+01	-.68+00
									.67+00	.54+00	.18+00	-.64-01	-.62-02	.29+00

.45+01	.21+01	-.12+01	-.20+01	-.93+00	.19+01	
.19+01	-.50+00	-.32+01	-.31+01	-.27+01	-.13+01	
-.67+00	-.10+01	-.14+01	-.12+01	-.16+01	-.75+00	

.33+01	.56+01	.21+01	-.15+00	.75+00	-.18+01	-.48-01
.75+00	.14+01	-.17+01	-.36+01	-.33+01	-.48+01	-.99+00
-.55+00	-.90+00	-.10+01	-.12+01	-.13+01	-.15+01	-.41+00

.22+01	.67+01	.54+01	.31+01	-.12+01	-.10+01	.12+01
-.12+00	.20+00	.29+00	-.79+00	-.57+01	-.61+01	-.44+01
-.37+00	-.11+01	-.91+00	-.73+00	-.11+01	-.13+01	-.12+01

.00+01	.45+01	.67+01	.44+01	.42+01	.20+01	-.19+00	.71+00	.25+01
.00+01	-.13+01	-.14+01	-.68+00	-.46+00	-.25+01	-.67+01	-.53+01	-.34+01
.00+01	-.71+00	-.10+01	-.72+00	-.70+00	-.69+00	-.11+01	-.10+01	-.70+00

.22+01	.33+01	.56+01	.44+01	.44+01	.53+01	.41+01	.48+01	-.28+01	.17+01	.63+00
-.17+01	-.22+01	-.29+01	-.19+01	-.16+01	-.17+01	-.11+01	-.15+01	-.76+01	-.49+01	-.46+00
-.23+00	-.41+00	-.78+00	-.66+00	-.68+00	-.84+00	-.66+00	-.75+00	-.61+00	-.67+00	-.71-01

.33+01	.44+01	.66+01	.33+01	.66+01	.44+01	.44+01	.53+01	.41+01	.48+01	.26+01	.47+01	.27+01
-.32+01	-.42+01	-.60+01	-.27+01	-.49+01	-.28+01	-.25+01	-.27+01	-.18+01	-.21+01	-.13+01	-.26+01	-.19+01
-.56-01	-.19+00	-.46+00	-.30+00	-.74+00	-.56+00	-.60+00	-.77+00	-.61+00	-.72+00	-.39+00	-.66+00	-.31+00

.55+01	.33+01	.33+01	.66+01	.32+01	.43+01	.52+01	.40+01	.48+01	.36+01	.48+01	.34+01
-.55+01	-.32+01	-.30+01	-.57+01	-.26+01	-.31+01	-.35+01	-.25+01	-.28+01	-.21+01	-.31+01	-.26+01
-.77-01	-.12+00	-.20+00	-.54+00	-.32+00	-.49+00	-.65+00	-.53+00	-.64+00	-.47+00	-.62+00	-.36+00

.43+01	.65+01	.32+01	.42+01	.42+01	.51+01	.39+01	.47+01	.34+01	.39+01	.34+01
-.43+01	-.63+01	-.30+01	-.38+01	-.36+01	-.41+01	-.30+01	-.34+01	-.25+01	-.29+01	-.28+01
-.42-01	-.21+00	-.17+00	-.30+00	-.36+00	-.50+00	-.42+00	-.53+00	-.39+00	-.42+00	-.33+00

.41+01	.41+01	.51+01	.30+01	.48+01	.37+01	.27+01	.49+01	.43+01	.25+01
-.41+01	-.40+01	-.49+01	-.28+01	-.43+01	-.32+01	-.22+01	-.41+01	-.37+01	-.23+01
-.35-01	-.10+00	-.23+00	-.18+00	-.36+00	-.31+00	-.25+00	-.43+00	-.34+00	-.17+00

.37+01	.37+01	.37+01	.45+01	.35+01	.41+01	.46+01	.33+01	.19+01
-.37+01	-.37+01	-.36+01	-.43+01	-.32+01	-.38+01	-.42+01	-.31+01	-.18+01
-.27-01	-.82-01	-.13+00	-.22+00	-.20+00	-.26+00	-.30+00	-.19+00	-.10+00

.32+01	.32+01	.39+01	.38+01	.42+01	.26+01	.12+01
-.32+01	-.32+01	-.39+01	-.37+01	-.41+01	-.25+01	-.12+01
-.19-01	-.56-01	-.11+00	-.13+00	-.15+00	-.10+00	-.45-01

.26+01	.26+01	.19+01	.12+01
-.26+01	-.26+01	-.19+01	-.12+01
-.96-02	-.28-01	-.31-01	-.25-01

TABLE 15

DISTRIBUTION FOR $\gamma = 0$, $\theta_0 = 30°$, AND $\sin \alpha_L = 0.8$

.37+01	.37+01	.37+01	.37+01	.18+01	.90+00			
-.37+01	-.37+01	-.36+01	-.35+01	-.16+01	-.81+00			
.27-01	.82-01	.13+00	.18+00	.11+00	.64-01			
.41+01	.41+01	.51+01	.30+01	.48+01	.37+01	.18+01		
-.41+01	-.40+01	-.49+01	-.28+01	-.43+01	-.32+01	-.15+01		
.35-01	.10+00	.23+00	.18+00	.36+00	.31+00	.17+00		
			.20+01	.20+01	.51+01	.39+01	.38+01	
			-.19+01	-.18+01	-.41+01	-.30+01	-.27+01	
			.14+00	.17+00	.50+00	.42+00	.44+00	
						.10+01	.99+00	.29+01
						-.74+00	-.64+00	-.17+01
						.12+00	.12+00	.38+00

						.10+01	.99+00	.29+01
						-.74+00	-.64+00	-.17+01
						-.12+00	-.12+00	-.38+00
			.20+01	.20+01	.51+01	.39+01	.38+01	
			-.19+01	-.18+01	-.41+01	-.30+01	-.27+01	
			-.14+00	-.17+00	-.50+00	-.42+00	-.44+00	
.41+01	.41+01	.51+01	.30+01	.48+01	.37+01	.18+01		
-.41+01	-.40+01	-.49+01	-.28+01	-.43+01	-.32+01	-.15+01		
-.35-01	-.10+00	-.23+00	-.18+00	-.36+00	-.31+00	-.17+00		
.37+01	.37+01	.37+01	.37+01	.18+01	.90+00			
-.37+01	-.37+01	-.36+01	-.35+01	-.16+01	-.81+00			
-.27-01	-.82-01	-.13+00	-.18+00	-.11+00	-.64-01			

TABLE 16

DISTRIBUTION FOR $\gamma = 0$, $\theta_0 = 60°$, AND $\sin \alpha_L = 0$

.14+01	.19+01	.10+01
-.10+01	.26+00	.79+00
.17-01	-.10-01	-.24-01

.97+01	.11+02	.11+02	.13+02	.12+02	.11+02	.78+01	.54+00
-.93+01	-.86+01	-.40+01	.98+00	.42+01	.58+01	.56+01	.50+00
.23+00	.69+00	.57+00	.51+00	.26-01	-.33+00	-.46+00	-.36-01

.12+02	.11+02	.13+02	.11+02	.13+02	.13+02	.16+02	.21+02	.16+02	.33+01	
-.12+02	-.98+01	-.81+01	-.35+01	-.96+00	.18+01	.57+01	.11+02	.10+02	.14+01	
.29+00	.66+00	.13+01	.12+01	.12+01	.88+00	.29+00	-.10+01	-.14+01	-.44+00	

.11+02	.15+02	.12+02	.15+02	.14+02	.14+02	.15+02	.15+02	.17+02	.23+02	.16+02	.15+01	
-.11+02	-.12+02	-.77+01	-.58+01	-.37+00	.16+01	.48+01	.48+01	.75+01	.11+02	.62+01	.56+00	
.30+00	.12+01	.13+01	.17+01	.17+01	.14+01	.11+01	.43+00	-.27+00	-.20+01	-.21+01	-.23+00	

.15+02	.15+02	.18+02	.15+02	.18+02	.18+02	.16+02	.17+02	.15+02	.16+02	.19+02	.25+02	.32+01	
-.14+02	-.12+02	-.92+01	-.39+01	.91+00	.52+01	.66+01	.95+01	.56+01	.67+01	.62+01	.61+01	.38+00	
.49+00	.13+01	.22+01	.20+01	.23+01	.20+01	.13+01	.93+00	.21+00	-.34+00	-.14+01	-.34+01	-.51+00	

.19+02	.21+02	.22+02	.21+02	.20+02	.21+02	.20+02	.15+02	.20+02	.16+02	.15+02	.18+02	.25+02	.24+01	
-.18+02	-.15+02	-.83+01	.51+00	.41+01	.91+01	.11+02	.73+01	.11+02	.76+01	.30+01	.33+01	-.97+00	-.59+00	
.75+00	.21+01	.31+01	.31+01	.24+01	.21+01	.14+01	.61+00	.26+00	-.31+00	-.73+00	-.17+01	-.36+01	-.39+00	

.24+02	.27+02	.30+02	.29+02	.30+02	.27+02	.18+02	.18+02	.18+02	.15+02	.15+02	.17+02	.19+02	.23+02	
-.22+02	-.17+02	-.58+01	.68+01	.14+02	.14+02	.12+02	.69+01	.10+02	.69+01	.34+01	.20+01	-.11+01	-.70+01	
.10+01	.32+01	.44+01	.40+01	.33+01	.19+01	.57+00	.52+00	-.22+00	-.52+00	-.58+00	-.11+01	-.20+01	-.32+01	

.44+02	.42+02	.49+02	.43+02	.32+02	.24+02	.19+02	.18+02	.16+02	.14+02	.13+02	.13+02	.16+02	.24+02	.11+02	
-.40+02	-.16+02	.10+02	.23+02	.20+02	.17+02	.11+02	.98+01	.53+01	.48+01	.20+01	-.24+00	-.51+00	-.83+01	-.65+01	
.25+01	.60+01	.72+01	.43+01	.17+01	.33+00	-.18+00	-.39+00	-.45+00	-.63+00	-.66+00	-.58+00	-.13+01	-.28+01	-.14+01	

.71+02	.11+03	.58+02	.26+02	.22+02	.16+02	.16+02	.14+02	.12+02	.12+02	.14+02	.14+02	.12+02	.18+02	.21+02	
-.43+02	.21+02	.35+02	.16+02	.10+02	.67+01	.39+01	.43+01	.26+01	.10+01	.75+00	.43+00	-.33+01	-.85+01	-.14+02	
.61+01	.15+02	.39+01	-.44+00	-.87+00	-.80+00	-.62+00	-.73+00	-.59+00	-.41+00	-.69+00	-.93+00	-.53+00	-.12+01	-.21+01	

.19+03	.24+02	.15+02	.14+02	.13+02	.13+02	.13+02	.12+02	.13+02	.11+02	.11+02	.13+02	.14+02	.15+02	.28+02	
-.11+03	-.88+01	-.41+01	-.33+01	-.15+01	-.83+00	-.98-01	-.38+00	.20+00	.33+00	-.12+01	-.21+01	-.42+01	-.73+01	-.22+02	
.22+02	.54+00	-.15+01	-.81+00	-.83+00	-.77+00	-.65+00	-.37+00	-.56+00	-.59+00	-.32+00	-.39+00	-.36+00	-.57+00	-.15+01	

.19+03	.11+02	.12+02	.12+02	.10+02	.11+02	.11+02	.13+02	.12+02	.12+02	.10+02	.12+02	.13+02	.14+02	.29+02	
-.16+03	-.97+01	-.90+01	-.66+01	-.43+01	-.36+01	-.23+01	-.23+01	-.16+01	-.15+01	-.16+01	-.25+01	-.39+01	-.70+01	-.24+02	
.12+02	-.40+00	-.37-01	-.66+00	-.71-01	-.34+00	-.36+00	-.21+00	-.37-01	-.40+00	-.49-01	-.73-01	-.59+00	-.30+00	-.56+00	

-.18+03	-.32+01	.16+01	-.35+01	.34+00	-.15+01	-.18+01	-.97+00	.10-01	-.21+01	-.77-01	-.11+00	-.32+01	-.11+01	.70+00
.15+03	.23+01	-.24+01	.27+00	-.17+01	-.11+01	-.14+01	-.18+01	-.19+01	-.15+01	-.13+01	-.15+01	-.51+00	-.96+00	-.23+01
.11+02	-.94+00	-.12+01	-.17+01	-.15+01	-.17+01	-.18+01	-.21+01	-.20+01	-.21+01	-.17+01	-.20+01	-.21+01	-.21+01	-.22+01

-.19+03	-.11+02	-.73+01	-.26+01	-.39+01	-.38+01	-.35+01	-.18+01	-.32+01	-.34+01	-.13+01	-.14+01	-.47+00	-.51+00	.64+01
.10+03	-.36+01	-.28+01	-.63+01	-.53+01	-.60+01	-.59+01	-.52+01	-.55+01	-.46+01	-.42+01	-.46+01	-.49+01	-.46+01	-.10+02
.21+02	-.57-01	-.20+01	-.18+01	-.19+01	-.20+01	-.20+01	-.18+01	-.20+01	-.18+01	-.18+01	-.21+01	-.20+01	-.20+01	-.14+01

-.63+02	-.10+03	-.50+02	-.18+02	-.12+02	-.85+01	-.55+01	-.62+01	-.44+01	-.25+01	-.38+01	-.49+01	-.66+00	-.10+01	.82+01
.35+02	-.29+02	-.43+02	-.23+02	-.19+02	-.13+02	-.11+02	-.10+02	-.83+01	-.81+01	-.81+01	-.77+01	-.62+01	-.85+01	-.97+01
.61+01	.15+02	.34+01	-.10+01	-.18+01	-.16+01	-.17+01	-.17+01	-.16+01	-.16+01	-.18+01	-.20+01	-.15+01	-.20+01	.13+00

-.22+02	-.34+02	-.43+02	-.33+02	-.21+02	-.17+02	-.11+02	-.10+02	-.60+01	-.61+01	-.41+01	-.20+01	-.51+01	.11+01	.94+01
.18+02	.85+01	-.16+02	-.33+02	-.29+02	-.23+02	-.18+02	-.16+02	-.13+02	-.11+02	-.10+02	-.99+01	-.99+01	-.11+02	-.60+01
.19+01	.58+01	.70+01	.37+01	.10+01	-.18+00	-.90+00	-.11+01	-.14+01	-.13+01	-.14+01	-.14+01	-.18+01	-.10+01	.11+01

.24+02	-.25+01	-.21+02	-.22+02	-.22+02	-.16+02	-.12+02	-.65+01	-.11+02	-.76+01	-.46+01	-.47+01	-.72+00	.19+02	
-.22+02	-.27+01	-.22+01	-.12+02	-.22+02	-.24+02	-.18+02	-.17+02	-.18+02	-.14+02	-.14+02	-.14+02	-.11+02	-.83+01	
-.10+01	.11+01	.41+01	.38+01	.29+01	.12+01	.20+00	-.29+00	-.70+00	-.95+00	-.10+01	-.12+01	-.83+00	.26+01	

.19+02	.21+02	.12+02	-.96+01	-.12+02	-.13+02	-.13+02	-.76+01	-.11+02	-.78+01	-.37+01	.40+01	.23+02	.24+01	
-.18+02	-.15+02	-.76+01	-.59+01	-.11+02	-.16+02	-.18+02	-.14+02	-.20+02	-.16+02	-.14+02	-.80+01	-.23+01	-.59+00	
-.75+00	-.21+01	-.14+01	.19+01	.22+01	.18+01	.11+01	.34+00	-.27-01	-.42+00	-.56+00	.23+00	.34+01	.39+00	

.15+02	.15+02	.18+02	.15+02	.13+02	.33+01	.11+01	-.22+01	.47+01	.76+01	.15+02	.25+02	.32+01	
-.14+02	-.12+02	-.92+01	-.39+01	-.62+00	-.43+01	-.56+01	-.88+01	-.51+01	-.18+01	.22+01	.61+01	.38+00	
-.49+00	-.13+01	-.22+01	-.20+01	-.16+01	-.77-01	.16+00	.32+00	.68-01	.25+00	.12+01	.34+01	.51+00	

.11+02	.15+02	.12+02	.15+02	.14+02	.14+02	.15+02	.15+02	.17+02	.23+02	.16+02	.15+01	
-.11+02	-.12+02	-.77+01	-.58+01	-.37+00	.16+01	.48+01	.48+01	.75+01	.11+02	.62+01	.56+00	
-.30+00	-.12+01	-.13+01	-.17+01	-.17+01	-.14+01	-.11+01	-.43+00	.27+00	.20+01	.21+01	.23+00	

.12+02	.11+02	.13+02	.11+02	.13+02	.13+02	.16+02	.21+02	.16+02	.33+01	
-.12+02	-.98+01	-.81+01	-.35+01	-.96+00	.18+01	.57+01	.11+02	.10+02	.14+01	
-.29+00	-.66+00	-.13+01	-.12+01	-.12+01	-.88+00	-.29+00	.10+01	.14+01	.44+00	

.97+01	.11+02	.11+02	.13+02	.12+02	.11+02	.78+01	.54+00
-.93+01	-.86+01	-.40+01	.98+00	.42+01	.58+01	.56+01	.50+00
-.23+00	-.69+00	-.57+00	-.51+00	-.26-01	.33+00	.46+00	.36-01

.14+01	.19+01	.10+01
-.10+01	.26+00	.79+00
-.17-01	.10-01	.24-01

TABLE 17

DISTRIBUTION FOR $\gamma = 0$, $\theta_0 = 60°$, AND $\sin \alpha_L = 0.2$

.71+01	.93+01	.86+01	.11+02	.12+02	.11+02	.78+01	.54+00	
-.69+01	-.75+01	-.44+01	-.38+00	.42+01	.58+01	.56+01	.50+00	
.96-01	.49+00	.36+00	.51+00	.26-01	-.33+00	-.46+00	-.36-01	

.12+02	.11+02	.13+02	.11+02	.13+02	.13+02	.16+02	.21+02	.16+02	.33+01
-.12+02	-.98+01	-.81+01	-.35+01	-.96+00	.18+01	.57+01	.11+02	.10+02	.14+01
.29+00	.66+00	.13+01	.12+01	.12+01	.88+00	.29+00	-.10+01	-.14+01	-.44+00

.11+02	.15+02	.12+02	.15+02	.14+02	.14+02	.15+02	.15+02	.17+02	.23+02	.16+02	.15+01
-.11+02	-.12+02	-.77+01	-.58+01	-.37+00	.16+01	.48+01	.48+01	.75+01	.11+02	.62+01	.56+00
.30+00	.12+01	.13+01	.17+01	.17+01	.14+01	.11+01	.43+00	-.27+00	-.20+01	-.21+01	-.23+00

.38+01	.92+01	.16+02	.15+02	.18+02	.18+02	.16+02	.17+02	.15+02	.16+02	.19+02	.25+02	.32+01	
-.38+01	-.78+01	-.83+01	-.39+01	.91+00	.52+01	.66+01	.95+01	.56+01	.67+01	.62+01	.61+01	.38+00	
.96-02	.61+00	.19+01	.20+01	.23+01	.20+01	.13+01	.93+00	.21+00	-.34+00	-.14+01	-.34+01	-.51+00	

.40+01	.40+01	.90+01	.19+02	.20+02	.21+02	.20+02	.15+02	.20+02	.16+02	.15+02	.18+02	.25+02	.24+01
-.40+01	-.40+01	-.49+01	.32+00	.41+01	.91+01	.11+02	.73+01	.11+02	.76+01	.30+01	.33+01	-.97+00	-.59+00
.19-01	.56-01	.91+00	.28+01	.24+01	.21+01	.14+01	.61+00	.26+00	-.31+00	-.73+00	-.17+01	-.36+01	-.39+00

.41+01	.41+01	.41+01	.41+01	.30+02	.27+02	.18+02	.18+02	.18+02	.15+02	.15+02	.17+02	.19+02	.23+02
-.41+01	-.41+01	-.40+01	-.25+01	.14+02	.14+02	.12+02	.69+01	.10+02	.69+01	.34+01	.20+01	-.11+01	-.70+01
.27-01	.82-01	.13+00	.28+00	.33+01	.19+01	.57+00	.52+00	-.22+00	-.52+00	-.58+00	-.11+01	-.20+01	-.32+01

.40+01	.40+01	.30+01	.51+01	.28+02	.24+02	.19+02	.18+02	.16+02	.14+02	.13+02	.13+02	.16+02	.24+02	.11+02	
-.40+01	-.40+01	-.29+01	-.48+01	.16+02	.17+02	.11+02	.98+01	.53+01	.48+01	.20+01	-.24+00	-.51+00	-.83+01	-.65+01	
.35-01	.10+00	.12+00	.28+00	.14+01	.33+00	-.18+00	-.39+00	-.45+00	-.63+00	-.66+00	-.58+00	-.13+01	-.28+01	-.14+01	

.39+01	.39+01	.40+01	.68+01	.22+02	.16+02	.16+02	.14+02	.12+02	.12+02	.14+02	.14+02	.12+02	.18+02	.21+02	
-.39+01	-.38+01	-.38+01	-.15+01	.10+02	.67+01	.39+01	.43+01	.26+01	.10+01	.75+00	.43+00	-.33+01	-.85+01	-.14+02	
.42-01	.13+00	.22+00	-.81-03	-.87+00	-.80+00	-.62+00	-.73+00	-.59+00	-.41+00	-.69+00	-.93+00	-.53+00	-.12+01	-.21+01	

.45+01	.82+01	.10+02	.14+02	.13+02	.13+02	.13+02	.12+02	.13+02	.11+02	.11+02	.13+02	.14+02	.15+02	.28+02	
-.44+01	-.72+01	-.48+01	-.33+01	-.15+01	-.83+00	-.98-01	-.38+00	.20+00	.33+00	-.12+01	-.21+01	-.42+01	-.73+01	-.22+02	
.77-01	.72-01	-.74+00	-.81+00	-.83+00	-.77+00	-.65+00	-.37+00	-.56+00	-.59+00	-.32+00	-.39+00	-.36+00	-.57+00	-.15+01	

.10+02	.11+02	.12+02	.12+02	.10+02	.11+02	.11+02	.13+02	.12+02	.12+02	.10+02	.12+02	.13+02	.14+02	.29+02	
-.10+02	-.97+01	-.90+01	-.66+01	-.43+01	-.36+01	-.23+01	-.23+01	-.16+01	-.15+01	-.16+01	-.25+01	-.39+01	-.70+01	-.24+02	
-.64-01	-.40+00	-.37-01	-.66+00	-.71-01	-.34+00	-.36+00	-.21+00	-.37-01	-.40+00	-.49-01	-.73-01	-.59+00	-.30+00	-.56+00	

-.10+01	-.32+01	.16+01	-.35+01	.34+00	-.15+01	-.18+01	-.97+00	.10-01	-.21+01	-.77-01	-.11+00	-.32+01	-.11+01	.70+00	
.98+00	.23+01	-.24+01	.27+00	-.17+01	-.11+01	-.14+01	-.18+01	-.19+01	-.15+01	-.13+01	-.15+01	-.51+00	-.96+00	-.23+01	
-.29+00	-.94+00	-.12+01	-.17+01	-.15+01	-.17+01	-.18+01	-.21+01	-.20+01	-.21+01	-.17+01	-.20+01	-.21+01	-.21+01	-.22+01	

.45+01	.48+01	-.25+01	-.26+01	-.39+01	-.38+01	-.35+01	-.18+01	-.32+01	-.34+01	-.13+01	-.14+01	-.47+00	-.51+00	.64+01	
-.44+01	-.52+01	-.20+01	-.63+01	-.53+01	-.60+01	-.59+01	-.52+01	-.55+01	-.46+01	-.42+01	-.46+01	-.49+01	-.46+01	-.10+02	
-.77-01	-.53+00	-.12+01	-.18+01	-.19+01	-.20+01	-.20+01	-.18+01	-.20+01	-.18+01	-.18+01	-.21+01	-.20+01	-.20+01	-.14+01	

.39+01	.39+01	.40+01	.12+01	-.12+02	-.85+01	-.55+01	-.62+01	-.44+01	-.25+01	-.38+01	-.49+01	-.66+00	-.10+01	.82+01	
-.39+01	-.38+01	-.38+01	-.57+01	-.19+02	-.13+02	-.11+02	-.10+02	-.83+01	-.81+01	-.81+01	-.77+01	-.62+01	-.85+01	-.97+01	
-.42-01	-.13+00	-.22+00	-.60+00	-.18+01	-.16+01	-.17+01	-.17+01	-.16+01	-.16+01	-.18+01	-.20+01	-.15+01	-.20+01	.13+00	

.40+01	.40+01	.30+01	.51+01	-.17+02	-.17+02	-.11+02	-.10+02	-.60+01	-.61+01	-.41+01	-.20+01	-.51+01	.11+01	.94+01	
-.40+01	-.40+01	-.29+01	-.48+01	-.26+02	-.23+02	-.18+02	-.16+02	-.13+02	-.11+02	-.10+02	-.99+01	-.99+01	-.11+02	-.60+01	
-.35-01	-.10+00	-.12+00	-.28+00	.75+00	-.18+00	-.90+00	-.11+01	-.14+01	-.13+01	-.14+01	-.14+01	-.18+01	-.10+01	.11+01	

.41+01	.41+01	.41+01	.20+01	-.22+02	-.16+02	-.12+02	-.65+01	-.11+02	-.76+01	-.46+01	-.47+01	-.72+00	.19+02		
-.41+01	-.41+01	-.40+01	-.34+01	-.22+02	-.24+02	-.18+02	-.17+02	-.18+02	-.14+02	-.14+02	-.14+02	-.11+02	-.83+01		
-.27-01	-.82-01	-.13+00	.35-01	.29+01	.12+01	.20+00	-.29+00	-.70+00	-.95+00	-.10+01	-.12+01	-.83+00	.26+01		

.40+01	.40+01	.90+01	-.76+01	-.12+02	-.13+02	-.13+02	-.76+01	-.11+02	-.78+01	-.37+01	.40+01	.23+02	.24+01		
-.40+01	-.40+01	-.49+01	-.57+01	-.11+02	-.16+02	-.18+02	-.14+02	-.20+02	-.16+02	-.14+02	-.80+01	-.23+01	-.59+00		
-.19-01	-.56-01	-.91+00	.16+01	.22+01	.18+01	.11+01	.34+00	-.27-01	-.42+00	-.56+00	.23+00	.34+01	.39+00		

.38+01	.92+01	.16+02	.15+02	.13+02	.33+01	.11+01	-.22+01	.47+01	.76+01	.15+02	.25+02	.32+01			
-.38+01	-.78+01	-.83+01	-.39+01	-.62+00	-.43+01	-.56+01	-.88+01	-.51+01	-.18+01	.22+01	.61+01	.38+00			
-.96-02	-.61+00	-.19+01	-.20+01	-.16+01	-.77-01	.16+00	.32+00	.68-01	.25+00	.12+01	.34+01	.51+00			

.11+02	.15+02	.12+02	.15+02	.14+02	.14+02	.15+02	.15+02	.17+02	.23+02	.16+02	.15+01				
-.11+02	-.12+02	-.77+01	-.58+01	-.37+00	.16+01	.48+01	.48+01	.75+01	.11+02	.62+01	.56+00				
-.30+00	-.12+01	-.13+01	-.17+01	-.17+01	-.14+01	-.11+01	-.43+00	.27+00	.20+01	.21+01	.23+00				

.12+02	.11+02	.13+02	.11+02	.13+02	.13+02	.16+02	.21+02	.16+02	.33+01						
-.12+02	-.98+01	-.81+01	-.35+01	-.96+00	.18+01	.57+01	.11+02	.10+02	.14+01						
-.29+00	-.66+00	-.13+01	-.12+01	-.12+01	-.88+00	-.29+00	.10+01	.14+01	.44+00						

.71+01	.93+01	.86+01	.11+02	.12+02	.11+02	.78+01	.54+00								
-.69+01	-.75+01	-.44+01	-.38+00	.42+01	.58+01	.56+01	.50+00								
-.96-01	-.49+00	-.36+00	-.51+00	-.26-01	.33+00	.46+00	.36-01								

TABLE 18

DISTRIBUTION FOR $\gamma = 0$, $\theta_0 = 60°$, AND $\sin \alpha_L = 0.4$

.20+01	.21+01	.21+01	.15+01	.15+01	.15+01	.49+00
-.20+01	-.20+01	-.20+01	-.14+01	-.10+01	-.83+00	-.54-03
-.23-01	-.69-01	-.11+00	-.11+00	-.17+00	-.20+00	-.82-01

.42+01	.42+01	.42+01	.33+01	.39+01	.43+01	.69+01	.18+02	.16+02	.33+01
-.42+01	-.42+01	-.42+01	-.32+01	-.37+01	-.36+01	-.17+01	.89+01	.10+02	.14+01
-.18-01	-.54-01	-.89-01	-.13+00	-.21+00	-.36+00	-.49+00	-.91+00	-.14+01	-.44+00

.34+01	.34+01	.34+01	.50+01	.32+01	.39+01	.77+01	.15+02	.17+02	.23+02	.16+02	.15+01
-.34+01	-.34+01	-.34+01	-.50+01	-.32+01	-.39+01	-.68+00	.48+01	.75+01	.11+02	.62+01	.56+00
.46-09	.91-08	.40-07	-.25-01	-.42-01	-.85-01	.25+00	.43+00	-.27+00	-.20+01	-.21+01	-.23+00

.38+01	.38+01	.38+01	.38+01	.36+01	.36+01	.35+01	.17+02	.15+02	.16+02	.19+02	.25+02	.32+01
-.38+01	-.38+01	-.38+01	-.37+01	-.36+01	-.36+01	-.35+01	.95+01	.56+01	.67+01	.62+01	.61+01	.38+00
.96-02	.28-01	.48-01	.66-01	.42-01	.52-01	.29-01	.93+00	.21+00	-.34+00	-.14+01	-.34+01	-.51+00

.40+01	.40+01	.40+01	.30+01	.39+01	.39+01	.38+01	.12+02	.20+02	.16+02	.15+02	.18+02	.25+02	.24+01
-.40+01	-.40+01	-.40+01	-.29+01	-.38+01	-.38+01	-.37+01	.51+01	.11+02	.76+01	.30+01	.33+01	-.97+00	-.59+00
.19-01	.56-01	.94-01	.92-01	.12+00	.15+00	.14+00	.50+00	.26+00	-.31+00	-.73+00	-.17+01	-.36+01	-.39+00

.41+01	.41+01	.41+01	.30+01	.41+01	.51+01	.30+01	.14+02	.18+02	.15+02	.15+02	.17+02	.19+02	.23+02
-.41+01	-.41+01	-.40+01	-.29+01	-.39+01	-.47+01	-.28+01	.24+01	.10+02	.69+01	.34+01	.20+01	-.11+01	-.70+01
.27-01	.82-01	.13+00	.12+00	.20+00	.32+00	.18+00	.38+00	-.22+00	-.52+00	-.58+00	-.11+01	-.20+01	-.32+01

.40+01	.40+01	.30+01	.51+01	.51+01	.31+01	.41+01	.16+02	.16+02	.14+02	.13+02	.13+02	.16+02	.24+02	.11+02
-.40+01	-.40+01	-.29+01	-.48+01	-.47+01	-.27+01	-.35+01	.78+01	.53+01	.48+01	.20+01	-.24+00	-.51+00	-.83+01	-.65+01
.35-01	.10+00	.12+00	.28+00	.36+00	.25+00	.35+00	-.29+00	-.45+00	-.63+00	-.66+00	-.58+00	-.13+01	-.28+01	-.14+01

.39+01	.39+01	.40+01	.40+01	.51+01	.41+01	.92+01	.14+02	.12+02	.12+02	.14+02	.14+02	.12+02	.18+02	.21+02
-.39+01	-.38+01	-.38+01	-.36+01	-.42+01	-.33+01	-.11+01	.43+01	.26+01	.10+01	.75+00	.43+00	-.33+01	-.85+01	-.14+02
.42-01	.13+00	.22+00	.29+00	.47+00	.41+00	.10+00	-.73+00	-.59+00	-.41+00	-.69+00	-.93+00	-.53+00	-.12+01	-.21+01

.45+01	.65+01	.38+01	.58+01	.49+01	.78+01	.13+02	.12+02	.13+02	.11+02	.11+02	.13+02	.14+02	.15+02	.28+02
-.44+01	-.62+01	-.34+01	-.48+01	-.34+01	-.30+01	-.98-01	-.38+00	.20+00	.33+00	-.12+01	-.21+01	-.42+01	-.73+01	-.22+02
.77-01	.30+00	.26+00	.53+00	.57+00	.16+00	-.65+00	-.37+00	-.56+00	-.59+00	-.32+00	-.39+00	-.36+00	-.57+00	-.15+01

.40+01	.40+01	.77+01	.70+01	.98+01	.11+02	.11+02	.13+02	.12+02	.12+02	.10+02	.12+02	.13+02	.14+02	.29+02
-.40+01	-.37+01	-.63+01	-.45+01	-.41+01	-.36+01	-.23+01	-.23+01	-.16+01	-.15+01	-.16+01	-.25+01	-.39+01	-.70+01	-.24+02
.10+00	.26+00	.52+00	.17+00	.63-01	-.34+00	-.36+00	-.21+00	-.37-01	-.40+00	-.49-01	.73-01	-.59+00	-.30+00	-.56+00

.40+01	.40+01	.60+01	.19+01	.11+01	-.15+01	-.18+01	-.97+00	.10-01	-.21+01	-.77-01	-.11+00	-.32+01	-.11+01	.70+00
-.40+01	-.37+01	-.51+01	-.17+01	-.19+01	-.11+01	-.14+01	-.18+01	-.19+01	-.15+01	-.13+01	-.15+01	-.51+00	-.96+00	-.23+01
-.10+00	-.26+00	-.72+00	-.87+00	-.14+01	-.17+01	-.18+01	-.21+01	-.20+01	-.21+01	-.17+01	-.20+01	-.21+01	-.21+01	-.22+01

.45+01	.65+01	.38+01	.58+01	.49+01	.23+01	-.35+01	-.18+01	-.32+01	-.34+01	-.13+01	-.14+01	-.47+00	-.51+00	.64+01
-.44+01	-.62+01	-.34+01	-.48+01	-.34+01	-.37+01	-.59+01	-.52+01	-.55+01	-.46+01	-.42+01	-.46+01	-.49+01	-.46+01	-.10+02
-.77-01	-.30+00	-.26+00	-.53+00	-.57+00	-.10+01	-.20+01	-.18+01	-.20+01	-.18+01	-.18+01	-.21+01	-.20+01	-.20+01	-.14+01

.39+01	.39+01	.40+01	.40+01	.51+01	.41+01	.12+01	-.62+01	-.44+01	-.25+01	-.38+01	-.49+01	-.66+00	-.10+01	.82+01
-.39+01	-.38+01	-.38+01	-.36+01	-.42+01	-.33+01	-.66+01	-.10+02	-.83+01	-.81+01	-.81+01	-.77+01	-.62+01	-.85+01	-.97+01
-.42-01	-.13+00	-.22+00	-.29+00	-.47+00	-.41+00	-.10+01	-.17+01	-.16+01	-.16+01	-.18+01	-.20+01	-.15+01	-.20+01	.13+00

.40+01	.40+01	.30+01	.51+01	.51+01	.31+01	.41+01	-.78+01	-.60+01	-.61+01	-.41+01	-.20+01	-.51+01	.11+01	.94+01
-.40+01	-.40+01	-.29+01	-.48+01	-.47+01	-.27+01	-.35+01	-.14+02	-.13+02	-.11+02	-.10+02	-.99+01	-.99+01	-.11+02	-.60+01
-.35-01	-.10+00	-.12+00	-.28+00	-.36+00	-.25+00	-.35+00	-.10+01	-.14+01	-.13+01	-.14+01	-.14+01	-.18+01	-.10+01	.11+01

.41+01	.41+01	.41+01	.30+01	.41+01	.51+01	.30+01	-.19+01	-.11+02	-.76+01	-.46+01	-.47+01	-.72+00	.19+02
-.41+01	-.41+01	-.40+01	-.29+01	-.39+01	-.47+01	-.28+01	-.13+02	-.18+02	-.14+02	-.14+02	-.14+02	-.11+02	-.83+01
-.27-01	-.82-01	-.13+00	-.12+00	-.20+00	-.32+00	-.18+00	-.43+00	-.70+00	-.95+00	-.10+01	-.12+01	-.83+00	.26+01

.40+01	.40+01	.40+01	.30+01	.39+01	.39+01	.38+01	-.53+01	-.11+02	-.78+01	-.37+01	.40+01	.23+02	.24+01
-.40+01	-.40+01	-.40+01	-.29+01	-.38+01	-.38+01	-.37+01	-.12+02	-.20+02	-.16+02	-.14+02	-.80+01	-.23+01	-.59+00
-.19-01	-.56-01	-.94-01	-.92-01	-.12+00	-.15+00	-.14+00	.23+00	-.27-01	-.42+00	-.56+00	.23+00	.34+01	.39+00

.38+01	.38+01	.38+01	.38+01	.36+01	.36+01	.35+01	-.22+01	.47+01	.76+01	.15+02	.25+02	.32+01
-.38+01	-.38+01	-.38+01	-.37+01	-.36+01	-.36+01	-.35+01	-.88+01	-.51+01	-.18+01	.22+01	.61+01	.38+00
-.96-02	-.28-01	-.48-01	-.66-01	-.42-01	-.52-01	-.29-01	.32+00	.68-01	.25+00	.12+01	.34+01	.51+00

.34+01	.34+01	.34+01	.50+01	.32+01	.39+01	.77+01	.15+02	.17+02	.23+02	.16+02	.15+01
-.34+01	-.34+01	-.34+01	-.50+01	-.32+01	-.39+01	-.68+00	.48+01	.75+01	.11+02	.62+01	.56+00
-.46-09	-.91-08	-.40-07	.25-01	.42-01	.85-01	-.25+00	-.43+00	.27+00	.20+01	.21+01	.23+00

.42+01	.42+01	.42+01	.33+01	.39+01	.43+01	.69+01	.18+02	.16+02	.33+01
-.42+01	-.42+01	-.42+01	-.32+01	-.37+01	-.36+01	-.17+01	.89+01	.10+02	.14+01
.18-01	.54-01	.89-01	.13+00	.21+00	.36+00	.49+00	.91+00	.14+01	.44+00

.20+01	.21+01	.21+01	.15+01	.15+01	.15+01	.49+00
-.20+01	-.20+01	-.20+01	-.14+01	-.10+01	-.83+00	-.54-03
.23-01	.69-01	.11+00	.11+00	.17+00	.20+00	.82-01

TABLE 19

DISTRIBUTION FOR $\gamma = 0$, $\theta_0 = 60°$, AND $\sin \alpha_L = 0.6$

.42+01	.42+01	.42+01	.27+01	.27+01	.20+01	.13+01	.65+00							
-.42+01	-.42+01	-.42+01	-.26+01	-.26+01	-.19+01	-.12+01	-.45+00							
-.18-01	-.54-01	-.89-01	-.99-01	-.12+00	-.11+00	-.10+00	-.76-01							
.34+01	.34+01	.34+01	.50+01	.32+01	.39+01	.38+01	.51+01	.49+01	.35+01	.35+01	.15+01			
-.34+01	-.34+01	-.34+01	-.50+01	-.32+01	-.39+01	-.37+01	-.47+01	-.41+01	-.24+01	-.11+01	.56+00			
.46-09	.91-08	.40-07	-.25-01	-.42-01	-.85-01	-.12+00	-.31+00	-.44+00	-.41+00	-.54+00	-.23+00			
.38+01	.38+01	.38+01	.38+01	.36+01	.36+01	.35+01	.34+01	.49+01	.48+01	.10+02	.25+02	.32+01		
-.38+01	-.38+01	-.38+01	-.37+01	-.36+01	-.36+01	-.35+01	-.34+01	-.49+01	-.46+01	-.20+01	.61+01	.38+00		
.96-02	.28-01	.48-01	.66-01	.42-01	.52-01	.29-01	.83-06	-.99-01	-.19+00	-.96+00	-.34+01	-.51+00		
.40+01	.40+01	.40+01	.30+01	.39+01	.39+01	.38+01	.37+01	.46+01	.44+01	.12+02	.18+02	.25+02	.24+01	
-.40+01	-.40+01	-.40+01	-.29+01	-.38+01	-.38+01	-.37+01	-.36+01	-.45+01	-.43+01	-.71-01	.33+01	-.97+00	-.59+00	
.19-01	.56-01	.94-01	.92-01	.12+00	.15+00	.14+00	.13+00	.14+00	.54-01	-.55+00	-.17+01	-.36+01	-.39+00	
.41+01	.41+01	.41+01	.30+01	.41+01	.51+01	.30+01	.60+01	.39+01	.38+01	.12+02	.17+02	.19+02	.23+02	
-.41+01	-.41+01	-.40+01	-.29+01	-.39+01	-.47+01	-.28+01	-.54+01	-.36+01	-.35+01	.35+00	.20+01	-.11+01	-.70+01	
.27-01	.82-01	.13+00	.12+00	.20+00	.32+00	.18+00	.41+00	.23+00	.21+00	-.36+00	-.11+01	-.20+01	-.32+01	
.40+01	.40+01	.30+01	.51+01	.51+01	.31+01	.41+01	.41+01	.51+01	.40+01	.13+02	.13+02	.16+02	.24+02	.11+02
-.40+01	-.40+01	-.29+01	-.48+01	-.47+01	-.27+01	-.35+01	-.33+01	-.41+01	-.33+01	.20+01	-.24+00	-.51+00	-.83+01	-.65+01
.35-01	.10+00	.12+00	.28+00	.36+00	.25+00	.35+00	.40+00	.48+00	.36+00	-.66+00	-.58+00	-.13+01	-.28+01	-.14+01
.39+01	.39+01	.40+01	.40+01	.51+01	.41+01	.52+01	.41+01	.41+01	.10+02	.14+02	.14+02	.12+02	.18+02	.21+02
-.39+01	-.38+01	-.38+01	-.36+01	-.42+01	-.33+01	-.38+01	-.29+01	-.28+01	-.49+00	.75+00	.43+00	-.33+01	-.85+01	-.14+02
.42-01	.13+00	.22+00	.29+00	.47+00	.41+00	.57+00	.49+00	.51+00	-.14+00	-.69+00	-.93+00	-.53+00	-.12+01	-.21+01
	.19+01	.28+01	.58+01	.49+01	.50+01	.51+01	.52+01	.63+01	.11+02	.11+02	.13+02	.14+02	.15+02	.28+02
	-.18+01	-.26+01	-.48+01	-.34+01	-.34+01	-.30+01	-.28+01	-.24+01	.33+00	-.12+01	-.21+01	-.42+01	-.73+01	-.22+02
	.86-01	.19+00	.53+00	.57+00	.62+00	.69+00	.73+00	.58+00	-.59+00	-.32+00	-.39+00	-.36+00	-.57+00	-.15+01
					.19+01	.30+01	.11+02	.12+02	.12+02	.10+02	.12+02	.13+02	.14+02	.29+02
					-.10+01	-.12+01	-.25+01	-.16+01	-.15+01	-.16+01	-.25+01	-.39+01	-.70+01	-.24+02
					.27+00	.45+00	.11+00	-.37-01	-.40+00	-.49-01	-.73-01	-.59+00	-.30+00	-.56+00

					.19+01	.30+01	.10+01	.10-01	-.21+01	-.77-01	-.11+00	-.32+01	-.11+01	.70+00
					-.10+01	-.12+01	-.16+01	-.19+01	-.15+01	-.13+01	-.15+01	-.51+00	-.96+00	-.23+01
					-.27+00	-.45+00	-.18+01	-.20+01	-.21+01	-.17+01	-.20+01	-.21+01	-.21+01	-.22+01
	.19+01	.28+01	.58+01	.49+01	.50+01	.51+01	.52+01	.42+01	-.34+01	-.13+01	-.14+01	-.47+00	-.51+00	.64+01
	-.18+01	-.26+01	-.48+01	-.34+01	-.34+01	-.30+01	-.28+01	-.28+01	-.46+01	-.42+01	-.46+01	-.49+01	-.46+01	-.10+02
	-.86-01	-.19+00	-.53+00	-.57+00	-.62+00	-.69+00	-.73+00	-.93+00	-.18+01	-.18+01	-.21+01	-.20+01	-.20+01	-.14+01
.39+01	.39+01	.40+01	.40+01	.51+01	.41+01	.52+01	.41+01	.41+01	-.35+00	-.38+01	-.49+01	-.66+00	-.10+01	.82+01
-.39+01	-.38+01	-.38+01	-.36+01	-.42+01	-.33+01	-.38+01	-.29+01	-.28+01	-.66+01	-.81+01	-.77+01	-.62+01	-.85+01	-.97+01
-.42-01	-.13+00	-.22+00	-.29+00	-.47+00	-.41+00	-.57+00	-.49+00	-.51+00	-.13+01	-.18+01	-.20+01	-.15+01	-.20+01	.13+00
.40+01	.40+01	.30+01	.51+01	.51+01	.31+01	.41+01	.41+01	.51+01	.40+01	-.41+01	-.20+01	-.51+01	.11+01	.94+01
-.40+01	-.40+01	-.29+01	-.48+01	-.47+01	-.27+01	-.35+01	-.33+01	-.41+01	-.33+01	-.10+02	-.99+01	-.99+01	-.11+02	-.60+01
-.35-01	-.10+00	-.12+00	-.28+00	-.36+00	-.25+00	-.35+00	-.40+00	-.48+00	-.36+00	-.14+01	-.14+01	-.18+01	-.10+01	.11+01
.41+01	.41+01	.41+01	.30+01	.41+01	.51+01	.30+01	.60+01	.39+01	.38+01	-.11+01	-.47+01	-.72+00	.19+02	
-.41+01	-.41+01	-.40+01	-.29+01	-.39+01	-.47+01	-.28+01	-.54+01	-.36+01	-.35+01	-.11+02	-.14+02	-.11+02	-.83+01	
-.27-01	-.82-01	-.13+00	-.12+00	-.20+00	-.32+00	-.18+00	-.41+00	-.23+00	-.21+00	-.84+00	-.12+01	-.83+00	.26+01	
.40+01	.40+01	.40+01	.30+01	.39+01	.39+01	.38+01	.37+01	.46+01	.44+01	-.48+00	.40+01	.23+02	.24+01	
-.40+01	-.40+01	-.40+01	-.29+01	-.38+01	-.38+01	-.37+01	-.36+01	-.45+01	-.43+01	-.11+02	-.80+01	-.23+01	-.59+00	
-.19-01	-.56-01	-.94-01	-.92-01	-.12+00	-.15+00	-.14+00	-.13+00	-.14+00	-.54-01	-.37+00	.23+00	.34+01	.39+00	
.38+01	.38+01	.38+01	.38+01	.36+01	.36+01	.35+01	.34+01	.49+01	.48+01	.10+02	.25+02	.32+01		
-.38+01	-.38+01	-.38+01	-.37+01	-.36+01	-.36+01	-.35+01	-.34+01	-.49+01	-.46+01	-.20+01	.61+01	.38+00		
-.96-02	-.28-01	-.48-01	-.66-01	-.42-01	-.52-01	-.29-01	-.83-06	.99-01	.19+00	.96+00	.34+01	.51+00		
.34+01	.34+01	.34+01	.50+01	.32+01	.39+01	.38+01	.51+01	.49+01	.35+01	.35+01	.15+01			
-.34+01	-.34+01	-.34+01	-.50+01	-.32+01	-.39+01	-.37+01	-.47+01	-.41+01	-.24+01	-.11+01	.56+00			
-.46-09	-.91-08	-.40-07	.25-01	.42-01	.85-01	.12+00	.31+00	.44+00	.41+00	.54+00	.23+00			
.42+01	.42+01	.42+01	.27+01	.27+01	.20+01	.13+01	.65+00							
-.42+01	-.42+01	-.42+01	-.26+01	-.26+01	-.19+01	-.12+01	-.45+00							
.18-01	.54-01	.89-01	.99-01	.12+00	.11+00	.10+00	.76-01							

TABLE 20

DISTRIBUTION FOR $\gamma = 0$, $\theta_0 = 60°$, AND $\sin \alpha_L = 0.8$

.34+01	.34+01	.34+01	.34+01	.16+01	.16+01	.84+00
-.34+01	-.34+01	-.34+01	-.34+01	-.16+01	-.16+01	-.84+00
.46-09	.91-08	.40-07	.10-06	-.10-01	-.13-01	-.73-02

.38+01	.38+01	.38+01	.38+01	.36+01	.36+01	.35+01	.34+01	.25+01	.16+01	.80+00
-.38+01	-.38+01	-.38+01	-.37+01	-.36+01	-.36+01	-.35+01	-.34+01	-.25+01	-.16+01	-.77+00
.96-02	.28-01	.48-01	.66-01	.42-01	.52-01	.29-01	.83-06	-.10-01	-.21-01	-.35-01

.40+01	.40+01	.40+01	.30+01	.39+01	.39+01	.38+01	.37+01	.46+01	.44+01	.59+01	.33+01	
-.40+01	-.40+01	-.40+01	-.29+01	-.38+01	-.38+01	-.37+01	-.36+01	-.45+01	-.43+01	-.58+01	-.31+01	
.19-01	.56-01	.94-01	.92-01	.12+00	.15+00	.14+00	.13+00	.14+00	.54-01	.87-01	-.13+00	

.41+01	.41+01	.41+01	.30+01	.41+01	.51+01	.30+01	.60+01	.39+01	.38+01	.55+01	.61+01	.59+01	.33+01
-.41+01	-.41+01	-.40+01	-.29+01	-.39+01	-.47+01	-.28+01	-.54+01	-.36+01	-.35+01	-.53+01	-.60+01	-.57+01	-.29+01
.27-01	.82-01	.13+00	.12+00	.20+00	.32+00	.18+00	.41+00	.23+00	.21+00	.24+00	.92-01	-.15+00	-.24+00

.20+01	.20+01	.20+01	.51+01	.51+01	.31+01	.41+01	.41+01	.51+01	.40+01	.49+01	.57+01	.54+01	.11+02	.11+02
-.20+01	-.20+01	-.20+01	-.48+01	-.47+01	-.27+01	-.35+01	-.33+01	-.41+01	-.33+01	-.42+01	-.51+01	-.52+01	-.82+01	-.65+01
.16-01	.50-01	.83-01	.28+00	.36+00	.25+00	.35+00	.40+00	.48+00	.36+00	.41+00	.41+00	.22+00	-.72+00	-.14+01

.20+01	.52+01	.41+01	.41+01	.51+01	.50+01	.49+01	.78+01	.16+02	.21+02
-.16+01	-.38+01	-.29+01	-.28+01	-.35+01	-.35+01	-.36+01	-.45+01	-.89+01	-.14+02
.21+00	.57+00	.49+00	.51+00	.62+00	.58+00	.56+00	.15+00	-.94+00	-.21+01

.21+01	.42+01	.42+01	.52+01	.61+01	.13+02	.15+02	.28+02
-.12+01	-.22+01	-.21+01	-.27+01	-.34+01	-.44+01	-.73+01	-.22+02
.28+00	.59+00	.60+00	.74+00	.85+00	-.19+00	-.57+00	-.15+01

.21+01	.42+01	.10+02	.13+02	.14+02	.29+02
-.79+00	-.13+01	-.26+01	-.39+01	-.70+01	-.24+02
.32+00	.67+00	.28+00	-.59+00	-.30+00	-.56+00

.21+01	.42+01	.20+01	-.32+01	-.11+01	.70+00
-.79+00	-.13+01	-.15+01	-.51+00	-.96+00	-.23+01
-.32+00	-.67+00	-.17+01	-.21+01	-.21+01	-.22+01

.21+01	.42+01	.42+01	.52+01	.61+01	.60+00	-.51+00	.64+01
-.12+01	-.22+01	-.21+01	-.27+01	-.34+01	-.46+01	-.46+01	-.10+02
-.28+00	-.59+00	-.60+00	-.74+00	-.85+00	-.19+01	-.20+01	-.14+01

.20+01	.52+01	.41+01	.41+01	.51+01	.50+01	.49+01	.36+01	.10+01	.82+01
-.16+01	-.38+01	-.29+01	-.28+01	-.35+01	-.36+01	-.36+01	-.51+01	-.81+01	-.97+01
-.21+00	-.57+00	-.49+00	-.51+00	-.62+00	-.58+00	-.56+00	-.84+00	-.16+01	.13+00

.20+01	.20+01	.20+01	.51+01	.51+01	.31+01	.41+01	.41+01	.51+01	.40+01	.49+01	.57+01	.54+01	.56+01	.94+01
-.20+01	-.20+01	-.20+01	-.48+01	-.47+01	-.27+01	-.35+01	-.33+01	-.41+01	-.33+01	-.42+01	-.51+01	-.52+01	-.81+01	-.60+01
-.16-01	-.50-01	-.83-01	-.28+00	-.36+00	-.25+00	-.35+00	-.40+00	-.48+00	-.36+00	-.41+00	-.41+00	-.22+00	-.22+00	.11+01

.41+01	.41+01	.41+01	.30+01	.41+01	.51+01	.30+01	.60+01	.39+01	.38+01	.55+01	.61+01	.59+01	.33+01
-.41+01	-.41+01	-.40+01	-.29+01	-.39+01	-.47+01	-.28+01	-.54+01	-.36+01	-.35+01	-.53+01	-.60+01	-.57+01	-.29+01
-.27-01	-.82-01	-.13+00	-.12+00	-.20+00	-.32+00	-.18+00	-.41+00	-.23+00	-.21+00	-.24+00	-.92-01	.15+00	.24+00

.40+01	.40+01	.40+01	.30+01	.39+01	.39+01	.38+01	.37+01	.46+01	.44+01	.59+01	.33+01	
-.40+01	-.40+01	-.40+01	-.29+01	-.38+01	-.38+01	-.37+01	-.36+01	-.45+01	-.43+01	-.58+01	-.31+01	
-.19-01	-.56-01	-.94-01	-.92-01	-.12+00	-.15+00	-.14+00	-.13+00	-.14+00	-.54-01	.87-01	.13+00	

.38+01	.38+01	.38+01	.38+01	.36+01	.36+01	.35+01	.34+01	.25+01	.16+01	.80+00	
-.38+01	-.38+01	-.38+01	-.37+01	-.36+01	-.36+01	-.35+01	-.34+01	-.25+01	-.16+01	-.77+00	
-.96-02	-.28-01	-.48-01	-.66-01	-.42-01	-.52-01	-.29-01	-.83-06	.10-01	.21-01	.35-01	

.34+01	.34+01	.34+01	.34+01	.16+01	.16+01	.84+00
-.34+01	-.34+01	-.34+01	-.34+01	-.16+01	-.16+01	-.84+00
-.46-09	-.91-08	-.40-07	-.10-06	.10-01	.13-01	.73-02

TABLE 21

DISTRIBUTION FOR $\gamma = 0$, $\theta_0 = 80°$, AND $\sin \alpha_L = 0$

.26+01	.40+01	.62+01	.59+01	.41+01										
-.14+01	.12+01	.38+01	.52+01	.38+01										
-.15-01	.11+00	-.33-01	-.57-01	-.74-01										
.12+02	.16+02	.18+02	.20+02	.30+02	.37+02	.40+02	.41+02	.35+02	.19+02					
-.11+02	-.91+01	-.17+01	.55+01	.16+02	.26+02	.33+02	.34+02	.30+02	.16+02					
.51+00	.13+01	.18+01	.12+01	.12+01	.51+00	-.29+00	-.14+01	-.21+01	-.16+01					
.17+02	.19+02	.24+02	.31+02	.38+02	.57+02	.52+02	.42+02	.47+02	.44+02	.47+02	.33+02			
-.16+02	-.11+02	-.47+01	.51+01	.16+02	.41+02	.42+02	.32+02	.35+02	.32+02	.30+02	.17+02			
.75+00	.19+01	.31+01	.36+01	.38+01	.43+01	.15+01	-.48+00	-.23+01	-.38+01	-.53+01	-.41+01			
.34+02	.42+02	.77+02	.10+03	.74+02	.39+02	.26+02	.23+02	.20+02	.23+02	.24+02	.32+02	.43+02	.62+01	
-.27+02	-.92+01	.22+02	.54+02	.52+02	.25+02	.13+02	.10+02	.86+01	.70+01	.77+01	.77+01	.89+01	.35+00	
.18+01	.42+01	.88+01	.10+02	.44+01	-.76+00	-.19+01	-.21+01	-.22+01	-.28+01	-.32+01	-.47+01	-.68+01	-.10+01	
.59+02	.10+03	.82+02	.87+01	.11+02	.10+02	.12+02	.12+02	.15+02	.11+02	.17+02	.17+02	.25+02	.40+02	.20+01
-.49+02	-.19+02	.97+01	-.52+01	-.49+01	-.40+01	-.29+01	-.30+01	-.24+01	-.17+01	-.40+01	-.16+01	-.56+01	-.83+01	-.83+00
.41+01	.15+02	.11+02	-.83+00	-.13+01	-.12+01	-.14+01	-.15+01	-.19+01	-.15+01	-.23+01	-.24+01	-.37+01	-.65+01	-.30+00
.13+03	.84+02	.84+01	.10+02	.87+01	.81+01	.83+01	.12+02	.10+02	.10+02	.12+02	.15+02	.17+02	.24+02	.26+02
-.90+02	-.24+02	-.78+01	-.84+01	-.71+01	-.61+01	-.58+01	-.68+01	-.65+01	-.59+01	-.75+01	-.70+01	-.89+01	-.13+02	-.15+02
.13+02	.11+02	-.37+00	-.67+00	-.60+00	-.59+00	-.65+00	-.13+01	-.10+01	-.10+01	-.12+01	-.18+01	-.20+01	-.31+01	-.34+01
.18+03	.74+01	.83+01	.84+01	.75+01	.85+01	.10+02	.78+01	.98+01	.10+02	.10+02	.14+02	.13+02	.20+02	.35+02
-.13+03	-.74+01	-.81+01	-.79+01	-.69+01	-.73+01	-.86+01	-.63+01	-.79+01	-.81+01	-.79+01	-.10+02	-.99+01	-.15+02	-.29+02
.17+02	-.76-01	-.18+00	-.25+00	-.24+00	-.39+00	-.63+00	-.50+00	-.53+00	-.73+00	-.56+00	-.11+01	-.97+00	-.16+01	-.31+01
.19+03	.77+01	.77+01	.77+01	.87+01	.88+01	.59+01	.99+01	.10+02	.90+01	.11+02	.12+02	.13+02	.18+02	.37+02
-.17+03	-.76+01	-.76+01	-.75+01	-.82+01	-.80+01	-.54+01	-.90+01	-.87+01	-.76+01	-.95+01	-.10+02	-.11+02	-.16+02	-.36+02
.10+02	-.25-01	-.42-01	-.58-01	-.14+00	-.17+00	-.67-01	-.14+00	-.20+00	-.25+00	-.14+00	-.29+00	-.35+00	-.53+00	-.12+01
-.14+03	.18+00	.17+00	.16+00	-.78+00	-.84+00	.42-01	.60-01	.45-01	-.99+00	.98+00	-.67-01	-.10+01	-.15+00	-.15+00
.12+03	-.20+00	-.23+00	-.26+00	.52+00	.46+00	-.22+00	-.46+00	-.68+00	.23+00	-.16+01	-.85+00	.42-01	-.11+01	-.91+00
.89+01	-.13+00	-.23+00	-.32+00	-.48+00	-.57+00	-.40+00	-.68+00	-.77+00	-.77+00	-.90+00	-.92+00	-.10+01	-.11+01	-.95+00
-.63+02	.54+00	-.32+00	-.41+00	.43+00	-.51+00	-.25+01	-.18+01	.10+00	-.98+00	-.79-01	-.22+01	-.12+01	-.47+00	.15+01
.32+02	-.60+00	.97-01	-.25-01	-.10+01	-.58+00	.74+00	.41+00	-.19+01	-.17+01	-.19+01	-.14+01	-.17+01	-.35+01	-.41+01
.94+01	-.13+00	-.28+00	-.39+00	-.41+00	-.60+00	-.87+00	-.63+00	-.75+00	-.93+00	-.78+00	-.11+01	-.97+00	-.11+01	-.62+00
.82+00	-.52+02	-.55+00	-.21+01	-.91+00	-.27+00	-.44+00	-.43+01	-.92+00	-.10+01	-.12+01	-.35+01	-.37+01	-.85+00	.56+01
-.12+02	.83+01	-.58-01	.58+00	-.74+00	-.17+01	-.20+01	-.91+00	-.29+01	-.36+01	-.39+01	-.37+01	-.35+01	-.53+01	-.54+01
.16+01	.79+01	-.37+00	-.67+00	-.60+00	-.59+00	-.65+00	-.12+01	-.79+00	-.86+00	-.98+00	-.12+01	-.12+01	-.83+00	.45+00
-.28+01	-.21+02	-.58+02	-.12+01	-.39+01	-.30+01	-.44+01	-.33+01	-.58+01	-.42+01	-.29+01	-.61+01	-.25+01	.71+01	.20+01
-.12+00	-.95+01	-.16+02	-.21+01	-.24+01	-.33+01	-.44+01	-.56+01	-.60+01	-.49+01	-.85+01	-.68+01	-.89+01	-.69+01	-.83+00
.34+00	.32+01	.83+01	-.70+00	-.11+01	-.10+01	-.12+01	-.10+01	-.13+01	-.10+01	-.10+01	-.12+01	-.87+00	.92+00	.30+00
.17+02	.12+02	-.68+01	-.62+02	-.62+02	-.29+02	-.16+02	-.13+02	-.11+02	-.99+01	-.10+02	-.69+01	.15+02	.40+01	
-.19+02	-.20+02	-.24+02	-.49+02	-.56+02	-.35+02	-.22+02	-.18+02	-.15+02	-.15+02	-.14+02	-.14+02	-.18+01	.52-01	
-.98+00	-.26+01	-.14+01	.55+01	.41+01	-.75-01	-.11+01	-.11+01	-.12+01	-.11+01	-.11+01	-.69+00	.25+01	.67+00	
.17+02	.19+02	.24+02	.31+02	.25+02	-.10+02	-.15+02	-.22+02	-.18+02	.17+01	.20+02	.24+02			
-.16+02	-.11+02	-.47+01	.51+01	.72+01	-.17+02	-.23+02	-.31+02	-.28+02	-.68+01	.77+01	.10+02			
-.75+00	-.19+01	-.31+01	-.36+01	-.22+01	.12+01	.11+01	.11+01	.54+00	.10+01	.29+01	.35+01			
.12+02	.16+02	.18+02	.20+02	.30+02	.37+02	.40+02	.41+02	.35+02	.19+02					
-.11+02	-.91+01	-.17+01	.55+01	.16+02	.26+02	.33+02	.34+02	.30+02	.16+02					
-.51+00	-.13+01	-.18+01	-.12+01	-.12+01	-.51+00	.29+00	.14+01	.21+01	.16+01					
.26+01	.40+01	.62+01	.59+01	.41+01										
-.14+01	.12+01	.38+01	.52+01	.38+01										
.15-01	-.11+00	.33-01	.57-01	.74-01										

TABLE 22

DISTRIBUTION FOR $\gamma = 0$, $\theta_0 = 80°$, AND $\sin \alpha_L = 0.2$

		.36+00	.35+01	.41+01										
		.42-01	.31+01	.38+01										
		-.59-01	-.81-01	-.74-01										
.35+01	.49+01	.44+01	.20+02	.30+02	.37+02	.40+02	.41+02	.35+02	.19+02					
-.34+01	-.42+01	-.31+01	.53+01	.16+02	.26+02	.33+02	.34+02	.30+02	.16+02					
-.83-01	-.38+00	-.49+00	.11+01	.12+01	.51+00	-.29+00	-.14+01	-.21+01	-.16+01					
.42+01	.42+01	.42+01	.23+02	.38+02	.57+02	.52+02	.42+02	.47+02	.44+02	.47+02	.33+02			
-.41+01	-.41+01	-.40+01	.32+01	.16+02	.41+02	.42+02	.32+02	.35+02	.32+02	.30+02	.17+02			
-.44-01	-.13+00	-.21+00	.24+01	.38+01	.43+01	.15+01	-.48+00	-.23+01	-.38+01	-.53+01	-.45+01			
.33+01	.33+01	.48+01	.15+02	.74+02	.39+02	.26+02	.23+02	.20+02	.23+02	.24+02	.32+02	.43+02	.62+01	
-.33+01	-.33+01	-.33+01	.48+01	.52+02	.25+02	.13+02	.10+02	.86+01	.70+01	.77+01	.77+01	.89+01	.35+00	
-.19-01	-.56-01	-.32+00	-.52+00	.44+01	-.76+00	-.19+01	-.21+01	-.22+01	-.28+01	-.32+01	-.47+01	-.68+01	-.10+01	
.84+01	.78+01	.93+01	.87+01	.11+02	.10+02	.12+02	.12+02	.15+02	.11+02	.17+02	.17+02	.25+02	.40+02	.20+01
-.83+01	-.69+01	-.69+01	-.52+01	-.49+01	-.40+01	-.29+01	-.30+01	-.24+01	-.17+01	-.40+01	-.16+01	-.56+01	-.83+01	-.83+00
-.17+00	-.44+00	-.79+00	-.83+00	-.13+01	-.12+01	-.14+01	-.15+01	-.19+01	-.15+01	-.23+01	-.24+01	-.37+01	-.65+01	-.30+00
.82+01	.10+02	.84+01	.10+02	.87+01	.81+01	.83+01	.12+02	.10+02	.10+02	.12+02	.15+02	.17+02	.24+02	.26+02
-.82+01	-.10+02	-.78+01	-.84+01	-.71+01	-.61+01	-.58+01	-.68+01	-.65+01	-.59+01	-.75+01	-.70+01	-.89+01	-.13+02	-.15+02
-.65-01	-.33+00	-.37+00	-.67+00	-.60+00	-.59+00	-.65+00	-.13+01	-.10+01	-.10+01	-.12+01	-.18+01	-.20+01	-.31+01	-.34+01
.74+01	.74+01	.83+01	.84+01	.75+01	.85+01	.10+02	.78+01	.98+01	.10+02	.10+02	.14+02	.13+02	.20+02	.35+02
-.74+01	-.74+01	-.81+01	-.79+01	-.69+01	-.73+01	-.86+01	-.63+01	-.79+01	-.81+01	-.79+01	-.10+02	-.99+01	-.15+02	-.29+02
-.25-01	-.76-01	-.18+00	-.25+00	-.24+00	-.39+00	-.63+00	-.50+00	-.53+00	-.73+00	-.56+00	-.11+01	-.97+00	-.16+01	-.31+01
.77+01	.77+01	.77+01	.77+01	.87+01	.88+01	.59+01	.99+01	.10+02	.90+01	.11+02	.12+02	.13+02	.18+02	.37+02
-.77+01	-.76+01	-.76+01	-.75+01	-.82+01	-.80+01	-.54+01	-.90+01	-.87+01	-.76+01	-.95+01	-.10+02	-.11+02	-.16+02	-.36+02
-.86-02	-.25-01	-.42-01	-.58-01	-.14+00	-.17+00	-.67-01	-.14+00	-.20+00	-.25+00	-.14+00	-.29+00	-.35+00	-.53+00	-.12+01

.18+00	.18+00	.17+00	.16+00	-.78+00	-.84+00	.42-01	.60-01	.45-01	-.99+00	.98+00	-.67-01	-.10+01	-.15+00	-.15+00
-.19+00	-.20+00	-.23+00	-.26+00	.52+00	.46+00	-.22+00	-.46+00	-.68+00	.23+00	-.16+01	-.85+00	.42-01	-.11+01	-.91+00
-.46-01	-.13+00	-.23+00	-.32+00	-.48+00	-.57+00	-.40+00	-.68+00	-.77+00	-.77+00	-.90+00	-.92+00	-.10+01	-.11+01	-.95+00
.56+00	.54+00	-.32+00	-.41+00	.43+00	-.51+00	-.25+01	-.18+01	.10+00	-.98+00	-.79-01	-.22+01	-.12+01	-.47+00	.15+01
-.56+00	-.60+00	.97-01	-.25-01	-.10+01	-.58+00	.74+00	.41+00	-.19+01	-.17+01	-.19+01	-.14+01	-.17+01	-.35+01	-.41+01
-.45-01	-.13+00	-.28+00	-.39+00	-.41+00	-.60+00	-.87+00	-.63+00	-.75+00	-.93+00	-.78+00	-.11+01	-.97+00	-.11+01	-.62+00
-.42+00	-.26+01	-.55+00	-.21+01	-.91+00	-.27+00	-.44+00	-.43+01	-.92+00	-.10+01	-.12+01	-.35+01	-.37+01	-.85+00	.56+01
.40+00	.23+01	-.58-01	.58+00	-.74+00	-.17+01	-.20+01	-.91+00	-.29+01	-.36+01	-.39+01	-.37+01	-.35+01	-.53+01	-.54+01
-.65-01	-.33+00	-.37+00	-.67+00	-.60+00	-.59+00	-.65+00	-.12+01	-.79+00	-.86+00	-.98+00	-.12+01	-.12+01	-.83+00	.45+00
-.10+01	-.44+00	-.19+01	-.12+01	-.39+01	-.30+01	-.44+01	-.33+01	-.58+01	-.42+01	-.29+01	-.61+01	-.25+01	.71+01	.20+01
.86+00	-.47+00	-.45+00	-.21+01	-.24+01	-.33+01	-.44+01	-.56+01	-.60+01	-.49+01	-.85+01	-.68+01	-.89+01	-.69+01	-.83+00
-.15+00	-.38+00	-.69+00	-.70+00	-.11+01	-.10+01	-.12+01	-.10+01	-.13+01	-.10+01	-.10+01	-.12+01	-.87+00	.92+00	.30+00
.33+01	.33+01	.19+01	-.88+01	-.62+02	-.29+02	-.16+02	-.13+02	-.11+02	-.99+01	-.10+02	-.69+01	.15+02	.40+01	
-.33+01	-.33+01	-.33+01	-.11+02	-.56+02	-.35+02	-.22+02	-.18+02	-.15+02	-.15+02	-.14+02	-.14+02	-.18+01	.52-01	
.19-01	.56-01	-.13+00	-.26+00	.41+01	-.75-01	-.11+01	-.11+01	-.12+01	-.11+01	-.11+01	-.69+00	.25+01	.67+00	
.42+01	.42+01	.42+01	.23+02	.25+02	-.10+02	-.15+02	-.22+02	-.18+02	.17+01	.20+02	.24+02			
-.41+01	-.41+01	-.40+01	.32+01	.72+01	-.17+02	-.23+02	-.31+02	-.28+02	-.68+01	.77+01	.10+02			
.44-01	.13+00	.21+00	-.24+01	-.22+01	.12+01	.11+01	.11+01	.54+00	.10+01	.29+01	.35+01			
.35+01	.49+01	.44+01	.20+02	.30+02	.37+02	.40+02	.41+02	.35+02	.19+02					
-.34+01	-.42+01	-.31+01	.53+01	.16+02	.26+02	.33+02	.34+02	.30+02	.16+02					
.83-01	.38+00	.49+00	-.11+01	-.12+01	-.51+00	.29+00	.14+01	.21+01	.16+01					
		.36+00	.35+01	.41+01										
		.42-01	.31+01	.38+01										
		.59-01	.81-01	.74-01										

TABLE 23

DISTRIBUTION FOR $\gamma = 0$, $\theta_0 = 80°$, AND $\sin \alpha_L = 0.4$

.11+01	.11+01	.17+01	.17+01	.30+01	.37+01	.60+01	.38+02	.35+02	.19+02					
-.11+01	-.10+01	-.15+01	-.12+01	-.15+01	-.94+00	.16+01	.31+02	.30+02	.16+02					
-.18-01	-.55-01	-.14+00	-.21+00	-.42+00	-.59+00	-.89+00	-.15+01	-.21+01	-.16+01					
.42+01	.42+01	.42+01	.50+01	.51+01	.44+01	.51+01	.33+02	.47+02	.44+02	.47+02	.33+02			
-.41+01	-.41+01	-.40+01	-.44+01	-.43+01	-.30+01	-.27+01	.23+02	.35+02	.32+02	.30+02	.17+02			
-.44-01	-.13+00	-.21+00	-.36+00	-.46+00	-.50+00	-.71+00	-.76+00	-.23+01	-.38+01	-.53+01	-.45+01			
.33+01	.33+01	.34+01	.34+01	.34+01	.50+01	.11+02	.22+02	.20+02	.23+02	.24+02	.32+02	.43+02	.62+01	
-.33+01	-.33+01	-.33+01	-.33+01	-.32+01	-.46+01	-.67+00	.92+01	.86+01	.70+01	.77+01	.77+01	.89+01	.35+00	
-.19-01	-.56-01	-.94-01	-.13+00	-.16+00	-.34+00	-.11+01	-.20+01	-.22+01	-.28+01	-.32+01	-.47+01	-.68+01	-.10+01	
.37+01	.37+01	.37+01	.44+01	.73+01	.98+01	.12+02	.12+02	.15+02	.11+02	.17+02	.17+02	.25+02	.40+02	.20+01
-.37+01	-.37+01	-.37+01	-.40+01	-.51+01	-.42+01	-.29+01	-.30+01	-.24+01	-.17+01	-.40+01	-.16+01	-.56+01	-.83+01	-.83+00
-.96-02	-.28-01	-.48-01	-.16+00	-.64+00	-.11+01	-.14+01	-.15+01	-.19+01	-.15+01	-.23+01	-.24+01	-.37+01	-.65+01	-.30+00
.82+01	.10+02	.84+01	.10+02	.87+01	.81+01	.83+01	.12+02	.10+02	.10+02	.12+02	.15+02	.17+02	.24+02	.26+02
-.82+01	-.10+02	-.78+01	-.84+01	-.71+01	-.61+01	-.58+01	-.68+01	-.65+01	-.59+01	-.75+01	-.70+01	-.89+01	-.13+02	-.15+02
-.65-01	-.33+00	-.37+00	-.67+00	-.60+00	-.59+00	-.65+00	-.13+01	-.10+01	-.10+01	-.12+01	-.18+01	-.20+01	-.31+01	-.34+01
.74+01	.74+01	.83+01	.84+01	.75+01	.85+01	.10+02	.78+01	.98+01	.10+02	.10+02	.14+02	.13+02	.20+02	.35+02
-.74+01	-.74+01	-.81+01	-.79+01	-.69+01	-.73+01	-.86+01	-.63+01	-.79+01	-.81+01	-.79+01	-.10+02	-.99+01	-.15+02	-.29+02
-.25-01	-.76-01	-.18+00	-.25+00	-.24+00	-.39+00	-.63+00	-.50+00	-.53+00	-.73+00	-.56+00	-.11+01	-.97+00	-.16+01	-.31+01
.77+01	.77+01	.77+01	.77+01	.87+01	.88+01	.59+01	.99+01	.10+02	.90+01	.11+02	.12+02	.13+02	.18+02	.37+02
-.77+01	-.76+01	-.76+01	-.75+01	-.82+01	-.80+01	-.54+01	-.90+01	-.87+01	-.76+01	-.95+01	-.10+02	-.11+02	-.16+02	-.36+02
-.86-02	-.25-01	-.42-01	-.58-01	-.14+00	-.17+00	-.67-01	-.14+00	-.20+00	-.25+00	-.14+00	-.29+00	-.35+00	-.53+00	-.12+01
.18+00	.18+00	.17+00	.16+00	-.78+00	-.84+00	.42-01	.60-01	.45-01	-.99+00	.98+00	-.67-01	-.10+01	-.15+00	-.15+00
-.19+00	-.20+00	-.23+00	-.26+00	.52+00	.46+00	-.22+00	-.46+00	-.68+00	.23+00	-.16+01	-.85+00	.42-01	-.11+01	-.91+00
-.46-01	-.13+00	-.23+00	-.32+00	-.48+00	-.57+00	-.40+00	-.68+00	-.77+00	-.77+00	-.90+00	-.92+00	-.10+01	-.11+01	-.95+00
.56+00	.54+00	-.32+00	-.41+00	.43+00	-.51+00	-.25+01	-.18+01	.10+00	-.98+00	-.79-01	-.22+01	-.12+01	-.47+00	.15+01
-.56+00	-.60+00	.97-01	-.25-01	-.10+01	-.58+00	.74+00	.41+00	-.19+01	-.17+01	-.19+01	-.14+01	-.17+01	-.35+01	-.41+01
-.45-01	-.13+00	-.28+00	-.39+00	-.41+00	-.60+00	-.87+00	-.63+00	-.75+00	-.93+00	-.78+00	-.11+01	-.97+00	-.11+01	-.62+00
-.42+00	-.26+01	-.55+00	-.21+01	-.91+00	-.27+00	-.44+00	-.43+01	-.92+00	-.10+01	-.12+01	-.35+01	-.37+01	-.85+00	.56+01
.40+00	.23+01	-.58-01	.58+00	-.74+00	-.17+01	-.20+01	-.91+00	-.29+01	-.36+01	-.39+01	-.37+01	-.35+01	-.53+01	-.54+01
-.65-01	-.33+00	-.37+00	-.67+00	-.60+00	-.59+00	-.65+00	-.12+01	-.79+00	-.86+00	-.98+00	-.12+01	-.12+01	-.83+00	.45+00
.37+01	.37+01	.37+01	.30+01	.83-01	-.23+01	-.44+01	-.33+01	-.58+01	-.42+01	-.29+01	-.61+01	-.25+01	.71+01	.20+01
-.37+01	-.37+01	-.37+01	-.33+01	-.23+01	-.31+01	-.44+01	-.56+01	-.60+01	-.49+01	-.85+01	-.68+01	-.89+01	-.69+01	-.83+00
.96-02	.28-01	.48-01	-.30-01	-.47+00	-.89+00	-.12+01	-.10+01	-.13+01	-.10+01	-.10+01	-.12+01	-.87+00	.92+00	.30+00
.33+01	.33+01	.34+01	.34+01	.34+01	.50+01	-.81+00	-.12+02	-.11+02	-.99+01	-.10+02	-.69+01	.15+02	.40+01	
-.33+01	-.33+01	-.33+01	-.33+01	-.32+01	-.46+01	-.83+01	-.17+02	-.15+02	-.15+02	-.14+02	-.14+02	-.18+01	.52-01	
.19-01	.56-01	.94-01	.13+00	.16+00	.34+00	-.33+00	-.11+01	-.12+01	-.11+01	-.11+01	-.69+00	.25+01	.67+00	
.42+01	.42+01	.42+01	.50+01	.51+01	.44+01	.51+01	-.13+02	-.18+02	.17+01	.20+02	.24+02			
-.41+01	-.41+01	-.40+01	-.44+01	-.43+01	-.30+01	-.27+01	-.22+02	-.28+02	-.68+01	.71+01	.10+02			
.44-01	.13+00	.21+00	.36+00	.46+00	.50+00	.71+00	.82+00	.54+00	.10+01	.29+01	.35+01			
.11+01	.11+01	.17+01	.17+01	.30+01	.37+01	.60+01	.38+02	.35+02	.19+02					
-.11+01	-.10+01	-.15+01	-.12+01	-.15+01	-.94+00	.16+01	.31+02	.30+02	.16+02					
.18-01	.55-01	.14+00	.21+00	.42+00	.59+00	.89+00	.15+01	.21+01	.16+01					

TABLE 24

DISTRIBUTION FOR $\gamma = 0$, $\theta_0 = 80°$, AND $\sin \alpha_L = 0.6$

.29+01	.29+01	.29+01	.30+01	.37+01	.23+01	.30+01	.39+01	.56+01	.93+01	.38+02	.33+02			
-.29+01	-.29+01	-.28+01	-.27+01	-.32+01	-.18+01	-.18+01	-.21+01	-.19+01	.56+00	.23+02	.17+02			
-.27-01	-.82-01	-.13+00	-.18+00	-.31+00	-.22+00	-.39+00	-.54+00	-.87+00	-.15+01	-.46+01	-.45+01			
.33+01	.33+01	.34+01	.34+01	.34+01	.50+01	.51+01	.51+01	.43+01	.12+02	.24+02	.32+02	.43+02	.62+01	
-.33+01	-.33+01	-.33+01	-.33+01	-.32+01	-.46+01	-.44+01	-.42+01	-.32+01	-.14+01	.77+01	.77+01	.89+01	.35+00	
-.19-01	-.56-01	-.94-01	-.13+00	-.16+00	-.34+00	-.39+00	-.46+00	-.47+00	-.15+01	-.32+01	-.47+01	-.68+01	-.10+01	
.37+01	.37+01	.37+01	.37+01	.37+01	.37+01	.37+01	.46+01	.10+02	.11+02	.17+02	.17+02	.25+02	.40+02	.20+01
-.37+01	-.37+01	-.37+01	-.37+01	-.37+01	-.36+01	-.36+01	-.43+01	-.40+01	-.17+01	-.40+01	-.16+01	-.56+01	-.83+01	-.83+00
-.96-02	-.28-01	-.48-01	-.66-01	-.85-01	-.10+00	-.12+00	-.23+00	-.12+01	-.15+01	-.23+01	-.24+01	-.37+01	-.65+01	-.30+00
.39+01	.39+01	.39+01	.39+01	.64+01	.56+01	.65+01	.12+02	.10+02	.10+02	.12+02	.15+02	.17+02	.24+02	.26+02
-.39+01	-.39+01	-.39+01	-.39+01	-.57+01	-.49+01	-.52+01	-.68+01	-.65+01	-.59+01	-.75+01	-.70+01	-.89+01	-.13+02	-.15+02
.21-10	.19-08	.91-08	.24-07	-.28+00	-.22+00	-.38+00	-.13+01	-.10+01	-.10+01	-.12+01	-.18+01	-.20+01	-.31+01	-.34+01
.74+01	.74+01	.83+01	.84+01	.75+01	.85+01	.10+02	.78+01	.98+01	.10+02	.10+02	.14+02	.13+02	.20+02	.35+02
-.74+01	-.74+01	-.81+01	-.79+01	-.69+01	-.73+01	-.86+01	-.63+01	-.79+01	-.81+01	-.79+01	-.10+02	-.99+01	-.15+02	-.29+02
-.25-01	-.76-01	-.18+00	-.25+00	-.24+00	-.39+00	-.63+00	-.50+00	-.53+00	-.73+00	-.56+00	-.11+01	-.97+00	-.16+01	-.31+01
.77+01	.77+01	.77+01	.77+01	.87+01	.88+01	.59+01	.99+01	.10+02	.90+01	.11+02	.12+02	.13+02	.18+02	.37+02
-.77+01	-.76+01	-.76+01	-.75+01	-.82+01	-.80+01	-.54+01	-.90+01	-.87+01	-.76+01	-.95+01	-.10+02	-.11+02	-.16+02	-.36+02
-.86-02	-.25-01	-.42-01	-.58-01	-.14+00	-.17+00	-.67-01	-.14+00	-.20+00	-.25+00	-.14+00	-.29+00	-.35+00	-.53+00	-.12+01
.18+00	.18+00	.17+00	.16+00	-.78+00	-.84+00	.42-01	.60-01	.45-01	-.99+00	.98+00	-.67-01	-.10+01	-.15+00	-.15+00
-.19+00	-.20+00	-.23+00	-.26+00	.52+00	.46+00	-.22+00	-.46+00	-.68+00	.23+00	-.16+01	-.85+00	.42-01	-.11+01	-.91+00
-.46-01	-.13+00	-.23+00	-.32+00	-.48+00	-.57+00	-.40+00	-.68+00	-.77+00	-.77+00	-.90+00	-.92+00	-.10+01	-.11+01	-.95+00
.56+00	.54+00	-.32+00	-.41+00	.43+00	-.51+00	-.25+01	-.18+01	.10+00	-.98+00	-.79-01	-.22+01	-.12+01	-.47+00	.15+01
-.56+00	-.60+00	.97-01	-.25-01	-.10+01	-.58+00	.74+00	.41+00	-.19+01	-.17+01	-.19+01	-.14+01	-.17+01	-.35+01	-.41+01
-.45-01	-.13+00	-.28+00	-.39+00	-.41+00	-.60+00	-.87+00	-.63+00	-.75+00	-.93+00	-.78+00	-.11+01	-.97+00	-.11+01	-.62+00
.39+01	.39+01	.39+01	.39+01	.14+01	.22+01	.12+01	-.43+01	-.92+00	-.10+01	-.12+01	-.35+01	-.37+01	-.85+00	.56+01
-.39+01	-.39+01	-.39+01	-.39+01	-.21+01	-.28+01	-.26+01	-.91+00	-.29+01	-.36+01	-.39+01	-.37+01	-.35+01	-.53+01	-.54+01
-.21-10	-.19-08	-.91-08	-.24-07	-.28+00	-.22+00	-.38+00	-.12+01	-.79+00	-.86+00	-.98+00	-.12+01	-.12+01	-.83+00	.45+00
.37+01	.37+01	.37+01	.37+01	.37+01	.37+01	.37+01	.46+01	-.10+01	-.42+01	-.29+01	-.61+01	-.25+01	.71+01	.20+01
-.37+01	-.37+01	-.37+01	-.37+01	-.37+01	-.36+01	-.36+01	-.43+01	-.44+01	-.49+01	-.85+01	-.68+01	-.89+01	-.69+01	-.83+00
.96-02	.28-01	.48-01	.66-01	.85-01	.10+00	.12+00	.23+00	-.65+00	-.10+01	-.10+01	-.12+01	-.87+00	.92+00	.30+00
.33+01	.33+01	.34+01	.34+01	.34+01	.50+01	.51+01	.51+01	.43+01	.17+01	-.10+02	-.69+01	.15+02	.40+01	
-.33+01	-.33+01	-.33+01	-.33+01	-.32+01	-.46+01	-.44+01	-.42+01	-.32+01	-.73+01	-.14+02	-.14+02	-.18+01	.52-01	
.19-01	.56-01	.94-01	.13+00	.16+00	.34+00	.39+00	.46+00	.47+00	.16+00	-.11+01	-.69+00	.25+01	.67+00	
.29+01	.29+01	.29+01	.30+01	.37+01	.23+01	.30+01	.39+01	.56+01	.93+01	.20+02	.24+02			
-.29+01	-.29+01	-.28+01	-.27+01	-.32+01	-.18+01	-.18+01	-.21+01	-.19+01	.56+00	.83+01	.10+02			
.27-01	.82-01	.13+00	.18+00	.31+00	.22+00	.39+00	.54+00	.87+00	.15+01	.29+01	.35+01			

TABLE 25

DISTRIBUTION FOR $\gamma = 0$, $\theta_0 = 80°$, AND $\sin \alpha_L = 0.8$

.17+01	.17+01	.17+01	.17+01	.17+01	.17+01	.17+01	.26+01	.17+01	.26+01	.18+01	.36+01	.88+01	.62+01	
-.17+01	-.17+01	-.17+01	-.17+01	-.17+01	-.16+01	-.16+01	-.22+01	-.14+01	-.20+01	-.11+01	-.19+01	-.12+01	.35+00	
-.83-02	-.25-01	-.41-01	-.57-01	-.73-01	-.88-01	-.10+00	-.21+00	-.17+00	-.28+00	-.22+00	-.51+00	-.14+01	-.10+01	
.37+01	.37+01	.37+01	.37+01	.37+01	.37+01	.37+01	.46+01	.46+01	.37+01	.74+01	.55+01	.19+02	.40+02	.20+01
-.37+01	-.37+01	-.37+01	-.37+01	-.37+01	-.36+01	-.36+01	-.43+01	-.42+01	-.33+01	-.62+01	-.42+01	-.69+01	-.83+01	-.83+00
-.96-02	-.28-01	-.48-01	-.66-01	-.85-01	-.10+00	-.12+00	-.23+00	-.28+00	-.25+00	-.63+00	-.60+00	-.28+01	-.65+01	-.30+00
.39+01	.39+01	.39+01	.39+01	.39+01	.39+01	.39+01	.38+01	.48+01	.48+01	.88+01	.13+02	.17+02	.24+02	.26+02
-.39+01	-.39+01	-.39+01	-.39+01	-.39+01	-.39+01	-.39+01	-.38+01	-.47+01	-.48+01	-.68+01	-.68+01	-.89+01	-.13+02	-.15+02
.21-10	.19-08	.91-08	.24-07	.50-07	.88-07	.13-06	-.35-01	-.10+00	-.99-01	-.61+00	-.14+01	-.20+01	-.31+01	-.34+01
.40+01	.40+01	.40+01	.40+01	.40+01	.40+01	.59+01	.59+01	.69+01	.10+02	.10+02	.14+02	.13+02	.20+02	.35+02
-.40+01	-.40+01	-.40+01	-.39+01	-.39+01	-.39+01	-.54+01	-.51+01	-.62+01	-.81+01	-.79+01	-.10+02	-.99+01	-.15+02	-.29+02
.96-02	.28-01	.48-01	.66-01	.85-01	.10+00	-.69-01	-.26+00	-.13+00	-.73+00	-.56+00	-.11+01	-.97+00	-.16+01	-.31+01
.58+01	.58+01	.58+01	.58+01	.87+01	.88+01	.59+01	.99+01	.10+02	.90+01	.11+02	.12+02	.13+02	.18+02	.37+02
-.58+01	-.58+01	-.58+01	-.57+01	-.82+01	-.80+01	-.54+01	-.90+01	-.87+01	-.76+01	-.95+01	-.10+02	-.11+02	-.16+02	-.36+02
.61-02	.18-01	.30-01	.43-01	-.14+00	-.17+00	-.67-01	-.14+00	-.20+00	-.25+00	-.14+00	-.29+00	-.35+00	-.53+00	-.12+01
.20+01	.20+01	.20+01	.20+01	-.78+00	-.84+00	.42-01	.60-01	.45-01	-.99+00	.98+00	-.67-01	-.10+01	-.15+00	-.15+00
-.20+01	-.20+01	-.20+01	-.20+01	.52+00	.46+00	-.22+00	-.46+00	-.68+00	.23+00	-.16+01	-.85+00	.42-01	-.11+01	-.91+00
-.31-01	-.95-01	-.15+00	-.21+00	-.48+00	-.57+00	-.40+00	-.68+00	-.77+00	-.77+00	-.90+00	-.92+00	-.10+01	-.11+01	-.95+00
.40+01	.40+01	.40+01	.40+01	.40+01	.40+01	.20+01	.79-01	.30+01	-.98+00	-.79-01	-.22+01	-.12+01	-.47+00	.15+01
-.40+01	-.40+01	-.40+01	-.39+01	-.39+01	-.39+01	-.24+01	-.80+00	-.36+01	-.17+01	-.19+01	-.14+01	-.17+01	-.35+01	-.41+01
-.96-02	-.28-01	-.48-01	-.66-01	-.85-01	-.10+00	-.31+00	-.38+00	-.34+00	-.93+00	-.78+00	-.11+01	-.97+00	-.11+01	-.62+00
.39+01	.39+01	.39+01	.39+01	.39+01	.39+01	.39+01	.38+01	.48+01	.48+01	.27+01	-.15+01	-.37+01	-.85+00	.56+01
-.39+01	-.39+01	-.39+01	-.39+01	-.39+01	-.39+01	-.39+01	-.38+01	-.47+01	-.48+01	-.45+01	-.40+01	-.35+01	-.53+01	-.54+01
-.21-10	-.19-08	-.91-08	-.24-07	-.50-07	-.88-07	-.13-06	.35-01	.10+00	.99-01	-.32+00	-.86+00	-.12+01	-.83+00	.45+00
.37+01	.37+01	.37+01	.37+01	.37+01	.37+01	.37+01	.46+01	.46+01	.37+01	.74+01	.55+01	.28+01	.71+01	.20+01
-.37+01	-.37+01	-.37+01	-.37+01	-.37+01	-.36+01	-.36+01	-.43+01	-.42+01	-.33+01	-.62+01	-.42+01	-.76+01	-.69+01	-.83+00
.96-02	.28-01	.48-01	.66-01	.85-01	.10+00	.12+00	.23+00	.28+00	.25+00	.63+00	.60+00	.32-02	.92+00	.30+00
.17+01	.17+01	.17+01	.17+01	.17+01	.17+01	.17+01	.26+01	.17+01	.26+01	.18+01	.36+01	.88+01	.40+01	
-.17+01	-.17+01	-.17+01	-.17+01	-.17+01	-.16+01	-.16+01	-.22+01	-.14+01	-.20+01	-.11+01	-.19+01	-.12+01	.52-01	
.83-02	.25-01	.41-01	.57-01	.73-01	.88-01	.10+00	.21+00	.17+00	.28+00	.22+00	.51+00	.14+01	.67+00	

TABLE 26

DISTRIBUTION FOR $\gamma = 0$, $\theta_0 = 90°$, AND $\sin \alpha_L = 0$

.24+02	.33+02	.50+02	.68+02	.11+03	.12+03	.10+03	.92+02	.87+02	.66+02	.43+02	.11+02			
-.18+02	.36-01	.20+02	.43+02	.87+02	.10+03	.97+02	.84+02	.79+02	.56+02	.34+02	.80+01			
.91+00	.24+01	.33+01	.40+01	.73+01	.49+01	.21+00	-.24+01	-.46+01	-.54+01	-.45+01	-.12+01			
.47+02	.62+02	.11+03	.95+02	.35+02	.14+02	.15+02	.21+02	.22+02	.26+02	.33+02	.61+02	.47+02	.85+01	
-.41+02	-.26+02	.11+02	.30+02	.60+01	-.58+01	-.34+01	-.16+01	.78+00	.42+01	.93+01	.22+02	.11+02	.99+00	
.26+01	.69+01	.14+02	.10+02	.87+00	-.21+01	-.24+01	-.34+01	-.36+01	-.41+01	-.52+01	-.91+01	-.75+01	-.14+01	
.64+02	.11+03	.49+02	.81+01	.83+01	.67+01	.86+01	.88+01	.90+01	.11+02	.11+02	.15+02	.22+02	.37+02	
-.54+02	-.25+02	-.48+00	-.73+01	-.71+01	-.53+01	-.60+01	-.57+01	-.49+01	-.53+01	-.45+01	-.50+01	-.51+01	-.68+01	
.48+01	.17+02	.67+01	-.56+00	-.71+00	-.69+00	-.10+01	-.11+01	-.12+01	-.16+01	-.17+01	-.24+01	-.36+01	-.61+01	
.14+03	.64+02	.71+01	.72+01	.72+01	.91+01	.73+01	.93+01	.94+01	.95+01	.11+02	.13+02	.15+02	.26+02	.24+02
-.97+02	-.21+02	-.70+01	-.69+01	-.68+01	-.82+01	-.64+01	-.79+01	-.74+01	-.73+01	-.82+01	-.88+01	-.96+01	-.14+02	-.13+02
.14+02	.90+01	-.23+00	-.32+00	-.40+00	-.64+00	-.59+00	-.82+00	-.96+00	-.10+01	-.13+01	-.17+01	-.21+01	-.36+01	-.34+01
.16+03	.76+01	.76+01	.77+01	.77+01	.77+01	.77+01	.97+01	.97+01	.78+01	.11+02	.11+02	.15+02	.18+02	.38+02
-.12+03	-.76+01	-.76+01	-.76+01	-.75+01	-.75+01	-.74+01	-.91+01	-.91+01	-.71+01	-.10+02	-.10+02	-.13+02	-.14+02	-.31+02
.15+02	-.86-01	-.14+00	-.19+00	-.25+00	-.30+00	-.35+00	-.53+00	-.56+00	-.51+00	-.85+00	-.93+00	-.13+01	-.16+01	-.36+01
.20+03	.79+01	.79+01	.79+01	.79+01	.79+01	.79+01	.79+01	.99+01	.99+01	.11+02	.11+02	.11+02	.20+02	.40+02
-.18+03	-.79+01	-.79+01	-.79+01	-.79+01	-.79+01	-.79+01	-.79+01	-.98+01	-.98+01	-.11+02	-.11+02	-.11+02	-.19+02	-.39+02
.11+02	-.29-01	-.48-01	-.67-01	-.85-01	-.10+00	-.12+00	-.13+00	-.21+00	-.19+00	-.29+00	-.32+00	-.34+00	-.62+00	-.13+01

TABLE 27

DISTRIBUTION FOR $\gamma = 0$, $\theta_0 = 90°$, AND $\sin \alpha_L = 0.2$

.25+01	.32+01	.62+01	.37+02	.11+03	.12+03	.10+03	.92+02	.87+02	.66+02	.43+02	.11+02
-.24+01	-.23+01	-.21+01	.23+02	.87+02	.10+03	.97+02	.84+02	.79+02	.56+02	.34+02	.80+01
-.10+00	-.36+00	-.93+00	.81+00	.73+01	.49+01	.21+00	-.24+01	-.46+01	-.54+01	-.45+01	-.12+01

.97+01	.13+02	.11+02	.21+02	.35+02	.14+02	.15+02	.21+02	.22+02	.26+02	.33+02	.61+02	.47+02	.85+01	
-.96+01	-.12+02	-.97+01	-.40+01	.60+01	-.58+01	-.34+01	-.16+01	.78+00	.42+01	.93+01	.22+02	.11+02	.99+00	
-.16+00	-.77+00	-.10+01	-.47+00	.87+00	-.21+01	-.24+01	-.34+01	-.36+01	-.41+01	-.52+01	-.91+01	-.75+01	-.14+01	

.63+01	.64+01	.64+01	.81+01	.83+01	.67+01	.86+01	.88+01	.90+01	.11+02	.11+02	.15+02	.22+02	.37+02	
-.63+01	-.63+01	-.61+01	-.73+01	-.71+01	-.53+01	-.60+01	-.57+01	-.49+01	-.53+01	-.45+01	-.50+01	-.51+01	-.68+01	
-.63-01	-.18+00	-.31+00	-.56+00	-.71+00	-.69+00	-.10+01	-.11+01	-.12+01	-.16+01	-.17+01	-.24+01	-.36+01	-.61+01	

.71+01	.71+01	.71+01	.72+01	.72+01	.91+01	.73+01	.93+01	.94+01	.95+01	.11+02	.13+02	.15+02	.26+02	.24+02	
-.71+01	-.71+01	-.70+01	-.69+01	-.68+01	-.82+01	-.64+01	-.79+01	-.74+01	-.73+01	-.82+01	-.88+01	-.96+01	-.14+02	-.13+02	
-.46-01	-.14+00	-.23+00	-.32+00	-.40+00	-.64+00	-.59+00	-.82+00	-.96+00	-.10+01	-.13+01	-.17+01	-.21+01	-.36+01	-.34+01	

.76+01	.76+01	.76+01	.77+01	.77+01	.77+01	.77+01	.97+01	.97+01	.78+01	.11+02	.11+02	.15+02	.18+02	.38+02	
-.76+01	-.76+01	-.76+01	-.76+01	-.75+01	-.75+01	-.74+01	-.91+01	-.91+01	-.71+01	-.10+02	-.10+02	-.13+02	-.14+02	-.31+02	
-.28-01	-.86-01	-.14+00	-.19+00	-.25+00	-.30+00	-.35+00	-.53+00	-.56+00	-.51+00	-.85+00	-.93+00	-.13+01	-.16+01	-.36+01	

.79+01	.79+01	.79+01	.79+01	.79+01	.79+01	.79+01	.79+01	.99+01	.99+01	.11+02	.11+02	.11+02	.20+02	.40+02	
-.79+01	-.79+01	-.79+01	-.79+01	-.79+01	-.79+01	-.79+01	-.79+01	-.98+01	-.98+01	-.11+02	-.11+02	-.11+02	-.19+02	-.39+02	
-.97-02	-.29-01	-.48-01	-.67-01	-.85-01	-.10+00	-.12+00	-.13+00	-.21+00	-.19+00	-.29+00	-.32+00	-.34+00	-.62+00	-.13+01	

TABLE 28

DISTRIBUTION FOR $\gamma = 0$, $\theta_0 = 90°$, AND $\sin \alpha_L = 0.4$

					.13+01	.12+02	.79+02	.87+02	.66+02	.43+02	.11+02
					.32+00	.70+01	.71+02	.79+02	.56+02	.34+02	.80+01
					-.22+00	-.15+01	-.27+01	-.46+01	-.54+01	-.45+01	-.12+01

.54+01	.79+01	.94+01	.95+01	.12+02	.14+02	.15+02	.21+02	.22+02	.26+02	.33+02	.61+02	.47+02	.85+01	
-.54+01	-.75+01	-.81+01	-.69+01	-.66+01	-.58+01	-.34+01	-.16+01	.78+00	.42+01	.93+01	.22+02	.11+02	.99+00	
-.77-01	-.41+00	-.77+00	-.10+01	-.17+01	-.21+01	-.24+01	-.34+01	-.36+01	-.41+01	-.52+01	-.91+01	-.75+01	-.14+01	

.63+01	.64+01	.64+01	.81+01	.83+01	.67+01	.86+01	.88+01	.90+01	.11+02	.11+02	.15+02	.22+02	.37+02	
-.63+01	-.63+01	-.61+01	-.73+01	-.71+01	-.53+01	-.60+01	-.57+01	-.49+01	-.53+01	-.45+01	-.50+01	-.51+01	-.68+01	
-.63-01	-.18+00	-.31+00	-.56+00	-.71+00	-.69+00	-.10+01	-.11+01	-.12+01	-.16+01	-.17+01	-.24+01	-.36+01	-.61+01	

.71+01	.71+01	.71+01	.72+01	.72+01	.91+01	.73+01	.93+01	.94+01	.95+01	.11+02	.13+02	.15+02	.26+02	.24+02	
-.71+01	-.71+01	-.70+01	-.69+01	-.68+01	-.82+01	-.64+01	-.79+01	-.74+01	-.73+01	-.82+01	-.88+01	-.96+01	-.14+02	-.13+02	
-.46-01	-.14+00	-.23+00	-.32+00	-.40+00	-.64+00	-.59+00	-.82+00	-.96+00	-.10+01	-.13+01	-.17+01	-.21+01	-.36+01	-.34+01	

.76+01	.76+01	.76+01	.77+01	.77+01	.77+01	.77+01	.97+01	.97+01	.78+01	.11+02	.11+02	.15+02	.18+02	.38+02	
-.76+01	-.76+01	-.76+01	-.76+01	-.75+01	-.75+01	-.74+01	-.91+01	-.91+01	-.71+01	-.10+02	-.10+02	-.13+02	-.14+02	-.31+02	
-.28-01	-.86-01	-.14+00	-.19+00	-.25+00	-.30+00	-.35+00	-.53+00	-.56+00	-.51+00	-.85+00	-.93+00	-.13+01	-.16+01	-.36+01	

.79+01	.79+01	.79+01	.79+01	.79+01	.79+01	.79+01	.79+01	.99+01	.99+01	.11+02	.11+02	.11+02	.20+02	.40+02	
-.79+01	-.79+01	-.79+01	-.79+01	-.79+01	-.79+01	-.79+01	-.79+01	-.98+01	-.98+01	-.11+02	-.11+02	-.11+02	-.19+02	-.39+02	
-.97-02	-.29-01	-.48-01	-.67-01	-.85-01	-.10+00	-.12+00	-.13+00	-.21+00	-.19+00	-.29+00	-.32+00	-.34+00	-.62+00	-.13+01	

TABLE 29

DISTRIBUTION FOR $\gamma = 0$, $\theta_0 = 90°$, AND $\sin \alpha_L = 0.6$

								.20+01	.37+02	.11+02
								.13+01	.28+02	.80+01
								-.25+00	-.40+01	-.12+01
					.31+01	.32+01	.84+01	.12+02	.22+02	.33+02	.61+02	.47+02	.85+01
					-.19+01	-.16+01	-.26+01	-.16+01	.26+01	.93+01	.22+02	.11+02	.99+00
					-.42+00	-.46+00	-.13+01	-.20+01	-.36+01	-.52+01	-.91+01	-.75+01	-.14+01

.63+01	.64+01	.64+01	.81+01	.83+01	.67+01	.86+01	.88+01	.90+01	.11+02	.11+02	.15+02	.22+02	.37+02	
-.63+01	-.63+01	-.61+01	-.73+01	-.71+01	-.53+01	-.60+01	-.57+01	-.49+01	-.53+01	-.45+01	-.50+01	-.51+01	-.68+01	
-.63-01	-.18+00	-.31+00	-.56+00	-.71+00	-.69+00	-.10+01	-.11+01	-.12+01	-.16+01	-.17+01	-.24+01	-.36+01	-.61+01	

.71+01	.71+01	.71+01	.72+01	.72+01	.91+01	.73+01	.93+01	.94+01	.95+01	.11+02	.13+02	.15+02	.26+02	.24+02	
-.71+01	-.71+01	-.70+01	-.69+01	-.68+01	-.82+01	-.64+01	-.79+01	-.74+01	-.73+01	-.82+01	-.88+01	-.96+01	-.14+02	-.13+02	
-.46-01	-.14+00	-.23+00	-.32+00	-.40+00	-.64+00	-.59+00	-.82+00	-.96+00	-.10+01	-.13+01	-.17+01	-.21+01	-.36+01	-.34+01	

.76+01	.76+01	.76+01	.77+01	.77+01	.77+01	.77+01	.97+01	.97+01	.78+01	.11+02	.11+02	.15+02	.18+02	.38+02	
-.76+01	-.76+01	-.76+01	-.76+01	-.75+01	-.75+01	-.74+01	-.91+01	-.91+01	-.71+01	-.10+02	-.10+02	-.13+02	-.14+02	-.31+02	
-.28-01	-.86-01	-.14+00	-.19+00	-.25+00	-.30+00	-.35+00	-.53+00	-.56+00	-.51+00	-.85+00	-.93+00	-.13+01	-.16+01	-.36+01	

.79+01	.79+01	.79+01	.79+01	.79+01	.79+01	.79+01	.79+01	.99+01	.99+01	.11+02	.11+02	.11+02	.20+02	.40+02	
-.79+01	-.79+01	-.79+01	-.79+01	-.79+01	-.79+01	-.79+01	-.79+01	-.98+01	-.98+01	-.11+02	-.11+02	-.11+02	-.19+02	-.39+02	
-.97-02	-.29-01	-.48-01	-.67-01	-.85-01	-.10+00	-.12+00	-.13+00	-.21+00	-.19+00	-.29+00	-.32+00	-.34+00	-.62+00	-.13+01	

TABLE 30

DISTRIBUTION FOR $\gamma = 0$, $\theta_0 = 90°$, AND $\sin \alpha_L = 0.8$

												.12+02	.85+01	
												.11+01	.99+00	
												-.20+01	-.14+01	
										.18+01	.57+01	.11+02	.22+02	.37+02
										-.10+01	-.28+01	-.40+01	-.51+01	-.68+01
										-.26+00	-.83+00	-.18+01	-.36+01	-.61+01
.36+01	.36+01	.36+01	.36+01	.37+01	.73+01	.73+01	.93+01	.94+01	.95+01	.11+02	.13+02	.15+02	.26+02	.24+02
-.36+01	-.36+01	-.36+01	-.35+01	-.35+01	-.66+01	-.64+01	-.79+01	-.74+01	-.73+01	-.82+01	-.88+01	-.96+01	-.14+02	-.13+02
-.21-01	-.63-01	-.10+00	-.14+00	-.18+00	-.51+00	-.59+00	-.82+00	-.96+00	-.10+01	-.13+01	-.17+01	-.21+01	-.36+01	-.34+01
.76+01	.76+01	.76+01	.77+01	.77+01	.77+01	.77+01	.97+01	.97+01	.78+01	.11+02	.11+02	.15+02	.18+02	.38+02
-.76+01	-.76+01	-.76+01	-.76+01	-.75+01	-.75+01	-.74+01	-.91+01	-.91+01	-.71+01	-.10+02	-.10+02	-.13+02	-.14+02	-.31+02
-.28-01	-.86-01	-.14+00	-.19+00	-.25+00	-.30+00	-.35+00	-.53+00	-.56+00	-.51+00	-.85+00	-.93+00	-.13+01	-.16+01	-.36+01
.79+01	.79+01	.79+01	.79+01	.79+01	.79+01	.79+01	.79+01	.99+01	.99+01	.11+02	.11+02	.11+02	.20+02	.40+02
-.79+01	-.79+01	-.79+01	-.79+01	-.79+01	-.79+01	-.79+01	-.79+01	-.98+01	-.98+01	-.11+02	-.11+02	-.11+02	-.19+02	-.39+02
-.97-02	-.29-01	-.48-01	-.67-01	-.85-01	-.10+00	-.12+00	-.13+00	-.21+00	-.19+00	-.29+00	-.32+00	-.34+00	-.62+00	-.13+01

TABLE 31

DISTRIBUTION FOR $\gamma = 1.5$, $\theta_0 = 0°$, AND $\sin \alpha_L = 0$

.39+01	.39+01	.31+01	.18+01	.91+00										
-.38+01	-.38+01	-.29+01	-.16+01	-.74+00										
.45-01	.13+00	.17+00	.13+00	.87-01										
.82+01	.82+01	.56+01	.69+01	.50+01	.39+01	.18+01	.89-01							
-.82+01	-.80+01	-.52+01	-.61+01	-.41+01	-.28+01	-.11+01	-.47-01							
.96-01	.28+00	.32+00	.54+00	.50+00	.45+00	.23+00	.12-01							
.93+01	.14+02	.93+01	.90+01	.72+01	.73+01	.55+01	.39+01	.91+00						
-.93+01	-.13+02	-.86+01	-.76+01	-.55+01	-.49+01	-.31+01	-.18+01	-.33+00						
.10+00	.56+00	.60+00	.80+00	.77+00	.90+00	.76+00	.57+00	.14+00						
.16+02	.16+02	.11+02	.16+02	.11+02	.10+02	.82+01	.83+01	.50+01	.18+01					
-.16+02	-.15+02	-.10+02	-.13+02	-.83+01	-.63+01	-.42+01	-.33+01	-.14+01	-.33+00					
.25+00	.69+00	.76+00	.15+01	.12+01	.13+01	.11+01	.12+01	.81+00	.29+00					
.22+02	.84+01	.23+02	.15+02	.19+02	.11+02	.12+02	.76+01	.68+01	.47+01	.32+01				
-.22+02	-.80+01	-.21+02	-.12+02	-.13+02	-.62+01	-.55+01	-.23+01	-.14+01	-.49+00	-.48-08				
.33+00	.38+00	.17+01	.15+01	.23+01	.15+01	.18+01	.12+01	.11+01	.78+00	.53+00				
.21+02	.31+02	.31+02	.18+02	.17+02	.19+02	.13+02	.15+02	.10+02	.68+01	.47+01	.18+01			
-.21+02	-.29+02	-.27+02	-.13+02	-.11+02	-.99+01	-.49+01	-.40+01	-.11+01	-.10-07	.49+00	.33+00			
.43+00	.16+01	.26+01	.20+01	.22+01	.27+01	.20+01	.25+01	.16+01	.11+01	.78+00	.29+00			
.41+02	.55+02	.19+02	.41+02	.26+02	.18+02	.25+02	.16+02	.12+02	.10+02	.68+01	.50+01	.91+00		
-.40+02	-.51+02	-.15+02	-.29+02	-.14+02	-.76+01	-.63+01	-.21+01	-.19-07	.11+01	.14+01	.14+01	.33+00		
.71+00	.31+01	.17+01	.48+01	.37+01	.28+01	.40+01	.27+01	.21+01	.16+01	.11+01	.81+00	.14+00		
.61+02	.33+02	.62+02	.29+02	.33+02	.35+02	.20+02	.21+02	.16+02	.15+02	.76+01	.83+01	.39+01	.89-01	
-.61+02	-.30+02	-.49+02	-.17+02	-.16+02	-.10+02	-.28+01	-.31-07	.21+01	.40+01	.23+01	.33+01	.18+01	.47-01	
.12+01	.22+01	.64+01	.38+01	.47+01	.55+01	.33+01	.35+01	.27+01	.25+01	.12+01	.12+01	.57+00	.12-01	
.52+02	.91+02	.68+02	.61+02	.46+02	.34+02	.36+02	.20+02	.25+02	.13+02	.12+02	.82+01	.55+01	.18+01	
-.51+02	-.82+02	-.51+02	-.33+02	-.16+02	-.52+01	-.54-07	.28+01	.63+01	.49+01	.55+01	.42+01	.31+01	.11+01	
.13+01	.64+01	.75+01	.85+01	.73+01	.57+01	.60+01	.33+01	.40+01	.20+01	.18+01	.11+01	.76+00	.23+00	
.12+03	.12+03	.92+02	.75+02	.66+02	.49+02	.34+02	.35+02	.18+02	.19+02	.11+02	.10+02	.73+01	.39+01	
-.12+03	-.11+03	-.59+02	-.31+02	-.13+02	-.72-07	.52+01	.10+02	.76+01	.99+01	.62+01	.63+01	.49+01	.28+01	
.37+01	.10+02	.11+02	.11+02	.10+02	.81+01	.57+01	.55+01	.28+01	.27+01	.15+01	.13+01	.90+00	.45+00	
.21+03	.15+03	.14+03	.11+03	.99+02	.66+02	.46+02	.33+02	.26+02	.17+02	.19+02	.11+02	.72+01	.50+01	.91+00
-.20+03	-.12+03	-.80+02	-.29+02	-.14-06	.13+02	.16+02	.16+02	.14+02	.11+02	.13+02	.83+01	.55+01	.41+01	.74+00
.80+01	.15+02	.19+02	.18+02	.16+02	.10+02	.73+01	.47+01	.37+01	.22+01	.23+01	.12+01	.77+00	.50+00	.87-01
.36+03	.29+03	.24+03	.11+03	.11+03	.75+02	.61+02	.29+02	.41+02	.18+02	.1[illegible]+02	.16+02	.90+01	.69+01	.18+01
-.33+03	-.20+03	-.76+02	-.17-06	.29+02	.31+02	.33+02	.17+02	.29+02	.13+02	.12+02	.13+02	.76+01	.61+01	.16+01
.17+02	.35+02	.37+02	.19+02	.18+02	.11+02	.8[illegible]+01	.38+01	.48+01	.20+01	.1[illegible]+01	.15+01	.80+00	.54+00	.13+00
.80+03	.60+03	.37+03	.24+03	.14+03	.92+02	.68+02	.62+02	.19+02	.31+02	.23+02	.11+02	.93+01	.56+01	.31+01
-.73+03	-.28+03	-.55-06	.76+02	.80+02	.59+02	.51+02	.49+02	.15+02	.27+02	.21+02	.10+02	.86+01	.52+01	.29+01
.44+02	.87+02	.62+02	.37+02	.19+02	.11+02	.75+01	.64+01	.17+01	.26+01	.17+01	.76+00	.60+00	.32+00	.17+00
.26+04	.11+04	.60+03	.29+03	.15+03	.12+03	.91+02	.33+02	.55+02	.31+02	.84+01	.16+02	.14+02	.82+01	.39+01
-.19+04	-.16-05	.28+03	.20+03	.12+03	.11+03	.82+02	.30+02	.51+02	.29+02	.80+01	.15+02	.13+02	.80+01	.38+01
.25+03	.18+03	.87+02	.35+02	.15+02	.10+02	.64+01	.22+01	.31+01	.16+01	.38+00	.69+00	.56+00	.28+00	.13+00
.96+06	.26+04	.80+03	.36+03	.21+03	.12+03	.52+02	.61+02	.41+02	.21+02	.22+02	.16+02	.93+01	.82+01	.39+01
-.54-02	.19+04	.73+03	.33+03	.20+03	.12+03	.51+02	.61+02	.40+02	.21+02	.22+02	.16+02	.93+01	.82+01	.38+01
.10+06	.25+03	.44+02	.17+02	.80+01	.37+01	.13+01	.12+01	.71+00	.43+00	.33+00	.25+00	.10+00	.96-01	.45-01

TABLE 32

DISTRIBUTION FOR $\gamma = 1.5$, $\theta_0 = 0°$, AND $\sin \alpha_L = 0.2$

.37+01	.37+01	.29+01	.16+01	.73+00									
-.37+01	-.36+01	-.27+01	-.14+01	-.60+00									
.43-01	.12+00	.16+00	.12+00	.70-01									
.82+01	.82+01	.56+01	.69+01	.50+01	.37+01	.16+01							
-.82+01	-.80+01	-.52+01	-.61+01	-.41+01	-.26+01	-.10+01							
.96-01	.28+00	.32+00	.54+00	.50+00	.43+00	.21+00							
.93+01	.14+02	.93+01	.90+01	.72+01	.73+01	.55+01	.38+01	.73+00					
-.93+01	-.13+02	-.86+01	-.76+01	-.55+01	-.49+01	-.31+01	-.18+01	-.26+00					
.10+00	.56+00	.60+00	.80+00	.77+00	.90+00	.76+00	.56+00	.11+00					
.16+02	.16+02	.11+02	.16+02	.11+02	.10+02	.82+01	.83+01	.50+01	.16+01				
-.16+02	-.15+02	-.10+02	-.13+02	-.83+01	-.63+01	-.42+01	-.33+01	-.14+01	-.30+00				
.25+00	.69+00	.76+00	.15+01	.12+01	.13+01	.11+01	.12+01	.80+00	.26+00				
.22+02	.84+01	.23+02	.15+02	.19+02	.11+02	.12+02	.76+01	.68+01	.47+01	.29+01			
-.22+02	-.80+01	-.21+02	-.12+02	-.13+02	-.62+01	-.55+01	-.23+01	-.14+01	-.49+00	-.43-08			
.33+00	.38+00	.17+01	.15+01	.23+01	.15+01	.18+01	.12+01	.11+01	.78+00	.49+00			
.21+02	.31+02	.31+02	.18+02	.17+02	.19+02	.13+02	.15+02	.10+02	.68+01	.47+01	.16+01		
-.21+02	-.29+02	-.27+02	-.13+02	-.11+02	-.99+01	-.49+01	-.40+01	-.11+01	-.10-07	.49+00	.30+00		
.43+00	.16+01	.26+01	.20+01	.22+01	.27+01	.20+01	.25+01	.16+01	.11+01	.78+00	.26+00		
.41+02	.55+02	.19+02	.41+02	.26+02	.18+02	.25+02	.16+02	.12+02	.10+02	.68+01	.50+01	.73+00	
-.40+02	-.51+02	-.15+02	-.29+02	-.14+02	-.76+01	-.63+01	-.21+01	-.19-07	.11+01	.14+01	.14+01	.26+00	
.71+00	.31+01	.17+01	.48+01	.37+01	.28+01	.40+01	.27+01	.21+01	.16+01	.11+01	.80+00	.11+00	
.61+02	.33+02	.62+02	.29+02	.33+02	.35+02	.20+02	.21+02	.16+02	.15+02	.76+01	.83+01	.38+01	
-.61+02	-.30+02	-.49+02	-.17+02	-.16+02	-.10+02	-.28+01	-.31-07	.21+01	.40+01	.23+01	.33+01	.18+01	
.12+01	.22+01	.64+01	.38+01	.47+01	.55+01	.33+01	.35+01	.27+01	.25+01	.12+01	.12+01	.56+00	
.52+02	.91+02	.68+02	.61+02	.46+02	.34+02	.36+02	.20+02	.25+02	.13+02	.12+02	.82+01	.55+01	.16+01
-.51+02	-.82+02	-.51+02	-.33+02	-.16+02	-.52+01	-.54-07	.28+01	.63+01	.49+01	.55+01	.42+01	.31+01	.10+01
.13+01	.64+01	.75+01	.85+01	.73+01	.57+01	.60+01	.33+01	.40+01	.20+01	.18+01	.11+01	.76+00	.21+00
.12+03	.12+03	.92+02	.75+02	.66+02	.49+02	.34+02	.35+02	.18+02	.19+02	.11+02	.10+02	.73+01	.37+01
-.12+03	-.11+03	-.59+02	-.31+02	-.13+02	-.72-07	.52+01	.10+02	.76+01	.99+01	.62+01	.63+01	.49+01	.26+01
.37+01	.10+02	.11+02	.11+02	.10+02	.81+01	.57+01	.55+01	.28+01	.27+01	.15+01	.13+01	.90+00	.43+00
.92+02	.10+03	.14+03	.11+03	.99+02	.66+02	.46+02	.33+02	.26+02	.17+02	.19+02	.11+02	.72+01	.50+01	.73+00
-.90+02	-.84+02	-.80+02	-.29+02	-.14-06	.13+02	.16+02	.16+02	.14+02	.11+02	.13+02	.83+01	.55+01	.41+01	.60+00
.31+01	.10+02	.19+02	.18+02	.16+02	.10+02	.73+01	.47+01	.37+01	.22+01	.23+01	.12+01	.77+00	.50+00	.70-01
		.61+02	.11+03	.11+03	.75+02	.61+02	.29+02	.41+02	.18+02	.15+02	.16+02	.90+01	.69+01	.16+01
		-.19+02	-.17-06	.29+02	.31+02	.33+02	.17+02	.29+02	.13+02	.12+02	.13+02	.76+01	.61+01	.14+01
		.97+01	.19+02	.18+02	.11+02	.85+01	.38+01	.48+01	.20+01	.15+01	.15+01	.80+00	.54+00	.12+00
			.61+02	.14+03	.92+02	.68+02	.62+02	.19+02	.31+02	.23+02	.11+02	.93+01	.56+01	.29+01
			.19+02	.80+02	.59+02	.51+02	.49+02	.15+02	.27+02	.21+02	.10+02	.86+01	.52+01	.27+01
			.97+01	.19+02	.11+02	.75+01	.64+01	.17+01	.26+01	.17+01	.76+00	.60+00	.32+00	.16+00
				.10+03	.12+03	.91+02	.33+02	.55+02	.31+02	.84+01	.16+02	.14+02	.82+01	.37+01
				.84+02	.11+03	.82+02	.30+02	.51+02	.29+02	.80+01	.15+02	.13+02	.80+01	.36+01
				.10+02	.10+02	.64+01	.22+01	.31+01	.16+01	.38+00	.69+00	.56+00	.28+00	.12+00
				.92+02	.12+03	.52+02	.61+02	.41+02	.21+02	.22+02	.16+02	.93+01	.82+01	.37+01
				.90+02	.12+03	.51+02	.61+02	.40+02	.21+02	.22+02	.16+02	.93+01	.82+01	.37+01
				.31+01	.37+01	.13+01	.12+01	.71+00	.43+00	.33+00	.25+00	.10+00	.96-01	.43-01

TABLE 33

DISTRIBUTION FOR $\gamma = 1.5$, $\theta_0 = 0°$, AND $\sin \alpha_L = 0.4$

.26+01	.26+01	.18+01	.54+00										
-.26+01	-.25+01	-.17+01	-.48+00										
.30-01	.91-01	.10+00	.39-01										
.82+01	.82+01	.56+01	.69+01	.47+01	.26+01	.54+00							
-.82+01	-.80+01	-.52+01	-.61+01	-.38+01	-.19+01	-.34+00							
.96-01	.28+00	.32+00	.54+00	.46+00	.30+00	.69-01							
.93+01	.14+02	.93+01	.90+01	.72+01	.73+01	.55+01	.24+01						
-.93+01	-.13+02	-.86+01	-.76+01	-.55+01	-.49+01	-.31+01	-.11+01						
.10+00	.56+00	.60+00	.80+00	.77+00	.90+00	.76+00	.35+00						
.16+02	.16+02	.11+02	.16+02	.11+02	.10+02	.82+01	.83+01	.44+01	.54+00				
-.16+02	-.15+02	-.10+02	-.13+02	-.83+01	-.63+01	-.42+01	-.33+01	-.12+01	-.11+00				
.25+00	.69+00	.76+00	.15+01	.12+01	.13+01	.11+01	.12+01	.71+00	.88-01				
.22+02	.84+01	.23+02	.15+02	.19+02	.11+02	.12+02	.76+01	.68+01	.47+01	.13+01			
-.22+02	-.80+01	-.21+02	-.12+02	-.13+02	-.62+01	-.55+01	-.23+01	-.14+01	-.49+00	-.19-08			
.33+00	.38+00	.17+01	.15+01	.23+01	.15+01	.18+01	.12+01	.11+01	.78+00	.22+00			
.21+02	.31+02	.31+02	.18+02	.17+02	.19+02	.13+02	.15+02	.10+02	.68+01	.47+01	.54+00		
-.21+02	-.29+02	-.27+02	-.13+02	-.11+02	-.99+01	-.49+01	-.40+01	-.11+01	-.10-07	.49+00	.11+00		
.43+00	.16+01	.26+01	.20+01	.22+01	.27+01	.20+01	.25+01	.16+01	.11+01	.78+00	.88-01		
.41+02	.55+02	.19+02	.41+02	.26+02	.18+02	.25+02	.16+02	.12+02	.10+02	.68+01	.44+01		
-.40+02	-.51+02	-.15+02	-.29+02	-.14+02	-.76+01	-.63+01	-.21+01	-.19-07	.11+01	.14+01	.12+01		
.71+00	.31+01	.17+01	.48+01	.37+01	.28+01	.40+01	.27+01	.21+01	.16+01	.11+01	.71+00		
.33+02	.33+02	.62+02	.29+02	.33+02	.35+02	.20+02	.21+02	.16+02	.15+02	.76+01	.83+01	.24+01	
-.32+02	-.30+02	-.49+02	-.17+02	-.16+02	-.10+02	-.28+01	-.31-07	.21+01	.40+01	.23+01	.33+01	.11+01	
.77+00	.22+01	.64+01	.38+01	.47+01	.55+01	.33+01	.35+01	.27+01	.25+01	.12+01	.12+01	.35+00	
			.33+02	.46+02	.34+02	.36+02	.20+02	.25+02	.13+02	.12+02	.82+01	.55+01	.54+00
			-.18+02	-.16+02	-.52+01	-.54-07	.28+01	.63+01	.49+01	.55+01	.42+01	.31+01	.34+00
			.45+01	.73+01	.57+01	.60+01	.33+01	.40+01	.20+01	.18+01	.11+01	.76+00	.69-01
				.83+01	.49+02	.34+02	.35+02	.18+02	.19+02	.11+02	.10+02	.73+01	.26+01
				-.17+01	-.72-07	.52+01	.10+02	.76+01	.99+01	.62+01	.63+01	.49+01	.19+01
				.13+01	.81+01	.57+01	.55+01	.28+01	.27+01	.15+01	.13+01	.90+00	.30+00
					.83+01	.46+02	.33+02	.26+02	.17+02	.19+02	.11+02	.72+01	.47+01
					.17+01	.16+02	.16+02	.14+02	.11+02	.13+02	.83+01	.55+01	.38+01
					.13+01	.73+01	.47+01	.37+01	.22+01	.23+01	.12+01	.77+00	.46+00
						.33+02	.29+02	.41+02	.18+02	.15+02	.16+02	.90+01	.69+01	.54+00
						.18+02	.17+02	.29+02	.13+02	.12+02	.13+02	.76+01	.61+01	.48+00
						.45+01	.38+01	.48+01	.20+01	.15+01	.15+01	.80+00	.54+00	.39-01
							.62+02	.19+02	.31+02	.23+02	.11+02	.93+01	.56+01	.18+01
							.49+02	.15+02	.27+02	.21+02	.10+02	.86+01	.52+01	.17+01
							.64+01	.17+01	.26+01	.17+01	.76+00	.60+00	.32+00	.10+00
							.33+02	.55+02	.31+02	.84+01	.16+02	.14+02	.82+01	.26+01
							.30+02	.51+02	.29+02	.80+01	.15+02	.13+02	.80+01	.25+01
							.22+01	.31+01	.16+01	.36+00	.69+00	.56+00	.28+00	.91-01
							.33+02	.41+02	.21+02	.22+02	.16+02	.93+01	.82+01	.26+01
							.32+02	.40+02	.21+02	.22+02	.16+02	.93+01	.82+01	.26+01
							.77+00	.71+00	.43+00	.33+00	.25+00	.10+00	.96-01	.30-01

TABLE 34

DISTRIBUTION FOR $\gamma = 1.5$, $\theta_0 = 0°$, AND $\sin \alpha_L = 0.6$

.34+01	.34+01											
-.34+01	-.33+01											
.39-01	.11+00											
.93+01	.14+02	.93+01	.76+01	.34+01	.17+01							
-.93+01	-.13+02	-.86+01	-.65+01	-.26+01	-.11+01							
.10+00	.56+00	.60+00	.67+00	.36+00	.20+00							
.16+02	.16+02	.11+02	.16+02	.11+02	.10+02	.55+01	.17+01					
-.16+02	-.15+02	-.10+02	-.13+02	-.83+01	-.63+01	-.28+01	-.69+00					
.25+00	.69+00	.76+00	.15+01	.12+01	.13+01	.79+00	.26+00					
		.10+02	.71+01	.19+02	.11+02	.12+02	.76+01	.17+01				
		-.96+01	-.58+01	-.13+02	-.62+01	-.55+01	-.23+01	-.35+00				
		.78+00	.70+00	.23+01	.15+01	.18+01	.12+01	.28+00				
					.10+02	.13+02	.15+02	.10+02	.17+01			
					-.57+01	-.49+01	-.40+01	-.11+01	-.25-08			
					.15+01	.20+01	.25+01	.16+01	.28+00			
						.35+01	.16+02	.12+02	.10+02	.17+01		
						-.99+00	-.21+01	-.19-07	.11+01	.35+00		
						.57+00	.27+01	.21+01	.16+01	.28+00		
							.35+01	.16+02	.15+02	.76+01	.17+01	
							-.54-08	.21+01	.40+01	.23+01	.69+00	
							.59+00	.27+01	.25+01	.12+01	.26+00	
								.35+01	.13+02	.12+02	.55+01	
								.99+00	.49+01	.55+01	.28+01	
								.57+00	.20+01	.18+01	.79+00	
									.10+02	.11+02	.10+02	.17+01
									.57+01	.62+01	.63+01	.11+01
									.15+01	.15+01	.13+01	.20+00
										.19+02	.11+02	.34+01
										.13+02	.83+01	.26+01
										.23+01	.12+01	.36+00
										.71+01	.16+02	.76+01
										.58+01	.13+02	.65+01
										.70+00	.15+01	.67+00
										.10+02	.11+02	.93+01
										.96+01	.10+02	.86+01
										.78+00	.76+00	.60+00
											.16+02	.14+02	.34+01
											.15+02	.13+02	.33+01
											.69+00	.56+00	.11+00
											.16+02	.93+01	.34+01
											.16+02	.93+01	.34+01
											.25+00	.10+00	.39-01

TABLE 35

DISTRIBUTION FOR $\gamma = 1.5$, $\theta_0 = 30°$, AND $\sin \alpha_L = 0$

.24+01	.28+01	.17+01	.10+01	.23+00										
-.24+01	-.25+01	-.11+01	-.20+00	.69-01										
.28-01	.11+00	.97-01	.52-01	-.14-02										
.75+01	.79+01	.75+01	.73+01	.52+01	.35+01	.17+01	.19+00							
-.75+01	-.74+01	-.63+01	-.49+01	-.20+01	-.25+00	.43+00	.41-01							
.12+00	.37+00	.53+00	.69+00	.48+00	.20+00	-.13-01	-.29-01							
.14+02	.97+01	.12+02	.88+01	.11+02	.86+01	.67+01	.56+01	.21+01						
-.14+02	-.91+01	-.10+02	-.65+01	-.62+01	-.31+01	-.97+00	.11+01	.29+00						
.25+00	.50+00	.10+01	.85+00	.13+01	.10+01	.73+00	.27+00	-.12+00						
.18+02	.18+02	.16+02	.19+02	.11+02	.12+02	.11+02	.94+01	.80+01	.48+01	.56+00				
-.18+02	-.17+02	-.13+02	-.13+02	-.61+01	-.46+01	-.26+01	-.47-01	.11+01	.11+01	.10+00				
.31+00	.10+01	.15+01	.22+01	.14+01	.16+01	.15+01	.11+01	.54+00	-.16+00	-.70-01				
.21+02	.30+02	.28+02	.20+02	.22+02	.16+02	.15+02	.12+02	.10+02	.71+01	.64+01	.89+00			
-.21+02	-.27+02	-.22+02	-.12+02	-.10+02	-.43+01	-.24+00	-.52+00	.13+01	.12+01	.76+00	.23+00			
.49+00	.19+01	.27+01	.26+01	.31+01	.24+01	.23+01	.15+01	.11+01	.46+00	-.27+00	-.11+00			
.43+02	.51+02	.39+02	.33+02	.29+02	.23+02	.19+02	.16+02	.14+02	.12+02	.90+01	.72+01	.44+00		
-.42+02	-.46+02	-.29+02	-.18+02	-.99+01	-.51+01	.28-01	.13+01	.43+01	.45+01	.11+01	.12+01	-.10+00		
.95+00	.34+01	.43+01	.44+01	.44+01	.35+01	.29+01	.23+01	.18+01	.11+01	.52+00	-.41+00	-.69-01		
.76+02	.65+02	.51+02	.47+02	.42+02	.37+02	.26+02	.23+02	.17+02	.11+02	.12+02	.93+01	.62+01		
-.74+02	-.57+02	-.34+02	-.24+02	-.89+01	-.70+00	.14+01	.66+01	.51+01	.43+01	.43+01	.17+01	.11+00		
.20+01	.49+01	.62+01	.64+01	.66+01	.58+01	.38+01	.32+01	.21+01	.12+01	.96+00	.27+00	-.55+00		
.13+03	.75+02	.10+03	.74+02	.58+02	.47+02	.33+02	.27+02	.22+02	.17+02	.11+02	.11+02	.94+01	.35+01	
-.12+03	-.62+02	-.63+02	-.25+02	-.85+01	.62+01	.87+01	.86+01	.11+02	.76+01	.71+01	.43+01	.13+01	-.10+01	
.44+01	.66+01	.12+02	.11+02	.90+01	.72+01	.48+01	.35+01	.25+01	.17+01	.76+00	.58+00	-.15+00	-.38+00	
.12+03	.18+03	.14+03	.88+02	.79+02	.57+02	.44+02	.33+02	.25+02	.19+02	.16+02	.12+02	.88+01	.77+01	
-.12+03	-.14+03	-.65+02	-.14+02	.78+01	.15+02	.19+02	.17+02	.12+02	.11+02	.89+01	.73+01	.29+01	-.20+01	
.43+01	.19+02	.20+02	.13+02	.12+02	.84+01	.59+01	.38+01	.26+01	.16+01	.11+01	.45+00	.92-01	-.52+00	
.31+03	.31+03	.22+03	.15+03	.10+03	.61+02	.63+02	.36+02	.30+02	.24+02	.14+02	.15+02	.93+01	.10+02	.23+01
-.30+03	-.22+03	-.68+02	.25+01	.32+02	.29+02	.37+02	.18+02	.21+02	.16+02	.89+01	.73+01	.27+01	.14-01	-.13+01
.14+02	.36+02	.35+02	.24+02	.15+02	.80+01	.69+01	.38+01	.23+01	.15+01	.88+00	.77+00	.38+00	-.20+00	-.24+00
.86+03	.62+03	.34+03	.20+03	.13+03	.10+03	.49+02	.35+02	.40+02	.22+02	.14+02	.16+02	.13+02	.91+01	.70+01
-.80+03	-.31+03	-.15+02	.66+02	.72+02	.69+02	.32+02	.25+02	.27+02	.13+02	.90+01	.97+01	.67+01	.16+01	-.32+01
.43+02	.86+02	.55+02	.31+02	.16+02	.10+02	.44+01	.26+01	.24+01	.14+01	.78+00	.53+00	.30+00	.33-01	-.43+00
.23+04	.12+04	.51+03	.26+03	.14+03	.91+02	.48+02	.53+02	.36+02	.24+02	.22+02	.15+02	.12+02	.10+02	.90+01
-.19+04	-.23+03	.16+03	.16+03	.10+03	.70+02	.32+02	.41+02	.26+02	.16+02	.15+02	.93+01	.60+01	.36+01	-.34+01
.17+03	.19+03	.75+02	.29+02	.12+02	.59+01	.33+01	.22+01	.13+01	.11+01	.71+00	.48+00	.43+00	-.18+00	-.40+00
.99+04	.20+04	.62+03	.25+03	.15+03	.99+02	.75+02	.38+02	.39+02	.24+02	.15+02	.18+02	.12+02	.11+02	.10+02
-.64+04	.72+03	.44+03	.20+03	.12+03	.80+02	.57+02	.26+02	.29+02	.18+02	.98+01	.11+02	.71+01	.38+01	-.43+01
.10+04	.29+03	.60+02	.13+02	.60+01	.26+01	.23+01	.16+01	.94+00	.14+00	.86+00	.10+00	-.17+00	-.68-01	-.89-01
.13+09	.11+04	.41+03	.19+03	.13+03	.90+02	.63+02	.50+02	.33+02	.30+02	.23+02	.16+02	.12+02	.10+02	.11+02

.15+07	-.15+02	-.34+02	-.19+02	-.13+02	-.98+01	-.66+01	-.54+01	-.37+01	-.35+01	-.27+01	-.21+01	-.17+01	-.15+01	-.15+01
-.98+04	-.19+04	-.51+03	-.15+03	-.79+02	-.47+02	-.21+02	-.19+01	-.80+01	-.72+01	.27+01	-.33+01	-.32+01	-.14+01	.15+01
.63+04	-.79+03	-.47+03	-.20+03	-.10+03	-.65+02	-.37+02	-.11+02	-.15+02	-.10+02	-.16+01	-.60+01	-.43+01	-.29+01	-.27+01
.10+04	.28+03	.43+02	-.32+01	-.56+01	-.55+01	-.60+01	-.38+01	-.37+01	-.25+01	-.19+01	-.21+01	-.16+01	-.16+01	-.11+01
-.22+04	-.11+04	-.43+03	-.21+03	-.92+02	-.52+02	-.13+02	-.26+02	-.12+02	-.33+01	-.52+01	-.18+01	-.13+00	-.26+01	.93+00
.19+04	.18+03	-.21+03	-.18+03	-.11+03	-.69+02	-.29+02	-.37+02	-.21+02	-.10+02	-.11+02	-.64+01	-.45+01	-.49+01	-.39+01
.17+03	.18+03	.64+02	.22+02	.39+01	-.49+00	-.24+01	-.20+01	-.25+01	-.22+01	-.20+01	-.17+01	-.16+01	-.15+01	-.89+00
-.81+03	-.57+03	-.30+03	-.17+03	-.10+03	-.71+02	-.24+02	-.16+02	-.14+02	-.42+01	-.26+01	-.39+01	-.26+01	-.65+00	.27+01
.75+03	.26+03	-.15+02	-.85+02	-.86+02	-.76+02	-.35+02	-.25+02	-.26+02	-.12+02	-.82+01	-.95+01	-.77+01	-.46+01	-.39+01
.41+02	.82+02	.50+02	.25+02	.11+02	.53+01	.23+00	-.53+00	-.17+01	-.15+01	-.12+01	-.15+01	-.15+01	-.12+01	-.26+00
-.28+03	-.27+03	-.19+03	-.11+03	-.76+02	-.41+02	-.40+02	-.11+02	-.16+02	-.11+02	-.41+01	-.23+01	-.10+00	.52-01	.23+01
.26+03	.18+03	.45+02	-.27+02	-.45+02	-.37+02	-.44+02	-.23+02	-.23+02	-.18+02	-.10+02	-.97+01	-.57+01	-.64+01	-.13+01
.13+02	.33+02	.32+02	.19+02	.11+02	.49+01	.33+01	-.16+00	-.19-01	-.56+00	-.90+00	-.13+01	-.10+01	-.10+01	.24+00
-.10+03	-.15+03	-.12+03	-.65+02	-.61+02	-.35+02	-.28+02	-.19+02	-.95+01	-.81+01	-.52+01	-.52+01	-.20+01	.46+01	
.96+02	.11+03	.44+02	-.28+01	-.19+02	-.27+02	-.26+02	-.23+02	-.17+02	-.14+02	-.12+02	-.10+02	-.70+01	-.40+01	
.37+01	.17+02	.18+02	.11+02	.10+02	.53+01	.35+01	.16+01	.16+00	-.15+00	-.60+00	-.64+00	-.80+00	.15+00	
-.11+03	-.58+02	-.75+02	-.62+02	-.35+02	-.32+02	-.19+02	-.11+02	-.12+02	-.57+01	-.59+01	-.24+01	.17+01	.35+01	
.11+03	.46+02	.40+02	.14+02	-.87+01	-.15+02	-.16+02	-.16+02	-.16+02	-.12+02	-.98+01	-.91+01	-.57+01	-.10+01	
.42+01	.59+01	.10+02	.10+02	.65+01	.53+01	.29+01	.12+01	.96+00	.76-02	-.56-01	-.61+00	-.32+00	.38+00	
-.56+02	-.53+02	-.40+02	-.26+02	-.32+02	-.24+02	-.11+02	-.13+02	-.65+01	-.42+01	-.31+01	.18+01	.50+01		
.55+02	.46+02	.24+02	.55+01	.11+01	-.86+01	-.11+02	-.13+02	-.11+02	-.89+01	-.10+02	-.56+01	-.97+00		
.17+01	.45+01	.55+01	.47+01	.56+01	.43+01	.20+01	.18+01	.69+00	.23+00	-.18+00	-.26+00	.49+00		
-.30+02	-.32+02	-.30+02	-.21+02	-.18+02	-.10+02	-.10+02	-.64+01	-.75+01	-.52+01	.40+01	.72+01	.44+00		
.29+02	.28+02	.20+02	.80+01	.13+00	-.52+01	-.70+01	-.88+01	-.93+01	-.98+01	-.36+01	.12+01	-.10+00		
.83+00	.27+01	.38+01	.36+01	.34+01	.22+01	.19+01	.11+01	.10+01	.44+00	-.25+00	.41+00	.69-01		
-.11+02	-.20+02	-.15+02	-.13+02	-.11+02	-.89+01	-.95+01	-.32+01	.13+01	.54+01	.64+01	.89+00			
.11+02	.17+02	.10+02	.53+01	.56+00	-.24+01	-.45+01	-.70+01	-.41+01	-.14+00	.76+00	.23+00			
.40+00	.16+01	.21+01	.21+01	.22+01	.18+01	.17+01	.76+00	.30-01	-.28+00	.27+00	.11+00			
-.10+02	-.10+02	-.88+01	-.10+02	-.21+01	-.25+01	.39+01	.71+01	.80+01	.48+01	.56+00				
.10+02	.96+01	.59+01	.42+01	-.10+01	-.25+01	-.35+01	-.71+00	.11+01	.11+01	.10+00				
.25+00	.90+00	.12+01	.17+01	.66+00	.80+00	-.26+00	-.76+00	-.54+00	.16+00	.70-01				
.71+01	.97+01	.95+01	.88+01	.11+02	.86+01	.67+01	.56+01	.21+01						
-.70+01	-.91+01	-.83+01	-.65+01	-.62+01	-.31+01	-.97+00	.11+01	.29+00						
-.14+00	-.50+00	-.68+00	-.85+00	-.13+01	-.10+01	-.73+00	-.27+00	.12+00						
.75+01	.79+01	.75+01	.73+01	.52+01	.35+01	.17+01	.19+00							
-.75+01	-.74+01	-.63+01	-.49+01	-.20+01	-.25+00	.43+00	.41-01							
-.12+00	-.37+00	-.53+00	-.69+00	-.48+00	-.20+00	.13-01	.29-01							
.24+01	.28+01	.17+01	.10+01	.23+00										
-.24+01	-.25+01	-.11+01	-.20+00	.69-01										
-.28-01	-.11+00	-.97-01	-.52-01	.14-02										

TABLE 36

DISTRIBUTION FOR $\gamma = 1.5$, $\theta_0 = 30°$, AND $\sin \alpha_L = 0.2$

.80+01	.88+01	.14+01	.68+00										
-.20+01	-.23+01	-.10+01	-.25+00										
.20-01	.11+00	.93-01	.51-01										
.75+01	.79+01	.75+01	.73+01	.51+01	.32+01	.17+01	.19+00						
-.75+01	-.74+01	-.63+01	-.49+01	-.21+01	-.45+00	.43+00	.41-01						
.12+00	.37+00	.53+00	.69+00	.48+00	.22+00	-.13-01	-.29-01						
.14+02	.97+01	.12+02	.88+01	.11+02	.86+01	.67+01	.56+01	.21+01					
-.14+02	-.91+01	-.10+02	-.65+01	-.62+01	-.31+01	-.97+00	.11+01	.29+00					
.25+00	.50+00	.10+01	.85+00	.13+01	.10+01	.73+00	.27+00	-.12+00					
.18+02	.18+02	.16+02	.19+02	.11+02	.12+02	.11+02	.94+01	.80+01	.48+01	.56+00			
-.18+02	-.17+02	-.13+02	-.13+02	-.61+01	-.46+01	-.26+01	-.47-01	.11+01	.11+01	.10+00			
.31+00	.10+01	.15+01	.22+01	.14+01	.16+01	.15+01	.11+01	.54+00	-.16+00	-.70-01			
.21+02	.30+02	.28+02	.20+02	.22+02	.16+02	.15+02	.12+02	.10+02	.71+01	.64+01	.89+00		
-.21+02	-.27+02	-.22+02	-.12+02	-.10+02	-.43+01	-.24+00	-.52+00	.13+01	.12+01	.76+00	.23+00		
.49+00	.19+01	.27+01	.26+01	.31+01	.24+01	.23+01	.15+01	.11+01	.46+00	-.27+00	-.11+00		
.43+02	.51+02	.39+02	.33+02	.29+02	.23+02	.19+02	.16+02	.14+02	.12+02	.90+01	.72+01	.44+00	
-.42+02	-.46+02	-.29+02	-.18+02	-.99+01	-.51+01	.28-01	.13+01	.43+01	.45+01	.11+01	.12+01	-.10+00	
.95+00	.34+01	.43+01	.44+01	.44+01	.35+01	.29+01	.23+01	.18+01	.11+01	.52+00	-.41+00	-.69-01	
.76+02	.65+02	.51+02	.47+02	.42+02	.37+02	.26+02	.23+02	.17+02	.11+02	.12+02	.93+01	.62+01	
-.74+02	-.57+02	-.34+02	-.24+02	-.89+01	-.70+00	.14+01	.66+01	.51+01	.43+01	.43+01	.17+01	.11+00	
.20+01	.49+01	.62+01	.64+01	.66+01	.58+01	.38+01	.32+01	.21+01	.12+01	.96+00	.27+00	-.55+00	
.13+03	.75+02	.10+03	.74+02	.58+02	.47+02	.33+02	.27+02	.22+02	.17+02	.11+02	.11+02	.94+01	.35+01
-.12+03	-.62+02	-.63+02	-.25+02	-.85+01	.62+01	.87+01	.86+01	.11+02	.76+01	.71+01	.43+01	.13+01	-.10+01
.44+01	.66+01	.12+02	.11+02	.90+01	.72+01	.48+01	.35+01	.25+01	.17+01	.76+00	.58+00	-.15+00	-.38+00
.14+02	.87+02	.13+03	.88+02	.79+02	.57+02	.44+02	.33+02	.25+02	.19+02	.16+02	.12+02	.88+01	.77+01
-.14+02	-.71+02	-.58+02	-.14+02	.78+01	.15+02	.19+02	.17+02	.12+02	.11+02	.89+01	.73+01	.29+01	-.20+01
.26+00	.81+01	.19+02	.13+02	.12+02	.84+01	.59+01	.38+01	.26+01	.16+01	.11+01	.45+00	.92-01	-.52+00
.16+02	.22+02	.13+02	.12+03	.10+03	.61+02	.63+02	.36+02	.30+02	.24+02	.14+02	.15+02	.93+01	.10+02	.23+01
-.16+02	-.20+02	-.11+02	.45+00	.32+02	.29+02	.37+02	.18+02	.21+02	.16+02	.89+01	.73+01	.27+01	.14-01	-.13+01
.32+00	.14+01	.13+01	.20+02	.15+02	.80+01	.69+01	.38+01	.23+01	.15+01	.88+00	.77+00	.38+00	-.20+00	-.24+00
.25+02	.25+02	.21+02	.48+02	.13+03	.10+03	.49+02	.35+02	.40+02	.22+02	.14+02	.16+02	.13+02	.91+01	.70+01
-.24+02	-.22+02	-.15+02	.36+01	.72+02	.69+02	.32+02	.25+02	.27+02	.13+02	.90+01	.97+01	.67+01	.16+01	-.32+01
.70+00	.20+01	.25+01	.72+01	.16+02	.10+02	.44+01	.26+01	.24+01	.14+01	.78+00	.53+00	.30+00	.33-01	-.43+00
.31+02	.35+02	.38+02	.24+02	.14+03	.91+02	.48+02	.53+02	.36+02	.24+02	.22+02	.15+02	.12+02	.10+02	.90+01
-.30+02	-.28+02	-.22+02	-.77+01	.10+03	.70+02	.32+02	.41+02	.26+02	.16+02	.15+02	.93+01	.60+01	.36+01	-.34+01
.10+01	.35+01	.52+01	.38+01	.12+02	.59+01	.33+01	.22+01	.13+01	.11+01	.71+00	.48+00	.43+00	-.18+00	-.40+00
		.32+02	.50+02	.93+02	.99+02	.75+02	.38+02	.39+02	.24+02	.15+02	.18+02	.12+02	.11+02	.10+02
		-.83+01	.18+00	.63+02	.80+02	.57+02	.26+02	.29+02	.18+02	.98+01	.11+02	.71+01	.38+01	-.43+01
		.52+01	.83+01	.51+01	.26+01	.23+01	.16+01	.94+00	.14+00	.86+00	.10+00	-.17+00	-.68-01	-.89-01

				-.50+01	-.13+02	-.98+01	-.66+01	-.54+01	-.37+01	-.35+01	-.27+01	-.21+01	-.17+01	-.15+01	-.15+01
			.32+02	.50+02	-.20+02	-.47+02	-.21+02	-.19+01	-.80+01	-.72+01	.27+01	-.33+01	-.32+01	-.14+01	.15+01
			-.83+01	.18+00	-.49+02	-.65+02	-.37+02	-.11+02	-.15+02	-.10+02	-.16+01	-.60+01	-.43+01	-.29+01	-.27+01
			-.52+01	-.83+01	-.65+01	-.55+01	-.60+01	-.38+01	-.37+01	-.25+01	-.19+01	-.21+01	-.16+01	-.16+01	-.11+01
.31+02	.35+02	.38+02	.24+02	-.92+02	-.52+02	-.13+02	-.26+02	-.12+02	-.33+01	-.52+01	-.18+01	-.13+00	-.26+01	.93+00	
-.30+02	-.28+02	-.22+02	-.77+01	-.11+03	-.69+02	-.29+02	-.37+02	-.21+02	-.10+02	-.11+02	-.64+01	-.45+01	-.49+01	-.39+01	
-.10+01	-.35+01	-.52+01	-.38+01	.39+01	-.49+00	-.24+01	-.20+01	-.25+01	-.22+01	-.20+01	-.17+01	-.16+01	-.15+01	-.89+00	
.25+02	.25+02	.21+02	-.11+02	-.10+03	-.71+02	-.24+02	-.16+02	-.14+02	-.42+01	-.26+01	-.39+01	-.26+01	-.65+00	.27+01	
-.24+02	-.22+02	-.15+02	-.22+02	-.86+02	-.76+02	-.35+02	-.25+02	-.26+02	-.12+02	-.82+01	-.95+01	-.77+01	-.46+01	-.39+01	
-.70+00	-.20+01	-.25+01	.18+01	.11+02	.53+01	.23+00	-.53+00	-.17+01	-.15+01	-.12+01	-.15+01	-.15+01	-.12+01	-.26+00	
.16+02	.22+02	.13+02	-.91+02	-.76+02	-.41+02	-.40+02	-.11+02	-.16+02	-.11+02	-.41+01	-.23+01	-.10+00	.52-01	.23+01	
-.16+02	-.20+02	-.11+02	-.25+02	-.45+02	-.37+02	-.44+02	-.23+02	-.23+02	-.18+02	-.10+02	-.97+01	-.57+01	-.64+01	-.13+01	
-.32+00	-.14+01	-.13+01	.15+02	.11+02	.49+01	.33+01	-.16+00	-.19-01	-.56+00	-.90+00	-.13+01	-.10+01	-.10+01	.24+00	
.14+02	-.58+02	-.10+03	-.65+02	-.61+02	-.35+02	-.28+02	-.19+02	-.95+01	-.81+01	-.52+01	-.52+01	-.20+01	.46+01		
-.14+02	.43+02	.37+02	-.28+01	-.19+02	-.27+02	-.26+02	-.23+02	-.17+02	-.14+02	-.12+02	-.10+02	-.70+01	-.40+01		
-.26+00	.66+01	.17+02	.11+02	.10+02	.53+01	.35+01	.16+01	.16+00	-.15+00	-.60+00	-.64+00	-.80+00	.15+00		
-.11+03	-.58+02	-.75+02	-.62+02	-.35+02	-.32+02	-.19+02	-.11+02	-.12+02	-.57+01	-.59+01	-.24+01	.17+01	.35+01		
.11+03	.46+02	.40+02	.14+02	-.87+01	-.15+02	-.16+02	-.16+02	-.16+02	-.12+02	-.98+01	-.91+01	-.57+01	-.10+01		
.42+01	.59+01	.10+02	.10+02	.65+01	.53+01	.29+01	.12+01	.96+00	.76-02	-.56-01	-.61+00	-.32+00	.38+00		
-.56+02	-.53+02	-.40+02	-.26+02	-.32+02	-.24+02	-.11+02	-.13+02	-.65+01	-.42+01	-.31+01	.18+01	.50+01			
.55+02	.46+02	.24+02	.55+01	.11+01	-.86+01	-.11+02	-.13+02	-.11+02	-.89+01	-.10+02	-.56+01	-.97+00			
.17+01	.45+01	.55+01	.47+01	.56+01	.43+01	.20+01	.18+01	.69+00	.23+00	-.18+00	-.28+00	.49+00			
-.30+02	-.32+02	-.30+02	-.21+02	-.18+02	-.10+02	-.10+02	-.64+01	-.75+01	-.52+01	.40+01	.72+01	.44+00			
.29+02	.28+02	.20+02	.80+01	.13+00	-.52+01	-.70+01	-.88+01	-.93+01	-.98+01	-.36+01	.12+01	-.10+00			
.83+00	.27+01	.38+01	.36+01	.34+01	.22+01	.19+01	.11+01	.10+01	.44+00	-.25+00	.41+00	.69-01			
-.11+02	-.20+02	-.15+02	-.13+02	-.11+02	-.89+01	-.95+01	-.32+01	.13+01	.54+01	.64+01	.89+00				
.11+02	.17+02	.10+02	.53+01	.56+00	-.24+01	-.45+01	-.70+01	-.41+01	-.14+00	.76+00	.23+00				
.40+00	.16+01	.21+01	.21+01	.22+01	.18+01	.17+01	.76+00	.30-01	-.28+00	.27+00	.11+00				
-.10+02	-.10+02	-.88+01	-.10+02	-.21+01	-.25+01	.39+01	.71+01	.80+01	.48+01	.56+00					
.10+02	.98+01	.59+01	.42+01	-.10+01	-.25+01	-.35+01	-.71+00	.11+01	.11+01	.10+00					
.25+00	.90+00	.12+01	.17+01	.66+00	.80+00	-.26+00	-.76+00	-.54+00	.16+00	.70-01					
.71+01	.97+01	.95+01	.88+01	.11+02	.86+01	.67+01	.56+01	.21+01							
-.70+01	-.91+01	-.83+01	-.65+01	-.62+01	-.31+01	-.97+00	.11+01	.29+00							
-.14+00	-.50+00	-.68+00	-.85+00	-.13+01	-.10+01	-.73+00	-.27+00	.12+00							
.75+01	.79+01	.75+01	.73+01	.51+01	.32+01	.17+01	.19+00								
-.75+01	-.74+01	-.63+01	-.49+01	-.21+01	-.45+00	.43+00	.41-01								
-.12+00	-.37+00	-.53+00	-.69+00	-.48+00	-.28+00	.13-01	.29-01								
.20+01	.26+01	.14+01	.62+00												
-.20+01	-.23+01	-.10+01	-.25+00												
-.20-01	-.11+00	-.93-01	-.51-01												

TABLE 37

DISTRIBUTION FOR $\gamma = 1.5$, $\theta_0 = 30°$, AND $\sin \alpha_L = 0.4$

.50+00	.50+00												
-.50+00	-.50+00												
-.67-03	-.20-02												
.53+01	.44+01	.51+01	.42+01	.17+01	.51+00								
-.53+01	-.43+01	-.46+01	-.33+01	-.13+01	-.50+00								
.77-01	.14+00	.27+00	.30+00	.92-01	-.72-02								
.14+02	.97+01	.12+02	.88+01	.11+02	.81+01	.51+01	.24+01	.25+00					
-.14+02	-.91+01	-.10+02	-.65+01	-.62+01	-.31+01	-.13+01	-.81-01	-.25+00					
.25+00	.50+00	.10+01	.85+00	.13+01	.99+00	.53+00	.22+00	-.53-02					
.12+02	.18+02	.16+02	.19+02	.11+02	.12+02	.11+02	.94+01	.64+01	.22+01				
-.12+02	-.17+02	-.13+02	-.13+02	-.61+01	-.46+01	-.26+01	-.47-01	.62+00	.11+00				
.24+00	.10+01	.15+01	.22+01	.14+01	.16+01	.15+01	.11+01	.55+00	.45-01				
.52+01	.13+02	.18+02	.20+02	.22+02	.16+02	.15+02	.12+02	.10+02	.71+01	.46+01	.27+00		
-.52+01	-.12+02	-.15+02	-.12+02	-.10+02	-.43+01	-.24+00	-.52+00	.13+01	.12+01	.21+00	.22+00		
.44-01	.74+00	.15+01	.26+01	.31+01	.24+01	.23+01	.15+01	.11+01	.46+00	-.29-01	-.26-01		
.64+01	.96+01	.47+01	.57+01	.15+02	.23+02	.19+02	.16+02	.14+02	.12+02	.90+01	.62+01	.22+00	
-.64+01	-.94+01	-.44+01	-.52+01	-.68+01	-.51+01	.28-01	.13+01	.43+01	.45+01	.11+01	.13+01	-.10+00	
.62-01	.32+00	.25+00	.41+00	.20+01	.35+01	.29+01	.23+01	.18+01	.11+01	.52+00	-.24+00	-.32-01	
.97+01	.55+01	.53+01	.10+02	.48+01	.32+02	.26+02	.23+02	.17+02	.11+02	.12+02	.93+01	.62+01	
-.97+01	-.54+01	-.50+01	-.92+01	-.38+01	-.12+01	.14+01	.66+01	.51+01	.43+01	.43+01	.17+01	.11+00	
.13+00	.20+00	.33+00	.87+00	.49+00	.50+01	.38+01	.32+01	.21+01	.12+01	.96+00	.27+00	-.55+00	
.65+01	.84+01	.12+02	.62+01	.11+02	.74+01	.33+02	.27+02	.22+02	.17+02	.11+02	.11+02	.94+01	.35+01
-.65+01	-.81+01	-.11+02	-.52+01	-.86+01	-.48+01	.87+01	.86+01	.11+02	.76+01	.71+01	.43+01	.13+01	-.10+01
.11+00	.37+00	.88+00	.57+00	.12+01	.94+00	.48+01	.35+01	.25+01	.17+01	.76+00	.58+00	-.15+00	-.38+00
.14+02	.14+02	.12+02	.11+02	.90+01	.10+02	.16+02	.33+02	.25+02	.19+02	.16+02	.12+02	.88+01	.77+01
-.14+02	-.13+02	-.10+02	-.87+01	-.59+01	-.57+01	.18+01	.17+02	.12+02	.11+02	.89+01	.73+01	.29+01	-.20+01
.26+00	.78+00	.10+01	.12+01	.11+01	.15+01	.22+01	.38+01	.26+01	.16+01	.11+01	.45+00	.92-01	-.52+00
.16+02	.22+02	.13+02	.18+02	.12+02	.10+02	.11+02	.36+02	.30+02	.24+02	.14+02	.15+02	.93+01	.10+02	.23+01
-.16+02	-.20+02	-.11+02	-.12+02	-.66+01	-.39+01	-.34+01	.18+02	.21+02	.16+02	.89+01	.73+01	.27+01	.14-01	-.13+01
.32+00	.14+01	.13+01	.23+01	.18+01	.15+01	.18+01	.38+01	.23+01	.15+01	.88+00	.77+00	.38+00	-.20+00	-.24+00
		.88+01	.12+02	.17+02	.16+02	.12+02	.26+02	.40+02	.22+02	.14+02	.16+02	.13+02	.91+01	.70+01
		-.64+01	-.63+01	-.68+01	-.36+01	-.15+01	.15+02	.27+02	.13+02	.90+01	.97+01	.67+01	.16+01	-.32+01
		.10+01	.17+01	.26+01	.27+01	.21+01	.22+01	.24+01	.14+01	.78+00	.53+00	.30+00	.33-01	-.43+00
				.63+01	.15+02	.17+02	.34+02	.36+02	.24+02	.22+02	.15+02	.12+02	.10+02	.90+01
				-.11+01	.24+00	.16+01	.23+02	.26+02	.16+02	.15+02	.93+01	.60+01	.36+01	-.34+01
				.10+01	.26+01	.29+01	.20+01	.13+01	.11+01	.71+00	.48+00	.43+00	-.18+00	-.40+00
						.23+02	.38+02	.39+02	.24+02	.15+02	.18+02	.12+02	.11+02	.10+02
						.81+01	.26+02	.29+02	.18+02	.98+01	.11+02	.71+01	.38+01	-.43+01
						.36+01	.16+01	.94+00	.14+00	.86+00	.10+00	-.17+00	-.68-01	-.89-01
							.18+02	.50+02	.33+02	.30+02	.23+02	.16+02	.12+02	.10+02	.11+02

							-.23+01	-.54+01	-.37+01	-.35+01	-.27+01	-.21+01	-.17+01	-.15+01	-.15+01
							.23+02	-.19+01	-.80+01	-.72+01	.27+01	-.33+01	-.32+01	-.14+01	.15+01
							.81+01	-.11+02	-.15+02	-.10+02	-.16+01	-.60+01	-.43+01	-.29+01	-.27+01
							-.36+01	-.38+01	-.37+01	-.25+01	-.19+01	-.21+01	-.16+01	-.16+01	-.11+01
					.63+01	.15+02	.17+02	-.74+01	-.12+02	-.33+01	-.52+01	-.18+01	-.13+00	-.26+01	.93+00
					-.11+01	.24+00	.16+01	-.18+02	-.21+02	-.10+02	-.11+02	-.64+01	-.45+01	-.49+01	-.39+01
					-.10+01	-.26+01	-.29+01	-.22+01	-.25+01	-.22+01	-.20+01	-.17+01	-.16+01	-.15+01	-.89+00
		.88+01	.12+02	.17+02	.16+02	.12+02	-.71+01	-.14+02	-.42+01	-.26+01	-.39+01	-.26+01	-.65+00	.27+01	
		-.64+01	-.63+01	-.68+01	-.36+01	-.15+01	-.16+02	-.26+02	-.12+02	-.82+01	-.95+01	-.77+01	-.46+01	-.39+01	
		-.10+01	-.17+01	-.26+01	-.27+01	-.21+01	-.92+00	-.17+01	-.15+01	-.12+01	-.15+01	-.15+01	-.12+01	-.26+00	
.16+02	.22+02	.13+02	.18+02	.12+02	.10+02	.11+02	-.11+02	-.16+02	-.11+02	-.41+01	-.23+01	-.10+00	.52-01	.23+01	
-.16+02	-.20+02	-.11+02	-.12+02	-.66+01	-.39+01	-.34+01	-.23+02	-.23+02	-.18+02	-.10+02	-.97+01	-.57+01	-.64+01	-.13+01	
-.32+00	-.14+01	-.13+01	-.23+01	-.18+01	-.15+01	-.18+01	-.16+00	-.19-01	-.56+00	-.90+00	-.13+01	-.10+01	-.10+01	.24+00	
.14+02	.14+02	.12+02	.11+02	.90+01	.10+02	-.15+00	-.19+02	-.95+01	-.81+01	-.52+01	-.52+01	-.20+01	.46+01		
-.14+02	-.13+02	-.10+02	-.87+01	-.59+01	-.57+01	-.88+01	-.23+02	-.17+02	-.14+02	-.12+02	-.10+02	-.70+01	-.40+01		
-.26+00	-.78+00	-.10+01	-.12+01	-.11+01	-.15+01	-.16+00	.16+01	.16+00	-.15+00	-.60+00	-.64+00	-.80+00	.15+00		
.65+01	.84+01	.12+02	.62+01	.11+02	.74+01	-.19+02	-.11+02	-.12+02	-.57+01	-.59+01	-.24+01	.17+01	.35+01		
-.65+01	-.81+01	-.11+02	-.52+01	-.86+01	-.48+01	-.16+02	-.16+02	-.16+02	-.12+02	-.98+01	-.91+01	-.57+01	-.10+01		
-.11+00	-.37+00	-.88+00	-.57+00	-.12+01	-.94+00	.29+01	.12+01	.96+00	.76-02	-.56-01	-.61+00	-.32+00	.38+00		
.97+01	.55+01	.53+01	.10+02	.48+01	-.20+02	-.11+02	-.13+02	-.65+01	-.42+01	-.31+01	.18+01	.50+01			
-.97+01	-.54+01	-.50+01	-.92+01	-.38+01	-.81+01	-.11+02	-.13+02	-.11+02	-.89+01	-.10+02	-.56+01	-.97+00			
-.13+00	-.20+00	-.33+00	-.87+00	-.49+00	.36+01	.20+01	.18+01	.69+00	.23+00	-.18+00	-.26+00	.49+00			
.64+01	.96+01	.47+01	.57+01	-.35+01	-.10+02	-.10+02	-.64+01	-.75+01	-.52+01	.40+01	.62+01	.22+00			
-.64+01	-.94+01	-.44+01	-.52+01	-.30+01	-.52+01	-.70+01	-.88+01	-.93+01	-.98+01	-.36+01	.13+01	-.10+00			
-.62-01	-.32+00	-.25+00	-.41+00	.10+01	.22+01	.19+01	.11+01	.10+01	.44+00	-.25+00	.24+00	.32-01			
.52+01	-.33+01	-.50+01	-.13+02	-.11+02	-.89+01	-.95+01	-.32+01	.13+01	.54+01	.46+01	.27+00				
-.52+01	.25+01	.23+01	.53+01	.56+00	-.24+01	-.45+01	-.70+01	-.41+01	-.14+00	.21+00	.22+00				
-.44-01	.47+00	.97+00	.21+01	.22+01	.18+01	.17+01	.76+00	.30-01	-.28+00	.29-01	.26-01				
-.47+01	-.10+02	-.88+01	-.10+02	-.21+01	-.25+01	.39+01	.71+01	.64+01	.22+01						
.45+01	.96+01	.59+01	.42+01	-.10+01	-.25+01	-.35+01	-.71+00	.62+00	.11+00						
.18+00	.90+00	.12+01	.17+01	.66+00	.80+00	-.26+00	-.76+00	-.55+00	-.45-01						
.71+01	.97+01	.95+01	.88+01	.11+02	.81+01	.51+01	.24+01	.25+00							
-.70+01	-.91+01	-.83+01	-.65+01	-.62+01	-.31+01	-.13+01	-.81-01	-.25+00							
-.14+00	-.50+00	-.68+00	-.85+00	-.13+01	-.99+00	-.53+00	-.22+00	.53-02							
.53+01	.44+01	.51+01	.42+01	.17+01	.51+00										
-.53+01	-.43+01	-.46+01	-.33+01	-.13+01	-.50+00										
-.77-01	-.14+00	-.27+00	-.30+00	-.92-01	.72-02										
.50+00	.50+00														
-.50+00	-.50+00														
.67-03	.20-02														

TABLE 38

DISTRIBUTION FOR $\gamma = 1.5$, $\theta_0 = 30°$, AND $\sin \alpha_L = 0.6$

.20+01	.20+01	.14+01	.89+00									
-.20+01	-.20+01	-.14+01	-.88+00									
.73-02	.22-01	.23-01	.17-01									
.29+01	.29+01	.35+01	.32+01	.33+01	.19+01	.87+00						
-.29+01	-.29+01	-.34+01	-.31+01	-.32+01	-.19+01	-.85+00						
.17-01	.51-01	.98-01	.11+00	.12+00	.77-01	.32-01						
.40+01	.40+01	.40+01	.48+01	.34+01	.38+01	.39+01	.25+01	.13+01				
-.40+01	-.40+01	-.39+01	-.45+01	-.32+01	-.35+01	-.36+01	-.24+01	-.13+01				
.29-01	.89-01	.14+00	.24+00	.20+00	.24+00	.26+00	.15+00	.75-01				
.52+01	.52+01	.65+01	.36+01	.56+01	.39+01	.28+01	.45+01	.35+01	.18+01			
-.52+01	-.51+01	-.63+01	-.34+01	-.50+01	-.34+01	-.23+01	-.37+01	-.30+01	-.17+01			
.44-01	.13+00	.29+00	.22+00	.41+00	.33+00	.26+00	.40+00	.28+00	.12+00			
.64+01	.96+01	.47+01	.57+01	.57+01	.64+01	.46+01	.52+01	.34+01	.34+01	.27+01		
-.64+01	-.94+01	-.44+01	-.52+01	-.49+01	-.52+01	-.35+01	-.37+01	-.24+01	-.26+01	-.22+01		
.62-01	.32+00	.25+00	.41+00	.50+00	.64+00	.50+00	.59+00	.38+00	.37+00	.26+00		
.97+01	.55+01	.53+01	.10+02	.48+01	.64+01	.72+01	.51+01	.55+01	.37+01	.44+01	.26+01	
-.97+01	-.54+01	-.50+01	-.92+01	-.38+01	-.46+01	-.48+01	-.31+01	-.33+01	-.22+01	-.28+01	-.20+01	
.13+00	.20+00	.33+00	.87+00	.49+00	.72+00	.89+00	.68+00	.73+00	.49+00	.57+00	.28+00	
.65+01	.84+01	.12+02	.62+01	.11+02	.74+01	.70+01	.78+01	.54+01	.57+01	.28+01	.43+01	.21+01
-.65+01	-.81+01	-.11+02	-.52+01	-.86+01	-.48+01	-.40+01	-.39+01	-.24+01	-.25+01	-.13+01	-.23+01	-.14+01
.11+00	.37+00	.88+00	.57+00	.12+01	.94+00	.96+00	.11+01	.80+00	.85+00	.41+00	.60+00	.24+00
			.44+01	.66+01	.10+02	.80+01	.72+01	.79+01	.55+01	.56+01	.12+02	.62+01	.45+00
			-.34+01	-.44+01	-.57+01	-.35+01	-.27+01	-.25+01	-.15+01	-.17+01	.73+01	.63+00	-.33+00
			.46+00	.82+00	.15+01	.11+01	.11+01	.12+01	.88+00	.88+00	.45+00	.30+00	.51-01
						.90+01	.12+02	.71+01	.63+01	.88+01	.15+02	.93+01	.46+01
						-.27+01	-.26+01	-.11+01	-.68+00	.28+01	.73+01	.27+01	-.65+00
						.14+01	.20+01	.11+01	.10+01	.87+00	.77+00	.38+00	.27+00
							.45+01	.12+02	.89+01	.82+01	.16+02	.13+02	.79+01
							-.24+00	.39+00	.48+00	.24+01	.97+01	.67+01	.10+01
							.75+00	.21+01	.14+01	.93+00	.53+00	.30+00	.19+00
								.68+01	.10+02	.15+02	.15+02	.12+02	.10+02	.21+01
								.15+01	.27+01	.81+01	.93+01	.60+01	.36+01	-.10+00
								.11+01	.16+01	.10+01	.48+00	.43+00	-.18+00	-.30-01
									.88+01	.15+02	.18+02	.12+02	.11+02	.37+01
									.38+01	.98+01	.11+02	.71+01	.38+01	-.49+00
									.13+01	.86+00	.10+00	-.17+00	-.68-01	.21+00

								-.61+00	-.27+01	-.21+01	-.17+01	-.15+01	-.86+00
								.88+01	.27+01	-.33+01	-.32+01	-.14+01	.13+01
								.38+01	-.16+01	-.60+01	-.43+01	-.29+01	-.10+01
								-.13+01	-.19+01	-.21+01	-.16+01	-.16+01	-.60+00
							.68+01	.10+02	.21+01	-.18+01	-.13+00	-.26+01	-.12+00
							.15+01	.27+01	-.40+01	-.64+01	-.45+01	-.49+01	-.78+00
							-.11+01	-.16+01	-.17+01	-.17+01	-.16+01	-.15+01	-.32+00
						.45+01	.12+02	.89+01	.39+01	-.39+01	-.26+01	.47+00	
						-.24+00	.39+00	.48+00	-.16+01	-.95+01	-.77+01	-.40+01	
						-.75+00	-.21+01	-.14+01	-.10+01	-.15+01	-.15+01	-.10+01	
					.90+01	.12+02	.71+01	.63+01	.19+01	-.23+01	-.10+00	.17+01	
					-.27+01	-.26+01	-.11+01	-.68+00	-.41+01	-.97+01	-.57+01	-.29+01	
					-.14+01	-.20+01	-.11+01	-.10+01	-.92+00	-.13+01	-.10+01	-.58+00	
		.44+01	.66+01	.10+02	.80+01	.72+01	.79+01	.55+01	.56+01	-.52+01	.58+00	.45+00	
		-.34+01	-.44+01	-.57+01	-.35+01	-.27+01	-.25+01	-.15+01	-.17+01	-.10+02	-.47+01	-.33+00	
		-.46+00	-.82+00	-.15+01	-.11+01	-.11+01	-.12+01	-.88+00	-.88+00	-.64+00	-.58+00	-.51-01	
.65+01	.84+01	.12+02	.62+01	.11+02	.74+01	.70+01	.78+01	.54+01	.57+01	.28+01	.43+01	.21+01	
-.65+01	-.81+01	-.11+02	-.52+01	-.86+01	-.48+01	-.40+01	-.39+01	-.24+01	-.25+01	-.13+01	-.23+01	-.14+01	
-.11+00	-.37+00	-.88+00	-.57+00	-.12+01	-.94+00	-.96+00	-.11+01	-.80+00	-.85+00	-.41+00	-.60+00	-.24+00	
.97+01	.55+01	.53+01	.10+02	.48+01	.64+01	.72+01	.51+01	.55+01	.37+01	.44+01	.26+01		
-.97+01	-.54+01	-.50+01	-.92+01	-.38+01	-.46+01	-.48+01	-.31+01	-.33+01	-.22+01	-.28+01	-.20+01		
-.13+00	-.20+00	-.33+00	-.87+00	-.49+00	-.72+00	-.89+00	-.68+00	-.73+00	-.49+00	-.57+00	-.28+00		
.64+01	.96+01	.47+01	.57+01	.57+01	.64+01	.46+01	.52+01	.34+01	.34+01	.27+01			
-.64+01	-.94+01	-.44+01	-.52+01	-.49+01	-.52+01	-.35+01	-.37+01	-.24+01	-.26+01	-.22+01			
-.62-01	-.32+00	-.25+00	-.41+00	-.50+00	-.64+00	-.50+00	-.59+00	-.38+00	-.37+00	-.26+00			
.52+01	.52+01	.65+01	.36+01	.56+01	.39+01	.28+01	.45+01	.35+01	.18+01				
-.52+01	-.51+01	-.63+01	-.34+01	-.50+01	-.34+01	-.23+01	-.37+01	-.30+01	-.17+01				
-.44-01	-.13+00	-.29+00	-.22+00	-.41+00	-.33+00	-.26+00	-.40+00	-.28+00	-.12+00				
.40+01	.40+01	.40+01	.48+01	.34+01	.38+01	.39+01	.25+01	.13+01					
-.40+01	-.40+01	-.39+01	-.45+01	-.32+01	-.35+01	-.36+01	-.24+01	-.13+01					
-.29-01	-.89-01	-.14+00	-.24+00	-.20+00	-.24+00	-.26+00	-.15+00	-.75-01					
.29+01	.29+01	.35+01	.32+01	.33+01	.19+01	.87+00							
-.29+01	-.29+01	-.34+01	-.31+01	-.32+01	-.19+01	-.85+00							
-.17-01	-.51-01	-.98-01	-.11+00	-.12+00	-.77-01	-.32-01							
.20+01	.20+01	.14+01	.89+00										
-.20+01	-.20+01	-.14+01	-.88+00										
-.73-02	-.22-01	-.23-01	-.17-01										

TABLE 39

DISTRIBUTION FOR $\gamma = 1.5$, $\theta_0 = 30°$, AND $\sin \alpha_L = 0.8$

.40+01	.40+01	.40+01	.40+01	.18+01	.92+00			
-.40+01	-.40+01	-.39+01	-.38+01	-.17+01	-.83+00			
.29-01	.89-01	.14+00	.20+00	.11+00	.65-01			
.52+01	.52+01	.65+01	.36+01	.56+01	.39+01	.19+01		
-.52+01	-.51+01	-.63+01	-.34+01	-.50+01	-.34+01	-.16+01		
.44-01	.13+00	.29+00	.22+00	.41+00	.33+00	.18+00		
			.27+01	.27+01	.64+01	.46+01	.43+01	
			-.24+01	-.23+01	-.52+01	-.35+01	-.30+01	
			.18+00	.23+00	.64+00	.50+00	.50+00	
						.13+01	.11+01	.33+01
						-.97+00	-.76+00	-.20+01
						.15+00	.14+00	.44+00

						.13+01	.11+01	.33+01
						-.97+00	-.76+00	-.20+01
						-.15+00	-.14+00	-.44+00
			.27+01	.27+01	.64+01	.46+01	.43+01	
			-.24+01	-.23+01	-.52+01	-.35+01	-.30+01	
			-.18+00	-.23+00	-.64+00	-.50+00	-.50+00	
.52+01	.52+01	.65+01	.36+01	.56+01	.39+01	.19+01		
-.52+01	-.51+01	-.63+01	-.34+01	-.50+01	-.34+01	-.16+01		
-.44-01	-.13+00	-.29+00	-.22+00	-.41+00	-.33+00	-.18+00		
.40+01	.40+01	.40+01	.40+01	.18+01	.92+00			
-.40+01	-.40+01	-.39+01	-.38+01	-.17+01	-.83+00			
-.29-01	-.89-01	-.14+00	-.20+00	-.11+00	-.65-01			

TABLE 40

DISTRIBUTION FOR $\gamma = 1.5$, $\theta_0, = 60°$, AND $\sin \alpha_L = 0$

.51+00	.73+00	.46+00											
-.40+00	.69-01	.36+00											
.11-01	.83-02	-.83-02											
.10+02	.13+02	.10+02	.12+02	.11+02	.10+02	.71+01	.50+00						
-.10+02	-.98+01	-.34+01	.11+01	.47+01	.62+01	.54+01	.45+00						
.32+00	.11+01	.96+00	.11+01	.48+00	.61-02	-.31+00	-.33-01						
.28+02	.23+02	.28+02	.24+02	.24+02	.23+02	.22+02	.24+02	.18+02	.32+01				
-.27+02	-.19+02	-.13+02	-.36+01	.33+01	.87+01	.13+02	.16+02	.12+02	.15+01				
.90+00	.20+01	.36+01	.34+01	.32+01	.23+01	.12+01	-.58+00	-.13+01	-.41+00				
.43+02	.59+02	.44+02	.51+02	.45+02	.38+02	.36+02	.29+02	.27+02	.28+02	.17+02	.15+01		
-.42+02	-.45+02	-.22+02	-.83+01	.81+01	.15+02	.21+02	.18+02	.19+02	.17+02	.78+01	.57+00		
.15+01	.60+01	.61+01	.77+01	.68+01	.48+01	.35+01	.15+01	.11+00	-.20+01	-.22+01	-.23+00		
.91+02	.97+02	.10+03	.81+02	.85+02	.73+02	.54+02	.47+02	.31+02	.27+02	.26+02	.27+02	.33+01	
-.87+02	-.71+02	-.41+02	-.45+01	.22+02	.38+02	.36+02	.37+02	.22+02	.19+02	.14+02	.92+01	.46+00	
.37+01	.10+02	.15+02	.13+02	.12+02	.94+01	.52+01	.29+01	.71+00	-.45+00	-.18+01	-.37+01	-.52+00	
.19+03	.21+03	.20+03	.17+03	.12+03	.11+03	.81+02	.46+02	.51+02	.32+02	.23+02	.24+02	.26+02	.24+01
-.18+03	-.14+03	-.50+02	.28+02	.55+02	.74+02	.64+02	.37+02	.42+02	.24+02	.11+02	.92+01	.81+00	-.54+00
.89+01	.26+02	.32+02	.28+02	.17+02	.12+02	.62+01	.19+01	.51+00	-.79+00	-.13+01	-.23+01	-.38+01	-.38+00
.40+03	.48+03	.43+03	.33+03	.24+03	.13+03	.73+02	.54+02	.48+02	.31+02	.26+02	.24+02	.23+02	.23+02
-.37+03	-.27+03	-.33+02	.11+03	.15+03	.11+03	.66+02	.41+02	.40+02	.21+02	.13+02	.86+01	.19+01	-.60+01
.21+02	.65+02	.70+02	.50+02	.28+02	.10+02	.20+01	.94+00	-.11+01	-.15+01	-.14+01	-.19+01	-.26+01	-.33+01
.16+04	.13+04	.98+03	.44+03	.19+03	.11+03	.65+02	.50+02	.35+02	.27+02	.22+02	.18+02	.20+02	.25+02	.10+02
-.14+04	-.43+03	.25+03	.30+03	.17+03	.10+03	.54+02	.38+02	.20+02	.15+02	.78+01	.32+01	.19+01	-.70+01	-.60+01
.10+03	.19+03	.15+03	.49+02	.11+02	.12+01	-.15+01	-.20+01	-.19+01	-.18+01	-.16+01	-.12+01	-.20+01	-.31+01	-.13+01
.50+04	.41+04	.70+03	.12+03	.77+02	.47+02	.37+02	.31+02	.24+02	.21+02	.21+02	.21+02	.14+02	.20+02	.20+02
-.37+04	.93+02	.42+03	.10+03	.55+02	.29+02	.17+02	.15+02	.91+01	.52+01	.40+01	.26+01	-.27+01	-.78+01	-.13+02
.48+03	.65+03	.77+02	-.11+01	-.41+01	-.33+01	-.26+01	-.25+01	-.18+01	-.13+01	-.16+01	-.17+01	-.92+00	-.16+01	-.20+01
.33+05	.54+03	.32+02	.29+02	.28+02	.27+02	.26+02	.22+02	.23+02	.19+02	.17+02	.18+02	.17+02	.16+02	.27+02
-.21+05	.34+01	-.55+01	-.27+01	-.21+00	.14+01	.24+01	.13+01	.21+01	.17+01	-.87+00	-.20+01	-.44+01	-.74+01	-.21+02
.36+04	.83+02	-.39+01	-.26+01	-.25+01	-.23+01	-.19+01	-.13+01	-.15+01	-.13+01	-.82+00	-.85+00	-.70+00	-.79+00	-.15+01
.34+09	.20+02	.20+02	.22+02	.19+02	.19+02	.20+02	.21+02	.19+02	.19+02	.15+02	.16+02	.16+02	.16+02	.27+02
-.34+09	-.17+02	-.15+02	-.11+02	-.75+01	-.59+01	-.37+01	-.35+01	-.22+01	-.21+01	-.21+01	-.32+01	-.46+01	-.75+01	-.23+02
.23+07	-.83+00	-.22+00	-.14+01	-.33+00	-.82+00	-.83+00	-.57+00	-.27+00	-.79+00	-.18+00	-.20+00	-.80+00	-.38+00	-.56+00
-.34+09	-.70+01	.14+01	-.79+01	-.77+00	-.42+01	-.45+01	-.29+01	-.13+01	-.44+01	-.82+00	-.86+00	-.44+01	-.16+01	.30+00
.34+09	.51+01	-.31+01	.13+01	-.26+01	-.16+01	-.21+01	-.27+01	-.28+01	-.21+01	-.18+01	-.18+01	-.46+00	-.85+00	-.19+01
.23+07	-.17+01	-.22+01	-.31+01	-.28+01	-.30+01	-.32+01	-.35+01	-.32+01	-.31+01	-.24+01	-.27+01	-.25+01	-.23+01	-.21+01
-.33+05	-.52+03	-.21+02	-.12+02	-.13+02	-.13+02	-.11+02	-.77+01	-.95+01	-.83+01	-.43+01	-.41+01	-.24+01	-.19+01	.49+01
.21+05	-.21+02	-.44+01	-.11+02	-.10+02	-.11+02	-.10+02	-.89+01	-.91+01	-.72+01	-.57+01	-.57+01	-.53+01	-.43+01	-.90+01
.36+04	.82+02	-.47+01	-.41+01	-.42+01	-.41+01	-.39+01	-.33+01	-.35+01	-.29+01	-.26+01	-.28+01	-.25+01	-.22+01	-.15+01
-.50+04	-.41+04	-.69+03	-.11+03	-.63+02	-.36+02	-.24+02	-.21+02	-.14+02	-.98+01	-.10+02	-.10+02	-.31+01	-.35+01	.67+01
.37+04	-.10+03	-.43+03	-.11+03	-.67+02	-.37+02	-.27+02	-.22+02	-.16+02	-.13+02	-.12+02	-.10+02	-.67+01	-.79+01	-.85+01
.48+03	.65+03	.77+02	-.19+01	-.54+01	-.44+01	-.41+01	-.37+01	-.31+01	-.28+01	-.29+01	-.29+01	-.19+01	-.23+01	.24-01
-.12+04	-.13+04	-.97+03	-.43+03	-.18+03	-.10+03	-.56+02	-.40+02	-.23+02	-.19+02	-.12+02	-.75+01	-.10+02	-.19+01	.87+01
.10+04	.42+03	-.26+03	-.31+03	-.18+03	-.10+03	-.63+02	-.46+02	-.30+02	-.22+02	-.16+02	-.13+02	-.11+02	-.10+02	-.55+01
.85+02	.19+03	.15+03	.48+02	.10+02	.60+00	-.23+01	-.29+01	-.30+01	-.26+01	-.25+01	-.21+01	-.25+01	-.13+01	.10+01
.40+03	-.16+03	-.42+03	-.32+03	-.23+03	-.12+03	-.67+02	-.41+02	-.40+02	-.23+02	-.15+02	-.13+02	-.51+01	.18+02	
-.37+03	.41+02	.23+02	-.12+03	-.16+03	-.12+03	-.72+02	-.53+02	-.47+02	-.28+02	-.23+02	-.19+02	-.12+02	-.75+01	
-.21+02	.28+02	.69+02	.49+02	.28+02	.96+01	.16+01	.69-01	-.15+01	-.19+01	-.19+01	-.21+01	-.13+01	.25+01	
.19+03	.21+03	.89+02	-.12+03	-.11+03	-.10+03	-.73+02	-.38+02	-.43+02	-.24+02	-.13+02	.61+00	.23+02	.24+01	
-.18+03	-.14+03	-.42+02	-.31+02	-.63+02	-.81+02	-.71+02	-.44+02	-.51+02	-.32+02	-.22+02	-.94+01	-.11+01	-.54+00	
-.89+01	-.26+02	-.13+02	.19+02	.17+02	.12+02	.59+01	.16+01	.23+00	-.90+00	-.11+01	-.44-01	.34+01	.38+00	
.91+02	.97+02	.10+03	.81+02	.57+02	-.31+01	-.89+01	-.17+02	.28+01	.83+01	.18+02	.27+02	.33+01		
-.87+02	-.71+02	-.41+02	-.45+01	.12+02	-.96+01	-.13+02	-.22+02	-.58+01	.20+00	.69+01	.92+01	.46+00		
-.37+01	-.10+02	-.15+02	-.13+02	-.86+01	.52+00	.10+01	.11+01	.30-01	.26+00	.14+01	.37+01	.52+00		
.43+02	.59+02	.44+02	.51+02	.45+02	.38+02	.36+02	.29+02	.27+02	.28+02	.17+02	.15+01			
-.42+02	-.45+02	-.22+02	-.83+01	.81+01	.15+02	.21+02	.18+02	.19+02	.17+02	.78+01	.57+00			
-.15+01	-.60+01	-.61+01	-.77+01	-.68+01	-.48+01	-.35+01	-.15+01	-.11+00	.20+01	.22+01	.23+00			
.28+02	.23+02	.28+02	.24+02	.24+02	.23+02	.22+02	.24+02	.18+02	.32+01					
-.27+02	-.19+02	-.13+02	-.36+01	.33+01	.87+01	.13+02	.16+02	.12+02	.15+01					
-.90+00	-.20+01	-.36+01	-.34+01	-.32+01	-.23+01	-.12+01	.58+00	.13+01	.41+00					
.10+02	.13+02	.10+02	.12+02	.11+02	.10+02	.71+01	.50+00						
-.10+02	-.98+01	-.34+01	.11+01	.47+01	.62+01	.54+01	.45+00						
-.32+00	-.11+01	-.96+00	-.11+01	-.48+00	-.61-02	.31+00	.33-01						
.51+00	.73+00	.46+00											
-.40+00	.69-01	.36+00											
-.11-01	-.83-02	.83-02											

TABLE 41

DISTRIBUTION FOR $\gamma = 1.5$, $\theta_0 = 60°$, AND $\sin \alpha_L = 0.2$

.81+01	.11+02	.87+01	.11+02	.11+02	.10+02	.71+01	.50+00								
-.78+01	-.90+01	-.37+01	.37+00	.47+01	.62+01	.54+01	.45+00								
.20+00	.96+00	.78+00	.10+01	.48+00	.61-02	-.31+00	-.33-01								
.28+02	.23+02	.28+02	.24+02	.24+02	.23+02	.22+02	.24+02	.18+02	.32+01						
-.27+02	-.19+02	-.13+02	-.36+01	.33+01	.87+01	.13+02	.16+02	.12+02	.15+01						
.90+00	.20+01	.36+01	.34+01	.32+01	.23+01	.12+01	-.58+00	-.13+01	-.41+00						
.43+02	.59+02	.44+02	.51+02	.45+02	.38+02	.36+02	.29+02	.27+02	.28+02	.17+02	.15+01				
-.42+02	-.45+02	-.22+02	-.83+01	.81+01	.15+02	.21+02	.18+02	.19+02	.17+02	.78+01	.57+00				
.15+01	.60+01	.61+01	.77+01	.68+01	.48+01	.35+01	.15+01	.11+00	-.20+01	-.22+01	-.23+00				
.37+01	.40+02	.86+02	.81+02	.85+02	.73+02	.54+02	.47+02	.31+02	.27+02	.26+02	.27+02	.33+01			
-.37+01	-.31+02	-.33+02	-.45+01	.22+02	.38+02	.36+02	.37+02	.22+02	.19+02	.14+02	.92+01	.46+00			
.95-02	.40+01	.12+02	.13+02	.12+02	.94+01	.52+01	.29+01	.71+00	-.45+00	-.18+01	-.37+01	-.52+00			
.43+01	.42+01	.50+02	.15+03	.12+03	.11+03	.81+02	.46+02	.51+02	.32+02	.23+02	.24+02	.26+02	.24+01		
-.42+01	-.42+01	-.12+02	.26+02	.55+02	.74+02	.64+02	.37+02	.42+02	.24+02	.11+02	.92+01	.81+00	-.54+00		
.20-01	.60-01	.75+01	.24+02	.17+02	.12+02	.62+01	.19+01	.51+00	-.79+00	-.13+01	-.23+01	-.38+01	-.38+00		
.47+01	.47+01	.47+01	.13+02	.24+03	.13+03	.73+02	.54+02	.48+02	.31+02	.26+02	.24+02	.23+02	.23+02		
-.47+01	-.47+01	-.46+01	.11+01	.15+03	.11+03	.66+02	.41+02	.40+02	.21+02	.13+02	.86+01	.19+01	-.60+01		
.32-01	.95-01	.15+00	.17+01	.28+02	.10+02	.20+01	.94+00	-.11+01	-.15+01	-.14+01	-.19+01	-.26+01	-.33+01		
.51+01	.51+01	.38+01	.62+01	.16+03	.11+03	.65+02	.50+02	.35+02	.27+02	.22+02	.18+02	.20+02	.25+02	.10+02	
-.51+01	-.50+01	-.37+01	-.59+01	.14+03	.10+03	.54+02	.38+02	.20+02	.15+02	.78+01	.32+01	.19+01	-.70+01	-.60+01	
.44-01	.13+00	.15+00	.34+00	.93+01	.12+01	-.15+01	-.20+01	-.19+01	-.18+01	-.16+01	-.12+01	-.20+01	-.31+01	-.13+01	
.53+01	.53+01	.53+01	.15+02	.77+02	.47+02	.37+02	.31+02	.24+02	.21+02	.21+02	.21+02	.14+02	.20+02	.20+02	
-.53+01	-.52+01	-.50+01	.28+01	.55+02	.29+02	.17+02	.15+02	.91+01	.52+01	.40+01	.26+01	-.27+01	-.78+01	-.13+02	
.57-01	.17+00	.30+00	-.64+00	-.41+01	-.33+01	-.26+01	-.25+01	-.18+01	-.13+01	-.16+01	-.17+01	-.92+00	-.16+01	-.20+01	
.67+01	.13+02	.19+02	.29+02	.28+02	.27+02	.26+02	.22+02	.23+02	.19+02	.17+02	.18+02	.17+02	.16+02	.27+02	
-.67+01	-.11+02	-.79+01	-.27+01	-.21+00	.14+01	.24+01	.13+01	.21+01	.17+01	-.87+00	-.20+01	-.44+01	-.74+01	-.21+02	
.11+00	-.43-01	-.18+01	-.26+01	-.25+01	-.23+01	-.19+01	-.13+01	-.15+01	-.13+01	-.82+00	-.85+00	-.70+00	-.79+00	-.15+01	
.18+02	.20+02	.20+02	.22+02	.19+02	.19+02	.20+02	.21+02	.19+02	.19+02	.15+02	.16+02	.16+02	.16+02	.27+02	
-.18+02	-.17+02	-.15+02	-.11+02	-.75+01	-.59+01	-.37+01	-.35+01	-.22+01	-.21+01	-.21+01	-.32+01	-.46+01	-.75+01	-.23+02	
-.14+00	-.83+00	-.22+00	-.14+01	-.33+00	-.82+00	-.83+00	-.57+00	-.27+00	-.79+00	-.18+00	.20+00	-.80+00	-.38+00	-.56+00	

-.28+01	-.70+01	.14+01	-.79+01	-.77+00	-.42+01	-.45+01	-.29+01	-.13+01	-.44+01	-.82+00	-.86+00	-.44+01	-.16+01	.30+00	
.27+01	.51+01	-.31+01	.13+01	-.26+01	-.16+01	-.21+01	-.27+01	-.28+01	-.21+01	-.18+01	-.18+01	-.46+00	-.85+00	-.19+01	
-.51+00	-.17+01	-.22+01	-.31+01	-.28+01	-.30+01	-.32+01	-.35+01	-.32+01	-.31+01	-.24+01	-.27+01	-.25+01	-.23+01	-.21+01	
.67+01	.59+01	-.88+01	-.12+02	-.13+02	-.13+02	-.11+02	-.77+01	-.95+01	-.83+01	-.43+01	-.41+01	-.24+01	-.19+01	.49+01	
-.67+01	-.71+01	-.21+01	-.11+02	-.10+02	-.11+02	-.10+02	-.89+01	-.91+01	-.72+01	-.57+01	-.57+01	-.53+01	-.43+01	-.90+01	
-.11+00	-.92+00	-.26+01	-.41+01	-.42+01	-.41+01	-.39+01	-.33+01	-.35+01	-.29+01	-.26+01	-.28+01	-.25+01	-.22+01	-.15+01	
.53+01	.53+01	.53+01	-.46+01	-.63+02	-.36+02	-.24+02	-.21+02	-.14+02	-.98+01	-.10+02	-.10+02	-.31+01	-.35+01	.67+01	
-.53+01	-.52+01	-.50+01	-.12+02	-.67+02	-.37+02	-.27+02	-.22+02	-.16+02	-.13+02	-.12+02	-.10+02	-.67+01	-.79+01	-.85+01	
-.57-01	-.17+00	-.30+00	-.14+01	-.54+01	-.44+01	-.41+01	-.37+01	-.31+01	-.28+01	-.29+01	-.29+01	-.19+01	-.23+01	.24-01	
.51+01	.51+01	.38+01	.62+01	-.15+03	-.10+03	-.56+02	-.40+02	-.23+02	-.19+02	-.12+02	-.75+01	-.10+02	-.19+01	.87+01	
-.51+01	-.50+01	-.37+01	-.59+01	-.15+03	-.10+03	-.63+02	-.46+02	-.30+02	-.22+02	-.16+02	-.13+02	-.11+02	-.10+02	-.55+01	
-.44-01	-.13+00	-.15+00	-.34+00	.84+01	.60+00	-.23+01	-.29+01	-.30+01	-.26+01	-.25+01	-.21+01	-.25+01	-.13+01	.10+01	
.47+01	.47+01	.47+01	-.69+01	-.23+03	-.12+03	-.67+02	-.41+02	-.40+02	-.23+02	-.15+02	-.13+02	-.51+01	.18+02		
-.47+01	-.47+01	-.46+01	-.77+01	-.16+03	-.12+03	-.72+02	-.53+02	-.47+02	-.28+02	-.23+02	-.19+02	-.12+02	-.75+01		
-.32-01	-.95-01	-.15+00	.14+01	.28+02	.96+01	.16+01	.69-01	-.15+01	-.19+01	-.19+01	-.21+01	-.13+01	.25+01		
.43+01	.42+01	.50+02	-.96+02	-.11+03	-.10+03	-.73+02	-.38+02	-.43+02	-.24+02	-.13+02	.61+00	.23+02	.24+01		
-.42+01	-.42+01	-.12+02	-.29+02	-.63+02	-.81+02	-.71+02	-.44+02	-.51+02	-.32+02	-.22+02	-.94+01	-.11+01	-.54+00		
-.20-01	-.60-01	-.75+01	.15+02	.17+02	.12+02	.59+01	.16+01	.23+00	-.90+00	-.11+01	-.44-01	.34+01	.38+00		
.37+01	.40+02	.86+02	.81+02	.57+02	-.31+01	-.89+01	-.17+02	.28+01	.83+01	.18+02	.27+02	.33+01			
-.37+01	-.31+02	-.33+02	-.45+01	.12+02	-.96+01	-.13+02	-.22+02	-.58+01	.20+00	.69+01	.92+01	.46+00			
-.95-02	-.40+01	-.12+02	-.13+02	-.86+01	.52+00	.10+01	.11+01	.30-01	.26+00	.14+01	.37+01	.52+00			
.43+02	.59+02	.44+02	.51+02	.45+02	.38+02	.36+02	.29+02	.27+02	.28+02	.17+02	.15+01				
-.42+02	-.45+02	-.22+02	-.83+01	.81+01	.15+02	.21+02	.18+02	.19+02	.17+02	.78+01	.57+00				
-.15+01	-.60+01	-.61+01	-.77+01	-.68+01	-.48+01	-.35+01	-.15+01	-.11+00	.20+01	.22+01	.23+00				
.28+02	.23+02	.28+02	.24+02	.24+02	.23+02	.22+02	.24+02	.18+02	.32+01						
-.27+02	-.19+02	-.13+02	-.36+01	.33+01	.87+01	.13+02	.16+02	.12+02	.15+01						
-.90+00	-.20+01	-.36+01	-.34+01	-.32+01	-.23+01	-.12+01	.58+00	.13+01	.41+00						
.81+01	.11+02	.87+01	.11+02	.11+02	.10+02	.71+01	.50+00								
-.78+01	-.90+01	-.37+01	.37+00	.47+01	.62+01	.54+01	.45+00								
-.20+00	-.96+00	-.78+00	-.10+01	-.48+00	-.61-02	.31+00	.33-01								

TABLE 42

DISTRIBUTION FOR $\gamma = 1.5$, $\theta_0 = 60°$, AND $\sin \alpha_L = 0.4$

.13+01	.14+01	.14+01	.10+01	.10+01	.10+01	.35+00
-.13+01	-.13+01	-.13+01	-.96+00	-.72+00	-.57+00	-.38-03
-.15-01	-.46-01	-.77-01	-.80-01	-.11+00	-.14+00	-.58-01

.33+01	.33+01	.34+01	.25+01	.30+01	.32+01	.69+01	.21+02	.18+02	.32+01
-.33+01	-.33+01	-.33+01	-.24+01	-.28+01	-.27+01	.22+00	.13+02	.12+02	.15+01
-.14-01	-.42-01	-.70-01	-.10+00	-.16+00	-.26+00	-.21+00	-.43+00	-.13+01	-.41+00

.31+01	.31+01	.31+01	.44+01	.27+01	.33+01	.13+02	.29+02	.27+02	.28+02	.17+02	.15+01	
-.31+01	-.31+01	-.31+01	-.44+01	-.27+01	-.33+01	.50+01	.18+02	.19+02	.17+02	.78+01	.57+00	
.49-04	.14-03	.24-03	-.20-01	-.36-01	-.71-01	.89+00	.15+01	.11+00	-.20+01	-.22+01	-.23+00	

.37+01	.37+01	.37+01	.37+01	.34+01	.34+01	.32+01	.47+02	.31+02	.27+02	.26+02	.27+02	.33+01	
-.37+01	-.37+01	-.37+01	-.37+01	-.34+01	-.33+01	-.32+01	.37+02	.22+02	.19+02	.14+02	.92+01	.46+00	
.95-02	.28-01	.47-01	.65-01	.40-01	.49-01	.28-01	.29+01	.71+00	-.45+00	-.18+01	-.37+01	-.52+00	

.43+01	.42+01	.42+01	.31+01	.40+01	.39+01	.38+01	.36+02	.51+02	.32+02	.23+02	.24+02	.26+02	.24+01	
-.42+01	-.42+01	-.42+01	-.31+01	-.39+01	-.38+01	-.37+01	.28+02	.42+02	.24+02	.11+02	.92+01	.81+00	-.54+00	
.20-01	.60-01	.10+00	.97-01	.12+00	.15+00	.14+00	.14+01	.51+00	-.79+00	-.13+01	-.23+01	-.38+01	-.38+00	

.47+01	.47+01	.47+01	.34+01	.45+01	.57+01	.32+01	.35+02	.48+02	.31+02	.26+02	.24+02	.23+02	.23+02
-.47+01	-.47+01	-.46+01	-.33+01	-.43+01	-.52+01	-.30+01	.22+02	.40+02	.21+02	.13+02	.86+01	.19+01	-.60+01
.32-01	.95-01	.15+00	.14+00	.22+00	.36+00	.19+00	.35+00	-.11+01	-.15+01	-.14+01	-.19+01	-.26+01	-.33+01

.51+01	.51+01	.38+01	.62+01	.62+01	.37+01	.48+01	.43+02	.35+02	.27+02	.22+02	.18+02	.20+02	.25+02	.10+02	
-.51+01	-.50+01	-.37+01	-.59+01	-.56+01	-.32+01	-.40+01	.31+02	.20+02	.15+02	.78+01	.32+01	.19+01	-.70+01	-.60+01	
.44-01	.13+00	.15+00	.34+00	.43+00	.31+00	.41+00	-.16+01	-.19+01	-.18+01	-.16+01	-.12+01	-.20+01	-.31+01	-.13+01	

.53+01	.53+01	.53+01	.53+01	.67+01	.53+01	.17+02	.31+02	.24+02	.21+02	.21+02	.21+02	.14+02	.20+02	.20+02	
-.53+01	-.52+01	-.50+01	-.48+01	-.56+01	-.42+01	.25+01	.15+02	.91+01	.52+01	.40+01	.26+01	-.27+01	-.78+01	-.13+02	
.57-01	.17+00	.30+00	.39+00	.63+00	.53+00	-.53+00	-.25+01	-.18+01	-.13+01	-.16+01	-.17+01	-.92+00	-.16+01	-.20+01	

.67+01	.95+01	.55+01	.84+01	.72+01	.12+02	.26+02	.22+02	.23+02	.19+02	.17+02	.18+02	.17+02	.16+02	.27+02	
-.67+01	-.91+01	-.50+01	-.69+01	-.51+01	-.41+01	.24+01	.13+01	.21+01	.17+01	-.87+00	-.20+01	-.44+01	-.74+01	-.21+02	
.11+00	.44+00	.37+00	.78+00	.84+00	-.68-01	-.19+01	-.13+01	-.15+01	-.13+01	-.82+00	-.85+00	-.70+00	-.79+00	-.15+01	

.66+01	.67+01	.12+02	.11+02	.17+02	.19+02	.20+02	.21+02	.19+02	.19+02	.15+02	.16+02	.16+02	.16+02	.27+02	
-.65+01	-.61+01	-.10+02	-.76+01	-.71+01	-.59+01	-.37+01	-.35+01	-.22+01	-.21+01	-.21+01	-.32+01	-.46+01	-.75+01	-.23+02	
.17+00	.44+00	.84+00	.22+00	-.68-01	-.82+00	-.83+00	-.57+00	-.27+00	-.79+00	-.18+00	-.20+00	-.80+00	-.38+00	-.56+00	

.66+01	.67+01	.97+01	.27+01	.85+00	-.42+01	-.45+01	-.29+01	-.13+01	-.44+01	-.82+00	-.86+00	-.44+01	-.16+01	.30+00	
-.65+01	-.61+01	-.83+01	-.26+01	-.30+01	-.16+01	-.21+01	-.27+01	-.28+01	-.21+01	-.18+01	-.18+01	-.46+00	-.85+00	-.19+01	
-.17+00	-.44+00	-.11+01	-.14+01	-.25+01	-.30+01	-.32+01	-.35+01	-.32+01	-.31+01	-.24+01	-.27+01	-.25+01	-.23+01	-.21+01	

.67+01	.95+01	.55+01	.84+01	.72+01	.14+01	-.11+02	-.77+01	-.95+01	-.83+01	-.43+01	-.41+01	-.24+01	-.19+01	.49+01	
-.67+01	-.91+01	-.50+01	-.69+01	-.51+01	-.55+01	-.10+02	-.89+01	-.91+01	-.72+01	-.57+01	-.57+01	-.53+01	-.43+01	-.90+01	
-.11+00	-.44+00	-.37+00	-.78+00	-.84+00	-.18+01	-.39+01	-.33+01	-.35+01	-.29+01	-.26+01	-.28+01	-.25+01	-.22+01	-.15+01	

.53+01	.53+01	.53+01	.53+01	.67+01	.53+01	-.41+01	-.21+02	-.14+02	-.98+01	-.10+02	-.10+02	-.31+01	-.35+01	.67+01	
-.53+01	-.52+01	-.50+01	-.48+01	-.56+01	-.42+01	-.12+02	-.22+02	-.16+02	-.13+02	-.12+02	-.10+02	-.67+01	-.79+01	-.85+01	
-.57-01	-.17+00	-.30+00	-.39+00	-.63+00	-.53+00	-.19+01	-.37+01	-.31+01	-.28+01	-.29+01	-.29+01	-.19+01	-.23+01	.24-01	

.51+01	.51+01	.38+01	.62+01	.62+01	.37+01	.48+01	-.33+02	-.23+02	-.19+02	-.12+02	-.75+01	-.10+02	-.19+01	.87+01	
-.51+01	-.50+01	-.37+01	-.59+01	-.56+01	-.32+01	-.40+01	-.39+02	-.30+02	-.22+02	-.16+02	-.13+02	-.11+02	-.10+02	-.55+01	
-.44-01	-.13+00	-.15+00	-.34+00	-.43+00	-.31+00	-.41+00	-.26+01	-.30+01	-.26+01	-.25+01	-.21+01	-.25+01	-.13+01	.10+01	

.47+01	.47+01	.47+01	.34+01	.45+01	.57+01	.32+01	-.22+02	-.40+02	-.23+02	-.15+02	-.13+02	-.51+01	.18+02		
-.47+01	-.47+01	-.46+01	-.33+01	-.43+01	-.52+01	-.30+01	-.34+02	-.47+02	-.28+02	-.23+02	-.10+02	-.12+02	-.75+01		
-.32-01	-.95-01	-.15+00	-.14+00	-.22+00	-.36+00	-.19+00	-.51+00	-.15+01	-.19+01	-.19+01	-.21+01	-.13+01	.25+01		

.43+01	.42+01	.42+01	.31+01	.40+01	.39+01	.38+01	-.29+02	-.43+02	-.24+02	-.13+02	.61+00	.23+02	.24+01		
-.42+01	-.42+01	-.42+01	-.31+01	-.39+01	-.38+01	-.37+01	-.35+02	-.51+02	-.32+02	-.22+02	-.94+01	-.11+01	-.54+00		
-.20-01	-.60-01	-.10+00	-.97-01	-.12+00	-.15+00	-.14+00	.11+01	.23+00	-.90+00	-.11+01	-.44-01	.34+01	.38+00		

.37+01	.37+01	.37+01	.37+01	.34+01	.34+01	.32+01	-.17+02	.28+01	.83+01	.18+02	.27+02	.33+01			
-.37+01	-.37+01	-.37+01	-.37+01	-.34+01	-.33+01	-.32+01	-.22+02	-.58+01	.20+00	.69+01	.92+01	.46+00			
-.95-02	-.28-01	-.47-01	-.65-01	-.40-01	-.49-01	-.28-01	.11+01	.30-01	.26+00	.14+01	.37+01	.52+00			

.31+01	.31+01	.31+01	.44+01	.27+01	.33+01	.13+02	.29+02	.27+02	.28+02	.17+02	.15+01				
-.31+01	-.31+01	-.31+01	-.44+01	-.27+01	-.33+01	.50+01	.18+02	.19+02	.17+02	.78+01	.57+00				
-.49-04	-.14-03	-.24-03	.20-01	.36-01	.71-01	-.89+00	-.15+01	-.11+00	.20+01	.22+01	.23+00				

.33+01	.33+01	.34+01	.25+01	.30+01	.32+01	.69+01	.21+02	.18+02	.32+01						
-.33+01	-.33+01	-.33+01	-.24+01	-.28+01	-.27+01	.22+00	.13+02	.12+02	.15+01						
.14-01	.42-01	.70-01	.10+00	.16+00	.26+00	.21+00	.43+00	.13+01	.41+00						

.13+01	.14+01	.14+01	.10+01	.10+01	.10+01	.35+00
-.13+01	-.13+01	-.13+01	-.96+00	-.72+00	-.57+00	-.38-03
.15-01	.46-01	.77-01	.80-01	.11+00	.14+00	.58-01

TABLE 43

DISTRIBUTION FOR $\gamma = 1.5$, $\theta_0 = 60°$, AND $\sin \alpha_L = 0.6$

.33+01	.33+01	.34+01	.21+01	.21+01	.16+01	.10+01	.50+00
-.33+01	-.33+01	-.33+01	-.20+01	-.20+01	-.15+01	-.93+00	-.35+00
-.14-01	-.42-01	-.70-01	-.76-01	-.98-01	-.92-01	-.79-01	-.59-01

.31+01	.31+01	.31+01	.44+01	.27+01	.33+01	.32+01	.41+01	.40+01	.28+01	.29+01	.15+01	
-.31+01	-.31+01	-.31+01	-.44+01	-.27+01	-.33+01	-.31+01	-.38+01	-.33+01	-.19+01	-.94+00	.57+00	
.49-04	.14-03	.24-03	-.20-01	-.36-01	-.71-01	-.10+00	-.25+00	-.35+00	-.33+00	-.46+00	-.23+00	

.37+01	.37+01	.37+01	.37+01	.34+01	.34+01	.32+01	.31+01	.43+01	.41+01	.11+02	.27+02	.33+01	
-.37+01	-.37+01	-.37+01	-.37+01	-.34+01	-.33+01	-.32+01	-.31+01	-.42+01	-.39+01	.47+00	.92+01	.46+00	
.95-02	.28-01	.47-01	.65-01	.40-01	.49-01	.28-01	.61-03	-.84-01	-.16+00	-.10+01	-.37+01	-.52+00	

.43+01	.42+01	.42+01	.31+01	.40+01	.39+01	.38+01	.36+01	.44+01	.40+01	.17+02	.24+02	.26+02	.24+01	
-.42+01	-.42+01	-.42+01	-.31+01	-.39+01	-.38+01	-.37+01	-.35+01	-.43+01	-.40+01	.55+01	.92+01	.81+00	-.54+00	
.20-01	.60-01	.10+00	.97-01	.12+00	.15+00	.14+00	.13+00	.14+00	.50-01	-.92+00	-.23+01	-.38+01	-.38+00	

.47+01	.47+01	.47+01	.34+01	.45+01	.57+01	.32+01	.63+01	.39+01	.37+01	.18+02	.24+02	.23+02	.23+02	
-.47+01	-.47+01	-.46+01	-.33+01	-.43+01	-.52+01	-.30+01	-.58+01	-.36+01	-.35+01	.62+01	.86+01	.19+01	-.60+01	
.32-01	.95-01	.15+00	.14+00	.22+00	.36+00	.19+00	.43+00	.24+00	.21+00	-.97+00	-.19+01	-.26+01	-.33+01	

.51+01	.51+01	.38+01	.62+01	.62+01	.37+01	.48+01	.48+01	.56+01	.42+01	.22+02	.18+02	.20+02	.25+02	.10+02	
-.51+01	-.50+01	-.37+01	-.59+01	-.56+01	-.32+01	-.40+01	-.39+01	-.46+01	-.35+01	.78+01	.32+01	.19+01	-.70+01	-.60+01	
.44-01	.13+00	.15+00	.34+00	.43+00	.31+00	.41+00	.46+00	.54+00	.38+00	-.16+01	-.12+01	-.20+01	-.31+01	-.13+01	

.53+01	.53+01	.53+01	.53+01	.67+01	.53+01	.66+01	.52+01	.51+01	.17+02	.21+02	.21+02	.14+02	.20+02	.20+02	
-.53+01	-.52+01	-.50+01	-.48+01	-.56+01	-.42+01	-.49+01	-.36+01	-.34+01	.19+01	.40+01	.26+01	-.27+01	-.78+01	-.13+02	
.57-01	.17+00	.30+00	.39+00	.63+00	.53+00	.73+00	.61+00	.62+00	-.79+00	-.16+01	-.17+01	-.92+00	-.16+01	-.20+01	

.27+01	.41+01	.84+01	.72+01	.71+01	.72+01	.71+01	.88+01	.19+02	.17+02	.18+02	.17+02	.16+02	.27+02		
-.26+01	-.37+01	-.69+01	-.51+01	-.48+01	-.42+01	-.38+01	-.31+01	.17+01	-.87+00	-.20+01	-.44+01	-.74+01	-.21+02		
.12+00	.28+00	.78+00	.84+00	.87+00	.97+00	.10+01	.69+00	-.13+01	-.82+00	-.85+00	-.70+00	-.79+00	-.15+01		

.30+01	.46+01	.18+02	.19+02	.19+02	.15+02	.16+02	.16+02	.16+02	.27+02						
-.16+01	-.19+01	-.39+01	-.22+01	-.21+01	-.21+01	-.32+01	-.46+01	-.75+01	-.23+02						
.41+00	.70+00	.45-01	-.27+00	-.79+00	-.18+00	-.20+00	-.80+00	-.38+00	-.56+00						

.30+01	.46+01	.80+00	-.13+01	-.44+01	-.82+00	-.86+00	-.44+01	-.16+01	.30+00						
-.16+01	-.19+01	-.23+01	-.28+01	-.21+01	-.18+01	-.18+01	-.46+00	-.85+00	-.19+01						
-.41+00	-.70+00	-.29+01	-.32+01	-.31+01	-.24+01	-.27+01	-.25+01	-.23+01	-.21+01						

.27+01	.41+01	.84+01	.72+01	.71+01	.72+01	.71+01	.50+01	-.83+01	-.43+01	-.41+01	-.24+01	-.19+01	.49+01		
-.26+01	-.37+01	-.69+01	-.51+01	-.48+01	-.42+01	-.38+01	-.38+01	-.72+01	-.57+01	-.57+01	-.53+01	-.43+01	-.90+01		
-.12+00	-.28+00	-.78+00	-.84+00	-.87+00	-.97+00	-.10+01	-.13+01	-.29+01	-.26+01	-.28+01	-.25+01	-.22+01	-.15+01		

.53+01	.53+01	.53+01	.53+01	.67+01	.53+01	.66+01	.52+01	.51+01	-.49+01	-.10+02	-.10+02	-.31+01	-.35+01	.67+01	
-.53+01	-.52+01	-.50+01	-.48+01	-.56+01	-.42+01	-.49+01	-.36+01	-.34+01	-.10+02	-.12+02	-.10+02	-.67+01	-.79+01	-.85+01	
-.57-01	-.17+00	-.30+00	-.39+00	-.63+00	-.53+00	-.73+00	-.61+00	-.62+00	-.22+01	-.29+01	-.29+01	-.19+01	-.23+01	.24-01	

.51+01	.51+01	.38+01	.62+01	.62+01	.37+01	.48+01	.48+01	.56+01	.42+01	-.12+02	-.75+01	-.10+02	-.19+01	.87+01	
-.51+01	-.50+01	-.37+01	-.59+01	-.56+01	-.32+01	-.40+01	-.39+01	-.46+01	-.35+01	-.16+02	-.13+02	-.11+02	-.10+02	-.55+01	
-.44-01	-.13+00	-.15+00	-.34+00	-.43+00	-.31+00	-.41+00	-.46+00	-.54+00	-.38+00	-.25+01	-.21+01	-.25+01	-.13+01	.10+01	

.47+01	.47+01	.47+01	.34+01	.45+01	.57+01	.32+01	.63+01	.39+01	.37+01	-.81+01	-.13+02	-.51+01	.18+02	
-.47+01	-.47+01	-.46+01	-.33+01	-.43+01	-.52+01	-.30+01	-.58+01	-.36+01	-.35+01	-.16+02	-.19+02	-.12+02	-.75+01	
-.32-01	-.95-01	-.15+00	-.14+00	-.22+00	-.36+00	-.19+00	-.43+00	-.24+00	-.21+00	-.14+01	-.21+01	-.13+01	.25+01	

.43+01	.42+01	.42+01	.31+01	.40+01	.39+01	.38+01	.36+01	.44+01	.40+01	-.66+01	.61+00	.23+02	.24+01	
-.42+01	-.42+01	-.42+01	-.31+01	-.39+01	-.38+01	-.37+01	-.35+01	-.43+01	-.40+01	-.15+02	-.94+01	-.11+01	-.54+00	
-.20-01	-.60-01	-.10+00	-.97-01	-.12+00	-.15+00	-.14+00	-.13+00	-.14+00	-.50-01	-.78+00	-.44-01	.34+01	.38+00	

.37+01	.37+01	.37+01	.37+01	.34+01	.34+01	.32+01	.31+01	.43+01	.41+01	.11+02	.27+02	.33+01	
-.37+01	-.37+01	-.37+01	-.37+01	-.34+01	-.33+01	-.32+01	-.31+01	-.42+01	-.39+01	.47+00	.92+01	.46+00	
-.95-02	-.28-01	-.47-01	-.65-01	-.40-01	-.49-01	-.28-01	-.61-03	.84-01	.16+00	.10+01	.37+01	.52+00	

.31+01	.31+01	.31+01	.44+01	.27+01	.33+01	.32+01	.41+01	.40+01	.28+01	.29+01	.15+01	
-.31+01	-.31+01	-.31+01	-.44+01	-.27+01	-.33+01	-.31+01	-.38+01	-.33+01	-.19+01	-.94+00	.57+00	
-.49-04	-.14-03	-.24-03	.20-01	.36-01	.71-01	.10+00	.25+00	.35+00	.33+00	.46+00	.23+00	

.33+01	.33+01	.34+01	.21+01	.21+01	.16+01	.10+01	.50+00
-.33+01	-.33+01	-.33+01	-.20+01	-.20+01	-.15+01	-.93+00	-.35+00
.14-01	.42-01	.70-01	.76-01	.98-01	.92-01	.79-01	.59-01

TABLE 44

DISTRIBUTION FOR $\gamma = 1.5$, $\theta_0 = 60°$, AND $\sin \alpha_L = 0.8$

.31+01	.31+01	.31+01	.31+01	.14+01	.14+01	.74+00								
-.31+01	-.31+01	-.31+01	-.31+01	-.14+01	-.14+01	-.73+00								
.49-04	.14-03	.24-03	.33-03	-.94-02	-.11-01	-.64-02								
.37+01	.37+01	.37+01	.37+01	.34+01	.34+01	.32+01	.31+01	.22+01	.14+01	.69+00				
-.37+01	-.37+01	-.37+01	-.37+01	-.34+01	-.33+01	-.32+01	-.31+01	-.22+01	-.14+01	-.66+00				
.95-02	.28-01	.47-01	.65-01	.40-01	.49-01	.28-01	.61-03	-.84-02	-.18-01	-.30-01				
.43+01	.42+01	.42+01	.31+01	.40+01	.39+01	.38+01	.36+01	.44+01	.40+01	.52+01	.28+01			
-.42+01	-.42+01	-.42+01	-.31+01	-.39+01	-.38+01	-.37+01	-.35+01	-.43+01	-.40+01	-.51+01	-.27+01			
.20-01	.60-01	.10+00	.97-01	.12+00	.15+00	.14+00	.13+00	.14+00	.50-01	-.74-01	-.11+00			
.47+01	.47+01	.47+01	.34+01	.45+01	.57+01	.32+01	.63+01	.39+01	.37+01	.52+01	.56+01	.52+01	.29+01	
-.47+01	-.47+01	-.46+01	-.33+01	-.43+01	-.52+01	-.30+01	-.58+01	-.36+01	-.35+01	-.50+01	-.55+01	-.51+01	-.25+01	
.32-01	.95-01	.15+00	.14+00	.22+00	.36+00	.19+00	.43+00	.24+00	.21+00	.23+00	.87-01	-.13+00	-.21+00	
.25+01	.25+01	.25+01	.62+01	.62+01	.37+01	.48+01	.48+01	.56+01	.42+01	.50+01	.56+01	.50+01	.10+02	.10+02
-.25+01	-.25+01	-.24+01	-.59+01	-.56+01	-.32+01	-.40+01	-.39+01	-.46+01	-.35+01	-.43+01	-.50+01	-.48+01	-.74+01	-.60+01
.20-01	.61-01	.10+00	.34+00	.43+00	.31+00	.41+00	.46+00	.54+00	.38+00	.42+00	.41+00	.21+00	-.76+00	-.13+01
					.26+01	.66+01	.52+01	.51+01	.60+01	.56+01	.53+01	.84+01	.17+02	.20+02
					-.20+01	-.49+01	-.36+01	-.34+01	-.41+01	-.40+01	-.39+01	-.44+01	-.83+01	-.13+02
					.26+00	.73+00	.61+00	.62+00	.72+00	.64+00	.60+00	.49-01	-.11+01	-.20+01
						.00-31	.27+01	.55+01	.53+01	.63+01	.71+01	.15+02	.16+02	.27+02
						.00-31	-.16+01	-.29+01	-.27+01	-.33+01	-.39+01	-.47+01	-.74+01	-.21+02
						.00-31	.36+00	.77+00	.77+00	.90+00	.98+00	-.45+00	-.79+00	-.15+01
									.28+01	.55+01	.13+02	.16+02	.16+02	.27+02
									-.10+01	-.17+01	-.32+01	-.46+01	-.75+01	-.23+02
									.43+00	.88+00	.30+00	-.80+00	-.38+00	-.56+00

.28+01	.55+01	.22+01	-.44+01	-.16+01	.30+00
-.10+01	-.17+01	-.18+01	-.46+00	-.85+00	-.19+01
-.43+00	-.88+00	-.21+01	-.25+01	-.23+01	-.21+01
.27+01	.55+01	.53+01	.63+01	.71+01	-.86+00	-.19+01	.49+01
-.16+01	-.29+01	-.27+01	-.33+01	-.39+01	-.50+01	-.43+01	-.90+01
-.36+00	-.77+00	-.77+00	-.90+00	-.98+00	-.23+01	-.22+01	-.15+01
.26+01	.66+01	.52+01	.51+01	.60+01	.56+01	.53+01	.29+01	-.88+00	.67+01
-.20+01	-.49+01	-.36+01	-.34+01	-.41+01	-.40+01	-.39+01	-.51+01	-.74+01	-.85+01
-.26+00	-.73+00	-.61+00	-.62+00	-.72+00	-.64+00	-.60+00	-.95+00	-.18+01	.24-01
.25+01	.25+01	.25+01	.62+01	.62+01	.37+01	.48+01	.48+01	.56+01	.42+01	.50+01	.56+01	.50+01	.43+01	.87+01
-.25+01	-.25+01	-.24+01	-.59+01	-.56+01	-.32+01	-.40+01	-.39+01	-.46+01	-.35+01	-.43+01	-.50+01	-.48+01	-.72+01	-.55+01
-.20-01	-.61-01	-.10+00	-.34+00	-.43+00	-.31+00	-.41+00	-.46+00	-.54+00	-.38+00	-.42+00	-.41+00	-.21+00	-.31+00	.10+01
.47+01	.47+01	.47+01	.34+01	.45+01	.57+01	.32+01	.63+01	.39+01	.37+01	.52+01	.56+01	.52+01	.29+01
-.47+01	-.47+01	-.46+01	-.33+01	-.43+01	-.52+01	-.30+01	-.58+01	-.36+01	-.35+01	-.50+01	-.55+01	-.51+01	-.25+01
-.32-01	-.95-01	-.15+00	-.14+00	-.22+00	-.36+00	-.19+00	-.43+00	-.24+00	-.21+00	-.23+00	-.87-01	.13+00	.21+00
.43+01	.42+01	.42+01	.31+01	.40+01	.39+01	.38+01	.36+01	.44+01	.40+01	.52+01	.28+01
-.42+01	-.42+01	-.42+01	-.31+01	-.39+01	-.38+01	-.37+01	-.35+01	-.43+01	-.40+01	-.51+01	-.27+01
-.20-01	-.60-01	-.10+00	-.97-01	-.12+00	-.15+00	-.14+00	-.13+00	-.14+00	-.50-01	.74-01	.11+00
.37+01	.37+01	.37+01	.37+01	.34+01	.34+01	.32+01	.31+01	.22+01	.14+01	.69+00
-.37+01	-.37+01	-.37+01	-.37+01	-.34+01	-.33+01	-.32+01	-.31+01	-.22+01	-.14+01	-.66+00
-.95-02	-.28-01	-.47-01	-.65-01	-.40-01	-.49-01	-.28-01	-.61-03	.84-02	.18-01	.30-01
.31+01	.31+01	.31+01	.31+01	.14+01	.14+01	.74+00
-.31+01	-.31+01	-.31+01	-.31+01	-.14+01	-.14+01	-.73+00
-.49-04	-.14-03	-.24-03	-.33-03	.94-02	.11-01	.64-02

TABLE 45

DISTRIBUTION FOR $\gamma = 1.5$, $\theta_0 = 80°$, AND $\sin \alpha_L = 0$

.15+01	.32+01	.64+01	.70+01	.55+01
-.95+00	.96+00	.40+01	.62+01	.52+01
.14-01	.18+00	.16+00	.29-01	-.58-01

.28+02	.39+02	.50+02	.52+02	.84+02	.94+02	.98+02	.93+02	.71+02	.35+02
-.25+02	-.19+02	.28+00	.20+02	.53+02	.75+02	.87+02	.84+02	.64+02	.29+02
.15+01	.49+01	.73+01	.61+01	.79+01	.52+01	.22+01	-.10+01	-.33+01	-.28+01

.97+02	.11+03	.16+03	.20+03	.24+03	.31+03	.21+03	.13+03	.12+03	.92+02	.81+02	.51+02
-.88+02	-.59+02	-.91+01	.70+02	.14+03	.25+03	.19+03	.12+03	.10+03	.75+02	.57+02	.28+02
.55+01	.15+02	.26+02	.30+02	.30+02	.28+02	.94+01	.87+00	-.40+01	-.68+01	-.87+01	-.69+01

.37+03	.42+03	.79+03	.96+03	.44+03	.12+03	.57+02	.45+02	.37+02	.39+02	.39+02	.47+02	.58+02	.78+01	
-.32+03	-.17+03	.17+03	.49+03	.32+03	.10+03	.41+02	.29+02	.21+02	.18+02	.17+02	.15+02	.13+02	.48+00	
.24+02	.60+02	.12+03	.13+03	.41+02	.16+01	-.36+01	-.39+01	-.39+01	-.47+01	-.50+01	-.67+01	-.92+01	-.13+01	

.11+04	.27+04	.15+04	.10+02	.14+02	.13+02	.15+02	.16+02	.20+02	.15+02	.22+02	.22+02	.30+02	.48+02	.22+01
-.92+03	-.38+03	.30+03	-.55+01	-.51+01	-.38+01	-.22+01	-.22+01	-.12+01	-.86+00	-.28+01	-.40+00	-.52+01	-.91+01	-.92+00
.94+02	.43+03	.24+03	-.10+01	-.17+01	-.16+01	-.20+01	-.21+01	-.27+01	-.21+01	-.30+01	-.33+01	-.45+01	-.77+01	-.33+00

.90+04	.41+04	.90+01	.11+02	.96+01	.90+01	.93+01	.14+02	.11+02	.12+02	.14+02	.17+02	.19+02	.26+02	.27+02
-.57+04	-.87+03	-.83+01	-.92+01	-.76+01	-.65+01	-.62+01	-.75+01	-.69+01	-.62+01	-.78+01	-.73+01	-.93+01	-.13+02	-.16+02
.10+04	.66+03	-.43+00	-.79+00	-.72+00	-.72+00	-.82+00	-.16+01	-.12+01	-.13+01	-.15+01	-.22+01	-.24+01	-.34+01	-.36+01

.43+05	.79+01	.89+01	.90+01	.81+01	.92+01	.11+02	.84+01	.10+02	.11+02	.10+02	.15+02	.13+02	.21+02	.36+02
-.33+05	-.78+01	-.86+01	-.84+01	-.73+01	-.79+01	-.93+01	-.68+01	-.83+01	-.86+01	-.83+01	-.10+02	-.10+02	-.16+02	-.30+02
.41+04	-.87-01	-.21+00	-.29+00	-.28+00	-.46+00	-.74+00	-.58+00	-.63+00	-.87+00	-.65+00	-.13+01	-.10+01	-.17+01	-.32+01

.43+09	.81+01	.82+01	.82+01	.93+01	.94+01	.63+01	.10+02	.10+02	.96+01	.11+02	.12+02	.13+02	.18+02	.37+02
-.43+09	-.81+01	-.80+01	-.79+01	-.87+01	-.86+01	-.57+01	-.95+01	-.92+01	-.81+01	-.99+01	-.10+02	-.11+02	-.16+02	-.35+02
.25+07	-.29-01	-.49-01	-.67-01	-.16+00	-.19+00	-.77-01	-.16+00	-.23+00	-.29+00	-.16+00	-.32+00	-.39+00	-.56+00	-.12+01

-.43+09	.81-01	.68-01	.49-01	-.10+01	-.10+01	-.43-01	-.13+00	-.21+00	-.12+01	.77+00	-.36+00	-.13+01	-.51+00	-.43+00
.43+09	-.10+00	-.12+00	-.15+00	.71+00	.65+00	-.16+00	-.31+00	-.50+00	.43+00	-.14+01	-.63+00	.26+00	-.85+00	-.64+00
.25+07	-.14+00	-.24+00	-.34+00	-.51+00	-.62+00	-.42+00	-.72+00	-.82+00	-.82+00	-.95+00	-.97+00	-.10+01	-.11+01	-.96+00

-.12+05	.25+00	-.75+00	-.87+00	.32-01	-.11+01	-.34+01	-.24+01	-.64+00	-.19+01	-.77+00	-.35+01	-.22+01	-.16+01	.80+00
.51+04	-.31+00	.49+00	.37+00	-.69+00	-.15+00	.13+01	.86+00	-.15+01	-.11+01	-.15+01	-.79+00	-.12+01	-.28+01	-.36+01
.21+04	-.14+00	-.30+00	-.43+00	-.46+00	-.67+00	-.99+00	-.70+00	-.85+00	-.10+01	-.87+00	-.13+01	-.10+01	-.12+01	-.71+00

.81+03	-.27+04	-.12+01	-.33+01	-.19+01	-.12+01	-.15+01	-.65+01	-.25+01	-.28+01	-.31+01	-.61+01	-.60+01	-.29+01	.54+01
-.16+04	.37+03	.58+00	.14+01	-.86-01	-.11+01	-.15+01	-.13+00	-.24+01	-.31+01	-.34+01	-.32+01	-.29+01	-.47+01	-.53+01
.82+02	.44+03	-.43+00	-.79+00	-.72+00	-.72+00	-.82+00	-.15+01	-.10+01	-.11+01	-.12+01	-.16+01	-.16+01	-.11+01	.40+00

.43+03	-.14+03	-.11+04	-.28+01	-.68+01	-.59+01	-.85+01	-.74+01	-.11+02	-.82+01	-.79+01	-.11+02	-.72+01	.58+01	.22+01
-.40+03	-.32+03	-.30+03	-.16+01	-.19+01	-.32+01	-.48+01	-.62+01	-.70+01	-.96+01	-.95+01	-.80+01	-.97+01	-.75+01	-.92+00
-.26+02	.33+02	.18+03	-.93+00	-.15+01	-.14+01	-.18+01	-.17+01	-.22+01	-.16+01	-.18+01	-.21+01	-.16+01	.69+00	.33+00

.33+03	.34+03	.17+03	-.53+03	-.39+03	-.11+03	-.47+02	-.35+02	-.28+02	-.25+02	-.24+02	-.18+02	.16+02	.50+01
-.30+03	-.19+03	-.10+03	-.35+03	-.31+03	-.11+03	-.50+02	-.37+02	-.27+02	-.26+02	-.24+02	-.22+02	-.26+01	.78-01
-.20+02	-.50+02	-.38+02	.64+02	.36+02	.22+01	-.28+01	-.30+01	-.30+01	-.29+01	-.29+01	-.22+01	.28+01	.84+00

.97+02	.11+03	.16+03	.20+03	.14+03	-.86+02	-.81+02	-.97+02	-.68+02	-.10+02	.24+02	.34+02
-.88+02	-.59+02	-.91+01	.70+02	.73+02	-.89+02	-.88+02	-.10+03	-.77+02	-.19+02	.92+01	.16+02
-.55+01	-.15+02	-.26+02	-.30+02	-.18+02	.58+01	.25+01	.19+01	-.83+00	.33+00	.38+01	.50+01

.28+02	.39+02	.50+02	.52+02	.84+02	.94+02	.98+02	.93+02	.71+02	.35+02
-.25+02	-.19+02	.28+00	.20+02	.53+02	.75+02	.87+02	.84+02	.64+02	.29+02
-.15+01	-.49+01	-.73+01	-.61+01	-.79+01	-.52+01	-.22+01	.10+01	.33+01	.28+01

.15+01	.32+01	.64+01	.70+01	.55+01
-.95+00	.96+00	.40+01	.62+01	.52+01
-.14-01	-.18+00	-.16+00	-.29-01	.58-01

TABLE 46

DISTRIBUTION FOR $\gamma = 1.5$, $\theta_0 = 80°$, AND $\sin \alpha_L = 0.2$

		.28+00	.41+01	.55+01									
		.32-01	.37+01	.52+01									
		-.46-01	-.49-01	-.58-01									
.26+01	.37+01	.36+01	.49+02	.84+02	.94+02	.98+02	.93+02	.71+02	.35+02				
-.26+01	-.32+01	-.26+01	.20+02	.53+02	.75+02	.87+02	.84+02	.64+02	.29+02				
-.61-01	-.28+00	-.39+00	.56+01	.79+01	.52+01	.22+01	-.10+01	-.33+01	-.28+01				
.37+01	.37+01	.37+01	.14+03	.24+03	.31+03	.21+03	.13+03	.12+03	.92+02	.81+02	.51+02		
-.36+01	-.36+01	-.36+01	.54+02	.14+03	.25+03	.19+03	.12+03	.10+03	.75+02	.57+02	.28+02		
-.39-01	-.11+00	-.19+00	.21+02	.30+02	.28+02	.94+01	.87+00	-.40+01	-.68+01	-.87+01	-.69+01		
.31+01	.31+01	.50+01	.47+02	.44+03	.12+03	.57+02	.45+02	.37+02	.39+02	.39+02	.47+02	.58+02	.78+01
-.31+01	-.31+01	-.31+01	.29+02	.32+03	.10+03	.41+02	.29+02	.21+02	.18+02	.17+02	.15+02	.13+02	.48+00
-.17-01	-.52-01	-.39+00	.15+01	.41+02	.16+01	-.36+01	-.39+01	-.39+01	-.47+01	-.50+01	-.67+01	-.92+01	-.13+01
.89+01	.84+01	.10+02	.10+02	.14+02	.13+02	.15+02	.16+02	.20+02	.15+02	.22+02	.22+02	.30+02	.48+02	.22+01
-.87+01	-.73+01	-.75+01	-.55+01	-.51+01	-.38+01	-.22+01	-.22+01	-.12+01	-.86+00	-.28+01	-.40+00	-.52+01	-.91+01	-.92+00
-.19+00	-.50+00	-.95+00	-.10+01	-.17+01	-.16+01	-.20+01	-.21+01	-.27+01	-.21+01	-.30+01	-.33+01	-.45+01	-.77+01	-.33+00
.88+01	.11+02	.90+01	.11+02	.96+01	.90+01	.93+01	.14+02	.11+02	.12+02	.14+02	.17+02	.19+02	.26+02	.27+02
-.87+01	-.11+02	-.83+01	-.92+01	-.76+01	-.65+01	-.62+01	-.75+01	-.69+01	-.62+01	-.78+01	-.73+01	-.93+01	-.13+02	-.16+02
-.73-01	-.38+00	-.43+00	-.79+00	-.72+00	-.72+00	-.82+00	-.16+01	-.12+01	-.13+01	-.15+01	-.22+01	-.24+01	-.34+01	-.36+01
.78+01	.79+01	.89+01	.90+01	.81+01	.92+01	.11+02	.84+01	.10+02	.11+02	.10+02	.15+02	.13+02	.21+02	.36+02
-.78+01	-.78+01	-.86+01	-.84+01	-.73+01	-.79+01	-.93+01	-.68+01	-.83+01	-.86+01	-.83+01	-.10+02	-.10+02	-.16+02	-.30+02
-.29-01	-.87-01	-.21+00	-.29+00	-.28+00	-.46+00	-.74+00	-.58+00	-.63+00	-.87+00	-.65+00	-.13+01	-.10+01	-.17+01	-.32+01
.81+01	.81+01	.82+01	.82+01	.93+01	.94+01	.63+01	.10+02	.10+02	.96+01	.11+02	.12+02	.13+02	.18+02	.37+02
-.81+01	-.81+01	-.80+01	-.79+01	-.87+01	-.86+01	-.57+01	-.95+01	-.92+01	-.81+01	-.99+01	-.10+02	-.11+02	-.16+02	-.35+02
-.99-02	-.29-01	-.49-01	-.67-01	-.16+00	-.19+00	-.77-01	-.16+00	-.23+00	-.29+00	-.16+00	-.32+00	-.39+00	-.56+00	-.12+01
.88-01	.81-01	.68-01	.49-01	-.10+01	-.10+01	-.43-01	-.13+00	-.21+00	-.12+01	.77+00	-.36+00	-.13+01	-.51+00	-.43+00
-.90-01	-.10+00	-.12+00	-.15+00	.71+00	.65+00	-.16+00	-.31+00	-.50+00	.43+00	-.14+01	-.63+00	.26+00	-.85+00	-.64+00
-.49-01	-.14+00	-.24+00	-.34+00	-.51+00	-.62+00	-.42+00	-.72+00	-.82+00	-.82+00	-.95+00	-.97+00	-.10+01	-.11+01	-.96+00
.27+00	.25+00	-.75+00	-.87+00	.32-01	-.11+01	-.34+01	-.24+01	-.64+00	-.19+01	-.77+00	-.35+01	-.22+01	-.16+01	.80+00
-.27+00	-.31+00	.49+00	.37+00	-.69+00	-.15+00	.13+01	.86+00	-.15+01	-.11+01	-.15+01	-.79+00	-.12+01	-.28+01	-.36+01
-.48-01	-.14+00	-.30+00	-.43+00	-.46+00	-.67+00	-.99+00	-.70+00	-.85+00	-.10+01	-.87+00	-.13+01	-.10+01	-.12+01	-.71+00
-.10+01	-.36+01	-.12+01	-.33+01	-.19+01	-.12+01	-.15+01	-.65+01	-.25+01	-.28+01	-.31+01	-.61+01	-.60+01	-.29+01	.54+01
.10+01	.32+01	.58+00	.14+01	-.86-01	-.11+01	-.15+01	-.13+00	-.24+01	-.31+01	-.34+01	-.32+01	-.29+01	-.47+01	-.53+01
-.73-01	-.38+00	-.43+00	-.79+00	-.72+00	-.72+00	-.82+00	-.15+01	-.10+01	-.11+01	-.12+01	-.16+01	-.16+01	-.11+01	.40+00
-.17+01	-.12+01	-.32+01	-.28+01	-.68+01	-.59+01	-.85+01	-.74+01	-.11+02	-.82+01	-.79+01	-.11+02	-.72+01	.58+01	.22+01
.16+01	.19+00	.36+00	-.16+01	-.19+01	-.32+01	-.48+01	-.62+01	-.70+01	-.56+01	-.95+01	-.80+01	-.97+01	-.75+01	-.92+00
-.17+00	-.45+00	-.86+00	-.93+00	-.15+01	-.14+01	-.18+01	-.17+01	-.22+01	-.16+01	-.18+01	-.21+01	-.16+01	.69+00	.33+00
.31+01	.31+01	.12+01	-.40+02	-.39+03	-.11+03	-.47+02	-.35+02	-.28+02	-.25+02	-.24+02	-.18+02	.16+02	.50+01	
-.31+01	-.31+01	-.31+01	-.35+02	-.31+03	-.11+03	-.50+02	-.37+02	-.27+02	-.26+02	-.24+02	-.22+02	-.26+01	.78-01	
.17-01	.52-01	-.22+00	.17+01	.36+02	.22+01	-.28+01	-.30+01	-.30+01	-.29+01	-.29+01	-.22+01	.28+01	.84+00	
.37+01	.37+01	.37+01	.14+03	.14+03	-.85+02	-.81+02	-.97+02	-.68+02	-.10+02	.24+02	.34+02			
-.36+01	-.36+01	-.36+01	.54+02	.73+02	-.89+02	-.88+02	-.10+03	-.77+02	-.19+02	.92+01	.16+02			
.39-01	.11+00	.19+00	-.21+02	-.18+02	.58+01	.25+01	.19+01	-.83+00	.33+00	.38+01	.50+01			
.26+01	.37+01	.36+01	.49+02	.84+02	.94+02	.98+02	.93+02	.71+02	.35+02					
-.26+01	-.32+01	-.26+01	.20+02	.53+02	.75+02	.87+02	.84+02	.64+02	.29+02					
.61-01	.28+00	.39+00	-.56+01	-.79+01	-.52+01	-.22+01	.10+01	.33+01	.28+01					
		.28+00	.41+01	.55+01									
		.32-01	.37+01	.52+01									
		.46-01	.49-01	.58-01									

TABLE 47

DISTRIBUTION FOR $\gamma = 1.5$, $\theta_0 = 80°$, AND $\sin \alpha_L = 0.4$

.92+00	.94+00	.14+01	.15+01	.27+01	.36+01	.68+01	.83+02	.71+02	.35+02					
-.91+00	-.89+00	-.12+01	-.10+01	-.13+01	-.87+00	.21+01	.74+02	.64+02	.29+02					
-.15-01	-.45-01	-.12+00	-.18+00	-.38+00	-.56+00	-.97+00	-.15+01	-.33+01	-.28+01					
.37+01	.37+01	.37+01	.44+01	.47+01	.41+01	.49+01	.98+02	.12+03	.92+02	.81+02	.51+02			
-.36+01	-.36+01	-.36+01	-.40+01	-.39+01	-.28+01	-.26+01	.88+02	.10+03	.75+02	.57+02	.28+02			
-.39-01	-.11+00	-.19+00	-.32+00	-.42+00	-.47+00	-.68+00	-.29+00	-.40+01	-.68+01	-.87+01	-.69+01			
.31+01	.31+01	.31+01	.32+01	.32+01	.47+01	.16+02	.43+02	.37+02	.39+02	.39+02	.47+02	.58+02	.78+01	
-.31+01	-.31+01	-.31+01	-.31+01	-.30+01	-.43+01	.32+01	.26+02	.21+02	.18+02	.17+02	.15+02	.13+02	.48+00	
-.17-01	-.52-01	-.87-01	-.12+00	-.15+00	-.32+00	-.17+01	-.38+01	-.39+01	-.47+01	-.50+01	-.67+01	-.92+01	-.13+01	
.35+01	.35+01	.35+01	.44+01	.82+01	.12+02	.15+02	.16+02	.20+02	.15+02	.22+02	.22+02	.30+02	.48+02	.22+01
-.35+01	-.35+01	-.35+01	-.39+01	-.53+01	-.42+01	-.22+01	-.22+01	-.12+01	-.86+00	-.28+01	-.40+00	-.52+01	-.91+01	-.92+00
-.92-02	-.27-01	-.46-01	-.18+00	-.80+00	-.14+01	-.20+01	-.21+01	-.27+01	-.21+01	-.30+01	-.33+01	-.45+01	-.77+01	-.33+00
.88+01	.11+02	.90+01	.11+02	.96+01	.90+01	.93+01	.14+02	.11+02	.12+02	.14+02	.17+02	.19+02	.26+02	.27+02
-.87+01	-.11+02	-.83+01	-.92+01	-.76+01	-.65+01	-.62+01	-.75+01	-.69+01	-.62+01	-.78+01	-.73+01	-.93+01	-.13+02	-.16+02
-.73-01	-.38+00	-.43+00	-.79+00	-.72+00	-.72+00	-.82+00	-.16+01	-.12+01	-.13+01	-.15+01	-.22+01	-.24+01	-.34+01	-.36+01
.78+01	.79+01	.89+01	.90+01	.81+01	.92+01	.11+02	.84+01	.10+02	.11+02	.10+02	.15+02	.13+02	.21+02	.36+02
-.78+01	-.78+01	-.86+01	-.84+01	-.73+01	-.79+01	-.93+01	-.68+01	-.83+01	-.86+01	-.83+01	-.10+02	-.10+02	-.16+02	-.30+02
-.29-01	-.87-01	-.21+00	-.29+00	-.28+00	-.46+00	-.74+00	-.58+00	-.63+00	-.87+00	-.65+00	-.13+01	-.10+01	-.17+01	-.32+01
.81+01	.81+01	.82+01	.82+01	.93+01	.94+01	.63+01	.10+02	.10+02	.96+01	.11+02	.12+02	.13+02	.18+02	.37+02
-.81+01	-.81+01	-.80+01	-.79+01	-.87+01	-.86+01	-.57+01	-.95+01	-.92+01	-.81+01	-.99+01	-.10+02	-.11+02	-.16+02	-.35+02
-.99-02	-.29-01	-.49-01	-.67-01	-.16+00	-.19+00	-.77-01	-.16+00	-.23+00	-.29+00	-.16+00	-.32+00	-.39+00	-.56+00	-.12+01

.88-01	.81-01	.68-01	.49-01	-.10+01	-.10+01	-.43-01	-.13+00	-.21+00	-.12+01	.77+00	-.36+00	-.13+01	-.51+00	-.43+00
-.90-01	-.10+00	-.12+00	-.15+00	.71+00	.65+00	-.16+00	-.31+00	-.50+00	.43+00	-.14+01	-.63+00	.26+00	-.85+00	-.64+00
-.49-01	-.14+00	-.24+00	-.34+00	-.51+00	-.62+00	-.42+00	-.72+00	-.82+00	-.82+00	-.95+00	-.97+00	-.10+01	-.11+01	-.96+00
.27+00	.25+00	-.75+00	-.87+00	.32-01	-.11+01	-.34+01	-.24+01	-.64+00	-.19+01	-.77+00	-.35+01	-.22+01	-.16+01	.80+00
-.27+00	-.31+00	.49+00	.37+00	-.69+00	-.15+00	.13+01	.86+00	-.15+01	-.11+01	-.15+01	-.79+00	-.12+01	-.28+01	-.36+01
-.48-01	-.14+00	-.30+00	-.43+00	-.46+00	-.67+00	-.99+00	-.70+00	-.85+00	-.10+01	-.87+00	-.13+01	-.10+01	-.12+01	-.71+00
-.10+01	-.36+01	-.12+01	-.33+01	-.19+01	-.12+01	-.15+01	-.65+01	-.25+01	-.28+01	-.31+01	-.61+01	-.60+01	-.29+01	.54+01
.10+01	.32+01	.58+00	.14+01	-.86-01	-.11+01	-.15+01	-.13+00	-.24+01	-.31+01	-.34+01	-.32+01	-.29+01	-.47+01	-.53+01
-.73-01	-.38+00	-.43+00	-.79+00	-.72+00	-.72+00	-.82+00	-.15+01	-.10+01	-.11+01	-.12+01	-.16+01	-.16+01	-.11+01	.40+00
.35+01	.35+01	.35+01	.27+01	-.10+01	-.48+01	-.85+01	-.74+01	-.11+02	-.82+01	-.79+01	-.11+02	-.72+01	.58+01	.22+01
-.35+01	-.35+01	-.35+01	-.31+01	-.17+01	-.29+01	-.48+01	-.62+01	-.70+01	-.56+01	-.95+01	-.80+01	-.97+01	-.75+01	-.92+00
.92-02	.27-01	.46-01	-.57-01	-.63+00	-.12+01	-.18+01	-.17+01	-.22+01	-.16+01	-.18+01	-.21+01	-.16+01	.69+00	.33+00
.31+01	.31+01	.31+01	.32+01	.32+01	.47+01	-.65+01	-.33+02	-.28+02	-.25+02	-.24+02	-.18+02	.16+02	.50+01	
-.31+01	-.31+01	-.31+01	-.31+01	-.30+01	-.43+01	-.11+02	-.35+02	-.27+02	-.26+02	-.24+02	-.22+02	-.26+01	.78-01	
.17-01	.52-01	.87-01	.12+00	.15+00	.32+00	-.99+00	-.29+01	-.30+01	-.29+01	-.29+01	-.22+01	.28+01	.84+00	
.37+01	.37+01	.37+01	.44+01	.47+01	.41+01	.49+01	-.61+02	-.68+02	-.10+02	.24+02	.34+02			
-.36+01	-.36+01	-.36+01	-.40+01	-.39+01	-.28+01	-.26+01	-.69+02	-.77+02	-.19+02	.92+01	.16+02			
.39-01	.11+00	.19+00	.32+00	.42+00	.47+00	.68+00	.74+00	-.83+00	.33+00	.38+01	.50+01			
.92+00	.94+00	.14+01	.15+01	.27+01	.36+01	.68+01	.83+02	.71+02	.35+02					
-.91+00	-.89+00	-.12+01	-.10+01	-.13+01	-.87+00	.21+01	.74+02	.64+02	.29+02					
.15-01	.45-01	.12+00	.18+00	.38+00	.56+00	.97+00	.15+01	.33+01	.28+01					

TABLE 48

DISTRIBUTION FOR $\gamma = 1.5$, $\theta_0 = 80°$, AND $\sin \alpha_L = 0.6$

.26+01	.26+01	.26+01	.27+01	.34+01	.21+01	.29+01	.38+01	.58+01	.10+02	.63+02	.51+02
-.26+01	-.25+01	-.25+01	-.25+01	-.30+01	-.17+01	-.17+01	-.21+01	-.19+01	.86+00	.41+02	.28+02
-.24-01	-.73-01	-.12+00	-.17+00	-.28+00	-.21+00	-.37+00	-.53+00	-.89+00	-.17+01	-.73+01	-.69+01

.31+01	.31+01	.31+01	.32+01	.32+01	.47+01	.48+01	.49+01	.42+01	.16+02	.39+02	.47+02	.58+02	.78+01
-.31+01	-.31+01	-.31+01	-.31+01	-.30+01	-.43+01	-.42+01	-.41+01	-.31+01	.86+00	.17+02	.15+02	.13+02	.48+00
-.17-01	-.52-01	-.87-01	-.12+00	-.15+00	-.32+00	-.37+00	-.44+00	-.46+00	-.21+01	-.50+01	-.67+01	-.92+01	-.13+01

.35+01	.35+01	.35+01	.35+01	.36+01	.36+01	.36+01	.44+01	.12+02	.15+02	.22+02	.22+02	.30+02	.48+02	.22+01
-.35+01	-.35+01	-.35+01	-.35+01	-.35+01	-.35+01	-.35+01	-.42+01	-.38+01	-.86+00	-.28+01	-.40+00	-.52+01	-.91+01	-.92+00
-.92-02	-.27-01	-.46-01	-.64-01	-.82-01	-.99-01	-.11+00	-.22+00	-.16+01	-.21+01	-.30+01	-.33+01	-.45+01	-.77+01	-.33+00

.38+01	.38+01	.38+01	.38+01	.68+01	.59+01	.71+01	.14+02	.11+02	.12+02	.14+02	.17+02	.19+02	.26+02	.27+02
-.38+01	-.38+01	-.38+01	-.38+01	-.60+01	-.51+01	-.54+01	-.75+01	-.69+01	-.62+01	-.78+01	-.73+01	-.93+01	-.13+02	-.16+02
.16-04	.49-04	.81-04	.11-03	-.34+00	-.27+00	-.47+00	-.16+01	-.12+01	-.13+01	-.15+01	-.22+01	-.24+01	-.34+01	-.36+01

.78+01	.79+01	.89+01	.90+01	.81+01	.92+01	.11+02	.84+01	.10+02	.11+02	.10+02	.15+02	.13+02	.21+02	.36+02
-.78+01	-.78+01	-.86+01	-.84+01	-.73+01	-.79+01	-.93+01	-.68+01	-.83+01	-.86+01	-.83+01	-.10+02	-.10+02	-.16+02	-.30+02
-.29-01	-.87-01	-.21+00	-.29+00	-.28+00	-.46+00	-.74+00	-.58+00	-.63+00	-.87+00	-.65+00	-.13+01	-.10+01	-.17+01	-.32+01

.81+01	.81+01	.82+01	.82+01	.93+01	.94+01	.63+01	.10+02	.10+02	.96+01	.11+02	.12+02	.13+02	.18+02	.37+02
-.81+01	-.81+01	-.80+01	-.79+01	-.87+01	-.86+01	-.57+01	-.95+01	-.92+01	-.81+01	-.99+01	-.10+02	-.11+02	-.16+02	-.35+02
-.99-02	-.29-01	-.49-01	-.67-01	-.16+00	-.19+00	-.77-01	-.16+00	-.23+00	-.29+00	-.16+00	-.32+00	-.39+00	-.56+00	-.12+01

.88-01	.81-01	.68-01	.49-01	-.10+01	-.10+01	-.43-01	-.13+00	-.21+00	-.12+01	.77+00	-.36+00	-.13+01	-.51+00	-.43+00
-.90-01	-.10+00	-.12+00	-.15+00	.71+00	.65+00	-.16+00	-.31+00	-.50+00	.43+00	-.14+01	-.63+00	.26+00	-.85+00	-.64+00
-.49-01	-.14+00	-.24+00	-.34+00	-.51+00	-.62+00	-.42+00	-.72+00	-.82+00	-.82+00	-.95+00	-.97+00	-.10+01	-.11+01	-.96+00

.27+00	.25+00	-.75+00	-.87+00	.32-01	-.11+01	-.34+01	-.24+01	-.64+00	-.19+01	-.77+00	-.35+01	-.22+01	-.16+01	.80+00
-.27+00	-.31+00	.49+00	.37+00	-.69+00	-.15+00	.13+01	.86+00	-.15+01	-.11+01	-.15+01	-.79+00	-.12+01	-.28+01	-.36+01
-.48-01	-.14+00	-.30+00	-.43+00	-.46+00	-.67+00	-.99+00	-.70+00	-.85+00	-.10+01	-.87+00	-.13+01	-.10+01	-.12+01	-.71+00

.38+01	.38+01	.38+01	.38+01	.93+00	.18+01	.62+00	-.65+01	-.25+01	-.28+01	-.31+01	-.61+01	-.60+01	-.29+01	.54+01
-.38+01	-.38+01	-.38+01	-.38+01	-.17+01	-.26+01	-.23+01	-.13+00	-.24+01	-.31+01	-.34+01	-.32+01	-.29+01	-.47+01	-.53+01
-.16-04	-.49-04	-.81-04	-.11-03	-.34+00	-.27+00	-.47+00	-.15+01	-.10+01	-.11+01	-.12+01	-.16+01	-.16+01	-.11+01	.40+00

.35+01	.35+01	.35+01	.35+01	.36+01	.36+01	.36+01	.44+01	-.35+01	-.82+01	-.79+01	-.11+02	-.72+01	.58+01	.22+01
-.35+01	-.35+01	-.35+01	-.35+01	-.35+01	-.35+01	-.35+01	-.42+01	-.44+01	-.56+01	-.95+01	-.80+01	-.97+01	-.75+01	-.92+00
.92-02	.27-01	.46-01	.64-01	.82-01	.99-01	.11+00	.22+00	-.10+01	-.16+01	-.18+01	-.21+01	-.16+01	.69+00	.33+00

.31+01	.31+01	.31+01	.32+01	.32+01	.47+01	.48+01	.49+01	.42+01	-.22+01	-.24+02	-.18+02	.16+02	.50+01	
-.31+01	-.31+01	-.31+01	-.31+01	-.30+01	-.43+01	-.42+01	-.41+01	-.31+01	-.96+01	-.24+02	-.22+02	-.26+01	.78-01	
.17-01	.52-01	.87-01	.12+00	.15+00	.32+00	.37+00	.44+00	.46+00	-.37+00	-.29+01	-.22+01	.28+01	.84+00	

.26+01	.26+01	.26+01	.27+01	.34+01	.21+01	.29+01	.38+01	.58+01	.10+02	.27+02	.34+02	
-.26+01	-.25+01	-.25+01	-.25+01	-.30+01	-.17+01	-.17+01	-.21+01	-.19+01	.86+00	.11+02	.16+02	
.24-01	.73-01	.12+00	.17+00	.28+00	.21+00	.37+00	.53+00	.89+00	.17+01	.38+01	.50+01	

TABLE 49

DISTRIBUTION FOR $\gamma = 1.5$, $\theta_0 = 80°$, AND $\sin \alpha_L = 0.8$

.16+01	.16+01	.16+01	.16+01	.16+01	.16+01	.17+01	.25+01	.17+01	.26+01	.18+01	.38+01	.10+02	.78+01	
-.16+01	-.16+01	-.16+01	-.16+01	-.16+01	-.16+01	-.15+01	-.22+01	-.13+01	-.20+01	-.11+01	-.20+01	-.14+01	.48+00	
-.78-02	-.23-01	-.39-01	-.54-01	-.69-01	-.84-01	-.98-01	-.20+00	-.16+00	-.27+00	-.22+00	-.53+00	-.16+01	-.13+01	

.35+01	.35+01	.35+01	.35+01	.36+01	.36+01	.36+01	.44+01	.44+01	.36+01	.73+01	.55+01	.23+02	.48+02	.22+01
-.35+01	-.35+01	-.35+01	-.35+01	-.35+01	-.35+01	-.35+01	-.42+01	-.41+01	-.32+01	-.61+01	-.42+01	-.70+01	-.91+01	-.92+00
-.92-02	-.27-01	-.46-01	-.64-01	-.82-01	-.99-01	-.11+00	-.22+00	-.27+00	-.25+00	-.62+00	-.60+00	-.33+01	-.77+01	-.33+00

.38+01	.38+01	.38+01	.38+01	.38+01	.38+01	.38+01	.38+01	.47+01	.47+01	.93+01	.14+02	.19+02	.26+02	.27+02
-.38+01	-.38+01	-.38+01	-.38+01	-.38+01	-.38+01	-.38+01	-.38+01	-.46+01	-.47+01	-.70+01	-.70+01	-.93+01	-.13+02	-.16+02
.16-04	.49-04	.81-04	.11-03	.14-03	.16-03	.18-03	-.34-01	-.10+00	-.97-01	-.70+00	-.17+01	-.24+01	-.34+01	-.36+01

.40+01	.40+01	.40+01	.40+01	.40+01	.40+01	.62+01	.62+01	.72+01	.11+02	.10+02	.15+02	.13+02	.21+02	.36+02
-.40+01	-.40+01	-.40+01	-.40+01	-.40+01	-.40+01	-.57+01	-.54+01	-.64+01	-.86+01	-.83+01	-.10+02	-.10+02	-.16+02	-.30+02
.98-02	.29-01	.48-01	.68-01	.87-01	.10+00	-.88-01	-.30+00	-.16+00	-.87+00	-.65+00	-.13+01	-.10+01	-.17+01	-.32+01

.61+01	.61+01	.61+01	.62+01	.93+01	.94+01	.63+01	.10+02	.10+02	.96+01	.11+02	.12+02	.13+02	.18+02	.37+02
-.61+01	-.61+01	-.61+01	-.60+01	-.87+01	-.86+01	-.57+01	-.95+01	-.92+01	-.81+01	-.99+01	-.10+02	-.11+02	-.16+02	-.35+02
.62-02	.18-01	.30-01	.43-01	-.16+00	-.19+00	-.77-01	-.16+00	-.23+00	-.29+00	-.16+00	-.32+00	-.39+00	-.56+00	-.12+01

.20+01	.20+01	.20+01	.20+01	-.10+01	-.10+01	-.43-01	-.13+00	-.21+00	-.12+01	.77+00	-.36+00	-.13+01	-.51+00	-.43+00
-.20+01	-.20+01	-.20+01	-.20+01	.71+00	.65+00	-.16+00	-.31+00	-.50+00	.43+00	-.14+01	-.63+00	.26+00	-.85+00	-.64+00
-.33-01	-.10+00	-.16+00	-.23+00	-.51+00	-.62+00	-.42+00	-.72+00	-.82+00	-.82+00	-.95+00	-.97+00	-.10+01	-.11+01	-.96+00

.40+01	.40+01	.40+01	.40+01	.40+01	.40+01	.19+01	-.26+00	.27+01	-.19+01	-.77+00	-.35+01	-.22+01	-.16+01	.80+00
-.40+01	-.40+01	-.40+01	-.40+01	-.40+01	-.40+01	-.22+01	-.55+00	-.34+01	-.11+01	-.15+01	-.79+00	-.12+01	-.28+01	-.36+01
-.98-02	-.29-01	-.48-01	-.68-01	-.87-01	-.10+00	-.33+00	-.42+00	-.38+00	-.10+01	-.87+00	-.13+01	-.10+01	-.12+01	-.71+00

.38+01	.38+01	.38+01	.38+01	.38+01	.38+01	.38+01	.38+01	.47+01	.47+01	.20+01	-.35+01	-.60+01	-.29+01	.54+01
-.38+01	-.38+01	-.38+01	-.38+01	-.38+01	-.38+01	-.38+01	-.38+01	-.46+01	-.47+01	-.42+01	-.35+01	-.29+01	-.47+01	-.53+01
-.16-04	-.49-04	-.81-04	-.11-03	-.14-03	-.16-03	-.18-03	.34-01	.10+00	.97-01	-.42+00	-.11+01	-.16+01	-.11+01	.40+00

.35+01	.35+01	.35+01	.35+01	.36+01	.36+01	.36+01	.44+01	.44+01	.36+01	.73+01	.55+01	.42+00	.58+01	.22+01
-.35+01	-.35+01	-.35+01	-.35+01	-.35+01	-.35+01	-.35+01	-.42+01	-.41+01	-.32+01	-.61+01	-.42+01	-.79+01	-.75+01	-.92+00
.92-02	.27-01	.46-01	.64-01	.82-01	.99-01	.11+00	.22+00	.27+00	.25+00	.62+00	.60+00	-.41+00	.69+00	.33+00

.16+01	.16+01	.16+01	.16+01	.16+01	.16+01	.17+01	.25+01	.17+01	.26+01	.18+01	.38+01	.10+02	.50+01	
-.16+01	-.16+01	-.16+01	-.16+01	-.16+01	-.16+01	-.15+01	-.22+01	-.13+01	-.20+01	-.11+01	-.20+01	-.14+01	.78-01	
.78-02	.23-01	.39-01	.54-01	.69-01	.84-01	.98-01	.20+00	.16+00	.27+00	.22+00	.53+00	.16+01	.84+00	

TABLE 50

DISTRIBUTION FOR $\gamma = 1.5$, $\theta_0 = 90°$, AND $\sin \alpha_L = 0$

.49+02	.79+02	.15+03	.26+03	.57+03	.52+03	.35+03	.25+03	.21+03	.14+03	.84+02	.20+02			
-.38+02	-.64+01	.58+02	.16+03	.42+03	.45+03	.33+03	.24+03	.19+03	.12+03	.66+02	.14+02			
.32+01	.10+02	.18+02	.27+02	.55+02	.35+02	.87+01	-.27+01	-.95+01	-.11+02	-.86+01	-.23+01			
.35+03	.54+03	.13+04	.97+03	.21+03	.16+02	.17+02	.25+02	.28+02	.34+02	.47+02	.90+02	.66+02	.11+02	
-.30+03	-.16+03	.28+03	.43+03	.11+03	-.62+01	-.37+01	-.15+01	.14+01	.63+01	.14+02	.34+02	.16+02	.13+01	
.25+02	.83+02	.21+03	.14+03	.24+02	-.24+01	-.28+01	-.41+01	-.46+01	-.55+01	-.73+01	-.13+02	-.10+02	-.18+01	
.15+04	.33+04	.10+04	.81+01	.85+01	.70+01	.90+01	.94+01	.98+01	.12+02	.13+02	.18+02	.26+02	.45+02	
-.12+04	-.54+03	.13+03	-.74+01	-.72+01	-.54+01	-.63+01	-.61+01	-.53+01	-.58+01	-.51+01	-.57+01	-.60+01	-.81+01	
.12+03	.54+03	.17+03	-.57+00	-.73+00	-.72+00	-.10+01	-.11+01	-.13+01	-.18+01	-.19+01	-.28+01	-.42+01	-.74+01	
.10+05	.35+04	.72+01	.72+01	.73+01	.92+01	.75+01	.96+01	.98+01	.10+02	.12+02	.14+02	.17+02	.29+02	.27+02
-.68+04	-.88+03	-.70+01	-.69+01	-.68+01	-.83+01	-.66+01	-.81+01	-.77+01	-.76+01	-.86+01	-.95+01	-.10+02	-.15+02	-.14+02
.11+04	.56+03	-.23+00	-.32+00	-.41+00	-.65+00	-.60+00	-.85+00	-.10+01	-.10+01	-.14+01	-.18+01	-.22+01	-.40+01	-.38+01
.41+05	.76+01	.77+01	.77+01	.77+01	.77+01	.78+01	.98+01	.99+01	.79+01	.12+02	.12+02	.16+02	.18+02	.39+02
-.30+05	-.76+01	-.76+01	-.76+01	-.75+01	-.75+01	-.75+01	-.92+01	-.92+01	-.73+01	-.10+02	-.10+02	-.14+02	-.15+02	-.32+02
.40+04	-.86-01	-.14+00	-.19+00	-.25+00	-.30+00	-.36+00	-.54+00	-.57+00	-.52+00	-.86+00	-.95+00	-.13+01	-.17+01	-.37+01
.44+09	.79+01	.79+01	.79+01	.79+01	.79+01	.79+01	.79+01	.99+01	.99+01	.12+02	.12+02	.12+02	.20+02	.40+02
-.44+09	-.79+01	-.79+01	-.79+01	-.79+01	-.79+01	-.79+01	-.79+01	-.98+01	-.99+01	-.11+02	-.11+02	-.11+02	-.19+02	-.39+02
.26+07	-.29-01	-.48-01	-.67-01	-.86-01	-.10+00	-.12+00	-.13+00	-.21+00	-.19+00	-.29+00	-.32+00	-.34+00	-.62+00	-.13+01

TABLE 51

DISTRIBUTION FOR $\gamma = 1.5$, $\theta_0 = 90°$, AND $\sin \alpha_L = 0.2$

.22+01	.29+01	.60+01	.13+03	.57+03	.52+03	.35+03	.25+03	.21+03	.14+03	.84+02	.20+02			
-.21+01	-.21+01	-.20+01	.85+02	.42+03	.45+03	.33+03	.24+03	.19+03	.12+03	.66+02	.14+02			
-.95-01	-.32+00	-.91+00	.11+02	.55+02	.35+02	.87+01	-.27+01	-.95+01	-.11+02	-.86+01	-.23+01			
.93+01	.12+02	.11+02	.86+02	.21+03	.16+02	.17+02	.25+02	.28+02	.34+02	.47+02	.90+02	.66+02	.11+02	
-.92+01	-.12+02	-.96+01	.33+02	.11+03	-.62+01	-.37+01	-.15+01	.14+01	.63+01	.14+02	.34+02	.16+02	.13+01	
-.15+00	-.74+00	-.10+01	.84+01	.24+02	-.24+01	-.28+01	-.41+01	-.46+01	-.55+01	-.73+01	-.13+02	-.10+02	-.18+01	
.63+01	.63+01	.64+01	.81+01	.85+01	.70+01	.90+01	.94+01	.98+01	.12+02	.13+02	.18+02	.26+02	.45+02	
-.63+01	-.62+01	-.61+01	-.74+01	-.72+01	-.54+01	-.63+01	-.61+01	-.53+01	-.58+01	-.51+01	-.57+01	-.60+01	-.81+01	
-.62-01	-.18+00	-.31+00	-.57+00	-.73+00	-.72+00	-.10+01	-.11+01	-.13+01	-.18+01	-.19+01	-.28+01	-.42+01	-.74+01	
.71+01	.71+01	.72+01	.72+01	.73+01	.92+01	.75+01	.96+01	.98+01	.10+02	.12+02	.14+02	.17+02	.29+02	.27+02
-.71+01	-.71+01	-.70+01	-.69+01	-.68+01	-.83+01	-.66+01	-.81+01	-.77+01	-.76+01	-.86+01	-.95+01	-.10+02	-.15+02	-.14+02
-.46-01	-.14+00	-.23+00	-.32+00	-.41+00	-.65+00	-.60+00	-.85+00	-.10+01	-.10+01	-.14+01	-.18+01	-.22+01	-.40+01	-.38+01
.76+01	.76+01	.77+01	.77+01	.77+01	.77+01	.78+01	.98+01	.99+01	.79+01	.12+02	.12+02	.16+02	.18+02	.39+02
-.76+01	-.76+01	-.76+01	-.76+01	-.75+01	-.75+01	-.75+01	-.92+01	-.92+01	-.73+01	-.10+02	-.10+02	-.14+02	-.15+02	-.32+02
-.28-01	-.86-01	-.14+00	-.19+00	-.25+00	-.30+00	-.36+00	-.54+00	-.57+00	-.52+00	-.86+00	-.95+00	-.13+01	-.17+01	-.37+01
.79+01	.79+01	.79+01	.79+01	.79+01	.79+01	.79+01	.79+01	.99+01	.99+01	.12+02	.12+02	.12+02	.20+02	.40+02
-.79+01	-.79+01	-.79+01	-.79+01	-.79+01	-.79+01	-.79+01	-.79+01	-.98+01	-.99+01	-.11+02	-.11+02	-.11+02	-.19+02	-.39+02
-.97-02	-.29-01	-.48-01	-.67-01	-.86-01	-.10+00	-.12+00	-.13+00	-.21+00	-.19+00	-.29+00	-.32+00	-.34+00	-.62+00	-.13+01

TABLE 52

DISTRIBUTION FOR $\gamma = 1.5$, $\theta_0 = 90°$, AND $\sin \alpha_L = 0.4$

						.17+01	.18+02	.21+03	.21+03	.14+03	.84+02	.20+02	
						.40+00	.11+02	.19+03	.19+03	.12+03	.66+02	.14+02	
						-.27+00	-.23+01	-.42+01	-.95+01	-.11+02	-.86+01	-.23+01	
.52+01	.77+01	.94+01	.97+01	.13+02	.16+02	.17+02	.25+02	.28+02	.34+02	.47+02	.90+02	.66+02	.11+02
-.52+01	-.73+01	-.81+01	-.70+01	-.70+01	-.62+01	-.37+01	-.15+01	.14+01	.63+01	.14+02	.34+02	.16+02	.13+01
-.75-01	-.40+00	-.77+00	-.10+01	-.18+01	-.24+01	-.28+01	-.41+01	-.46+01	-.55+01	-.73+01	-.13+02	-.10+02	-.18+01
.63+01	.63+01	.64+01	.81+01	.85+01	.70+01	.90+01	.94+01	.98+01	.12+02	.13+02	.18+02	.26+02	.45+02
-.63+01	-.62+01	-.61+01	-.74+01	-.72+01	-.54+01	-.63+01	-.61+01	-.53+01	-.58+01	-.51+01	-.57+01	-.60+01	-.81+01
-.62-01	-.18+00	-.31+00	-.57+00	-.73+00	-.72+00	-.10+01	-.11+01	-.13+01	-.18+01	-.19+01	-.28+01	-.42+01	-.74+01
.71+01	.71+01	.72+01	.72+01	.73+01	.92+01	.75+01	.96+01	.98+01	.10+02	.12+02	.14+02	.17+02	.29+02	.27+02
-.71+01	-.71+01	-.70+01	-.69+01	-.68+01	-.83+01	-.66+01	-.81+01	-.77+01	-.76+01	-.86+01	-.95+01	-.10+02	-.15+02	-.14+02
-.46-01	-.14+00	-.23+00	-.32+00	-.41+00	-.65+00	-.60+00	-.85+00	-.10+01	-.10+01	-.14+01	-.18+01	-.22+01	-.40+01	-.38+01
.76+01	.76+01	.77+01	.77+01	.77+01	.77+01	.78+01	.98+01	.99+01	.79+01	.12+02	.12+02	.16+02	.18+02	.39+02
-.76+01	-.76+01	-.76+01	-.76+01	-.75+01	-.75+01	-.75+01	-.92+01	-.92+01	-.73+01	-.10+02	-.10+02	-.14+02	-.15+02	-.32+02
-.28-01	-.86-01	-.14+00	-.19+00	-.25+00	-.30+00	-.36+00	-.54+00	-.57+00	-.52+00	-.86+00	-.95+00	-.13+01	-.17+01	-.37+01
.79+01	.79+01	.79+01	.79+01	.79+01	.79+01	.79+01	.79+01	.99+01	.99+01	.12+02	.12+02	.12+02	.20+02	.40+02
-.79+01	-.79+01	-.79+01	-.79+01	-.79+01	-.79+01	-.79+01	-.79+01	-.98+01	-.99+01	-.11+02	-.11+02	-.11+02	-.19+02	-.39+02
-.97-02	-.29-01	-.48-01	-.67-01	-.86-01	-.10+00	-.12+00	-.13+00	-.21+00	-.19+00	-.29+00	-.32+00	-.34+00	-.62+00	-.13+01

TABLE 53

DISTRIBUTION FOR $\gamma = 1.5$, $\theta_0 = 90°$, AND $\sin \alpha_L = 0.6$

									.34+01	.70+02	.20+02		
									.22+01	.53+02	.14+02		
									-.43+00	-.75+01	-.23+01		
						.33+01	.35+01	.96+01	.14+02	.29+02	.47+02	.90+02	.66+02	.11+02
						-.20+01	-.18+01	-.30+01	-.18+01	.40+01	.14+02	.34+02	.16+02	.13+01
						-.44+00	-.50+00	-.15+01	-.24+01	-.47+01	-.73+01	-.13+02	-.10+02	-.18+01
.63+01	.63+01	.64+01	.81+01	.85+01	.70+01	.90+01	.94+01	.98+01	.12+02	.13+02	.18+02	.26+02	.45+02
-.63+01	-.62+01	-.61+01	-.74+01	-.72+01	-.54+01	-.63+01	-.61+01	-.53+01	-.58+01	-.51+01	-.57+01	-.60+01	-.81+01
-.62-01	-.18+00	-.31+00	-.57+00	-.73+00	-.72+00	-.10+01	-.11+01	-.13+01	-.18+01	-.19+01	-.28+01	-.42+01	-.74+01
.71+01	.71+01	.72+01	.72+01	.73+01	.92+01	.75+01	.96+01	.98+01	.10+02	.12+02	.14+02	.17+02	.29+02	.27+02
-.71+01	-.71+01	-.70+01	-.69+01	-.68+01	-.83+01	-.66+01	-.81+01	-.77+01	-.76+01	-.86+01	-.95+01	-.10+02	-.15+02	-.14+02
-.46-01	-.14+00	-.23+00	-.32+00	-.41+00	-.65+00	-.60+00	-.85+00	-.10+01	-.10+01	-.14+01	-.18+01	-.22+01	-.40+01	-.38+01
.76+01	.76+01	.77+01	.77+01	.77+01	.77+01	.78+01	.98+01	.99+01	.79+01	.12+02	.12+02	.16+02	.18+02	.39+02
-.76+01	-.76+01	-.76+01	-.76+01	-.75+01	-.75+01	-.75+01	-.92+01	-.92+01	-.73+01	-.10+02	-.10+02	-.14+02	-.15+02	-.32+02
-.28-01	-.86-01	-.14+00	-.19+00	-.25+00	-.30+00	-.36+00	-.54+00	-.57+00	-.52+00	-.86+00	-.95+00	-.13+01	-.17+01	-.37+01
.79+01	.79+01	.79+01	.79+01	.79+01	.79+01	.79+01	.79+01	.99+01	.99+01	.12+02	.12+02	.12+02	.20+02	.40+02
-.79+01	-.79+01	-.79+01	-.79+01	-.79+01	-.79+01	-.79+01	-.79+01	-.98+01	-.99+01	-.11+02	-.11+02	-.11+02	-.19+02	-.39+02
-.97-02	-.29-01	-.48-01	-.67-01	-.86-01	-.10+00	-.12+00	-.13+00	-.21+00	-.19+00	-.29+00	-.32+00	-.34+00	-.62+00	-.13+01

TABLE 54

DISTRIBUTION FOR $\gamma = 1.5$, $\theta_0 = 90°$, AND $\sin \alpha_L = 0.8$

												.16+02	.11+02		
												.14+01	.13+01		
												-.27+01	-.18+01		
										.20+01	.63+01	.13+02	.26+02	.45+02	
										-.11+01	-.31+01	-.46+01	-.60+01	-.81+01	
										-.29+00	-.92+00	-.21+01	-.42+01	-.74+01	
.36+01	.36+01	.36+01	.37+01	.37+01	.74+01	.75+01	.96+01	.98+01	.10+02	.12+02	.14+02	.17+02	.29+02	.27+02	
-.36+01	-.36+01	-.36+01	-.35+01	-.35+01	-.67+01	-.66+01	-.81+01	-.77+01	-.76+01	-.86+01	-.95+01	-.10+02	-.15+02	-.14+02	
-.21-01	-.63-01	-.10+00	-.14+00	-.18+00	-.52+00	-.60+00	-.85+00	-.10+01	-.10+01	-.14+01	-.18+01	-.22+01	-.40+01	-.38+01	
.76+01	.76+01	.77+01	.77+01	.77+01	.77+01	.78+01	.98+01	.99+01	.79+01	.12+02	.12+02	.16+02	.18+02	.39+02	
-.76+01	-.76+01	-.76+01	-.76+01	-.75+01	-.75+01	-.75+01	-.92+01	-.92+01	-.73+01	-.10+02	-.10+02	-.14+02	-.15+02	-.32+02	
-.28-01	-.86-01	-.14+00	-.19+00	-.25+00	-.30+00	-.36+00	-.54+00	-.57+00	-.52+00	-.86+00	-.95+00	-.13+01	-.17+01	-.37+01	
.79+01	.79+01	.79+01	.79+01	.79+01	.79+01	.79+01	.79+01	.99+01	.99+01	.12+02	.12+02	.12+02	.20+02	.40+02	
-.79+01	-.79+01	-.79+01	-.79+01	-.79+01	-.79+01	-.79+01	-.79+01	-.98+01	-.99+01	-.11+02	-.11+02	-.11+02	-.19+02	-.39+02	
-.97-02	-.29-01	-.48-01	-.67-01	-.86-01	-.10+00	-.12+00	-.13+00	-.21+00	-.19+00	-.29+00	-.32+00	-.34+00	-.62+00	-.13+01	