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THE HIGHLY COUPLED SYSTEM - A GENERAL APPROACH TO THE PASSIVE ATTITUDE STABILIZATION OF SPACE VEHICLES

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THE HIGHLY COUPLED SYSTEM - A GENERAL APPROACH TO THE PASSIVE ATTITUDE STABILIZATION OF SPACE VEHICLES[1]

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SUMMARY

Implicit in the design philosophy of most vehicle control and stabilization systems is freedom of choice to promote the types of configuration symmetry which simplify the task of mathematical analysis. Such an approach can unduly restrict the class of acceptable systems and may result in a greater than necessary mechanical complication.

This report supports the above observation by showing that space vehicles having certain types of unsymmetrical mass distribution can be passively stabilized using fewer of the same components than is required with a symmetrical mass distribution. The saving is a direct result of the inertial coupling between the degrees of freedom. This coupling, when used to the best advantage, ensures mutual dependence of all the degrees of freedom and usually precludes mathematical simplification.

Examples are given which show that the highly coupled system approach can be used in the design of space vehicle passive attitude stabilization systems when any of the three major ambient force fields (solar electromagnetic, earth magnetic, and gravitation) are used to provide the prime stabilizing torques. Some of the unique synthesis problems arising from the mathematical complexity are discussed.

INTRODUCTION

Implicit in the design philosophy of most vehicle control and stabilization systems is freedom of choice to promote advantageous types of vehicle configuration symmetry. For example, symmetry of mass and of force or moment distributions are often employed to simplify a control system design by minimizing control interaction. Almost without exception, a convenient result of symmetry is both conceptual and analytical simplification. It is important, however, to recognize that conceptual and analytical simplification cannot be regarded, in themselves, as fundamental reasons for the adoption of configuration symmetry. Indeed, if such an assumption is made, it can unduly restrict the class of acceptable systems and may result in a greater than necessary hardware complication.

This report supports the above observation by showing that space vehicles having certain types of asymmetrical mass distribution can be passively

[1]Presented at the Third Congress of International Federation of Automatic Control, London, June 1966.

stabilized with fewer of the same components than are required with a symmetrical mass distribution. This is easily understood when it is recognized that symmetrical mass distributions are usually associated with dynamic decoupling of some of the degrees of freedom of the system. Since, by definition, mechanical energy cannot flow between decoupled degrees of freedom, the complete removal of all energy resulting from disturbances can only be accomplished by providing an energy conversion device for each of the decoupled degrees of freedom. On the other hand, asymmetrical mass distributions can always be found which do not permit decoupling and these provide the possibility of removing all mechanical energy from the system with a single energy conversion device. Although the principle of this approach is relatively simple, there remains the vital question of whether or not it is possible to find an asymmetrical mass distribution for which the vehicle, including the energy conversion device, is stable. If this is possible, then it is further required to know whether the degree of stability is compatible, at least in the broad sense, with the mission requirements. Although it is not possible, as yet, to give a general answer to these questions, even for the relatively simple class of passively stabilized vehicles, it is possible to demonstrate the feasibility of the technique for particular systems of practical interest. This is the main aim of this report.

The report starts with a description of the essential features of coupled passive stabilization systems which may use one or more of the three major ambient force fields (solar electromagnetic, earth magnetic, and earth gravitational). Two of these systems are then analyzed, the first in considerably more detail than the second. The first uses the gravitational field to provide both the stabilizing and damping torques for an earth-pointing satellite. The second uses the solar electromagnetic field to provide both the stabilizing and damping torques for a sun-pointing interplanetary probe in a heliocentric orbit. A brief description is given of the type of multivariable analysis used, which in this case is a method of finding the system which has optimum damping.

NOMENCLATURE

General

$\bar{o}_j$ $(j=1,2,3)$ unit vectors defining orthogonal reference frame fixed in the satellite orbit such that $\bar{o}_3$ is directed towards the center of the orbit, $\bar{o}_1$ is in the direction of motion, and $\bar{o}_2$ completes the right-hand set

$\bar{v}_j$ $(j=1,2,3)$ unit vectors defining an orthogonal reference frame fixed in the main satellite body; in steady state (no disturbances) $\bar{v}_j = \bar{o}_j$ $(j=1,2,3)$. $\bar{v}_j$ $(j=1,2,3)$ is related to $\bar{o}_j$ $(j=1,2,3)$ by small angle rotations of φ about $\bar{o}_1$, θ about $\bar{o}_2$, and ψ about $\bar{o}_3$

$\overline{b}_j$ $(j = 1,2,3)$ — unit vectors along principal inertial axes of the main satellite body

I_j $(j = 1,2,3)$ — principal inertias of main satellite body

I_d — inertia of damper rod

φ,θ,ψ — small roll, pitch, and yaw angles relating $\overline{v}_j$ $(j = 1,2,3)$ to $\overline{o}_j$ $(j = 1,2,3)$

D — damping constant associated with angular rate damper

K — spring constant associated with spring that restrains motion of damper relative to main satellite

Applicable to Gravity-Oriented, Gravity-Damped System

$\overline{c}_j$ $(j = 1,2,3)$ — unit vectors defining orthogonal reference frame fixed in main satellite body; $\overline{c}_j$ $(j = 1,2,3)$ frame oriented relative to $\overline{b}_j$ $(j = 1,2,3)$ frame by Euler angle sequence Ψ about $\overline{b}_3$, Θ about $\overline{b}_2$, and Φ about $\overline{b}_1$

$\overline{d}_j$ $(j = 1,2,3)$ — unit vectors defining orthogonal reference frame fixed in damper rod such that $\overline{d}_1$ is along axis of the rod, and $\overline{d}_2$ along the hinge axis. In steady state (no disturbances) $\overline{d}_j = \overline{c}_j$ $(j = 1,2,3)$

δ — angle between $\overline{c}_1$ and $\overline{v}_1$ for configurations such that $\Theta = 0$ and $\Phi = 0$ (positive value of δ is positive rotation about $\overline{v}_3$)

γ — angle between $\overline{b}_1$ and $\overline{v}_1$ for configurations such that $\Theta = 0$ and $\Phi = 0$ (positive value of γ is negative rotation about $\overline{v}_3$)

B — dimensionless damper constant $D/\omega_o I_d$

C — dimensionless spring constant $K/\omega_o^2 I_d$

Applicable to Solar-Oriented, Solar-Damped System

$\overline{c}_j$ $(j = 1,2,3)$ — unit vectors defining an orthogonal reference frame fixed in main satellite body such that $\overline{c}_1$ makes an angle ζ to $\overline{b}_1\overline{b}_3$ plane and η to $\overline{b}_1\overline{b}_2$ plane (positive ζ and η are positive rotations about $\overline{b}_2$ and $\overline{b}_3$, respectively)

$\overline{d}_j$ $(j = 1,2,3)$ — unit vectors defining an orthogonal reference frame fixed in the damper body such that $\overline{d}_1$ is along the axis of the rod. In steady state (no disturbances) $\overline{d}_j = \overline{c}_j$ $(j = 1,2,3)$

$\overline{p}_j$ $(j = 1,2,3)$ unit vectors defining an orthogonal reference frame fixed in the damper such that in steady state (no disturbances) $\overline{p}_3 = \overline{o}_3$

$\overline{l}$ unit vector in direction of hinge axis: lies in the $\overline{b}_1\overline{b}_3$ plane and makes an angle ξ relative to $\overline{b}_1$ axis (positive is positive rotation about $\overline{b}_2$)

k_{bj} $(j = 1,2,3)$ solar spring constants of main satellite body about $\overline{b}_j$ $(j = 1,2,3)$

k_{pj} $(j = 1,2,3)$ solar spring constants of damper body about $\overline{p}_j$ $(j = 1,2,3)$ axes, respectively

THE TECHNIQUE OF PASSIVE ATTITUDE STABILIZATION

For many important missions, requiring attitude stabilization relative to well-defined directions in space, it is possible to choose an ambient force field and arrange the satellite interaction with it to provide adequate stabilizing torques relative to the desired directions. With few exceptions, the resulting dynamical behavior is typified by the existence of energy-type invariants which, in turn, imply the existence of undamped modes of motion. A general solution to the problem of damping these modes of motion, in a passive manner, is to connect to the satellite, through viscous dampers and possibly springs, one or more auxiliary bodies. These bodies are designed to interact with force fields which need not be the same as those which interact with the satellite. The satellite and auxiliary bodies are so configured that their interaction with the selected force fields results in relative motion between them whenever the system is disturbed from equilibrium. This permits the mechanical energy associated with any disturbance to be removed from the system by the viscous dampers. If any one of the auxiliary bodies, in its design equilibrium position, is unstable (when not connected to the satellite), then it must be connected to the satellite by means of a spring in addition to a viscous damper.

It is important to recognize, at this stage, that passive satellite attitude systems are subject to three types of error classified according to the type of source. The first is caused by unwanted interaction of the satellite with force fields other than those which provide the stabilizing and damping torques, and by micrometeorite bombardment. These can be termed errors due to external disturbances. The second type of error is that due to any time varying accelerations of the complete system, such as occurs when the satellite orbit is eccentric. The third type of error is that which exists even in the absence of external disturbances and nonuniform accelerations and is, in large measure, inherent in the design. Thus, if both the satellite and its auxiliary bodies are designed to interact with the same force field, then errors occur because the lines of force may not coincide exactly with the directions about which attitude stabilization is required. If, on the other hand, the auxiliary bodies are designed to interact with a force field other than that which interacts with the satellite, then, since none of the principal ambient fields have identical spatial distribution, the equilibrium position of the auxiliary bodies relative to the satellite varies according to position in the orbit. The resulting motion of the auxiliary bodies relative to the satellite imposes torques on the satellite, which are transmitted to it through the springs and dampers, and cause additional inherent attitude errors.

The practical mechanization of the technique described above is illustrated in the next section, where several possible configurations are described. Each of these dynamically coupled stabilization systems utilizes a single auxiliary body, which will be referred to as the damping body or, simply, the damper.

METHOD OF SYSTEM ANALYSIS

Undoubtedly the two most important performance characteristics of any satellite attitude stabilization system are the damping time and the steady-state pointing error. The damping dictates how long it takes the system to acquire the steady state after injection into orbit, and how long it takes to reacquire after being disturbed. The steady-state pointing error is intimately connected with the ultimate mission requirements of the satellite. The analysis technique adopted here is not so much aimed at determining the best system to satisfy a given set of requirements, as in establishing the broad feasibility of using coupling to simplify system mechanization and to demonstrate the kind of system performance to be expected. The specific procedure used depends primarily on the evaluation of system parameters yielding maximum damping for various fixed values of one or more of the system parameters. The response of these best damped systems to frequencies corresponding to those of the principal disturbances is then used to obtain some idea of their relative steady-state behavior. These parts of the analysis are based on the linear autonomous equations approximating the system behavior. The rationale justifying the use of the linear equations is that if the system has any practical value, then, apart from the period of initial acquisition, it is certain to be operating in the region where the linear equations are valid. The system or systems which appear to have acceptable damping and frequency response are evaluated to see how sensitive they are to parameter variations. In addition, they are simulated with a large angle digital computer program to check their acquisition behavior.

The main difficulty involved in the adopted method of analysis is that of finding the system which has the greatest damping in the presence of various system constraints. The constraints are either imposed by the physics of the problem or are arbitrary, as when a parameter is fixed at a given value. Suppose σ_i, $i = 1 \ldots m$, are the distinct real parts of the roots of the characteristic stability polynomial. The problem is to find the set of system parameters (n in number) which minimize, over the parameter space R^n, the maximum of the set of real numbers σ_i. Since the σ_i are all negative for a stable system, and it is usually less confusing to deal with a set of positive numbers, the objective function can be written as $\max\limits_{R^n}\left[\min\limits_{i}(\sigma_i')\right]$ where $\sigma_i' = -\sigma_i$.

An application of the method of steepest ascent is used to solve this problem, although some modifications have to be made to overcome a difficulty which arises when a set of functions (σ_i') is involved rather than just a single function. To understand the nature of this difficulty, consider the case where, for a given set of parameters, no two of the σ_i' have the same value. Then the conventional method of steepest ascent may be applied directly to increase the σ_i' with the smallest value. This process may be continued until two or more of the σ_i' become equal. It is then no longer clear which is the best direction - in the parameter space - along which to continue the steepest ascent calculation. Herein lies the difficulty. It is resolved in the appendix, where it is shown that the best direction along which to proceed corresponds to the minimum distance to the smallest convex polygonal figure formed by the tips of the vector derivatives of those functions that are equal. In the degenerate case of only one function, this coincides with the usual steepest ascent rule. The minimum distance from a point to a convex polygonal figure, and the corresponding direction, is a particular example of a quadratic programming problem and can be solved by any of the standard techniques. The method used to obtain the results given in this report is essentially that of reference 1.

HIGHLY COUPLED PASSIVE STABILIZATION SYSTEMS

Gravity-Oriented, Gravity-Damped System for Earth Satellites

Due to the radial variation of the earth's gravitational field, any satellite with unequal principal moments of inertia experiences torques which act to aline its axis of minimum moment of inertia with the gravitational vertical. In addition, gyroscopic torques exist on a satellite in orbit which act to aline the axis of maximum inertia with a normal to the orbital plane. When the angular deviations from equilibrium are small, the torque about any principal axis of inertia is directly proportional to the difference of the moments of inertia about the remaining two principal axes.

The success of satellite attitude stabilization systems based on gravity and gyroscopic torques depends on the practicability of creating satellites and auxiliary bodies with large moments of inertia and large differences of inertia for only a small weight penalty. Practical techniques for overcoming this problem and for creating suitable "frictionless" hinges to connect the auxiliary body to the satellite are presented in references 2, 3, and 4.

A sketch of a dynamically coupled satellite configuration is shown in figure 1(a). It should be noted that the auxiliary body or damper rod has only one rotational degree of freedom relative to the satellite. The only constraint on the mass distribution of the satellite and the mass distribution and orientation of the damper is that, in equilibrium, the principal axes of inertia of the complete satellite (including the damper) must lie along the $\overline{\sigma}_j$ $(j = 1,2,3)$ or orbital axes shown in figure 1(a). This condition is dictated by the dynamical behavior of the complete satellite system. A necessary condition for all axes of the system to be dynamically coupled is that the mass distribution of the satellite alone be asymmetrical with respect to the orbital axes. This condition implies a nonzero value of at least one cross product of inertia of the satellite relative to the orbital axes and gives rise to the term "inertially coupled" to describe this type of dynamically coupled system.

System parameters corresponding to the best damping (minimum value of σ_{max}) have been determined for a range of damper rod inertias ranging from 0 to 25 percent of the moment of inertia of the satellite about the roll axis. The parameters which were allowed to vary during the damping optimization procedure were: the Euler angles, Ψ, Θ, Φ, defining the position of the damper rod and hinge axis relative to the principal inertia axes of the satellite, the satellite inertia ratios I_2/I_1 and I_3/I_1, the spring constant C and the damping constant B. One general result obtained is that, in all the cases considered, the configurations with the best damping are those which have Φ and Θ both equal to zero. From the definition of the Euler angle sequence (see Nomenclature) it follows that the best damped configurations have both the damper rod axis and the hinge axis in the $\bar{b}_1\bar{b}_2$ plane. Thus, the principal axis of minimum inertia of the whole satellite (including the damper rod) must lie along the $\bar{b}_3$ axis. This in turn implies that the products of inertia I_{13} and I_{23} are both equal to zero. It follows from the dynamics of the whole satellite system that, in equilibrium, the $\bar{b}_3$ axis is coincident with the local vertical $\bar{o}_3$. These configurations are similar to those studied in references 5 and 6. However, in reference 5 the inertia ratio I_3/I_d was maintained constant at an arbitrary value of 1.5, whereas in the present study no such restriction was imposed on the mass distribution. A second general result obtained from the damping analysis is that all optimum configurations have a satellite (less damper rod) mass distribution which is planar, that is, J_3 is unity $[J_3 = (I_2 - I_1)/I_3]$. In reference 5 a similar result was obtained for many, but not all, of the configurations studied. While in practice a planar mass distribution is impossible, the satellite with rods extended usually has a J_3 value close to unity.

The optimum damping in terms of the minimum σ_{max} is shown in figure 2 along with the associated system parameters. The maximum damping of 0.47 orbit to 36.8 percent of the initial amplitude[2] (min $\sigma_{max} = -0.336$) occurs at a damper rod inertia ratio I_d/I_1 of about 0.18.

The variation with I_d/I_1 of the roots (both real and imaginary) of the characteristic equation corresponding to the maximum damping (min σ_{max}) is shown in figure 3. This reveals two interesting features which appear to be true at least to within the limits of accuracy of the calculation procedure. The first is that for values of I_d/I_1 less than 0.08 or greater than 0.18, three out of a total of four distinct real parts of the roots of the characteristic equation are equal in value. The second is that for values of I_d/I_1 between 0.08 and 0.18, not only are all four of the distinct real parts equal but two pairs of complex roots are also equal (both real and imaginary parts). In reference 7 Zajac was able to show, analytically, that for the case of a simple pitch damper ($\Psi = 0$), all the real parts of the roots are equal at the optimum damping point.

[2]The damping time constant in terms of number of orbits to 36.8 percent (1/e) of the initial amplitude is given by $-1/2\pi\sigma_{max}$.

The principal external disturbance torques acting on a satellite of this type are due to solar radiation and the magnetic field (ref. 8). Solar pressure torques occur when the center of solar pressure differs from the center of mass. This can be due first to the basic geometry and surface characteristics of the satellite, and secondly to changes of geometry due to thermal distortion, particularly of the extended rods. In practice, it is important to distinguish between these two causes since, while the basic geometry effects have a single periodic component, of orbital frequency, ω_o, the effects due to thermal distortion contain, in addition, significant periodic components of twice and three times orbital frequency. The influence of solar pressure disturbances increases with satellite altitude, since the restoring gravity-gradient torques vary as the inverse cube of the orbital radius, while the solar torques remain roughly constant. Hence, at very high altitudes (above about two earth radii) it becomes essential to design the satellite so that the center of pressure coincides with the center of mass for all satellite orientations relative to the sun and to minimize the thermal distortion of the rods by minimizing thermal gradients. The disturbances caused by the earth's magnetic field are due to its interaction with any residual magnetism of the satellites. The magnitude of this disturbance is dependent upon altitude and the frequency upon orbital inclination.

The remaining significant source of attitude error is orbital eccentricity. In this case the attitude errors are functions of the eccentricity of the orbit only and are, therefore, independent of altitude.

Clearly, the response of the satellite will depend upon the proximity of the disturbance frequency to the modal frequencies. Thus, figure 3 shows that the response of all optimum-damped systems to disturbances of frequency $3\omega_o$ is likely to be far less than the response to disturbances of frequencies ω_o and $2\omega_o$. Detailed calculations show that not only is the response of these systems to torques of frequency $3\omega_o$ small but the anticipated amplitudes of these torques is smaller than those of frequencies ω_o and $2\omega_o$. Torques of frequency $3\omega_o$ may, therefore, be neglected in a preliminary analysis. Figure 3 also suggests that the smaller the value of I_d/I_1 the greater may be the response to torques of frequency $2\omega_o$, while the higher the value of I_d/I_1 the greater may be the response to torques of frequency ω_o. This conjecture is to some extent supported by figure 4 which shows the response to orbital eccentricity, and figure 5, which shows the response to oscillatory pitch torques of frequency ω_o and $2\omega_o$. Perhaps the most significant feature of figures 4 and 5 is that the responses of systems with I_d/I_1 between 0.08 and 0.25 do not differ widely.

Once a system has been found which has good damping and steady-state pointing accuracy, the question of sensitivity to off-design conditions arises. Such off-design conditions can result, for example, from temperature variations, manufacturing tolerances, or deterioration with time. The sensitivity of the damping of the least damped mode to change of spring constant, damper constant, and angle Ψ, is illustrated in figure 6. This sensitivity is illustrated for two damper-rod inertia ratios. The largest is $I_d/I_1 = 0.18$ which is the best damped system, and the smallest is $I_d/I_1 = 0.08$. It is clear that the critical parameter is the spring constant. When $I_d/I_1 = 0.18$ a reduction of spring constant by more than about 8 percent is sufficient to cause instability. The case $I_d/I_1 = 0.08$ is significantly less sensitive to changes of spring constant and this may be a compelling reason for adopting a system with a value of I_d/I_1 considerably less than that which gives the best damping. It is important to recognize that the instability produced by a weak spring is of the static type and merely means that the destabilizing gravity torque on the damper rod is sufficient to overcome the spring torque. When the true nonlinear nature of the gravity-gradient torque is considered, the damper rod will merely be found to have a stable equilibrium position other than horizontal. One method of alleviating the spring constant sensitivity problem might be to design the spring to be stronger than the optimum value so that any reduction within the anticipated tolerances does not cause instability.

Even if the linear damping and frequency response of a passive satellite appear satisfactory, there is no guarantee that the large angle motion, such as might occur at injection into the orbit, is well damped. In the case of coupled systems of the type considered here, no general results relating to large angle motion have been discovered and recourse must be made to the study of particular systems with particular injection errors. A considerable number of time histories of motion from various initial conditions have been made for coupled systems. The conclusions from this work is that systems which have good linear damping also have good acquisition characteristics. Two examples of typical time histories for a coupled system having an $I_d/I_1 = 0.08$ are given in reference 5.

To sum up, from the point of view of pointing accuracy, there seems to be little to choose between any system with I_d/I_1 values in the range 0.08 to 0.18. In this range the greatest errors of a system without an orbital control capability are probably due to orbital eccentricity. However, the extreme sensitivity to changes of spring constant of systems with I_d/I_1 values close to 0.18 may make it preferable to use systems with I_d/I_1 values close to 0.08, even though the damping is reduced. If still smaller damping can be tolerated, systems with I_d/I_1 less than 0.08 may have better pointing accuracy.

Solar-Oriented, Solar-Damped System for Interplanetary Probes

In this system the torques required both for orientation and damping are obtained by erecting solar sails on the satellite and the auxiliary body. The auxiliary body, or solar damper, is allowed to have one rotational degree of freedom with respect to the satellite, as shown in figure 1(c). The orientation of the hinge axis and the mass distribution of the satellite and solar damper are arranged to promote the desired coupling between the degrees of freedom of the system.

It is important to recognize that, in practice, this system is capable of providing only a sun-pointing capability, or two axis stabilization. The reason is that even at the minimum practical orbital radius, as dictated by satellite temperature, the orbital angular velocity is too low to yield usable information about the orientation of the normal to the orbital plane. In other words the gyroscopic torques due to orbital angular velocity are too small to be of value in providing stabilization about the sun-pointing axis. This situation should be contrasted with that of the previous earth-pointing system where full three axis stabilization is possible as a result of the relatively larger orbital angular velocity and correspondingly larger gyroscopic torques.

It is difficult to visualize any sources of significantly large errors for this system. Errors from such external sources as the solar magnetic field, the gravity gradient of the sun, and gravitational perturbations caused by the planets are very small. There may possibly be significant transient errors due to micrometeorites. Unfortunately it is not possible to state with certainty whether this constitutes a problem, since there is little knowledge of micrometeorite statistics, particularly in regions of space far distant from the earth.

Unlike the previous example, the orientation and damping torques acting on a solar-stabilized satellite are independent of its mass distribution. It follows immediately that if the system has no practical design constraints, there is no optimum damping, since the damping can always be improved by reducing the satellite inertias or by increasing the solar-stabilizing torques. Therefore, since design constraints must be included to make the problem realistic, it is impossible, in the short space available here, to treat the system with the same degree of generality as the previous system. The best that can be done to demonstrate the feasibility of a coupled system is to choose a reasonable set of design constraints and evaluate the remaining system parameters corresponding to the optimum damping.

Consider a satellite such as that shown in figure 7 which is required to operate in a circular orbit at a distance of 1 astronomical unit from the sun and be stabilized relative to the sun line $\overline{o}_3$. The configuration is assumed to have symmetry of mass, geometry, and solar reflection properties relative to the $\overline{b}_3$ axis,[3] so that $I_1 = I_2$ and $k_{b_1} = k_{b_2}$ (k_{b_1} is the solar spring constant for rotations about the $\overline{b}_1$ axis). Furthermore, in equilibrium, it is required that $\overline{b}_3 = \overline{o}_3$ and, in this condition, that the solar torques acting on the satellite (less damper) be zero. The assumed moments of inertia and solar spring constants of the satellites are given in table 1 of figure 7. Finally, it is assumed that the mass distribution of the damper body is approximately that of a rod and has symmetry of geometry and solar reflection properties relative to the $\overline{p}_3$ axis. Thus, $I_{d_2} = I_{d_3} = I_d$, $I_{d_1} = 0$, $k_{p_2} = k_{p_3}$ and, in equilibrium ($\overline{p}_3 = \overline{o}_3$), the damper body has no solar torques acting on it. The optimization procedure previously described was used to determine the inertia ratio I_d/I_1 and solar spring constant k_{p_1} of the damper, the orientation of the damper rod and its hinge line relative to the satellite and the spring and damper constants corresponding to the best damped system. In addition, the damper rod inertia ratio was fixed at several values less than the optimum and the system reoptimized to find the best damping. The results are summarized in figure 8. The time constant of the best damped system is 13 hours and occurs with $I_d/I_1 = 0.4115$. A reduction of damper inertia ratio to 0.1 increases the time constant to 120 hours. Figure 7 shows how the damper might be attached to the main satellite body. In this case the hinge point is located at the end of a spike attached rigidly to the main satellite body. Some such arrangement is required to avoid having the damper rod foul

[3]Note that in this example $\overline{v}_j = \overline{b}_j$ $(j = 1,2,3)$.

the main satellite structure. Probably a more efficient alternative is to replace the spike by a rod rigidly attached to the damper cone and hinged to the satellite structure. This would violate the assumption that the damper mass distribution approximates that of a rod, but the optimum damping should be of the same order as those quoted. It should be noted that orbital eccentricity should have very little effect on the pointing accuracy of these satellites because the time constant of the system is several orders of magnitude less than the orbital period.

The sensitivity of the system with the best damping ($I_d/I_1 = 0.4115$) to changes of spring or damper constants is shown in figure 9. Changes of ±20 percent in either spring or damper constants do not reduce the damping to less than half its optimum value. It is concluded, therefore, that manufacturing tolerances are not particularly critical. It is conceivable, however, that different design constraints may result in more sensitive systems than the one demonstrated here.

Other Coupled Systems

Sketches of two other possible coupled systems are shown in figure 1. One is a gravity-oriented, magnetically damped system (fig. 1(b)) in which the damper rod of the gravity-damped system is replaced by a magnet and damping torques are obtained through the interaction of this magnet with the earth's magnetic field. The other coupled system (fig. 1(d)) is gravity oriented, gyroscopically damped. In this case the damper rod of the gravity-damped system is replaced by a suitably oriented, single degree of freedom, constant speed flywheel. Although this stabilization system does not strictly fall within the domain of this report, since it contains an active element (the flywheel), it is included as an example because it clearly illustrates the thesis of the report. A complete analysis is given in reference 9.

CONCLUDING REMARKS

It has been demonstrated that it is feasible to passively stabilize earth and sun-pointing satellites using a single auxiliary body with a single degree of freedom. In addition, it has been indicated that the underlying principle of dynamic coupling may be useful in reducing the number of components in passive attitude stabilization systems of all types. It may be conjectured, further, that the introduction of dynamic coupling may offer some advantages in the design of stabilization systems for a wide variety of aerospace vehicles. It is not the intention here, however, to claim that dynamic coupling provides the ultimate answer to all vehicle stabilization systems. Rather the intention is to show that in considering systems to meet a given requirement, the arbitrary adoption of configuration symmetry on some vague aesthetic grounds is a needless restriction, particularly if mechanical simplicity is an all important factor.

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National Aeronautics and Space Administration
Moffett Field, Calif., Feb. 25, 1966

APPENDIX

THE DIRECTION OF STEEPEST ASCENT WHEN TWO OR MORE FUNCTIONS ARE EQUAL

The problem is to find the direction $\hat{r}\epsilon R^n$ (parameter space) that yields the maximum simultaneous rate of increase of those functions (of the set σ_i', $i = 1 \ldots m$,) which have the same value. Suppose the functions σ_i', $i = 1 \ldots m$, have been renumbered so that those that are equal occur first. Let their number be p. Then each of these p functions has a derivative, along a chosen direction $\hat{r}$, of the form

$$\frac{d\sigma_i'}{ds} = \nabla\sigma_i' \cdot \hat{r}, \quad i = 1 \ldots p$$

where s is the distance measured along the direction $\hat{r}$. The required direction $\hat{r}$ is that which maximizes the minimum value of $\nabla\sigma_i' \cdot \hat{r}$, $i = 1 \ldots p$. The formal solution to this problem is provided by the following two theorems.

Theorem 1: Let $B\{\bar{y}\}$ be a compact, convex subspace of R^n and the vector $\bar{Y}\epsilon B$ be such that $\|\bar{Y}\| = \min_{\bar{y}\epsilon B} \|\bar{y}\|$. Then

$$\|\bar{Y}\| = \max_{\hat{r}\epsilon R^n}\left[\min_{\bar{y}\epsilon B}(\bar{y} \cdot \hat{r})\right]$$

where $\hat{r}$ is a unit vector in R^n. Furthermore, the corresponding unique unit vector $\hat{r}$ is given by $\hat{r} = \bar{Y}/\|\bar{Y}\|$.

Proof: The equation $\mu\hat{r} \cdot (\bar{y} - \mu\hat{r}) = 0$, where μ is any real scalar, defines a hyperplane in R^n which is distance μ from the origin. The hyperplane divides the space R^n into two subspaces R_1^n and R_2^n defined as

$$R_1^n = \{\bar{y}:\mu\hat{r} \cdot (\bar{y} - \mu\hat{r}) \geq 0\}$$

$$R_2^n = \{\bar{y}:\mu\hat{r} \cdot (\bar{y} - \mu\hat{r}) < 0\}$$

Since B is a bounded subspace of R^n, a value of μ can always be found such that $B\epsilon R_1^n$ with the tip of at least one vector in B in the hyperplane. Thus, μ can be chosen such that every vector $\bar{y}\epsilon B$ satisfies the inequality.

$$\mu\hat{r} \cdot (\bar{y} - \mu\hat{r}) \geq 0 \tag{1}$$

and at least one vector $\bar{y}\epsilon B$ satisfies the equation

$$\mu\hat{r} \cdot (\bar{y} - \mu\hat{r}) = 0 \tag{2}$$

Inequality (1) and equation (2) can be rewritten

$$\bar{y} \cdot \hat{r} \geq \mu \quad \text{for all} \quad \bar{y}\epsilon B \tag{3}$$

(since $\hat{r} \cdot \hat{r} = 1$ by definition) and

$$\bar{y} \cdot \hat{r} = \mu \quad \text{for at least one} \quad \bar{y}\epsilon B \tag{4}$$

It follows that

$$\mu = \min_{\bar{y}\epsilon B}(\bar{y} \cdot \hat{r}) \tag{5}$$

Since $\bar{y} \cdot \hat{r} \leq ||\bar{y}|| \; ||\hat{r}|| = ||\bar{y}||$, it follows from inequality (3) that

$$||\bar{y}|| \geq \mu \quad \text{for all} \quad \bar{y}\epsilon B \tag{6}$$

Inequality (6) shows that, provided inequality (3) and equation (4) can be satisfied

$$\max_{\hat{r}\epsilon R^n} \mu = \min_{\bar{y}\epsilon B}||\bar{y}|| = ||\bar{Y}||$$

Equation (4) is then satisfied with $\bar{y} = \bar{Y}$ and $\hat{r} = \bar{Y}/||\bar{Y}||$. It now remains to show that inequality (3) also holds when $\mu = ||\bar{Y}||$ and $\hat{r} = \bar{Y}/||\bar{Y}||$. Assume first that inequality (3) does not hold, that is

$$(\bar{y} \cdot \bar{Y} - ||\bar{Y}||^2) < 0$$

Since B is a convex set and $\bar{y}, \bar{Y}\epsilon B$, it follows that if $0 \leq \lambda \leq 1$ then $\bar{y}' = \lambda\bar{y} + (1 - \lambda)\bar{Y}$ is also in B. Thus

$$\begin{aligned}
||\bar{Y}||^2 - ||\bar{y}'||^2 &= ||\bar{Y}||^2 - ||\lambda\bar{y} + (1 - \lambda)\bar{Y}||^2 \\
&= 2\lambda(||\bar{Y}||^2 - \bar{y} \cdot \bar{Y}) - \lambda^2(\bar{y} - \bar{Y})^2 \\
&= \lambda[2(||\bar{Y}||^2 - \bar{y} \cdot \bar{Y}) - (\bar{y} - \bar{Y})^2\lambda]
\end{aligned}$$

if

$$0 < \lambda < 2 \, \frac{(||\bar{Y}||^2 - \bar{y} \cdot \bar{Y})}{(\bar{y} - \bar{Y})^2}$$

$$||\bar{Y}||^2 - ||\bar{y}'||^2 > 0$$

or

$$\|\bar{Y}\| > \|\bar{y}'\|$$

which violates the definition of $\bar{Y}$. Hence, (3) must hold and the first part of the theorem is proved. The uniqueness of the minimum vector to a convex set is proved in most texts on linear vector spaces (ref. 10).

Theorem 2: If $\nabla\sigma_i'$ $(i = 1 \ . \ . \ . \ p)$ are a set of vectors in R^n and $\bar{Y}$ is the vector of the set

$$B = \left\{ \bar{y} : \bar{y} = \sum_{i=1}^{p} \alpha_i \nabla\sigma_i', \quad \sum_{i=1}^{p} \alpha_i = 1, \quad \alpha_i \geq 0 \ (i = 1 \ . \ . \ . \ p) \right\}$$

such that $\|\bar{Y}\| = \min_{\bar{y} \in B} \|\bar{y}\|$ then,

$$\|\bar{Y}\| = \max_{\hat{r} \in R^n} \left[\min_i (\nabla\sigma_i' \cdot \hat{r}) \right] \tag{7}$$

and the corresponding unique unit vector $\hat{r}$ is given by $\hat{r} = \bar{Y}/\|\bar{Y}\|$

Proof: Let

$$k_{\hat{r}} = \min_i (\nabla\sigma_i' \cdot \hat{r})$$

then

$$k_{\hat{r}} \leq \nabla\sigma_i' \cdot \hat{r}, \qquad i = 1 \ . \ . \ . \ p$$

and

$$k_{\hat{r}} = \nabla\sigma_j' \cdot \hat{r} \qquad \text{for some} \quad j$$

thus

$$\alpha_i k_{\hat{r}} \leq \nabla\sigma_j' \cdot \hat{r}, \qquad i = 1 \ . \ . \ . \ p$$

(since $\alpha_i \geq 0 \quad i = 1 \ . \ . \ . \ p$) and

$$\sum_{i=1}^{p} \alpha_i k_{\hat{r}} \leq \sum_{i=1}^{p} \alpha_i \nabla\sigma_i' \cdot \hat{r}$$

or

$$k_{\hat{r}} \leq \bar{y} \cdot \hat{r} \quad \text{for all} \quad \bar{y} \in B$$

$\left(\text{since } \bar{y} = \sum_{i=1}^{p} \alpha_i \nabla\sigma_i' \text{ and } \sum_{i=1}^{p} \alpha_i = 1\right)$. It follows that $k_{\hat{r}} = \min_{\bar{y}\in B}(\bar{y} \cdot \hat{r})$ since if this were not so, it would be possible to find a $k_{\hat{r}}' > k_{\hat{r}}$ for which

$$k_{\hat{r}}' \leq \bar{y} \cdot \hat{r} \quad \text{for all} \quad \bar{y}\in B$$

but since $\nabla\sigma_j' \in B$ this would mean that

$$k_{\hat{r}}' \leq \nabla\sigma_j' \cdot \hat{r} = k_{\hat{r}}$$

which contradicts the assumption that $k_{\hat{r}}' > k_{\hat{r}}$. Thus, $\min_i(\nabla\sigma_i' \cdot \hat{r}) = \min_{\bar{y}\in B}(\bar{y} \cdot \hat{r})$ and since the foregoing arguments are for an arbitrary $\hat{r}$

$$\max_{\hat{r}\in R^n}\left[\min_i(\nabla\sigma_i' \cdot \hat{r})\right] = \max_{\hat{r}\in R^n}\left[\min_{\bar{y}\in B}(\bar{y} \cdot \hat{r})\right] \tag{8}$$

and both must occur at the same $\hat{r}$. Since it follows from the definition that B is compact and convex, theorem 1 can be applied to the right side of (8) and the theorem is proved.

The geometrical interpretation of theorems 1 and 2 is that the sought after direction $\hat{r}$ corresponds to the minimum distance from the origin to the smallest convex polygonal figure defined by the tips of the vectors $\nabla\sigma_i'$ $(i = 1 \ . \ . \ . \ p)$. It should be noted that there is no restriction on the size of p relative to n and, therefore, it is not necessary that the $\nabla\sigma_i'$ $(i = 1 \ . \ . \ . \ p)$ be independent. If $p \leq n$, the tips of all the vectors $\nabla\sigma_i'$ define the bounding edges of the polygonal figure. If, however, $p > n$, then, in general, the tips of n of the vectors $\nabla\sigma_i'$ define the bounding edges while those of the remaining $p - n$ vectors are located in the interior of the polygonal figure.

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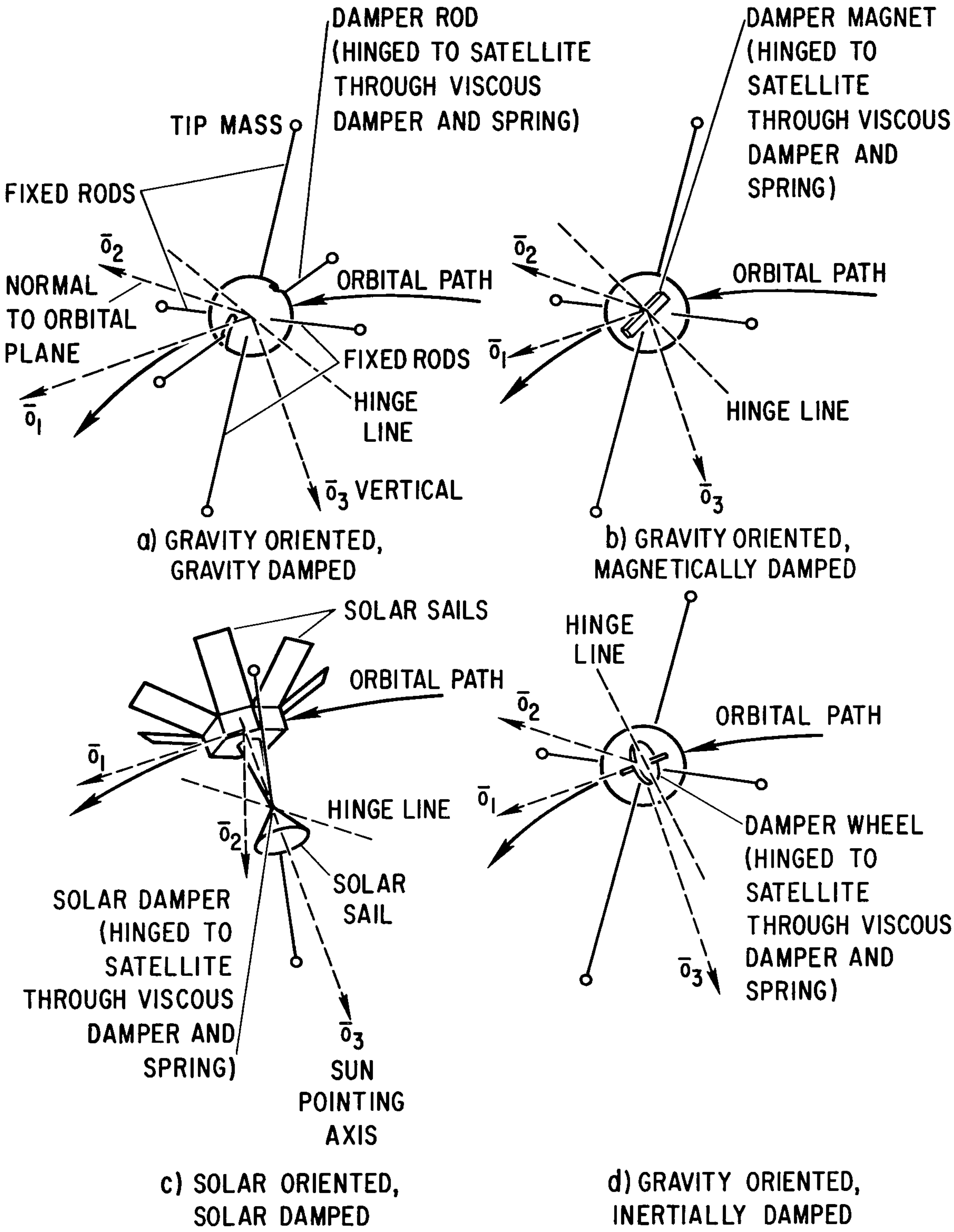


Figure 1.- Dynamically coupled passive and semipassive satellite stabilization systems.

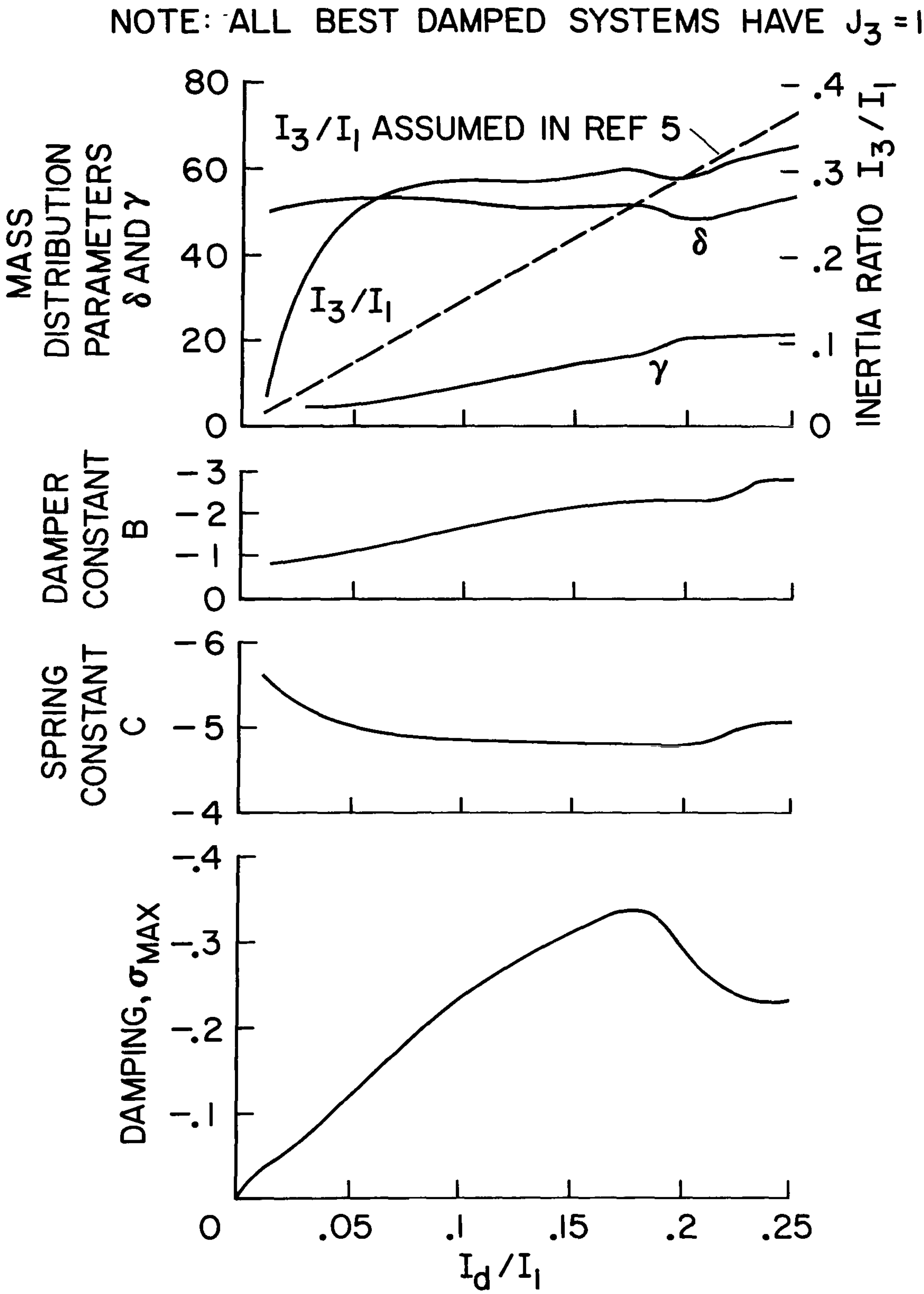


Figure 2.- Damping of least-damped mode and associated system parameters.

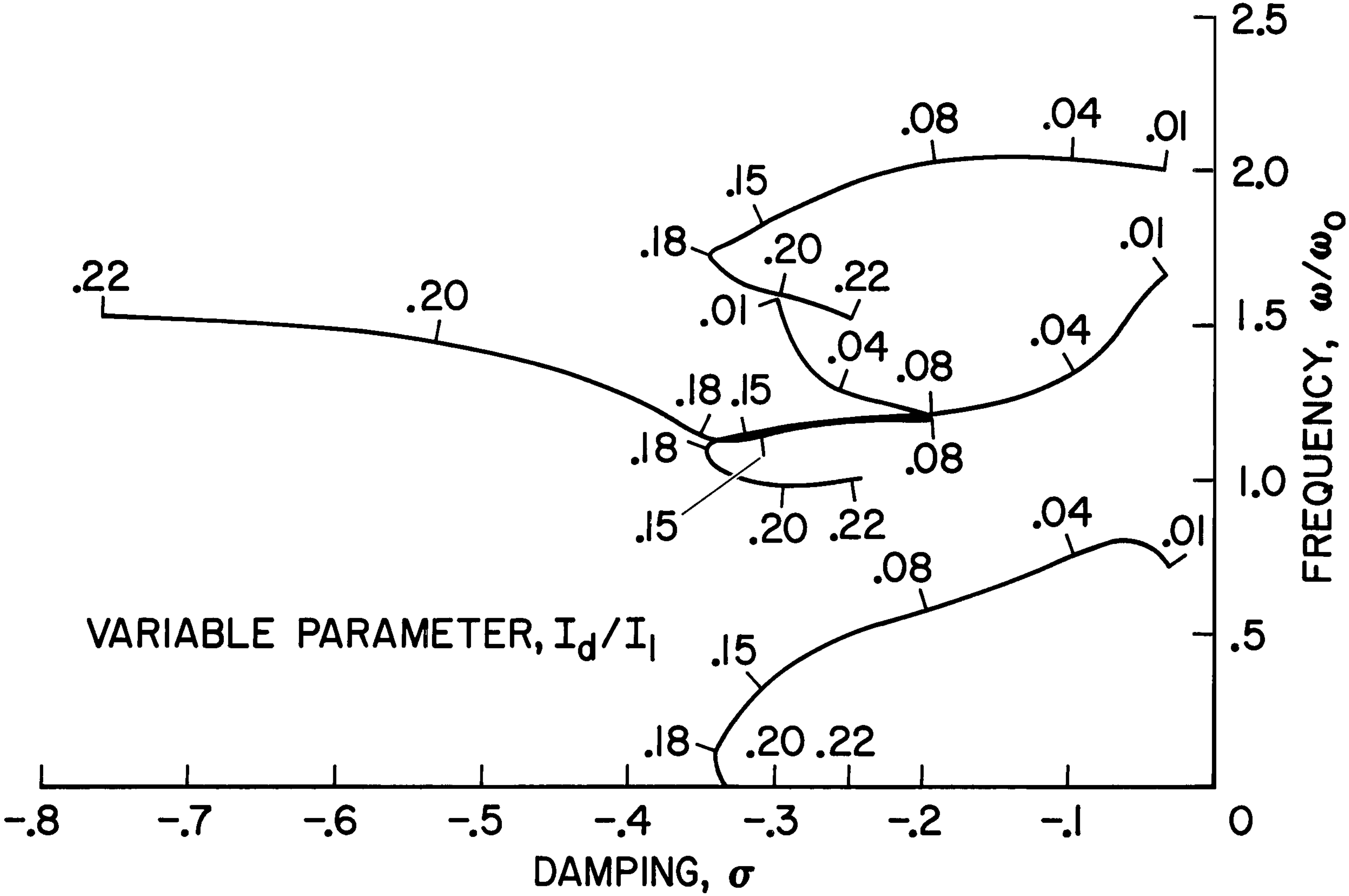


Figure 3.- Variation of frequency and damping with I_d/I_1 for best damped systems.

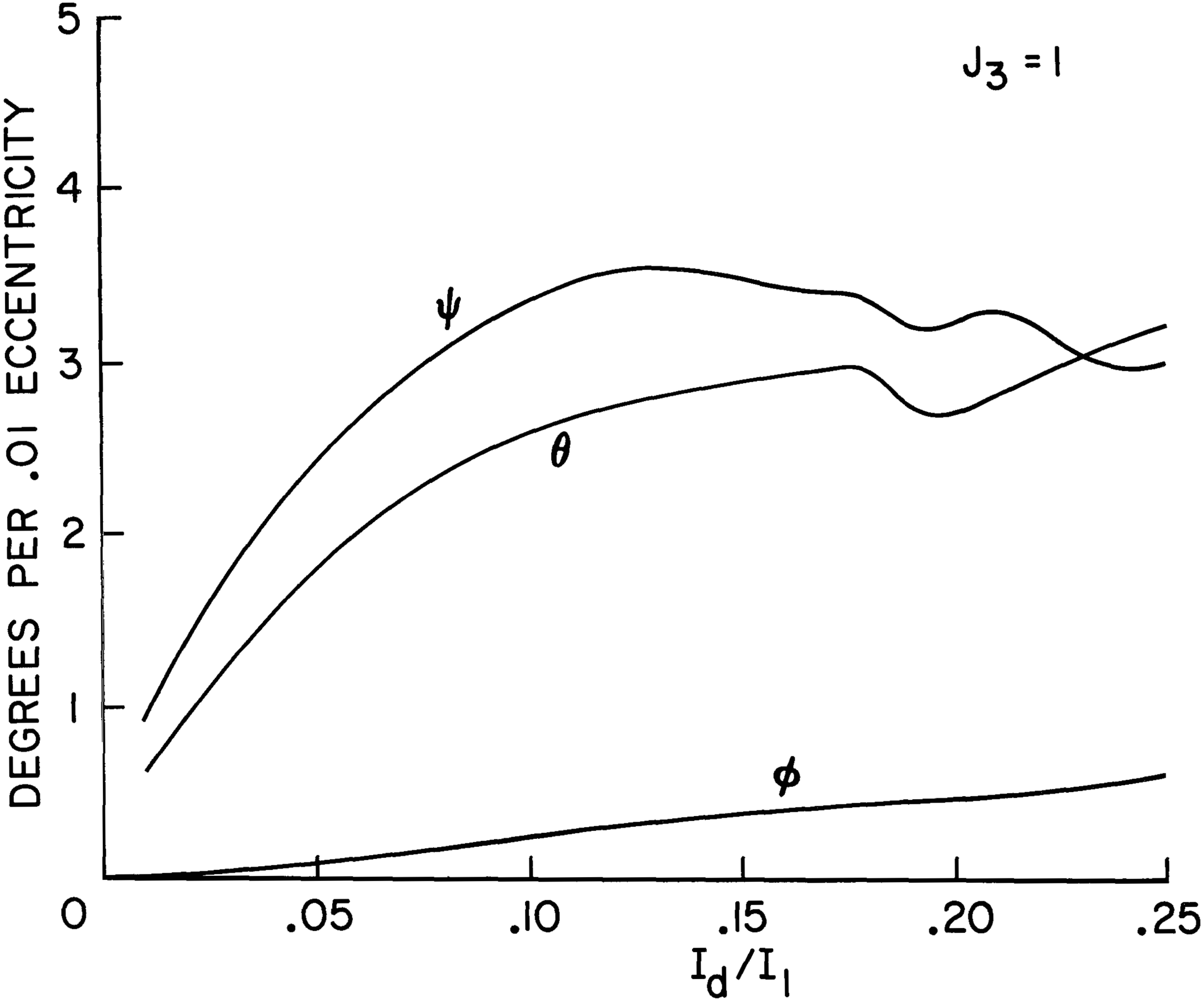


Figure 4.- Steady-state response to orbital eccentricity.

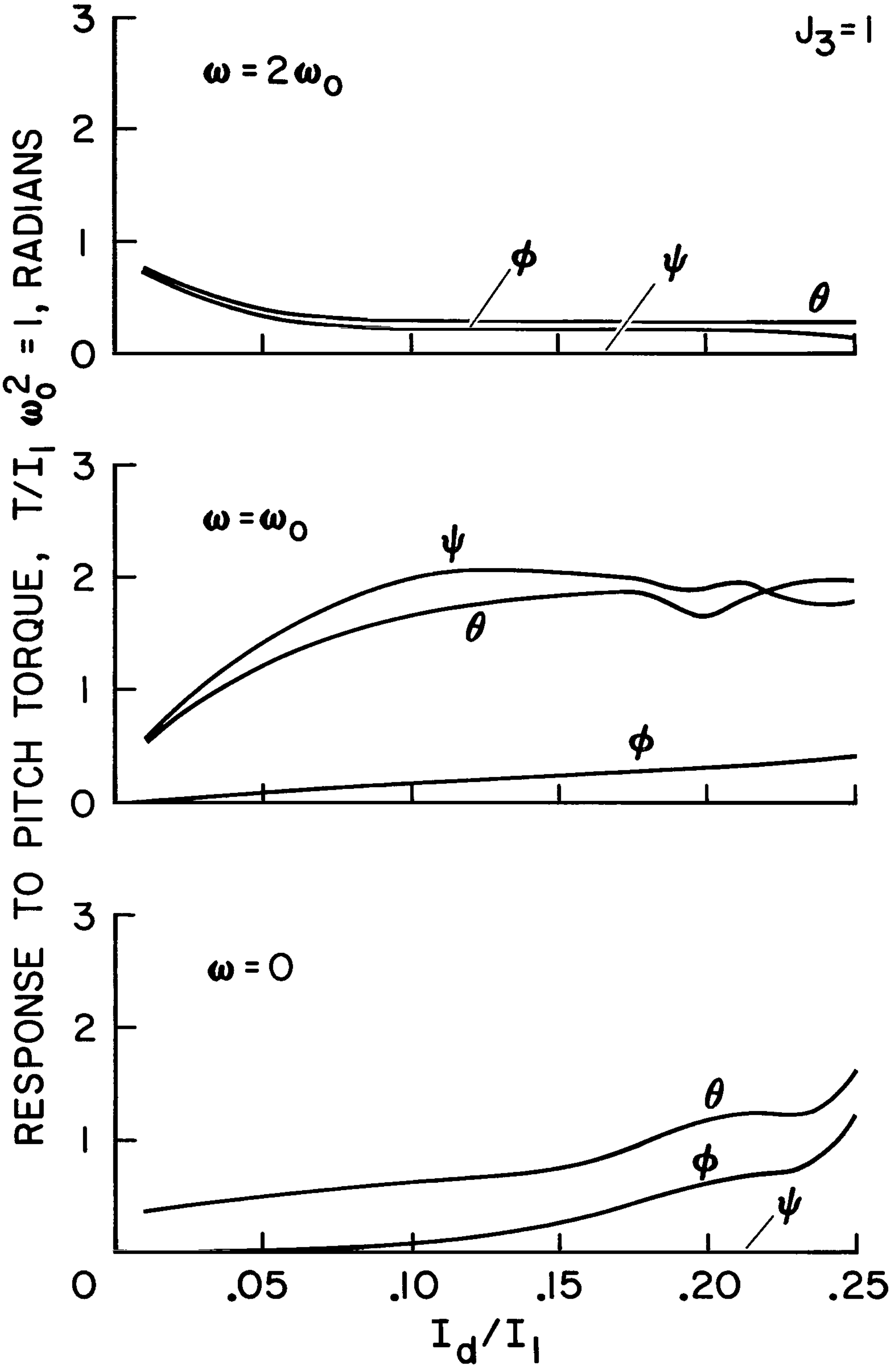


Figure 5.- Response to disturbance torques due to solar pressure.

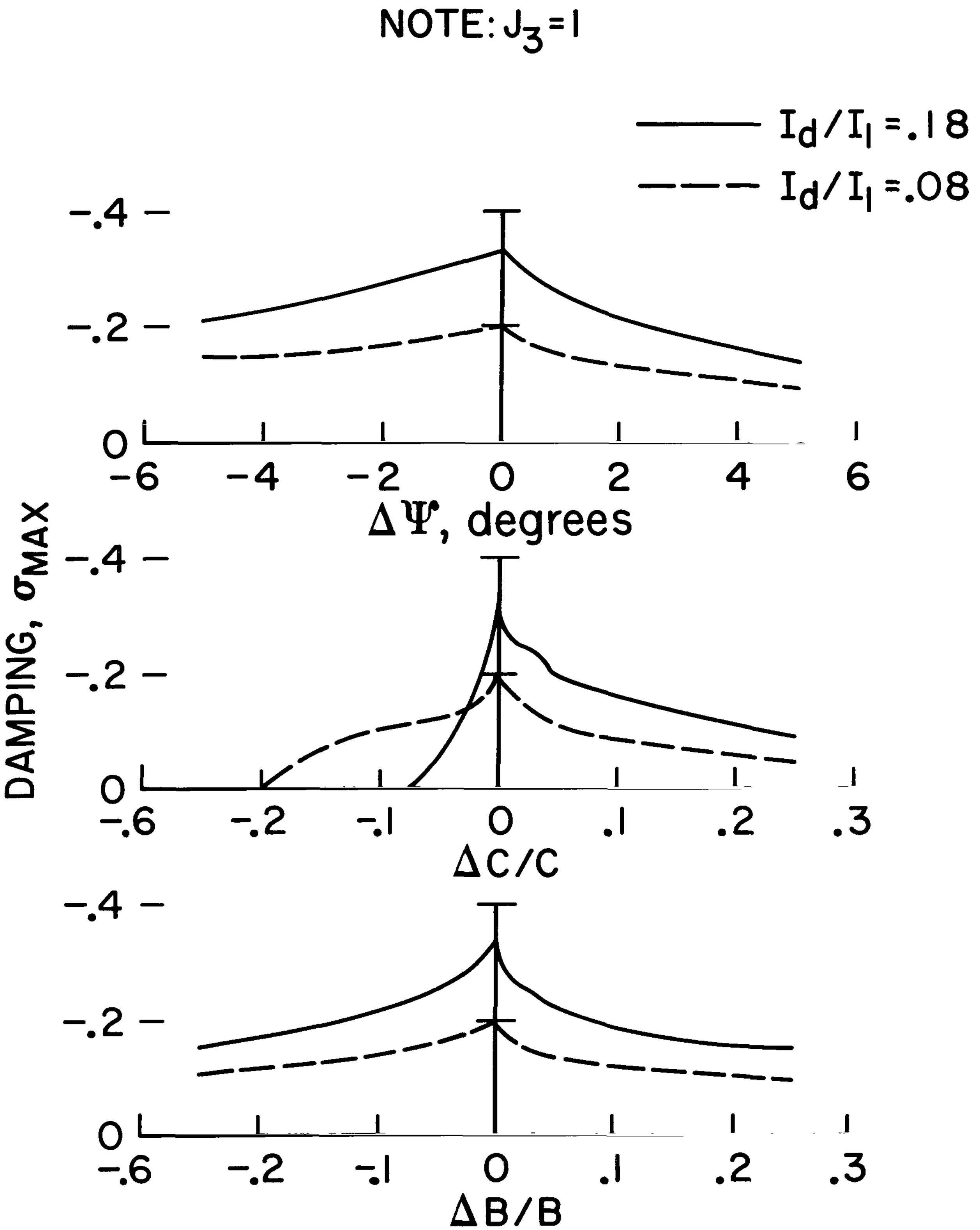
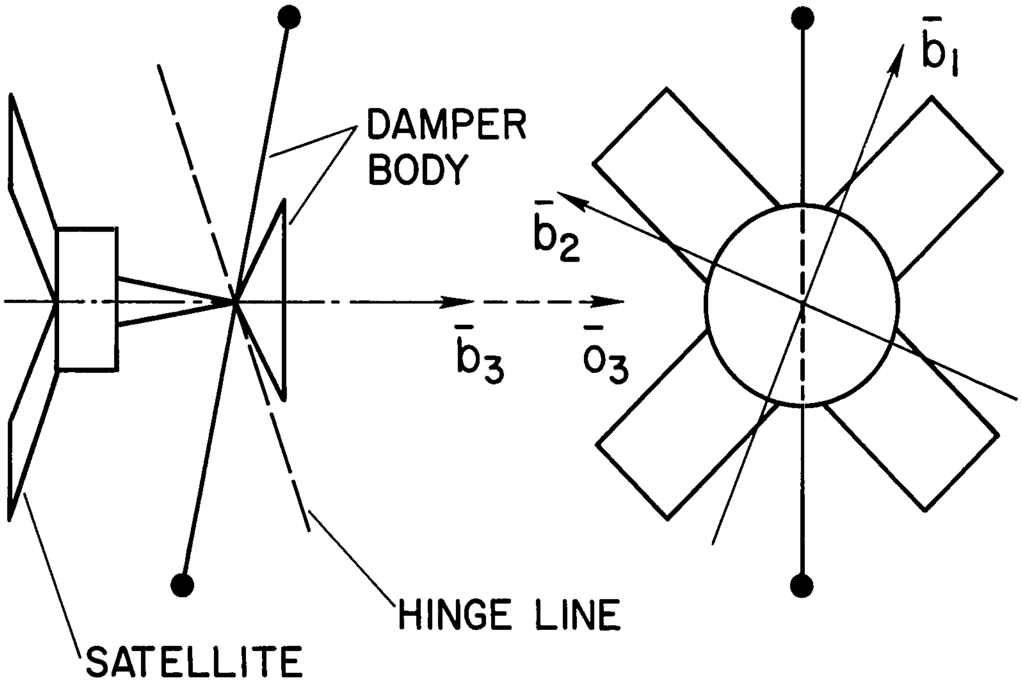
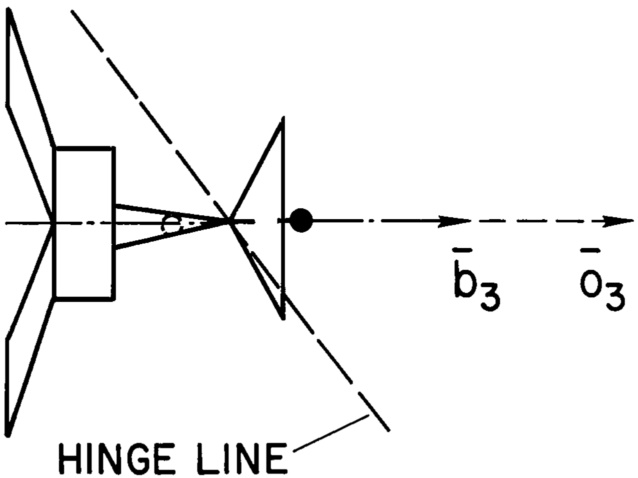


Figure 6.- Sensitivity of least-damped mode to off-design conditions.





DAMPING TIME-CONSTANT
13 HOURS

TABLE I

ASSUMED DESIGN PARAMETERS	
$I_1 = I_2$	77.3×10^7 gm cm^2
I_3	138.3×10^7 gm cm^2
$k_{b_1} = k_{b_2}$	67.8 dyne cm/rad

TABLE 2

PARAMETERS CALC$^{\underline{D}}$ FOR OPT DAMPING	
I_d	31.7×10^7 gm cm^2
$k_{p_1} = k_{p_2}$	14.9 dyne cm/rad
K	24.4 dyne cm/rad
B	8.79 dyne cm sec/rad
ζ	24.9°
η	-11.12°
ξ	17.7°

Figure 7.- Example of dynamically coupled solar-oriented solar-damped system.

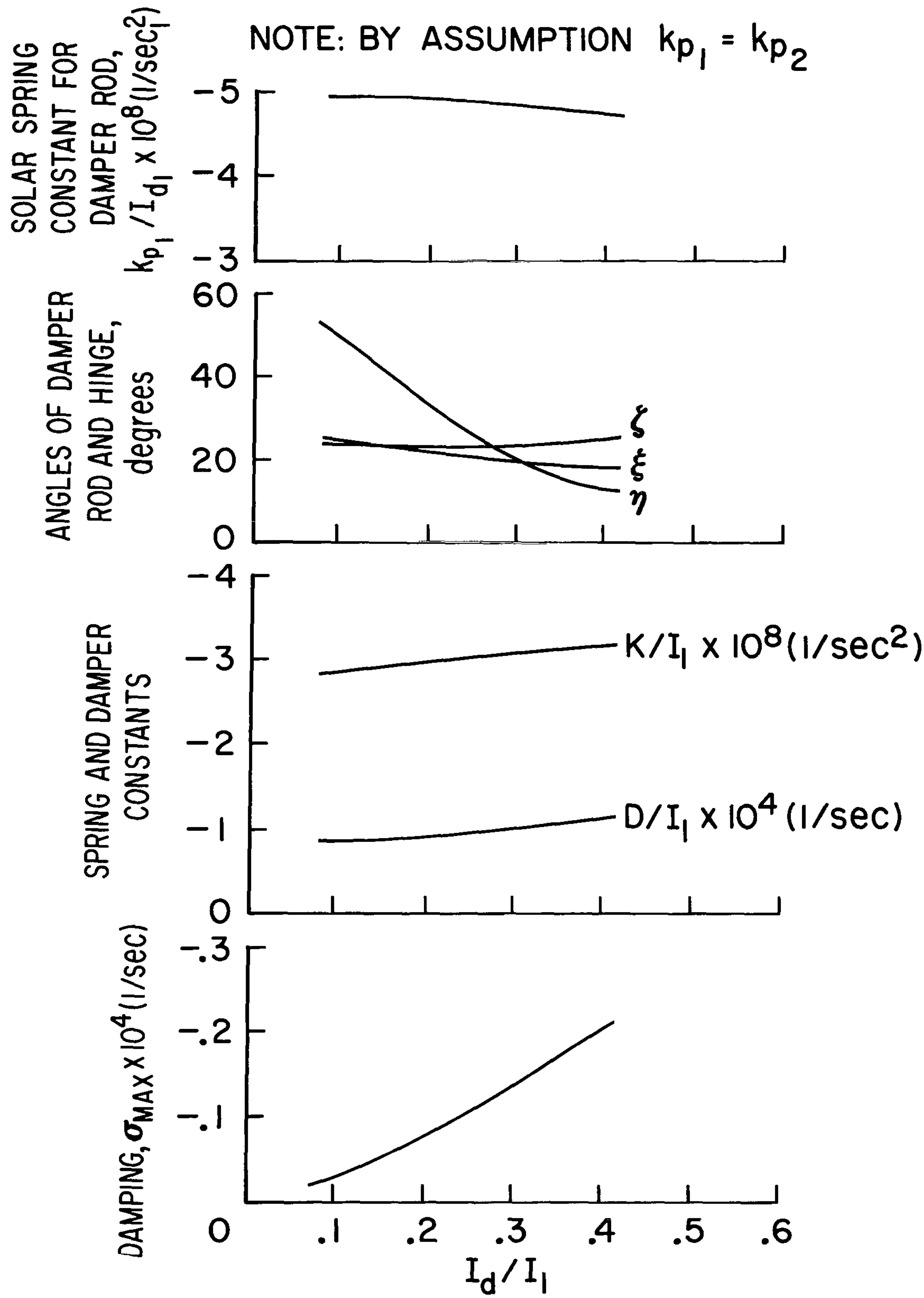


Figure 8.- Damping of least-damped mode and associated system parameters.

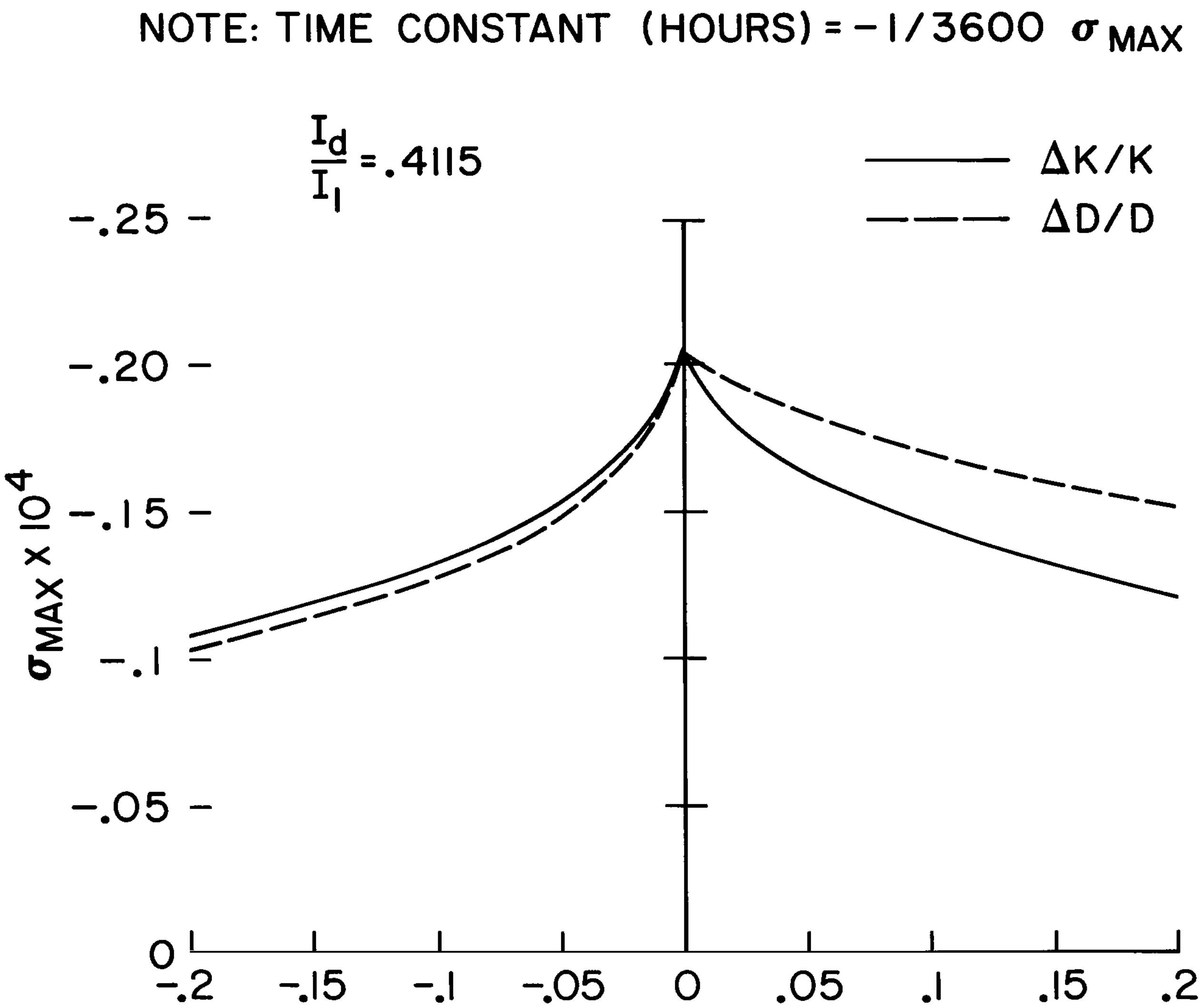


Figure 9.- Sensitivity of damping of least-damped mode to variations of spring and damper constants.

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