A COMPUTATIONAL METHOD FOR TWO-IMPULSE ORBITAL RENDEZVOUS AND TRANSFER PROBLEMS

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Page 64: In the FORTRAN notation in a write statement 14 lines from the bottom of the page, $H$ should be changed to $AH$. The corrected notation would be as follows:

\[ \text{WRITE}(6,126) \text{DTTI,DTTF,DTWI,DTWF,RMIN,AH} \]

Page 68: Insert the following statements after FORTRAN statement 402 (before 8th line from bottom of page):

```
C TEST FOR ZERO VALUED DENOMINATOR
CKDEN=RHO1*COS(TH1)-RHO2*COS(TH2)
IF(CKDEN.GE.1.0E-10)410,405
410 CONTINUE
```

Page 70: Insert the following statements after FORTRAN statement 602 (after 7th line from top of page):

```
C TEST FOR ZERO VALUED DENOMINATOR
CKDEN=RHO1*COS(TH1)-RHO2*COS(TH2)
IF(CKDEN.GE.1.0E-10)614,604
614 CONTINUE
```

Page 72: The absolute value of $i_1$ and $i$ should be used in Subroutine Six which evaluates the transformation matrix. Therefore, the second statement on page 72 should be deleted and replaced by the following two statements:

```
RAII=AII
CALL SIX(A11,A12,A21,A22,A31,A32,OMI,OMEGAI,RAII)
```

Also, the present one-line FORTRAN statement 900 should be deleted and replaced by the following two statements:

```
900 RBI=BI
CALL SIX(B11,B12,B21,B22,B31,B32,OM,OMEGA,RBI)
```
Page 78: Add the following statement between lines 1 and 2:

\[ U3 = |U3| \]

Page 83 (table III): Under the heading DV1Z (12th column), delete the minus signs before the first six numbers and insert minus signs before the last nine numbers; under the heading DV2Z (15th column), insert minus signs before all fifteen numbers.
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SUMMARY

A detailed derivation of exact equations and an associated computer program with which the basic parameters involved in transfer or rendezvous between two arbitrary elliptic orbits may be calculated are presented. The computations are exact in the sense that the Kepler solutions for bodies orbiting in a point-gravity field are used. This method deviates from the "classical" approach based on the theorem of Lambert, inasmuch as it uses the true anomaly and the Kepler equations for iterating to the desired rendezvous transfer time. The method has a unique feature in that definite boundaries, dependent on the problem input, are used which limit the range of the true anomaly, and thereby reduce the search effort required in the iteration procedure. The program provides for a solution to problems where the transfer angle is less than 360°. Examples are given for three particular uses of the program: (1) interplanetary transfer between massless planets, (2) near-planet orbit rendezvous, and (3) orbital transfer.

INTRODUCTION

It seems appropriate to provide a useful technique for the computation of velocity increments and other important parameters involved in the problems of orbital rendezvous and transfer by use of the solution to the exact equations of motion. Various programs and techniques exist at this time; however, they are either unpublished or are not of a sufficiently general nature to be used in the variety of orbital problems one might desire. (For instance, see refs. 1 to 4.) It is desirable, therefore, to have a simple and yet general computational method which will solve the problems of: (a) interplanetary transfer between massless planets, (b) planet orbit rendezvous, and (c) orbital transfer. The purpose of this paper is to give the description of the analysis required for the solution of the rendezvous problem from the exact Keplerian relations. The fundamental problem is that of determining the velocity increment required to rendezvous from some initial interceptor orbit to some final target orbit as a function of transfer time. Also determined are eccentricity, semi-major axis, initial and final anomalies, and other parameters associated with the transfer orbit. Although the problem, as stated, is a rendezvous problem, it
is also possible to interpret the results for use in studying the orbital transfer problem. It is assumed that the Keplerian orbital quantities are known in advance for the target vehicle and that the initial interceptor orbit is known either from its Keplerian orbital elements or from relative coordinate data. The coordinates chosen are referenced to the target orbit, and another axis transformation will be necessary if the user desires the results and input referenced to some other axis system (such as the ecliptic). Figures 1, 2, and 3 show the coordinates and position symbols used to describe the orbits and figures 4 and 5 show the (input) information needed on the position of the orbits.

A solution to the rendezvous problem may be obtained by specifying the transfer time for the interceptor to travel from its initial orbit to its final (target) orbit which, along with the initial conditions of the problem, gives the initial and final positions in space through which the interceptor must pass. The transfer orbit must then be found which passes through the known initial and final positions. An iteration is required for this calculation inasmuch as the transfer orbit which will yield the proper transfer time is not yet known. The procedure begins by choosing some orbit which passes through the initial and final position vectors and then the corresponding transfer time is computed and compared with the real (desired) transfer time. If the computed transfer time is not the same as the desired time, another orbit must be chosen and the time again computed and compared. This process is repeated until the conic section which provides the true rendezvous transfer orbit is found.

There are (at least) two methods which have been used for this iteration. The solution by use of Lambert's theorem was used by Battin (ref. 1) and Breakwell (ref. 2). However, iteration of the true anomaly, as was considered by Lascody (ref. 3), is more readily visualized since it does not require such artificial devices as "flattening" the transfer orbit used in geometrically describing the transfer problem deduced from the theorem of Lambert (ref. 1). It is also possible to find definite regions within which the true anomaly must be chosen and therefore shorten computational (convergence) time to the extent that a direct search routine may be used in the iteration; this routine does not involve the derivatives necessary in the method of iteration as presented in reference 3.

In programing this problem, an effort has been made to provide a sufficiently general program with a minimum amount of difficulty in reading input and printing output information for the particular problem. For instance, problem input information may be provided in two fundamentally different ways: (1) as Keplerian data (eccentricity, semi-major axis, longitude of ascending node, etc.) and (2) as relative coordinate input referenced to rectangular coordinate axes fixed with the target vehicle.
SYMBOLS

Unless otherwise noted all quantities are nondimensional. The nondimensional forms are derived as follows:

\[ \text{Nondimensional length} = \frac{\text{Dimensional length}}{\text{Semi-major axis of target orbit}, \ a_T} \]

\[ \text{Nondimensional velocity} = \frac{\text{Dimensional velocity}}{\text{Mean circular velocity of target}, \ V_{CT}} \]

\[ \text{Nondimensional angular rate} = \frac{(\text{Dimensional angular rate}) \times \text{Target orbital period}, \ P_T}{2\pi} \]

\[ \text{Nondimensional time} = \frac{\text{Dimensional time}}{\text{Target orbital period}, \ P_T} \]

- \( a \) \quad \text{semi-major axis of orbit}
- \( a_{ij} \) \quad \text{element of transformation matrix, } x,y,z \text{ to } X,Y,Z
- \( b_{ij} \) \quad \text{element of transformation matrix, } x',y',z' \text{ to } X,Y,Z
- \( c \) \quad \text{chord joining } \rho_1 \text{ to } \rho_2
- \( e \) \quad \text{eccentricity of orbit}
- \( E \) \quad \text{eccentric anomaly of vehicle in orbit}
- \( F \) \quad \text{functional relationship}
- \( F_1,F_2 \) \quad \text{equivalent eccentric anomalies for hyperbolic orbit}
- \( h \) \quad \text{increment for iteration}
- \( H \) \quad \text{angular momentum}
- \( i \) \quad \text{inclination of orbital plane to target orbit plane}
- \( i,j,k \) \quad \text{unit vectors along } x,y,z; \ x'',y'',z'' \text{ axes as indicated}
- \( \hat{i},\hat{j},\hat{k} \) \quad \text{unit vectors along } x',y',z' \text{ axes}
- \( \hat{I},\hat{J},\hat{K} \) \quad \text{unit vectors along } X,Y,Z \text{ axes}
M = mean anomaly
p = semi-latus rectum

\( P_T \) = target orbital period, \( 2\pi \sqrt{\frac{a_T^3}{\mu}} \), seconds

r = radial distance to target

\( \dot{r}_f(X), \dot{r}_f(Y), \dot{r}_f(Z) \) = velocity components after final impulse

\( t \) = time

\( t_0 \) = time at observation of input

\( t_i \) = time of initial impulse

\( t_f \) = time of final impulse

\( \Delta t_w \) = wait time before initial impulse, \( t_i - t_0 \)

\( \Delta t \) = interceptor transfer time, \( t_f - t_i \)

\( T_1, T_2 \) = computed times from periapse when the interceptor is at \( \rho_1 \) and \( \rho_2 \) in transfer orbit

\( \Delta T \) = computed transfer time, \( T_2 - T_1 \), for comparison

V = speed

\( V_{CT} \) = mean circular speed of target orbit, \( \sqrt{\frac{\mu}{a_T}} \) ft/sec

\( V(X), V(Y), V(Z) \) = velocity components along \( X, Y, Z \) coordinate axes

\( \Delta V(X), \Delta V(Y), \Delta V(Z) \) = velocity increment components

\( \Delta V_i, \Delta V_f \) = initial velocity impulse; final velocity impulse

\( \Delta V \) = total velocity impulse

\( x, y, z \) = coordinates fixed to interceptor initial orbit (inertial)

\( x', y', z' \) = coordinates fixed to interceptor transfer orbit (inertial)

\( x'', y'', z'' \) = coordinates fixed to target vehicle (rotating)
$X,Y,Z$ coordinates fixed to target orbit (inertial)

$\alpha, \beta, \gamma$ auxiliary quantities

$\gamma$ constant determining hyperbolic or elliptic computation

$\Delta \theta$ transfer angle

$\theta$ true anomaly of interceptor in transfer orbit

$\delta \theta$ increment of $\theta_1$

$\delta t$ increment in transfer time

$\delta t_w$ increment in wait time

$\delta \nu$ increment in true anomaly of interceptor in initial orbit

$\mu$ gravitational constant, ft$^3$/sec$^2$

$\nu$ true anomaly of interceptor in initial orbit

$\xi$ radial distance to interceptor in initial orbit

$\dot{\xi}_i(x), \dot{\xi}_1(y), \dot{\xi}_i(z)$ velocity components of interceptor immediately before initial impulse

$\rho$ radial distance to interceptor in target orbit

$\dot{\rho}(x'), \dot{\rho}(y'), \dot{\rho}(z')$ velocity components of interceptor in transfer orbit

$\phi$ true anomaly of target

$\psi$ auxiliary angle

$\omega$ longitude of periapsis measured from node in plane in question

$\Omega$ longitude of ascending node measured in target plane

Subscripts:

$c$ circular orbits

$f$ final time $t = t_f$

$i$ initial time $t = t_i$ unless specified differently

$I$ initial interceptor orbit
The coordinate systems are chosen with the target orbit plane as the reference plane, the periapsis of the target defining the reference position vector in space. Figure 1 is presented to emphasize the parameters involved in describing the target orbit and the target vehicle. The target vehicle radius is denoted by \( r \) and its true anomaly by \( \phi \). Input is given to define the geometry of the target orbit \( e_T, a_T \) and the true anomaly of the target vehicle \( \phi_o \) at time \( t = t_o \). The coordinate system \( X, Y, \) and \( Z \) is defined with \( X \) piercing the target orbit periapse, \( Z \) pointing along the positive rotation vector of the target, and \( Y \) completing a right-handed triad.
Unless otherwise noted, the quantities in this study are all nondimensional for generality. This condition allows the deletion of two parameters \( \mu \), and \( a_T \) from the pertinent equations. The semi-major axis \( a_T \) is not needed when dimensionless quantities are considered inasmuch as it is used for the normalizing and is only necessary when certain dimensional information is required. Neither is the central-body gravitational constant \( \mu \) required for dimensionless studies but must be used if the times are required to be dimensional (minutes, days).

The parameters required to describe the initial interceptor plane are shown in figure 2. These parameters are the usual Keplerian elements describing the initial plane and the periapse position of the interceptor orbit \( \Omega_I \), \( \omega_I \), \( i_I \) and the geometrical elements of the interceptor ellipse \( e_I \), \( a_I \). The position vector of the interceptor in the initial interceptor orbit is given by the magnitude \( \xi \) and the true anomaly \( \nu \). The coordinates \( x, y, \) and \( z \) are also defined in this system; \( x \) being directed through the periapse, \( z \) perpendicular to the orbital plane (see fig. 2), and \( y \) completing a right-handed triad.

It is not necessary to specify the six parameters for the interceptor position in the Keplerian form \( (\Omega_I, \omega_I, i_I, e_I, a_I, \nu) \) as the program provides an alternate form for this input. The position and velocity \( (x''_O, y''_O, z''_O, \dot{x''}_O, \dot{y''}_O, \dot{z''}_O) \) of the interceptor relative to the target at time \( t = t_O \) may be used in place of the Keplerian parameters. (See fig. 5.) A derivation of the relationship between these parameters is given in appendix A.

The transfer orbit plane is specified entirely by the geometry of the initial and final position vectors of the interceptor.
The time for the initial impulse is denoted by $t = t_1$ and the final impulse $t = t_4$. The quantities $\phi_0$ and $\nu_0$ are the true anomalies of the target and interceptor at some time $t = t_0$ but the initial impulse may not come until some time later $t_1$; thus, another quantity is introduced, the wait time $\Delta t_w$ where $\Delta t_w = t_1 - t_0$. It is then clear that the transfer time $\Delta t$ is $\Delta t = t_4 - t_1$. The initial position vector $\hat{s}_1$ is determined by the orbital quantities and the wait time $e_I$, $a_I$, $\Omega_T$, $i_I$, $\omega_I$, $\nu_0$, and $\Delta t_w$ or the relative coordinate data $x_0''$, $y_0''$, $z_0''$, $\dot{x}_0''$, $\dot{y}_0''$, $\dot{z}_0''$, and $\Delta t_w$. To determine the final position vector $\hat{r}_f$, the position vector of the target at $t = t_f$, the quantities $e_T$, $\phi_0$, and $\Delta t_w + \Delta t$ are needed. The total time from $t_0$ to $t_f$ is the sum of the wait time and the transfer time.

The initial position vector $\hat{s}_1$ is determined by first computing the eccentric anomaly $E_{I0}$ at $t = t_0$ of the interceptor in its initial orbit from the equation

$$\tan \frac{E_{I0}}{2} = \frac{1 - e_I}{1 + e_I} \tan \frac{\nu_0}{2} \quad (1)$$

The mean anomaly at $t = t_0$ is found directly from Kepler's equation as

$$M_{I0} = E_{I0} - e_I \sin E_{I0} \quad (2)$$

and at $t = t_1$ the mean anomaly will be $M_{II}$ where

$$M_{II} = M_{I0} + 2\pi \frac{1}{a_I^3} \Delta t_w \quad (3)$$

The eccentric anomaly $E_{II}$ at $t = t_1$ is found from iteration of the Kepler equation as described in reference 5.
\[ M_{II} = E_{II} + e_I \sin E_{II} \] (4)

After obtaining \( E_{II} \) from equation (4), the true anomaly \( \nu_1 \) may be found from

\[ \tan \frac{\nu_1}{2} = \frac{1 + e}{1 - e} \tan \frac{E_{II}}{2} \]

After the true anomaly of the interceptor \( \nu_1 \) preceding the first impulse is found, the radius vector magnitude is obtained from the well-known solution

\[ \xi_1 = \frac{a_1(1 - e_1^2)}{1 + e_1 \cos \nu_1} \] (5)

The rates may be easily computed from the relations

\[ \dot{\nu}_1 = \frac{a_1(1 - e_1^2)}{(\xi_1)^2} \] (6)

and

\[ \dot{\xi}_1 = \frac{e_1 \sin \nu_1}{a_1(1 - e_1^2)} \] (7)

The initial vector \( \hat{\xi}_1 \) written in the \( x,y,z \) coordinate system (fig. 2) becomes

\[ \hat{\xi}_1 = \xi_1 (\cos \nu_1 \hat{i} + \sin \nu_1 \hat{j}) \] (8)

The final position vector is determined from the initial conditions \( e_T \) and \( \phi_o \) along with the elapsed time \( \Delta t_w + \Delta t \) in the same manner as the initial position vector. These computations are shown in appendix B. By repeating these steps for the target vehicle at \( t = t_f \) the true anomaly \( \phi_f \) may be obtained and may be used to find the final radius vector written in the \( X,Y,Z \) system as

\[ \hat{r}_f = r_f (\cos \phi_f \hat{i} + \sin \phi_f \hat{j}) \] (9)

where

\[ r_f = \frac{1 - e_T^2}{1 + e_T \cos \phi_f} \] (10)
Also

\[ \dot{\phi}_f = \frac{\sqrt{1 - e_T^2}}{(r_f)^2} \]  
(11)

\[ \dot{r}_f = \frac{e_T \sin \phi_f}{\sqrt{1 - e_T^2}} \]  
(12)

The interceptor velocity immediately preceding the initial impulse at \( t = t_i \) is obtained from

\[ \dot{\xi}_i = \left( \xi_i \cos \nu_i - \xi_i \dot{\nu}_i \sin \nu_i \right) \hat{i} + \left( \xi_i \sin \nu_i + \xi_i \dot{\nu}_i \cos \nu_i \right) \hat{j} \]  
(13)

and after the final impulse the interceptor velocity must be the same as that of the target or

\[ \dot{r}_f = (\dot{r}_f \cos \phi_f - r_f \dot{\phi}_f \sin \phi_f) \hat{i} + (\dot{r}_f \sin \phi_f + r_f \dot{\phi}_f \cos \phi_f) \hat{j} \]  
(14)

The transfer orbital plane and some of the properties of the transfer orbit are found by noting that the interceptor must leave vector \( \dot{\xi}_i \) at time \( t_i \) and arrive at vector \( \dot{r}_f \) at time \( t_f \). Since \( \dot{\xi}_i \) and \( \dot{r}_f \) are known, the transfer plane properties and the transfer angle \( \Delta \theta \) may be determined.

In equations (8) and (9), \( \dot{\xi}_i \) was specified in terms of its components in the \( x,y,z \) coordinate system whereas \( \dot{r}_f \) was specified in the \( X,Y,Z \) system. In order to manipulate with these vectors, their components must be referenced to the same coordinate system. The target coordinate system \( X,Y,Z \) is used here so that the vector \( \dot{\xi}_i \) must be transformed. This transformation can be accomplished by the common Euler angle matrix. The elements of this matrix are dependent upon the angles \( \Omega_I, \omega_I, \) and \( \iota_I \) and are given in appendix B. The matrix will here be simply noted as \( \left[ a_{ij} \right] \). Then \( \dot{\xi}_i \) is written with components in \( X,Y,Z \) as

\[ \dot{\xi}_i(\hat{I},\hat{J},\hat{K}) = \left[ a_{ij} \right] \dot{\xi}_i(\hat{i},\hat{j},\hat{k}) \]  
(15)

or letting

\[ \dot{\xi}_i(\hat{I},\hat{J},\hat{K}) = \xi_i(X)\hat{I} + \xi_i(Y)\hat{J} + \xi_i(Z)\hat{K} \]  
(16)
where \( \xi_1(X), \xi_1(Y), \xi_1(Z) \) are functions of \( \xi_i, \nu_i, \omega_i, \Omega_i, \) and \( i_0 \) and are found from equation (15). These relations are written out in appendix B (eqs. (B21)).

With both \( \hat{\xi}_1 \) and \( \hat{r}_f \) in the \( X,Y,Z \) coordinate system, the transfer angle \( \Delta \theta \) and the transfer-orbit plane inclination \( i \) may be derived from the vector identities

\[
\hat{\xi}_1 \cdot \hat{r}_f = \xi_1 r_f \cos \Delta \theta
\]

\[
\hat{\xi}_1 \times \hat{r}_f = \xi_1 r_f \sin \Delta \theta \hat{k}'
\]

where

\[
\hat{k}' = -\sin i \sin \phi \hat{i} + \sin i \cos \phi \hat{j} + \cos i \hat{k}
\]

A complete derivation of \( \Delta \theta \) and \( i \) is given in appendix B.

The velocity components of equation (13) may also be transformed to the \( X,Y,Z \) system

\[
\hat{\xi}_1(\hat{i},\hat{j},\hat{k}) = [a_{ij}] \hat{\xi}(\hat{i},\hat{j},\hat{k})
\]

which gives expressions for \( \xi_1(X), \xi_1(Y), \) and \( \xi_1(Z) \) as shown in appendix B (eqs. (B44)).

The problem now becomes one of finding an arc of a conic section which will pass through the initial vector \( \hat{\xi}_1 \) and the final vector \( \hat{r}_f \) and which also has the property that a body traversing this arc will do so in the desired transfer time \( \Delta t \). For future convenience, the following convention is defined: Let \( \rho_1 \) be the minimum of \( r_f \) and \( \xi_1 \) and \( \rho_2 \) the maximum; then the transfer orbit must be the conic section passing through the radii \( \rho_1 \) and \( \rho_2 \), separated by an angle \( \Delta \theta \), in the time \( \Delta t \).

If an angle \( \theta_1 \) of periapse to \( \rho_1 \) is guessed, the eccentricity for the conic section which has radii \( \rho_1 \) and \( \rho_2 \) separated by \( \Delta \theta \) may be found from

\[
\rho_1 = \frac{p}{1 + e \cos \theta_1}
\]

\[
\rho_2 = \frac{p}{1 + e \cos \theta_2}
\]

\[
\theta_2 = \theta_1 + \Delta \theta
\]
and

\[ p = a(1 - e^2) \]  

(18c)

from which \( p \) is eliminated to find

\[ e = \frac{\rho_2 - \rho_1}{\rho_1 \cos \theta_1 - \rho_2 \cos \theta_2} \]  

(19)

The eccentric anomalies may also be computed from

\[ \tan \frac{E}{2} = \sqrt{\frac{1 - e}{1 + e}} \tan \frac{\theta}{2} \]

and the mean anomalies from

\[ M = E - e \sin E \]

The time of transfer for this particular choice of \( \theta_1 \) is found from the mean anomalies as

\[ \Delta T = \frac{a^3}{2 \pi} \Delta M \]

The semi-major axis \( a \) is determined from \( e \) and \( \theta_1 \) or \( \theta_2 \) in equation (18c).

The computed transfer time \( \Delta T \) will not, in general, correspond to the desired transfer time \( \Delta t \); therefore, it will be necessary to assume a new value for \( \theta_1 \) and continue with this process until the desired agreement is obtained.

It is possible to find certain regions from which to choose \( \theta_1 \) and thereby shorten the iteration considerably. It is easy to see in which conic section the transfer orbit must be by computing the time required to transfer by a parabolic orbit. This time may be determined without iteration by setting \( e = 1.0 \) and following the procedure outlined in appendix B. If this parabolic time \( \Delta T_p \) is less than the desired transfer time \( \Delta t \), the orbit must be elliptic (\( e < 1.0 \)) whereas if the parabolic time \( \Delta T_p \) is greater than the time \( \Delta t \), the orbit must be hyperbolic (\( e > 1.0 \)). The procedure is to find the type of orbit, hyperbolic or elliptic, and then set \( \theta_1 \) equal to the parabolic anomaly \( \theta_p \) plus or minus some increment \( \pm \delta \theta_1 \) so that \( \theta_1 \) lies in the hyperbolic or elliptic region, whichever is appropriate. The size of this increment \( \delta \theta_1 \) is important in convergence of the iteration. It is governed by two other limiting values, one each for the elliptic and hyperbolic regions. Then a straightforward incrementation process is used directly until the correct value of \( \theta_1 \) is found. A complete description of this process is found in appendix B.
After the properties e, a, \( \theta_1 \), \( \Delta \theta \), p, \( \Omega \), \( \omega \), and i of the transfer orbit are found, the next step is to compute the velocity increments required for the rendezvous. The velocities before the initial impulse and after the final impulse are given in equations (17) and (14). The velocities after the initial impulse and before the final impulse are described in the transfer orbit. If \( \hat{i}' \), \( \hat{j}' \), and \( \hat{k}' \) are the unit vectors along the \( x', y', z' \) coordinate axes previously defined in the transfer orbit system, these velocities are written as

\[
\hat{\rho}_1 = (\hat{\rho}_1 \cos \theta_1 - \rho_1 \dot{\theta}_1 \sin \theta_1)\hat{i}' + (\hat{\rho}_1 \sin \theta_1 + \rho_1 \dot{\theta}_1 \cos \theta_1)\hat{j}'
\]

\[
\hat{\rho}_f = (\hat{\rho}_f \cos \theta_f - \rho_f \dot{\theta}_f \sin \theta_f)\hat{i}' + (\hat{\rho}_f \sin \theta_f + \rho_f \dot{\theta}_f \cos \theta_f)\hat{j}'
\]

These velocities are then transformed to the \( X, Y, Z \) system for ease of manipulation. This transformation is done by a matrix \([b_{ij}]\) identical functionally to \([a_{ij}]\) but with the transfer orbit angular parameters \( \Omega \), \( \omega \), and i replacing the parameters for the interceptor initial orbit \( \Omega_I \), \( \omega_I \), and \( i_I \). After these transformations, the velocity increments are easily found from

\[
\Delta \hat{V}_1 = \hat{\rho}_1 - \hat{s}_1
\]

\[
\Delta \hat{V}_f = \hat{\rho}_f - \hat{\rho}_f
\]

and the total velocity increment required becomes

\[
\Delta V = \sqrt{\left| \Delta \hat{V}_1 \right|^2 + \left| \Delta \hat{V}_f \right|^2}
\]

A thorough discussion of this method and the pertinent mathematics of the problem is found in appendix B.

**COMPUTER INPUT AND OUTPUT**

The problem was programed in the FORTRAN IV language for the IBM 7094 computer installation at Langley Research Center. Certain options were incorporated in the input of the program and are described. A more thorough description of the entire program is found in appendix C. It was felt that the present writeup is sufficient for most uses and if changes are desired, they are easily incorporated through the FORTRAN IV language.
The program automatically increments the anomaly \( \nu_0 \) of the interceptor at \( t = t_0 \), the wait time in orbit \( \Delta t_w \), and the transfer time \( \Delta t \). The incrementation begins with \( \Delta t \) or \( \Delta t_w \) depending on the value of the option control variable GUIDE in the input. The second parameter incremented is \( \Delta t_w \) or \( \Delta t \), whichever is not incremented first, and the third incrementation is \( \nu_0 \).

An option is also available for using the input and output times \( \Delta t \) and \( \Delta t_w \) as dimensional quantities. The input quantity OPTION determines whether the quantities \( \Delta t \) and \( \Delta t_w \) are dimensionless or dimensioned as minutes or as days. Because of the nondimensional parameters used in the program, the input \( \mu \) and \( a_T \) may be any convenient values as long as the time is dimensionless and \( P_T \) and \( V_{CT} \) are not desired; otherwise, \( \mu \) and \( a_T \) must be properly dimensioned.

The input quantity DATAS determines whether the initial interceptor orbit data are Keplerian or relative Cartesian.

Other necessary input quantities are \( h \), the increment constant for the \( \theta_1 \) iteration (in the examples solved a value of 3 was used), \( \delta t \) and \( \delta t_w \), the transfer and wait time incrementation for the time grid. Also, input is the quantity \( r_{min} \) which is the minimum radius the interceptor may take as it travels the transfer arc. This value does not affect the computations in any way; however, if the radial distance of the transfer arc falls below the value \( r_{min} \), an asterisk is printed out at the right-hand side of the output sheet.

The variables output in the program described herein were those which were considered to be of general use. The output is printed as shown in tables I, II, and III which are output data for the three examples following this discussion. The first block of data is a reprint of the input where the following correlation between symbols may be observed:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Corresponding value</th>
</tr>
</thead>
<tbody>
<tr>
<td>ET</td>
<td>( e_T )</td>
</tr>
<tr>
<td>EI</td>
<td>( e_1 )</td>
</tr>
<tr>
<td>AI</td>
<td>( a_I )</td>
</tr>
<tr>
<td>OMEGA I</td>
<td>( \Omega_I )</td>
</tr>
<tr>
<td>OMI</td>
<td>( \omega_I )</td>
</tr>
<tr>
<td>I</td>
<td>( i_1 )</td>
</tr>
<tr>
<td>PHIO</td>
<td>( \phi_0 ) (first)</td>
</tr>
<tr>
<td>NU01</td>
<td>( \nu_0 ) (first)</td>
</tr>
<tr>
<td>NU0L</td>
<td>( \nu_0 ) (last)</td>
</tr>
<tr>
<td>MU</td>
<td>( \mu )</td>
</tr>
<tr>
<td>AT</td>
<td>( a_T ) (ft)</td>
</tr>
<tr>
<td>TPER</td>
<td>( P_T ) (sec)</td>
</tr>
<tr>
<td>VCT</td>
<td>( V_{CT} ) (ft/sec)</td>
</tr>
<tr>
<td>DTTT</td>
<td>( \Delta t ) (first)</td>
</tr>
<tr>
<td>DTTF</td>
<td>( \Delta t ) (last)</td>
</tr>
<tr>
<td>DTTFw</td>
<td>( \Delta t_w ) (first)</td>
</tr>
<tr>
<td>DTTFw</td>
<td>( \Delta t_w ) (last)</td>
</tr>
<tr>
<td>RMIN</td>
<td>( r_{min} )</td>
</tr>
<tr>
<td>H</td>
<td>( h )</td>
</tr>
</tbody>
</table>

In the case that the input is in the form of relative coordinate data, these Keplerian quantities for the initial interceptor orbit are computed from the relative coordinate data as has been explained. An example of this printout is shown in table II where a fourth row of variables is noted and \( X_O, Y_O, \) and \( Z_O \) correspond to \( x''_O, y''_O, \) and \( z''_O \) and \( XODT, YODT, \) and \( ZODT \) correspond to \( \dot{x}_O, \dot{y}_O, \) and \( \dot{z}_O \).

After this output which records the data for the terminal orbits, the data are computed for the rendezvous problem and the following description of the printout nomenclature should be noted:
EXAMPLES

Example 1 – Interplanetary Transfer Between Massless Planets

To compute velocity increments required for interplanetary trajectories the input for the planet ephemeris is needed. This input can be found from reference 6. As an example, consider a trip from Earth to Mars which is to be accomplished by a two-impulse orbit. As the observation time \( t_0 \), January 4, 1964 (Julian date, 243-8398.5) is selected. From the 1964 ephemeris tables (pp. 18, 50, 172, 176), data for this problem are found which are referenced to the ecliptic plane. From the geometry of the ecliptic coordinate system (see refs. 1 and 2) and the definitions presented here, the following program input is determined:

\[
e_T = 0.093372 \quad \quad \quad \quad v_0 = 0.37^\circ
\]
\[
a_T = 7.4737 \times 10^{11} \quad \quad \quad \quad \Omega_I = 253.88^\circ
\]
\[ \phi_0 = 324.4^{\circ} \quad \omega_1 = 233.02^{\circ} \]
\[ e_1 = 0.0167242 \quad i_1 = 1.850^{\circ} \]
\[ a_1 = 0.656301 \]

In order to be notified in the output whether the transfer arc is less than 0.6 astronomical units, let \( r_{\text{min}} = 0.6a_1 \). Also let

\[ \mu = 4.679 \times 10^{21} \text{ ft}^3/\text{sec}^2 \]
\[ h = 3.0 \]

then,

\[ r_{\text{min}} = 0.393781 \]

Since \( \nu_0 \) is not to be incremented (because it is fixed by the physics of the solar system), let \( \nu_0 \) (first), \( \nu_0 \) (last) be \( \nu_0 \) as shown and the increment \( \delta \nu_0 = 0 \). Let it be desirable to collect data in increments of 20 days with transfer times from 160 to 260 days and waiting times of 0 to 40 days, and then use

\[ \Delta t(\text{first}) = 160.0 \quad \Delta t(\text{last}) = 260.0 \quad \delta t = 20.0 \]
\[ \Delta t_w(\text{first}) = 0.0 \quad \Delta t_w(\text{last}) = 40.0 \quad \delta t_w = 20.0 \]

If the data are to be analyzed with \( \Delta t \) as the primary independent variable and \( \Delta t_w \) as the parameter, \( \Delta t \) is incremented first. Therefore, set GUIDE at 2.0. The times are to be input and output in days so that OPTION is 3.0, and since Keplerian input is used, set DATAS as 1.0. Some output data are given in table I with time in days and all other quantities except \( \mu \), \( a_T \), \( P_T \), and \( V_{CT} \) are nondimensional.

Example 2 – Earth-Orbit Rendezvous

There are many approximate schemes which have been derived and investigated for determining the velocity increments required to rendezvous between similar orbits. Such investigations have been mainly concerned with circular orbits or first-order representations of elliptic orbits (near circular orbits). The simplest case of rendezvous with a circular target orbit (as done by Clohessy and Wiltshire, ref. 7) gives a closed-form solution to the linearized equations of motion for the velocity increment required to rendezvous in a given amount of time. This linearized solution is accurate for the terminal phases of a general rendezvous problem but the accuracy is soon lost for large initial interceptor-target ranges. It should be noticed that even in the docking phase, this solution may not be
too accurate if the target orbit is highly eccentric. Further attempts to increase the applicability of the Clohessy-Wiltshire results usually require an iteration scheme (Anthony and Sasaki, ref. 8) or some other process to get a solution of the velocity increment required to rendezvous.

It is noted that the solution of the exact equations presented herein is not too involved once a computer program is available, and, of course, offers the advantage of giving correct results for both large eccentricities and inclinations.

As an example of the use of this program for earth-orbit rendezvous, assume that the following elements are used for the target orbit:

\[
e_T = 0.0234
\]
\[
a_T = 2.248 \times 10^7 \text{ ft}
\]

Also let

\[
\phi_o = 0.0^0
\]
\[
r_{\text{min}} = 0.954
\]
\[
\mu = 1.408 \times 10^{16} \text{ ft}^3/\text{sec}^2
\]
\[
h = 3.0
\]

The interceptor position and velocity are given in relative coordinates:

\[
x_O'' = -0.01692 \quad x_O'' = -0.00376
\]
\[
y_O'' = 0.0376 \quad y_O'' = 0.1526
\]
\[
z_O'' = 0.0 \quad z_O'' = 0.0
\]

Note that \(e_I, a_I, \nu_o, i_I, \Omega_I, \text{ and } \omega_I\) are computed from these relations. Let the time be input and output in minutes:

\[
\Delta t(\text{first}) = 2.0 \quad \Delta t(\text{last}) = 40.0 \quad \delta t = 10.0
\]
\[
\Delta t_w(\text{first}) = 0.0 \quad \Delta t_w(\text{last}) = 30.0 \quad \delta t_w = 10.0
\]

If a plot of \(\Delta t_w\) as abscissa and \(\Delta t\) as a constant parameter is desired, set GUIDE at 1.0. Since Input time is in minutes, OPTION is 2.0, and since relative coordinate data are used, set DATAS at 2.0.

The output of the computer program for this input is given in table II.
Example 3 – Transfer Orbits

The problem of orbital transfer between two arbitrary elliptic orbits may also be accomplished by this computational procedure. The use of this method in orbital transfer problems is indicated in this example where the minimum two-impulse transfer orbit is desired between two fixed terminal orbits. The transfer orbit requiring minimum velocity increment may be found by proper interpretation of the data and the use of a plotter. The program as presented here does not have a gradient or other optimization scheme to find the minimum velocity transfer directly. However, an example is shown of one way of obtaining the optimum (two-impulse) orbital transfer.

For the terminal orbits the following input data are used:

\[ e_T = 0.5 \quad a_T = 1.0 \quad \text{(arbitrary)} \]
\[ \Omega_I = 90^\circ \quad e_I = 0.2 \]
\[ i_I = 30^\circ \quad a_I = 0.9 \]
\[ \omega_I = -90^\circ \]

Also let

\[ r_{\text{min}} = 0.45 \]
\[ h = 3.0 \]
\[ \mu = 1.0 \quad \text{(arbitrary)} \]

The true anomaly \( \nu_0 \) of the interceptor at \( t = t_0 \) is chosen in the following range:

\[ \nu_0(\text{first}) = 0.0 \]
\[ \nu_0(\text{last}) = 350^\circ \]
\[ \delta \nu_0 = 10^\circ \]

which is incremented automatically by the program. To obtain all the possible transfer arcs, it is necessary to increment either \( \Delta t_w \) or \( \nu_0 \), but not both. As \( \nu_0 \) is chosen to be the incremented parameter, the wait time is set equal to zero.

The true anomaly \( \phi_0 \) is chosen, and when the transfer time \( \Delta t \) is varied, a set of curves for the variation of \( \Delta V \) with \( \Delta t \) with \( \nu_0 \) as the parameter are obtained for a specified value of \( \phi_0 \). The program does not increment \( \phi_0 \) automatically as it does for \( \Delta t \), \( \Delta t_w \), and \( \nu_0 \). However, it always returns to read the first data card of a new
set after running through one complete set of data. If there are no more sets of data, the computation ends; if there is another complete data set, it starts anew. To increment \( \phi_o \), it is necessary to reproduce all data cards for every new value of \( \phi_o \) and to place these one behind the other. (See appendix C.)

As set out above, each complete data set gives one complete set of curves for a given \( \phi_o \) with \( \nu_o \) as the parameter. (See sketch 1.)

As an aid, sketch 1 has been drawn with \( \phi_f \) as the abscissa value rather than \( \Delta t \) and is possible because of the functional relation between \( \Delta t \) and \( \phi_f \) or

\[
\phi_f = \phi_o + F(\Delta t)
\]

since \( \Delta t_w = 0 \).

The transfer time \( \Delta t \) is read in as follows:

\[
\begin{align*}
\Delta t(\text{first}) &= 0.09 \\
\Delta t(\text{last}) &= 1.008 \\
\delta t &= 0.009
\end{align*}
\]

These quantities are dimensionless ratios to the target orbit period. The waiting time is input as

\[
\Delta t_w(\text{first}) = \Delta t_w(\text{last}) = \delta t_w = 0
\]

Some sample computer output is shown in table III for \( \phi_o = 0.0^\circ \).

From a Beckman automatic plotter, the characteristics of one of the lower \( \Delta V \) transfer orbits are found to be

\[
\begin{align*}
\nu_1 &= 55^\circ & \Delta \theta &= 132.47^\circ \\
\phi_f &= 193.58^\circ & i &= -22.88^\circ \\
\Delta t &= 0.64 & \rho_{\text{min}} &= 0.78 \\
e &= 0.4361 & \Delta V_i &= 0.0808 \\
a &= 1.086 & \Delta V_f &= 0.2563 \\
\theta_i &= 71.96^\circ & \Delta V &= 0.3371
\end{align*}
\]
CONCLUDING REMARKS

This report presents a technique for solving three-dimensional orbital problems in a straightforward manner using the exact solutions or Kepler solutions to the equations of motion. Basically the method developed is an iteration on the Kepler equation using the true anomaly as the iteration parameter and the mean anomaly or transfer time compared with a prespecified transfer time as the stopping criteria. To aid in the choice of the true anomaly to begin the iteration, certain boundaries are devised within which the solution must lie. The iteration is performed directly on the Kepler equations and no derivatives are necessary. This method works very well and the computation time compares favorably with other methods, the typical run times being about 0.003 minute per transfer. The generality of the program format presented allows rapid computations, with simple engineering input parameters, of interplanetary rendezvous or near-planet rendezvous cases. For instance the program has proven useful in studies concerning fuel requirements for abort missions during lunar letdown of the lunar excursion module and command module.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Station, Hampton, Va., February 14, 1966.
APPENDIX A

OBTAINING KEPLERIAN ORBITAL ELEMENTS FROM RELATIVE COORDINATE DATA

In many problems concerning rendezvous, the initial conditions are given in terms of the relative position and velocity of the interceptor with respect to the target. If the Keplerian equations are used for the exact solution of the motion, the Keplerian elements in terms of the relative coordinates must be obtained. Let \( x''_0, y''_0, z''_0 \) be the relative position vector (see fig. 5) of the interceptor with respect to the \( x'', y'', z'' \) coordinate system attached to the target; \( x'' \) points away from the gravitational center, \( y'' \) is in the positive direction of angular motion, and \( z'' \) is along \( Z \) normal to the \( x''y'' \) plane.

Suppose that the following are given in dimensionless form: \( e_T, \phi_0, x''_0, y''_0, z''_0, \dot{x}''_0, \dot{y}''_0, \) and \( \dot{z}''_0 \). It is desired to find the Keplerian constants \( e_I, a_I, \nu_0, \Omega_I, \alpha_I \), and \( i_I \). By using the formal application of vector analysis, let \( \hat{i}, \hat{j}, \hat{k} \) be unit vectors along \( x'', y'', \) and \( z'' \), and obtain the radius vector for the interceptor, and since the rotational rate of the \( (x'', y'', z'') \) coordinate system is \( \dot{\phi}\hat{k} \), the velocity is

\[
\dot{\xi} = (r + x'')\hat{i} + y''\hat{j} + z''\hat{k}
\]

\( (A1) \)

where

\[
\hat{k} \times \dot{\xi} = (r + x'')\hat{j} - y''\hat{i}
\]

\( (A3) \)

The velocity of the interceptor becomes:

\[
\dot{\nu} = \dot{\xi} = (\dot{r} + \dot{x}'')\hat{i} + (\dot{y}'' + \dot{\phi}(r + x''))\hat{j} + \dot{z}''\hat{k}
\]

\( (A4) \)

The fundamental relations of orbital mechanics show that

\[
\begin{align*}
p_T &= 1 - e_T^2 \\
r &= \frac{p_T}{1 + e_T \cos \phi} \\
\dot{r} &= \frac{e_T \sin \phi}{\sqrt{p_T}} \\
\dot{\phi} &= \frac{\sqrt{p_T}}{r^2}
\end{align*}
\]

\( (A5) \)
APPENDIX A

From these expressions the velocity of the interceptor in its initial orbit can be written in terms of the desired quantities along the axes of the moving \( x', y', z' \) coordinate system. Quantities considered here are at the time \( t = 0 \) and are subscripted accordingly. The velocity components at \( t = t_0 \) along the \( x', y', z' \) axes will be denoted by \( V_{o1}, V_{o2}, \text{ and } V_{o3} \), respectively; thus, the interceptor velocity becomes

\[
\hat{V} = V_{o1}\hat{i} + V_{o2}\hat{j} + V_{o3}\hat{k} \quad \text{(A6)}
\]

By using equations (A4), (A5), and (A6), the components are written in terms of known quantities as

\[
\begin{align*}
V_{o1} &= \frac{e_T \sin \phi_0}{\sqrt{p_T}} + x''_0 - y''_0 \frac{\sqrt{p_T}}{r_0^2} \\
V_{o2} &= \frac{\dot{x}_0'' + (r_0 + x''_0) \frac{\sqrt{p_T}}{r_0^2}}{r_0^2} \\
V_{o3} &= \frac{\dot{z}_0''}{r_0^2} \\
\end{align*}
\]

\[
\text{(A7)}
\]

The energy equation for the interceptor at time \( t_0 \) may be written (nondimensionally) as

\[
V_0^2 - \frac{2}{\xi_o} = -\frac{1}{a_I} \quad \text{(A8)}
\]

where \( \xi_o \), \( V_o \) are magnitudes of the defined vectors \( \xi, \hat{V} \) at time \( t_0 \) or

\[
\xi_o = \sqrt{(r_o + x''_0)^2 + (y''_0)^2 + (z''_0)^2} \quad \text{(A9)}
\]

\[
V_o = \sqrt{V_{o1}^2 + V_{o2}^2 + V_{o3}^2} \quad \text{(A10)}
\]

By using equation (A8), solve for the semi-major axis \( a_I \)

\[
a_I = \frac{\xi_o}{2 - \xi_o V_o^2} \quad \text{(A11)}
\]

To determine the eccentricity of the initial interceptor orbit, write the angular momentum at time \( t_0 \) of the interceptor,

\[
\hat{H}_o = \xi_o \times \hat{V}_o \quad \text{(A12)}
\]
APPENDIX A

where \( \hat{\xi}_o \) and \( \hat{V}_o \) are dimensionless quantities. If

\[
\hat{H}_o = H_{o1}\hat{i} + H_{o2}\hat{j} + H_{o3}\hat{k}
\]

performing the indicated operations yields

\[
\begin{align*}
H_{o1} &= y_o''V_{o3} - z_o''V_{o2} \\
H_{o2} &= z_o''V_{o1} - \langle r_o + x_o'' \rangle V_{o3} \\
H_{o3} &= \langle r_o + x_o'' \rangle V_{o2} - y_o''V_{o1}
\end{align*}
\]

and

\[
H_o = \sqrt{(H_{o1})^2 + (H_{o2})^2 + (H_{o3})^2}
\]

(A14)

In the nondimensional form, the angular momentum and semi-latus rectum may easily be shown to be related as

\[
p_I = H_o^2
\]

(A15)

and therefore the well-known relationship \( p_I = a_I(1 - e_I^2) \) gives with equation (A15):

\[
e_I = \sqrt{1 - \frac{H_o^2}{a_I}}
\]

(A16)

Sketch 2 and figure 5 are given as aids in describing how the angular measures \( \iota_I, \Omega_I, \) and \( \omega_I \) are determined. The unit vector \( \hat{N} \) directed along the line of nodes of the initial interceptor orbit and pointing toward the ascending node is defined by the relation

\[
\hat{k} \times \hat{H}_o = |H_o| \sin \iota_I \hat{N}
\]

(A17)

The inclination \( \iota_I \) is also described by the relation

\[
\hat{k} \cdot \hat{H}_o = H_o \cos \iota_I
\]

(A18)
APPENDIX A

and equation (A18) gives

\[ i = \cos^{-1} \frac{H_o^3}{H_o} \quad (0 < i < \pi) \]  

(A19)

From figure 5, it is geometrically evident that

\[ \hat{I} = \hat{i} \cos \phi_o - \hat{j} \sin \phi_o \]  

(A20)

and also from the definition of the angle \( \Omega_I \)

\[ \cos \Omega_I = \hat{i} \cdot \hat{N} \]  

(A21)

\[ \hat{k} \sin \Omega_I = \hat{i} \times \hat{N} \]  

(A22)

and expanding the vector manipulation of equation (A17) gives

\[ \hat{N} = \frac{H_o^1}{H_o \sin i} \hat{i} - \frac{H_o^2}{H_o \sin i} \hat{j} \]  

(A23)

From the well-known relationship

\[ \xi_o = \frac{p_I}{1 + e_i \cos \nu_o} \]

obtain

\[ \cos \nu_o = \frac{1}{e_i} \frac{p_I}{\xi_o} - 1 \]  

(A24)

A basic orbital relationship which is of value here is

\[ \sin \nu_o = \frac{\sqrt{p_I}}{e_i} \xi_o \]  

(A25)

and, as \( \xi_o \) is the component of \( \hat{V}_o \) along \( \xi_o \),

\[ \xi_o = \hat{V}_o \cdot \xi_o \]  

(A26)

In order to find the angle \( \omega_i \), note that another geometric relation is

\[ \hat{\xi}_o \cdot \hat{N} = \xi_o \cos (\omega_i + \nu_o) \]  

(A27)
APPENDIX A

and also

\[ \hat{N} \times \hat{\xi}_o = \xi_o \sin (\omega_I + \nu_o) \frac{\hat{H}_o}{H_o} \]  

(A28)

Note also that \( \nu_o \) may be found from equations (A24) and (A25) as

\[ \nu_o = \tan^{-1}\left(\frac{\sin \nu_o}{\cos \nu_o}\right) \quad (0 \leq \nu_o < 2\pi) \]  

(A29)

From equations (A20), (A21), and (A23),

\[ \cos \Omega_1 = -\frac{H_{o2} \cos \phi_o + H_{o1} \sin \phi_o}{H_o \sin i_I} \]  

(A30)

and the proper vector manipulations with equations (A20), (A22), and (A23) yield

\[ \sin \Omega_1 = \frac{H_{o1} \cos \phi_o - H_{o2} \sin \phi_o}{H_o \sin i_I} \]  

(A31)

From equations (A30) and (A31),

\[ \Omega_1 = \tan^{-1}\left(\frac{\sin \Omega_1}{\cos \Omega_1}\right) \quad (0 \leq \Omega_1 < 2\pi) \]  

(A32)

Similarly, performing the indicated vector manipulations on equations (A27) and (A28) leads to the scalar equations

\[ \cos (\omega_I + \nu_o) = \xi_o \frac{H_{o2} \cos \phi_o - H_{o1} \sin \phi_o}{\xi_o H_o \sin i_I} \]  

(A33)

\[ \sin (\omega_I + \nu_o) = \xi_o \frac{H_{o2} \cos \phi_o - H_{o1} \sin \phi_o}{\xi_o \sin i_I} \]  

(A34)

If the inclination \( i_I \) is zero, \( \Omega_1 \) is arbitrary; thus, let

\[ \Omega_1 = 0 \]  

(A35)

Also, equations (A27) and (A28) yield

\[ \cos (\omega_I + \nu_o) = \frac{\xi_o \cos \phi_o - \xi_o \sin \phi_o}{\xi_o} \]  

(A36)
APPENDIX A

\[
\sin (\omega_I + \nu_o) = \frac{\xi_{o1} \sin \phi_o + \xi_{o2} \cos \phi_o}{\xi_o}
\]  \hspace{1cm} (A37)

Whether \( i_I \) is zero or not, equations (A33), (A34), or (A36), (A37) give

\[
\omega_I + \nu_o = \tan^{-1} \left[ \frac{\sin (\omega_I + \nu_o)}{\cos (\omega_I + \nu_o)} \right]
\]  \hspace{1cm} (A38)

and equation (A38) with equation (A29) give the desired expression

\[
\omega_I = (\omega_I + \nu_o) - \nu_o
\]  \hspace{1cm} (A39)

Equations (A11), (A16), (A19), (A29), (A32), and (A39) give the information necessary to compute the desired Keplerian elements.
APPENDIX B

MATHEMATICAL DESCRIPTION OF PROBLEM

The following description is a logical flow of the problem as it is programmed.

Description of Initial and Final Properties

If the initial and terminal states of an orbit referred to two other elliptic orbits, namely, the interceptor initial orbit and the target orbit, and also the time required to transfer from orbit to orbit are known, the velocity increments required to establish the transfer orbit and the elements of this transfer orbit may be computed. The initial and terminal states may be found from the input as follows. Suppose that the following data are given: \( \mu, a_T, e_T, \phi_o, e_I, a_I, \omega_I, \Omega_I, i_I, \nu_o, \Delta t, \) and \( \Delta t. \) Recall that all quantities except \( \mu \) and \( a_T \) are dimensionless.

Compute the semi-latus rectum of the initial interceptor orbit and the target orbit from

\[
\begin{align*}
p_I &= a_I \left(1 - e_I^2\right) \\
p_T &= 1 - e_T^2
\end{align*}
\]

The target orbital period in seconds is

\[
P_T = 2\pi \sqrt{\frac{a_T^3}{\mu}}
\]

The eccentric anomaly of the target at \( t = t_o \) is found from \( e_T \) and \( \phi_o \) by the defined relationship

\[
\tan \frac{E_{To}}{2} = \sqrt{\frac{1 - e}{1 + e}} \tan \frac{\phi_o}{2}
\]

and then functionally as

\[
E_{To} = F(e_T, \phi_o)
\]

For computational purposes, define the function \( F(\alpha, \beta) \) as

\[
F(\alpha, \beta) = 2\pi + 2 \tan^{-1} \left( \frac{1 - \alpha}{\gamma \tan \frac{\beta}{2}} \right)
\]

1st and 4th quadrants
APPENDIX B

where

\[ \begin{align*}
\alpha &= e_T \\
\beta &= \phi_0
\end{align*} \]  

(B5b)

and

\[ n = \text{Integer}\left(\frac{\beta + \pi}{2\pi}\right) \text{lowest integer value} \]  

(B5c)

\[ \begin{align*}
\gamma &= 1.0 & (\alpha < 1.0) \\
\gamma &= -1.0 & (\alpha > 1.0)
\end{align*} \]  

(B5d)

Note that allowance is made here for the possibility of hyperbolic orbits because \( F(\alpha, \beta) \) will also be used for transfer orbits.

The mean anomaly of the target orbit at \( t = t_0 \) is found directly from Kepler's equation

\[ M_{To} = E_{To} - e_T \sin E_{To} \]  

(B6)

The mean anomaly of the target at the final time \( t = t_f \) is

\[ M_{Tf} = M_{To} + 2\pi(\Delta t + \Delta t_w) \]  

(B7)

The eccentric anomaly of the target at \( t = t_f \) is required so that the true anomaly at \( t = t_f \) may ultimately be obtained. It is possible to solve for the eccentric anomaly by use of an expansion in terms of the infinite Bessels series (see ref. 9) but it was found to be more rapid to iterate the Kepler equation by using the method of differential correction as shown in reference 5 or 9. The steps in the iteration procedure are as follows:

1. Estimate the initial value \( E_1 \) from truncated series solution of the Kepler equation

\[ E_1 = M_0 + e \sin M_0 + \frac{1}{2} e^2 \sin 2M_0 \quad \left( M_0 = M_{Tf}; \quad e = e_T \right) \]  

(B8)

2. Compute the following sequence:
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\[ i = 1 \]
\[ M_i = E_i - e_T \sin E_i \]
\[ \Delta M_i = M_{Tf} - M_i \]
\[ \Delta E_i = \Delta M_i / (1 - e_T \cos E_i) \]
\[ E_{i+1} = E_i + \Delta E_i \]

Compare \( E_{i+1} \) and \( E_i \):
\[ \begin{cases} 
E_{i+1} \neq E_i \\
E_{i+1} = E_i 
\end{cases} \]

which gives after the completed iteration:

\[ E_{Tf} = E_{i+1} \]  \hspace{1cm} (B9)

At \( t = t_i \) the eccentric anomaly of the interceptor \( E_{li} \) is obtained from \( \nu_o, e_i, \) and \( \Delta t_w \) in the same manner as \( E_{Tf} \). The expressions are rewritten here for completeness with eccentric anomaly of interceptor at \( t = t_o \) as

\[ E_{Io} = F(e_i, \nu_o) \]

where \( F(e_i, \nu_o) \) is the function previously defined (eq. (B5)) with \( \alpha = e_i \) and \( \beta = \nu_o \). Also

\[ M_{Io} = E_{Io} - e_i \sin (E_{Io}) \]

At \( t = t_i \) the interceptor mean anomaly is found from

\[ M_{li} = M_{Io} + 2\pi / a_i^3 \Delta t_w \]

The iteration steps are shown in equations (B8) and (B9) with the following replacements: \( e_T \) by \( e_i \); \( M_{Tf} \) by \( M_{li} \); \( E_{Tf} \) by \( E_{li} \).

With this information the true anomalies at the initial and final times of the interceptor in its initial orbit and the target in its orbit may be computed. The routine given by equations (B4) and (B5) will give these values if \( -\alpha \) is replaced by the corresponding \( e \). Thus, the true anomaly of the interceptor in its initial orbit at \( t = t_i \) is, \( \alpha \) and \( \beta \) in \( F \) being replaced by the parameters \( -e_i \) and \( E_{li} \)

\[ \nu_i = F(-e_i, E_{li}) \]  \hspace{1cm} (B10)

and the target in its plane at \( t = t_f \) is
With these quantities it is possible to compute the properties $\xi_1$, $\dot{\xi}_1$, $\dot{\nu}_1$, $r_f$, $\dot{r}_f$, and $\phi_f$ which will be required for computation of the velocity increments. The initial radius vector is

$$\xi_1 = \frac{p_1}{1 + e_1 \cos \nu_1}$$  \hspace{1cm} (B12)

The rate of change of true anomaly at $t = t_1$ is

$$\dot{\nu}_1 = \sqrt{p_1} \frac{1}{(\xi_1)^2}$$  \hspace{1cm} (B13)

Also the rate of change of the radial distance is

$$\dot{\xi}_1 = \frac{e_1 \sin \nu_1}{\sqrt{p_1}}$$  \hspace{1cm} (B14)

For the final time $t = t_f$, the properties of the target orbit are

$$r_f = \frac{p_T}{1 + e_T \cos \phi_f}$$  \hspace{1cm} (B15)

$$\phi_f = \frac{\sqrt{p_T}}{r_f^2}$$  \hspace{1cm} (B16)

$$\dot{r}_f = \frac{e_T \sin \phi_f}{\sqrt{p_T}}$$  \hspace{1cm} (B17)

If an interceptor is to transfer (or rendezvous) from one orbit to another, the two orbits being determined by this information, it must begin at $t = t_1$ in the initial orbit, change velocity (instantaneously) to the transfer orbit at $t = t_1$, and travel until $t = t_f$ in the transfer orbit. Then it instantaneously changes its velocity to satisfy the terminal or target orbit properties. Enough information is now available to compute the initial and final terminal velocities but further discussion of this procedure will be delayed until the initial and final transfer orbit properties have been computed.

**Computation of Transfer Orbit Properties**

Information giving initial and final terminal vectors in space $\hat{\xi}_1$, $\hat{r}_f$ is available, and it is now necessary to determine the transfer conic which connects these two vectors.
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in the specified transfer time \( \Delta t \). Reference 3 shows that the solution to this problem is unique. However, because of the implicit nature of the Kepler problem, it is necessary to perform an iteration in order to obtain the solution. The method used here is a straightforward iteration of the Keplerian orbital equations as was also described in reference 3.

The plane of the transfer orbit is specified by the plane containing the initial and final position vectors and the center of attraction. The transfer plane inclination \( i \) and the transfer angle \( \Delta \theta \) are determined with the aid of vector representation. Let \( \hat{I}, \hat{J}, \) and \( \hat{K} \) be unit vectors along \( X, Y, \) and \( Z \) axes; let \( \hat{i}, \hat{j}, \) and \( \hat{k} \) be unit vectors along \( x, y, \) and \( z \) axes as shown in figures 1, 2, and 3. Let \([a_{ij}]\) be the \( x \) to \( X \) transformation matrix so that a general vector \( \hat{A} \) transforms as:

\[
\hat{A}(IJK) = [a_{ij}] \hat{A}(ijk)
\]

The initial vector may be written as

\[
\hat{\xi}_i = \hat{\xi}_i (\cos \nu \hat{i} + \sin \nu \hat{j})
\]

By letting \( \xi_{i1}, \xi_{i2}, \xi_{i3} \) be the components of \( \hat{\xi}_i \) along the \( X, Y, Z \) axes, the following relations are obtained:

\[
\hat{\xi}_i = \xi_{i1} \hat{I} + \xi_{i2} \hat{J} + \xi_{i3} \hat{K}
\]

\[
\hat{r}_f = r_f (\cos \phi \hat{i} + \sin \phi \hat{j})
\]

In order to examine the vectors in the \( \hat{I}, \hat{J}, \hat{K} \) system, a transformation of components is required; thus,

\[
\hat{\xi}_1(I,J,K) = [a_{ij}] \hat{\xi}_1(ijk)
\]

where

\[
[a_{ij}] = \begin{bmatrix}
\cos \omega \cos \Omega - \cos i \sin \Omega \sin \omega & - \sin \omega \cos \Omega - \cos i \sin \Omega \cos \omega & \sin \Omega \sin i \\
\cos \omega \sin \Omega + \cos i \cos \Omega \sin \omega & - \sin \omega \sin \Omega + \cos i \cos \Omega \cos \omega & \cos \Omega \sin i \\
\sin i \cos \omega & \sin i \sin \omega & \cos i
\end{bmatrix}
\]

From equations (B18), (B19), and (B20),
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\[\begin{align*}
\xi_{11} &= \xi_1 \left[ \cos \Omega_i \cos (\nu_i + \omega_i) - \cos \nu_i \sin \Omega_i \sin (\nu_i + \omega_i) \right] \\
\xi_{12} &= \xi_1 \left[ \sin \Omega_i \cos (\nu_i + \omega_i) + \cos \nu_i \cos \Omega_i \sin (\nu_i + \omega_i) \right] \\
\xi_{13} &= \xi_1 \left[ \sin \nu_i \sin (\nu_i + \omega_i) \right]
\end{align*}\]  

(B21)

To determine \( \Delta \theta \) and \( i \), note the vector identities:

\[\hat{\xi}_i \cdot \hat{r}_f = \xi_1 r_f \cos \Delta \theta \]  

(B22)

\[\hat{\xi}_i \times \hat{r}_f = \xi_1 r_f \sin \Delta \theta \hat{k'}\]  

(B23)

Note that the unit vector \( \hat{k'} \) along the \( z' \)-axis is written as

\[\hat{k'} = \hat{i} \sin i \sin \Omega - \hat{j} \sin i \cos \Omega + \hat{k} \cos i\]  

(B24)

and since

\[\Omega + \pi = \phi_f\]

\[
\hat{k'} = - \sin i \sin \phi_f \hat{i} + \sin i \cos \phi_f \hat{j} + \cos i \hat{k}
\]

From equations (B18), (B19), (B23), and (B24) find

\[
\cos \Delta \theta = \frac{\xi_{11} \cos \phi_f + \xi_{12} \sin \phi_f}{\xi_1}
\]

\[
\sin \Delta \theta \sin i = \frac{\xi_{13}}{\xi_1}
\]

\[
\tan i = \frac{\xi_{13}}{\xi_{11} \sin \phi_f - \xi_{12} \cos \phi_f}
\]

Combining these equations and solving for \( i \) yields the following expression:

\[
i = \arctan \frac{\xi_{13}}{\xi_{11} \sin \phi_f - \xi_{12} \cos \phi_f} \quad \left( -\frac{\pi}{2} < i < \frac{\pi}{2} \right)
\]  

(B25)
APPENDIX B

and also

\[
\cos \Delta \theta = \frac{\xi_{11} \cos \phi_f + \xi_{12} \sin \phi_f}{\xi_1} \quad (B26)
\]

\[
\sin \Delta \theta = \frac{\xi_{11} \sin \phi_f - \xi_{12} \cos \phi_f}{\xi_1 \cos i} \quad (B27)
\]

\[
\Delta \theta = \arctan \frac{\sin \Delta \theta}{\cos \Delta \theta} \quad (0 \leq \Delta \theta \leq 2\pi) \quad (B28)
\]

The nodal angle \( \Omega \) may be expressed by

\[
\begin{align*}
\Omega &= \phi_f + \pi & (i > 0) \\
\Omega &= \phi_f & (i < 0)
\end{align*}
\]

(B29)

An inherent symmetry in the equations exists so that it makes no difference in the iteration whether the transfer from \( \xi_1 \) to \( \xi_f \) or \( \xi_f \) to \( \xi_1 \) is considered, and advantage of this symmetry is taken by considering transfers only from the shortest of \( \xi_1 \), \( \xi_f \) to the longest. Therefore, let \( \rho_1 \) be the minimum of \( \xi_1 \), \( r_f \) and \( \rho_2 \) be the maximum of \( \xi_1 \), \( r_f \). (If \( r_f = \xi_1 \), \( \rho_1 = \rho_2 = \rho = r_f = \xi_1 \).)

The chord \( c \) is given by

\[
c = \sqrt{\rho_1^2 + \rho_2^2 - 2\rho_1\rho_2 \cos \Delta \theta} \quad (B30)
\]

A geometric picture is shown in the following sketch:

![Sketch 3.]- The quantities \( \rho_1 \), \( \rho_2 \), \( \Delta \theta \), and \( c \).

By assuming a periapse angle of \( \theta_1 \) in the transfer plane, an orbit between \( \rho_1 \) and \( \rho_2 \) can be determined as shown in the following sketch:
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For each properly chosen angle \( \theta_1 \), there is a conic section which passes through \( \rho_1, \rho_2 \) and an associated time of passage \( T_1, T_2 \). Hence, for a given \( \theta_1 \), a given transfer time \( T_2 - T_1 \) is generated. In general, this time will not be the desired transfer time \( \Delta t \), and thus an iteration is necessary. The necessary calculations for the determination of the orbit by this method are now developed. Define

\[
\theta_2 = \theta_1 + \Delta \theta \tag{B31}
\]

By eliminating \( p \) from the well-known expressions

\[
\rho_1 = \frac{p}{1 + e \cos \theta_1} \tag{B32}
\]

\[
\rho_2 = \frac{p}{1 + e \cos \theta_2}
\]

the unknown eccentricity of the transfer orbit is found from

\[
e = \frac{\rho_2 - \rho_1}{\rho_1 \cos \theta_1 - \rho_2 \cos \theta_2} \tag{B33}
\]

and the semilatus rectum

\[
p = \rho_1(1 + e \cos \theta_1) \tag{B34}
\]

The semi-major axis is computed as

\[
a = \frac{p}{1 - e^2} \tag{B35}
\]

All the orbital parameters may be fixed by choice of the periapse angle \( \theta_1 \).

A distinctive value of \( \theta_1 \) is the periapse angle \( \theta_p \) for parabolic transfer from \( \rho_1 \) to \( \rho_2 \) and the associated parabolic transfer time \( \Delta T_p = T_2 - T_1 \). This value of \( \theta_1 \) is
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easily found since \( e = 1.0 \) for parabolic orbits. Letting \( \theta_1 = \theta_p \) and using equation (B32) yields

\[
\rho_1(1 + \cos \theta_p) = \rho_2(1 + \cos \theta_2) = \rho_2 \left[ 1 + \cos(\Delta \theta + \theta_p) \right]
\]

or

\[
(\rho_1 - \rho_2 \cos \Delta \theta) \cos \theta_p + (\rho_2 \sin \Delta \theta) \sin \theta_p = \rho_2 - \rho_1
\]

Then \( \theta_p \) is found by trigonometric identity

\[
\sin(\theta_p + \psi) = \frac{\rho_2 - \rho_1}{c} > 0 \quad (B37a)
\]

where the angle \( \psi \) is defined by

\[
\sin \psi = \frac{\rho_1 - \rho_2 \cos \Delta \theta}{c}
\]

\[
\cos \psi = \frac{\rho_2 \sin \Delta \theta}{c}
\]

\[
\psi = \tan^{-1}\left(\frac{\sin \psi}{\cos \psi}\right) \quad \left( -\frac{\pi}{2} \leq \psi \leq \frac{3\pi}{2} \right)
\]

Let

\[
\bar{\alpha} = \sin^{-1}\left(\frac{\rho_2 - \rho_1}{c}\right) \quad \left(0 \leq \bar{\alpha} \leq \frac{\pi}{2} \right)
\]

and then note both values of \( \theta_p \) in an interval of \( 2\pi \)

\[
\theta_p = \theta_p|_{\text{minimum}} = \bar{\alpha} - \psi \quad (B37b)
\]

\[
\theta_p = \theta_p|_{\text{maximum}} = \pi - (\bar{\alpha} + \psi) \quad (B37c)
\]

A geometrical description of \( \psi \) is shown in the following sketch:
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Sketch 5.- Geometric interpretation of $\psi$.

The semi-latus rectum corresponding to $\theta_p$ is

$$p_p = \rho_1 (1 + \cos \theta_p)$$

The parabolic transfer time is then found as follows: The time to travel from periapse to $\rho_1$ is found in reference 10 and is

$$T_{p1} = \frac{1}{2\pi} \sqrt{p_p^3 \left( \frac{1}{6} \tan^3 \frac{\theta_p}{2} + \frac{1}{2} \tan \frac{\theta_p}{2} \right)}$$

and from periapse to $\rho_2$ is

$$T_{p2} = \frac{1}{2\pi} \sqrt{p_p^3 \left( \frac{1}{6} \tan^3 \left( \frac{\theta_p + \Delta \theta}{2} \right) + \frac{1}{2} \tan \left( \frac{\theta_p + \Delta \theta}{2} \right) \right)}$$

so the parabolic transfer time is

$$\Delta T_p = T_{p2} - T_{p1}$$

(B38)

This value may be used to compare the desired time $\Delta t$ of transfer and to determine whether the orbit is elliptic ($\Delta t > \Delta T_p$) or hyperbolic ($\Delta t < \Delta T_p$).

There are certain definite regions in which $\theta_1$ must remain for a solution to exist. These regions are found by use of equation (B33) and the positive sign of $e$ and $\rho_2 - \rho_1$. These equations imply

$$\rho_1 \cos \theta_1 > \rho_2 \cos \theta_2$$

which leads to $\sin(\theta_1 + \psi) > 0$ or

$$\pi - \psi > -\psi$$
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where \( \psi \) is defined in equation (B37). It is seen that the lower limit for \( \theta_1 \) is \(-\psi\) for all cases. The angle \( \theta_p \) is greater than \(-\psi\) as it is given by equation (B37). For an arbitrary \( e \), \( \theta_p \) is replaced by \( \theta_1 \) and equations (B37a) are written

\[
\theta_1 = \sin^{-1}\left(\frac{\rho_2 - \rho_1}{ec}\right)\left|_{\text{First quadrant}} \right. - \psi
\]

From equations (B39) and (B37), the following inequalities are obtained:

\[
\begin{align*}
\theta_1 &> \theta_p \bigg|_{\text{minimum}} & (e < 1) \\
\theta_1 &< \theta_p \bigg|_{\text{minimum}} & (e > 1)
\end{align*}
\]

It may also be shown that the true maximum value for \( \theta_1 \) is less than \( \pi - \psi \) and is the second parabolic solution

\[
\theta_1 \bigg|_{\text{maximum}} = \theta_p \bigg|_{\text{maximum}}
\]

Hence, the following regions for choice of the iterative periapse angles \( \theta_1 \) are determined from these relations:

1. If \( \Delta t < \Delta T_p \), then \( e > 1 \) (hyperbolic transfer) and \(-\psi < \theta_1 < \theta_p \)
2. If \( \Delta t > \Delta T_p \), then \( e < 1 \) (elliptic transfer) and \( \theta_p < \theta_1 < \pi - 2(\psi + \theta_p) \)
3. If \( \Delta t = \Delta T_p \), then \( e = 1 \) and the solution is \( \theta_1 = \theta_p \)

A geometric interpretation may be given to these regions and an example is shown in the following sketch:

![Sketch of regions for periapse \( \theta_1 \).](image)

Sketch 6.- Regions of possible choice for the periapse \( \theta_1 \).
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Elliptic transfer orbits.- In the case that $\Delta t > \Delta T_p$, the transfer orbit for rendezvous must be elliptic and hence $\theta_p < \theta_1 < \pi - 2\psi - \theta_p$. The iteration scheme used here is a straightforward computation of the values $T_2 - T_1$ for values of $\theta_1$ starting with $\theta_p$ and continuing until $T_2 - T_1$ becomes equal to $\Delta t$. That is, the interval of $\theta_1$ is divided into $h$ smaller intervals $\delta \theta$ where

$$\delta \theta = \frac{\pi - 2(\psi + \theta_p)}{h} = \frac{\theta_p \max - \theta_p \min}{h}$$

The first estimation of $\theta_1$ is $\theta_p + \delta \theta$ and with this value compute

$$\theta_2 = \theta_1 + \Delta \theta$$

$$e = \frac{\rho_2 - \rho_1}{\rho_1 \cos \theta_1 - \rho_2 \cos \theta_2}$$

$$p = \rho_1 (1 + e \cos \theta_1)$$

$$a = \frac{p}{1 - e^2}$$

and the eccentric anomalies $E_1$ and $E_2$ from the $F(\alpha,\beta)$ function defined previously to give:

$$E_1 = F(e, \theta_1)$$

$$E_2 = F(e, \theta_2)$$

and the elliptic time

$$T_2 - T_1 = \frac{1}{2\pi} a^3 \left[ E_2 - E_1 - e(\sin E_2 - \sin E_1) \right]$$

This value of $T_2 - T_1$ is compared with $\Delta t$ and if still too small, another increment $\delta \theta$ is added to $\theta_1$ and the process repeated. When $T_2 - T_1$ is larger than $\Delta t$, then the interval $\delta \theta$ is halved and subtracted from the last value of $\theta_1$. This method of iteration is free from the singularities involved in methods using derivatives and is not significantly longer. The value of $h$ used affects the time of iteration; however, an arbitrary value of $h = 3$ seems to produce sufficiently fast convergence.
Hyperbolic transfer orbits. - A value of $\Delta t < \Delta T_p$ gives hyperbolic transfer orbits and hence $-\psi < \theta_1 < \theta_p$. The method of iteration is identical to that for the elliptic case although the equations differ slightly. These equations appear as

$$
\delta \theta = \frac{\psi + \theta_p}{h} = \frac{\theta - (\psi)}{h} \\
\theta_1 = \theta_p - \delta \theta \\
\theta_2 = \theta_1 + \Delta \theta
$$

The iteration begins at $\theta_p$ as in the elliptic case but now proceeds into the region of smaller $\theta_1$

$$
e = \frac{\rho_2 - \rho_1}{\rho_1 \cos \theta_1 - \rho_2 \cos \theta_2} \\
p = \rho_1(1 + e \cos \theta_1) \\
a = \frac{p}{1 - e^2}
$$

The hyperbolic time functions must now be used and equation (B5) is applicable if $\gamma$ is set equal to -1.0; thus, $\sqrt{\frac{\alpha - 1}{1 + \alpha}}$ is obtained. Proceed with the usual hyperbolic "eccentric anomalies" which are the $F(\alpha, \beta)$ functions with the preceding replacements.

$$
F_1 = F'(e, \theta_1) \\
F_2 = F'(e, \theta_2)
$$

and the times from $\theta = 0$ to $\theta = \theta_1$ and $\theta = \theta_2$ as

$$
T_1 = \frac{1}{2\pi} \sqrt{-a^3} \left\{ e \tan F_1 - \ln \left[ \tan \left( \frac{F_1}{2} + \frac{\pi}{4} \right) \right] \right\}
$$

$$
T_2 = \frac{1}{2\pi} \sqrt{-a^3} \left\{ e \tan F_2 - \ln \left[ \tan \left( \frac{F_2}{2} + \frac{\pi}{4} \right) \right] \right\}
$$
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Compare $T_2 - T_1$ with $\Delta t$. If it is still larger, compute again with a new $\theta_1$ decreased from the last by $\delta \theta$. If $T_2 - T_1$ is greater, increase $\theta_1$ by $\frac{\delta \theta}{2}$ and proceed in this manner until the desired agreement is obtained.

Special cases. In the case that $\rho_1 = \rho_2$ in either the elliptic or hyperbolic case, special computations are necessary. For instance, the iteration can no longer be accomplished by incrementing $\theta_1$ as $\theta_1$ becomes a fixed value $-\psi$. The problem is solved by iterating the eccentricity in a straightforward manner and is easily followed in the flow chart in appendix D.

In the case that the time $\Delta T_p$ happens to be equal to $\Delta t$, then there is no further iteration as the solution is the parabolic case.

Computation of the properties. After obtaining $\theta_1$, information on the rates of change is obtained. Also the quantities $\rho_1$ and $\rho_2$ must be reassigned with $r_f$ and $\xi_i$. The properties of the transfer orbit occurring at $t = t_i$, $t = t_f$ are defined as $\rho_1$, $\rho_f$, $\theta_i$, and $\theta_f$ where

\[
\begin{align*}
\rho_1 &= \rho_1 & (\xi_i < r_f) \\
\rho_1 &= \rho_2 & (\xi_i > r_f) \\
\rho_f &= \rho_2 & (r_f > \xi_i) \\
\rho_f &= \rho_1 & (r_f < \xi_i) \\
\theta_i &= \theta_1 & (\xi_i < r_f) \\
\theta_i &= -\theta_2 & (\xi_i > r_f) \\
\theta_f &= \theta_2 & (r_f > \xi_i) \\
\theta_f &= -\theta_1 & (r_f < \xi_i)
\end{align*}
\]

These relations follow directly from the geometry of the transfer orbit and the definition that $\rho_1$ is the minimum of $r_f$ and $\xi_i$ and $\rho_2$ is the maximum of $r_f$ and $\xi_i$. The rates of change at $t = t_i$, $t = t_f$ in the plane of the transfer orbit may now be computed. Immediately after the impulse at $t = t_i$,

\[
\dot{\theta}_1 = \sqrt{p} \frac{1}{\rho_1^2}
\]

\[
\dot{\rho}_1 = \sqrt{\frac{1}{p}} e \sin \theta_1
\]

and immediately before the impulse at $t = t_f$,
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\[ \dot{\theta}_f = \sqrt{\mu} \frac{1}{\mu_f^2} \]

\[ \dot{\rho}_f = \sqrt{\mu} e \sin \theta_f \]

The transfer orbit plane is defined in exactly the same way as the initial interceptor orbit with the elements \( \Omega, \omega, i, \) and \( \theta_1. \) The anomaly \( \theta_1 \) and inclination \( i \) were found previously. Once \( \Omega \) and \( \omega \) have been determined, the transformation matrix \([b_{ij}]\) for transforming the position and velocity vectors in \( x', y', \) and \( z' \) (see fig. 3) to components in \( X,Y,Z \) is needed. This transformation matrix is identical with the \([a_{ij}]\) transformation matrix where \( \omega_1, \Omega_1, \) and \( i_1 \) are replaced by \( \omega, \Omega, \) and \( i. \)

The velocity vector at \( t = t_i \) in the \( x' \) direction is \( \dot{\rho}_1(x'), \) \( y' \) is \( \dot{\rho}_1(y'), \) \( z' \) is \( \dot{\rho}_1(z') \) where

\[
\begin{align*}
\dot{\rho}_1(x') &= \dot{\rho}_1 \cos \theta_1 - \rho_1 \dot{\theta}_1 \sin \theta_1 \\
\dot{\rho}_1(y') &= \dot{\rho}_1 \sin \theta_1 + \rho_1 \dot{\theta}_1 \cos \theta_1 \\
\dot{\rho}_1(z') &= 0
\end{align*}
\]

(B42)

The velocity components at \( t = t_f \) in the \( x', y', z' \) coordinate directions are

\[
\begin{align*}
\dot{\rho}_f(x') &= \dot{\rho}_f \cos \theta_f - \rho_f \dot{\theta}_f \sin \theta_f \\
\dot{\rho}_f(y') &= \dot{\rho}_f \sin \theta_f + \rho_f \dot{\theta}_f \cos \theta_f \\
\dot{\rho}_f(z') &= 0
\end{align*}
\]

(B43)

These velocity components may be transformed to components in the \( X,Y,Z \) coordinate system by use of the \([b_{ij}]\) matrix above.

From these results the velocities may be determined at \( t = t_i \) in the initial interceptor orbit and \( t = t_f \) in the target plane. Then at \( t = t_i \) in the \( x,y,z \) coordinate system:

\[
\begin{align*}
\dot{\xi}_1(x) &= \dot{\xi}_1 \cos \nu_1 - \xi_1 \dot{\nu}_1 \sin \nu_1 \\
\dot{\xi}_1(y) &= \dot{\xi}_1 \sin \nu_1 + \xi_1 \dot{\nu}_1 \cos \nu_1 \\
\dot{\xi}_1(z) &= 0
\end{align*}
\]

(B44)

which may be transformed to \( X,Y,Z \) coordinates by the matrix \([a_{ij}]\).
Finally, obtain the velocity after the final impulse directly from the components $X,Y,Z$ without transformation

$$\begin{align*}
\dot{r}_f(X) &= \dot{r}_f \cos \phi_f - r_f \dot{\phi}_f \sin \phi_f \\
\dot{r}_f(Y) &= \dot{r}_f \sin \phi_f + r_f \dot{\phi}_f \cos \phi_f \\
\dot{r}_f(Z) &= 0
\end{align*}$$

(B45)

The velocities in $X$, $Y$, and $Z$

The matrix operations on these vectors give the desired velocity components in the Newtonian frame $X,Y,Z$. These components are denoted by $V_k(j)$ where the subscript $k$ refers to the time (where $k$ equals 1 and 2 just before and after the initial impulse and $k$ equals 3 and 4 just before and after the final impulse) and $j$ to the component $X,Y,Z$. These velocity components, in terms of the components shown in equations (B42), (B43), (B44), (B45), and the elements of the $[a_{ij}]$ and $[b_{ij}]$ matrices, are given in the flow diagram of appendix D.

The velocity increments required to perform the rendezvous maneuver are easily found by subtracting these components. For the velocity increment at $t = t_1$, it is necessary to subtract the corresponding components of state (1) from state (2) as

$$\begin{align*}
\Delta V_1(X) &= V_2(X) - V_1(X) \\
\Delta V_1(Y) &= V_2(Y) - V_1(Y) \\
\Delta V_1(Z) &= V_2(Z) - V_1(Z)
\end{align*}$$

and for the terminal maneuver subtract state (3) from state (4) to give the desired increments

$$\begin{align*}
\Delta V_1(X) &= V_4(X) - V_3(X) \\
\Delta V_1(Y) &= V_4(Y) - V_3(Y) \\
\Delta V_1(Z) &= V_4(Z) - V_3(Z)
\end{align*}$$

The total velocity increment for the maneuver is the sum of the maneuvers at $t = t_1(\Delta V_1)$ and $t = t_f(\Delta V_2)$

$$\Delta V_1 = \sqrt{\Delta V_1(X)^2 + \Delta V_1(Y)^2 + \Delta V_1(Z)^2}$$
APPENDIX B

\[ \Delta V_f = \sqrt{\Delta V_f(X)^2 + \Delta V_f(Y)^2 + \Delta V_f(Z)^2} \]

and

\[ \Delta V = \Delta V_i + \Delta V_f \]
APPENDIX C

DESCRIPTION OF PROGRAM

This program was written in the FORTRAN IV (Ibsys version 9) language for the IBM 7094 computer at the Langley Research Center. Throughout the program there is an emphasis on simplicity, but capability has been provided for several uses and for a freedom of choice on input and output.

The method of solving the problem can readily be obtained by following the flow diagram (appendix D) and the description given in appendix B. The description in this appendix gives additional information about the options available, types of input and output, criteria for testing variables for transfer in the program, criteria for testing the convergence in the iterative processes, and criteria for incrementing the times. The methods used in the iterative processes are described fully in appendix B and can be followed on the flow diagram (appendix D). The flow diagram also shows the methods of incrementing $\Delta t_w$, $\Delta t$, and $\nu_0$.

A complete listing of the FORTRAN IV program is given as appendix E.

Subprograms

Six small subprograms are used in addition to the main program. They are called: AAA, VELOC, SIX, ANOM, SPACE, and CONVRT. An explanation of the computations, provided by AAA and ANOM, are included in appendix B. Equations (B5) are contained in AAA and equations (B9) are contained in ANOM. The other subprograms are self-explanatory; however, a brief description of the uses of all six subprograms is given here.

AAA is used to compute the eccentric anomaly if the true anomaly is known or vice versa.

VELOC is used to compute the velocity components.

SIX is used to compute the elements of the $[a_{ij}]$ and $[b_{ij}]$ matrices.

ANOM is used to compute the eccentric anomaly if the mean anomaly is known.

SPACE is used to test the line count and to skip pages and print column headings when necessary.

CONVRT is used to convert the times from dimensionless quantities to units corresponding to input for printing purposes.
APPENDIX C

Options Available

The program offers several options for choosing the variable which will be incremented initially and for choosing the type of input. This choice is made by reading in three control factors: GUIDE, OPTION, and DATAS.

GUIDE determines whether $\Delta t_w$ or $\Delta t$ varies initially in the program and gives the appropriate output format regarding the choice. The program provides that the time $\Delta t_w$ or $\Delta t$ not incremented initially will be incremented secondly, after which $\nu_0$ will be incremented. This procedure works for any number of $\Delta t$, $\Delta t_w$, and $\nu_0$ values. The quantity incremented initially will appear as the abscissa for ease in plotting or analyzing the data. Hence,

- if GUIDE = 1, $\Delta t_w$ varies initially;
- if GUIDE = 2, $\Delta t$ varies initially.

OPTION provides a choice of three types of input for times:

- OPTION = 1, $\Delta t_w$, $\Delta t$ input dimensionless;
- OPTION = 2, $\Delta t_w$, $\Delta t$ input in minutes; and
- OPTION = 3, $\Delta t_w$, $\Delta t$ input in days.

Restrictions are placed on maximums for $\Delta t_w$ and $\Delta t$ because of the six spaces allowed by the output format for printout of the quantity varying initially. The restrictions are due only to the output format, and may easily be changed.

(a) If either time exceeds 9999.0 minutes, they must be read in either dimensionless or in days.

(b) If either exceeds 9999.0 days, they must be read in dimensionless.

(c) If either exceeds 9.9999 in dimensionless time and is restricted by (a) or (b), the leftmost characters of the value will not be printed on output.

DATAS provide a choice of input:

- if DATAS = 1, use Keplerian input;
- if DATAS = 2, use relative orbital input.

Input

Information on input may be found in the "Computer Input and Output" section of the paper and in the immediately preceding paragraphs. Input is to be made in units according to the following criteria:
APPENDIX C

(1) The quantities $\mu$ and $a_T$ are dimensioned, unless time is dimensionless in which case they may be in any units desired. However, in order to get values for $P_T$ and $V_{CT}$, $\mu$ and $a_T$ should be dimensioned.

(2) All angles are in degrees.

(3) Times are in units described under "Options Available" in this appendix.

(4) All other quantities are dimensionless.

Input cards in the order to be read in and the proper FORTRAN formats for each are listed in the following table:

<table>
<thead>
<tr>
<th>Order of cards</th>
<th>Variable names</th>
<th>FORTRAN format</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>GUIDE, OPTIØN, DATAS</td>
<td>4E18.8</td>
</tr>
<tr>
<td>2</td>
<td>AMU, AT</td>
<td>4E18.8</td>
</tr>
<tr>
<td>3</td>
<td>AH, RMIN, ET, PHI0</td>
<td>4E18.8</td>
</tr>
<tr>
<td>4</td>
<td>DTWI, DTWF, DTTI, DTTF</td>
<td>4E18.8</td>
</tr>
<tr>
<td>5</td>
<td>DELTW, DELTT</td>
<td>4E18.8</td>
</tr>
<tr>
<td>6</td>
<td>Keplerian input:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>EI, AI, ØMEGAI, ØMI</td>
<td>4E18.8</td>
</tr>
<tr>
<td>7</td>
<td>Relative orbital input:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>XO, YO, ZO</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Keplerian input:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>AII, ANUO1, ANUOL, DELNUO</td>
<td>4E18.8</td>
</tr>
<tr>
<td>7</td>
<td>Relative orbital input:</td>
<td></td>
</tr>
<tr>
<td></td>
<td>XODT, YODT, ZODT</td>
<td></td>
</tr>
</tbody>
</table>

Output

All output will be in units corresponding to input. The output format will vary as $\Delta t$ and $\Delta t_w$ are varied initially. When a set of data is read in, those initial conditions are printed. As $v_o$ and $\Delta t$ or $\Delta t_w$ are incremented, they are printed. The computed values are then printed in columns. Special notation and messages which may be printed are

(1) If relative orbital input is used and $a_I$ is computed as less than zero, a message to this effect will be written and the program will transfer to the initial input section.

(2) If $i \geq 90^\circ - 0.1^\circ$, the message "$i = +/-90$ degrees not acceptable data" is written.

(3) If the orbit is parabolic, the value for $a$ will appear as 99.999 instead of $\infty$. 

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APPENDIX C

(4) If $\rho_{\text{min}} < r_{\text{min}}$, an asterisk is printed at the extreme right-hand end of the line of data.

Testing Criteria

There are several places in the program where tolerance factors or allowances for computational inaccuracies must be defined to insure proper flow through the program. An explanation of the values chosen in these tests follows:

<table>
<thead>
<tr>
<th>Values compared</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_03/H_0$ with 0.9999995</td>
<td>This test compares the value of the $\cos \theta$, $(H_03/H_0)$, to 1.0, and restricts the minimum value of a computed $\theta$ to be 0.0001 radian; otherwise, $\theta$ is set equal to 0.</td>
</tr>
<tr>
<td>$i$ with $90^\circ - 0.1^\circ$</td>
<td>Data which result in an $i$ of $90^\circ$ are not acceptable in this program. The absolute value of $i$ is tested against $90^\circ$ with a margin of $0.1^\circ$.</td>
</tr>
<tr>
<td>$\Delta T_{p}$ with $\Delta t$</td>
<td>This test of the parabolic time against the transfer time to determine the type of orbit results in a parabolic orbit only if the values are equal. A tolerance of $0.5 \times 10^{-6}(\Delta t)$ defined as CRIT was allowed at the point of equivalence for computational error.</td>
</tr>
<tr>
<td>$i$ with $0.5 \times 10^{-7}$</td>
<td>For this test a margin of tolerance of $0.5 \times 10^{-7}$ was allowed for machine inaccuracy.</td>
</tr>
<tr>
<td>$\Delta T$ with $\Delta t$</td>
<td>This test is made to determine when the convergence is sufficient to leave the loop in the iteration process. A value CRIT = $0.5 \times 10^{-6}(\Delta t)$ was defined to give a tolerance margin based on the value of the transfer time and to cover any computational error.</td>
</tr>
<tr>
<td>$\delta \phi$ with $1.0 \times 10^{-7}$</td>
<td>The appropriate test is made in each iteration scheme to test for the effectiveness of the increment on the variable. If the increment is equal to or less than the value tested against, its effect on the variable is negligible and the iteration is ended. This procedure acts as a safety check for ending the iteration in the event that $\Delta T$ never gets within the prescribed range (CRIT) of $\Delta t$.</td>
</tr>
<tr>
<td>$\delta e$ with $1.0 \times 10^{-7}$</td>
<td></td>
</tr>
<tr>
<td>$\rho_1$ with $\rho_2$</td>
<td>A tolerance margin of $1.0 \times 10^{-6}(\rho_1)$, defined as CRIRH0, provides for transfer to the special iteration necessary when $\rho_1 = \rho_2$.</td>
</tr>
<tr>
<td>$\Delta \theta$ with $0.0001^\circ$</td>
<td>Restrictions are put on a margin of $0.0001^\circ$ around $\Delta \theta = 0^\circ(360^\circ)$ because of computational sensitivity. For these cases, transfer orbit properties are set equal to the initial target orbit properties and the iterations are omitted.</td>
</tr>
<tr>
<td>$\Delta \theta$ with $360^\circ - 0.0001^\circ$</td>
<td></td>
</tr>
</tbody>
</table>
APPENDIX C

The quantities used for testing in incrementing $\Delta t_w$, $\Delta t$, and $v_o$ are as follows:

$$TWFMDT = \Delta t_w(last) - 1.5(\delta t_w) - TESTTW$$

$$TTFMDT = \Delta t(last) - 1.5(\delta t) - TESTTT$$

$$ANLMDN = v_o(last) - 1.5(\delta v_o) - 1.0 \times 10^{-7}$$

$$DTWFT = \Delta t_w(last) - TESTTW$$

$$DTTFT = \Delta t(last) - TESTTT$$

$$ANUOLT = v_o(last) - 1.0 \times 10^{-7}$$

$$TESTTW = 0.5 \times 10^{-6}[\Delta t_w(last)], \text{ or } = 1.0 \times 10^{-7}$$

$$TESTTT = 0.5 \times 10^{-6}[\Delta t(last)], \text{ or } = 1.0 \times 10^{-7}$$

The TWFMDT is used to test against $\Delta t_w$ and allows it to be incremented by $\delta t_w$ until $\Delta t_w$ is greater than or equal to $\Delta t_w(last) - 1.5 \delta t_w$. A value TESTTW is arbitrarily chosen as $1.0 \times 10^{-7}$ or $0.5 \times 10^{-6}[\Delta t_w(last)]$ depending upon whether $\delta t_w$ is 0 or is greater than 0, respectively. The TESTTW value is included to provide a tolerance for computational error in the value of TWFMDT if the $\delta t_w$ is equal to 0 and to insure that control is transferred out of this loop after computation with $\Delta t_w(first)$. The DTWFT is used to test against $\Delta t_w$ and always provides the computation for $\Delta t_w(last)$, even if $\Delta t_w(last) - \Delta t_w(first)$ is not an integer. This separate control for the final point also insures ease in incorporating any additional desired statements or controls, such as those for plotting routines, in the program at the end of a series of incrementations. The TESTTW in the expression for DTWFT provides tolerance for machine error.

The same explanation applies for TTFMDT, DTTFT, and TESTTT, if these variable names replace TWFMDT, DTWFT, and TESTTW in the preceding paragraph. These variables are used for testing in the incrementing procedure for $\Delta t$.

A similar explanation for ANLMDN and ANUOLT, used for incrementing $v_o$, holds with the exception that the tolerance margin or computational error is chosen as $1.0 \times 10^{-7}$ for both values, instead of being a ratio as in TESTTW and TESTTT, because $v_o$ is in radians (in the form $X.XXXXXXXX$) and the $1.0 \times 10^{-7}$ will always be effective on the number of places used in the computation.
APPENDIX C

The mathematical symbol and its FORTRAN equivalent are given in the following table:

<table>
<thead>
<tr>
<th>Mathematical symbol</th>
<th>Fortran symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>AT, AL, A</td>
</tr>
<tr>
<td>a_{ij}</td>
<td>A11, A12, A21, A22, A31, A32</td>
</tr>
<tr>
<td>b_{ij}</td>
<td>B11, B12, B21, B22, B31, B32</td>
</tr>
<tr>
<td>c</td>
<td>C</td>
</tr>
<tr>
<td>e</td>
<td>E</td>
</tr>
<tr>
<td>E</td>
<td>E</td>
</tr>
<tr>
<td>F</td>
<td>F1, F2</td>
</tr>
<tr>
<td>h</td>
<td>AH</td>
</tr>
<tr>
<td>H</td>
<td>AHO1, AHO2, AHO3, AHO</td>
</tr>
<tr>
<td>i</td>
<td>BI, AIM1, II</td>
</tr>
<tr>
<td>M</td>
<td>AMTO, AMTF, AMIO, AMII, AM1</td>
</tr>
<tr>
<td>p</td>
<td>PI, PT, P, PP</td>
</tr>
<tr>
<td>P_T</td>
<td>TPER</td>
</tr>
<tr>
<td>r</td>
<td>R0, RODT, RF, RDTF, RDTF1, RDTF2, RMIN</td>
</tr>
<tr>
<td>\Delta t_w</td>
<td>DTWI, DTFW, DELTTW</td>
</tr>
<tr>
<td>\Delta t</td>
<td>DTII, DTF, DELTTC</td>
</tr>
<tr>
<td>T_1 , T_2</td>
<td>T1, T2, TP1, TP2</td>
</tr>
<tr>
<td>\Delta T</td>
<td>DT, DELTTP, DELTTTC</td>
</tr>
<tr>
<td>V</td>
<td>VO1, VO2, VO3, VO</td>
</tr>
<tr>
<td>V_C T</td>
<td>VCT</td>
</tr>
<tr>
<td>V_k (i)</td>
<td>V11, V12, V13, V21, V22, V23, V31, V32, V33, V41, V42, V43</td>
</tr>
<tr>
<td>\Delta V_v (i), \Delta V_f (i)</td>
<td>DV11, DV12, DV13, DV21, DV22, DV23</td>
</tr>
<tr>
<td>\Delta V_v , \Delta V_f</td>
<td>DELV1, DELV2</td>
</tr>
<tr>
<td>\Delta \gamma</td>
<td>DELV</td>
</tr>
<tr>
<td>x', y'', z'' , x''', y''' , z'''</td>
<td>XO, YO, ZO, YODT, ZODT</td>
</tr>
<tr>
<td>\beta</td>
<td>ALPHA</td>
</tr>
<tr>
<td>\gamma</td>
<td>GAMMA</td>
</tr>
<tr>
<td>\tilde{\gamma}</td>
<td>GAMBAR</td>
</tr>
<tr>
<td>\Delta \theta</td>
<td>DELTTH</td>
</tr>
<tr>
<td>\theta</td>
<td>TH1, TH2, THP, THI, THF, THDTI, THDTF, THPMAX</td>
</tr>
<tr>
<td>\delta \theta</td>
<td>DELTH</td>
</tr>
<tr>
<td>\mu</td>
<td>AMU, MU</td>
</tr>
<tr>
<td>\nu</td>
<td>ANUO1, ANUOL, ANUO, ANUI, ANUDTI, NUO1, NUOL, NUO, NUI</td>
</tr>
<tr>
<td>\xi</td>
<td>XIO1, XIO2, XIO3, XIO, XIODT, XI, XIDTI, XI1, XI2, XI3, XIIDT1, XIIDT2</td>
</tr>
<tr>
<td>\rho</td>
<td>RH\Omega1, RH\Omega2, RH\Omega, RH\OmegaF, RH\OmegaDTI, RH\OmegaDTF, R\OmegaDTI1, R\OmegaDTI2, R\OmegaDF1, R\OmegaDF2, RH\OmegaM</td>
</tr>
<tr>
<td>\phi</td>
<td>PHI0, PHIODT, PHIF, PHIDTF</td>
</tr>
<tr>
<td>\psi</td>
<td>PSI</td>
</tr>
<tr>
<td>\omega</td>
<td>\OmegaM, \Omega</td>
</tr>
<tr>
<td>\Omega</td>
<td>\OmegaEGA, \OmegaEGA</td>
</tr>
</tbody>
</table>

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APPENDIX D

COMPUTATIONAL FLOW DIAGRAM

The computational flow diagram is given in this appendix. All quantities are dimensionless unless otherwise specified. Provisions for control of spacing on output are not included on this diagram.
\[ P_T = 2 \left( \frac{3}{4} \right) \frac{G_{33}}{\mu} (\text{sec}) \]
\[ V_{CT} = \frac{E}{\pi} (\text{sec}) \]

Write:
\[ \phi_{0}, \Omega \phi_{0} \Omega, \phi_{0} \Omega, \phi_{0} \Omega, \nu_{0} \Omega, \nu_{0} \Omega, \Delta t(\text{first}), \Delta t(\text{last}), \Delta \omega(\text{first}), \Delta \omega(\text{last}) \]
\[ r_{\text{min,h}} \]

DATAS: 1.5

Write:
\[ \nu_{0} \Omega(\text{first}) \]

\[ \Delta t(\text{first}) > 0 \]

GUIDE: 1.5

Write:
\[ \Delta t(\text{first}) \text{ choosing format depending upon the value of OPTION} \]

DATAS: 1.5

Change degrees to radians:
\[ \phi_{0}, \Omega \phi_{0} \Omega, \phi_{0} \Omega, \phi_{0} \Omega, \nu_{0} \Omega, \nu_{0} \Omega \]

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APPENDIX D

\[ I_1 = \arccos \frac{H_0}{H_0} \quad \text{for} \quad 0 < I_1 \leq \pi \]
\[ \cos \Omega_I = \frac{H_0 - H_0 \cos \phi_0 + H_0 \sin \phi_0}{H_0 \sin I_1} \]
\[ \sin \Omega_I = \frac{H_0 \cos \phi_0 - H_0 \sin \phi_0}{H_0 \sin I_1} \]
\[ \Omega_I = \arctan \left( \frac{\sin \Omega_I}{\cos \Omega_I} \right) \quad 0 \leq \Omega_I \leq 2\pi \]
\[ \cos(\omega_I + \nu_0) = \frac{\cos \Omega_I - \cos \nu_0}{\cos \Omega_I} \]
\[ \sin(\omega_I + \nu_0) = \frac{\sin \Omega_I}{\cos \Omega_I} \]

relationship to radians for printing \((\phi_0, \Omega_I, I_1, \nu_0, \nu_0, \text{first}, \nu_0, \text{last}, \nu_0)\)

\[ (\omega_I + \nu_0) = \arctan \left( \frac{\sin(\omega_I + \nu_0)}{\cos(\omega_I + \nu_0)} \right) \quad 0 < (\omega_I + \nu_0) < 2\pi \]
\[ \omega_I = (\omega_I + \nu_0) - \nu_0 \]

Compute quantities to use in testing for the completion of incrementing \(\Delta \nu, \Delta t, \) and \(\nu_0:\)

\(\text{TESTTT, TESTTT, TWFFMDT, TTFMDT, ANLMDN, DTWFT, DTYFT, ANULOIT}\)

\[
\text{GUIDE: 1}\]

\[ \nu_0 = \nu_0\text{(first)} \]

\[
\text{GUIDE: 2}\]

\[ \nu_0 = \nu_0\text{(first)} \]

\[
\text{2a}\]

\[ \Delta t = \Delta t(\text{first}) \]

\[
\text{2b}\]

\[ \Delta t_w = \Delta t_w(\text{first}) \]

\[ \Delta t = \Delta t(\text{first}) \]

\[ \Delta t_w = \Delta t_w(\text{first}) \]

\[ E_{10} = E_1(e_1, \nu_0, \gamma) \]
\[ M_0 = E_{10} - e_1 \sin E_{10} \]
\[ M_{11} = M_0 + \frac{2r(\Delta \Delta t)}{P_t} \]
\[ E_{11} = S_2(e_1, M_{11}) \]
\[ \nu_1 = \omega_1(-e_1, E_{11}, \gamma) \]
\[ \xi_1 = \frac{P_t}{1 + e_1 \cos \nu_1} \]
\[ \nu_1 = \frac{\xi_1}{P_t} \]
\[ \xi_1 = \frac{\nu_1 \sin \nu_1}{P_t} \]

\*\(S_1\) in this diagram refers to Function Subprogram AAA.
\*\(S_2\) in this diagram refers to Subroutine Subprogram ANOM.

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APPENDIX D

Determination of $\Delta \theta$, $i$ of transfer orbit

\[
\xi_{11} = \xi_{1} \left[ \cos \Omega_{1} \cos (v_{1} + w_{1}) - \cos i \sin \Omega_{1} \sin (v_{1} + w_{1}) \right]
\]

\[
\xi_{12} = \xi_{1} \left[ \sin \Omega_{1} \cos (v_{1} + w_{1}) + \cos i \cos \Omega_{1} \sin (v_{1} + w_{1}) \right]
\]

\[
\xi_{13} = \xi_{1} \left[ \sin i \sin (v_{1} + w_{1}) \right]
\]

\[
i = \arctan \frac{\xi_{13}}{\xi_{1} \sin \phi_{1} - \xi_{12} \cos \phi_{1}} \quad -\frac{\pi}{2} < i < \frac{\pi}{2}
\]

\[
\cos \Delta \theta = \frac{\xi_{11} \cos \phi_{1} + \xi_{12} \sin \phi_{1}}{\xi_{1} i}
\]

\[
\sin \Delta \theta = \frac{\xi_{11} \sin \phi_{1} - \xi_{12} \cos \phi_{1}}{\xi_{1} \cos i}
\]

\[
\Delta \theta = \arctan \frac{\sin \Delta \theta}{\cos \Delta \theta} \quad 0 \leq \theta \leq 2\pi
\]

Computation of parabolic time

\[\rho_{2} = \max (r_{T}, \xi_{1}) \quad \rho_{1} = \min (r_{T}, \xi_{1})\]

If $r_{T} = \xi_{1}$, then $\rho_{1} = \rho_{2}$

\[\Delta \theta = 0.001^\circ \]

\[\Delta \theta \leq 0.001^\circ\]

Write message

\[a = 1.0\]

\[p = p_{T}\]

\[\phi_{1} = \phi_{T} - \Delta \theta\]

\[\phi_{2} = \phi_{T}\]

\[e = e_{T}\]

\[\Delta \theta > 0.001^\circ\]

\[\Delta \theta \leq 360^\circ - 0.001^\circ\]

\[\Delta \theta \geq 360^\circ - 0.001^\circ\]

\[c = \sqrt{\rho_{1}^{2} + \rho_{2}^{2} - 2\rho_{1}\rho_{2} \cos \Delta \theta}\]

\[\sin \psi = \frac{\rho_{1} - \rho_{2} \cos \Delta \theta}{c}\]

\[\cos \psi = \frac{\rho_{2} \sin \Delta \theta}{c}\]

\[\psi = \arctan \frac{\sin \psi}{\cos \psi} \quad -\frac{\pi}{2} \leq \psi \leq \frac{3\pi}{2}\]

\[\tilde{\alpha} = \sin^{-1} \frac{\rho_{2} - \rho_{1}}{c} \quad 0 \leq \tilde{\alpha} \leq \frac{\pi}{2}\]

\[\phi_{p} = \tilde{\alpha} - \psi \quad \text{(Note: use of } \phi_{p} = (\phi_{p})_{\min}\text{)}\]

\[p_{p} = \rho_{1} (1 + \cos \phi_{p})\]

\[T_{p1} = \frac{1}{2\pi} \left[ a \left( \tan^{3} \left( \frac{\phi_{p}}{2} \right) + \frac{1}{2} \tan \left( \frac{\phi_{p}}{2} \right) \right) \right]
\]

\[T_{p2} = \frac{1}{2\pi} \left[ a \left( \tan^{3} \left( \frac{\phi_{p} + \Delta \theta}{2} \right) + \frac{1}{2} \tan \left( \frac{\phi_{p} + \Delta \theta}{2} \right) \right) \right]
\]

\[\Delta T_{p} = T_{p2} - T_{p1}\]

\[(\phi_{p})_{\max} = \pi - (\alpha + \psi)\]
APPENDIX D

\[ \Delta T_p : \Delta t \]

\[ \Delta t > \Delta T_p \]

\[ \Delta t = \Delta T_p \]

\[ \Delta t < \Delta T_p \]

hyperbolic

parabolic

elliptic

\[ \rho_1 : \rho_2 \]

\[ \rho_1 = \rho_2 \]

\[ \rho_1 \neq \rho_2 \]

\[ \delta \theta = \frac{\pi - 2(\theta_p + \psi)}{h} \]

\[ \theta_1 = \theta_0 + \delta \theta \]

\[ \theta_2 = \theta_1 + \Delta \theta \]

\[ \frac{p_2 - p_1}{\rho_1 \cos \theta_1 - \rho_2 \cos \theta_2} \]

\[ e = 1.0 \]

\[ p = P \]

\[ \delta_1 = \delta_0 \]

\[ \theta_2 = \theta_1 + \Delta \theta \]

\[ a = \infty (99.999 for printing purposes) \]

\[ \Delta T = \Delta t \]

\[ \Delta T < \Delta t \]

\[ \Delta T > \Delta t \]

\[ \delta \theta = \frac{\delta \theta}{2} \]

\[ \delta \theta = 1.0 \times 10^{-7} \]

\[ \delta \theta > 1.0 \times 10^{-7} \]

\[ \delta \theta > 1.0 \times 10^{-7} \]

\[ \delta \theta > 1.0 \times 10^{-7} \]

\[ \theta_1 = \theta_1 - \delta \theta \]

\[ \theta_1 = \theta_1 + \delta \theta \]

\[ \theta_1 = \theta_1 + \delta \theta \]

This precaution is necessary in cases where \( \theta_1 = \theta_{\text{pmax}} \) and machine computes \( e \) to be (false) greater than 1.
elliptic ($P_1 = P_2$)

\[ \Delta T_c = \frac{\Delta \theta}{2\pi} P_1^3 \]
\[ \delta e = \frac{1}{2} \]
\[ e = \delta e \]

\[ \Delta T_c > \Delta t \]
\[ \Delta T_c < \Delta t \]

\[ p = \rho_1(1 + e \cos \psi) \]
\[ a = \frac{p}{1 - e^2} \]
\[ \theta_1 = \pi - \psi \]
\[ \theta_2 = \theta_1 + \Delta \theta \]
\[ E_1 = S_1(e, \theta_1, \gamma) \]
\[ \Delta T = -\frac{1}{2} \sqrt{3} (E_1 - e \sin E_1) \]

\[ \Delta T > \Delta t \]
\[ \Delta T = \Delta t \]
\[ \Delta T < \Delta t \]

\[ \delta \epsilon = \frac{\delta e}{2} \]
\[ \delta e : 1.0 \times 10^{-7} \]
\[ e = e - \delta e \]

\[ \delta \epsilon \leq 1.0 \times 10^{-7} \]

\[ \delta e : 1.0 \times 10^{-7} \]

\[ e = e + \delta e \]

\[ \delta \epsilon \leq 1.0 \times 10^{-7} \]
hyperbolic loop \( p_1 \neq p_2 \)

\[ \delta \theta = \frac{\theta_0 \cdot \psi}{h} \]

\[ \theta_1 = \theta_0 - \delta \theta \]

\[ \theta_1 = \theta_1 + \Delta \theta \]

\[ e = \frac{p_2 - p_1}{p_1 \cos \theta_1 - p_2 \cos \theta_2} \]

\[ \delta \theta > 0.1 \times 10^{-8} \]

\[ \delta \theta : 0.1 \times 10^{-8} \]

\[ \delta \theta \leq 0.1 \times 10^{-8} \]

Write message

\[ p = p_1(1 + e \cos \theta_1) \]

\[ a = \frac{p}{1 - e^2} \]

\[ F_1 = S_1(\theta; \theta_1; \gamma; \gamma = -1) \]

\[ F_2 = S_1(\theta; \theta_2; \gamma; \gamma = -1) \]

\[ T_1 = \frac{1}{2r} \left\{ a^2 \tan F_1 - \tan \left[ \tan \left( \frac{F_1 + \gamma}{2} \right) \right] \right\} \]

\[ T_2 = \frac{1}{2r} \left\{ a^2 \tan F_2 - \tan \left[ \tan \left( \frac{F_2 + \gamma}{2} \right) \right] \right\} \]

\[ \Delta T = T_2 - T_1 \]

\[ \Delta T : \Delta t \]

\[ \Delta T \geq \Delta t \]

\[ \theta_1 = \theta_1 - \delta \theta \]

\[ \theta_1 > 1.0 \times 10^{-7} \]

\[ \theta_1 = \theta_1 + \delta \theta \]

\[ \delta \theta > 1.0 \times 10^{-7} \]

\[ \delta \theta : 1.0 \times 10^{-7} \]

\[ \delta \theta \leq 1.0 \times 10^{-7} \]

\[ \Delta T = \Delta t \]
APPENDIX D

hyperbolic loop \( (\rho_1 = \rho_2) \)

\( \Delta T_p > \Delta t \)
\( \rho_1 = \rho_2 \)

\[
\delta e = \frac{1}{\hbar} ; \quad \theta_1 = -\psi
\]
\[
e = 1 + \delta e ; \quad \theta_2 = \theta_1 + \Delta \theta
\]

\[
p = \rho_1(1 + e \cos \theta_1)
\]

\[
a = \frac{-p}{1 - e^2}
\]

\[
F_1 = S_1(e, \theta_1, \gamma)(\gamma = -1)
\]

\[
\Delta T = -\frac{1}{\pi}\sqrt{-a^3}\left\{ e \tan F_1 - \ln\left[ \tan\left(\frac{F_1}{2} + \frac{\pi}{4}\right)\right] \right\}
\]

\[
\Delta T : \Delta t
\]
\[
\Delta T > \Delta t \quad e = e + \delta e
\]

\[
\Delta T < \Delta t \quad \delta e = \frac{\delta e}{2}
\]

\[
\delta e : 1.0 \times 10^{-7}
\]
\[
\delta e > 1.0 \times 10^{-7} \quad e = e - \delta e
\]

\[
\delta e \leq 1.0 \times 10^{-7}
\]
\[ \rho_{\text{min}} = \frac{p}{1 + e} \]

\[ \rho_{\text{min}} = \rho_1 \]

\[ \rho_{\text{min}} : r_{\text{min}} \]

Print out * in output

\[ r_f \geq \xi_i \]

\[ r_f < \xi_i \]

\[ r_f : \xi_i \]

\[ \rho_1 = \rho_2 \]

\[ \rho_f = \rho_1 \]

\[ \theta_1 = 2\pi - \theta_2 \]

\[ \theta_f = \theta_1 + \Delta \theta \]

\[ \dot{\theta}_f = \frac{\sqrt{p}}{\rho_1^2} \]

\[ \dot{\rho}_f = \frac{e \sin \theta_1}{\sqrt{p}} \]

\[ \dot{\theta}_i = \frac{\sqrt{p}}{\rho_1^2} \]

\[ \dot{\rho}_i = \frac{e \sin \theta_i}{\sqrt{p}} \]

\[ a_{11} = \cos \omega_I \cos \Omega_I - \cos \xi_I \sin \Omega_I \sin \omega_I \]

\[ a_{12} = -\sin \omega_I \cos \Omega_I - \cos \xi_I \sin \Omega_I \cos \omega_I \]

\[ a_{21} = \cos \omega_I \sin \Omega_I + \cos \xi_I \cos \Omega_I \sin \omega_I \]

\[ a_{22} = -\sin \omega_I \sin \Omega_I + \cos \xi_I \cos \Omega_I \cos \omega_I \]

\[ a_{31} = \sin \xi_I \sin \omega_I \]

\[ a_{32} = \sin \xi_I \cos \omega_I \]

\[ i = 0 \]

\[ i > 0 \]

\[ i < 0 \]

\[ i = 0 \]

\[ \Omega = \phi_I + \pi \]

\[ \omega = \pi - \theta_f \]

\[ \Omega = \phi_I \]

\[ \omega = 2\pi - \theta_f \]

\[ \Omega = 0 \]

\[ \omega = \phi_I - \theta_f \]
Matrix $[b_{ij}]$ same as $[a_{ij}]$

with $\Omega$ replacing $\Omega_I$

$\omega$ replacing $\omega_I$

$i$ replacing $i_I$

\[
\begin{align*}
\dot{x}_1(x) &= \dot{x}_1 \cos \nu_1 - \dot{x}_1 \dot{v}_1 \sin \nu_1 \\
\dot{x}_1(y) &= \dot{x}_1 \sin \nu_1 + \dot{x}_1 \dot{v}_1 \cos \nu_1 \\
\dot{\rho}_1(x') &= \dot{\rho}_1 \cos \theta_1 - \rho_1 \dot{\theta}_1 \sin \theta_1 \\
\dot{\rho}_1(y') &= \dot{\rho}_1 \sin \theta_1 + \rho_1 \dot{\theta}_1 \cos \theta_1 \\
\dot{\rho}_f(x') &= \dot{\rho}_f \cos \phi_f - \rho_f \dot{\phi}_f \sin \phi_f \\
\dot{\rho}_f(y') &= \dot{\rho}_f \sin \phi_f + \rho_f \dot{\phi}_f \cos \phi_f \\
V_1(X) &= a_{11} \dot{x}_1(x) + a_{12} \dot{x}_1(y) \\
V_1(Y) &= a_{21} \dot{x}_1(x) + a_{22} \dot{x}_1(y) \\
V_1(Z) &= a_{31} \dot{x}_1(x) + a_{32} \dot{x}_1(y) \\
V_2(X) &= b_{11} \dot{\rho}_1(x') + b_{12} \dot{\rho}_1(y') \\
V_2(Y) &= b_{21} \dot{\rho}_1(x') + b_{22} \dot{\rho}_1(y') \\
V_2(Z) &= b_{31} \dot{\rho}_1(x') + b_{32} \dot{\rho}_1(y') \\
V_3(X) &= b_{11} \dot{\rho}_f(x') + b_{12} \dot{\rho}_f(y') \\
V_3(Y) &= b_{21} \dot{\rho}_f(x') + b_{22} \dot{\rho}_f(y') \\
V_3(Z) &= b_{31} \dot{\rho}_f(x') + b_{32} \dot{\rho}_f(y') \\
V_4(X) &= \dot{r}_f(X) \\
V_4(Y) &= \dot{r}_f(Y) \\
V_4(Z) &= 0
\end{align*}
\]
\[
\Delta V_1(X) = V_2(X) - V_1(X) \\
\Delta V_1(Y) = V_2(Y) - V_1(Y) \\
\Delta V_1(Z) = V_2(Z) - V_1(Z) \\
\Delta V_f(X) = V_4(X) - V_3(X) \\
\Delta V_f(Y) = V_4(Y) - V_3(Y) \\
\Delta V_f(Z) = V_4(Z) - V_3(Z)
\]

\[
\Delta V_I = \sqrt{\Delta V_1(X)^2 + \Delta V_1(Y)^2 + \Delta V_1(Z)^2} \\
\Delta V_f = \sqrt{\Delta V_f(X)^2 + \Delta V_f(Y)^2 + \Delta V_f(Z)^2} \\
\Delta V = \Delta V_I + \Delta V_f
\]

Convert radians to degrees for printing: \(\phi_f, \nu_1, \theta_1, \Delta \theta, i\)

Convert times (\(\Delta t\) and \(\Delta t_w\)) into units corresponding to input

Write computed quantities according to appropriate format

Convert times (\(\Delta t\) and \(\Delta t_w\)) back to dimensionless times

GUIDE : 1.5

GUIDE = 2

\[11b\]

GUIDE = 1

\[11a\]
GUIDE = 1

\( \Delta t_w : \text{TWFMDT} \)

\( \Delta t_w \leq \text{TWFMDT} \)

\( \Delta t_w : \text{DTWFT} \)

\( \Delta t_w \leq \text{DTWFT} \)

\( \Delta t : \text{TTFMDT} \)

\( \Delta t \leq \text{TTFMDT} \)

\( \Delta t : \text{DTTFT} \)

\( \Delta t \leq \text{DTTFT} \)

\( \nu_0 : \text{ANLMDN} \)

\( \nu_0 < \text{ANLMDN} \)

\( \nu_0 \geq \text{ANLMDN} \)

\( \nu_0 : \text{ANUOLT} \)

\( \nu_0 < \text{ANUOLT} \)

\( \nu_0 \geq \text{ANUOLT} \)

Read in new input, beginning with input 1

\( \Delta t < \text{TTFMDT} \)

\( \Delta t = \Delta t + \delta t \)

Convert \( \Delta t \) into units corresponding to input and write

\( \Delta t = \Delta t \text{ (last)} \)

Convert \( \Delta t \) into units corresponding to input and write

\( \nu_0 = \nu_0 + \delta \nu_0 \)

Convert \( \Delta t \text{ (first)} \) into units corresponding to input; write \( \nu_0 \) and \( \Delta t \text{ (first)} \)

\( \nu_0 = \nu_0 \text{ (last)} \)

Convert \( \Delta t \text{ (first)} \) into units corresponding to input; write \( \nu_0 \) and \( \Delta t \text{ (first)} \)
APPENDIX D

GUIDE = 2

Δt : TTFMDT

Δt < TTFMDT

Δt = Δt + 0

Δt = Δt (last)

Δt ≥ TTFMDT

Δt : DTTFT

Δt < DTTFT

Δt = Δt (last)

Δt ≥ DTTFT

Δm : TWFMOT

Δm < TWFMOT

Δm = Δm + Δm

Convert Δm into units corresponding to input and write

Δm = Δm (last)

Convert Δm into units corresponding to input and write

Δm ≥ TWFMOT

Δm : DTWFT

Δm < DTWFT

Δm = Δm (last)

Convert Δm into units corresponding to input and write

Δm ≥ DTWFT

ν0 : ANLMOT

ν0 < ANLMOT

ν0 = ν0 + Δν0

Convert Δν0 into units corresponding to input; write ν0 and Δν0 (first)

ν0 = ν0 (last)

Convert Δν0 (first) into units corresponding to input; write ν0 and Δν0 (first)

ν0 ≥ ANLMOT

ν0 : ANUIOT

ν0 < ANUIOT

ν0 = ν0 (last)

Convert Δν0 into units corresponding to input; write ν0 and Δν0 (first)

ν0 ≥ ANUIOT

Read in new input, beginning with input 1
APPENDIX E

FORTRAN PROGRAM

The FORTRAN program to determine the velocity increment required for rendezvous between two arbitrary elliptic orbits is as follows:

```fortran
WRITE(6,104)
WRITE(6,102)
J=7
NJ=2
READ(5,100)G,.defineProperty,DTAS
READ(5,100)ANU,AT
READ(5,100)A1,PHI0
READ(5,100)TPer,DTTF,DTT1,DTTF
READ(5,100)DELTV,DELTT
IF(NJ.GT.1)GO TO 48
J=3
GO TO 49
48 J=J
49 NJ=0
IF(DTAS.GT.1.5)GO TO 11
READ(5,100)E1,A1,CHEGA1,OM1
READ(5,100)A11,AHU1,ANU01,DELNU0
24 TPer=0.2831853*SQRT((A1**2/ANU1)*N)
VCT=SQRT(ANU1/AT)
WRITE(6,168)
WRITE(6,106)ET,E1,A1,CHEGA1,OM1,A11
WRITE(6,110)
WRITE(6,106)PHI0,AHU01,ANU01,NU01,AT,TPer,VCT
WRITE(6,122)
WRITE(6,126)DTTI,DTTF,DTT1,DTTF,RIH,AH
J=J+5
IF(DTAS.GT.1.5)GO TO 25
WRITE(6,124)
J=J+2
55 WRITE(6,132)ANU01
C IF DTTI IS INPUT AS ZERO, IT IS REASSIGNED A VALUE EQUIVALENT TO
C THE INCREMENT SIZE.
IF(DTTI.GT.0.0)GO TO 54
ISG=49
CALL SPACE
J=J+2
DTTI=DELTV
WRITE(6,162)DTTI
54 CONTINUE
IF(G,DEFINE.GT.1.5)GO TO 30
IF(OPTION.LT.1.5)GO TO 50
WRITE(6,140)DTTI
GO TO 51
50 WRITE(6,130)DTTI
51 WRITE(6,116)
GO TO 51
30 CONTINUE
IF(OPTION.LT.1.5)GO TO 53
WRITE(6,133)DTTI
GO TO 52
53 WRITE(6,128)DTTI
52 WRITE(6,118)
51 J=J+4
IF(DTAS.GT.1.5)GO TO 26
```

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APPENDIX E

\[ P_I = AI \times (1.0 - EI^{**2}) \]
\[ PT = 1.0 - ET^{**2} \]

26 \[ \text{PHI0} = \text{PHI0} \times 1.7453293E-01 \]
\[ \text{OMEGA1} = \text{OMEGA1} \times 1.7453293E-01 \]
\[ \text{OM1} = \text{OM1} \times 1.7453293E-01 \]
\[ \text{AI} = \text{AI} \times 1.7453293E-01 \]
\[ \text{ANU01} = \text{ANU01} \times 1.7453293E-01 \]
\[ \text{ANU0L} = \text{ANU0L} \times 1.7453293E-01 \]
\[ \text{DELNU0} = \text{DELNU0} \times 1.7453293E-01 \]

GO TO 200

11 READ(5,100)X0,Y0,Z0
READ(5,100)X0DT,Y0DT,Z0DT
DELTU0=0.0
\[ \text{PHI0} = \text{PHI0} \times 1.7453293E-01 \]
\[ \text{PT} = 1.0 - ET^{**2} \]
\[ \text{CPHIO} = \cos(\text{PHI0}) \]
\[ \text{SPHIO} = \sin(\text{PHI0}) \]
\[ R0 = \text{PT}/(1.0 + ET \times \text{CPHIO}) \]
\[ \text{PHIO} = \text{PHIO} \times \sqrt{\text{PT}}/(R0^{**2}) \]
\[ \text{RODT} = (ET \times \text{SPHIO})/\sqrt{\text{PT}} \]
\[ \text{VO1} = \text{RODT} \times X0DT - \text{PHIO} \times Y0 \]
\[ \text{VO2} = \text{Y0DT} + \text{PHIO} \times Y0 \]
\[ \text{V03} = \text{Z0DT} \]
\[ X101 = R0 + X0 \]
\[ X102 = Y0 \]
\[ X103 = Z0 \]
\[ X10 = \sqrt{(X101**2 + X102**2 + X103**2)} \]
\[ \text{VO} = \sqrt{(\text{VO1}**2 + \text{VO2}**2 + \text{VO3}**2)} \]
\[ \text{AI} = \text{X10}/(2.0 - \text{X10} \times \text{VO}**2) \]

C IF AI IS LESS THAN OR EQUAL TO ZERO, A MESSAGE TO THIS EFFECT IS
C WRITTEN AND TRANSFER IS TO THE BEGINNING OF THE PROGRAM,
C WHERE A NEW SET OF DATA MAY BE READ IN.

17 \[ \text{TPER} = 6.2831853 \times \sqrt{((\text{AT}**2/\text{AHU}) \times \text{AT})} \]
\[ \text{VCT} = \sqrt{((\text{ANU} / \text{AT})} \]
WRITE(6,148)
WRITE(6,106)PHI0,ET,ANU,AT,TPER,VCT
WRITE(6,112)
WRITE(6,126)DTI,DTDF,DTU,DTUF,RHIN,AIH
WRITE(6,114)
WRITE(6,106)X0,Y0,Z0,X0DT,Y0DT,Z0DT
WRITE(6,124)
WRITE(6,144)
WRITE(6,104)
GO TO 10

18 \[ \text{AH01} = X102 \times \text{VO3} - X103 \times \text{VO2} \]
\[ \text{AH02} = X103 \times \text{VO1} - X101 \times \text{VO3} \]
\[ \text{AH03} = X101 \times \text{VO2} - X102 \times \text{VO1} \]
\[ \text{PI} = \text{AH01}**2 + \text{AH02}**2 + \text{AH03}**2 \]
\[ \text{AH0} = \sqrt{\text{PI}} \]
\[ \text{X10DT} = (\text{VO1} \times X101 + \text{VO2} \times X102 + \text{VO3} \times X103) / \text{X10} \]
\[ \text{PIDAI} = \text{PI} / \text{AI} \]
APPENDIX E

TEST TO INSURE THAT EI IS NOT EQUAL TO THE SQRT OF A NEG QUANTITY DUE TO SLIGHT COMPUTING INACCURACY ON PI/AI. ASSIGN VALUES FOR COSNU0) AND SINNU0) IF THE QUANTITY UNDER THE RADICAL IS NEGATIVE.

IF(PIDAI.LT.1.0)GO TO 19
EI=0.0
CUU0=1.0
SNU0=0.0
GO TO 28
19 CONTINUE
EI=SQR(1.0-(PI/AI))
CUO=((PI/X10)-1.0)/EI
SNU0=(SQR(PI))*X10DT/EI
28 CONTINUE
ANU0=ATAN2(SNU0,CNU0)
IF(SNU0.LT.0.0.AND.CNU0.GT.0.0)ANU0=6.2831853+ANU0
IF(SNU0.LT.0.0.AND.CNU0.LT.0.0)ANU0=6.2831853+ANU0
13 CONTINUE
ANU01=ANU0
ANU0L=ANU0
15 CII=AH03/Al
IF(CII.GE.0.99999995)GO TO 16
AII=ATAN(SQR(1.0-CII**2)/CII)
IF(CII.LT.0.0)AII=3.1415927+AII
SAII=SINC(AII)
C0MEGI=-(AH02*CHII0+AH01*SPHI0)/(AH0*SAII)
S0MEGI=(AH01*CPII0-AH02*SPHI0)/(AH0*SAII)
OMEGAI=ATAN2(SOMEGI,COMEII)
IF(SOMEGI.LT.0.0.AND.COMEII.GT.0.0)OMEGAI=6.2831853+OMEGAI
IF(SOMEGI.LT.0.0.AND.COMEII.LT.0.0)OMEGAI=6.2831853+OMEGAI
20 CONTINUE
COPN=(X102*AII01-X101*AH02)/(X10*AH0*SAII)
SOPN=X103/(X10*SAII)
GO TO 21
16 AII=0.0
OMEGAI=0.0
COPN=(X101*CPII0-X102*SPHI0)/X10
SOPN=(X101*PHI0+X102*PHI0)/X10
21 OPI=ATAN2(SOPN,COPN)
IF(SOPN.LT.0.0.AND.COPN.GT.0.0)OPN=6.2831853+OPN
IF(SOPN.LT.0.0.AND.COPN.LT.0.0)OPN=6.2831853+OPN
23 CONTINUE
OMI=OPN-ANU0
C
CONVERT ANGLES IN RADIANS TO DEGREES FOR PRINTING
PHI0=PHI0/.17453293E-01
OMEGAI=OMEGAI/.17453293E-01
OMI=OMI/.17453293E-01
AII=AII/.17453293E-01
ANU01=ANU01/.17453293E-01
ANU0L=ANU0L/.17453293E-01
DELUO=DELUO/.17453293E-01
GO TO 24
25 WRITE(6,114)
WRITE(6,106)X0,Y0,Z0,X0DT,Y0DT,Z0DT

65
APPENDIX E

WRITE(6,124)
J=J+4
GO TO 55

CHANGE TIMES TO DIMENSIONLESS QUANTITIES

C
IF(OPTION.LT.1.5)GO TO 201
IF(OPTION.LT.2.5)GO TO 202
GO TO 203

CMIN=60./TPER
DTWI=DTWI*CMIN
DTWF=DTWF*CMIN
DELTW=DELTW*CMIN
DTTI=DTTI*CMIN
DTTF=DTTF*CMIN
DELT=DELT*CMIN
GO TO 201

CDAYS=86400./TPER
DTWI=DTWI*CDAYS
DTWF=DTWF*CDAYS
DELTW=DELTW*CDAYS
DTTI=DTTI*CDAYS
DTTF=DTTF*CDAYS
DELT=DELT*CDAYS

C
CONTINUE
IF(DTWF.GT.0.0)GO TO 204
TESTTI=.0000001
GO TO 205

TESTTT=.5E-06*DTWF

IF(DTTF.GT.0.0)GO TO 206
TESTTT=.0000001
GO TO 207

TESTTT=.5E-06*DTTF

C
CONTINUE
TTF=DTF-1.5*DELTW-TESTTT
TTF=DTF-1.5*DELT-TESTTT
ANLHDN=ANUO-1.5*DELNUO-.0000001
DTWF=DTWF-TESTTT
DTTF=DTTF-TESTTT
ANUO=ANUO-.0000001
IF(GUIDE-1.5)39,43,43

ANUO=ANUO1

DELT=DELTI

GO TO 42

ANUO=ANUO1

DELT=DELTI

GO TO 45

C
CRIT IS A VALUE USED TO TEST THE ITERATION SCHEMES FOR CONVERGENCE

CRIT=.5E-06*DELT

CRJRH=1.0E-06*AI

COMMON J, GUIDE, SIG

ET0=A.AA(ET, PHI0, GAMMA)
AMTO=ET0-ET*(SIN(ET0))
AMTF=AMTO+6.2831853*(DELT+DELTW)
CALL ANON(ET, AMTF, ETF)

63 PHIF = AAA(-ET, ETF, GAMMA)
CPHIF = COS(PHIF)
SPHIF = SIN(PHIF)
RF = PT/(1.0 + ET * CPHIF)
PHIDTF = (SRT(PT))/(RF**2)
RDTF = ET * SPHIF / SQRT(PT)

65 E10 = AAA(E1, ANU0, GAMMA)
AM10 = E10 - E1 * (SIN(E10))
AM11 = AM10 + ((6.2831853*DELTTH)/(AI**1.5))
CALL ANON(E1, AM11, E11)

67 ANUI = AAA(-E1, E11, GAMMA)
XI1 = PI/(1.0 + E1 * COS(ANUI))
ANUD1 = SQRT(PI)/(XI1**2)
XID1 = E1 * SIN(ANUI)/SQRT(PI))

C DETERMINATION OF DELTTH AND 1 OF TRANSFER ORBIT

300 O1PN1 = ANUI + OM1
C01PN1 = COS(O1PN1)
S01PN1 = SIN(O1PN1)
C0MEG1 = COS(OMEGAI)
S0MEG1 = SIN(OMEGAI)
C11 = COS(A11)
S11 = SIN(A11)
XI1 = XI1 * (C0MEG1*C01PN1 - C11*S0MEG1*S01PN1)
X12 = XI1 * (S0MEG1*C01PN1 + C11*C0MEG1*S01PN1)
X13 = XI1 * (S11*S01PN1)
SUB1 = XI1/(XI1*SPHIF-X12*CPHIF)
BI = ATAN(SUB1)
ASOL1 = ABS(B1)
IF(ASOL1 - 1.5690510) 306, 305, 305

305 CONTINUE
ISIG = 48
CALL SPACE
WRITE(6, 150)
J = J + 2
GO TO 1028

306 CONTINUE
CDTH = (XI11*CPHIF + XI12*SPHIF)/X11
SDTH = (XI11*SPHIF - XI12*CPHIF)/(XI11*COS(B1))
DELTTH = ATAN2(SDTH, CDTH)
IF(SDTH, LT, 0.0, AND,CDDH, GT, 0.0)DELTTH = 6.2831853 + DELTTH
IF(SDTH, LT, 0.0, AND,CDTH, LT, 0.0)DELTTH = 6.2831853 + DELTTH

302 CONTINUE

C COMPUTATION OF PARABOLIC TIME
RH02 = AMAX1(RF, XI1)
RH01 = AMIN1(RF, XI1)
IF(DELTTH, GT, 0.1745E-04, AND, DELTTH, LT, 6.2851679)GO TO 308
DEGTH = DELTTH/1.7453293E-01
ISIG = 47
CALL SPACE
J = J + 1
WRITE(6, 160) DEGTH
A = 1.0
P = PT
TH1=PH1-F-DELTTH
TH2=PH1
E=ET
GO TO 800

308 CONTINUE
C=SQRT(RH01**2+RH02**2-2.0*RH01*RH02*CDTH)
SPS1=(RH01-RH02*CDTH)/C
CPS1=(RH02*SDTH)/C
PS1=ATAN2(SPS1,CPS1)
IF(SPS1.LT.0.0.AND.CPS1.LT.0.0)PS1=6.2831853+PS1

304 CONTINUE
SALP=(RH02-RH01)/C
IF(SALP.GE.1.0)GO TO 311
ALPHA=ATAN(SALP/SQRT(1.0-SALP**2))
GO TO 312

311 ALPHA=1.5707963

312 CONTINUE
THP=ALPHA-PS1
PP=RH01*(1.0+COS(THP))
THP=(SIN(THP/2.))/(COS(THP/2.))
THPDT=(SIN((THP+DELTTH)/2.))/(COS((THP+DELTTH)/2.))
TP1=((PP**1.5)*((THPDT**3)/6.0+THP/2.))/G.2831853
TP2=((PP**1.5)*((THPDT**3)/6.0+THPDT/2.))/G.2831853
DELTTP=TP2-TP1
THP:AX=3.1415927-(ALPHA+PS1)
SUB12=DELTTP-DELTTH
ASUB12=ABS(SUB12)
IF(ASUB12-GRT)37,37,38

C IF DELTTP EQUALS DE LTTH, THE ORBIT IS PARABOLIC
37 E=1.0
P=PP
TH1=THP
TH2=TH1+DELTTH
A IS INFINITE, IS SET EQUAL TO 99.999 FOR PRINTING PURPOSES
A=99.999
GO TO 800

C IF DELTTP IS GREATER THAN DE LTTH, THE ORBIT IS HYPERBOLIC
C IF DELTTP IS LESS THAN DE LTTH, THE ORBIT IS ELLIPTIC
38 IF(DELTTP-DELTTH)400,37,600

400 SUB13=RH01-RH02
ASUB13=ABS(SUB13)
IF(SUB13-GRT)RH00,500,401

C ELLIPTIC LOOP RHO2 NOT EQUAL TO RH01
401 DELTH=(3.1415927-2.0*(THP+PS1))/AH
TH1=THP+DELTTH
TH2=TH1+DELTTH
E=(RH02-RH01)/(RH01*COS(TH1)-RH02*COS(TH2))
DIFF=THP:AX-TH1
IF(DIFF.LE.0.0)GO TO 405
IF(E.GE.1.0)GO TO 405
P=RHO1*(1.0+E*COS(TH1))
A=P/(1.0-E**2)
GAMMA=1.0
E1=AAA(E,TH1,GAMMA)
APPENDIX E

E2=AAA(E,TH2,GA1)
DT=((A**1.5)*(E2-E1-E*(SIN(E2)-SIN(E1))))/6.2831853
SUB14=DT-DELTTH
ASUB14=ABS(SUB14)
IF(ASUB14-CRIT)800,800,403
403 IF(SUB14)404,800,405
404 TH1=TH1+DELTTH
GO TO 402
405 DELTH=DELTH/2.
IF(DELTH-.0000001)800,800,408
408 TH1=TH1-DELTH
GO TO 402
C ELLIPTIC LOOP RH02 EQUAL TO RH01
500 DELTTC=(DELTTH*(RH01**1.5))/6.2831853
DELE=1.0/AH
E=DELE
SUB15=DELTTC-DELTTH
IF(SUB15)502,501,501
501 P=RH01*(1.0+E*(COS(PSI)))
A=P/(1.0-E**2)
TH1=-1.0*PSI
TH2=TH1+DELTTH
GA1=1.0
E1=AAA(E,TH1,GA1)
DT=-(A**1.5)*(E1-E*SIN(E1))/3.1415927
SUB16=DT-DELTTH
ASUB16=ABS(SUB16)
IF(ASUB16-CRIT)800,800,503
503 IF(SUB16)504,800,505
504 DELE=DELE/2.
IF(DELE-.0000001)800,800,512
512 E=E-DELE
GO TO 501
505 E=E+DELE
GO TO 501
502 P=RH01*(1.0-E*(COS(PSI)))
A=P/(1.0-E**2)
TH1=3.1415927-PSI
TH2=TH1+DELTTH
GA1=1.0
E1=AAA(E,TH1,GA1)
AM1=E1-E*(SIN(E1))
DT=-(A**1.5)*(3.1415927-AM1))/3.1415927
SUB17=DT-DELTTH
ASUB17=ABS(SUB17)
IF(ASUB17-CRIT)800,800,506
506 IF(SUB17)507,800,508
507 E=E+DELE
GO TO 502
508 DELE=DELE/2.
IF(DELE-.0000001)800,800,514
514 E=E-DELE
GO TO 502
APPENDIX E

600 SUB18=RHO1-RHO2
ASUB18=ABS(SUB18)
IF(ASUB18-CT1(RHO)700,700,601
C HYPERBOLIC LOOP RHO2 NOT EQUAL TO RHO1

601 DELTH=(THP+PSI)/AH
610 TH1=THP-DELTH
602 TH2=TH1+DELTH
E=(RHO2-RHO1)/(RHO1*(COS(TH1))-RHO2*(COS(TH2)))
CTH1=COS(TH1)
OPROC=1.0+E*CTH1
IF(OPROC.GT.0.0)GO TO 608
609 DELTH=DELTH/2.
IF(DELTH.GT.0.1E-08)GO TO 612

C IF THE CASE IS A LIMITING HYPERBOLIC ORBIT WITH E APPROACHING
C (-1.0)/COS(DELTH/2.0) WRITE A MESSAGE AND DISCONTINUE
C COMPUTATION FOR THAT CASE.
ISIG=49
CALL SPACE
J=J+2
E=(-1.0)/COS(DELTH/2.0)
IF(GUIDE.GT.1.5)GO TO 611
CALL CONVT(DELTH,CTH,OPROC,TPER)
WRITE(6,164)CTH,E
GO TO 1028
611 CALL CONVT(DELTTT,CTTT,OPROC,TPER)
WRITE(6,166)CTTT,E
GO TO 1028
612 CONTINUE
TH1=TH1+DELTH
GO TO 602
608 IF(E.LE.1.0)GO TO 604
P=RHO1*(1.0+E*COS(TH1)))
A=P/(1.0-E**2)
GAMMA=-1.0
F1=AAA(E,TH1,GAMMA)
F2=AAA(E,TH2,GAMMA)
TANF1=(SIN(F1))/(COS(F1))
SU19=F1/2.+3.1415927/4.
TS10=(SIN(SU19))/(COS(SU19))
T1=(((-A)**1.5)*(E*TANF1-ALOS(TS19))/S.2831853
TANF2=(SIN(F2))/(COS(F2))
SUB10=F2/2.+3.1415927/4.
TS20=(SIN(SUB10))/(COS(SUB10))
T2=(((-A)**1.5)*(E*TANF2-ALOS(TS20))/S.2831853
DT=T2-T1
SUB21=DT-DELTTT
ASUB21=ABS(SUB21)
IF(ASUB21-CT1)800,800,603
APPENDIX E

503 IF(SUB21) 304, 604, 605
604 DELT = DELT / 2.
605 IF(DEL T <= .0000001) 606, 608, 607
607 CONTINUE
608 TTI = TTI * DELT:
609 GEO 602
605 TTI = TTI - DELT:
610 GEO 602
C

100 DELE = 1. / AI:
101 E = E + DELE
102 TTI = TTI * PS1
103 CTH1 = COS(TTI)
104 TLI = TTI * DELT:
105 P = SH1 * (1. + E * CTH1)
106 A = P / (1. - E ** 2)
107 USU23 = -1.0
108 F1 = COS(TLI) / CTH1
109 TF1 = SIN(F1) / (COS(F1))
110 SUB22 = F1 / 2. + 3.1415927 / 4.
111 TS22 = (SIN(SUB22)) / (COS(SUB22))
112 DT = (((A) ** 1.5) * (R*TF1 - 1.0) * (TS22)) / 3.1415927
113 SUB23 = DT - DELT:
114 ASU23 = ABS(SUB23)
115 IF (ASU23 < .01) 707, 705, 706
702 IF (SUB23) 703, 800, 705
703 DELE = DELE / 2.
705 IF (DELE <= .0000001) 800, 800, 707
707 E = E - DELE
708 GO TO 701
704 E = E + DELE
709 GO TO 701
800 IF (TTI) 301, 802, 802
801 RHOC1 = P / (1. + E)
802 GEO 803
803 RHOC1 = RHOC1
804 GEO 803
803 K = 0
805 IF (RHOC1 < .0000001) 304, 305, 305
804 K = 2
805 IF (RF - XI1) 305, 307, 307
806 RHOC1 = RHOC1
807 RHOF = RHOC1
808 TH1 = 5.2831853 - TTI:
809 RH1 = RHOC1
807 RHOF = RHOF
808 TTH1 = TTI:
809 TTF = TTI:
808 SQP = SQRT(P)
809 TTH1 = SQP / (RHOC1 ** 2)
809 RHOT1 = E * SIN(TH1) / SQP
809 TTHF = SQP / (RHOF ** 2)
APPENDIX E

RHODTF=E*SIN(THDF)/SCP

CALL SIX(A11,A12,A21,A22,A31,A32,O11,O12,O13,A11)

809 ASI=ABS(D11)

IF(A11=5.E-08)810,810,811

810 OMEGA=0.0
O11=PI*IF-TIF
GOTO 900

811 IF(B11)813,810,812

812 OMEGA=PI*IF+3.1415927
O11=3.1415927-TIF
GOTO 900

813 OMEGA=PI*IF
O11=6.2831853-TIF

900 CALL SIX(B11,B12,B21,B22,B31,B32,O11,OMEGA,B1)

XI1DI1=XI1DI1*(COS(A11))-(XI1DI1+XI1DI1)*(SIN(A11))

XI1DI1=XI1DI1*(COS(A11))+(XI1DI1+XI1DI1)*(SIN(A11))

RODT1=RODT1*(COS(TH1T))-(RODT1+RODT1)*(SIN(TH1T))

RODT2=RODT1*(SIN(TH1T))+(RODT1+RODT1)*(COS(TH1T))

RODT1=RODT1*(SIN(TH1T))+(RODT1+RODT1)*(COS(TH1T))

RODT2=RODT1*(SIN(TH1T))+(RODT1+RODT1)*(COS(TH1T))

RDTF1=RDTF1*(COS(PHIF))-RF*PH1DF*(SIN(PHIF))

RDTF2=RDTF2*(SIN(PHIF))-RF*PH1DF*(COS(PHIF))

901 CALL VELOC(V11,V12,V13,A11,A12,A21,A22,A31,A32,X11DI1,X11DI2)

CALL VELOC(V21,V22,V23,B11,B12,B21,B22,B31,B32,RODT1,RODT2)

CALL VELOC(V31,V32,V33,B11,B12,B21,B22,B31,B32,RODT1,RODT2)

V41=RDTF1

V42=RDTF2

V43=0,0

1001 DV11=V21-V11

DV12=V22-V12

DV13=V23-V13

DV21=V41-V31

DV22=V42-V32

DV23=V43-V33

1002 DELV1=SQRT(DV11**2+DV12**2+DV13**2)

DELV2=SQRT(DV21**2+DV22**2+DV23**2)

DELV=DELV1+DELV2

PHIF=PHIF/.17453293E-01

A11=A11/.17453293E-01

TH1=TH1/.17453293E-01

DELTTH=DELTTH/.17453293E=01

B1=B1/.17453293E-01

CALL CONVRT(DELTTH,DELTTH,OPTION,TPER)

CALL CONVRT(DELTTH,DELTTH,OPTION,TPER)

SIG=49

CALL SPACE

J=J+2

IF(GUIDE.GT.1.5)GO TO 1030

IF(K.GT.1)GO TO 1032

IF(OPTION.GT.1.5)GO TO 1040

WRITE(6,120)DELTTH,PHIF,A11,E1,B1,TH1,DELTTH,RHO1H,DV11,DV12,DV13,DV21,DV22,DV23,DELV1,DELV2,DELV

GO TO 1031
APPENDIX E

1040 WRITE(6,134)DELTTH,PHIF,ANUI,E,A,B1,TH1,DELTTH,RHCITH,DV11,DV12,DV13,DV21,DV22,DV23,DELV1,DELV2,DELV
GO TO 1031

1032 IF(OPTION.GT.1.5)GO TO 1041
WRITE(6,122)DELTTH,PHIF,ANUI,E,A,B1,TH1,DELTTH,RHCITH,DV11,DV12,DV13,DV21,DV22,DV23,DELV1,DELV2,DELV
GO TO 1031

1041 WRITE(6,136)DELTTH,PHIF,ANUI,E,A,B1,TH1,DELTTH,RHCITH,DV11,DV12,DV13,DV21,DV22,DV23,DELV1,DELV2,DELV
GO TO 1031

1030 IF(K.GT.1)GO TO 1033
IF(OPTION.GT.1.5)GO TO 1042
WRITE(6,120)DELTTH,PHIF,ANUI,E,A,B1,TH1,DELTTH,RHCITH,DV11,DV12,DV13,DV21,DV22,DV23,DELV1,DELV2,DELV
GO TO 1031

1042 WRITE(6,134)DELTTH,PHIF,ANUI,E,A,B1,TH1,DELTTH,RHCITH,DV11,DV12,DV13,DV21,DV22,DV23,DELV1,DELV2,DELV
GO TO 1031

1033 IF(OPTION.GT.1.5)GO TO 1043
WRITE(6,122)DELTTH,PHIF,ANUI,E,A,B1,TH1,DELTTH,RHCITH,DV11,DV12,DV13,DV21,DV22,DV23,DELV1,DELV2,DELV
GO TO 1031

1043 WRITE(6,136)DELTTH,PHIF,ANUI,E,A,B1,TH1,DELTTH,RHCITH,DV11,DV12,DV13,DV21,DV22,DV23,DELV1,DELV2,DELV
GO TO 1031

1031 CONTINUE
C
CONVERT BACK TO DIMENSIONLESS TIME
IF(OPTION.LT.1.5)GO TO 1028
IF(OPTION.LT.2.5)GO TO 1072
DELTTH=85400.*DELTTH/TPER
DELTTH=85400.*DELTTH/TPER
GO TO 1028

1072 DELTTH=60.*DELTTH/TPER
DELTTH=60.*DELTTH/TPER
1023 CONTINUE
IF(GUISE-1.5)1003,1015,1915
1003 IF(DELTTH-GT.0.1)1004,1005,1005
1004 DELTTH=DELTTH+DELTTH
NO=6
GO TO 1042

1005 IF(K.GT.1)GO TO 1007
IF(DELTTH-LT.0.1)1006,1007,1007
1006 DELTTH=DTUF
NO=2
GO TO 42

1007 IF(DELTTH-LT.0.1)1003,1000,1000
1008 DELTTH=DELTTH+DELTTH
NO=6
NO=0
ISIC=48
CALL SPACE
WRITE(6,124)
call convert(DELTTH,COTT,OPTION,TPER)
IF(OPTION.GT.1.5)GO TO 1060
WRITE(6,130)COTT
APPENDIX E

GO TO 1061
1060 WRITE(6,140)CDTT
1061 J=J+3
GO TO 41
1069 IF(NO1.GT.1)GO TO 1011
   IF(DELTTF-DELTTF)1010,1011,1011
1010 DELTTT=DTTF
   NO=0
   NO1=2
   SIG=48
   CALL SPACE
   WRITE(6,124)
   CALL CONVRT(DELTTT,CDTT,OPTION,TPER)
   IF(OPTION.GT.1.5)GO TO 1062
   WRITE(6,130)CDTT
   GO TO 1063
1062 WRITE(6,140)CDTT
1063 J=J+3
GO TO 41
1011 IF(ANUO=ANLHDN)1012,1013,1013
1012 ANUO=ANUO+DELNUO
   NO=0
   NO1=0
   NO2=0
   SIG=47
   CALL SPACE
   WRITE(6,124)
   ANUO=ANUO/.17453293E-01
   WRITE(6,132)ANUO
   CALL CONVRT(DTTI,CDTT,OPTION,TPER)
   IF(OPTION.GT.1.5)GO TO 1064
   WRITE(6,130)CDTT
   GO TO 1065
1064 WRITE(6,140)CDTTI
1065 ANU0=ANU0*.17453293E-01
   J=J+3
GO TO 40
1013 IF(NO2.GT.1)GO TO 12
   IF(ANUO=ANUOLT)1014,12,12
1014 ANU0=ANU0L
   NO=0
   NO1=0
   NO2=2
   SIG=47
   CALL SPACE
   WRITE(6,124)
   ANU0=ANU0/.17453293E-01
   WRITE(6,132)ANU0
   CALL CONVRT(DTTI,CDTT,OPTION,TPER)
   IF(OPTION.GT.1.5)GO TO 1066
   WRITE(6,130)CDTTI
   GO TO 1067
1066 WRITE(6,140)CDTTI
1067 ANU0=ANU0*.17453293E-01

74
J=J+3
GO TO 40
1015 IF(DELTTT=TTFMTT)1016,1017,1017
1016 DELTTT=DELTIlT+DELT
NO=0
GO TO 42
1017 IF(NO,GT,1)GO TO 1019
IF(DELTTT=DFTFT)1018,1019,1019
1018 DELTTT=DTTF
NO=2
GO TO 42
1019 IF(DELTTT=TDFTFD)1020,1021,1021
1020 DELTTT=DELTIlT+DELT
J=J+3
GO TO 40
1015 IF(DELTTT=TTFMTT)1016,1017,1017
1016 DELTTT=DELTIlT+DELT
NO=0
GO TO 42
1017 IF(NO,GT,1)GO TO 1019
IF(DELTTT=DFTFT)1018,1019,1019
1018 DELTTT=DTTF
NO=2
GO TO 42
1019 IF(DELTTT=TDFTFD)1020,1021,1021
1020 DELTTT=DELTIlT+DELT
J=J+3
GO TO 40
1015 IF(DELTTT=TTFMTT)1016,1017,1017
1016 DELTTT=DELTIlT+DELT
NO=0
GO TO 42
1017 IF(NO,GT,1)GO TO 1019
IF(DELTTT=DFTFT)1018,1019,1019
1018 DELTTT=DTTF
NO=2
GO TO 42
1019 IF(DELTTT=TDFTFD)1020,1021,1021
1020 DELTTT=DELTIlT+DELT
NO=0
NO1=0
ISIG=48
CALL SPACE
WRITE(6,124)
CALL CONVRT(DELTTT,CDTW,OPTICH,TPE)
IF(OPTICH,GT,1.5)GO TO 1080
WRITE(6,128)CDTW
GO TO 1081
1080 WRITE(6,138)CDTW
1081 J=J+2
GO TO 45
1022 DELTTT=DTTF
NO=0
NO1=2
ISIG=48
CALL SPACE
WRITE(5,124)
CALL CONVRT(DELTTT,CDTW,OPTICH,TPE)
IF(OPTICH,GT,1.5)GO TO 1082
WRITE(6,128)CDTW
GO TO 1083
1082 WRITE(6,138)CDTW
1083 J=J+2
GO TO 45
1024 ANUO=ANUO+DENUO
NO=0
NO1=0
NO2=0
ISIG=47
CALL SPACE
WRITE(6,124)
ANUO=ANUO/.17453293E-01
WRITE(6,132)ANUO
CALL CONVRT(DTW1,CDTW1,OPTCH,TPE)
IF(OPTCH,GT,1.5)GO TO 1084
WRITE(6,128)CDTW1
1024 ANUO=ANUO+DENUO
NO=0
NO1=0
NO2=0
ISIG=47
CALL SPACE
WRITE(6,124)
ANUO=ANUO/.17453293E-01
WRITE(6,132)ANUO
CALL CONVRT(DTW1,CDTW1,OPTCH,TPE)
IF(OPTCH,GT,1.5)GO TO 1084
WRITE(6,128)CDTW1
1024 ANUO=ANUO+DENUO
NO=0
NO1=0
NO2=0
ISIG=47
CALL SPACE
WRITE(6,124)
ANUO=ANUO/.17453293E-01
WRITE(6,132)ANUO
CALL CONVRT(DTW1,CDTW1,OPTCH,TPE)
IF(OPTCH,GT,1.5)GO TO 1084
WRITE(6,128)CDTW1
APPENDIX E

GO TO 1085
1084 WRITE (6, 138) CDTWI
1085 ANU0 = ANU0 * 1.7453293E-01
   J = J + 3
   GO TO 44
1025 IF (NO2 .GT. 1) GO TO 12
   IF (ANU0 .LT. ANU0) 1026, 12, 12
1026 ANU0 = ANU0
   NO = 0
   NO1 = 0
   NO2 = 2
   SIG = 47
   CALL SPACE
   WRITE (6, 124)
   ANU0 = ANU0 / 1.7453293E-01
   WRITE (6, 132) ANU0
   CALL CONVERT (DTWI, CDTWI, OPTION, TPER)
   IF (OPTION .GT. 1.5) GO TO 1086
   WRITE (6, 128) CDTWI
   GO TO 1087
1086 WRITE (6, 138) CDTWI
1087 ANU0 = ANU0 * 1.7453293E-01
   J = J + 3
   GO TO 44
12 WRITE (6, 104)
   GO TO 10
100 FORMAT (4E18.8)
102 FORMAT (//4X42HVELOCITY INCREMENT REQUIRED FOR RENDEZVOUS IN 37H BETWEEN TWO ARBITRARY ELLIPTIC ORBITS//)
104 FORMAT (1H1)
106 FORMAT (7E18.8)
108 FORMAT (1X2XHET16X2HE116X2HAI12X6H010E115X3H01115X6H0111)
110 FORMAT (1X2XHHPH1014X4HNU1114X4HNU0116X2HNU16X2HAT14X4HTPER15X3HVCT)
112 FORMAT (1X4HDHT114X4HDTH114X4HDTH114X4HDTH114X4HDTH114X4HDTH114X4HDTH114X4HDTH114X4HDTH114X4HDTH114X4HDTH114X4HDTH114X4HDTH114X4HDTH114X4HDTH114X4HDTH114X4HDTH114X4HDTH114X4HDTH114X4HDTH114X4HDTH114X4HDTH114X4HDTH114X4HDTH)
114 FORMAT (1X4HDDELTT114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL)
116 FORMAT (1X4HDDELTT114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL)
118 FORMAT (1X4HDDELTT114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL114X4HDDEL)
120 FORMAT (F7.4, 2F7.2, F7.4, F7.3, F7.2, F8.2, F7.2, F5.2, F6.8.5, 3F7.4/)
122 FORMAT (F7.4, 2F7.2, F7.4, F7.3, F7.2, F8.2, F7.2, F5.2, F6.8.5, 3F7.4, 1H*//)
124 FORMAT (1X//)
126 FORMAT (4E18.8, E36.8, E18.8)
128 FORMAT (1X4HDDELTT114X4HDDEL)
130 FORMAT (1X4HDDELTT114X4HDDEL)
132 FORMAT (4XSNHNUO114X4HNU)
134 FORMAT (F7.1, 2F7.2, F7.4, F7.3, F7.2, F8.2, F7.2, F5.2, F6.8.5, 3F7.4/)
136 FORMAT (F7.1, 2F7.2, F7.4, F7.3, F7.2, F8.2, F7.2, F5.2, F6.8.5, 3F7.4, 1H*//)
138 FORMAT (1X4HDDELTT114X4HDDEL)
140 FORMAT (1X4HDDELTT114X4HDDEL)
142 FORMAT (6X20HAI IS LESS THAN ZERO//)
144 FORMAT (10X4HPH1016X2HET16X2HNU16X2HAT14X4HTPER15X3HVCT)
APPENDIX E

150 FORMAT(4X41HI EQUAL +/-90 DEGREES NOT ACCEPTABLE DATA//)
160 FORMAT(10X19HDELTATHEETA EQUALS F6.1,1X39HDEGREES. DO NOT CONSIDER UNLESS ORBITS 1X25HINTERSECT, IN WHICH CASE,)
162 FORMAT(6X48HDTTI INPUT EITHER AS ZERO OR A NEGATIVE QUANTITY 1X23HBUT EQUATED TO DELTT ( F10.4,1X33H) BY PROGRAM. RECONSIDER INPUT. 2//)
164 FORMAT(10X20HTHIS CASE (DELTTH = F10.4,1X37H) IS A LIMITING HYPERBOLIC ORBIT WITH1X14HE APPROACHING F10.5//)
166 FORMAT(10X20HTHIS CASE (DELTTH = F10.4,1X37H) IS A LIMITING HYPERBOLIC ORBIT WITH1X14HE APPROACHING F10.5//)
END

FUNCTION AAA(ALPHA,BETA,GAMMA)
GAMMA EQUAL 1,EXCEPT FOR HYPERBOLIC ORBITS WHEN IT IS -1
PINUM=3.1415927
BETA2=BETA/2.0
ABETA2=ABS(BETA2)
SBETA2=SIN(BETA2)
IF(SBETA2.GT.0.0.AND.ABETA2.GT.1.5707789.AND.ABETA2.LT.1.5708370)G 10 TO 1
IF(SBETA2.LT.0.0.AND.ABETA2.GT.1.5707789.AND.ABETA2.LT.1.5708137)G 10 TO 2
SUB1=(SQRT(GAMMA*(1.0-ALPHA)/(1.0+ALPHA)))*(SBETA2/(COS(BETA2)))
APR=2.*ATAN(SUB1)
GO TO 3
1 APR=3.1415927
GO TO 3
2 APR=-3.1415927
3 N=(BETA+PINUM)/(2.*PINUM)
AN=N
AAA=APR+2.*AN*PINUM
RETURN
END

SUBROUTINE VELOC(Y11,Y12,Y13,H11,H12,H21,H22,H31,H32,S1,S2)
Y11=H11*S1+H12*S2
Y12=H21*S1+H22*S2
Y13=H31*S1+H32*S2
RETURN
END
APPENDIX E

SUBROUTINE SIX(W11, W12, W21, W22, W31, W32, U1, U2, U3)
CU1 = COS(U1)
SU1 = SIN(U1)
CU2 = COS(U2)
SU2 = SIN(U2)
CU3 = COS(U3)
SU3 = SIN(U3)
W11 = CU1*CU2 - CU3*SU2*SU1
W12 = -SU1*CU2 - CU3*SU2*CU1
W21 = CU1*SU2 + CU3*CU2*SU1
W22 = -SU1*SU2 + CU3*CU2*CU1
W31 = SU3*SU1
W32 = SU3*CU1
RETURN
END

SUBROUTINE ANOM(ECCEN, AMANOM, EAANOM)
BARM = AMANOM
E = ECCEN
EA0 = BARM + E*SIN(BARM) + .5*E**2*SIN(2.*BARM)
EA = EA0
16 AMANOM = EA - E*SIN(EA)
DELMA = BARM - AMANOM
DELEA = DELMA / (1. - E*COS(EA))
EA1 = EA + DELEA
ADELEA = ABS(DELEA)
CONST = (EA1 - EA) / EA1
ACT = ABS(CONST)
IF (ACT < 1E-06) 17, 15, 15
15 EA = EA1
GO TO 16
17 EAANOM = EA
RETURN
END

SUBROUTINE SPACE
COMMON J, GUIDE, ISIG
IF (ISIG = J) 10, 10, 11
10 WRITE(6, 101)
APPENDIX E

```fortran
IF(GUIDE.GT.1.5)GO TO 12
WRITE(6,103)
GO TO 13
12 WRITE(6,104)
13 J=5
RETURN
11 CONTINUE
RETURN
101 FORMAT(1H1)
103 FORMAT(1X6HDELTTH3X4HPHI4X3HNU16X1HE6X1MA6X1H15X3HTH11X6HDELTTH1X
14HRHON4X4HDV1X4X4HDV1Y4X4HDV1Z4X4HDV2X4X4HDV2Y4X4HDV2Z2X5HDELV12X
25HDELV23X4HDELV//)
104 FORMAT(1X6HDELTTH3X4HPHI4X3HNU16X1HE6X1MA6X1H15X3HTH11X6HDELTTH1X
14HRHON4X4HDV1X4X4HDV1Y4X4HDV1Z6X4HDV2X4X4HDV2Y4X4HDV2Z2X5HDELV12X
25HDELV23X4HDELV//)
END

SUBROUTINE CONVRT(TIME,CTIME,OPTION,TPER)
C TO CONVERT TIME FROM DIMENSIONLESS QUANTITIES TO UNITS
C CORRESPONDING TO INPUT
C
1 IF(OPTION.LT.1.5)GO TO 1
2 IF(OPTION.LT.2.5)GO TO 2
3 CTIME=TIME*TPER/86400.
RETURN
1 CTIME=TIME
RETURN
2 CTIME=TIME*TPER/60.
RETURN
END
```

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REFERENCES


TABLE I
COMPUTER PRINTOUT OF EXAMPLE 1

VELOCITY INCREMENT REQUIRED FOR RENDEZVOUS BETWEEN TWO ARBITRARY ELLIPTIC ORBITS

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### TABLE II

**COMPUTER PRINTOUT OF EXAMPLE 2**

**VELOCITY INCREMENT REQUIRED FOR RENDEZVOUS BETWEEN TWO ARBITRARY ELLIPTIC ORBITS**

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<td>0.22480000E 08</td>
<td>0.56438179E 04</td>
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<td>0.20000000E 02</td>
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**NUO** = 0.49

**DELTTH** = 20.0

**DELTTH PHIF NUI E A I THI DELTTH RHQH DV1X DV1Y DV1Z DV2X DV2Y DV2Z DELV1 DELV2 DELV**

| ET     | 0. | 0.175 | 0.49 | 0.0499 | 0.948 | 0. | 0. | 107.97 | 76.93 | 0.96 | 0.05227 | 0.14489 | 0. | -0.02179 | 0.03203 | 0. | 0.1540 | 0.0387 | 0.1928 |
|--------|----|-------|------|--------|------|---|---|-------|-------|------|-----------|-----------|---|----------|----------|---|--------|--------|---|--------|--------|
| 0.117.22 | 44.59 | 0.0735 | 0.964 | 0. | 0. | 105.04 | 70.87 | 0.01 | 0.01005 | 0.20010 | 0. | -0.05237 | 0.05712 | 0. | 0.2004 | 0.0775 | 0.2779 |
| 0.154.27 | 80.01 | 0.1979 | 1.271 | 0. | 0. | 280.13 | 72.50 | 0.02 | 0.05508 | 0.43009 | 0. | 0.01498 | 0.10739 | 0. | 0.4336 | 0.1084 | 0.5420 |
| 0.190.85 | 106.69 | 0.6942 | 3.300 | 0. | 0. | 292.34 | 82.40 | 0.10 | 0.09842 | 0.74862 | 0. | 0.08038 | 0.32180 | 0. | 0.7551 | 0.3317 | 1.0867 |

**DELTTH** = 30.0

| ET     | 0. | 0.117 | 0.49 | 0.0379 | 0.974 | 0. | 0. | 71.09 | 114.98 | 0.96 | 0.03930 | 0.13126 | 0. | -0.02346 | 0.01591 | 0. | 0.1370 | 0.0283 | 0.1654 |
|--------|----|-------|------|--------|------|---|---|-------|-------|------|-----------|-----------|---|----------|----------|---|--------|--------|---|--------|--------|
| 0.154.27 | 44.59 | 0.0312 | 1.006 | 0. | 0. | 132.66 | 107.92 | 1.02 | 0.03276 | 0.19063 | 0. | -0.03231 | 0.01876 | 0. | 0.1934 | 0.0374 | 0.2308 |
| 0.190.85 | 80.01 | 0.1500 | 1.197 | 0. | 0. | 266.85 | 109.08 | 1.02 | 0.03019 | 0.39549 | 0. | 0.02658 | 0.08689 | 0. | 0.3966 | 0.0909 | 0.4875 |
| 0.227.63 | 106.69 | 0.4355 | 1.713 | 0. | 0. | 273.43 | 119.18 | 0.97 | 0.15056 | 0.60254 | 0. | 0.01612 | 0.27834 | 0. | 0.6211 | 0.2788 | 0.8999 |

**DELTTH** = 40.0

| ET     | 0. | 0.154 | 0.49 | 0.0367 | 0.987 | 0. | 0. | 45.21 | 152.03 | 0.96 | 0.02890 | 0.12482 | 0. | -0.02205 | 0.02860 | 0. | 0.1281 | 0.0222 | 0.1504 |
|--------|----|-------|------|--------|------|---|---|-------|-------|------|-----------|-----------|---|----------|----------|---|--------|--------|---|--------|--------|
| 0.190.85 | 44.59 | 0.0196 | 1.019 | 0. | 0. | 113.84 | 144.49 | 1.02 | 0.04107 | 0.18982 | 0. | -0.01610 | 0.00656 | 0. | 0.1942 | 0.0174 | 0.2116 |
| 0.227.63 | 80.01 | 0.1204 | 1.137 | 0. | 0. | 245.09 | 145.86 | 1.00 | 0.00549 | 0.36346 | 0. | 0.00835 | 0.09499 | 0. | 0.3635 | 0.0954 | 0.4589 |

**DELTTH PHIF NUI E A I THI DELTTH RHQH DV1X DV1Y DV1Z DV2X DV2Y DV2Z DELV1 DELV2 DELV**

<p>| ET     | 0. | 0.256 | 106.69 | 0.3124 | 1.344 | 0. | 0. | 250.68 | 156.78 | 0.92 | 0.17237 | 0.48852 | 0. | -0.08178 | 0.24000 | 0. | 0.5180 | 0.2535 | 0.7716 |</p>
<table>
<thead>
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**TABLE III**

**COMPUTER PRINTOUT OF EXAMPLE 3**

**VELOCITY INCREMENT REQUIRED FOR KINEMATICS BETWEEN TWO ARBITRARY ELLIPTIC ORBITS**

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—National Aeronautics and Space Act of 1958

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