NONGREY RADIATION EFFECTS ON THE BOUNDARY LAYER OF AN ABSORBING GAS OVER A FLAT PLATE

by A. M. Smith and H. A. Hassan

Prepared by
RESEARCH TRIANGLE INSTITUTE
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FOREWORD

This report was prepared for the Langley Research Center, National Aeronautics and Space Administration, under the contract entitled "A Research Study of Radiative Energy Transfer Considerations for a Reentry Vehicle with an Absorbing and Emitting Boundary Layer". The Contract No. was NAS1-4270. The technical monitor for NASA was W. B. Olstad of the Gas Physics Section, Aerophysics Division. The work documented herein was performed by A. M. Smith of the Research Triangle Institute with Dr. H. A. Hassan of North Carolina State University at Raleigh serving as a consultant.
A study of the nongrey radiation effects on the laminar boundary layer of an absorbing gas over a flat plate is presented. The nongrey model employed assumes an absorption coefficient with a stepwise frequency dependence. Such a model gives a reasonable fit to the published data on monochromatic absorption coefficient for air.

Examination of the optical thicknesses corresponding to the various step functions shows that the maximum optical thickness based on boundary layer thickness and maximum absorption coefficient, from the data of Sewell, is less than unity in the boundary layer over a flat plate. This conclusion is based on a consideration of missions such as a reentry of a space probe from Mars.

For typical temperatures and Eckert numbers at the edge of the boundary layer, it is shown that, for high external flow emissivities the nongrey radiative flux to the wall is greater than the grey while the grey convective flux is greater than the nongrey. The total heat transfer, based on the nongrey model, is found to be greater than that based on the grey model, for a combination of high wall and high external flow emissivities or, low wall and low external flow emissivities. Also, for high external flow emissivities, the ratio of the radiative to the convective flux is always greater than unity; while for low external flow emissivities, the ratio is greater or less than unity depending on the wall emissivity and external flow Eckert number. These conclusions are based on numerical calculations for a temperature of 10,000° K and higher at the edge of the boundary layer.

INTRODUCTION

The advent of space flight at velocities greater than the escape velocities focused increased attention on the problem of radiative heating. For such velocities, the transfer of energy by radiation is

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comparable to that by convection and therefore, the coupling between
the radiative transfer and other energy transport processes may have
a large influence on the flow field around the reentry vehicle.

The object of this work is to consider the influence of the
nongrey radiation effects on the heat transfer in the boundary layer
of an absorbing gas over a flat plate. The wall and the external flow
are not considered to be perfect absorbers and the influence of their
optical properties on heat transfer is investigated. This study is
motivated by the fact that results based on the grey approximation
may, under certain conditions, be of little practical use as a result
of the large variation of the absorption coefficient with frequency,
especially at the higher temperatures of interest here.

An attempt at considering the effects of a nongrey model on the
heat transfer of a boundary layer of an absorbing gas over a flat
plate has been made by Cess (ref. 1). He ignored the viscous dissipation
term in the energy equation and assumed, in addition to a Prandtl
number of unity and the usual linear viscosity law, that the monochro-
matic absorption coefficient is independent of temperature and the
Planck mean is a reciprocal function of temperature. His results for the
nongrey model differed substantially from those for the grey model.
Although the assumptions introduced by Cess regarding the temperature
dependence of the monochromatic and total absorption coefficients are
unrealistic for most gases, especially for air, his model, nevertheless,
serves to illustrate the importance of nongrey effects.

A much improved nongrey model has been introduced by Olstad (ref. 2)
in his analysis of the inviscid stagnation shock layer. He assumed
an absorption coefficient with a stepwise dependence on the wavelength.
To illustrate the importance of the nongrey effects, he assumed, for
the sake of simplicity, that the pressure and temperature dependence in
each wavelength interval is given by that of the Planck mean. His
results also demonstrate the significance of the nongrey effects.

Examination of the data on monochromatic absorption coefficients
given by Sewell (ref. 3) shows that it can be closely approximated by
a stepwise frequency dependence. The pressure and temperature depen-
dence in each frequency interval do not correspond, however, to that
of the Planck mean or Rosseland mean. Using such a representation the
divergence of the monochromatic radiative flux vector can be integrated
in closed form to yield the divergence of the radiative flux vector.
As a result of this, it is shown, in general, that the divergence of
the radiative flux vector can be represented as a linear combination
of terms corresponding to the usual thin, self-absorbing, and thick
approximations. Using a reentry of a space probe from Mars and from
the far solar system (ref. 4) as typical missions, it is shown that, under
the most pessimistic estimates, the maximum optical thickness based on
a characteristic length equal to the maximum boundary layer thickness
on a ten meter plate is less than unity. It is concluded, therefore,
that the boundary layer on a flat plate for such missions, is optically
thin.
The thin approximation coupled with a two-step monochromatic absorption coefficient is employed in analyzing the boundary layer of an absorbing gas on a flat plate. The wall is assumed to be opaque, diffuse reflecting, and emitting; and the external flow is assumed to be emitting and nonreflecting. The wall is assumed to be grey and the emissivity of the external flow is assumed to be frequency dependent with the dependence being, for the sake of convenience, identical to that of the absorption coefficient.

Since the radiative flux is negligible for low external flow temperatures (ref. 5) the calculations were carried out for a $T_e = 10,000^\circ K$. The results show that, for high external flow emissivities, the ratio of the radiative flux to the convective flux is always greater than unity. However, for low external flow emissivities, this ratio is influenced by the wall emissivity, being smaller than one for low wall emissivities and high Eckert numbers and about one or greater for high wall emissivities and low Eckert numbers.

For the higher Eckert numbers, the nongrey radiative flux is greater than the grey and the grey convective flux is greater than the nongrey if the external flow emissivity is large. When the external flow emissivity is low, the reverse is true. The total nongrey heat transfer is found to be greater than the grey for a combination of high wall and high external flow emissivities or, a combination of low wall and low external flow emissivities.

SYMBOLS

- $B_\lambda$ Planck's distribution function (eq. (11))
- $B_\nu$ Planck's distribution function (eq. (9))
- $R_0$ Boltzmann number
- $c$ speed of light
- $E$ specific internal energy
- $F_i^R$ radiation energy flux vector
- $F_{\nu i}$ divergence of the monochromatic radiative flux vector
- $g_{m(\nu)}$ function defined in eq. (11)
- $H_\infty$ freestream total enthalpy
- $h$ specific enthalpy, Planck's constant or altitude
- $h_*$ reference enthalpy in eq. (73)
\( \bar{h}_o \), solution of eq. (70) in the absence of radiation

\( I_v \), specific intensity of radiation

\( J_\lambda \), spectral radiance in eqs. (C8) and (C9)

\( J_T \), total radiance in eqs. (C3), (C4), (C10), (C11), and (C12)

\( j_\lambda \), spectral emission coefficient

\( K \), mean linear absorption coefficient in eqs. (C7), (C10), and (C11)

\( K_v, K_\lambda \), linear spectral absorption coefficients

\( K_p \), Planck mean linear absorption coefficient

\( K_R \), Rosseland mean linear absorption coefficient

\( K_m(x_i) \), height of the mth step function in eq. (11) (see fig. 2)

\( k \), coefficient of conductivity or Boltzmann's constant

\( L \), characteristic length

\( M_\infty \), freestream Mach number

\( P_{ij} \), pressure tensor

\( P \), pressure

\( Pr \), Prandtl number

\( q_i \), heat flux (other than radiative) vector

\( q_R \), characteristic value of radiation energy flux vector

\( q_v \), spectral radiative flux

\( q_R \), global radiative flux

\( q_{cw} \), convective wall heat flux for a radiating gas

\( q_{cwo} \), convective wall heat flux for a nonradiating gas

\( q_{tw} \), total wall heat flux

\( Re_\infty \), freestream Reynolds number

\( r \), reflectivity

\( T \), temperature
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<thead>
<tr>
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<th>Definition</th>
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<tr>
<td>$T_s$</td>
<td>stagnation temperature</td>
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<tr>
<td>$u_1$</td>
<td>velocity vector</td>
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<td>$u_e^2/h_e$</td>
<td>Eckert number</td>
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<tr>
<td>$z$</td>
<td>compressibility factor</td>
</tr>
<tr>
<td>$\alpha_j$</td>
<td>function in eq. (13) and (A9)</td>
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<td>$\alpha_j(\beta_j)$</td>
<td>function in eq. (A13)</td>
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<td>$\Gamma$</td>
<td>radiation-convection parameter</td>
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<td>$\Delta_m$</td>
<td>frequency interval or intervals associated with $K_m(x_i)$ (see fig. 2)</td>
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<tr>
<td>$\delta$</td>
<td>boundary layer thickness</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>emissivity</td>
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<tr>
<td>$\varepsilon/L$</td>
<td>emissivity per cm</td>
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<td>$\varepsilon_v,\varepsilon_\lambda$</td>
<td>spectral gas emissivity in eqs. (C2) and (C8)</td>
</tr>
<tr>
<td>$\kappa_p$</td>
<td>Planck mean mass absorption coefficient</td>
</tr>
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<td>$\kappa_j$</td>
<td>mass absorption coefficients in eqs. (D10) and (D11)</td>
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<tr>
<td>$\lambda$</td>
<td>wavelength</td>
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<tr>
<td>$\mu$</td>
<td>coefficient of viscosity</td>
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<tr>
<td>$\nu$</td>
<td>frequency</td>
</tr>
<tr>
<td>$\rho$</td>
<td>density</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Stefan–Boltzmann constant</td>
</tr>
<tr>
<td>$\tau_\lambda$</td>
<td>characteristic optical thickness or Bouguer number for a grey gas</td>
</tr>
<tr>
<td>$\tau_{jo}$</td>
<td>Bouguer number for a nongrey gas</td>
</tr>
<tr>
<td>$\tau'_{jo}$</td>
<td>effective Bouguer number for optically thin nongrey gas</td>
</tr>
<tr>
<td>$\tau''_{jo}$</td>
<td>effective Bouguer number for optically thick nongrey gas</td>
</tr>
<tr>
<td>$\omega$</td>
<td>wavenumber or solid angle</td>
</tr>
</tbody>
</table>
Subscripts

e external flow conditions  
g grey gas  
o reference state  
s.l. sea level conditions  
w wall conditions  
v frequency dependent quantity

Superscripts

- dimensionless or normalized quantity

ANALYSIS

The conservation equations for the laminar boundary layer of a radiating gas may be obtained from the general equations of radiation gas dynamics derived by Goulard (ref. 6), which can be expressed as

\[ \frac{D\rho}{Dt} + \rho u_i, i = 0 \quad (1) \]

\[ \rho \frac{Du_i}{Dt} + P_{ij}, j = 0 \quad (2) \]

\[ \rho \frac{DE}{Dt} + q_{i,i} + F_{i,i}^R + P_{ij} u_{j,i} = 0 \quad (3) \]

where \( \rho, E, u_i, q_i, F_i^R, P_{ij} \) denote, respectively, the density, specific internal energy, velocity, heat flux (other than radiative) vector, radiation energy flux vector and pressure tensor. In writing eqs. (2) and (3) the radiation pressure tensor and the radiation energy density were ignored compared to the (molecular) pressure tensor and specific internal energy. As has been indicated by Goulard (ref. 7) this is justified except for temperatures greater than \( 10^7 \)K or particle densities approaching zero particles/cm\(^3\).

Except for the term \( F_{i,i}^R \) appearing in eq. (3), the conservation equations are identical with those of classical gas dynamics. As a
result of this, the boundary layer equations of a radiating gas are identical with the usual boundary layer equations except that an additional term $F_i^{R}$ appears in the energy equation. Thus, the boundary layer equations for a steady flow of a radiating gas over a flat plate can be written as

\begin{align*}
\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) &= 0 \quad (4) \\
\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} &= \frac{\partial}{\partial y}(\mu \frac{\partial u}{\partial y}) \quad (5) \\
\rho u \frac{\partial h}{\partial x} + \rho v \frac{\partial h}{\partial y} &= \frac{\partial}{\partial y}(k \frac{\partial T}{\partial y}) + \mu(\frac{\partial u}{\partial y})^2 - F_{i,i}^{R} \quad (6)
\end{align*}

where $u$ and $v$ are the velocity components in the $x$ and $y$ directions, $h$ is the specific enthalpy, $T$ is the temperature, and $\mu$ and $k$ are the coefficients of viscosity and conductivity. It should be pointed out that eqs. (4) through (6) assume a flow in chemical equilibrium and, in the case where Lewis number is different from unity, $k$ is to be interpreted as the total heat conduction coefficient which includes the effects of both conduction and diffusion.

The general expression for $F_{i,i}^{R}$ can be obtained from the divergence of the monochromatic radiative flux vector as

\begin{align*}
F_{i,i}^{R} &= \int_{0}^{\infty} F_{vi,i}(\omega) \, d\omega 
\end{align*}

(7)

A general expression for $F_{vi,i}(\xi)$ has been given by Goulard (ref. 6). For nonscattering gases in local thermodynamic equilibrium and perfectly absorbing surfaces, this relation can be expressed as (see Appendix A and fig. 1)

\begin{align*}
F_{vi,i}(\xi) = 4\pi \left[ K_{v}(\xi)B_{v}(\xi) - K_{v}(\xi) \int_{\eta_{1}}^{\xi} K_{v}(\eta_{1})B_{v}(\eta_{1}) \exp \left( - \int_{\eta_{1}}^{\xi} K_{v} \, ds \right) \frac{d\omega}{4\pi} \right] \\
&\quad - K_{v}(\xi) \int_{\eta_{1}}^{\xi} B_{v}(\eta_{1}) \exp \left( - \int_{\eta_{1}}^{\xi} K_{v} \, ds \right) \frac{d\omega}{4\pi}
\end{align*}

(8)

where $K_{v}$ is the spectral linear absorption coefficient and $B_{v}$ is Planck's distribution function.
\[ B_\nu = \frac{2h\nu}{c^2} \frac{1}{\exp(h\nu/kT) - 1} \]  

(9)

It is to be noted that the integral

\[ \int K_\nu \, ds \]  

(10)

which is usually referred to as the optical thickness (for frequency \( \nu \)) is evaluated along a line \( S(\xi_1) \) with direction cosines \( \xi_1 \). It is evident from eq. (8) that, before one can carry out the integration in eq. (7) the frequency dependence of \( K_\nu \) should be specified. The simplest case, where \( K_\nu \) is independent of frequency, leads to the well-known grey case. In general, arbitrary dependence of \( K_\nu \) on frequency leads to a rather complicated integration which would have to be carried out numerically. The desirability of choosing a \( K_\nu \) which would represent a fair approximation to physical reality and which would make it possible to carry out the integration in closed form is evident. One way in which this can be achieved is to approximate \( K_\nu \) by a number of step functions (see fig. 2), i.e.,

\[ K_{\nu}(x_i, \nu) = \sum_{m=1}^{n} K_m(x_i)g_m(\nu) \]

where

\[ g_m(\nu) = \begin{cases} 
1 \text{ if } \nu \in \Delta_m \\
0 \text{ otherwise} 
\end{cases} \]  

(11)

with \( \Delta_m \) the frequency interval or intervals in which \( K_\nu \) is \( K_m(x_i) \). Substitution of eq. (11) into eq. (8), (see Appendix A), yields

\[ F_{1,1}^R = 4\pi \sum_{j=1}^{n} \left[ K_j \alpha_j T^4 - K_j \int_{4\pi}^{X_i} \int_{\eta_1}^{x_i} K_j(\xi_1)\alpha_j(\xi_1)T^4(\xi_1) \exp\left(-\int_{\xi_1}^{x_i} K_j \, ds\right) \, ds \, \frac{d\omega}{4\pi} \right. \\
\left. - K_j \int_{4\pi}^{X_1} \alpha_j(\eta_1)T^4(\eta_1) \exp\left(-\int_{\eta_1}^{X_1} K_j \, ds\right) \, d\omega \right] \frac{4\pi}{4\pi} \]  

(12)

where
\[
\alpha_j = \frac{\int_{\Delta_v} R_v \, dv}{\sigma T^4/\pi}; \quad \sum_{j=1}^{n} \alpha_j = 1
\]  

(13)

As a result of using the approximation indicated in eq. (11), it is seen from eq. (12) that the contributions of the various frequency intervals to the divergence of the radiative flux vector add up linearly. The implication of this interesting property will be discussed below.

The grey gas approximation follows from eq. (11) by choosing \( n = 1, \beta_1 = 1 \) and \( \Delta_1 \) the frequency interval \((0, \infty)\). This rule is to be followed in reducing the nongrey expressions derived here to the corresponding grey expressions. In this connection, it is seen from eq. (12) that there is no grey equivalent to a nongrey model; this may be compared with eq. (4.23) of ref. 6 where such an equivalence is implied.

SIMILARITY PARAMETERS

Since eq. (6) is frequency independent, the similarity parameters which characterize radiation transfer in relation to other modes of energy transfer are identical with those derived by Goulard (ref. 7) for a grey gas. The dimensionless quantities appropriate for boundary layer analysis are

\[
\bar{x} = \frac{x}{L}, \quad \bar{y} = \frac{y}{\delta}, \quad \bar{u} = \frac{u}{u_0}, \quad \bar{v} = \frac{v}{u_0}, \quad \bar{h} = \frac{h}{h_0},
\]

\[
\bar{T} = \frac{T}{T_0}, \quad \bar{\rho} = \frac{\rho}{\rho_0}, \quad \bar{k} = \frac{k}{k_0}, \quad \bar{\mu} = \frac{\mu}{\mu_0}, \quad \bar{F}_R = \frac{F_R}{F_R_0},
\]

(14)

where the subscript 'o' designates some reference state, \( \delta \) is the boundary layer thickness and \( L \) is a characteristic length which may be chosen as the length of the plate. Introducing eq. (14) into eq. (6) one finds that the ratio of energy transfer by radiation to that by convection is characterized by the dimensionless number

\[
\Gamma = \frac{q_R}{\rho_0 u_0 h_0} \frac{L}{\delta}
\]

(15)

first introduced by Unsöld (ref. 8). If the characteristic value of the radiation energy flux vector can be chosen as
\[ \mathcal{R}_o = \sigma T_o^4 \]  

(16)

\( r \) reduces to the inverse of the Boltzmann number \( \text{Bo} \)

\[ \Gamma = \frac{\sigma T_o^4}{\rho_o u_o h_o \delta} = \text{Bo}^{-1} \]  

(17)

In addition to the above "extrinsic" dimensionless parameters, there exists "intrinsic" dimensionless parameters which govern the structure of the radiation field. These can be deduced from examination of the various forms of \( F_i^R \). It may be recalled that, for a grey gas, the appropriate intrinsic parameter is the optical thickness or Bouguer number, \( \tau \), defined as the product of a characteristic absorption coefficient and a characteristic length. In the presence of nongrey radiation, it is very difficult to define a characteristic absorption coefficient. In this connection, it may be mentioned that Olstad (ref. 2) employed a nongrey absorption coefficient given by

\[ K_\nu = \gamma_j K_p ; \nu_j \leq \nu \leq \nu_{j+1} ; j = 1, \ldots, m \]  

(18)

where the constants \( \gamma \) satisfy the relation

\[ \sum_{j=1}^{m} \gamma_j b_j = 1 \]

with

\[ b_j = \frac{\int_{\nu_j}^{\nu_{j+1}} B_\nu \, d\nu}{\int_{\nu}^{\infty} B_\nu \, d\nu} \]  

(19)

and defined an optical thickness based on \( K_p \) where \( K_p \) is the Planck mean. Using the same idea, one can define
\[ K_v = \delta_j K_R ; \ \nu_j \leq \nu \leq \nu_{j+1} ; \ j = 1, \ldots, m \]  

(20)

with \( K_R \) being the Rosseland mean, and employ an optical thickness based on \( K_R \). In this case, the \( \delta_j \)'s satisfy the relation

\[ \sum_{j=1}^{m} \frac{a_j}{\delta_j} = 1 \]

with

\[ a_j = \int_{\nu_j}^{\nu_{j+1}} \frac{d\nu}{dT} \int_{0}^{\infty} \frac{d\nu}{dT} d\nu , \ T = T_0 \]  

(21)

As has been recognized in ref. 2, definitions like eqs. (18) and (20), have one drawback in common, namely the \( K \) dependence on pressure and temperature is the same as that of the respective \( \delta \) mean which, in a true nongrey representation, is not the case. Because of such difficulties, the concept of a representative or an effective optical thickness for nongrey representations, other than the ones mentioned above, is not feasible.

Writing eq. (12) in dimensionless form shows that one can define a characteristic optical thickness for each frequency interval \( \Delta \). Thus, letting

\[ \bar{x}_i - \bar{x}_i^d / d, \ \bar{s} = s / d, \ \bar{\omega} = \omega / 4\pi, \ \bar{T} = T / T_0 \]

\[ \bar{F}_1^R = \frac{F_1}{4\sigma T_o^4}, \ \bar{K}_j = K_j / K_{j_0}, \ \bar{\alpha}_j = \alpha_j / \alpha_{j_0} \]  

(22)

and substituting into eq. (12), one finds

\[ \bar{F}_1^{n_1} = \sum_{j=1}^{n} \tau_{j_0} \alpha_j \bar{K}_j \bar{x}_j \left[ \bar{\alpha}_j \bar{T}^4 - \tau_{j_0} \int_{0}^{\bar{x}_i} \bar{K}_j \bar{\alpha}_j \bar{T}^4 \exp \left( - \tau_{j_0} \int_{0}^{\bar{x}_i} \bar{K}_j ds \right) ds d\bar{\omega} \right. 

- \left. \int_{0}^{\bar{x}_i} \bar{\alpha}_j \bar{T}^4 \exp \left( - \tau_{j_0} \int_{0}^{\bar{x}_i} \bar{K}_j ds \right) ds d\bar{\omega} \right] \]  

(23)
where

$$\tau_{jo} = K_{jo}d$$ (24)

is the Bouguer number for the interval \(\Delta_j\) and \(d\) is a characteristic geometric length of the radiating gas. Since \(\tau_{jo}\) may have different values for different \(\Delta_j\), it is possible that in some \(\Delta_j\), \(\tau_{jo} < 1\), while in another \(\Delta_j\), \(\tau_{jo}\) may be of order unity and finally, in still another \(\Delta_j\), it is possible that \(\tau_{jo} > 1\). The fact that \(\tau_{jo}\) can have arbitrary values shows the difficulty of defining an effective optical thickness.

The role played by \(\tau_{jo}\) in determining the asymptotic form of the jth bracket of eq. (23) is identical to the role played by the optical thickness in the case of grey radiation (ref. 7). Thus, for the thin case, where \(\tau_{jo} < 1\), the jth bracket reduces to

$$a_{jo}\tau_{jo}\left[\alpha_j^{-1}\bar{K}^{-4} - \bar{K}_j\int_0^1 a_j^{-1}\eta_1^{-4}\eta_1 d\omega\right]$$ (25)

It is seen from eq. (25) that an effective Bouguer number can be defined, for the thin case, by the relation

$$\tau'_{jo} = a_{jo}\tau_{jo}$$ (26)

Since \(0 < a_{jo} < 1\), this means that the gas may be optically thinner in this \(\Delta_j\) than actually indicated by \(\tau_{jo}\).

In the case where \(\tau_{jo}\) is of order unity, all the terms of the jth bracket should be retained. For the thick case, where \(\tau_{jo} > 1\), it is shown in Appendix B that the jth bracket reduces to

$$-\frac{1}{3} \frac{\alpha_{io}}{\tau_{jo}} \sum_{i=1}^{3} \frac{\partial}{\partial x_i} \left\{\frac{1}{\bar{K}_j} \frac{\partial}{\partial x_i} \left[\alpha_j^{-1}\bar{K}^{-4} \eta_1^{-4}\right]\right\}$$ (27)

Similar to eq. (26), an effective Bouguer number \(\tau'_{jo}\) can be defined by the relation
Thus the gas is optically thicker than indicated by $\tau_{jo}$.

The above discussion shows that, in general, eq. (23) can be written as

$$\bar{R}^{i}_{i,1} = \sum_{j=1}^{p} a_{jo} \tau_{jo} J_{j} K_{j} T_{j}^{4} - \sum_{j=1}^{q} a_{jo} \tau_{jo} K_{j} T_{j}^{4} - \frac{1}{3} \sum_{j=1}^{r} a_{jo} \frac{3}{2} \frac{\partial}{\partial x_{1}} \left( \frac{1}{K_{j}} \frac{\partial}{\partial x_{1}} \right)$$

$$- \int_{-\delta_{j}}^{\delta_{j}} \frac{1}{K_{j}} a_{jo} \left( T_{j} + a_{jo} K_{j} \right) d\delta d\varpi - \int_{-\delta_{j}}^{\delta_{j}} \frac{1}{K_{j}} a_{jo} \left( T_{j} + a_{jo} K_{j} \right) d\delta d\varpi$$

Equation (29) shows a basic difference between grey and nongrey radiation. Since there is one characteristic optical thickness for a grey gas, it is possible to characterize the whole flow field as thin, self-absorbing or thick depending on whether $\tau_{jo} < 1$. On the other hand, in the nongrey case, such characterization is, in general, not possible unless $\tau_{jo} < 1$, $\tau_{jo} \approx 1$ or $\tau_{jo} > 1$ for all $j$.

**ESTIMATE OF $\tau_{jo}$ FOR RADIATING AIR IN THE BOUNDARY LAYER**

To determine $p$, $q$, and $r$ in eq. (29), it is necessary to estimate the order of magnitude of the various Bouguer numbers for the radiating boundary layer on a flat plate. The characteristic geometric length of the radiating gas may be taken as the boundary layer thickness $\delta$ and, therefore, $\tau_{jo}$ can be expressed as

$$\tau_{jo} = K_{jo} \delta$$
Since it is expected that most of the $\tau_{ij0}$'s will be much less than unity for the radiating air boundary layer, the maximum value of $\tau_{ij0}$ will be estimated for a set of representative flight conditions encountered upon reentry into the earth's atmosphere. The reentry flight paths chosen are those given by Howe and Viegas (ref. 4) for reentry from a Mars mission and reentry from a far solar system mission (see fig. 3).

To estimate $(\tau_{ij0})_{\text{max}}$ one needs to estimate the maximum $K_{ij0}$ and $\delta$ for the flight conditions existing along these reentry trajectories. From the calculations of Van Driest (ref. 9) and for the range of Mach numbers considered there, the correlation

$$\frac{\delta}{\sqrt{Re_\infty}} = 2.1(M_\infty - 15) + 28, \ M_\infty > 15$$

may be used to calculate an upper limit for the boundary layer thickness on a flat plate for $M_\infty > 15$ and all calculated wall to free stream temperature ratios. Another method for estimating the boundary layer thickness can be inferred from the usual order of magnitude analysis employed in deriving the boundary layer equations. As a result of such analysis, one finds

$$\frac{\delta}{x \sqrt{Re_\infty}} = 0(1)$$

or

$$\frac{\delta}{x} = 0(\sqrt{\frac{\mu_a}{\rho_a u_e x}})$$

(33)

where $\mu_a$ and $\rho_a$ are representative average values of the viscosity and density in the boundary layer while $u_e$ is the velocity at the edge of the boundary layer. Letting

$$\frac{\mu_a}{\mu_\infty} = \frac{T_a}{T_\infty}, \ \rho = \frac{P}{ZRT}$$

(34)

where $P$ is the pressure, $Z$ the compressibility factor and $R$ is the gas constant, and assuming weak interactions, i.e.,

$$u_e \approx u_\infty, \ P_e \approx P_\infty$$
one finds that eq. (33) reduces to

\[
\frac{\delta}{x} = 0 \left( \sqrt{\frac{Z_a \frac{T_a}{R \infty}}{T_\infty}} \right)
\]  

Comparison of eqs. (32) and (35) shows that the two equations give similar results. For a given freestream total enthalpy \( H_\infty \) and pressure \( P_\infty \), all that is necessary to calculate the boundary layer thickness is a knowledge of the plate length and flight conditions. These conditions were determined from the altitude-velocity curves of fig. 3 in conjunction with the standard atmospheric data presented by COESA (ref. 10). The data of Neel and Lewis (ref. 11) for high temperature air was used in calculating the various thermodynamic properties for a given \( H_\infty \) and \( P_\infty \). As suggested by Dorrance (ref. 12) and others, \( T_a \) is usually taken as the stagnation temperature, \( T_s \).

The maximum \( K_{10} \) occurring at flight conditions corresponding to the trajectories of fig. 3 were calculated from the data of Sewell (ref. 3) which give the spectral emission coefficient of air versus wave number for a wide range of temperature and density. The maximum \( K_{10} \) were calculated using the values of \( T_s \) and \( \rho_a \) (see eq. (34)) corresponding to \( (H_\infty,P_\infty) \) for equilibrium air. Now choosing \( T_a = T_s \) underestimates \( \rho_a \) because, for a boundary layer along a flat plate,

\[
P = \text{const or } Z\rho T = \text{const or } \rho T = \text{const}
\]  

Since \( K_j = K_j(\rho,T) \), the question arises whether such choice of temperature would lead to \( (K_{10})_{\text{max}} \). This can be seen from the consideration that \( K_j \) can be approximated as (ref. 13)

\[
K_j = c_j \rho^\alpha T^\beta, \quad c_j, \alpha, \beta = \text{const}, \quad \beta > \alpha
\]

\[
= c_j (\rho T)^{\alpha \beta - \alpha}
\]  

It is seen from eq. (36) that using \( T = T_s \) in eq. (37) results in the highest value of \( K_j \) for a flat plate.

Knowing \( \delta \) and \( (K_{10})_{\text{max}} \), it is possible to estimate \( (\tau_{10})_{\text{max}} \) which occurs in the boundary layer during reentry into the earth's atmosphere along the flight paths shown in fig. 3. The results for a plate three meters long are shown in fig. 4. For a plate of length \( L \) meters, the ordinate should be multiplied by \( \sqrt{L/3} \). It is seen that \( (\tau_{10})_{\text{max}} < 1 \). Therefore, \( p = n \) in eqs. (29) and (30) and the flat plate laminar boundary layer of radiating air is optically thin.
SURFACE EFFECTS ON RADIATION

The expression for $F_{i,i}$ given in eq. (8) assumes that the external flow and the wall are perfect absorbers. A relaxation of this restriction requires replacing the last integral of eq. (8) by

$$K_v(x_i) \int_{\eta_i} \frac{1}{4\pi} I_v(\eta_i) \exp \left( - \int_{\eta_i}^{x_i} K_v \, ds \right) \frac{d\omega}{4\pi} \quad (38)$$

or, $B_v$ is replaced by $I_v$, where $I_v$ is the specific intensity of radiation. A general expression for $I_v$ can be derived from the general transfer equation. For a nonscattering medium in local thermodynamic equilibrium, the transfer equation can be written as

$$\frac{1}{K_v} \frac{dI_v}{ds} = -I_v + B_v \quad (39)$$

Integration of eq. (39) yields (ref. 6)

$$I_v(x_i, \eta_i) = \int_{\eta_i}^{x_i} B_v(\xi_i) \exp \left( - \int_{\eta_i}^{\xi_i} K_v \, ds \right) K_v(\xi_i) \, ds + I_v(\eta_i) \exp \left( - \int_{\eta_i}^{x_i} K_v \, ds \right) \quad (40)$$

or

$$I_v(x_i, \zeta_i) = \int_{\zeta_i}^{x_i} B_v(\eta_i) \exp \left( - \int_{\zeta_i}^{\eta_i} K_v \, ds \right) K_v(\eta_i) \, ds + I_v(\zeta_i) \exp \left( - \int_{\zeta_i}^{x_i} K_v \, ds \right) \quad (41)$$

when $\eta_i$ refers to the wall surface and $\zeta_i$ to the edge of the boundary layer.

A general treatment of eq. (38) is extremely difficult. However, since it has been established that the air boundary layer on a flat plate is optically thin, a treatment of eq. (38) is possible by utilizing the fact that second order terms in optical thickness can be ignored. Thus, for an optically thin gas, one may write
Hence, keeping first order terms in the optical thickness, eq. (38) reduces to

\[ K_v(x_1) \left( \int_{\eta_1}^{x_1} K_v \, ds \right) = 1 - \int_{\eta_1}^{x_1} K_v \, ds \]  \hspace{1cm} (42)

A general expression for the radiation intensity at an interface has been given by Goulard (eq. (4.16) of ref. 6). Assuming that the interface properties are independent of direction and, diffuse reflection, one finds, at an interface

\[ I_v = \varepsilon_v B_v + r_v \int \frac{I_v'(\cos \theta) \, d\omega'}{\pi} \]  \hspace{1cm} (44)

where \( \varepsilon_v \) and \( r_v \) are the emissivity and reflectivity. Thus, assuming an emitting but nonreflecting external flow and a reflecting opaque wall, one obtains

\[ I_v,e = I_v(\xi_1) = \varepsilon_v,e B_v(T_e) \]

and

\[ I_v,w = I_v(\eta_1) = \varepsilon_v,w B_v(T_w) + (1 - \varepsilon_v,w) \int \frac{I_v(\eta_1, \xi_1') \, \cos \theta' \, d\omega'}{\pi} \]  \hspace{1cm} (45)

Keeping first order terms in the optical thickness, \( I_v(\eta_1, \xi_1) \) is calculated from eqs. (41) and (45) as

\[ I_v(\eta_1, \xi_1) = \varepsilon_v,e B_v(T_e) + \int_{\xi_1}^{\eta_1} (B_v(\xi_1') - \varepsilon_v,e B_v(T_e)) K_v(\xi_1) \, ds \]  \hspace{1cm} (46)

Substituting eqs. (45) and (46) into eq. (43) and retaining first order terms, one finds, for a grey wall

\[ K_v(x_1) \left( \int_{\eta_1}^{x_1} I_v(\eta_1) \frac{d\omega}{4\pi} \right) = K_v(x_1) \left[ \frac{\varepsilon_w}{2} B_v(T_w) + \left( \frac{2 - \varepsilon_w}{2} \right) \varepsilon_v,e B_v(T_e) \right] \]  \hspace{1cm} (47)
Thus, using eqs. (8) and (47), \( F_{vi,i} \) for an optically thin gas reduces to

\[
F_{vi,i} = 4\pi K_v \left[ B_v(x_i) - \frac{\epsilon_k}{2} B_v(T_w) - \left( \frac{2 - \epsilon_k}{2} \right) \epsilon_v(T_e) B_v(T_e) \right]
\]  

(48)

Assuming that \( \epsilon_v \) has the same frequency dependence as \( K_v(x_i, \nu) \) (eq. (11)) and integrating over the entire frequency spectrum, one obtains

\[
F_{i,i}^R = 4\sigma \left[ K_p(x_i) T^4(x_i) - \frac{\epsilon_k}{2} T^4_w \sum_{j=1}^{n} K_j(x_i) \alpha_j(T_w) \right. \\
\left. - \left( \frac{2 - \epsilon_k}{2} \right) T^4_e \sum_{j=1}^{n} \epsilon_j \alpha_j(T_e) K_j(x_i) \right]
\]  

(49)

where

\[
K_p = \sum_{j=1}^{n} \alpha_j(x_i) K_j(x_i)
\]

is the Planck’s mean, and

\[
\alpha_j(T_\nu) = \int \frac{B_\nu(T_\nu) d\nu}{\Delta_j} \left( \sigma T^4_\nu / \pi \right), \ t = w, e
\]  

(50)

If the contribution of the wall and external flow to the divergence of the flux vector is ignored, then eq. (49) reduces to

\[
F_{i,i}^R = 4\sigma K_p(x_i) T^4(x_i)
\]  

(51)

Thus, grey heat transfer calculations for an optically thin gas with the absorption coefficient equal to the Planck’s mean are identical to the nongrey calculations regardless of the frequency dependence of \( K_v(x_i, \nu) \).

The grey equivalent of eq. (49) can be written as

\[
F_{i,i}^R = 4\sigma K_p T^4 \left[ \left( \frac{T}{T_e} \right)^4 - \frac{\epsilon_k}{2} \left( \frac{T_w}{T_e} \right)^4 - \epsilon_e \left( \frac{2 - \epsilon_k}{2} \right) \right]
\]  

(52)
THE RADIATIVE HEAT FLUX AT THE WALL

The radiative heat flux at the wall can be derived from the general relations given by Goulard (ref. 6) for the radiation flux at an arbitrary point in a radiating gas. These relations are the expression for the spectral radiative flux crossing a surface of unit normal vector $\mathbf{n}_i$ at the point $\xi_i$,

$$q_v(\xi_i, \mathbf{n}_i) = \int_{4\pi} I_v \cos \theta d\omega \quad (53)$$

where $\theta$ is the angle between $\mathbf{n}_i$ and the ray cone specified by $\omega$ (see fig. 1), and the definition of global radiative flux

$$q_R = \int_0^\infty q_v d\nu \quad (54)$$

The radiative flux at an arbitrary surface $x_i$ between $\mathbf{n}_i$ and $\zeta_i$ can be calculated from the radiation intensity directed upwards $I_v(x_i, \mathbf{n}_i)$ and the radiation intensity directed downward $I_v(x_i, \zeta_i)$ as

$$q_v = \int_0^{2\pi} \int_0^{\pi/2} I_v(x_i, \zeta_i) \cos \theta \sin \phi d\phi d\theta + \int_0^{2\pi} \int_0^{\pi/2} I_v(x_i, \mathbf{n}_i) \cos \theta \sin \phi d\phi d\theta \quad (55)$$

where, in writing eq. (55), $d\omega$ was replaced by

$$d\omega = \sin \theta d\phi \quad (56)$$

In particular, eq. (55) shows that the radiative heat flux at the wall where $x_i = \mathbf{n}_i$ is given by

$$q_{vw} = \int_0^{2\pi} \int_0^{\pi/2} I_v(\mathbf{n}_i, \zeta_i) \cos \theta \sin \phi d\phi d\theta + \int_0^{2\pi} \int_0^{\pi/2} I_v(\mathbf{n}_i) \cos \theta \sin \phi d\phi d\theta \quad (57)$$

Now $I_v(\mathbf{n}_i)$ is independent of $\theta$ and $\phi$ as a result of assuming diffuse reflection. If it is assumed further that $I_v(\mathbf{n}_i, \zeta_i)$ is independent of $\phi$, eq. (57) reduces to

$$q_{vw} = 2\pi \int_0^{\pi/2} I_v(\mathbf{n}_i, \zeta_i) \cos \theta \sin \phi d\phi - \pi I_v(\mathbf{n}_i) \quad (58)$$

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For an optically thin gas, $I_{v}(n_{i}, \xi_{i})$ is given by eq. (46). Substitution of eq. (46) into eqs. (45) and (58) yields, for a grey wall

$$q_{vw} = \pi \varepsilon \left\{ B_{v}(T_{e}) - B_{v}(T_{w}) \right\}$$

$$+ 2 \int_{0}^{\pi/2} \int_{\zeta_{i}}^{n_{i}} \left[ (B_{v}(\xi_{i}) - \varepsilon_{v,e} B_{v}(T_{e})) K_{v}(\xi_{i}) ds \right] \cos \theta \sin \phi d\phi$$

(59)

The spectral radiative flux at a given $x$ on a flat plate can be obtained from eq. (59) by letting

$$ds = - \cos \theta dy, \quad \zeta_{i} = \delta, \quad n_{i} = 0$$

(60)

The resulting expression can be written as

$$q_{vw} = \pi \varepsilon \left\{ B_{v}(T_{e}) - B_{v}(T_{w}) \right\}$$

$$+ 2 \int_{0}^{\pi/2} \int_{0}^{\delta} \left[ B_{v}(\theta, y) - \varepsilon_{v,e} B_{v}(T_{e}) \right] K_{v}(\theta, y) \sin \theta d \theta$$

(61)

The global radiative flux to the surface is

$$q_{Rw} = \int_{0}^{\infty} q_{vw} d\nu$$

$$= \sigma \varepsilon \left\{ \sum_{j=1}^{n} \alpha_{j}(T_{e}) \varepsilon_{j} \frac{T_{e}^{4}}{T_{w}^{4}} - \frac{T_{e}^{4}}{T_{w}^{4}} \right\}$$

$$+ 2 \int_{0}^{\pi/2} \int_{0}^{\delta} \left[ K_{v} \frac{T_{e}^{4}}{T_{w}^{4}} - \sum_{j=1}^{n} K_{j} \alpha_{j}(T_{e}) \varepsilon_{j} \frac{T_{e}^{4}}{T_{w}^{4}} \right] \sin \theta d \theta$$

(62)

where, again, the frequency dependence of $\varepsilon_{v,e}$ was assumed identical to that of $K_{v}$. 

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TRANSFORMATION OF THE GOVERNING EQUATIONS

The governing equations are equations (4) through (6) with $F_{i,j}^R$ given by eq. (49). The appropriate boundary conditions can be written as

\[ u = v = 0, \quad h = h_w \text{ at } y = 0 \]
\[ u \to u_e, \quad h \to h_e \text{ as } y \to \infty \]
\[ u = u_e, \quad h = h_e \text{ at } x = 0 \]

(63)

where $u_e$, $h_e$, and $h_w$ are assumed to be constants. The continuity equation is satisfied identically if one introduces the stream function $\psi$ defined by

\[ \rho u = \frac{\partial \psi}{\partial y}, \quad \rho v = -\frac{\partial \psi}{\partial x} \]

(64)

The momentum and energy equations will now be transformed from the x-y to an s-\(\eta\) coordinate system. Letting

\[ s = \rho_e \mu_e x, \quad \eta = \frac{\rho_e u_e}{\sqrt{C_s}} \int_0^y \rho dy, \quad \psi = \sqrt{C_s} f(s, \eta) \]

(65)

with

\[ \bar{h} = \frac{h}{h_e}, \quad \bar{\rho} = \frac{\rho}{\rho_e}, \quad C = \frac{\mu_e}{\mu_e \rho_e} = \text{const.} \]

(66)

eqs. (5) and (6) reduce to, respectively.

\[ \frac{3f}{\eta^3} + f \frac{2f}{\eta^2} = s \left( \frac{\partial f}{\partial \eta} \frac{\partial^2 f}{\partial \eta \partial s} - \frac{\partial f}{\partial s} \frac{\partial^2 f}{\partial \eta^2} \right) \]

(67)

and

\[ ^1 \text{See Appendix F.} \]

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where $\bar{x} = x/L$ and it has been assumed that the Prandtl number $Pr = \text{const}$. As is seen from eqs. (49) and (68), true similarity does not exist in the boundary layer of an optically thin gas.

In order to simplify the numerical calculations, the assumption of local similarity will be introduced. Thus, eqs. (67) and (68) reduce to, respectively,

$$ f'''' + \frac{f}{2} f''' = 0 \quad (69) $$

and

$$ \bar{h}'' + Pr \frac{f}{2} \bar{h}' = -Pr \frac{u_e^2}{h_e} f''/2 + Pr \frac{\bar{x}L}{\rho_e u_e h_e} \frac{R}{f'''} \quad (70) $$

where the prime denotes differentiation with respect to $\eta$. Equation (69) is the well-known Blasius equation. Therefore, the solution of the problem reduces to the solution of eq. (70) with the boundary conditions

$$ \bar{h} = \bar{h}_w \text{ at } \eta = 0 \text{ and } \bar{h} \rightarrow 1 \text{ as } \eta \rightarrow \infty \quad (71) $$

The convective heat transfer at the wall $q_{cw}$ is given by

$$ q_{cw} = -k \left( \frac{\partial T}{\partial y} \right)_{y=0} = -\frac{u_w}{Pr} \left( \frac{\partial h}{\partial y} \right)_{y=0} $$

$$ = -\sqrt{C_0 \frac{u_e}{e} \frac{h_e}{L} \frac{\partial \bar{h}}{\partial \eta}} \left( \frac{\partial \bar{h}}{\partial \eta} \right)_{\eta=0} \quad (72) $$

where $C$ is evaluated, as suggested by Eckert (ref. 14), at the reference enthalpy $h_*$ which, for $Pr = 0.70$, can be written as

$$ h_* = \frac{1}{2} (1 + h_w) + 0.092 \frac{u_e^2}{h_e} \quad (73) $$
The radiative heat flux at a given station \( x \) is obtained from eq. (62) as

\[
q_{R_w} = \varepsilon_w \sigma T^4 \left\{ \sum_{j=1}^{n} \alpha_j(T_e) \varepsilon_j - \frac{T^4}{T_w} \right\}
\]

\[
+ 2 \sqrt{\frac{C_s}{u_e}} \int_0^{\eta_0} \left[ \frac{K_p(T_e, P_e)}{\varepsilon} - \frac{K_i}{\varepsilon} \right] \frac{T^4}{T_w^4} - \sum_{j=1}^{n} \frac{K_i(T_e, T_e)}{\varepsilon} \alpha_j(T_e) \varepsilon_j \right] d\eta \right\}
\]

(74)

where \( \eta_0 \) is the value of \( \eta \) at which \( f'(\eta) = 0.9999 \). The corresponding grey expression can be expressed as

\[
q_{R_w, g} = \varepsilon_w \sigma T^4 \left\{ \varepsilon_e - \frac{T^4}{T_w} + 2 \sqrt{\frac{C_s}{u_e}} \int_0^{\eta_0} \left[ \frac{K_p(T_e, P_e)}{\varepsilon} - \frac{K_i}{\varepsilon} \right] \frac{T^4}{T_w^4} - \varepsilon_e \right] d\eta \right\}
\]

(75)

Method of Solution

The method of solution to be employed here is to transfer the differential eq. (70) into an integral equation and to solve the resulting integral equation by iteration. A formal integration of eq. (70) yields

\[
\bar{h} = \bar{h}_w + c_1 g_1(\eta) - Pr \frac{u_e^2}{h_e} g_2(\eta) + \lambda g_3(\eta)
\]

(76)

where

\[
g_1(\eta) = \int_0^\eta \exp \left( - \frac{Pr}{2} \int_0^{\eta_1} f d\eta_2 \right) d\eta_1
\]

\[
g_2, g_3 = \int_0^\eta \left\{ \exp \left( - \frac{Pr}{2} \int_0^{\eta_1} f d\eta_2 \right) \left[ \int_0^{\eta_1} \chi \exp \left( \frac{Pr}{2} \int_0^{\eta_2} f d\eta_3 \right) d\eta_2 \right] \right\} d\eta_1
\]

\[
\chi = f'/\bar{w}, \quad \bar{w}
\]

\[
\bar{w} = \frac{R_i^4}{\rho_e n_n} = \frac{K_p(T_e, P_e)}{\varepsilon} - \frac{K_i}{\varepsilon} \frac{T^4}{T_w^4} - \sum_{j=1}^{n} \frac{K_i(T_e, T)}{\varepsilon} \alpha_j(T_e)
\]

\[
+ \frac{(2 - \varepsilon_w)}{2} \varepsilon_j \alpha_j(T_e)
\]

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and

\[ \lambda = \frac{4Pr_0 T e^4 L x}{u e} \]

(77)

\( C_1 \) is a "constant" determined from the condition that

\[ \bar{h} = 1 \text{ when } \eta = \eta_6 \]

(78)

The resulting expression for \( C_1 \) is

\[ C_1 = \left[ 1 - \bar{h}_w + Pr \frac{u^2}{h e} g_2(\eta_6) - \lambda g_3(\eta_6) \right] / g_1(\eta_6) \]

(79)

It is seen from eqs. (76) and (77) that, in order to calculate \( g_3(\eta) \) one needs to know the temperature or enthalpy distribution in the boundary layer at a given \( x \). In the scheme employed here, the iteration has been started by assuming that the enthalpy distribution needed to calculate \( \bar{W} \) and \( g_3 \) is given by the solution of eq. (70) for the case of no radiation. This enables one to calculate \( g_3 \) and an improved value of \( \bar{h} \). This improved value of \( \bar{h} \) is employed in calculating \( g_3 \) and so on.

Designating the solution of eq. (70) in the absence of radiation by \( \bar{h}_o \), one obtains

\[ \bar{h}_o = \bar{h}_w + C_0 g_1(\eta) - Pr \frac{u^2}{h e} g_2(\eta) \]

(80)

where \( C_0 \) is determined from eqs. (78) and (80) as

\[ C_0 = \left[ 1 - \bar{h}_w + Pr \frac{u^2}{h e} g_2(\eta_6) \right] / g_1(\eta_6) \]

(81)

Examination of eqs. (79) and (81) shows that

\[ C_1 = C_0 - \lambda \frac{g_3(\eta_6)}{g_1(\eta_6)} \]

(82)
and therefore,

$$\tilde{h} = \tilde{h}_0 + \lambda g_3(\eta_0) \left[ \frac{g_3(\eta)}{g_1(\eta)} - \frac{g_1(\eta)}{g_1(\eta_0)} \right]$$

(83)

The calculations were carried out for values of $\tilde{x}$ ranging from 0.1 to 1.0. The function for $\tilde{h}_0$ was employed to start the iteration scheme for the case where $\tilde{x} = 0.1$. For other values of $\tilde{x}$ the calculated value of $\tilde{h}$ at the preceding $\tilde{x}$ station was employed to start the iteration. In the course of the calculations, it was noticed that convergence is achieved rapidly if one chooses for the input of the $n$th iteration the mean of the input and output of the previous iteration, i.e.,

$$\frac{(\tilde{h}_{in})_{n-1} + (\tilde{h}_{out})_{n-1}}{2} = \frac{(\tilde{h}_{in})_{n-1} + (\tilde{h}_{out})_{n-1}}{2}$$

(84)

The ratio of the convective heat flux with radiation $q_{cw}$ to that without radiation $q_{cw0}$ follows from eqs. (72), (76), (80), and (82) as

$$\frac{q_{cw}}{q_{cw0}} = \frac{C_1}{C_0} = 1 - \frac{\lambda}{C_0} \frac{g_3(\eta_0)}{g_1(\eta_0)}$$

(85)

RESULTS AND DISCUSSION

The influence of the nongrey radiation effects on the air boundary layer over a flat plate has been investigated. The monochromatic absorption coefficient employed assumes a stepwise frequency dependence; as a result of this assumption it is shown that the contributions of the various frequency intervals to the divergence of the radiative flux vector add up linearly. This marks a departure from the grey case in the sense that for certain conditions the various contributions to the divergence of the radiative flux vector may be a combination of the usual thin, self-absorbing and thick contributions. Because of this it is not possible to define an effective optical thickness for the whole flow field and hence there is no grey equivalent to a nongrey flow model.

It is shown that for a typical reentry mission from Mars and other similar missions, the air boundary layer over a flat plate is optically thin. Therefore the results presented here are for an optically thin boundary layer on a flat plate. The calculations were carried out for Eckert numbers, $u_0^2/h_e$, of approximately 16, 10, 1 and 0.7; external flow
temperatures of 2000°, 4000°, 10 000°, and 12 000° K; external flow velocities of 32, 24.4 and 7.7 Km/sec; wall temperatures of 1000°, 2000° and 3000° K; pressures of 1.0 and 0.1 atmospheres; and plate lengths of 3 and 10 meters. The Prandtl number was chosen as 0.7. The monochromatic absorption coefficient model employed is shown in fig. 5 with \( K_1 \) and \( K_2 \) given, respectively, by eqs. (D2) and (D5). The Planck mean absorption coefficient employed in the grey calculations is given by eq. (D4). The calculations are aimed at providing an increased knowledge of the physical effects of radiation (and the various parameters which characterize the nature of that radiation) on the behavior of the boundary layer.

The calculated results presented are for the cases of high and low Eckert number. The specific results given do not pertain to a particular reentry vehicle or trajectory but serve only as examples. However, in order to better illustrate the meaning of these results it is worthwhile to make some connection between the Eckert number and a vehicle geometry with specified freestream flight conditions. For example, the Eckert number for a wedge with a semi-apex angle of 53° traveling at 45 000 ft/sec at an altitude of 150 000 ft is about 0.7 while the Eckert number for a wedge of 28° semi-apex angle moving at a velocity of 39 000 ft/sec at an altitude of 150 000 ft is around 7.0 (ref. 28).

Figures 6(a) and 6(b) show the effects of grey radiation on the boundary layer enthalpy distribution for high and low Eckert numbers, respectively. The enthalpy profile refers to the similar enthalpy distribution for a non-radiating gas; the other profiles are for a grey radiating gas at \( \bar{x} = 1.0 \). From fig. 6(a) it is seen that for high Eckert number flows and all values of wall and external flow emissivities the energy loss by emission significantly reduces the boundary layer enthalpy below its value for a non-radiating gas. In particular the peak enthalpy is reduced a considerable amount. The results in fig. 6(a) also indicate that the enthalpy profile for a high Eckert number flow is relatively insensitive to the magnitude of the wall and external flow emissivities. This may be explained by the fact that for the high Eckert number case the enthalpy in the boundary layer exceeds that in any other region of the flow by a considerable amount and, consequently, the gas in the boundary layer will emit energy at a much greater rate than it will absorb. Thus, the amount of radiant energy incident on the boundary layer, which depends on the magnitude of the external flow and wall emissivities, is of secondary importance in determining the boundary layer enthalpy profile.

From fig. 6(b) the results indicate that the enthalpy distribution for low Eckert number flows is affected considerably different by radiation than is the enthalpy distribution for high Eckert number flows. In particular, the magnitudes of the wall and external flow emissivities influence the enthalpy distribution in such a manner that the boundary layer enthalpy may be greater or less than its value for a nonradiating gas in some or all parts of the boundary layer. For instance, the results given in fig. 6(b) indicate that for a combination of high external flow and low wall emissivity the enthalpy is somewhat greater than its value for a nonradiating gas in that part of the boundary layer nearest the wall. This sensitivity of the enthalpy profile to the external flow and wall emissivities is due to the fact that for the low Eckert number case the enthalpy in the boundary layer
is equal or less than that in any other portion of the flow and hence the gas in the boundary layer absorbs energy at approximately the same rate as it emits. Thus the amount of radiant energy incident on the boundary layer, which is governed by the magnitude of the wall and external flow emissivities, strongly influences the enthalpy distribution in the boundary layer.

From fig. 6(b) it is noted that there is no appreciable effect of the magnitude of the wall emissivity on the boundary layer enthalpy distribution for a low external flow emissivity but a quite significant effect for a high external flow emissivity. These results point out the fact that the photons emitted by the relatively cold wall have a negligible influence on the boundary layer enthalpy profile and emphasize that the major effect of the wall emissivity on the enthalpy profiles is due to reflection from the surface of incident photons which originated in the radiating external flow.

Plots of \( \frac{\Delta h}{h} \) and \( \frac{\Delta h}{h} \) vs. \( \eta \) for high Eckert number flow are shown in figs. 7(a) and 7(b) for various wall and external flow emissivities. It is seen that the nongrey effects are more pronounced at the high external flow emissivity and/or low wall emissivity. The results also indicate that the nongrey profiles are less sensitive than the corresponding grey profiles to the various values of external flow and wall emissivities. In general the results of fig. 7 show that the enthalpy profiles for a nongrey gas are not significantly different from those for a grey gas. This is explained by the combination of facts that for high Eckert number flows the radiation process is emission dominated and the Planck mean absorption coefficient used for the grey gas is the proper mean absorption coefficient for the nongrey emission process.

Plots of grey and nongrey enthalpy profiles for the low Eckert number case are presented in figs. 8(a) and 8(b). It is seen that for all combinations of wall and external flow emissivities the enthalpy for a nongrey gas significantly exceeds its value for a grey gas. The curves in figs. 8(a) and 8(b) also indicate that for low Eckert number flows the nongrey enthalpy profiles are more sensitive to the wall and external flow emissivities than the grey profiles. Both results are explained by noting that absorption is always more significant for a nongrey gas than for a grey gas. In fact, it is observed from fig. 8(a) that at a high external flow emissivity the effect of absorption in a nongrey gas is so significant that the geometrical boundary layer thickness is markedly reduced below its value for a non-radiating gas.

Figures 9(a) and 9(b) show a comparison of the grey convective heat flux and the convective heat flux in the absence of radiation for high and low Eckert numbers, respectively. The results presented indicate that the ratio of these two fluxes increases with an increase in the external flow emissivity and/or decrease in the wall emissivity. From fig. 9(a) it is seen that for a high Eckert number flow the convective heat flux is reduced appreciably below its value for a nonradiating gas and also there is little influence of the wall or external flow emissivities on the heat flux. Both results, of course, are explained by the previous discussion pertaining to the enthalpy profiles for high Eckert number flows. For low Eckert number flows the results in fig. 9(b) indicate that the effect of radiation on the
grey convective heat flux is quite significant and is strongly influenced
by the magnitude of the wall and external flow emissivities. In particular,
for the combination of high external flow and low wall emissivities, the
grey convective heat flux exceeds the convective heat flux for a non-
radiating gas. This result is explained by the fact that near the wall the
absorption of radiant energy incident on the boundary layer exceeds the
energy loss by emission and hence the enthalpy gradient near the wall in-
creases over its nonradiating value.

The ratio of the nongrey convective heat flux to the corresponding
grey heat flux is shown in figs. 10(a) and 10(b) for high and low Eckert
numbers, respectively. This ratio is generally greater than unity except
for a combination of high Eckert number and high external flow emissivity.
The high Eckert number results in fig. 10(a) indicate that the convective
heat fluxes for a nongrey gas are not significantly different from those
for a grey gas. The previous discussion on nongrey enthalpy profiles for
high Eckert numbers explains these results. For low Eckert number flows
the results in fig. 10(b) show that the effect of nongrey absorption on the
convective heat flux is quite significant. In particular, the nongrey con-
vective heat flux is appreciably greater (up to 27%) than the grey convective
heat flux. Also, the influence of the wall emissivity on the results is so
strong that the ratio of the nongrey to grey convective heat flux is greater
for a combination of low wall and low external flow emissivity than for a
combination of high wall and high external flow emissivity.

In figs. 11(a) and 11(b) the nongrey radiative heat flux is compared to
the corresponding grey radiative heat flux for high and low Eckert numbers,
respectively. The ratio of these two fluxes is generally less than unity
except for the combination of a high Eckert number and high external flow
emissivity. An analysis of the numerical results from which fig. 11 was
derived indicates that for high Eckert number flows the effect of the op-
tically thin boundary layer on the radiative wall heat flux is to reinforce
the radiant heat flux directed from the inviscid flow to the wall. This
result is explained by noting that the boundary layer in a high Eckert number
flow emits more energy than it absorbs and hence increases the total radiant
heat transfer. However, this increase in the radiative wall heat flux is
negligible except for low values of external flow emissivity where the
boundary-layer contribution to the radiative wall heat flux is a maximum of
17% of the external flow contribution. For these low values of external
flow emissivity, the nongrey absorption effect on the boundary-layer con-
tribution is found to be small (less than 6%). At low Eckert number flow
conditions, the numerical results indicate that the effect of the optically
thin boundary layer on the radiative heat flux directed from the external flow
to the wall is to inhibit it for a high external flow emissivity and reinforce
it for a low external flow emissivity. This reduction (increase) of the total
radiant heat transfer is explained by noting that the low Eckert number
boundary layer will absorb (emit) more energy than it emits (absorbs) for a
high (low) external flow emissivity. However, from the numerical results for
low Eckert number, it is found that the reduction in radiant wall heat flux
at high external flow emissivities is negligible while the increase in
radiant wall heat flux at low external flow emissivities is small (less than
2.5%).

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The radiative heat flux and the convective heat flux are compared in figs. 12(a) and 12(b). It is seen that, for the high external flow temperatures considered here, 10 000° and 12 000° K, the radiative heat flux is larger than the convective heat flux with the ratio of the two fluxes increasing with an increase in the external flow and/or wall emissivities and a decrease in the Eckert number. The results also show that the ratio of the radiative to convective heat flux increases with an increase in the external flow temperature and plate length and a decrease in the external flow pressure and wall temperature.

The ratio of the total (convective + radiative) heat flux to the heat flux in the absence of radiation is shown in figs. 13(a) and 13(b) for high and low Eckert numbers, respectively. This ratio, which is always greater than unity, increases with an increase in wall and/or external flow emissivities with the influence of the external flow emissivity being more pronounced. It is also seen that a significant reduction in the total heat flux can be achieved by a reduction in both wall and external flow emissivities.

Figures 14(a) and 14(b) compare the nongrey total heat flux to the grey total heat flux. From the high Eckert number results in fig. 14(a), it is seen that the nongrey heat flux is slightly larger than the grey heat flux for a combination of low external flow and low wall or high external flow and high wall emissivities. For low Eckert number flow conditions, the results in fig. 14(b) indicate that the nongrey heat flux exceeds the grey heat flux for the lower wall emissivities. However, in either the high or low Eckert number case, the results show that the difference between the grey and nongrey total heat flux is small regardless of the values of the wall and external flow emissivities.

It is noticed from the results that the differences between the various grey and the nongrey heat fluxes are small except for the convective heat flux at low Eckert number where the dominant heat flux is radiative. It follows from eqs. (65), (74), and (75) that for given external flow emissivity and wall temperature the dimensionless radiative heat flux depends explicitly on the effective Bouguer numbers (for an optically thin gas) \( \tau'_j \) where the \( \tau'_j \) are related to the grey Bouguer number \( \tau_{pe} (= K_{pe} \delta) \) by the expression

\[
\tau_{pe} = \sum_{j=1}^{N} \tau'_j \]

The difference between the nongrey and grey radiative heat flux is proportional to the \( \tau'_j \) and they are small (of the order of 10^{-2}) for the cases under consideration.
CONCLUSIONS

The analysis carried out here shows that in the presence of nongrey radiation it is not, in general, possible to define an effective optical thickness for the whole flow field. This, in turn, suggests that there is no grey equivalent to a nongrey flow field.

Optical thickness estimates for a typical reentry mission from Mars suggest that the boundary layer over a flat plate is optically thin. The results obtained in the analysis of the optically thin boundary layer indicate that the effect of radiative transfer on the boundary-layer characteristics depends primarily on the Bouguer-Boltzmann number ratio as appropriately defined for boundary-layer flow geometry. From the numerical results for air, it is found that the effect of radiative transfer on the boundary-layer enthalpy profile and convective heat flux becomes quite significant when the Bouguer-Boltzmann number ratio is of the order of magnitude of 0.1. Other explicit parameters found to appreciably influence the effect of radiation on the enthalpy profile and dimensionless convective heat flux are the Eckert number, the wall emissivity, and the emissivity of the external flow. The numerical results also indicate that the optically thin boundary layer has little influence on the radiative wall heat flux except for low values of external flow emissivity at high Eckert number where the boundary-layer contribution to the radiative wall heat flux is roughly 15% of the external flow contribution. Thus, by far the major portion of the radiative wall heat flux comes from the external flow region of the shock layer.

The nongrey heat flux results obtained in the optically thin boundary-layer calculations show that the total nongrey heat flux for Eckert numbers of interest is greater than the corresponding grey heat flux for a combination of high external flow and high wall or low external flow and low wall emissivities. These results also indicate that the differences between the various grey and nongrey heat fluxes are small except for the convective heat flux at low Eckert number where the nongrey heat flux is up to 27% greater than the grey heat flux. The numerical results also show that the total wall heat flux may be drastically reduced by lowering both wall and external flow emissivities. This suggests a scheme for decreasing the total heat flux to the wall. It should be noted, however, that the emissivity of the external flow depends on the optical properties of the gases in the shock layer.

Since the pressure in the boundary layer over a flat plate is constant, the dependence of the absorption coefficient on temperature and density should be consistent with the requirement that $\rho T \sim \text{const}$. Thus, it is not possible to simultaneously have both high temperature and high density in the flat plate boundary layer. This is why the air boundary layer over a flat plate is optically thin. In the stagnation point region no such restriction exists. Therefore, it is expected that the various contributions to the divergence of the radiation flux vector in the stagnation point region are a combination of the usual thin, self-absorbing and thick contributions.
APPENDIX A

THE DIVERGENCE OF THE RADIATION ENERGY FLUX VECTOR FOR NONGREY GASES

The general expression for \( F_{v_1,1}(x_1) \), the divergence of the spectral radiation energy flux vector \( F_{v_1} \) at the point \( x_1 \), has been formulated previously by Goulard (ref. 6). For nonscattering gases in local thermodynamic equilibrium and perfectly absorbing surfaces, this relation is

\[
F_{v_1,1}(x_1) = 4\pi K_v(x_1)B_v(x_1) - K_v(x_1)\int \frac{dV(\xi_1)}{\Sigma [x_1 - \xi_1]^2} - K_v(x_1)\int_A B_v(\eta_1)\exp\left[-\int_{\eta_1}^{x_1} K_v ds\right] \cos \theta dA(\eta_1) \tag{A1}
\]

where \( K_v(x_1) \) is the spectral linear absorption coefficient which is related to the spectral mass absorption coefficient \( \kappa_v(x_1) \) by the relation

\[
K_v(x_1) = \rho(x_1)\kappa_v(x_1)
\]

The physical meaning of the three terms on the right hand side of eq. (A1) are illustrated most easily by use of fig. 1 which depicts a volume \( V \) of radiating gas enclosed in the boundary surface \( A \). The first term represents the radiation energy emitted by the gas at point \( x_1 \) where \( B_v(x_1) \) is the well-known Planck distribution function

\[
B_v(x_1) = \frac{2h^3}{c^2} \frac{1}{\exp\left(\frac{h\nu}{kT(x_1)}\right) - 1} \tag{A2}
\]

The second term represents that energy emitted by the gas in all the elementary volumes \( dV(\xi_1) \) around the points \( \xi_1 \) in the entire volume \( V \) which is absorbed by the gas at point \( x_1 \). In this term the point \( \xi_1 \) is a "running" point moving along the directed line \( S(\xi_1) \) from the point \( \eta_1 \) on the elemental surface area \( dA(\eta_1) \) to the point \( x_1 \). The angle between the unit normal vector \( n_1 \) of \( dA(\eta_1) \) and the directed line \( S(\xi_1) \) is \( \theta \). It should be noted that the integral

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which is called the optical length from $\xi_1$ to $x_1$, is evaluated by integrating along the directed line $S(\xi_1)$ whose direction cosines are $\xi_1$. Hence the optical length from $\xi_1$ to $x_1$ is a function of direction $S(\xi_1)$. Finally the third term in eq. (Al) represents that energy emitted by all the elemental surfaces $dA(\eta_i)$ around the points $\eta_i$ on the boundary $A$ which is absorbed by the gas at point $x_1$. Here the optical length from $\eta_i$ to $x_1$,

\[
\int_{\eta_i}^{x_1} K_\nu ds
\]

is evaluated by integrating along $S(\xi_1)$ and hence it is a function of the direction $S(\xi_1)$.

Now eq. (Al) can be simplified by introducing the relations

\[
dV(\xi_1) = \sum_{i} [x_1 - \xi_1_i]^2 d\omega ds
\]

\[
dA(\eta_i) = \frac{[x_1 - \eta_1_i]^2 d\omega}{\cos \theta}
\]

derived from the geometry of fig. 1, and applying the appropriate integration limits. Thus $F_{\nu i, i}(x_1)$ becomes

\[
F_{\nu i, i}(x_1) = 4\pi \left\{ K_\nu(x_1)B_\nu(x_1) - K_\nu(x_1) \int_{4\pi}^{x_1} K_\nu(\xi_1)B_\nu(\xi_1) \exp \left[ - \int_{\xi_1}^{x_1} K_\nu ds \right] \frac{d\omega}{4\pi} \right\} ds
\]

\[
- K_\nu(x_1) \int_{4\pi}^{x_1} B_\nu(\eta_1) \exp \left[ - \int_{\eta_1}^{x_1} K_\nu ds \right] \frac{d\omega}{4\pi}
\]

(A3)

where \( \int_{4\pi}^{x_1} \) denotes integration over a solid angle of \( 4\pi \) steradians and the \( \int_{\eta_1}^{x_1} \) is carried out along the directed line $S(\xi_1)$.

Now it is necessary to integrate $F_{\nu i, i}$ over frequency $\nu$ to obtain the divergence of the radiation flux vector $F_{i, i}^R$ which appears in the general energy equation of radiation gas dynamics. Thus
can be evaluated by integrating eq. (A3) over \( \nu \). However, before this integration can be carried out it is necessary that the frequency dependence of \( K_\nu \) be specified. As has been indicated in the analysis, this has been chosen as

\[
K_\nu (x, \nu) = \sum_{m=1}^{n} K_m(x_1) g_m(\nu)
\]  

(A5)

with

\[
g_m(\nu) = \begin{cases} 
1 & \text{if } \nu \in \Delta_m \\
0 & \text{otherwise}
\end{cases}
\]

where \( \Delta_m \) corresponds to the frequency interval or intervals in which the value of \( K_\nu \) is \( K_m(x_1) \). The \( \Delta_m \) do not overlap and the \( K_m(x_1) \) are not functions of frequency. Hence the frequency and position dependence of \( K_\nu \) is assumed to be separable. An example of a nongrey absorption coefficient model which might be specified by eq. (A5) is shown in fig. 2.

If the absorption coefficient model of eq. (A5) is now introduced into the \( F_{\nu i, i} \) of eq. (A3) the resulting first term on the right hand side of eq. (A4) becomes

\[
4\pi \int_0^\infty K_\nu (x_1) B_\nu (x_1) d\nu = 4\pi \sum_{j=1}^{n} \int_{\Delta_j} \sum_{m=1}^{n} K_m(x_1) g_m(\nu) B_\nu (x_1) d\nu
\]

\[
= 4\pi \sum_{j=1}^{n} K_j(x_1) \int_{\Delta_j} B_\nu (x_1) d\nu
\]

since by definition of \( g_j \) the sum

\[
\sum_{m=1}^{n} K_m(x_1) g_m(\nu)
\]

reduces to \( K_j(x_1) \) as the integration on \( \Delta_j \) is carried out. If the definition
\[
\int_{\Delta_j} B_v(x_i) dv = \alpha_j[T] \int_0^{\infty} B_v(x_i) dv = \alpha_j[T(x_i)] \frac{\sigma T^4(x_i)}{\pi} = \alpha_j(x_i) \frac{\sigma T^4(x_i)}{\pi}
\]

is now employed the first term takes the final form

\[
4\pi \int_0^{\infty} K_v(x_i) B_v(x_i) dv = 4\sigma T^4(x_i) \sum_{j=1}^{n} K_j(x_i) \alpha_j(x_i)
\]  

(A6)

It should be noted at this point that \( \int_{\Delta_j} B_v dv \) may actually represent a sum of integrals as, for example, in the case of \( \Delta_1 \) in Fig. 2 where

\[
\int_{\Delta_1} B_v dv = \int_{v_1}^{v_2} B_v dv + \int_{v_5}^{v_6} B_v dv + \int_{v_9}^{v_{10}} B_v dv
\]

since \( \Delta_1 \) contains the frequency intervals \((v_1,v_2), (v_5,v_6)\) and \((v_9,v_{10})\).

It is also obvious that from the definition of \( \alpha_j[T(x_i)] \) that

\[
\sum_{j=1}^{n} \alpha_j(x_i) = 1
\]

(A7)

Employing similar techniques as above the second term on the right-hand side of eq. (A4) becomes

\[
-4\pi \int_0^{\infty} K_v(x_i) \int_{4\pi \eta_1}^{x_1} K_v(\xi_1) B_v(\xi_1) \exp \left[ - \int_{\xi_1}^{x_1} K_v ds \right] ds \frac{d\omega}{4\pi} dv
\]

\[
= -4\pi \sum_{j=1}^{n} \int \int \int \int 4\pi \eta_1 \int_{4\pi \eta_1}^{x_1} \int_{m=1}^{n} \sum_{j=1}^{n} \sum_{m=1}^{n} K_m(\xi_1) \frac{d\omega}{4\pi} dv
\]

\[
\times \exp \left[ - \int_{\xi_1}^{x_1} \sum_{m=1}^{n} K_m ds \right] ds \frac{d\omega}{4\pi} dv
\]

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Finally the third term of eq. (A4) becomes

\[-4\pi \int_\infty^0 K_v(x_i) \int_{4\pi} B_v(\eta_1) \exp \left[ -\int_{\eta_1}^{x_i} K_v dv \right] \frac{d\omega}{4\pi} dv\]

\[-4\pi \sum_{j=1}^{n} \int_{\Delta_j}^{x_i} K_j(\xi_i) \alpha_j(\xi_i) T^4(\xi_i) \exp \left[ -\int_{\xi_i}^{x_i} K_j dv \right] \frac{d\omega}{4\pi}\]

Thus when the absorption coefficient model of eq. (A5) is substituted into the \( F_{vi,i} \) of eq. (A3) and the indicated integration over frequency is carried out, \( F_{i,i}^R \) of eq. (A4) takes the final form

\[F_{i,i}^R(x_i) = 4\sigma \left\{ \sum_{j=1}^{n} K_j(x_i) \alpha_j(x_i) T^4(x_i) \right\}
\]

\[-\sum_{j=1}^{n} K_j(x_i) \int_{4\pi}^{\Delta_j} K_j(\xi_i) \alpha_j(\xi_i) T^4(\xi_i) \exp \left[ -\int_{\xi_i}^{x_i} K_j dv \right] \frac{d\omega}{4\pi}\]

\[-\sum_{j=1}^{n} K_j(x_i) \alpha_j(\eta_1) T^4(\eta_1) \exp \left[ -\int_{\eta_1}^{x_i} K_j dv \right] \frac{d\omega}{4\pi} \]  

(A8)

When the function \( \alpha_j \) applies to a single frequency interval \((v_j, v_{j+1})\) it may be written as
Introducing the wavelength \( \lambda \) by the relation

\[ \nu = \frac{c}{\lambda} \]

eq. (A9) reduces to

\[ \alpha_j = \frac{\int_1^{\nu_{j+1}} B_\nu d\nu}{\int_0^{\infty} B_\nu d\nu} \tag{A9} \]

where

\[ B_\lambda = \frac{2hc^2}{\lambda^5} \frac{1}{\exp(hc/\lambda kT) - 1} \tag{A11} \]

Letting

\[ \beta = \frac{hc}{k\lambda T} \]

and replacing the definite integral in the denominator of eq. (A10) by its value of \( \pi^4/15 \), eq. (A10) reduces to

\[ \alpha_j = \frac{15}{4} \int_{\beta_j}^{\beta_{j+1}} \frac{\beta^3}{\exp(\beta) - 1} d\beta \tag{A12} \]
or, by defining $a_j(\beta_j)$ as

$$a_j(\beta_j) = \frac{15}{4} \int_{\beta_j}^{\infty} \frac{\beta^3}{\exp(\beta) - 1} \, d\beta$$  \hspace{1cm} (A13)

eq. (A12) reduces to

$$a_j = a_j(\beta_j) - a_j(\beta_{j+1})$$  \hspace{1cm} (A14)

A plot of $a_j(\beta_j)$ vs $\lambda_jT$ is shown in fig. 15 using the tabulated data given in Tribus' (ref. 15).
APPENDIX B

ASYMPTOTIC FORM OF $\mathbb{R}_{i,j}$ FOR LARGE $\tau_{jo}$

Factoring $\tau_{jo}$ outside of the jth bracket of eq. (23), the remaining terms can be expressed as

$$\left\{ \tilde{a}_j(x_1)\bar{K}_j(x_1)\bar{T}^4(x_1) - \tau_{jo}\bar{K}_j(x_1) \int_0^1 \tilde{a}_j(\xi_1)\bar{T}^4(\xi_1) \exp \left[ -\tau_{jo} \left( \int_0^{\xi_1} \bar{K}_j ds - \int_0^{\eta_1} \bar{K}_j ds \right) \right] d\xi_1 \right\}$$

Now if the following change of variables

$$t_j = \int_0^{\bar{x}_1} \bar{K}_j ds, t_j = \int_0^{\eta_1} \bar{K}_j ds, t_j' = \int_0^{\xi_1} \bar{K}_j ds$$

are introduced into the last two terms of the jth bracket above the resulting equation is

$$\left\{ \tilde{a}_j(x_1)\bar{K}_j(x_1)\bar{T}^4(x_1) - \tau_{jo}\bar{K}_j(x_1) \int_0^1 \tilde{a}_j(t_j)\bar{T}^4(t_j) \exp \left[ -\tau_{jo}(t_j - t_j') dt_j d\tilde{\omega} \right] \right\}$$

From this relation it is seen that the exponential term,

$$\exp[-\tau_{jo}(t_j - t_j')]$$

in the integrand of the second term is the governing factor for the case of $\tau_{jo} > 1$. For the points $\xi_1$ removed from the neighborhood of point
\( \bar{x}_i, (t_j - t'_j) \) has a finite value. Hence if \( \tau_{j0} \) increases indefinitely this exponential term approaches zero faster than \( \tau_{j0} \) approaches infinity, so that the second term tends to zero. The same argument applies to the third term except that it approaches zero much faster than the second term as \( \tau_{j0} \) is increased since it is not multiplied by \( \tau_{j0} \). Thus the third term in the jth bracket can be omitted. Hence, for \( \tau_{j0} > 1 \) the only radiation energy contributions to the point \( \bar{x}_i \) come from points \( \bar{x}_i \) that are very close to \( \bar{x}_i \). Thus the lower integration limit \( t_{j1} \) in the second term may be replaced by \(-\infty\) since the points \( \bar{n}_i \) behave like points at \(-\infty\) with respect to \( \bar{x}_i \) in terms of their radiation energy contributions to \( \bar{x}_i \). (This argument breaks down if \( \bar{x}_i \) is very close to \( n_i \), i.e., in the immediate vicinity of the wall.)

Now since the only radiation energy contributions to the point \( \bar{x}_i \) come from the points \( \bar{x}_i \) that are very close to it the variation of \( \alpha_j \bar{T}^4 \) between these points \( \bar{x}_i \) and point \( \bar{x}_i \) cannot be too large and hence \( \alpha_j(t_j)T^4(t_j) \) in the integrand of the second term of eq. (23) can be replaced by the Taylor series expansion around its value at \( \bar{x}_i \) \( \alpha_j(\bar{x}_i)T^4(\bar{x}_i) \). Thus, \( \alpha_j(t_j)T^4(t_j) \) in the integrand of the second term of eq. (B1) can be replaced by its Taylor series expansion around \( t_j \)

\[
\alpha_j(t_j')T^4(t_j') = \alpha_j(t_j)T^4(t_j) + (t_j' - t_j) \frac{d}{dt_j} \left[ \alpha_j(t_j)T^4(t_j) \right] + \frac{(t_j' - t_j)}{2!} \frac{d^2}{dt_j^2} \left[ \alpha_j(t_j)T^4(t_j) \right] + \cdots
\]

Then the integration over \( t_j' \) in the second term of eq. (B1) becomes

\[
\int_{-\infty}^{t_j} \alpha_j(t_j')T^4(t_j') \exp[-\tau_{j0}(t_j - t_j')] dt_j' = \alpha_j(t_j)T^4(t_j) \int_{-\infty}^{t_j} \exp[\tau_{j0}(t_j' - t_j)] dt_j' + \frac{d}{dt_j} \left[ \alpha_j(t_j)T^4(t_j) \right] \int_{-\infty}^{t_j} (t_j' - t_j) \exp[\tau_{j0}(t_j' - t_j)] dt_j' + \frac{1}{2!} \frac{d^2}{dt_j^2} \left[ \alpha_j(t_j)T^4(t_j) \right] \int_{-\infty}^{t_j} (t_j' - t_j)^2 \exp[\tau_{j0}(t_j' - t_j)] dt_j' + \cdots
\] (B2)

Introducing the change of variable...
\[ Y = t_j' - t_j \]

into this expression, and using the relations

\[ \frac{1}{b!} \int_{-\infty}^{\infty} Y^b \exp(\tau_{jo} Y) dY = (-1)^b / (\tau_{jo})^{b+1} , b = 0, 1, 2, \ldots \]

and

\[ \frac{\partial}{\partial t_j} = \frac{1}{\hat{k}_{j}} \sum_{i=1}^{3} \frac{\partial}{\partial \bar{x}_i} \]

the resulting eq. (B2) reduces to

\[ \int_{-\infty}^{t_j} a_j(t_j') \bar{T}^4(t_j') \exp[- \tau_{jo}(t_j - t_j')] dt_j' \]

\[ = \bar{a}_j(\bar{x}_i) \bar{T}^4(\bar{x}_i) + \frac{1}{\tau_{jo} \hat{k}_j} \sum_{m=1}^{3} \frac{\partial}{\partial \bar{x}_m} \left[ \frac{1}{\hat{k}_j} \sum_{p=1}^{3} \frac{\partial}{\partial \bar{x}_p} \left[ \bar{a}_j(\bar{x}_i) \bar{T}^4(\bar{x}_i) \right] \right] \ldots \ldots \tau_{jo} > 1 \] (B3)

is obtained where the \( \bar{x}_i \) are the direction cosines of the directed line \( S(\bar{x}_i) \). If eq. (B3) is now substituted into the second term of eq. (B1) and the integration over the solid angle carried out the resulting expression can be written as

\[ \tau_{jo} \hat{k}_j(\bar{x}_i) \int_{-\infty}^{t_j} a_j(t_j') \bar{T}^4(t_j') \exp[- \tau_{jo}(t_j - t_j')] dt_j' d\bar{w} \]

\[ = \bar{k}_j(\bar{x}_i) \bar{a}_j(\bar{x}_i) \bar{T}^4(\bar{x}_i) + \frac{1}{3\tau_{jo}} \sum_{m=1}^{3} \frac{\partial}{\partial \bar{x}_m} \left[ \frac{1}{\hat{k}_j} \sum_{p=1}^{3} \frac{\partial}{\partial \bar{x}_p} \left[ \bar{a}_j(\bar{x}_i) \bar{T}^4(\bar{x}_i) \right] \right] + \text{terms } 0(1/\tau_{jo}^4) + \ldots \ldots \] (B4)
In obtaining this expression the properties

\[ \int_{0}^{1} \ell_{i} \, d\omega = 0, \quad \int_{0}^{1} \ell_{i} \ell_{j} \, d\omega = \delta_{ij}/3, \quad \sum_{j=1}^{3} \ell_{i} \ell_{j} \ell_{i} = 0, \quad i, j = 1, 2, 3 \]

were employed where \( \delta_{ij} \) is Kronecker's delta.

Now if terms of \( O(1/\tau_{jo}) \) and lower are neglected in eq. (B4) and eq. (B4) is then substituted into eq. (B1) the resulting expression inside the jth bracket is

\[ - \frac{1}{3\tau_{jo}^2} \sum_{j=1}^{3} \frac{\partial}{\partial x_{j}} \left( \frac{1}{\kappa_{j}} \frac{\partial}{\partial x_{j}} \left[ \alpha_{j} \tau_{jo}^{4} \right] \right) \]  

(B5)

Thus the form of \( \Pi_{i, j}^{R} \) for a \( \Delta_{j} \) where \( \tau_{jo} > 1 \) is given by the relation

\[ - \frac{1}{3} \alpha_{jo} \sum_{i=1}^{3} \frac{\partial}{\partial x_{i}} \left( \frac{1}{\kappa_{i}} \frac{\partial}{\partial x_{i}} \left[ \alpha_{j} \tau_{jo}^{4} \right] \right) \]  

(B6)

which is a diffusion-type expression.
APPENDIX C

REVIEW OF THE AVAILABLE ABSORPTION PROPERTY DATA
FOR HIGH-TEMPERATURE AIR

In the course of the research reported here, a survey of the data available for the absorption properties of high-temperature air was conducted and a brief evaluation of this data was carried out with regard to its extensiveness over wide ranges of frequency, temperature, and density and hence its usefulness in radiation gas dynamics.

Kivel and Bailey (ref. 16) were probably the first to present extensive data on the radiative properties of high-temperature air. They presented graphical and tabular data on the emissivity per cm, $E/L$, of optically thin air in a temperature range from 1000° to 18 000° K and a density range from $10^1$ to $10^{-6}$ of normal sea level density. However, very little data was presented on the frequency dependence of the radiative properties. It should be noted that the data given for $E/L$ in this reference can be converted to a Planck mean linear absorption coefficient by the relation

$$K_p = \frac{1}{2} \frac{E}{L} \quad (C1)$$

Armstrong et. al., (ref. 17) presented graphical data for the linear spectral absorption coefficient of air at 12 000° K and sea level density over a wavelength range of 1167 to 19 837 Å. Other graphical data on the frequency dependence of the absorption coefficient was given at a pressure of 1 atm for kT ranging from 2 to 20 ev and $h\nu$ from 1 to 1000 ev. Armstrong et. al., also plotted the absorption coefficient at a wavelength of 3967 Å over the temperature range 3000° to 13 000° K for sea level density and $10^{-6}$ of sea level density. Tabular data was also presented for the Planck mean linear absorption coefficient over a range of temperature from 1000° to 18 000° K and a range of densities from $10^1$ to $10^{-6}$ of sea level density. This data has been displayed graphically by Davis (ref. 18). Graphical data for the Planck mean linear absorption coefficient was also given by Armstrong et.al., for densities ranging from $10^1$ to $10^{-6}$ of sea level density and temperatures ranging from approximately 6000° to 250 000° K. The preceding reference is a good survey article on the absorption coefficients of high temperature air. However, little data is given on the frequency dependence of the absorption coefficient for the temperature and pressure ranges of interest in reentry calculations.

Breene and Nardone (ref. 19) have presented curves of the total spectral emissivity for a 1 cm thick isothermal layer of air. The wave number range covered is from 1000 to 62 000 cm$^{-1}$ while the density ranges from $10^{-3}$ to $10^2$ of normal sea level density for temperatures of 3000° to 9000° K.
The emissivity data of this reference can be converted to linear absorption coefficient data by use of the relation

\[ \varepsilon_\nu = 1 - \exp(-\sigma K \nu x) \]  

(C2)

where \( x = 1 \text{ cm} \) and \( \sigma \) is a geometrical factor taken as 1.8.

Breene and Nardone also presented data on the total radiance of air for the same temperature and density ranges considered in the case of the spectral emissivity. The quantity plotted is apparently derived from the relation

\[ J_T = [1 - \exp(-\sigma K x)] \frac{\sigma T^4}{\pi} \]  

(C3)

with \( x \) taken as unity. For an optically thin layer, where \( [1 - \exp(-\sigma K x)] < 0.1 \), eq. (C3) reduces to

\[ J_T = \sigma K x \frac{\sigma T^4}{\pi} \]  

(C4)

with \( x \) equal unity. The major drawbacks of the radiative property data given by Breene and Nardone are that temperatures higher than 9000° K are not considered and also the data is presented in a form which is difficult to use in general radiation gas dynamics where the absorption coefficient is the radiative property of primary interest.

Thomas (ref. 20) presented graphical data on the total emissivity per cm, \( c/L \), for high-temperature air. The range of temperatures covered was 6000° to 23 000° K and the range of density levels was \( 10^{-6} \) to \( 10^1 \) of sea level density. This emissivity data can be converted to Planck mean linear absorption coefficients by eq. (C1). The data in this reference would be quite useful in obtaining the Planck mean absorption coefficient. Its main limitation is that no data was given on the frequency dependence of the absorption coefficient.

Sewell (ref. 3) has presented graphical data on the spectral emission coefficient of air over a wave number range of 6000 to 202 000 cm\(^{-1}\), a density range of \( 10^1 \) to \( 10^{-6} \) of sea level density and a temperature range of 4000° to 20 000° K. The linear spectral absorption coefficient \( K_\lambda \) can be calculated from the emission coefficient \( j_\lambda \) by use of the simple relation

\[ K_\lambda = \frac{j_\lambda}{B_\lambda} \]  

(C5)
where

\[ B_\lambda = \frac{2hc^2}{\lambda^5} \frac{1}{\exp(hc/k\lambda T) - 1} \]  \hspace{1cm} (C6)

Sewell has also presented graphical data on the mean linear absorption coefficient \( K \) for the same temperature and density ranges as those mentioned earlier in the case of the spectral emission coefficient. It should be noted that the Planck mean absorption coefficient \( K_P \) may be related to mean absorption coefficient of Sewell by the expression

\[ K_P = \pi K \]  \hspace{1cm} (C7)

The data on the absorption coefficient of air which is given in this reference is much more extensive than that of any other reference reviewed during this survey. Also the data is presented in a form from which the spectral and Planck mean absorption coefficients can be obtained with ease. Hence Sewell's absorption coefficient data for air should be quite usable and useful in radiation gas dynamics problems.

Nardone et al. (ref. 21) have presented graphically the spectral radiance \( J_\lambda \) for a 1 cm thick slab of equilibrium air over a wavelength range of 600 to 100 000 Å at a density of \( 10^{-1} \) of sea level density for temperatures of 3000°, 10 000°, and 25 000° K. The quantity plotted was

\[ J_\lambda = \varepsilon_\lambda B_\lambda = [1 - \exp(-K_\lambda L)]B_\lambda \]  \hspace{1cm} (C8)

where \( L = 1 \) cm. For an optically thin layer, where \([1 - \exp(-K_\lambda L)] < 0.1\), eq. (C8) becomes

\[ J_\lambda = K_\lambda LB_\lambda \]  \hspace{1cm} (C9)

with \( L = 1 \) cm.

Nardone et al., also presented graphical data for the total radiance of a 1 cm slab of air over a density range of \( 10^{-4} \) to \( 10^2 \) of sea level density and a temperature range from 3000° to 25 000° K. In this case the quantity plotted was

\[ J_T = [1 - \exp(-KL)] \frac{\sigma T^4}{\pi} \]  \hspace{1cm} (C10)
where $L = 1$ cm. For an optically thin layer, where $[1 - \exp(-KL)] < 0.1$, eq. (C10) reduces to

$$J_T = KL \frac{\sigma T^4}{\pi}$$

with $L = 1$ cm.

The spectral radiative property data given in this reference extends over a wide wavelength range but is restricted to a single density level and three different temperatures. It is also presented in a form which is difficult to use in general radiation gas dynamics where the absorption coefficient $K_\lambda$ is of primary interest. The frequency averaged radiative property data given by Nardone et al., is quite extensive with regard to the density and temperature ranges covered. However it is also presented in a form which is difficult to use when the Planck mean absorption coefficient is of primary interest.

Archer (ref. 22) has presented tabular data for the spectral distribution of energy radiated by equilibrium air in the temperature range from 3000° to 9000° K for densities of $10^{-4}$, $10^{-5}$, and $10^{-6}$ of sea level density. These results for the spectral radiant emission are an extension of the work of Breene and Nardone (ref. 19) to the lower density levels and cover the wave number range from 1250 to 56 000 $cm^{-1}$. The data given in this reference are presented in a form from which it would be difficult to obtain the spectral absorption coefficient $K_\lambda$. Thus its usefulness in general radiation gas dynamics is limited.

Churchill et al., (ref. 23) defined an average absorption coefficient for an isothermal, homogeneous, gas layer of thickness $x$. Graphical results for heated air were plotted versus wavenumber with temperature, density, and thickness $x$ as parameters. The wavenumber range covered was 21 500 to 49 000 $cm^{-1}$ while the temperature range was 1000° to 8000° K and the density range was $10^0$ to $10^{-4}$ of sea level density.

Gilmore (ref. 24) presented tabular data for the linear spectral absorption coefficient of air over a wavelength range of 0.1167 to 1.9836 microns. The temperature range covered was 2000° to 8000° K and the density range was $10^1$ to $10^{-4}$ of sea level density. Gilmore also presented tabular data on the Planck mean absorption coefficient for the same temperature and density range. The absorption coefficient data given in this reference is considered to be quite accurate but its usefulness in radiation gas dynamics is limited by the fact that temperatures above 8000° K are not considered.

Bowen (ref. 25) presented graphical data on the spectral radiance of high-temperature air over a wavelength range of 0.2 to 1.0 micron for density levels from $10^{-6}$ to $10^0$ of sea level density and temperatures from 3000° to 12 000° K. The quantity plotted is apparently derived from eq. (C8) with $L$ taken as unity. Bowen also presented graphical data on the total radiance of air for the wavelength interval $\delta \lambda$ of 0.35 to 0.55
microns over the same temperature and density ranges as those of the spectral data. The quantity plotted is apparently

\[ J_T = \int_{\delta \lambda} J_\lambda \, d\lambda \]  

(C12)

where \( \delta \lambda \) is 0.35 to 0.55 microns and \( J_\lambda \) is given by eq. (C8). The total radiance of air for the wavelength interval of 0.35 to 0.75 microns was also given over the same temperature and density ranges as those above. The quantity plotted is apparently the same as that given by eq. (C12) except \( \delta \lambda \) is 0.35 to 0.75 microns.

Ashley (ref. 26) has presented graphical data on the linear spectral absorption coefficient of high-temperature equilibrium air for optical wavelengths within the range of 3800 to 6500 Å. The temperature range covered was 1000° to 24 000° K while the density levels ranged from \( 10^{-6} \) to \( 10^0 \) of sea level density. The mean linear absorption coefficient averaged over the wavelength interval \( \delta \lambda \) from 3800 to 6500 Å, \( \mu_{\delta \lambda} \), where

\[ \mu_{\delta \lambda} = \int_{\delta \lambda} \frac{\mu_\lambda \, d\lambda}{\delta \lambda} \]

has also been presented for the same temperature and density ranges. Tabular data has also been given over the same wavelength, temperature, and density ranges for the apparent spectral absorption coefficient which is the true absorption coefficient corrected for induced emission. The absorption coefficient data given in the above reference is quite extensive over large temperature and density ranges. However the wavelength range considered is limited to the optical band of 0.38 to 0.65 microns.

Gilmore (ref. 27) gives graphical data on the linear absorption coefficient of air at a wavelength of 1270 Å for a temperature range of 2000° to 8000° K and density levels of \( 10^{-3} \) and \( 10^0 \) of sea level density. Graphical data is also presented on the mean absorption coefficients in the spectral regions of 4100 to 4500 Å and 5500 to 6200 Å for the same temperature range and density levels.

In the calculations reported here, Sewell's data was employed because it is extensive and is presented in a form suitable for nongrey calculation.
In Appendix C it was concluded that the data presented by Sewell (ref. 3) for the absorption properties of air was the most extensive with respect to the frequency, temperature, and density and was in a form from which both the spectral and Planck mean absorption coefficients could be obtained with ease. Hence Sewell's data has been used to determine the linear spectral absorption coefficient of air for several representative combinations of density level and temperature. These results have been plotted versus wavenumber and are given in fig. 16.

For comparison purposes the spectral absorption coefficient of optically thin air has also been determined using the spectral radiance data at 10,000° K and a density of $10^{-1}$ normal sea level density given by Nardone et.al., (ref. 21). The results have been plotted versus wavelength and are shown in fig. 17 along with the spectral absorption coefficient derived from Sewell's data for the same temperature and density condition. It is seen from fig. 17 that Sewell's spectral absorption coefficient data and the data of Nardone et.al., are in fair agreement with regard to order of magnitude and gross spectral behavior.

From the curves of spectral absorption coefficient in figs. 16 and 17 with particular emphasis on those curves displayed in fig. 17 it is seen that the nongrey absorption coefficient of air could be mathematically represented with substantial accuracy by the step function model shown in fig. 5.

The step function model in fig. 5 appears to be a quite reasonable mathematical representation of the spectral absorption coefficient of air for most thermodynamic conditions except combinations of high temperatures on the order of 18,000° to 20,000° K and low densities of $10^{-3}$ to $10^{-6}$ of sea level density such as those of figs. 16(f) and 16(i). However at these thermodynamic conditions of low density and high temperature the assumption of local thermodynamic equilibrium which applies throughout the analysis presented here is probably invalid (ref. 27).

Now the nongrey absorption coefficient model in fig. 5 represents quite well the spectral absorption coefficient data for air as given in fig. 17 at a temperature of 10,000° K and a density of $10^{-1}$ of sea level density which are typical of thermodynamic conditions expected to exist in the boundary layer when the effect of energy transfer by air radiation is appreciable. Thus the temperature and density dependence used for $K_1(T,p)$ in fig. 5 will be obtained by first determining from Sewell's data the average absorption coefficient for the spectral interval associated with
K_l and then correlating this average absorption coefficient as a function of temperature and density.

Using Sewell's graphical data and the relation

$$K_1 = \frac{\int K_\lambda d\omega}{\delta\omega} , \omega = 1/\lambda$$  \hspace{1cm} (D1)

the average absorption coefficient $K_1$ for the wavenumber interval $\delta\omega$ of 74 000 cm$^{-1}$ to 122 000 cm$^{-1}$ has been determined first for temperature of 8000$^\circ$, 10 000$^\circ$, 12 000$^\circ$, and 14 000$^\circ$ K at a density of 10$^{-1}$ of sea level density and then for densities of 10$^{-4}$, 10$^{-3}$, 10$^{-2}$, 10$^{-1}$, and 10$^0$ of sea level density at a temperature of 10 000$^\circ$ K. Next assuming that the functional dependence of $K_1$ on temperature $T$ and density $\rho$ can be expressed in power law form (see eq. (37)) the aforementioned temperature and density data for the average absorption coefficient has been correlated using the least-mean-square method to obtain the following relation for $K_1(\rho,T)$

$$K_1(\rho,T) = 4.370(\rho/\rho_{s.l.})^{1.009}(T/10^4)^{2.850}$$  \hspace{1cm} (D2)

where $\rho_{s.l.}$ is the normal sea level density of 1.225 × 10$^{-3}$ gm·cm$^{-3}$.

Once $K_1(\rho,T)$ is obtained the relation $K_2(\rho,T)$ for the mean absorption coefficient associated with the other spectral interval of fig. 5 may be determined from the following expression for the Planck mean absorption coefficient

$$K_p(\rho,T) = K_1(\rho,T)\alpha_1(T) + K_2(\rho,T)\alpha_2(T)$$  \hspace{1cm} (D3)

provided $K_p(\rho,T)$ and the $\alpha_j(T)$, $j = 1, 2$, are known. The $\alpha_j(T)$ for the spectral intervals of fig. 5 can be determined at any given temperature from fig. 15 by use of eq. (A14). The functional relation used for $K_p(\rho,T)$ is the correlation formula determined by Olstad (ref. 28) for Planck mean absorption coefficient data derived from the results of Kivel and Bailey (ref. 16), Sewell (ref. 3) and Nardone et.al. (ref. 21). This relation is

$$K_p(\rho,T) = 7.943 \times 10^{-2}(\rho/\rho_{s.l.})^{1.10}(T/10^4)^{6.95}$$  \hspace{1cm} (D4)

It should be noted that the density and temperature dependence of $K_p$ in
eq. (D4) is in excellent agreement with that proposed by Goulard (ref. 13) for the grey absorption coefficient of high-temperature air.

Numerical values of $K_2(\rho, T)$ have been determined from eq. (D3) for temperatures ranging from 4000° to 14 000° K and densities ranging from $10^{-4}$ to $10^0$ of sea level density. These data for $K_2(\rho, T)$ were then correlated as a function of temperature and density in the same manner as that employed for $K_1(\rho, T)$. The resulting correlation formula for $K_2(\rho, T)$ is

$$K_2(\rho, T) = 4.985 \times 10^{-2}(\rho/\rho_{s,1})^{1.205}(T/10^4)^{5.47}$$

(D5)

Now the linear absorption coefficients $K_p(\rho, T), K_1(\rho, T)$, and $K_2(\rho, T)$ given respectively by eqs. (D4), (D2), and (D5) may be expressed as mass absorption coefficients $\kappa_p(T, P), \kappa_1(T, P)$, and $\kappa_2(T, P)$ by using the relation between linear and mass absorption coefficient

$$K = \rho \kappa$$

(D6)

and then employing the equation of state for a perfect gas

$$\rho = \frac{P}{ZRT}$$

(D7)

where $Z$ is expressed as a function of temperature and pressure by the correlation formula

$$Z = 2.11(T/10^4)^{0.775}(P/P_o)^{-0.065}$$

(D8)

This correlation formula for $Z$ was determined by Olstad (ref. 28) from the thermodynamic data of high-temperature air given by Ahyte and Peng (ref. 29). The resulting equations for $\kappa_p, \kappa_1, \kappa_2$ are, respectively,

$$\kappa_p(T, P) = 42.22(T/10^4)^{6.772}(P/P_o)^{0.1065}$$

(D9)

$$\kappa_1(T, P) = 3.431 \times 10^3(T/10^4)^{2.834}(P/P_o)^{0.0096}$$

(D10)

and
\[ \kappa_2(T, P) = 16.89(T/10^4)^5.106(P/P_o)^{0.2182} \]  \hspace{1cm} (D11)

where \( T \) is in °K and \( P \) is in atm with \( P_o \) equal 1 atm. The units of \( \kappa_p, \kappa_1, \) and \( \kappa_2 \) are \( \text{cm}^2\cdot\text{gm}^{-1} \).

It is necessary to know the enthalpy as a function of temperature and pressure, \( h(T, P) \), before the solution to the integral form of the energy equation can be obtained. The functional relation used for \( h(T, P) \) was

\[ h(T, P) = 5.201 \times 10^{11}(T/10^4)^{1.7699}(P/P_o)^{-0.0920} \]  \hspace{1cm} (D12)

which is a modified form of a correlation formula determined by Olstad (ref. 28) from the high-temperature air data of Ahyte and Peng (ref. 29). The units of \( h(T, P) \) in eq. (D12) are \( \text{erg} \cdot \text{gm}^{-1} \).

If eq. (D12) is written for external flow conditions the resulting relation is

\[ h_e = 5.201 \times 10^{11}(T_e/10^4)^{1.7699}(P_e/P_o)^{-0.0920} \]  \hspace{1cm} (D13)

Then from eqs. (D12) and (D13) the expression for \( \tilde{h} = h/h_e \) is

\[ \tilde{h} = \frac{1}{T^{1.7699}} \]  \hspace{1cm} (D14)

where the boundary layer condition of \( P/P_e \) equal to unity has been employed. Equation (D14) is the \( \tilde{h} = \tilde{h}(T) \) relation employed in the calculation.

The only remaining property value needed for the calculation of eq. (76) is the Prandtl number of air. This has been taken as 0.70.

Once the solution \( \tilde{h}(\xi, \eta) \) for the energy equation, eq. (70), is obtained the convective and radiative wall heat fluxes may be calculated using eqs. (72) and (74) provided that the values of \( \rho_e \mu_e \) and \( C \) are known. The values of \( \rho_e \mu_e \) can be determined from the relation

\[ \rho_e \mu_e = \frac{2.80 \times 10^{-4}(P_e)^{0.992}}{(h_e)^{0.3329} - 119.9} \]  \hspace{1cm} (D15)
which was derived from the correlation formulas given by Cohen (refs. 30, 31) for high-temperature air. The units of $\rho e^{\mu_e}$ in eq. (D15) are $\text{gm}^2\cdot\text{cm}^{-4}\cdot\text{sec}^{-1}$ when $h_e$ is expressed in erg $\cdot$ gm$^{-1}$ and $P_e$ in atm. The values of $C$ may be determined from the relation

$$C = \frac{\rho c^{\mu_c}}{\rho e^{\mu_e}} = \frac{(h_e)^{0.3329} - 119.9}{(h_e)^{0.3329} - 119.9}$$

(D16)

where $h_*$ is the reference enthalpy given by eq. (73).
The solution to the boundary layer energy equation has been determined by the method described previously using an IBM 7072 digital computer. From these results the computer subsequently calculated both the convective and radiative wall heat fluxes. The computer program necessary to make the computer carry out these actions was written in FORTRAN II programming language. The variable names which were assigned to the mathematical relations and symbols used in the text and appendices are presented below immediately preceding the program listing.

**Variable Names and Mathematical Symbols**

- **ALPHA**\( \alpha_j(T_\text{e}) \) as given by eq. (A12)
- **BE2** \( \beta_{j+1} \) in eq. (A12)
- **BE1** \( \beta_j \) in eq. (A12)
- **N** number of intervals used in subroutine for \( \alpha_j \)
- **ETA** \( \eta \) as given by eq. (65)
- **W** \( W \) as given by eq. (77)
- **AMESS** integrand of eq. (A12)
- **MMN** number of \( \eta \)'s used
- **PR** \( \text{Pr} \)
- **UE** \( u_\text{e}, \text{cm}\cdot\text{sec}^{-1} \)
- **TE** \( T_\text{e}, \text{°K} \)
- **PE** \( P_\text{e}, \text{atm} \)
- **TBARW** \( \bar{T}_\text{w} \)
- **EPSW** \( \varepsilon_\text{w} \)
- **EPS** a predetermined small number used in the convergence test
SIGMA
Stefan–Boltzmann constant, erg·cm\(^{-2}\)·sec\(^{-1}\)·K\(^{-4}\)

FPP1
\(f''(\eta)\) at \(\eta = 0.05\)

FPP2
\(f''(\eta)\) at \(\eta = 0.10\)

XL
\(L\), cm

POW1
the exponent on \((T/10^4)\) in the relation for \(\kappa_p(T,P)\)

POW2
the exponent on \((P/P_o)\) in the relation for \(\kappa_p(T,P)\)

CE
the coefficient of \((T/10^4)^{POW1}(P/P_o)^{POW2}\) in the relation for \(\kappa_p(T,P)\)

C
\(C\) as given by eq. (D16)

FA(I)
the values of \(f(\eta)\) at \(\eta = \eta(I)\)

ETA(I)
the values of \(\eta = \eta(I)\)

FPP(I)
the values of \(f''(\eta)\) at \(\eta = \eta(I)\)

FFA(I)
the values of \(f(\eta)\) at \(\eta = 0, 0.025, 0.05, 0.10, 0.20\)

X(I)
the values of \(\bar{x}\)

E1E(I)
the values of \(\varepsilon_j e\)

POWER(I)
the exponents on \((T/10^4)\) in the relations for \(\kappa_j(T,P)\)

POW(I)
the exponents on \((P/P_o)\) in the relations for \(\kappa_j(T,P)\)

CON(I)
the coefficients of \((T/10^4)^{POWER(I)}(P/P_o)^{POW(I)}\) in the relations for \(\kappa_j(T,P)\)

JUNK
variable which equals zero for grey case and unity for nongrey case

ALAM1(I)
\(\lambda_j\) used in eq. (A10), microns

ALAM2(I)
\(\lambda_{j+1}\) used in eq. (A10), microns

A1E(K)
\(\alpha_j(T_e)\)

A1W(K)
\(\alpha_j(T_w/T_e)\)

BET1(K)
\(\beta_j\) for \(\alpha_j(T_e)\) calculation

BET2(K)
\(\beta_{j+1}\) for \(\alpha_j(T_e)\) calculation

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BETW1(K) \[ \beta_j \text{ for } \alpha_j(T_w, T_e) \] calculation

BETW2(K) \[ \beta_{j+1} \text{ for } \alpha_j(T_w, T_e) \] calculation

XKPE \[ \kappa_p(T_e, P_e), \text{ cm}^2 \cdot \text{gm}^{-1} \]

FACT \[ c_w \sigma T_e^4, \text{ erg.cm}^{-2} \cdot \text{sec}^{-1} \]

ACT \[ \sum_{j=1}^{N} \epsilon_j a_j(T_e) - T_w^4 \]

TEMP1(K) \[ - \alpha_j(T_e) \epsilon_j \]

TEMP2(K) \[ - 0.5 \left[ c_w a_j(T_w T_e) T_w^4 + (2 - \epsilon_w) \epsilon_j a_j(T_e) \right] \]

AA1 \[ 1/0.565 \]

AHE \[ h_e \text{ as given by eq. (D13), erg.gm}^{-1} \]

RHOMU \[ \rho_e \mu_e \text{ as given by eq. (D15), gm}^2 \cdot \text{cm}^{-4} \cdot \text{sec}^{-1} \]

AA2 \[ Pr u_e^2/h_e \]

APOWR \[ \text{POWER(K)} \]

APOW \[ \text{POW(K)} \]

XK(K) \[ \kappa_j(T_e, P_e), \text{ cm}^2 \cdot \text{gm}^{-1} \]

AHBRW \[ \bar{n}_w \]

G1(1) \[ f_0^n f d n_1 \text{ at } \eta = 0.0 \]

G1(2) \[ f_0^n f d n_1 \text{ at } \eta = 0.2 \]

G1(I) \[ f_0^n f d n_1 \text{ at } \eta = \eta(I) \]

PR2 \[ \frac{Pr}{2} \]

G2(1) \[ \exp \left[ - \frac{Pr}{2} \right] G1(1) \]

G2(I) \[ \exp \left[ - \frac{Pr}{2} \right] G1(I) \]

G4(1) \[ G2(1) \]

G \[ f_0^n f d n_1 \text{ at } \eta = 0.1 \]

G4(2) \[ \exp \left[ - \frac{Pr}{2} \right] G \]

G4(3) \[ \exp \left[ - \frac{Pr}{2} \right] G1(2) \text{ at } \eta = 0.2 \]
\begin{align*}
G3(1) & \quad \int_0^n G^4(n_1) \, dn_1 \text{ at } n = 0 \\
G3(2) & \quad \int_0^n G^4(n_1) \, dn_1 \text{ at } n = 0.2 \\
G3(3) & \quad \int_0^n G^4(n_1) \, dn_1 \text{ at } n = \eta(I) \\
G5(1) & \quad \exp \left[ \frac{\eta(1)}{2} \right] \text{ at } n = 0 \\
G5(2) & \quad \exp \left[ \frac{\eta(1)}{2} \right] \\
G6(1) & \quad \left( \frac{\eta''}{\eta} \right)^2 G5(1) \text{ at } n = 0 \\
G6(2) & \quad f^2 \text{ at } n = 0.05 \\
G6(3) & \quad f^2 \exp \left( \frac{\eta(1)}{2} \right) \text{ at } n = 0.05 \\
G7(1) & \quad \int_0^n G^6(n_1) \, dn_1 \text{ at } n = 0 \\
G7(2) & \quad \int_0^n G^6(n_1) \, dn_1 \text{ at } n = 0.1 \\
G8(1) & \quad G6(1) \\
G8(2) & \quad G6(3) \\
G8(3) & \quad \left( \frac{\eta''}{\eta} \right)^2 \exp \left[ \frac{\eta(1)}{2} \right] \text{ at } n = 0.2 \\
G7(3) & \quad \int_0^n G^8(n_1) \, dn_1 \text{ at } n = 0.2 \\
G9(1) & \quad G4(1)G7(1) \\
G9(2) & \quad G4(2)G7(2) \\
G9(3) & \quad G4(3)G7(3) \\
G12(1) & \quad \left( \frac{\eta''}{\eta} \right)^2 G5(I) \text{ at } n = \eta(I) \\
G11(1) & \quad G9(1) \\
G11(2) & \quad G9(3) \\
G13(1) & \quad G7(1) \\
G13(2) & \quad G7(3) \\
G13(3) & \quad \int_0^n G^12(n_1) \, dn_1 \text{ at } n = \eta(I) \\
G11(3) & \quad G13(I)G2(I)
\end{align*}
G10(1) \quad \int_{0}^{\eta} G{(\eta_1)} d\eta_1 \text{ at } \eta = 0.0

G10(2) \quad \int_{0}^{\eta} G{(\eta_1)} d\eta_1 \text{ at } \eta = 0.2

G10(I) \quad \int_{0}^{\eta} G_{11}{(\eta_1)} d\eta_1 \text{ at } \eta = \eta(I)

CZERO \quad C_o \text{ as given by eq. (81)}

G10(MMN) \quad G10(I) \text{ at } \eta = \eta_\delta

G3(MMN) \quad G3(I) \text{ at } \eta = \eta_\delta

AHBR1(I) \quad \left(\bar{h}_{in}\right)_{n} \text{ in eq. (84), } \eta = \eta(I)

AHBRO(I) \quad \bar{h}_o(\eta) \text{ as given by eq. (80), } \eta = \eta(I)

CONST \quad -4Pr_0 T^4_{el}/ue e

CONS \quad \text{CONST} \cdot \kappa_p(T_e, P_e)

TBAR(I) \quad \left[\left(\bar{h}_{in}\right)_{n}\right]^{1/AA_1}, \eta = \eta(I)

TO(I) \quad \left(\bar{h}_o\right)^{1/AA_1}, \eta = \eta(I)

DONST \quad \text{CONST} \cdot \kappa_j(T_e, P_e) \alpha_j(T_e)

ABC(I) \quad \text{CONST} \cdot G3(I)/G3(MMN)

APOWR \quad \text{POWER(K)}

XK1(I,K) \quad \bar{\kappa}_j(T) \text{ at } \eta = \eta(I)

AB(I) \quad \text{ABC(I)} \cdot X(J)

XKP(I) \quad \bar{\kappa}_p(T) \text{ at } \eta = \eta(I)

W(I) \quad W \text{ of eq. (77) at } \eta = \eta(I)

QW(I) \quad \text{the integrand of the integral in eq. (74) at } \eta = \eta(I)

G15(I) \quad W(I) \cdot G5(I)

XT \quad \bar{T} \text{ at } \eta = 0.1

XK1A(K) \quad \bar{\kappa}_j(T) \text{ at } \eta = 0.1

XKP1 \quad \bar{\kappa}_p(T) \text{ at } \eta = 0.1

WT \quad W \text{ at } \eta = 0.1
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Equation/Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>QT</td>
<td>$QW$ at $\eta = 0.1$</td>
</tr>
<tr>
<td>AGAIN(1)</td>
<td>$\int_{0}^{\eta} QW(\eta_1) d\eta_1$ at $\eta = 0$</td>
</tr>
<tr>
<td>AGAIN(2)</td>
<td>$\int_{0}^{\eta} QW(\eta_1) d\eta_1$ at $\eta = 0.2$</td>
</tr>
<tr>
<td>AGAIN(I)</td>
<td>$\int_{0}^{\eta} QW(\eta_1) d\eta_1$ at $\eta = \eta(I)$</td>
</tr>
<tr>
<td>G17(1)</td>
<td>$\int_{0}^{\eta} G15(\eta_1) d\eta_1$ at $\eta = 0$</td>
</tr>
<tr>
<td>G17(2)</td>
<td>$\int_{0}^{\eta} W(\eta_1) \exp\left(\frac{Pr}{2} \int_{0}^{\eta} f d\eta_2\right) d\eta_1$ at $\eta = 0.2$</td>
</tr>
<tr>
<td>G17(I)</td>
<td>$\int_{0}^{\eta} G15(\eta_1) d\eta_1$ at $\eta = \eta(I)$</td>
</tr>
<tr>
<td>G19(I)</td>
<td>$G17(1) G2(1)$</td>
</tr>
<tr>
<td>G19(1)</td>
<td>$G17(1) G2(1)$</td>
</tr>
<tr>
<td>G19(2)</td>
<td>$G17(2) G2(2)$</td>
</tr>
<tr>
<td>G20(1)</td>
<td>$G19(1)$</td>
</tr>
<tr>
<td>G20(3)</td>
<td>$G19(2)$</td>
</tr>
<tr>
<td>XTl</td>
<td>$\bar{T}$ at $\eta = 0.05$</td>
</tr>
<tr>
<td>XK1B(K)</td>
<td>$\kappa_j(\bar{T})$ at $\eta = 0.05$</td>
</tr>
<tr>
<td>XKp11</td>
<td>$\kappa_p(\bar{T})$ at $\eta = 0.05$</td>
</tr>
<tr>
<td>WT1</td>
<td>$W$ at $\eta = 0.05$</td>
</tr>
<tr>
<td>WTG1</td>
<td>$WT1 \exp\left(\frac{Pr}{2} G\phi\right)$</td>
</tr>
<tr>
<td>GTX</td>
<td>$\int_{0}^{\eta} W(\eta_1) \exp\left(\frac{Pr}{2} \int_{0}^{\eta} f d\eta_2\right) d\eta_1$ at $\eta = 0.1$</td>
</tr>
<tr>
<td>G20(2)</td>
<td>$G4(2) GTX$</td>
</tr>
<tr>
<td>TERM2(1)</td>
<td>$\int_{0}^{\eta} G19(\eta_1) d\eta_1$ at $\eta = 0$</td>
</tr>
<tr>
<td>TERM2(2)</td>
<td>$\int_{0}^{\eta} G20(\eta_1) d\eta_1$ at $\eta = 0.2$</td>
</tr>
<tr>
<td>CX</td>
<td>$\text{CONST} \cdot X(J)$</td>
</tr>
<tr>
<td>TERM2(I)</td>
<td>$\int_{0}^{\eta} G19(\eta_1) d\eta_1$ at $\eta = \eta(I)$</td>
</tr>
<tr>
<td>AHBR2(I)</td>
<td>$\bar{h}_{out}^{n}$ in eq. (84)</td>
</tr>
<tr>
<td>SS</td>
<td>$\sum_{I=1}^{\text{MMN}} [\text{AHBR2}(I) - \text{AHBR1}(I)]$</td>
</tr>
</tbody>
</table>

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SUMA

\[ \sum_{I=1}^{MMN} \left| \left[ AHBRI(I) - AHBRI1(I) \right] \right| \]

AHBAR(I,J)

the solution for \( \bar{h} \), \( \eta = \eta(I) \), \( \bar{x} = X(J) \)

T(I,J)

the solution for \( \bar{T} \), \( \eta = \eta(I) \), \( \bar{x} = X(J) \)

C1(J)

\( C_1 \) as given by eq. (82), \( \bar{x} = X(J) \)

TERM2(MMN)

TERM2(I) at \( \eta = \eta_\delta \)

S(J)

\( s \) as given by eq. (65), \( \bar{x} = X(J) \)

QWR(J)

\( q_{RW} \) as given by eq. (74), \( \bar{x} = X(J) \)

AGAIN(MMN)

AGAIN(I) at \( \eta = \eta_\delta \)

QWCO(J)

\( q_{CWO} \) as given by eq. (72), \( \bar{x} = X(J) \)

QWC1(J)

\( q_{CW} \) as given by eq. (72), \( \bar{x} = X(J) \)
PROGRAM WITH EXPANDED ETA AND EXPANDED W
SUBROUTINE ALPHA(BE2,BE1,N,VALUE)
DIMENSION AMESS(400),Z(400)
DO 1 I = 1,400
AMESSI = 0.0
Z(I) = 0.0
1 CONTINUE
AN = N
NN = N + 1
Z(1) = BE1
IF(BE1) 6,3,7
6 STOP
7 IF(Z(1) - 112.) 16,16,17
16 AMESS(I) = (Z(I)**3)/(EXPF(BE1) - 1.)
GO TO 18
17 VALUE = 0.0
RETURN
18 CONTINUE
5 CONTINUE
IF(BE2 - 112.) 19,19,20
20 DZ = (112. - BE1)/AN
GO TO 21
19 DZ = (BE2 - BE1)/AN
21 CONTINUE
DO 10 JJ = 2,NN
Z(JJ) = Z(JJ - 1) + DZ
10 CONTINUE
NM = N - 2
NM = NM
SA = 0.0
SB = 0.0
DO 4 L = 2,NM,2
SA = SA + AMESS(L)
SB = SB + AMESS(L + 1)
4 CONTINUE
SA = SA + AMESS(N)
VALUE = (.15398J*(DZ/3.J*(AMESS(1) + 4.*SA + 2.*SB + AMESS(NN))
VALUE = VALUE
RETURN
END
700 DIMENSION G1(45),FFA(5),G2(45),G3(45),G4(3),FA(45),ETA(45),G5(45),
70016(3),G7(3),G8(3),G9(3),G10(45),G11(45),G12(45),G13(45),AHBR0(45),
7002AHBR1(45),TBAR(45),AB(45),G15(45),G17(45),G19(45),G20(3),TERM2(45),
7003,AHBR2(45),X(10),AHBAR(45,10),ABC(45),W(45),FPP(45),XK1B(10),ALAM1
7004(10),ALAM2(10),POWER(10),PWD(10),CON(10),A1E(10),A1W(10),E
7005E(10),TEMP2(10),XK(10),BET1(10),BET2(10),BETW1(10),BETW2(10),XKP(45),QWR(10
7006)
DIMENSION TEMP1(10),G1(10),QW(45),AGAIN(45),S(10),QWCO(10),QWG1(10
1)
DIMENSION T(45,10),TO(45),BETW2(10),XK1(45,10),DONST(10),XKIA(10)
701 READ 20,NUM
PRINT 21,NUM
NUM = NUM
READ 20,MMN
PRINT 136,MMN
READ 100, PR,UE,TE,PE,TBARW, EPSW, EPS
490 READ 100, SIGMA, FPP1, FPP2, XL
READ 100, POW1, POW2, CE
POW1 = POW1
POW2 = POW2
READ 100, C
702 READ 101, (FA(I), I = 1, MMN)
703 READ 101, (ETA(I), I = 1, MMN)
704 READ 101, (FPP(I), I = 1, MMN)
705 READ 101, (FFA(I), I = 1, 5)
706 READ 101, (X(I), I = 1, 10)
READ 100, (E1E(I), I = 1, NUM)
READ 100, (POWER(I), I = 1, NUM)
READ 100, (POW(I), I = 1, NUM)
READ 100, (CON(I), I = 1, NUM)
READ 55, JUNK
IF JUNK = 0, GREY CASE, IF JUNK = 1, NON-GREY CASE
IF(JUNK)56,56,57
57 READ 100, (ALAM1(I), I = 1, NUM)
READ 100, (ALAM2(I), I = 1, NUM)
GO TO 58
56 PRINT 59
58 CONTINUE
970 PRINT 920, PR
971 PRINT 921, UE
972 PRINT 922, TE
973 PRINT 923, PE
974 PRINT 924, TBARW
975 PRINT 925, EPSW
978 PRINT 928, EPS
979 PRINT 929, XL
PRINT 50, POW1
PRINT 51, POW2
PRINT 52, CE
977 PRINT 927, (E1E(I), I = 1, NUM)
980 PRINT 930, (POWER(I), I = 1, NUM)
PRINT 450, (POW(I), I = 1, NUM)
PRINT 451, (CON(I), I = 1, NUM)
PRINT 32, C
IF(JUNK)60,60,62
62 PRINT 926, (ALAM1(I), I = 1, NUM)
PRINT 976, (ALAM2(I), I = 1, NUM)
60 CONTINUE
IF(JUNK)63,63,64
63 DO 65 K = 1, NUM
A1E(K) = 1.0
AIW(K) = 1.0
65 CONTINUE
GO TO 66
64 DO 150 K = 1, NUM
BET1(K) = (14388.)/(TE*ALAM1(K))
BET2(K) = (14388.)/(TE*ALAM2(K))
BETW1(K) = BET1(K)/TBARW
BETW2(K) = BET2(K)/TBARW
BAT1 = BET1(K)
BAT2 = BET2(K)
call alphabat2, bat1, 398, ans
AIE(K) = ans
BAT1 = BETW1(K)

60
BAT2 = BETWZ(K)
CALL ALPHA(BAT2, BAT1, 398, ANS)
AIW(K) = ANS

150 CONTINUE
66 CONTINUE
454 PRINT 404, (A1E(K), K = 1, NUM)
455 PRINT 405, (AIW(I), I = 1, NUM)
XKE = CE*(TI/T0000.)*PWI*(PE**PWI)
PRINT 71, XKPE
FACT = EPSW*(TE**4)*SIGMA
ACT = 0.0
DO 41 K = 1, NUM
ACT = ACT + E1E(K)*A1E(K)
41 CONTINUE
ACT = ACT - TBWR**4
DO 151 K = 1, NUM
TEM(1) = -E1E(K)*A1E(K)
TEMP(2)(K) = -**5*(EPSW*AIW(K)*TBWR**4) + (2. - EPSW)*E1E(K)*A1E(K)
151 CONTINUE
PRINT 360, FACT
PRINT 361, ACT
PRINT 179, (TEMP(1)(K), K = 1, NUM)
456 PRINT 406, (TEMP(2)(K), K = 1, NUM)
14 AI1 = 1/1.565
715 AHE = ((TE/002401)**AA1)*PE*(-AA1*0.052)
457 PRINT 407, AHE
RHOMU = (.0002796)*PE**(.992)/(AHE**(.3329) - 119.9526)
PRINT 33, RHOMU
716 AA2 = (PR*(UE**2))/AHE
PRINT 67, AA2
DO 152 K = 1, NUM
APOWR = POWER(K)
APOW = POW(K)
APOWR = APOWR
APOW = APOW
XK(I) = CON(K)*((TE/10000.)*APOWR)*PE*APOW
152 CONTINUE
458 PRINT 408, (XK(I), I = 1, NUM)
718 AHBWR = TBWR**AA1
459 PRINT 409, AHBWR
690 G1(1) = 0.0
720 G1(2) = (.03333333)*(FA(1) + 4.*FFA(4) + FA(2))
721 DO 202 I = 3, MMN
722 G1(I) = (.06666667)*FA(I-2) + 4.*FA(I-1) + FA(I) + G1(I-2)
202 CONTINUE
724 G2(1) = 1.0
725 PR2 = PR/2.
726 DO 203 I = 2, MMN
727 G2(I) = EXPF(-PR2*G1(I))
203 CONTINUE
729 G4(1) = G2(1)
730 G = (.05/3.)*(FA(1) + 4.*FFA(3) + FFA(4))
731 G4(2) = EXPF(-PR2*G)
732 G4(3) = G2(2)
735 G3(1) = 0.0
736 G3(2) = (.03333333)*(G4(1) + 4.*G4(2) + G4(3))
737 DO 204 I = 3, MMN
738 G3(I) = (.06666667)*(G2(I-2) + 4.*G2(I-1) + G2(I)) + G3(I-2)

61
204 CONTINUE)
740 G5(1) = 1.0
    DO 205 I = 2, MMN
742 G5(I) = EXPF(PR2*G1(I))
205 CONTINUE
744 G6(1) = FPP(1)**2
745 GG = (.025/3.)*(FFA(1) + 4.*FFA(2) + FFA(3))
746 G6(2) = (FPP1**2)*EXPF(PR2*GG)
747 G6(3) = (FPP2**2)*EXPF(PR2*G)
750 G7(1) = 0.0
751 G7(2) = (.05/3.)*(G6(1) + 4.*G6(2) + G6(3))
752 G8(1) = G6(1)
753 G8(2) = G6(3)
748 G8(3) = (FPP(2)**2)*G5(2)
756 G7(3) = (.03333333)*(G8(1) + 4.*G8(2) + G8(3))
758 G9(1) = 0.0
759 G9(2) = G4(2)*G7(2)
760 G9(3) = G2(2)*G7(3)
764 DO 206 I = 1, MMN
765 G12(I) = (FPP(I)**2)*G5(I)
206 CONTINUE
767 G11(1) = 0.0
768 G11(2) = G9(3)
769 G13(1) = 0.0
770 G13(2) = G7(3)
771 DO 207 I = 3, MMN
772 G13(I) = (.06666667)*(G12(I-2) + 4.*G12(I-1) + G12(I)) + G13(I-2)
773 G11(I) = G13(I)*G2(I)
207 CONTINUE
776 G10(1) = 0.0
777 G10(2) = (.03333333)*(G9(1) + 4.*G9(2) + G9(3))
778 DO 208 I = 3, MMN
779 G10(I) = (.06666667)*(G11(I-2) + 4.*G11(I-1) + G11(I)) + G10(I-2)
208 CONTINUE
781 CZERO = (1.0 - AHBW + AA2*G10(MMN))/G3(MMN)
782 DO 209 I = 1, MMN
783 AHBRI(I) = AHBW + CZERO*G3(I) - AA2*G10(I)
784 AHBRO(I) = AHBRI(I)
209 CONTINUE
785 const = - 4.*PR*SIGMA*(TE**4)*XL/(UE*AHE)
788 PRINT 117, CZERO
789 PRINT 142, const
CONS = CONS*XTKE
PRINT 70, CONS
CONS = CONS
DO 115 I = 1, MMN
TBAR(I) = AHBRO(I)**.555
TO(I) = TBAR(I)
115 CONTINUE
DO 153 K = 1, NUM
DONST(K) = CONS*XTK(K)*A1E(K)
153 CONTINUE
PRINT 102, (DONST(K), K = 1, NUM)
790 DO 300 I = 1, MMN
791 ADC(I) = CONS*G3(I)/G3(MMN)
300 CONTINUE
900 DO 250 J = 1, 10
907 MM = 1
291 CONTINUE
DO 211 I = 1,MMN
DO 154 K = 1,NUM
APOWR = POWER(K)
XK1(I,K) = TRAR(I) * APOWR
154 CONTINUE

602 ABI(I) = ABC(I) * X(J)
XKP(I) = TBAR(I) * POW1
211 CONTINUE

DO 156 I = 1,MMN
W(I) = XKPE * XKP(I) * (TBAR(I) ** 4)
QW(I) = XKPE * XKP(I) * (TBAR(I) ** 4)
156 CONTINUE

DO 157 I = 1,MMN
W(I) = W(I) + XK(I,K) + XKP(I,J) + TEMPl(K)
QW(I) = QW(I) + XK(I,K) + XKP(I,J) + TEMPl(K)
157 CONTINUE

DO 159 I = 1,MMN
G15(I) = W(I) * G5(I)
159 CONTINUE

623 XT = (TBAR(2) + TBAR(1)) * (.5)
624 DO 160 K = 1,NUM
APOWR = POWER(K)
XK1A(K) = XT ** APOWR
160 CONTINUE

XKP1 = XT ** POW1
WT = XKPE * XKP1 * (XT ** 4)
QT = XKPE * XKP1 * (XT ** 4)
625 DO 161 K = 1,NUM
WT = WT + XK(K) * XK1A(K) * TEMP2(K)
QT = QT + XK(K) * XK1A(K) * TEMP1(K)
161 CONTINUE

AGAIN(1) = 0.0
AGAIN(2) = (.03333333) * (QW(1) + 4. * QT + QW(2))
626 DO 48 I = 3,MMN
AGAIN(I) = (.06666667) * (QW(I - 2) + 4. * QW(I - 1) + QW(I)) + AGAIN(I - 2)
48 CONTINUE

635 G17(1) = 0.0
636 G17(2) = (.03333333) * (G15(1) + 4. * WT * EXPF(G2 * G17) + G15(2))
637 DO 212 I = 3,MMN
638 G17(I) = (.06666667) * (G15(I - 2) + 4. * G15(I - 1) + G15(I)) + G17(I - 2)
639 G19(1) = G17(I) * G2(I)
212 CONTINUE

640 G19(2) = G17(2) * G2(2)
641 G20(1) = G19(1)
642 G20(2) = G19(2)
643 G20(3) = G19(2)
644 XT1 = (XT + TBAR(11)) * (.5)
645 DO 162 K = 1,NUM
APOWR = POWER(K)
XK1B(K) = XT1 ** APOWR
162 CONTINUE

XKP11 = XT1 ** POW1
WT1 = XKP11 * XKPE * (XT1 ** 4)
DO 163 K = 1,NUM
WT1 = WT1 + XK(K) * XK1B(K) * TEMP2(K)
163 CONTINUE

648 WTG1 = WT1 * EXPF(PR2 * GG)
650 GTX = (.05/3.)*(G15(1) + 4.*WTG1 + WT*EXPF(PR2*G))
651 G20(2) = G4(2)*GTX
666 TERM2(I) = 0.0
   TERM2(2) = (.03333333)*(G20(1) + 4.*G20(2) + G20(3))
668 CX = CONST*X(I)
669 DO 260 I = 3,MMN
670 TERM2(I) = (.06666667)*(G19(I-2) + 4.*G19(I-1) + G19(I)) + TERM2(I
6701-2)
260 CONTINUE
   DU 672 I = 1,MMN
671 AHBR2(I) = AHBR0(I) + AB(I)*TERM2(MMN) - CX*TERM2(I)
   IF(AHBR2(I)) 844,846,846.
844 STOP
846 CONTINUE
672 CONTINUE
   SS = 0.0
676 SUMA = 0.0
677 DO 261 I = 1,MMN
   SS = SS + (AHBR2(I) - AHBR1(I))
   SUMA = SUMA + ABSF(AHBR2(I) - AHBR1(I))
261 CONTINUE
981 PRINT 932,MM
   PRINT 673,SS
679 PRINT 931, SUMA
680 IF(SUMA-EPS) 280,280,281
281 CONTINUE
681 DO 282 I = 1,MMN
   AHBR1(I) = (AHBR1(I) + AHBR2(I))*(.5)
683 TRAR(I) = AHBR1(I)**(.565)
282 CONTINUE
904 MM = MM + 1
684 GO TO 291
280 CONTINUE
685 DO 285 I = 1,MMN
686 AHBAR(I,J) = AHBR2(I)
   TBAR(I) = AHBAR(I,J)**(.565)
688 AHBR1(I) = AHBR2(I)
   TI(J) = TBAR(I)
285 CONTINUE
983 PRINT 933, MM
   CI(J) = CZERO + CONST*X(J)*TERM2(MMN)/G3(MMN)
   S(J) = RHOOM*UE*X(J)*XL
   QWR(J) = FACT*(ACT + (2.*SQRTF(C*S(J))/(UE))*AGAIN(MMN))
   QWR(J) = QWR(J)
   QWCO(J) = -(SQRTF(C*RHOOM*UE)/(XL*X(J)))*AHE*CZERO/PR
   QWCJ(J) = -(SQRTF(C*RHOOM*UE)/(XL*X(J)))*AHE*CI(J)/PR
250 CONTINUE
   DONST(I) = DONST(I)
   PKNI 170
   PRINT 171
   PRINT 170
   PRINT 174
   PRINT 173,(ETA(I),AHBR0(I),(AHBAR(I,J),J=1,10),I=1,MMN)
   PRINT 170
   PRINT 170
   PRINT 172
   PRINT 170
   PRINT 175
   PRINT 173,(ETA(I),TO(I),(T(I,J),J=1,10),I=1,MMN)

64
PRINT 170
PRINT 170
PRINT 177
PRINT 176,(X(J),C1(J),S(J),QWCO(J),QWC1(J),QWR(J),J=1,10)
PRINT 170
PRINT 170
GO TO 701
20 FORMAT(12)
21 FORMAT(29H THE NUMBER OF TERMS IN W IS 14)
32 FORMAT(5H C = E16.8)
33 FORMAT(9H RHOMU = E16.8)
50 FORMAT(8H POW1 = E16.8)
51 FORMAT(8H POW2 = E16.8)
52 FORMAT(6H CE = E16.8)
55 FORMAT(I1)
59 FORMAT(10H GREY CASE)
67 FORMAT(15H PROUE**2/HE = E16.8)
70 FORMAT(8H CONS = E16.8)
71 FORMAT(7H KPE = E16.8)
100 FORMAT(E14.8)
101 FORMAT(5E14.8)
102 FORMAT(9H DONST = E16.8)
117 FORMAT(9H CZERO = E16.8)
136 FORMAT(28H THE NUMBER OF ETAS USED IS 14)
142 FORMAT(9H COST = E16.8)
170 FORMAT(2H )
171 FORMAT(54H)
172 FORMAT(54H)
173 FORMAT(F4.1,11F10.6)
174 FORMAT(114H ETA HBAR XBAR =.1 XBAR =.2 XBAR =.3 XBAR =.4
174 XBAR =.5 XBAR =.6 XBAR =.7 XBAR =.8 XBAR =.9 XBAR=1.0)
175 FORMAT(114H ETA TBARO XBAR =.1 XBAR =.2 XBAR =.3 XBAR =.4
175 XBAR =.5 XBAR =.6 XBAR =.7 XBAR =.8 XBAR =.9 XBAR=1.0)
176 FORMAT(F5.1,5E16.8)
177 FORMAT(86H XBAR C1 S QWCO
1771 QW1 QWR )
179 FORMAT(12H TEMP1(K) = E16.8)
360 FORMAT(8H FACT = E16.8)
361 FORMAT(7H ACT = E16.8)
404 FORMAT(7H A1E = E16.8)
405 FORMAT(7H A1W = E16.8)
406 FORMAT(9H TEMP2 = E16.8)
407 FORMAT(7H AHE = E16.8)
408 FORMAT(6H XK = E16.8)
409 FORMAT(9H AHBWR = E16.8)
450 FORMAT(7H POW = E16.8)
451 FORMAT(7H CON = E16.8)
673 FORMAT(6H SS = E16.8)
920 FORMAT(6H PK = E16.8)
921 FORMAT(6H UE = E16.8)
922 FORMAT(6H TE = E16.8)
923 FORMAT(6H PE = E16.8)
924 FORMAT(9H TBARW = E16.8)
925 FORMAT(8H EPSW = E16.8)
926 FORMAT(9H ALAM1 = E16.8)
927 FORMAT(7H E1E = E16.8)
928 FORMAT(7H EPS = E16.8)
929 FORMAT(6H XL = E16.8)
930 FORMAT(9H POWER = E16.8)
931 FORMAT(8H SUMA = E16.8)
932 FORMAT(26H THESE ARE THE RESULTS OF 14,11H ITERATIONS)
933 FORMAT(28H CONVERGENCE OBTAINED AFTER 14, 11H ITERATIONS)
976 FORMAT(9H ALAM2 = E16.8)
END
APPENDIX F

THE OUTER-EDGE BOUNDARY CONDITION FOR THE ENTHALPY

In previous investigations dealing with the radiating boundary layer on a flat plate the enthalpy boundary condition which has generally been specified for the outer-edge of the boundary layer is the one given in eq. (63),

\[ h \rightarrow h_e \quad \text{as} \quad y \rightarrow \infty \]

However, there appears to be some question as to whether this is the proper outer-edge boundary condition for the enthalpy. A brief study of this outer-edge boundary condition problem indicates that it is not sufficient to only require that the enthalpy in the outer part of the boundary layer be asymptotic to some arbitrary value of the external flow enthalpy \( h_e \). The conclusions of the study, which agree with the comments of Olstad (ref. 28), show that, in addition, it is necessary to also insist that the value of \( h_e \) be consistent with the condition that the divergence of the radiation flux vector in the outer part of the boundary layer be asymptotic to the value of the divergence of the radiation flux vector for the external flow in the vicinity of the wall. In this manner, the enthalpy solution to the boundary layer energy equation (6) will have asymptotic character and its value on the outer-edge of the boundary layer will be equal to the value of the enthalpy for the radiating external flow in the vicinity of the wall. Thus, the proper value for \( h_e \) in eq. (63) is the wall enthalpy for the radiating external flow.

The outer-edge boundary condition which was employed in this investigation is the arbitrary value of \( h_e \) given in eq. (63). Of course this boundary condition for the enthalpy is only a first approximation to the proper outer-edge boundary condition discussed in the previous paragraph. However, the resulting enthalpy profiles in fig. 6(a) indicate that this approximation is quite good for the high Eckert number flows considered in the investigation. For the low Eckert number flows investigated, the resulting enthalpy profiles in figs. 6(b) and 8 indicate that this approximation is quite good for a combination of high external flow and low wall emissivities, but only fair for high wall and/or low external flow emissivities.

From figs. 6 and 8, it appears that the first-order approximation for the outer-edge boundary condition is poorest for a combination of low Eckert number, low external flow emissivity, and high wall emissivity. Hence, the effect of this approximation on the wall heat flux results will be greatest for the aforementioned emissivity and Eckert number conditions. The quantitative effect of the boundary condition approximation on the heat transfer results has been estimated for the combination of low Eckert number, low external flow emissivity, and high wall emissivity. The estimates yield the following values for the magnitude of the effect of the boundary condition approximation on the heat flux results at these specific Eckert number and emissivity conditions:
4.0% for $q_{cw,g}/q_{cwo}$ in fig. 9(b),
1.5% for $q_{cw}/q_{cw,g}$ in fig. 10(b),
0.8% for $q_{Rw}/q_{Rw,g}$ in fig. 11(b),
2.8% for $q_{Rw}/q_{cw}$ in fig. 12(b),
0.1% for $q_{tw}/q_{cwo}$ in fig. 13(b),
0.7% for $q_{tw}/q_{tw,g}$ in fig. 14(b).
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dynamic and Transport Properties of High-Temperature Nitrogen with

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Some Transport Properties of Equilibrium Dissociating Air for Use

Equations for Laminar Heat-Transfer Distribution in Equilibrium Air
at Velocities up to 41,100 feet per second. NASA TR R-118, 1961.
Figure 1. - Radiation contributions to the point $x_1$ from the elementary cone of solid angle $d\omega$ whose axis is the directed line $S(\xi_1)$. 
Figure 2.- Example of a nongrey absorption coefficient model obeying eq. (11).
Figure 3. - Flight paths for reentry into the earth's atmosphere from a Mars mission and a far solar system mission as given in ref. 4.
Figure 4.— The maximum boundary-layer Bouguer numbers for a 3 meters long flat plate where the reference thermodynamic states along the typical earth reentry trajectories are taken to be \((H_\infty, P_\infty)\).
Figure 5.-Step function model of the air absorption coefficient.
Figure 6.- Enthalpy profiles at $\bar{x} = 1.0$ for the grey radiating air boundary layer on a flat plate; $P_e = 0.1$ atm, $T_w = 2000^\circ$ K.

(a) $u_e^2/h_e = 16$, $T_e = 10 000^\circ$K, $L = 10$ m
Figure 6.- Concluded.
Figure 7. Normalized enthalpy profiles at $x = 1.0$ for the radiating air boundary layer on a flat plate; $u_e^2/h_e = 16$, $T_e = 10000^\circ K$, $P_e = 0.1$ atm, $L = 10$ m, $T_w = 2000^\circ K$. 

(a) $\varepsilon_w = 0.20$
Figure 7.- Concluded.
Figure 8.- Enthalpy profiles at $\bar{x} = 1.0$ for the radiating air boundary layer on a flat plate; $u^2/\nu_e = 0.7$, $T_e = 12000^\circ$ K, $P_e = 0.1$ atm, $L = 3$ m, $T_w = 2000^\circ$ K.
\( \varepsilon_1 e = 1.0, \varepsilon_2 e = 0.9 \)

\( \varepsilon_e = 0.902 \) (grey)

\( \varepsilon_1 e = 0.5, \varepsilon_2 e = 0.01 \)

\( \varepsilon_e = 0.020 \) (grey)

Figure 8. Concluded.
Figure 9.- Ratio of the convective wall heat flux for grey radiating air to that for nonradiating air for the flat plate boundary layer; 

\( P_e = 0.1 \) atm, \( T_w = 2000^\circ K \), \( L = 10 \) m.
Figure 9.- Concluded.

(b) \( u_e^2 / h_e = 0.7, \ T_e = 12000^\circ\ K, \ L = 3\ m \)
Figure 10.- Ratio of the nongrey to grey convective wall heat flux for the radiating air boundary layer on a flat plate; $P_e = 0.1 \text{ atm}$, $T_w = 2000^\circ \text{ K}$. 

(a) $u_e^2/h_e = 16$, $T_e = 10000^\circ \text{ K}$, $L = 10 \text{ m}$
Figure 10. Concluded.

(b) \( u_e^2/h_e = 0.7, T_e = 12\,000^\circ K, L = 3\, m \)
Figure 11.- Ratio of the nongrey to grey radiative wall heat flux for the radiating air boundary layer on a flat plate; \( P_e = 0.1 \) atm, \( T_w = 2000^\circ \) K.

\[ \frac{q_{Rw}}{q_{Rw, g}} \]

- \( \varepsilon_w = 0.95, \varepsilon_{le} = 1.0, \varepsilon_{2e} = 0.9, \varepsilon_e = 0.901 \)
- \( \varepsilon_w = 0.2, \varepsilon_{le} = 0.5, \varepsilon_e = 0.01, \varepsilon_e = 0.013 \)

(a) \( \frac{u_e^2}{h_e} = 16, T_e = 10000^\circ \) K, \( L = 10 \) m
(b) $u_e^2/h_e = 0.7$, $T_e = 12000^\circ K$, $L = 3$ m

Figure 11.—Concluded.
Figure 12.- Ratio of the radiative to convective wall heat flux for the nongrey radiating air boundary layer on a flat plate; $P_e = 0.1$ atm, $T_w = 2000^\circ$ K.

(a) $\frac{u_e^2}{h_e} = 16$, $T_e = 10,000^\circ$ K, $l = 10$ m
Figure 12.-- Concluded.

(b) $u_e^2 h_e = 0.7$, $T_e = 12000^\circ$ K, $L = 3$ m

Figure 12.-- Concluded.
Figure 13.- Ratio of the total wall heat flux for nongrey radiating air to the total wall heat flux for nonradiating air for the flat plate boundary layer; $u_e^2/h_e = 16$, $T_e = 10000^\circ K$, $L = 10$ m

(a) $u_e^2/h_e = 16$, $T_e = 10000^\circ K$, $L = 10$ m

Figure 13.- Ratio of the total wall heat flux for nongrey radiating air to the total wall heat flux for nonradiating air for the flat plate boundary layer; $P_e = 0.1$ atm, $T_w = 2000^\circ K$. 

90
(b) $u_e^2/h_e = 0.7$, $T_e = 12,000\,^\circ K$, $L = 3\, m$

Figure 13.- Concluded.
Figure 14. - Ratio of the nongrey to grey total wall heat flux for the radiating air boundary layer on a flat plate; $P_e = 0.1$ atm, $T_w = 2000^\circ$ K.

(a) $u_e^2/h_e = 16$, $T_e = 10000^\circ$ K, $L = 10$ m
Figure 14.— Concluded.
Figure 15 - $\alpha_j(\beta_j)$ as a function of $\lambda_jT$ where the numerical values plotted were taken from ref. 15.
Figure 15.— Continued.

(b) \(0.05 \leq \lambda_j T \leq 0.085\)

Figure 15.— Continued.
95
(c) \(0.085 \leq \lambda_j T \leq 0.225\)

Figure 15.- Concluded.
Figure 16.- The linear spectral absorption coefficient of air as derived from ref. 3.
Figure 16.—Continued.

(b) $T = 12000^\circ K$, $\rho/\rho_{s.l.} = 10^0$

Figure 16.—Continued.
Figure 16. - Continued.

(c) $T = 20000^\circ K$, $\rho/\rho_{s.l.} = 10^0$
(d) $T = 8000^\circ K$, $\rho/\rho_{s.i.} = 10^{-3}$

Figure 16.—Continued.

100
(e) $T = 12000^\circ K$, $\rho/\rho_{s.i.} = 10^{-3}$

Figure 16.—Continued.
Figure 16.— Continued.

Absorption coefficient, $K_{\lambda}$, cm$^{-1}$

Wavenumber, $\omega$, cm$^{-1}$

(f) $T = 20000^\circ$ K, $\rho/\rho_{s.l.} = 10^{-3}$

Figure 16.— Continued.
(g) $T = 8000 \degree K$, $\rho/\rho_{s.1.} = 10^{-6}$

Figure 16.- Continued.
Figure 16.—Continued.

(h) \( T = 12\,000^\circ \text{K}, \rho/\rho_{s,1} = 10^{-6} \)
Absorption coefficient, $K_A$, cm$^{-1}$

Wavenumber, $\omega$, cm$^{-1}$

(i) $T = 20,000^\circ K$, $\rho/\rho_{s\, l.} = 10^{-6}$

Figure 16.- Concluded.
Figure 17. - Comparison of the spectral absorption coefficients derived from refs. 3 and 21, 
$T = 10^4$ $\text{K}$, $\rho/\rho_{\text{sl.}} = 10^{-1}$. 
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—National Aeronautics and Space Act of 1958

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