RADIANT ENERGY TRANSFER MEASUREMENTS IN AIR

by H. Hoshizaki, A. D. Wood, J. C. Andrews, and K. H. Wilson

Prepared by
LOCKHEED AIRCRAFT CORPORATION
Palo Alto, Calif.

for Western Operations Office

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FOREWORD

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NOTATION

A  area of active element of thin film gage

A_p  area of aperture

A^*  area ratio (Sec. 4.8)

B_ν  Planckian intensity (spectral), W/cm^2-sr sec^{-1}

c  velocity of light in vacuo, 3.00 × 10^{10} cm/sec

C_0  constant in spectral absorption coefficient equation, cgs units

C_p  specific heat of gage substrate

C_{p_i}  i^{th} species specific heat at constant pressure

E  total (AC + DC) voltage

E_b  constant voltage

e  time-varying component of E

e_o  differential output voltage of bridge

\overline{e_D}_i  i^{th} species dissociation energy

\overline{e_I}_i  i^{th} species ionization energy

f_i  oscillator strength of i^{th} bound-bound transition, dimensionless

\overline{H}  total enthalpy

h  Planck constant, 6.62 × 10^{-27} erg sec

\hat{h}(x)  local streamline static enthalpy

I  radiant intensity, W/cm^2-sr or current through gage

I_L  current in reference leg of bridge
\( I_0, I_1 \) Bessel functions, Eq. (3.11)

\( I_\nu \) spectral radiant intensity

\( K \) gage calibration constant Eq. (4.7)

\( k \) Boltzmann constant, \( 1.38 \times 10^{-16} \text{ erg/k or thermal conductivity} \)

\( k' \) grouping of gage properties Eq. (4.3)

\( l \) total path length for intensity determination, cm

\( m \) index used in Sec. 4.5

\( N_i \) number density of \( i^{th} \) species in continuum absorption coefficient equation, particles/cm\(^3\)

\( n \) running index (Sec. 4.5)

\( m_i \) Molar fraction of \( i^{th} \) species

\( P \) power dissipation in gage

\( p(x) \) local streamline static pressure

\( q_r \) radiative flux, W/cm\(^2\)

\( q \) heat flux absorbed by gage

\( q^* \) constant value of a step function heat flux

\( q' \) reduced heat flux, \( q^* = q/K \)

\( R \) resistance of gage

\( R_p \) resistor in bridge or gage box circuit used to approximate a constant current source

\( R_1 \) resistors in bridge (Fig. 24)

\( R^* \) resistance ratio (Sec. 4.8)

\( S_i \) integrated line strength

\( \bar{T}(x) \) local streamline kinetic temperature

\( T \) absolute temperature (°K) or surface temperature excess (Sec. 4)
\( t \)  
**time or dummy optical length variable**

\( u \)  
**dimensionless frequency variable, \( hν/kT \)**

\( \bar{u}(x) \)  
**local streamline fluid velocity**

\( W_i \)  
**effective emission width of an isolated spectral line, \( \text{sec}^{-1} \)**

\( \dot{w}_i \)  
**rate of production of \( i^{\text{th}} \) component (species)**

\( y \)  
**coordinate in planar radiative transfer equation**

\( z_i \)  
**charge on residual ion resulting from photoionization process**

\( α \)  
**temperature coefficient of resistivity**

\( α_ν \)  
**spectral absorptance of gage surface**

\( \bar{α} \)  
**weighted gage absorptance, Eq. (4.12)**

\( β \)  
**bridge constant, Eq. (4.16)**

\( Γ_i \)  
**term structure constant**

\( γ_i \)  
**half half-width of electron impact broadened line, \( \text{sec}^{-1} \)**

\( θ \)  
**angular variable in planar radiative transfer equation or angle between gage-aperture line and aperture normal**

\( λ \)  
**dummy time variable**

\( μ^c_ν \)  
**continuum spectral absorption coefficient, \( \text{cm}^{-1} \)**

\( μ_ν \)  
**discrete spectral absorption coefficient, \( \text{cm}^{-1} \)**

\( μ'_ν \)  
**spectral absorption coefficient including induced emission, \( \text{cm}^{-1} \)**

\( ν \)  
**spectral frequency**

\( ν_i \)  
**frequency of line center for bound-bound transitions**

\( ν_o \)  
**quantum defect correction factor**

\( ρ \)  
**density, \( \text{gm/cm}^3 \) or distance between gage and aperture**
\[ \rho_o \] density of air at standard temperature and pressure, \( 1.29 \times 10^{-3} \text{ gm/cm}^3 \)

\[ \sum_{0}^{i}, \sum_{1}^{i} \] partition function of the parent and residual atom of \( i^{th} \) species involved in photoionization

\[ \tau \] optical length variable

\[ \phi_i \] summed photoionization cross section for all low lying states of a complex atom, \( \text{cm}^2 \)

\[ \omega \] solid angle subtended by aperture at gage

Subscripts

\( a, b \) refer to the two part gage (Sec. 4.8)

\( i \) refers to \( i^{th} \) bound-bound transition or refers to \( i^{th} \) chemical species

\( G \) refers to gage under ambient conditions

\( \nu \) refers to frequency variable

Superscripts

\( C \) refers to continuum absorption process

\( L \) refers to discrete absorption process
Section 1
INTRODUCTION

The transfer of radiant energy plays a fundamental role in the physical phenomena associated with entry into planetary atmospheres at superorbital flight velocities. It determines not only the radiative heat transfer but can also have an important effect on the convective heating. The convective heating is affected by radiant energy transfer through the radiation cooling effect and by the transfer of radiant energy between the inviscid and viscous flow regions in the shock layer. At entry velocities between two to three times satellite velocities, radiative heating and the proper evaluation of the radiation-convective coupling effects become of primary importance. The understanding of the physical phenomenon of superorbital entry and our ability to correctly predict the combined radiative and convective heating hinges on our knowledge of radiant energy transfer.

Early theoretical work on the fundamental aspects of radiant energy transfer in air consisted primarily of calculations of integrated emission coefficients (Refs. 1 and 2) which are related to the Planck mean absorption coefficient. These calculations of the emission coefficient considered only the continuum free-free and free-bound transitions over a limited frequency range. Discrete transitions (bound-bound) were not included. The frequency range considered excluded the vacuum ultraviolet. Subsequent theoretical work (Refs. 3, 4, and 5) has shown that free-bound transitions to the ground state which produce photons with frequencies in the vacuum ultraviolet can easily dominate the integrated emission coefficient. The magnitude of the absorption coefficients in this frequency range indicate that for most superorbital entry conditions of interest, self-absorption cannot be neglected.

Experimental investigations have been carried out in shock tubes (Refs. 6 and 7) to measure the integrated emission coefficient of air. These investigations employed
models in which a radiation gage was located behind a quartz window. The gage measured the radiation from the shock layer gas. The quartz window serves as a filter in that it absorbs the vacuum ultraviolet portion of the spectrum. The wavelengths which are passed by the quartz window are sufficiently long to be unattenuated by the shock layer gas. Thus the results of these experiments do not include the vacuum ultraviolet nor the effect of self-absorption; two very important aspects of superorbital entry radiation transfer.

The purpose of the present investigation is to obtain experimental data on the integrated intensity of high temperature air, including the vacuum ultraviolet, under conditions similar to those encountered during superorbital entry by full scale vehicles. The gas path lengths considered varies between 1 and 12 cm. For these values of the gas path length and the thermodynamic conditions considered, the effect of self-absorption is very important. The experiments were carried out in a 12-in. I.D. arc-driven shock tube. A gage-in-cavity model was used to measure the total intensity as a function of gas path length. This model enables the total radiation from the gas in the reflected shock region to be observed through a small windowless aperture. The radiation data are recorded before the test gas arrives at the radiation gage. Thus the problem of separating the radiative heating from the convective heating is avoided.
The primary objective of this investigation was to obtain integrated intensity data which includes the vacuum ultraviolet. This imposes the requirement that the radiation detector observe the test gas sample without any intervening media which can absorb or emit radiation in the vacuum ultraviolet. This requirement precludes the use of windows. The basic idea employed to carry out these measurements is to place a radiation detector in a cavity and measure the radiative flux through a small windowless aperture before the test gas arrives at the detector sensing surface.

Figure 1 is a schematic of the gage-in-cavity model showing some of its details and its location in the shock tube to enable the observation of the gas in the reflected shock region. Four thin film heat transfer gages are located in each cavity, approximately 2 in. from a small windowless aperture. Optical stops are located on the floor which allows selection of the gas path length. Flow details in the cavity during data acquisition are shown schematically in Fig. 2. The purpose of the cavity is to allow the test gas which flows through the aperture to expand quickly, thereby minimizing the absorption and emission from the expanding gas.

The chemical processes associated with the gas expansion in the cavity are not in equilibrium. In order to properly assess the effect of the expanding gas on the data, the number density and temperature distribution through the plume need to be determined. By making the assumption of radiative equilibrium, the absorption and emission by the plume can be evaluated using the number densities and kinetic temperature obtained from the chemical non-equilibrium calculation. These calculations have been carried out and are discussed in Sec. 3.

In addition to the effect of the plume, one must also consider the effect of test gas outflow through the aperture. This outflow will create a small, time dependent,
Fig. 1 Windowless Gage-in-Cavity Model
Fig. 2 Flow Details in Cavity During Radiation Data Acquisition
non-uniform region near the aperture which could have a significant effect for short
test gas path lengths. This possibility has also been investigated and is discussed in
Sec. 3. Other factors discussed in Sec. 3 are the possible effect of radiation cooling
on the thermodynamic state of the test case and possible non-equilibrium radiation
behind the reflected shock.
3.1 CONTINUUM RADIATION

For a plane parallel slab of gas with temperature gradients in the normal direction only, the expression for the net radiant flux is (Ref. 8)

\[ q_\tau = 2\pi \int_0^\infty \int_0^\infty I_\nu(\tau_\nu, \theta) \sin \theta \cos \theta \, d\theta \, d\nu \]  

(3.1)

where

\[ I_\nu(\tau_\nu, \theta) = \int_{\tau_\nu}^{\infty} B_\nu(t, \nu) e^{-(t-\tau_\nu) \sec \theta} \sec \theta \, dt - \int_0^{\tau_\nu} B_\nu(t, \nu) e^{-(\tau_\nu-t) \sec \theta} \sec \theta \, dt \]

\[ \tau_\nu = \int_0^y \mu_\nu \, dy \]

\[ B_\nu = \frac{2h\nu^3}{c^2} \frac{1}{e^{h\nu/kT} - 1} \]

The coordinate system employed is illustrated below.
In the current experiment, \( \theta \) is restricted to small values so that we can set \( \sec \theta \approx 1 \) and assume that the monochromatic intensity \( I_\nu \) is only a function of the monochromatic optical depth \( \tau_\nu \). The integration over the angle \( \theta \) can now be carried out from which the intensity can be expressed as

\[
I = \frac{q_\nu}{\omega} = \int_0^\infty \left[ \int_0^{\tau_\nu} B_\nu(t, \nu) e^{-(t-\tau_\nu)} dt - \int_0^{\tau_\nu} B_\nu(t, \nu) e^{-(\tau_\nu-t)} dt \right] d\nu \quad (3.2)
\]

where the solid angle \( \omega \) is given by

\[
\omega = 2\pi \int_0^\theta \sin \theta d\theta
\]

Before the intensity can be evaluated for a particular situation, the absorption coefficient of the gas must be known as a function of the thermodynamic properties. The temperatures which are of interest are sufficiently high that we can consider the gas to be composed of nitrogen and oxygen atoms and ions only. The continuum absorption coefficients (i.e., photon absorption resulting from bound-free and free-free transitions) are calculated from equations set down by Biberman (Ref. 9). Biberman's equations are based on an approximate treatment of the quantum defect method developed by Burgess and Seaton (Ref. 10) for calculation of the basic photon absorption cross sections. This approximate theory is applicable to free-free transitions and bound-free transitions from excited states for complex atoms such as nitrogen and oxygen. The photon cross sections for bound-free transitions from the ground and low-lying excited states are taken from Armstrong's work (Ref. 3). Armstrong has performed a very detailed cross section calculation using the quantum defect method for photon frequencies lying near the threshold value and a high energy theory for photon energies well above threshold. Armstrong's calculations are felt to be quite accurate and are used as a standard against which Biberman's approximate
equations are compared. Such comparisons at typical plasma conditions of interest are shown in Figs. 3 and 4. The particular equations used to calculate the continuum absorption coefficient based on the methods summarized above are described in Appendix A.

The intensity incident on the gage surface was calculated by means of Eq. (3.2) using the absorption coefficient discussed above. The results are presented in Fig. 5 as a function of gas path length for several temperatures about the nominal experimental values. The variation of intensity with temperature indicated by these results were used to adjust the experimental data to a common temperature (17,000 and 14,700°K for the two experimental conditions considered). Also shown on Fig. 5 are the intensity obtained by using Armstrong's value of the absorption coefficient directly rather than the curve fits. The agreement between the hand calculation and the computer results using the analytical expressions for the absorption coefficient is very good.

The monochromatic intensities for the two test conditions are shown in Figs. 6 and 7. Note that for the present conditions, the vacuum ultraviolet is very close to the black-body curve.

3.2 LINE RADIATION

The number of transitions between various atomic states in a plasma of complex atoms and ions which give rise to discrete emission and absorption processes is typically on the order of several hundred or more. Hence a systematic accounting of all these transitions in a transport calculation appears impractical. Biberman (Ref. 11) and Vorobev (Ref. 12) point out, however, that only a relatively small number of transitions, namely those involving the ground or low lying excited states need be considered individually. The remaining transitions can be considered by aggregate formulas. We have performed a somewhat more complete analysis of the line transport problem than that reported by Biberman and his colleagues. In particular, we have considered
Fig. 3 Comparison of Biberman and Armstrong Continuum Absorption Coefficients for Air

P = 1.0 atm
T = 14,000°K
Fig. 4 Comparison of Biberman and Armstrong Continuum Absorption Coefficients for Nitrogen

$T = 16,000^\circ\text{K}$

$\rho/\rho_0 = 10^{-2}$
Fig. 5: Integrated Continuum Intensity
Fig. 6 Monochromatic Continuum Intensity: $T = 14,700^\circ K; \rho/\rho_o = 3.45 \times 10^{-2}$
Fig. 7 Monochromatic Continuum Intensity: $T = 17,000^\circ K; \rho/\rho_o = 3.18 \times 10^{-2}$
more transitions* and used the oscillator strengths and half-widths for each individual transition as calculated in detail by Armstrong (Ref. 3). For example, while Biberman considers only transitions involving the lowest energy configuration of the parent core electrons, Armstrong's tabulation includes a large number of excited core parentages. While certainly not dominant, these excited core transitions are not entirely negligible and we have included those judged significant. On the whole, however, we have maintained the spirit of Biberman's approach by retaining only those transitions which contribute significantly to the overall line transport.

The overall line transport problem can be examined in terms of the transport by a single isolated line (or isolated groups of lines where overlapping is important). For an isothermal plasma the total energy intensity including both continuum and discrete processes, emergent in a given direction from a path length $\ell$, is

$$I_{\text{total}} = \int_0^\infty B(\nu) \left( 1 - \exp \left( -\left( \mu^c(\nu) + \sum_{i} \mu_i^L(\nu) \right) \right) \right) d\nu$$  \hspace{1cm} (3.3)

where $\mu^c(\nu)$ is the continuum contribution to the absorption coefficient and $\sum_{i} \mu_i^L(\nu)$ the sum of all discrete contributions to the absorption coefficient.

Equation (3.3) may be rewritten as,

$$I = \int_0^\infty B(\nu) \left( 1 - \exp \left( -\mu^c(\nu) \ell \right) \right) d\nu + \int_0^\infty B(\nu) \exp \left( -\mu^c(\nu) \ell \right) \left( 1 - \exp \left( -\sum_{i} \mu_i^c(\nu) \ell \right) \right) d\nu$$  \hspace{1cm} (3.4)

*The word transition is used in this report to define a multiplet array. The oscillator strengths and half-widths calculated by Armstrong are for the totality of individual line transitions within a given multiplet.
The first term on the right-hand side of Eq. (3.4) is simply the continuum portion of the total intensity,

$$I_{\text{cont.}} = \int_0^\infty B(\nu) \left[ 1 - \exp \left( -\mu_c(\nu) \ell \right) \right] d\nu$$  (3.5)

If the discrete contribution consists of a series of isolated lines (or groups of isolated lines) for which $\mu_1^L(\nu)$ is finite only over a small interval $\Delta\nu_i$, then the second term on the right-hand side of Eq. (3.4) becomes

$$I_{\text{line}} = \sum_i \exp \left( -\mu_1^c \ell \right) I_i^L$$  (3.6a)

where

$$I_i^L = B_i \int_{\Delta\nu_i} \left\{ 1 - \exp \left( -\mu_1^I(\nu) \ell \right) \right\} d\nu$$  (3.6b)

In Eq. (3.6) we have assumed that $\mu_1^c$ and $B_i$ are constant over each line interval $\Delta\nu_i$. Combining Eqs. (3.5) and (3.6) with Eq. (3.3) yields

$$I_{\text{total}} = I_{\text{cont.}} + \sum_i \exp \left( -\mu_1^c \ell \right) I_i^L$$  (3.7)

Hence we can calculate the total intensity by separately calculating the continuum contribution and each isolated line contribution and combining the contributions according to Eq. (3.7). The basic problem in determining the energy transported by line emission
and absorption processes is thus the evaluation of $I_1^L$ from Eq. (3.6b). Consider an isolated line for which we represent the line absorption coefficient as

$$\mu_1^L(\nu) = S_1 b_1(\nu)$$  \hspace{1cm} (3.8)

where $S_1$ is the integrated absorptivity

$$S_1 = \int_{0}^{\infty} \mu_1^L(\nu) d\nu = \frac{\pi e^2}{mc} N n f_{nn}$$  \hspace{1cm} (3.9)

and $b_1(\nu)$ is the normalized line shape defined such that

$$\int b_1(\nu) d\nu = 1$$

The solution to the problem of line transport rests upon a knowledge of the oscillator strength $f_{nn}$, and line shape $b_1(\nu)$.

Armstrong and his colleagues have calculated values of the oscillator strength for a large number of multiplet transitions (including all significant transitions) in nitrogen and oxygen atoms. It should be emphasized that Armstrong's work represents the culmination of a large research effort directed towards a precise theoretical determination of oscillator strengths using the best current quantum mechanical calculational methods. The soundness of Armstrong's calculations can be judged by comparison with recently published experimental data. Both the experimental data obtained by Lawrence and Savage (Ref. 14), and the NBS tabulation (Ref. 15) of the "best" experimental and theoretical values of atomic transition probabilities agree well with Armstrong's work. In particular, for the important "resonance" transitions in the UV from the low-lying excited states to the ground states of nitrogen, Armstrong's f-numbers all agree within 25% of the measured values. The f-number calculation for
these "resonance" transitions are the most difficult and uncertain from a theoretical viewpoint. Hence agreement with experiment for these transitions gives confidence to the theoretical method.

It is of interest to compare Armstrong's results with those of Vorobev (Ref. 12) and Griem (Ref. 16). For both the transitions which lie in the UV and in the visible--ir regions of the spectrum, the Armstrong and Vorobev values for the f-numbers agree well; for most lines within 25% or less. Also, Griem's values for the f-numbers for transitions which lie in the visible--ir region of the spectral agree quite well with those calculated by Armstrong. However, for the important UV transitions, Griem's values for the f-numbers are considerably lower than those calculated by Armstrong (and Biberman) being lower by, typically, a factor of 30. This large discrepancy is due to Griem's use of Coulomb wave functions in evaluating the f-numbers for the transitions to the ground states. Finally, it is important to note, that Griem's tabulation of transitions in the UV is not (nor was it intended to be) a complete list of the significance transitions. Indeed, the \( nd \rightarrow 2p \), \( n \geq 4 \), transitions which are not listed in Griem, carry as much as 40% of the total radiative energy transported in lines at conditions which are of interest from both experimental and reentry investigations, (see Ref. 12). Hence the use of Griem's f-number data in calculating radiative line transport will lead to a large underprediction.

Griem (Ref. 16) has made detailed line shape calculations for isolated lines of low atomic number elements including nitrogen and oxygen. Griem's calculations lead to the conclusion that, for the relatively dense plasmas of interest where the electron number density is at least as large as \( 10^{16} \) particles/cm\(^3\), electron impact broadening dominates. The contribution from ion broadening under the quasi--static approximation is on the order of a few percent and can be neglected for the purposes of transport calculations. Hence, the lines acquire a Lorentz (dispersion) shape,

\[
b_1(\nu) = \frac{\gamma_1/\pi}{(\nu - \nu_1 - d_1)^2 + \gamma_1^2} \tag{3.10}
\]
with a (half) half-width $\gamma_1$ and a line shift $d_1$. Griem's calculations show that the line shift is of the same order as the half-width. Since the magnitude of the line shift is not sufficiently great to produce overlapping on a multiplet basis, then for the isothermal conditions of immediate interest, we can neglect the line shift term, $d_1$, in Eq. (3.10). Then $b_i(\nu)$ is given by the somewhat more simple expression,

$$b_i(\nu) = \frac{\gamma_1/\pi}{(\nu - \nu_i)^2 + \gamma_1^2} \quad (3.11)$$

In terms of Eq. (3.11) the line shape is determined by the half-width $\gamma_1$. We have used the half-widths calculated by Armstrong (Ref. 3) whose method follows the work of Stewart and Pyatt (Ref. 17). A comparison of Armstrong’s half-widths with those presented by Griem shows Armstrong’s values to be consistently large by, roughly, a factor of 5, in the temperature range of interest. Griem’s work is the most detailed treatment currently available. As Griem points out, only the wave functions for the upper state are required in the half-width calculation. Then the use of Coulomb wave functions half-widths to calculate should be applicable without a serious degradation of accuracy. Hence, the half-widths tabulated by Griem for transitions will be reasonably accurate. Unfortunately, Griem tabulates half-widths for only three significant transitions in the UV and a limited number for the visible–IR transitions. The only complete tabulation of half-widths are those provided by Armstrong, which, from necessity, are used in our analysis. The use of Armstrong’s $\gamma_1$ values represents the largest source of uncertainty in the theoretical line transport predictions.

Proceeding with the solution to the transport equation, we evaluated Eq. (3.6b) using the line shape given by Eq. (3.11). Following the analysis of Plass (Ref. 18) for the case of an isolated line with a Lorentz shape, the effective (emission) width of the $i^{th}$ line is defined by
\[ W_i = \frac{I_i^L}{B_i} = \int_{\Delta \nu_i} \left\{ 1 - \exp \left[ -\mu_i^L(\nu) \right] \right\} d\nu \]  

(3.12)

and inserting the Lorentz shape in Eq. (3.12) yields

\[ \frac{W_i}{2\pi \gamma_i} = \frac{\tau_i}{2} \exp \left( -\frac{\tau_i}{2} \right) \left[ J_0 \left( \frac{\tau_i}{2} \right) + J_1 \left( \frac{\tau_i}{2} \right) \right] \]  

(3.13)

where \( \tau_i \) is the optical depth, \( \mu_i^L(\nu = \nu_i) \) based on the absorption coefficient at the line center, \( \nu = \nu_i \),

\[ \mu_i^L(\nu = \nu_i) = \frac{s_i}{\pi \gamma_i} \]

and \( J_0, J_1 \) are Bessel functions of an imaginary argument. The exact solution for the effective width afforded by Eq. (3.13) is cumbersome as it involves the use of Bessel functions. However, two useful limiting expressions for \( W_i \) are available. One is the thin limit \( \tau_i \ll 1 \), for which

\[ \frac{W_i}{2\pi \gamma_i} = \frac{\tau_i}{2} \]  

(3.14a)

and the other is the thick limit \( \tau_i \gg 1 \), for which

\[ \frac{W_i}{2\pi \gamma_i} = \left( \frac{\tau_i}{\tau^2} \right)^{1/2} \]  

(3.14b)

The "square-root" law of Eq. (3.14b) can be substantiated physically by realizing that Eq. (3.14b) follows directly from Eq. (3.12) if we neglect the term \( \gamma_i^2 \) in comparison
to \((\nu - \nu_i)^2\) in the denominator in Eq. (3.11). Deleting \(\gamma_i^2\) causes the absorption coefficient to become infinitely large at the line center. However, if the actual absorption coefficient is sufficiently large at the center, i.e., \(\tau_i \gg 1\), then the emission is already blackbody limited and no substantial error is encountered in neglecting the \(\gamma_i^2\) term. A comparison of the limiting solutions from Eqs. (3.14a) and (3.14b) with the exact solution from Eq. (3.13) is shown in Fig. 8. The line transport is evaluated from either Eq. (3.14a) for \(\tau_i < 1\) or Eq. (3.14b) for \(\tau_i > 1\). Reference to Fig. 8 shows that the maximum error of about 25% introduced by these approximate relations is roughly at \(\tau_i = 1\). Since most of the transitions are at conditions where \(\tau_i \ll 1\) or \(\tau_i \gg 1\), the aggregate error is much less than the maximum of 25%.

Calculations were made of the radiative intensity resulting from line transitions from an isothermal nitrogen plasma. The following thermodynamic conditions were considered corresponding to the nominal experimental target conditions:

\[
\begin{align*}
T &= 14,700^\circ K & T &= 17,000^\circ K \\
\rho/\rho_o &= 3.45 \times 10^{-2} & \rho/\rho_o &= 3.18 \times 10^{-2}
\end{align*}
\]

where \(\rho_o\) is the density at standard conditions, \(\rho_o = 1.29 \times 10^{-3}\) g/cm\(^3\).

The spectral distribution of the total intensity (continuum plus lines) is shown in Figs. 9a and 9b for the 14,700\(^\circ\)K temperature condition at pathlengths of 3 and 12 cm. The monochromatic continuum intensity is taken from Fig. 6. The line intensity calculated for each group of transitions is the total energy (i.e., spectrally integrated) within that spectral group. To display the spectral location of the energy emitted in lines, it is convenient to plot the equivalent width of blackbody intensity for each group of lines. Thus the shaded areas limited by the blackbody intensity shown in Figs. 9a and 9b represent schematically the line intensity. Each shaded area is equal to the total line radiation from the transitions located within the isolated spectral intervals shown. The numbers listed adjacent to each spectral group give the percent
Fig. 8 Effective Width of a Lorentz Line as a Function of Optical Depth
Fig. 9  Monochromatic Intensity From an Isothermal Air Plasma: (a) $T = 14,700^\circ K$, $\rho/\rho_0 = 3.45 \times 10^{-2}$, $l = 3$ cm; (b) $T = 14,700^\circ K$, $\rho/\rho_0 = 3.45 \times 10^{-2}$, $l = 12$ cm
of the total (continuum plus line) energy carried by each group of lines. For the case
where the lines within a spectral group are black and overlap (as occurs for most of
the important transitions at these dense plasma conditions), then the shaded areas in
Figs. 9a and 9b reflect the actual monochromatic intensity resulting from line
transitions.

Note the predominance of the UV transitions in the line transport. We list below
some of the important transitions within the various spectral groups (see Moore's
tables of energy levels (Ref. 19) for the spectroscopic notation):

<table>
<thead>
<tr>
<th>Spectral Group</th>
<th>Multiplets</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h\nu = 7.11$ eV</td>
<td>$2p^3 2p^0 \rightarrow 3s^2p$</td>
</tr>
<tr>
<td>$h\nu = 8.30$ eV</td>
<td>$2p^3 2D^0 \rightarrow 3s^2p$</td>
</tr>
<tr>
<td>$h\nu = 9.4$ eV</td>
<td>$2p^3 2p^0 \rightarrow 3d^2D$</td>
</tr>
<tr>
<td>$h\nu = 10.4$ eV</td>
<td>$2p^3 4S^0 \rightarrow 3s^4p$</td>
</tr>
<tr>
<td></td>
<td>$2p^3 2D^0 \rightarrow 4s$</td>
</tr>
<tr>
<td></td>
<td>$2p^3 2p^0 \rightarrow nd (n \geq 4)$</td>
</tr>
<tr>
<td></td>
<td>$2p^3 2D^0 \rightarrow 3d^2F$</td>
</tr>
</tbody>
</table>

Except for the upper members of the $2p^3 2p^0 \rightarrow nd$ series, the above transitions are
low lying with relatively small half-widths. As a result, these lines are strongly
reabsorbed so that a knowledge of the line shape, particularly in the wings, is impor-
tant. Additionally, in the important spectral region from 10.0 eV to 10.8 eV, the
above lines overlap in terms of the effective emission widths. This overlapping must
be considered in order to calculate meaningful intensity values.

In the infrared portion of the spectrum, lines are relatively unimportant (in terms of
the total line transport) at short pathlengths but increase in importance at longer
pathlengths as will be observed by comparing Figs. 9a and 9b. In the infrared, the important transitions are, primarily, the various multiplets in the $3s \rightarrow 3p$ and $3p \rightarrow 3d$ series.

The integrated line intensity is obtained by summing the contributions from the spectral groups shown in Figs. 9a and 9b with the continuum attenuation included. The resultant total line intensity versus pathlength prediction is the solid curves in Fig. 10a. The region of transmittance of a Suprasil window and a Wratten bandpass filter is shown in Figs. 9a and 9b. By summing only contributions which lie in these spectral regions, the prediction of the line intensity transmitted by a window and filter given in Fig. 10b is obtained.

A comparison of the line intensity predictions of Biberman, et. al (Ref. 20) at the two thermodynamic conditions and at pathlengths of 1 and 10 cm is included in Fig. 10a. There are two sources of uncertainty in extracting an intensity value from Biberman's data. First, Biberman tabulates in Ref. 20 only the total (lines plus continuum) flux emitted by an isothermal slab. An estimate is made of the relative contribution of lines and continuum to the total flux based on graphs in Refs. 12 and 20. Finally, the line flux values thus obtained must be converted into an intensity by estimating the effective solid angle for the energy transported in lines.

As mentioned above, Armstrong's half-width values are larger than Griem's values by roughly a factor of five at the temperatures of interest for the transitions compared. From a theoretical viewpoint, Griem's calculations are more accurate since Griem has performed a more detailed solution. In order to assess the effect of uncertainty in Armstrong's half-width data, the line transport at $T = 14,700^\circ$K was re-evaluated using half-widths which were uniformly reduced by a factor of five from Armstrong's values. The resultant line intensity is given by the dashed curve in Fig. 10a. It is seen that the line intensity is reduced by about 30% uniformly with pathlength, considerably less than the factor of five decrease in the half-widths. This relative insensitivity of the total line intensity to changes in the half-width values is a result
Fig. 10 Line Intensity From an Isothermal Nitrogen Plasma: (a) All Lines; (b) Lines Transmitted Through a Window and Window Plus Filter
of three factors. First, the intensity from line transitions which are optically thin is independent of the half-width. Second, the intensity from line transitions which are optically thick vary as $\gamma^{1/2}$. Third, strong overlapping of optically thick lines results in a $\gamma$ dependence which is much less than a one-half power variation.

3.3 GAGE SURFACE ABSORPTANCE

The radiative intensity measurements were carried out by means of platinum thin film gages. The platinum films were coated with silicon monoxide or aluminum black. The silicon monoxide provided an insulating surface over the platinum film which was found necessary to prevent the gage from being shorted by the gas in the cavity. The radiation from the test gas in the windowless configuration photo-dissociates and/or photoionizes the gas in the cavity which causes the gages to short to their surroundings if they are not insulated. At the lower of the two temperatures investigated, nominally 14,700°K, the intensity incident upon the gages located behind a quartz window were measured with platinum films coated with aluminum black. The aluminum black has a very uniform (with frequency) absorptance close to unity.

Unfortunately, unlike aluminum black, the silicon monoxide coating does not absorb uniformly over the radiation frequency range of interest. The surface absorptance for several specimens, which were prepared in the same manner as the thin film gages, were measured on a Beckman DK-2 spectrophotometer. The results of three such measurements are presented in Fig. 11. These silicon monoxide absorptance curves exhibit interference patterns in the near infrared. In the visible and ultraviolet, the surface absorptance is more uniform and approaches unity at the higher frequencies, (shorter wave lengths). Although these absorptance curves differ in detail, when the monochromatic continuum intensity absorbed by the gage surface is integrated over frequency, the variation in the integrated continuum intensity from one absorptance curve to another is quite small as shown in Figs. 12 and 13. The surface absorptance of the silicon monoxide coating reduces the radiative intensity absorbed by the gage by 10 to 20% from the value assuming a surface absorptance
Fig. 11 Silicon Monoxide Coated Gage Surface Absorptance
Fig. 12 Effect of Surface Absorptance and Window and Window Plus Filter Transmittance on Continuum Intensity:  $T = 14,700^\circ K$; $\rho/\rho_o = 3.45 \times 10^{-2}$
Fig. 13 Effect of Surface Absorptance and Window Transmittance on Continuum Intensity: $T = 17,000^\circ$K; $\rho/\rho_0 = 3.18 \times 10^{-2}$
coefficient of unity. As the gas path length is increased, the infrared portion of the spectrum becomes increasingly more important since it is not nearly absorbed as much as the ultraviolet. This causes the silicon monoxide surface absorptance to have a larger effect in reducing the intensity absorbed by the gage.

Also shown in Fig. 12 are the intensities incident upon the gage surface when a quartz window or a quartz window in combination with a filter is placed between the radiating gas and the gage. Similar results are shown in Fig. 13 for the case of the quartz window alone. The transmittance curves for the quartz window and the filter are presented in Fig. 14. These transmittance curves were obtained from measurements carried out as part of the current investigation.

3.4 EFFECT OF CAVITY PLUME

When data without a window are being taken, the test gas expands into the cavity, forming a time dependent plume which is at a lower temperature and density than the test gas and hence is not expected to significantly absorb or emit radiation. However, to verify this assumption, detailed calculations were carried out. The chemical processes associated with this rapid expansion of the test gas are not in equilibrium. The number density of the atoms and electrons and the kinetic temperature were calculated using a one-dimensional, one-equilibrium streamtube analysis. This streamtube analysis requires the pressure distribution along the streamline as input data. The pressure distribution along the axial streamline was obtained from characteristic solutions for steady state plumes in chemical equilibrium. The details of the streamtube analysis are presented in Appendix B.

In carrying out these non-equilibrium calculations it was assumed that the test gas first expands to sonic conditions at the orifice. This expansion to sonic conditions was assumed to be in chemical equilibrium. The non-equilibrium streamtube analysis was then used to calculate the number densities and temperature along the axial streamline. The results of these calculations are presented in Fig. 15. These number densities and temperature profiles were used to calculate the absorption coefficient
Fig. 14 Transmittance of Quartz (Suprasil) Window and Window Plus Wratten No. 25 Filter
PLUME EXPANSION, NON-EQUILIBRIUM PARTICLE DENSITY
INITIAL CONCENTRATIONS SET AT SONIC ORIFICE CONDITIONS

RESERVOIR CONDITIONS (EQUILIBRIUM)
T = 17,000 K, P = 6 atm

SONIC ORIFICE CONDITIONS (EQUILIBRIUM)
T = 15,900 K, P = 3.3 atm

STARTING CONDITIONS AT x = 5 \times 10^{-4} \text{ cm (EQUILIBRIUM)}
T = 15,800 K, P = 3.29 atm

DISTANCE DOWNSTREAM FROM SONIC ORIFICE (cm)

Fig. 15 Plume Flow Properties
and the resultant intensity to the gage surface. These results are shown in Fig. 12, where it is seen that the absorption by the plume is negligible. In these calculations the intensity was evaluated along the plume axis so that the emission from the plume itself was not included. The number density and temperature profiles normal to the plume axis are not known and hence a precise evaluation of the emission from the plume cannot be carried out. However, the temperature along the axis decreases rather rapidly and it is expected that the temperature will decrease in a similar manner along the neighboring streamlines. If this is the case, the emission from the plume will be negligible since only a small fraction of the plume near the orifice will emit to any significant degree. Furthermore, the maximum plume temperature is the temperature of the gas at the orifice so that the emission per unit mass from the plume is significantly less than that in the test gas.

3.5 ORIFICE OUTFLOW EFFECT

When the test gas expands into the cavity an expansion wave will be propagated into the test gas. This expansion wave will lower both the number densities and the temperature in the region near the orifice. This non-uniform region near the orifice is, of course, time dependent. It has been estimated that the expansion wave will travel approximately 4 cm in 10 μsec. The change in the number density and temperature caused by the passage of the expansion wave has not been precisely evaluated. The possible effect of this expansion wave in reducing the radiative intensity was investigated experimentally by obtaining intensity data behind a window with the window placed at the aperture and on subsequent shots, with the window moved back from the aperture and placed in front of the gage. With the window at the aperture, there is no gas expansion into the cavity. With the window placed immediately in front of the gage, which is located approximately 5.6 cm from the aperture, the test gas will expand into the cavity. The data obtained with the windows placed as described above, show no discernible differences. The intensity data obtained are discussed in Sec. 5. From these results it was concluded that the expansion wave in the test gas has no measurable effect on the intensity data.
3.6 RADIATION COOLING AND NON-EQUILIBRIUM RADIATION

In the current experiment, the temperature behind the incident and reflected shocks are sufficiently high so that the possible effect of energy loss by radiation must be considered. Radiation cooling can affect shock tube flows by two means. The first is by attenuation of the incident shock caused by radiation cooling of the gas behind the incident shock. The second is by cooling of the test gas behind the incident shock and in the reflected shock region.

A simplified analysis of the effect of radiation cooling on the flow properties behind the incident shock was carried out in Ref. (21). In this analysis, the gas was assumed to be optically thin and simple approximate expressions were used for the integrated emissivity. The results indicate that for the shock velocities (9.5–10.5 mm/μsec) and initial pressures (0.200 Torr) of this experiment, radiation cooling will have a negligible effect on the gas properties behind the incident shock. The analysis, however, neglects the vacuum ultraviolet and self-absorption. These two effects compensate each other and the results of the analysis should be indicative of the actual condition.

The effect of radiation cooling on the test gas in the reflected shock region is much more difficult to assess. The reason being that neither the quantity of test gas in the reflected shock region is precisely known nor is the emission from the driver gas. The cooling of the test gas is dependent on the net exchange of radiant energy between the test gas, the driver gas and the surrounding surfaces. An estimate of the energy lost by radiation indicates that if one neglects the radiant energy transfer from the driver gas, approximately 10% of the internal energy will be lost in 10 μsec. This decrease in internal energy will result in approximately a 10% decrease in temperature and a 10 to 20% decrease in the radiant intensity. This slight decrease in the radiant intensity is evident in some of the photomultiplier records which are used to monitor the test gas (see Sec. 4.2). The resolution of the thin film heat transfer gages, however, is not sufficient to consistently detect this radiation cooling effect.
One of the fortunate circumstances of the present experimental setup is that the non-equilibrium region will move out of the field of view of the radiation gages. Based on the ionization rate data presented in Ref. 22, the time for the ionization relaxation zone to move out of the field of view of the gages has been estimated to be between 2 and 4 μsec. This time is comparable to the time necessary for the reflected shock to move across the field of view of the gages and is an inherent initial transient in this experiment. It appears, therefore, that non-equilibrium radiation is not an important consideration.
Section 4
EXPERIMENTAL METHOD

4.1 SHOCK TUBE

The LMSC 12 in. arc-driven shock tube was used exclusively in this research. The shock tube driver had an inside diameter of 2-3/4 in. and was 30 in. long. The driver gas was helium at 100 psig initially and was heated by a 30-in. axially-concentric arc struck between a beryllium-copper anode and a stainless steel diaphragm. The driver contained a Lexan liner with a 1/8 in. thick wall and which lasted about 25 shots on the average.

The energy for the arc was supplied by a 4800-μF capacitor bank capable of being charged to 20 kV. The first of the two experimental shock tube conditions used in this research was obtained by charging the full bank to 12 kV resulting in an available energy of 350,000 J. This would drive the incident shock at about Mach 30 (10.5 mm/μsec). The other and lower temperature condition was obtained by disconnecting 3/8 of the capacitor bank and holding all other parameters constant. Here incident shock Mach numbers of about 25.4 (8.9 mm/μsec) were obtained. In both cases the residual bank voltage of typically 4.5 kV meant that about 85% of the available energy was utilized.

The driver and driven sections were connected by a 4-ft shock formation section with a 3-in. inside diameter followed by a conical transition section. This section expanded the inside diameter from 3 to 12 in. and had a 10-deg half-angle with both the entrance and exit regions contoured to minimize the effect of secondary shock waves.

The driven section was 38 ft long with a nominal inside diameter of 12 in. and is shown in Fig. 16. The stainless steel driven section had a 32-μin. finish and the concentricity
Fig. 16 The 12-in. Arc-Driven Shock Tube at the Lockheed Palo Alto Research Laboratory
of the three 12 ft sections was held to ±0.002 in. to minimize disturbances at the joints. The driven section was pumped through a large contour valve in a 2-ft section located at the driver end of the tube.

The test gas was bottled air obtained from the Matheson Company and labeled 21% O\textsubscript{2}, balance N\textsubscript{2}. This gas was purified by passing it first through a dry ice-acetone cold trap, then through a 6-ft length of tubing filled with Ascarite to remove CO\textsubscript{2} as a possible impurity, and finally through 6 ft of Drierite (CaSO\textsubscript{4}) to remove any residual water vapor. All experiments were performed at an initial driven section pressure, P\textsubscript{1}, within 1% of 200 μ. The tube was filled through a small storage reservoir and the entire filling system was calibrated with a McLeod gage. Prior to each shot the tube was evacuated to below 1 μ and purged several times with test gas. The leak rate was always below 1 μ/min and a time of 1 min was required between closing the contour valve and firing the tube.

The shock tube was contaminated after each shot by a very fine black dust from the Lexan liner. This was removed by pulling a dry rotating brush through the tube followed by several passes with dry cloth wipers wrapped around the brush. Then all joints and instrumentation parts were opened and wiped clean. On several occasions the tube was thoroughly cleaned with soap and water but there was no noticeable change in the radiation data after this more extensive cleaning.

All the shock tube instrumentation was located in the last 12 ft of the driven section. This included a vacuum gage (NRC Equipment Corp., Alphatron), the test gas reservoir, an incident shock profile phototube, and three ionization-type shock speed detectors. Most of these items are visible near the top of Fig. 16.

The shock speed detectors were spaced 1 ft apart and the last one was located slightly behind the leading edge of the model splitter plates. The outputs of the shock speed detectors were displayed on oscilloscopes using Textronix type 53/54D plug-in units operated in the A-B mode. Thus the outputs of two adjacent detectors were displayed
on the same calibrated sweep trace as shown in the top of Fig. 17. It was estimated that the time interval could be determined to ± 0.5% and since the distance between the detectors was much better known, the shock speed was measured to ± 0.5%. The difference between the two shock speed measurements obtained from the three detectors was always within this uncertainty. The temperature and density of the test gas behind the reflected shock were obtained from the recent report by Laird and Heron (Ref. 23).

The quality of each shot was assessed from the output of a collimated phototube located about 4 ft upstream of the model and which measured the radiation profile along the incident shock. Examples of this profile are given on Fig. 17 for the two test conditions used in this research. Here the incident shock and the interface arrival are clearly seen. The non-equilibrium radiation overshoot was clearly visible at $M_s = 25.4$ conditions but was not apparent at $M_s = 30.1$. The precursor radiation apparent on these traces is attributed to scattering off the opposite wall of the shock tube. The quality of the shot was a much stronger function of the diaphragm parameters (thickness and depth of scribe) than had heretofore been suspected. Optimum values were stainless stcl 0.093-in. thick and scribed 0.053-in. dcep.

4.2 GAGE-IN-CAVITY MODEL

The gage-in-cavity model is best described from the three views shown in Fig. 18. The photograph at the lower left shows the model ready for insertion into the end of the shock tube with the exception of a top ring which is discussed in Sec. 4.10. The view at the upper-left shows the model rotated 90 deg and with a cavity cover removed. Here four thin film gages with taped and magnetically shielded leads are visible. These gages were pointed toward the windowless aperture which is also visible immediately above the gages. The object between the gages is a Pyrex light pipe which was collimated to give the same field of view as the gages. The light was fed to a cavity profile phototube through a flexible fiber optic bundle. The output of this phototube was used to provide a time reference and also to determine the time variation of the radiation (at least the visible part of it) from the test gas. Typical phototube outputs
Fig. 17 Examples of Shock Speed Measurement and Outputs of Profile Phototubes
Rear view of gages and light pipe

Model complete with exception of top ring

Gas-side view showing stops and aperture

Fig. 18 Gage-in-Cavity Model
are shown in Figs. 17 and 31. In all cases a decrease with time is apparent. This
effect was attributed to radiation cooling and is discussed in Sec. 3.6.

The view at the upper-right of Fig. 18 shows a gas-side view of a windowless aperture
and the elevated floor of the splitter plates with the stops used to fix the geometric path
length of the various gage channels. As shown, the apertures were mounted flush with
the gas-side of the splitter plates. The window-type apertures (not shown) contained
studs on the gage side which were used to clamp the Suprasil windows against the rear
of the aperture. All apertures were cut from 0.017-in. thick nickel alloy steel using
an ultrasonic cutter to provide a rectangular opening. The dimensions of this opening
were nominally 0.047-in. wide by 0.222-in. long. The area of each aperture was
measured on an optical comparator. The model splitter plates were blackened by a
chemically-applied diffuse black oxide to minimize spurious reflections. The interior
of the gage cavity was painted with flat black paint for the same reason.

The field of view of the gages was restricted in the direction normal to the incident
and reflected shock waves both to minimize any possibility of interference from the
driver gas interface and to reduce the transient caused by the passage of the reflected
shock across this field of view. In a direction parallel with the shock waves, the field
of view was much larger to provide a fairly high heat flux incident on the gages. Two
views of the actual field of view are sketched in Fig. 19 for the 12.2 cm path length
case. The other path length cases were obtained by moving the stop toward the aper-
ture. This geometry created a transient period of about 3 μsec and made any inter-
ference a remote possibility. The side view of Fig. 19 also shows the splitter plate
floor in an alternate and lowered position which was used to determine whether the
close proximity of the original position had an effect on the data. The results of this
check are discussed in Sec. 5.

The gage to aperture distance determined the test time in the windowless configuration.
The distance of 5.6 cm yielded a test time of about 15 μsec before the inrushing test
gas struck the gages and obliterated the radiation signals by a higher rate of convective
ALIGNMENT: BOTTOM OF GAGE EVEN WITH TOP OF APERTURE

HORIZONTAL SCALE: ACTUAL SIZE
VERTICAL SCALE: 10x (DISTORTS ANGLES)

Fig. 19 Gage Field of View
4.3 THIN-FILM GAGE CONSTRUCTION

The platinum thin film heat transfer gages, used as thermal energy detectors for this research, were constructed following the standard procedures described by Rabinowicz, et al., (Ref. 24) and will merely be outlined here.

The basic gage element was constructed on a fused quartz substrate approximately 4.8 × 2.5 × 1 mm thick as sketched in Fig. 20. The ends and a portion of the top were painted with Hanovia Liquid Platinum Alloy 130-A (NEW) and baked following the manufacturer's instructions to provide means for attaching leads. A platinum strip was then sputtered over the entire length of the gage creating an active element 1 × 3 mm as sketched. This element was then baked at 1100°F for 8 hr to age and harden the film. It then had a resistance of 10 to 20 Ω and was slightly transparent to visible light indicating a film thickness of less than 500 Å. The element was then glued to a convenient base, wires were soldered to the ends, and these solder joints were painted with red Glyptal enamel.
The next step for most gages was the evaporation of a dielectric layer of silicon monoxide (SiO) for reasons to be discussed later. This was done in a 10⁻⁵ Torr vacuum using an Allen-Jones Electronics Corporation S13 chimney-type source at a temperature nominally 1200°C. The gages were heated to 350°F and were located 5 in. away resulting in a deposition rate of about 1500 Å/min.

The last step was the application of a layer of aluminum black used as a blackening agent. The aluminum black was applied in a manner quite similar to that used for coating mirrors except that the evaporation was performed in a nitrogen atmosphere at a pressure of 100 to 200 μ. The result was a diffuse, black surface with a measured absorptance of 0.98 ± 0.01 in the visible and near IR spectrum. This coating was used during the calibration of the silicon monoxide-coated gages but could not be used for the shock tube measurements for reasons discussed in Sec. 4.6.

Gage Thermal Analysis. The thermal analysis of a thin film gage is essentially the problem of one-dimensional transient heat conduction into a semi-infinite medium (i.e., the quartz substrate). The inverse equation relating the surface temperature excess T and the surface heat flux q is readily available in the literature [see Rabinowicz et al., (Ref. 24) or Vidal (Ref. 25)] and is

\[
T(t) = \frac{1}{\sqrt{\pi k \rho c_p}} \int_0^t \frac{q(\lambda)}{\sqrt{t - \lambda}} \, d\lambda. \tag{4.1}
\]

where \( k, \rho, \) and \( c_p \) are respectively the thermal conductivity, density, and specific heat of the substrate, \( t \) is time and \( \lambda \) is a dummy variable. This equation is applicable to a thin film gage provided that the heat pulse does not completely penetrate the substrate and provided the calorimetric effect of the thin film is small. It is well to note here that \( q \) is the heat flux absorbed by the gage and not the flux incident on the gage.
The surface temperature excess is determined by measuring the change of resistance of the gage element. These are related by

\[ R = R_G(1 + \alpha T) \]  

(4.2)

where \( \alpha \) is the temperature coefficient of resistivity of the sputtered platinum film.

A particularly convenient grouping of the gage physical properties is

\[ k' \triangleq \frac{\sqrt{\pi k \rho c}}{2\alpha} \]  

(4.3)

because it will be shown below that this is the desired result of any gage calibration procedure. Thus Eq. (4.1) can be written as

\[ \alpha T = \frac{1}{2k'} \int_0^t \frac{q(\lambda)}{\sqrt{t - \lambda}} \, d\lambda \]  

(4.4)

For the special case of a constant heat flux, \( \bar{q} \), beginning at \( t = 0 \) this equation becomes

\[ \alpha T = \frac{\bar{q}}{k' \sqrt{t}} \]  

(4.5)

4.4 GAGE BOX CIRCUIT ANALYSIS

The common method of determining a resistance change is to measure the voltage change at constant current. This condition is approximated by the gage box circuit shown in Fig. 21. Eight such gage boxes were used — one for each thin film gage in the model. Typical component values are \( E_b = 6 \text{ V} \) and \( I = 50 \text{ mA} \) which requires \( R_p \) to be about 100 \( \Omega \).
Fig. 21 Schematic of Gage Box Circuit

The total (AC+DC) voltage across the gage is given by

\[ E = IR = \frac{E_b R_G (1 + \alpha T)}{R_p + R_G (1 + \alpha T)} \]

The AC component of this is

\[ e = E(T) - E(0) = \frac{E_b R_p}{[R_p + R_G][R_p + R_G (1 + \alpha T)]} (R_G \alpha T) \]
Now since \( \alpha T \) is the order of \( 10^{-6} \) or less, it is neglected in favor of unity in the denominator whereupon the above equation reduces to

\[
e = \frac{E_b R_p}{(R_p + R_G)^2} \left( R_G \alpha T \right) = \frac{I_G R_p}{R_p + R_G} \left( R_G \alpha T \right)
\]

(4.6)

where

\[
I_G = \frac{E_b}{R_p + R_G}
\]

The factor \( \frac{R_p}{R_p + R_G} \) accounts for slight variations in the gage current and should be included unless, of course, \( R_p >> R_G \).

Now combining Eqs. (4.4) and (4.6) results in

\[
e = \frac{R_p}{R_p + R_G} \frac{1}{2k'} \int_0^t \frac{q(\lambda)}{\sqrt{t - \lambda}} \, d\lambda
\]

and defining a calibration constant as

\[
K = \frac{k'(R_p + R_G)}{I_G R_G R_p}
\]

(4.7)

yields

\[
e = \frac{1}{2K} \int_0^t \frac{q(\lambda)}{\sqrt{t - \lambda}} \, d\lambda
\]

(4.8)
For the special case of a constant heat flux, \( \bar{q} \), beginning at \( t = 0 \), this equation reduces to

\[
\bar{q} = K \frac{e}{\sqrt{t}}
\]  

Equation (4.7) can be rewritten as

\[
I_{G}K = \frac{k'}{R_{G}} \left( 1 + \frac{R_{G}}{R_{p}} \right)
\]  

and we note that \( k' \) is a property of the gage itself [Eq. (4.3)] and that the factor \( 1 + \frac{R_{G}}{R_{p}} \) is relatively insensitive to changes in \( R_{p} \) since \( R_{p} > R_{G} \). Now because a significant change in \( R_{G} \) is sufficient reason to discard a gage, the product \( I_{G}K \) tends also to be a property of the gage and this fact was used in the data reduction to account for slight (10%) changes in \( I_{G} \) caused by battery aging or different values of \( R_{p} \) in the various gage box circuits.

4.5 DATA REDUCTION

The first part of the data reduction procedure consists of obtaining the unknown heat-flux history \( q(t) \) from a given arbitrary voltage signal \( e(t) \). If the time variation of \( q(t) \) is known, then Eq. (4.8) may be integrated as for example in Eq. (4.9). Several forms of the inverse of Eq. (4.8) [where \( q(t) \) is given explicitly in terms of \( e(\lambda) \)] are given by Vidal (Ref. 25), but this approach eventually results in the same kind of mathematical manipulations as in the approach below.

The approach taken was to assume that the heat flux varied linearly over any arbitrarily small (but uniform for simplicity) time interval \( \delta t \) and further was continuous. This assumption is in accord with the expected radiative history in the reflected shock region (including the starting transient).
Defining a reduced heat flux by

\[ q^*(t) = \frac{q(t)}{K} \]

the assumed variation is given by

\[ q^*_m(t) = a_m(t - t_m) + b_m \quad \text{for} \quad t_m \leq t \leq t_m + \delta t \]

\[ t_m = m \delta t \]

\[ m = 0, 1, 2, \ldots \]

Continuity is given by

\[ b_m = a_m \delta t + b_{m-1} \quad m = 1, 2, 3, \ldots \]

and \( b_0 \) is arbitrary.

At this point inserting the above relations into Eq. (4.8) results in a sum of \( m \) integrable integrals. The solution now becomes a matter of algebraic manipulations and keeping track of the special cases for small \( m \). The continuity relation allows one to build up the solution because the heat flux at any point in time depends on previous history. Introducing the shorthand notation \( q^*_m = q(t = t_m) \), the solution at the discrete times \( t_m = m \delta t \) is given by:

\[ q^*_o = \text{arbitrary (usually zero for experimental traces)} \]

\[ q^*_1 = \frac{3}{2} \frac{e(t_1)}{\sqrt{\delta t}} - \frac{1}{2} q^*_o \]

\[ q^*_2 = \frac{3}{2} \frac{e(t_2)}{\sqrt{\delta t}} - 2(\sqrt{2} - 1)q^*_1 - \left( 1 - \frac{1}{\sqrt{2}} \right) q^*_o \]
\begin{align*}
q_m^* &= \frac{3}{2} \frac{e(t_m)}{\sqrt{\delta t}} - \frac{3}{2} \sqrt{m} \ q_0^* - (m)^{3/2} \left(q_1^* - q_0^*\right) + 2q_{m-1}^* - q_{m-2}^* \\
&- \sum_{n=2}^{m-1} (m - n + 1)^{3/2} \left(q_n^* - 2q_{n-1}^* + q_{n-2}^*\right) \quad m = 3, 4, 5, \ldots
\end{align*}

This system of equations has been programmed for an IBM 7094 digital computer. The \( e(t_m) \) input data is read from a Polaroid photograph of the oscilloscope display of the gage output voltage. Ordinarily twenty to thirty input data points are read. An initial condition on Eq. (4.1) is that the surface temperature (and hence the AC output voltage) must be zero at time zero. Thus the time origin must be chosen so that at least the initial voltage is zero.

The computer output format included a plot of both the output and the input on the same graph. Examples of these will be found in later sections of the report.

The heat flux absorbed by the gage, \( q \), is related to the intensity at the gage, \( I \), by the following relation

\[ I = \frac{q_r}{\omega} = \frac{q}{\bar{\alpha} \left( A_p \cos \theta / \rho^2 \right)} \quad (4.11) \]

because the dimensions of the aperture are small compared to the separation distance, \( \rho \). Here \( A_p \) is the area of the aperture, \( \theta \) the angle between the line \( \rho \) and the normal to the aperture (the gages were installed normal to this line), and \( \bar{\alpha} \) a weighted gage absorptance defined by

\[ \bar{\alpha} = \int_0^\infty \alpha_\nu \frac{I}{I} \ d\nu \quad (4.12) \]
As discussed earlier in this report, it was decided to include the absorptance in the theoretical predictions of the intensity so that the above equations might well be written

\[ \int_{0}^{\infty} \alpha \nu \, d\nu = \bar{\alpha} I = \frac{q}{(A_p \cos \theta / \rho)^2} \]  

(4.13)

where each term represents the intensity insofar as the data reduction procedure is concerned. The bracketed factor represents the solid angle subtended by the aperture and was nominally \(2 \times 10^{-3}\) sr \((A_p = 1.2 \times 5.6 \text{ mm}, \rho = 5.6 \text{ cm}, \theta = 13\) or 26 deg).

4.6 GAGE CALIBRATION — HEAT LAMP

The thin film gages were calibrated by two independent methods — one using a heat lamp and the other an electrical bridge circuit. In the heat lamp method a gage was exposed to a radiation pulse of known intensity and the aforementioned computer program used to calculate \(K\).

This method requires that the absorptance of the gage surface be known. Because the lamp radiation was mostly in the visible and near infrared spectral region where the surface absorptance of the SiO-coated gages varied (Fig. 11), a coating of aluminum black was evaporated on the surface of these gages. This coating was adequate for the 20 msec time scale of a lamp calibration but it was thermally thick and could not be used for the shock tube measurements where the time scale was about 10 \(\mu\)sec. Fortunately metallic black coatings are quite easily removed. It was relatively simple to apply a thermally thin layer of aluminum black to a bare platinum gage element and thus this coating was used for both calibration and shock tube measurements for this type of gage.

A schematic of the lamp calibration apparatus is shown by Fig. 22. The heat lamp was a commercial quartz lamp with a gold plated reflector. The water cell filter blocked much of the IR radiation and the aperture provided a reference point for gage
Fig. 22 Heat Lamp Calibration Apparatus and Typical Output Traces
location. The radiant flux at this aperture was measured with three different U.S. Navy Radiological Defense Laboratory slug-type calorimeters and was found to be 2.52 W/cm² after a 2-sec lamp warm-up.

A typical gage output trace is shown by Fig. 22 together with a phototube monitor of the heat pulse. Figure 23 shows the output of the computer program for this same gage. The ordinate is \( q^* = q/K \) for the solid curve. The time variation agrees well with the phototube monitor and since the constant portion of the absorbed heat flux is 0.98 (2.52) W/cm², the calibration constant is obtained directly as shown.

The heat lamp calibration subjected the gages to a heat pulse about 10% of the actual shock tube values and this occurred over a time scale 2000 times longer. The result was an output signal voltage (or \( \alpha T \) product) about four times that realized in the shock tube.

4.7 GAGE CALIBRATION – BRIDGE

The second and independent calibration method used was to heat the gage internally by the Joule heating from a current pulse passing through the sputtered platinum element. This requires a bridge circuit to separate the large steady heating voltage from the small signal caused by the gage element temperature (and hence resistance) change. The bridge calibration could easily simulate the experimental heat fluxes over a time scale about five times longer. This method is independent because the analysis requires an entirely different set of assumptions and measurement parameters and hence provides an absolute check on the validity of the lamp calibration.

The analysis of the bridge calibration will utilize the smaller schematic of Fig. 24 which includes the salient features of the actual circuit. Here the bridge as drawn is initially balanced because balancing is relatively simple to do on the actual circuit. Although it is not essential that both legs be identical, this will prove to be useful feature.
Fig. 23 Computer Output for Gage No. 282

\[ K = \frac{q}{q/K} = \frac{(0.98)(2.52)}{0.0244} = 101 \]
TO DELAYED PULSE

2D21

MERCUROY WETTED CONTACT RELAY

100 Ω (8 W)

0.005 μF

250 μF

25

THIN FILM (RESISTANCE RG)

2.5K

390

10K

B

A

R* (2 W)

TO DIFFERENTIAL PREAMPLIFIER OF OSCILLOSCOPE (DISPLAYS A - B)

*RESISTANCE SELECTED TO BE SLIGHTLY LARGER THAN RG

SALIENT PORTIONS OF CIRCUIT

Fig. 24 Bridge Calibration Circuit
The following expressions for the current in each leg may be derived in a straightforward manner:

\[
I = \frac{E_b (R_1 + R_G)}{R_p (R_1 + R_G) + (R_1 + R_G + R_p) (R_1 + R)}
\]

\[
I_L = \frac{E_b (R_1 + R)}{R_p (R_1 + R_G) + (R_1 + R_G + R_p) (R_1 + R)}
\]

The differential output voltage, \(e_0\) is given by

\[
e_0 = IR - I_L R_G
\]

and by neglecting \(\alpha T\) (of order \(10^{-6}\)) in favor of \(1 + R_1/ R_G\) (can be made of order 10) in the denominator, we get upon eliminating the currents and simplifying

\[
e_0 = E_b \left( \frac{R_1}{R_1 + R_G} \right) \frac{R_G \alpha T}{R_1 + R_G + 2R_p} \quad (4.14)
\]

The power dissipated in the gage is given by

\[
P = I^2 R = \frac{E_b^2 R_G (1 + \alpha T)}{(R_1 + R_G + 2R_p)^2} \quad (4.15)
\]

where the denominator has been simplified using the above argument. But here it is consistent to neglect the \((\alpha T)\) product in the numerator in favor of unity with the result that the power dissipation is essentially constant. Noting that \(q = P/A\) (where \(A\) is the area of the gage element) provided that the sputtered film is uniform (more will be said about this shortly), we now have a condition of constant heat flux and Eq. (4.5) is applicable.
Substituting Eqs. (4.14) and (4.15) into Eq. (4.5) yields

\[ k' = \frac{\beta G^2}{e_0\sqrt{t}} \]  

(4.16)

where

\[ \beta = \frac{(F_d)^3 R_1}{A(R_1 + R_2)(R_1 + R_G + 2R_p)^3} \]

By making \( R_1 > R_2 \), the factor \( \beta \) becomes a very weak function of \( R_2 \) and so is readily tabulated for a given \( R_p \), a nominal \( A \), and certain fixed values of \( E_b \).

The actual bridge circuit is also shown in Fig. 24. The large energy storage capacitor maintains a constant voltage over the 80-\( \mu \)sec test period. Switching is accomplished by actuating the mercury wetted contact relay which closes in nanoseconds. It is necessary to terminate the current pulse to prevent possible overheating and destruction of the gage. This is accomplished by firing the thyratron 80 \( \mu \)sec or so after the relay is closed by using a delay generator.

No inductance problems were encountered in building the bridge and composition resistors were used throughout. The performance of the entire bridge was checked using a bifilar-wound (low inductance) nichrome (\( \alpha \) essentially zero) resistor in place of a gage and the expected zero trace was achieved.

It is difficult to measure \( R_2 \) directly because of the attendant heating that must occur. This measurement is easily done on the balanced bridge by removing the gage and measuring the value of the adjustable resistors in the other leg. It can be shown that a bridge unbalance of 0.01 \( \Omega \) will result in a zero shift of 1.5 mV at \( E_b = 125 \) V and such a zero shift is readily noticed.
An example of a typical bridge calibration is shown in Fig. 25. The inset shows the actual oscilloscope trace from which eight points were read and plotted versus the square root of time. The linearity of the plotted points proves that the output voltage had the parabolic behavior indicative of a constant heat flux [Eq. (4.5)] and the bridge was properly balanced because an extrapolation of the data points will fall quite near the plot origin.

The bridge calibration is amenable to checking the linearity of the gages because the simulated heat flux is readily varied over a wide range by merely changing $E_b$. A typical result of a linearity check on a gage was that $k'$ varied by about $\pm 2\%$ over a simulated heat flux range of 5 to 42 W/cm$^2$. No linearity problems were anticipated because the experimental heat fluxes were never large enough to cause a $1^\circ$C rise in gage surface temperature in the available test time.

4.8 COMPARISON OF GAGE CALIBRATION RESULTS

A comparison of the results of the heat lamp and bridge calibration for eighteen gages is given in the upper half of Fig. 26. Here we note that while the results for six gages agree within $\pm 5\%$ and five more agree within $\pm 12\%$, the remaining seven all show a rather large, one-sided discrepancy. The lower half of Fig. 26 shows the lamp results and it is seen that these are fairly consistent and the average is somewhat less than twice the value calculated using handbook bulk property values — a common occurrence with sputtered platinum thin film gages. Thus it would appear that the lamp calibration is correct and that the discrepancy lies in the results obtained from the bridge calibration.

This discrepancy is explained by considering the effect of a non-uniform gage element in both calibration methods. The bridge is especially susceptible to effects caused by non-uniformities in the sputtered film because here the local heating depends on the local resistance. A microscopic examination of many gage elements revealed that occasionally the sputtered platinum film tended to pull away or thin out at the ends of
THYRATRON FIRES

AMPLIFIER

TRANSIENT

GAGE NO. 140

$R_G = 20.38 \, \Omega$

$E_p = 125.0 \, V$

$A = 0.033 \, cm^2$

$\beta = 152.7 \, mV \, W/cm^2$

$k' = \frac{152.7 \times (20.38)^2}{0.663}$

$= 0.96 \times 10^5 \, \frac{W \sqrt{\mu sec}}{cm^2}$

SIMULATED HEAT FLUX = 18.5 W/cm²

Fig. 25 Example of Bridge Calibration Including Actual Oscilloscope Trace and Data Reduction
Fig. 26 Comparison of Results of Lamp and Bridge Calibration
the active element during the aging process. A general analysis of the bridge calibration method for a non-uniform gage is tedious and it will suffice here to postulate that the gage element consists of two parts differing in resistance and area.

We assume that the gage active element consists of two parts, \( a \) and \( b \), that are electrically in series so that the total gage resistance is given by

\[
R = R_a (1 + \alpha T)_a + R_b (1 + \alpha T)_b = R_G + (R\alpha T)_a + (R\alpha T)_b \tag{4.17}
\]

where

\[
R_G = R_a + R_b
\]

The analysis of the effect of a two-part gage on the bridge calibration is quite similar to that done earlier and the result is

\[
e_0 = \left(\frac{E_b}{R_G}\right)^3 \frac{R_1}{R_1 + R_G} \frac{\sqrt{t}}{(R_1 + R_G + 2R_p)^3} \left[\left(\frac{R^2}{Ak^2}\right)_a + \left(\frac{R^2}{Ak^2}\right)_b\right]
\]

Now we further assume that the physical constant, \( k' \), is identical in both parts. This is reasonable considering the relative constancy in the lamp calibration results for many gages. The above expression becomes

\[
k' = \frac{A\beta}{e_0\sqrt{t}} \left[\left(\frac{R^2}{A}\right)_a + \left(\frac{R^2}{A}\right)_b\right]
\]

where \( \beta \) is given by Eq. (4.16). By introducing

\[
R^* = \frac{R_a}{R_G}, A^* = \frac{A_a}{A} \text{ where } A = A_a + A_b
\]
the above expression becomes

\[ k' = \frac{\beta(R_G)^2}{e_0\sqrt{t}} \left[ \frac{(R^*)^2}{A^*} + \frac{(1 - R^*)^2}{1 - A^*} \right] \]  \hspace{1cm} (4.18)

and except for the factor in brackets, this equation is identical to Eq. (4.16), the corresponding result for a uniform gage. Since the bridge calibration method tacitly assumed a uniform gage, this bracketed factor represents an unknown correction that should have been applied, but was not. This factor has a minimum value of unity for \( A^* = R^* \) (i.e., a uniform gage) and is greater than unity for all other possible values of \( A^* \) and \( R^* \) between zero and unity. A thin spot in the sputtered film would mean that it is possible for a large fraction of the resistance to occur over a relatively small fraction of the total area and hence the bracketed factor could easily be as large as a factor of two. Thus any amount of gage non-uniformity will make the value of \( k' \) obtained by the bridge calibration too small. A reduction in \( k' \) by one-half is not unreasonable.

The results of the lamp calibration are not affected by a non-uniform gage element. This can be proven by using Eq. (4.17) instead of Eq. (4.2) in the gage box circuit analysis and again assuming that the physical constant \( k' \) is identical in both parts. We will not go through the equations here because a logical argument will yield the same result.

The heating of the gage surface in the lamp method is uniform even though the gage resistance is not because here the heat source is external to the gage and not the gage element itself as it is in the bridge method. Thus the surface temperature rise of each part of the gage is identical [Eq. (4.1)] and the parts act in series. If \( k' \) is identical then all the equations in the gage box circuit analysis are unchanged and hence the lamp calibration is valid. Even if the values of \( k' \) were different, there would be no effect in the data reduction procedure. Admittedly Eq. (4.10) would no longer hold, but one could merely treat Eq. (4.8) as the defining equation for \( K \) and since the heating
mode is the same in both the lamp calibration and the shock tube experiments, the data reduction procedures using the $K$ so defined would be valid.

Thus the conclusion is reached that the discrepancy in the comparison of the gage calibration results stems from uncorrected gage non-uniformities which caused the bridge results to be too low while having no important effect on the lamp calibration results and no effect on the data reduction provided (as was done) the heat lamp calibration results are used. These conclusions are borne out by Fig. 26.

The bridge calibration was employed as an independent check on the lamp calibration and since from Fig. 26 the values of $k_{lamp}'/k_{bridge}'$ are either essentially unity or greater, the absolute validity of the lamp calibration is verified. The bridge calibration is still a valid gage linearity check because the unknown factor of Eq. (4.18) is not a function of the magnitude of Joule heating but rather its distribution.

4.9 GAGE TIME RESPONSE CHECK

The only yet unsolved facet of the gage calibration procedures is a check of the time response since the time scales used in the lamp and bridge calibrations were both greater than that used in the shock tube.

The gage time response was obtained by irradiation with a 15-μsec pulse of visible light obtained from a xenon flash tube. The gage output was fed into the computer data reduction program and the computer output compared against the pulse shape obtained using a photomultiplier tube. Examples of this are given by Figs. 27 and 28 where the solid lines are the computer output while the dashed lines are the normalized pulse shape plotted on the same time scale. Figure 27 shows the time response of a bare gage (i.e., no coating on the platinum element) and here the pulse shape is reproduced quite well with a time lag of about 1 μsec. Since this time is equivalent to the risetime of the oscilloscope differential amplifiers, it is considered quite acceptable. Figure 28 shows the results of a gage coated with a 4-min evaporation of SiO and which exhibited the largest time lag of a number of gages with various coatings.
Fig. 27 Xenon Flash Results for Bare Gage
Fig. 28 Xenon Flash Results for Gage With 4 min. of Silicon Monoxide
Here the delay is about 2 or 3 $\mu$sec and some distortion of the pulse shape is apparent. However, this is not serious when compared to the 15-$\mu$sec test time and such performance was considered acceptable.

An EG&G.FX-12 xenon flash tube was pulsed by using a lumped parameter delay line with a total energy storage of 1.4 J (8 $\mu$F, 600 V). No circuit schematic is given here because the required parameters vary with individual flash tubes and tube life and one must resort to trial and error adjustments to obtain the desired pulse shape. The flash tube light output was gathered by a parabolic mirror and concentrated by a condensing lens with the result that the peak heat flux was about 500 W/cm$^2$. This flux varied from flash to flash and so this time response check was not used as a calibration method. This variation could be minimized by increasing the total energy in the delay line but at a considerable sacrifice in flash tube life.

4.10 NOISE PROBLEMS

Noise is a perennial problem in any experimental investigation and this research was no exception. Several noise sources were identified and eliminated while at least one other source was suppressed but not entirely eliminated. The noise problems can be divided into two areas: those that occurred when windows were used and those which occurred only when no windows were used. We shall logically begin with the former.

The effects of one large and erratic noise source are given by Fig. 29 which shows noise traces from three thin film gage channels together with a sketch of the model configuration at that time. A copper disc was used instead of a window to ensure that no radiation or test gas entered the gage cavity. The noise on the three traces correlated quite well with the incident shock position as shown in the figure. The important thing to note is that there was no noise associated with event D – the time when the incident shock struck the splitter plate floor. Noting that this was the only event where the incident shock reflected back uniformly, it was suspected that the noise was caused by magnetic pickup from vortices generated by asymmetries in the flow. It might be added that the thin film gages and associated circuitry were already supposedly well-shielded magnetically with Netic and Co-Netic metal.
<table>
<thead>
<tr>
<th>CHANNEL</th>
<th>VERTICAL SENSITIVITY (mV/cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>0.2</td>
</tr>
<tr>
<td>5</td>
<td>0.2</td>
</tr>
<tr>
<td>6</td>
<td>0.5</td>
</tr>
</tbody>
</table>

EVENT A : SHOCK STRIKES
EVENT C : SHOCK STRIKES
EVENT D : SHOCK STRIKES
EVENT E : SHOCK STRIKES

SHOT:  284
WINDOW: COPPER (NO TEST GAS OR RADIATION ENTERED GAGE CAVITY)

Fig. 29 Correlation of Noise Traces With Incident Shock Position
The solution was to mount a ring around the model in the manner sketched in Fig. 30 to stop the flow uniformly in the region outside the splitter plates. The results are shown in the upper-half of Fig. 30 where a dramatic noise reduction was realized.

Later in the experiment some erratic noise became evident in the traces at event D when higher (to 50 μV/cm) oscilloscope gains were used. These were cured by physically relocating the gage circuit cable connectors to a point where they could be magnetically shielded more effectively. At this time the noise level of the thin film traces was reduced to a value very nearly the inherent preamplifier noise. This is shown by Fig. 31 which shows traces from six channels during one of the last window shots. The missing two channels were off-scale as a result of an occasional disturbance from the trigger bank of the arc tube driver which was later completely eliminated by reducing the current loops in the trigger bank circuit.

Thus it is concluded that the noise caused by magnetic pickup was greatly reduced by a model designed to eliminate asymmetries in the flow but that careful attention to magnetic shielding was still necessary to eliminate the erratic vestige that remained at very high instrumentation gains. It was not necessary to employ the SiO dielectric layer for gages used behind a window and indeed over half of the window data presented in the next section was taken with the aluminum black on platinum type of gage.

We now turn our attention to the noise problems uniquely associated with the model in a windowless configuration and Fig. 32 is indicative of the early noise problems. Here it is seen that only one channel (Channel 4) appears acceptable while the other seven all show varying degrees of excessive noise. The traces of Channels 1, 3, and 8 went completely off-scale at the time the incident shock struck the bottom of the splitter plates (event D of Fig. 29) and this is also the initial time when radiation from the reflected shock region was incident on the gages. The gains of 0.5 to 2.0 mV/cm were at least ten times less sensitive than the maximum gains ultimately employed.

Heretofore the effectiveness of the SiO insulation was determined by dipping the gage into mercury and measuring the resistance to the active element. This provided no
<table>
<thead>
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<td>1.0</td>
</tr>
<tr>
<td>8</td>
<td>0.2</td>
</tr>
</tbody>
</table>

A SHOCK STRIKES
B SHOCK STRIKES
RING POSITION

SHOT: 285
WINDOW: COPPER (NO TEST GAS OR RADIATION ENTERED GAGE CAVITY)

Fig. 30 Noise Traces After Addition of Ring
SHOT 355, \( M_s = 25.2 \), \( P_1 = 200 \mu \)
ALL GAGE CHANNELS AT 50 \( \mu V/cm \)

Fig. 31 Examples of Traces Obtained Behind Suprasil Window
<table>
<thead>
<tr>
<th>CHANNEL</th>
<th>VERTICAL SENSITIVITY (mV/cm)</th>
</tr>
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<tbody>
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<td>1.0</td>
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<tr>
<td>8</td>
<td>0.5</td>
</tr>
</tbody>
</table>

SHOT: 288  
WINDOW: NONE

Fig. 32 Examples of Traces Obtained Without Window
information about pinholes or fissures in the SiO layer because the mercury did not wet this layer. Therefore a plasma check, shown schematically by Fig. 33, was instigated to provide a more meaningful test. The method of performing this check was to determine the plasma potential of the gage (i.e., the voltage at \( I = 0 \), typically \(-10 \) V) and then to bias the gage more positive and note the current.

The results of this plasma check on the gages used in Shot 288 (Fig. 32) were as follows:

- The gages in all channels except Channel 4 drew 10 to 70 times as much current as Channel 4 when biased \(+10\) V from their plasma potential.
- The gages corresponding to the traces that showed high frequency noise (Channels 1, 2, 5, and 6) were erratic during the plasma check.

At this time these results indicated that the plasma check could determine the effectiveness of the SiO insulation and hence the SiO evaporation procedure was much modified until gages that drew less than \(5 \times 10^{-9}\) A at 12-V bias were commonplace. The final evaporation procedure was the one already described.

Unfortunately only about 20% of the gages of this generation yielded relatively noise-free traces in the windowless model at the higher (to \(50 \mu V/cm\)) oscilloscope gains ultimately employed. Therefore, the plasma check must be considered as a necessary but not sufficient test. This then is the state of the art at the time of this writing. Many unsuccessful attempts were made to correlate the construction details or variation of the SiO evaporation parameters. These parameters included source temperature, separation distance, evaporation time and two separate evaporations (an artifice used to minimize pinholes caused by foreign materials). No notable improvement in this 20% acceptance figure was realized during any of these perturbations.

Further the gages in general were not durable and often a steady increase in noise was experienced. Some lasted for only several shots while a few yielded consistently good results for a dozen shots or more. Many gage failures were attributed to a separation
Fig. 33 Schematic of Plasma Insulation Check Apparatus
of the SiO and the platinum from the quartz substrate. Before the SiO evaporation the sputtered platinum film was hard and could be removed only with great difficulty, but after the SiO evaporation the film was rather easily removed by scraping with a sharp instrument. Possible causes for this degradation include stresses caused by the condensing SiO, free silicon embrittlement of the platinum, or as was suggested, reduction of an intermediate platinum oxide compound which provided the initial strong bonding. Therefore, while this state of the art leaves something to be desired, nevertheless by building many gages and subjecting each one to a final test in the shock tube it was possible to obtain meaningful results. Figure 34 shows examples of gage traces that yielded good data in the windowless model. We now turn to a discussion and possible explanation of the noise source(s) of the 80% of the gages that were rejected.

Figure 35 shows examples of the traces from rejected gages in the windowless model. The top five traces indicate the common initially-negative behavior while the bottom two show the opposite and much less common initially-positive noise bump. All these bumps occurred at the time the incident shock struck the floor of the splitter plates and one would suspect that they were caused either by some interaction with the ionized gas in the gage cavity at $P_1$ or by some residual magnetic pickup. The latter was eliminated by noting that this behavior was not seen when windows were used and further was cured several times by merely painting the surface of the rejected gage with red Glyptal enamel and observing the noise trace on the ensuing shot.

The gas in the gage cavity becomes ionized by photoionization at the onset of radiation as a result of the vacuum ultraviolet radiation that was formerly blocked by the Suprasil window. That the gas becomes ionized at this time was proven by placing two small plates at the gage location and about 1 cm apart. With 22 V across the plates, several milliamperes of current were drawn at a time beginning when the incident shock struck the floor of the splitter plates. Also this is the time when faulty gages went off-scale (Channels 1, 3, and 8 of Fig. 32). Therefore the gages must operate in a transient, room-temperature plasma while exposed to vacuum ultraviolet radiation. This is the essence of the windowless problem and because it is a difficult situation to create in a bench-type experiment, one must resort to a trial and error experimentation in the shock tube itself.
<table>
<thead>
<tr>
<th>SHOT</th>
<th>CHANNEL</th>
<th>GAIN (mV/cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>291</td>
<td>2</td>
<td>0.2</td>
</tr>
<tr>
<td>292</td>
<td>3</td>
<td>0.2</td>
</tr>
<tr>
<td>304</td>
<td>8</td>
<td>0.2</td>
</tr>
<tr>
<td>308</td>
<td>4</td>
<td>0.2</td>
</tr>
<tr>
<td>310</td>
<td>6</td>
<td>0.2</td>
</tr>
<tr>
<td>342</td>
<td>5</td>
<td>0.2</td>
</tr>
<tr>
<td>359</td>
<td>6</td>
<td>0.1</td>
</tr>
<tr>
<td>366</td>
<td>6</td>
<td>0.1</td>
</tr>
<tr>
<td>373</td>
<td>5</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Fig. 34 Examples of Traces From Acceptable Gages in the Windowless Model
GAIN
SHOT CHANNEL (mV/cm)

<table>
<thead>
<tr>
<th>SHOT</th>
<th>CHANNEL</th>
<th>GAIN (mV/cm)</th>
</tr>
</thead>
<tbody>
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<td>0.2</td>
</tr>
<tr>
<td>315</td>
<td>5</td>
<td>0.2</td>
</tr>
<tr>
<td>357</td>
<td>1</td>
<td>0.1</td>
</tr>
<tr>
<td>367</td>
<td>4</td>
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<tr>
<td>373</td>
<td>1</td>
<td>0.05</td>
</tr>
<tr>
<td>342</td>
<td>6</td>
<td>0.2</td>
</tr>
<tr>
<td>376</td>
<td>5</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Fig. 35 Examples of Traces From Rejected Gages in the Windowless Model
The windowless noise phenomena discussed above are not unique to this experiment and several authors have expressed their views on the subject. Gaydon and Hurle (Ref. 26) note this effect when thin film gages are exposed to an ionized flow and conclude that the initially-negative behavior is caused by charge acquisition rather than a shorting effect. In a similar situation Marrone and Hartunian (Ref. 27) considered the problem as a shorting effect and cured it by an evaporated layer of SiO. It is noted here that a 0.1 mV negative signal could be caused by a decrease in gage resistance of only 0.01% for a 20-Ω gage at 50 mA and this corresponds to suddenly paralleling the gage with 200 kΩ.

Clearly more work needs to be done on the problem of making total radiation measurements (including the vacuum ultraviolet) in a transient and cold plasma.
DISCUSSION OF RESULTS

Experimental radiative intensity data have been obtained at two shock tube conditions. These conditions are (1) $T = 14,700^\circ K$, $\rho/\rho_o = 3.18 \times 10^{-2}$ where $\rho_o = 1.29 \times 10^{-3}$ g/cm$^3$. At the first condition, data were obtained with and without a window and also with a window in combination with a filter which transmits only in the infrared. At the second condition data were obtained with and without a window. The data are presented in Figs. 36 through 40. The test conditions in the shock tube were difficult to control precisely and the data presented in these figures were adjusted to the temperature indicated on the respective figures. The adjustment to the data were made according to the variation in intensity with temperature as indicated by the results in Fig. 5. Any data which required an adjustment greater than 50% were not considered. The adjustment to the data on the average was about 15%.

The data obtained behind a window at the first test condition are presented in Fig. 36. The transmission curve for the window used to obtain these data is presented in Fig. 14. The data presented in Fig. 36 were obtained with three different model configurations. The first condition is the normal one as depicted in Fig. 19. For the second condition the floor plate was lowered by 0.10 in. The reason for lowering the floor plate was to check the possible effect of truncating the field of view of the gages and possible cooling of the test gas by conduction to the floor plate. Lowering the floor plate decreases both the field of view truncation and thermal cooling. For the third condition the plate was kept in the lowered position and the quartz window at the aperture was moved back and placed immediately in front of the gage. With this configuration it was possible to obtain data behind a window with the gas expanding into the cavity. A comparison of the data obtained with this configuration to the data obtained with the original configuration provides an experimental check on the effect of expansion wave in the test gas. The effect of field of view truncation, thermal cooling, and the
Fig. 36  Radiant Intensity Data Obtained Behind a Quartz Window:  $T = 14,700^\circ$K
$\rho/\rho_0 = 3.45 \times 10^{-2}$
propagation of the expansion wave into the test gas is not discernable in the data presented in Fig. 36. From these results it is concluded that the above effects are not important with the present data scatter.

The gages used to obtain these data were coated with aluminum black so that the surface absorption was uniform and at a value of approximately 0.98. The continuum prediction was obtained from Fig. 12 while the line prediction was obtained from Fig. 10b. A comparison of the theoretical prediction of the lines plus continuum intensity show that the predictions are approximately a factor of two higher than the data. On the other hand, the continuum only prediction is in good agreement with the data. The variation of the intensity data with geometric path length is in good agreement with the prediction. Furthermore, the intensity variation is nearly linear which indicates that for the frequencies transmitted by the window, the gas is optically thin. The discrepancy between the theoretical prediction and the experimental data was consistently present in all of the data obtained (Figs. 36 through 40). Possible reasons for this discrepancy will be discussed later.

In Fig. 37 are presented data which were obtained behind a window in series with a filter. The transmission of the filter is shown in Fig. 14. The corresponding theoretical predictions were obtained by taking into account the transmission characteristics of both the window and the filter. Again, it is seen that the data are somewhat below the lines plus continuum prediction.

Data obtained without a window are presented in Fig. 38. As the geometric path length is increased the data increases and then apparently levels off. At the present time it is believed that the data at the longer path lengths (l > 9 cm) are not valid through no explanation can be offered at this time. The data obtained at the shorter path lengths are seen to be in better agreement with the continuum only prediction than the total lines plus continuum prediction. These predictions employed the surface absorptance values presented in Fig. 11 (No. 2). A comparison of the windowless data with the window data of Fig. 36 shows the importance of the vacuum ultraviolet. The nonlinear behavior of the intensity with the geometric path length indicates the importance of self absorption.
Fig. 37 Radiant Intensity Obtained Behind a Quartz Window and Filter: $T = 14,700^\circ$K

$\rho/\rho_o = 3.45 \times 10^{-2}$
Fig. 38 Radiant Intensity Obtained Without a Window: $T = 14,700^\circ K$; 
$\rho/\rho_0 = 3.45 \times 10^{-2}$
Similar window and windowless data for the second shock tube test condition are presented in Figs. 39 and 40. The window data are in good agreement with the continuum only prediction. The windowless data at the long path lengths appear to be somewhat low. The total theoretical prediction for this case is consistently higher than the data. A comparison of the window and windowless again indicates the importance of the vacuum ultraviolet.

There are several possibilities for explaining the discrepancies between the data and the theoretical predictions. The first is that the shock tube temperatures are somewhat lower than those obtained from the shock tube using the measured value of the incident shock speed. If the gas temperature is approximately 5 to 7% lower than the values given by the shock tube tables, the agreement between theory and data would be quite good. The possibility that the temperature of the gas in the reflected shock region is less than expected is currently being examined. An alternate explanation to explain the discrepancy between theory and data is that the line radiation is overpredicted. As discussed in Sec. 3.2, there is considerable uncertainty on the half-widths of the line profiles. This uncertainty in the half-widths and the oscillator strengths results in uncertainties in the line radiation predictions of about 30 to 50%. It is currently thought that the line predictions may be too high but the uncertainty in the line prediction is not large enough to account for the discrepancy between theory and experiment.
Fig. 39 Radiant Intensity Data Obtained Behind a Quartz Window:

$T = 17,000^\circ K; \frac{\rho}{\rho_0} = 3.18 \times 10^{-2}$
Fig. 40 Radiant Intensity Data Obtained Without a Window: $T = 17,000^\circ K$; $\rho/\rho_0 = 3.18 \times 10^{-2}$
Section 6
CONCLUSIONS

From the results of this investigation we conclude first of all that the vacuum ultraviolet is a significant contributor to the integrated radiative heating to entry vehicles. Secondly, we conclude that self-absorption is an important mechanism in reducing the radiative heating. Finally, we conclude that our present knowledge of radiative energy transfer is such that we can predict the integrated intensity to within a factor of two. Although this is a significant improvement to the order of magnitude uncertainty which existed previously, additional theoretical and experimental work is necessary to reduce this uncertainty to acceptable levels.
Appendix A

CONTINUUM ABSORPTION COEFFICIENTS OF NITROGEN AND OXYGEN

A. 1 INTRODUCTION

We list below formulas for the continuum absorption coefficients due to the bound-free and free-free transitions for the neutral and singly ionized atoms of nitrogen and oxygen. These formulas result from the theory of Biberman (Ref. 9) for free-free transitions and bound-free transitions from the higher excited states. In addition, the detailed quantum mechanical calculations of Armstrong, Kelley, and colleagues at LMSC (Ref. 3) are used to provide cross-sections for photon absorption due to transitions from the ground and low lying excited states. The absorption coefficients $\mu'_\nu$, given below may be generalized to include induced emission (we denote by $\mu'_\nu$, the absorption coefficient including induced-emission) by the relationship

$$\mu'_\nu = \mu_\nu \left[ 1 - \exp \left( -\frac{\hbar \nu}{kT} \right) \right]$$  (A. 1)

A. 2 ABSORPTION COEFFICIENT FORMULAS

The general absorption coefficient formula for a particular absorbing specie $i$ is,

$$\mu_i(\nu) = C \cdot N_i \cdot \Gamma_i \cdot T \cdot Z^2_i \cdot \exp \left[ -\left( u'_o - u_i \right) \right] \frac{\xi_i(h\nu)}{(h\nu)^3} \cdot \text{cm}^{-1} \; \text{; } h\nu < h\nu_{ig}$$  (A.2a)

$$\mu_i(\nu) = C \cdot N_i \cdot \Gamma_i \cdot T \cdot Z^2_i \cdot \left[ \exp \left( -u'_o - u'_g \right) \right] \frac{\xi_i(h\nu)}{(h\nu)^3} + N_i \phi_i(h\nu) \cdot \text{cm}^{-1} \; \text{; } h\nu \geq h\nu_{ig}$$  (A.2b)
where the frequency $\left(h\nu^i_g\right)$ at which we break from Eq. (A.2a) to Eq. (A.2b) is
dependent on the absorbing species. The various terms in Eqs. (A.2) are defined as
follows:

- $C_0 =$ constant $= 6.26 \times 10^{-20}$
- $N_i =$ number density of species $i$, particles/cm$^3$
- $T =$ gas temperature, °K
- $Z_i =$ ionic charge on the residual ion, dimensionless
- $u =$ $h\nu/kT$, dimensionless
- $(h\nu) =$ frequency variable, eV
- $\xi_i(h\nu) =$ Biberman function for species $i$ (plotted in Fig. 41).

The parameter $\Gamma_i$ is defined by

$$\Gamma_i = \frac{2\sum^i_0}{\sum^i_1}$$

where $\sum^i_0$ and $\sum^i_1$ are the partition function for the parent atom and residual atom of
species $i$, respectively. Formulas for $\sum^i_0$ and $\sum^i_1$ are given below. We have found
somewhat better agreement between Biberman's equation and the Armstrong data by
using a constant value of $\Gamma_i$ based on the ground state statistical weights of the parent
and residual atom. The function $\phi_i(h\nu)$ accounts for absorption from the ground and
low excited states. Formulas for $\phi_i$ are given for the various species.

A.2.1 Nitrogen

Neutral Atom, N. For neutral nitrogen atoms, the function $\xi(h\nu)$ may be found
plotted in Fig. 41.
The remaining parameters and functions pertinent to this species are listed below:

\[
\Gamma_N = 4.5 \quad ; \quad Z_N = 1.0
\]
\[
\sum_0^N = 4 + 10 \exp \left( -\frac{2.38}{kT} \right) + 6 \exp \left( -\frac{3.57}{kT} \right)
\]
\[
\sum_1^N = -9 + 5 \exp \left( -\frac{1.90}{kT} \right)
\]

The function \( \phi_N(h\nu) \) is conveniently defined in terms of the following auxiliary functions

\[
\phi_1 = 0
\]
\[
\phi_2 = 8.6 \times 10^{-18} \left( \frac{6}{\sum_0^N} \right) \exp \left[ -(u_0 - u_2) \right]
\]
\[
\phi_3 = 6.4 \times 10^{-18} \left( \frac{10}{\sum_0^N} \right) \exp \left[ -(u_0 - u_1) \right]
\]
\[
\phi_4 = 7.9 \times 10^{-18} \left( \frac{4}{\sum_0^N} \right)
\]

Then

\[
\phi_N = \phi_1 \quad \text{for} \quad h\nu_g \leq h\nu < h\nu_2
\]
\[
\phi_N = \phi_1 + \phi_2 \quad \text{for} \quad h\nu_2 < h\nu < h\nu_1
\]
\[
\phi_N = \phi_1 + \phi_2 + \phi_3 \quad \text{for} \quad h\nu_1 < h\nu < h\nu_0
\]
\[
\phi_N = \phi_1 + \phi_2 + \phi_3 + \phi_4 \quad \text{for} \quad h\nu > h\nu_0
\]
The boundary frequencies which define \( \nu_g, \nu_2, \nu_1, \nu_0 \) (and, correspondingly, \( u_g, u_2, u_1, u_0 \)) are

\[
\begin{align*}
\nu_g &= 4.22 \text{ eV} \\
\nu_2 &= 10.8 \text{ eV} \\
\nu_1 &= 12.0 \text{ eV} \\
\nu_0 &= 14.3 \text{ eV}
\end{align*}
\]

Singly Ionized Atom, \( \text{N}^+ \). The function \( \xi_{\text{N}^+}(\nu) \) for singly ionized nitrogen atoms is plotted in Fig. 41. The remaining parameters and functions pertinent to this species are listed below:

\[
\begin{align*}
\Gamma_{\text{N}^+} &= 1.33; \quad Z_{\text{N}^+} = 2.0 \\
\sum_{0}^{N^+} = \sum_{1}^{N} &= 9 + 5 \exp \left( - \frac{1.90}{kT} \right) \\
\sum_{1}^{N^+} &= 6 + 12 \exp \left( - \frac{7.10}{kT} \right)
\end{align*}
\]

Again we define the functions,

\[
\begin{align*}
\phi_1 &= 0 \\
\phi_2 &= 9.5 \times 10^{-18} \left( \frac{1}{\sum_{0}^{N^+}} \right) \exp \left[ -(u_0 - u_2) \right] \\
\phi_3 &= 7.4 \times 10^{-18} \left( \frac{5}{\sum_{0}^{N^+}} \right) \exp \left[ -(u_0 - u_1) \right] \\
\phi_4 &= 5.8 \times 10^{-18} \left( \frac{9}{\sum_{0}^{N^+}} \right)
\end{align*}
\]
Then

\[ \phi_{N^+} = \phi_1 \quad \text{for} \quad \nu_g \leq \nu < \nu_2 \]
\[ \phi_{N^+} = \phi_1 + \phi_2 \quad \text{for} \quad \nu_2 < \nu < \nu_1 \]
\[ \phi_{N^+} = \phi_1 + \phi_2 + \phi_3 \quad \text{for} \quad \nu_1 < \nu < \nu_0 \]
\[ \phi_{N^+} = \phi_1 + \phi_2 + \phi_3 + \phi_4 \quad \text{for} \quad \nu < \nu_0 \]

The boundary frequencies which define \( \nu_g, \nu_2, \nu_1, \nu_0 \) (and, correspondingly, \( u_g, u_2, u_1, u_0 \)) are

\[ \nu_g = 11.2 \text{ eV} \]
\[ \nu_2 = 25.2 \text{ eV} \]
\[ \nu_1 = 27.7 \text{ eV} \]
\[ \nu_0 = 29.6 \text{ eV} \]

A. 2.2 Oxygen

Neutral Atom, \( O \). The function \( \xi_O(\nu) \) for neutral oxygen atoms is plotted in Fig. 41.

The remaining parameters and functions are listed below:

\[ Z_0 = 1.0 \; ; \; \Gamma_0 = \frac{8}{9} \]
\[ \sum_0^0 = 9 + 5 \exp \left( - \frac{1.98}{kT} \right) + \exp \left( - \frac{4.18}{kT} \right) \]
\[ \sum_1^0 = 4 + 10 \exp \left( - \frac{3.33}{kT} \right) + 6 \exp \left( - \frac{5.03}{kT} \right) \]
Again we define the functions

\[
\begin{align*}
\phi_1 &= 0 \\
\phi_2 &= 0 \\
\phi_3 &= 0 \\
\phi_4 &= 4.0 \times 10^{-18} \left( \frac{9}{\sum^0} \right)
\end{align*}
\]

Then

\[
\begin{align*}
\phi_0 &= \phi_1 & \text{for } & \nu_g \leq \nu < \nu_2 \\
\phi_0 &= \phi_2 & \text{for } & \nu_2 < \nu < \nu_1 \\
\phi_0 &= \phi_3 & \text{for } & \nu_1 < \nu < \nu_0 \\
\phi_0 &= \phi_4 & \text{for } & \nu > \nu_0
\end{align*}
\]

The boundary frequencies which define \( \nu_g, \nu_2, \nu_1, \nu_0 \), (and, correspondingly, \( u_g, u_2, u_1, u_0 \)), are

\[
\begin{align*}
\nu_g &= 4.25 \text{ eV} \\
\nu_2 &= 8.43 \text{ eV} \\
\nu_1 &= 11.6 \text{ eV} \\
\nu_0 &= 13.6 \text{ eV}
\end{align*}
\]

Singly Ionized Atom, \( \text{O}^+ \). The function \( \xi_{\text{O}^+}(\nu) \) for singly ionized oxygen atoms is plotted in Fig. 41. The remaining functions and parameters are listed as follows.
$Z_{0^+} = 2.0; \Gamma_{0^+} = 4.5$

$$\sum_{0}^{0^+} = \sum_{1}^{0} = 4 + 10 \exp\left(-\frac{3.33}{kT}\right) + 6 \exp\left(-\frac{5.03}{kT}\right)$$

$$\sum_{1}^{0^+} = 9 + 5 \exp\left(-\frac{2.51}{kT}\right) + \exp\left(-\frac{5.35}{kT}\right)$$

Again we list

$$\phi_1 = 0$$

$$\phi_2 = 0$$

$$\phi_3 = 0$$

$$\phi_4 = 1.0 \times 10^{-18}\left(\frac{4}{\sum_{0}^{0^+}}\right)$$

Then

$$\phi_{0^+} = \phi_1 \quad h \nu_g \leq h \nu < h \nu_2$$

$$\phi_{0^+} = \phi_2 \quad h \nu_2 < h \nu < h \nu_1$$

$$\phi_{0^+} = \phi_3 \quad h \nu_1 < h \nu < h \nu_0$$

$$\phi_{0^+} = \phi_4 \quad h \nu > h \nu_0$$

The boundary frequencies which define $h \nu_g, h \nu_2, h \nu_1, h \nu_0$, (and, correspondingly, $u_g, u_2, u_1, u_0$) are

$$h \nu_g = 12.1 \text{ eV}$$

$$h \nu_2 = 20.0 \text{ eV}$$

$$h \nu_1 = 31.8 \text{ eV}$$

$$h \nu_0 = 35.1 \text{ eV}$$
Fig. 41 Quantum Mechanical Correction Factor $\xi(h\nu)$ Versus Frequency
The rate-governed production of species was calculated using a Lockheed numerical computer code on a Univac 1107 digital computer. The computer code, itself, is a version of the more or less standard stream-tube rate chemistry method. For detailed description the reader is referred to J. G. Hall et al. (Ref. 28) one of several references describing the method and application. A fundamental assumption underlying this method is that the principle evolution of specie components takes place in the direction of the streaming fluid motion (streamline directions) and that the small transverse contributions (across streamlines) can be neglected in many kinds of inviscid non-equilibrium flow processes.

The coupled array of differential species production processes is numerically integrated along a streamline subject to a prescribed pressure or velocity distribution. This distribution is obtained from a prior (equilibrium) flow calculation. The motion of the fluid element is thus described by the designated distribution in conjunction with the one dimensional stream-line momentum equation:

$$u \frac{du}{dx} = - \frac{1}{\rho} \frac{dp}{dx}$$

where x is the streamline coordinate. Hence, the motion of the fluid element (or, in general, the position in space and time of the evolving chemical constituent species) is governed by the results of an auxiliary calculation. This considerably simplifies the code problem to one of forward integration of the coupled chemical rate processes within a fluid element following a designated trajectory in space and time.

The specific version of the code used in the present calculations is described in considerable detail in Vinokur et al. (Ref. 29). Briefly an array of 10 component species
of "air" (considered, initially, as an undisturbed binary mixture of 79 parts molecular nitrogen to 21 parts molecular oxygen) is treated using 7 basic differential rate processes and 3 algebraic species conservation statements. In formulation the system follows closely the work of G. Emanuel (Ref. 30). The 7 basic rate processes are augmented by consideration of multiple third body collision partners of variable efficiency. This consideration essentially increases the number of differential rate processes to 20 (Ref. 29).

The state of the gas at any position is described by a simple equation of state appropriate for a multi-component mixture of ideal constituents,

$$\tilde{\rho}(x) = \tilde{\rho}(x) \tilde{T}(x) \sum_{i=1}^{10} \tilde{n}_i(x)$$

along with an energy (enthalpy) relation,

$$\tilde{h}(x) = \sum_{i=1}^{10} \tilde{n}_i(x) \left[ \tilde{C}_{p_i} \tilde{T}(x) + \tilde{e}_{D_i} + \tilde{e}_{I_i} \right]$$

$$\tilde{h}(x) = \tilde{H}(\text{total}) - \frac{\tilde{u}(x)^2}{2}$$

which completes the system along with the array of differential species production processes written symbolically,

$$\tilde{\omega}_i(x) \cdot \frac{1}{\tilde{u}(x)} = \frac{\text{d}\tilde{n}_i(x)}{\text{d}x}$$

In the foregoing equations, p, \( \rho \), and T are the stream pressure, density and kinetic temperature, \( n_i \) the \( i^{th} \) constituent molar ratio, \( C_{p_i} \) the specific heat capacity of
the $i^{th}$ constituent at constant pressure, $eD_i$ and $eI_i$ the $i^{th}$ component dissociation and ionization energies, $u$ the streaming velocity and $\dot{\omega}_i$ the usual rate of production term for the $i^{th}$ species. The barred designation indicates appropriate non-dimensional form of the variables (Ref. 29). The gas itself is considered instantaneously fully excited in rotational and vibrational degrees of freedom, equilibrated to the translational mode.

Forward rate constants were adopted from the lists in J. G. Hall et al. (Ref. 28) and M. H. Bortner (Ref. 31). Equilibrium constants were developed from the spectroscopic data of Herzberg (Ref. 32), Moore (Ref. 19) and listed energy states of Gilmore (Ref. 33).

Program output includes temperature, density, pressure, enthalpy, velocity and the molar ratios for the 10 component species all as a function of streamline position. The components considered for air are: $O_2$, $N_2$, NO, O, N, NO$^+$, $N_2^+$, O$^+$, N$^+$, and free electrons.
REFERENCES


22. J. Wilson, "Ionization Rate of Air Behind High Speed Shock Waves," AVCO Research Laboratory, Research Report 222, Oct 1965


"The aeronautical and space activities of the United States shall be conducted so as to contribute . . . to the expansion of human knowledge of phenomena in the atmosphere and space. The Administration shall provide for the widest practicable and appropriate dissemination of information concerning its activities and the results thereof."

—NATIONAL AERONAUTICS AND SPACE ACT OF 1958

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