EXPERIMENTAL NEUTRON FLUX ALBEDO FOR CONCRETE

by James A. Kish, Ira T. Myers, and John J. Smithbrick
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SUMMARY

Thermal neutron flux albedo for concrete due to an incident fast-neutron beam is of interest in predicting activation of a nuclear-rated space chamber. Some measurements of this quantity were made for limestone concrete by using plutonium-beryllium and reactor-beam neutron sources. In both cases, measurements were made by using a moving-detector technique to perform the integration of emergent thermal neutron flux. The accuracy of results for the reactor source was limited by lack of beam-spectrum data, but a value of flux albedo for the PuBe source was determined to be 0.13±0.04.

INTRODUCTION

Neutron albedo from thick concrete slabs is of concern in testing of nuclear-powered vehicles in a large space environment chamber now under construction at the NASA Plum Brook Station. The test chamber, which is 100 feet in diameter and 122 feet high, will be surrounded by a 6-foot-thick concrete biological shield, parts of whose surface are to be borated. Of particular concern, from the materials activation standpoint, is the flux of thermal neutrons being returned by the concrete walls that resulted from incident fast neutrons from a test reactor in the chamber.

Thermal neutron flux albedo factors resulting from fast neutrons are difficult to calculate, and there is a general lack of experimental data on the subject. Spinney (refs. 1 and 2) has used an age-diffusion theory approach to predict the albedo of a lightly capturing infinite-slab medium. This calculation tends to be unreliable since age-diffusion theory is questionable in boundary regions or media that have a noticeable hydrogen-atom density. Allen (ref. 3) and Leimdörfer (ref. 4) performed Monte Carlo calculations of neutron flux and current albedo factors for concrete slabs for source energies ranging over 0.1 to 14 MeV. Because chemical binding effects and moderator-atom thermal motions are difficult to take into account and because machine time would become excessive, results of these calculations were cut off at 1 to 10 eV. Furthermore, only hydrogen, oxygen, silicon, and aluminum atoms are considered, and other elements
in an assumed typical composition are replaced by silicon atoms. If the replaced atoms or others not known to be present cause appreciable absorption, particularly at low energies, an accurate result cannot be predicted.

Measurements of thermal-from-fast flux albedo factors resulting from fast neutrons were made for ordinary and borated concrete slabs by using a reactor beam at the Plum Brook Reactor Facility. Measurements were also made by using a collimated plutonium-beryllium source at the Lewis Research Center. A detector sweep method was used in both cases, foil detectors for the reactor source and a lithium 6 semiconductor sandwich detector for the plutonium-beryllium measurement.

DEFINITION OF FLUX ALBEDO

A distinction should be made as to the quantity of interest in consideration of albedo factors. In general the neutron albedo for a given material and geometry may be specified in terms of current (or number), particle flux, energy flux, or dose. A neutron detector that is uniformly swept over a test slab surface has a response that is proportional to total emergent current, flux, or dose, depending upon detector characteristics. Thus foils that are thin (compared with the absorption mean free path) may be used to measure flux albedo and thick foils for current albedo.

The quantity of interest is the thermal neutron flux albedo for a plane-collimated fast neutron beam incident upon an effectively infinite concrete slab. The thermal-from-fast neutron flux albedo $\beta_{TF}$ is defined as follows:

$$\beta_{TF} \equiv \lim_{S \to \infty} \frac{\frac{1}{S} \int_S \varphi_T \, dS'}{\frac{1}{S} \int_S \varphi_F \, dS'} = \frac{\int_S \varphi_T \, dS'}{\int_S \varphi_F \, dS'}$$

where an area, or surface, integration of emitted thermal (subcadmium) neutron flux $\varphi_T$ over an area $S$ is taken that corresponds to the surface integral of incident fast (epicadmium) neutron flux $\varphi_F$. In practice, these surface integrals should be performed over a surface region large enough to account for most of the total emergent thermal neutrons.
EXPERIMENTAL PROCEDURE

Reactor-Beam Source Experiment

The first experiment on neutron flux albedo utilized the underwater beam room facility at the NASA 60 megawatt Plum Brook reactor. The basic arrangement of equipment is shown in figure 1. A boral-filtered beam was incident upon the concrete slab surface over which gold and sulfur foils were swept. The concrete blocks that composed the test slab were poured according to the mix in table I. An approximate composition is given in table II, which is based upon the materials and proportions in table I. As is shown in
TABLE I. - MIX PROPERTIES FOR CONCRETE USED IN REACTOR-BEAM EXPERIMENT

<table>
<thead>
<tr>
<th>Material</th>
<th>Nonborated A</th>
<th>Borated A</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cement (type II grey)</td>
<td>470</td>
<td>470</td>
</tr>
<tr>
<td>Sand (Lake Erie)</td>
<td>1357</td>
<td>978</td>
</tr>
<tr>
<td>Coarse aggregate (crushed limestone)</td>
<td>2032</td>
<td>2032</td>
</tr>
<tr>
<td>Water</td>
<td>272</td>
<td>272</td>
</tr>
<tr>
<td>Boron frit (Chicago Vitreous #S-85)</td>
<td>---</td>
<td>379</td>
</tr>
</tbody>
</table>

TABLE II. - NONBORATED A CONCRETE COMPOSITION

<table>
<thead>
<tr>
<th>Constituent</th>
<th>Composition, weight percent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calcium carbonate</td>
<td>40.6</td>
</tr>
<tr>
<td>Silicon dioxide</td>
<td>36.8</td>
</tr>
<tr>
<td>Calcium oxide</td>
<td>7.3</td>
</tr>
<tr>
<td>Watera</td>
<td>6.6</td>
</tr>
<tr>
<td>Magnesium carbonate</td>
<td>5.9</td>
</tr>
<tr>
<td>Ferric oxide</td>
<td>.7</td>
</tr>
<tr>
<td>Aluminum oxide</td>
<td>1.2</td>
</tr>
<tr>
<td>Magnesium oxide</td>
<td>.3</td>
</tr>
<tr>
<td>Sulfur trioxide</td>
<td>.3</td>
</tr>
<tr>
<td>Sodium oxide</td>
<td>.1</td>
</tr>
<tr>
<td>Potassium oxide</td>
<td>.1</td>
</tr>
<tr>
<td>Sulfur</td>
<td>.1</td>
</tr>
</tbody>
</table>

*aWater retention assumed to be 100 percent.

figure 1, the gold and sulfur foils were mounted on a wand that was mechanically driven by two synchronous motors across an area approximately 18 inches square with an area resolution of about 0.5 inch. Motor speed was such that a single sweep across the test area required a time of about 1 hour. Since the beam intensity was somewhat low, many sweeps (about 20) were required to obtain adequate sulfur pellet counting statistics. Also, since the beam intensity was subject to noticeable change over a reactor power cycle, all data were normalized to the activity of the sulfur pellet beam monitors placed in front of the boral shield (fig. 1).

The necessary slab thickness required for saturation back diffusion of thermal neutrons was determined by placing bare and cadmium-covered gold foils on the slab surface and by varying the number of concrete blocks in the slab. A typical plot is given in figure 2 for the nonborated blocks, which indicates that a slab thickness of 12 inches or greater is effectively infinite for test simulation purposes.

The thermal-from-fast flux albedo $\beta_{TF}$ is related to the foil activities by the following equation:

$$\beta_{TF} = K_1 \frac{A_g}{A_s}$$  \hspace{1cm} (2)
Figure 2. - Gold foil activity as function of slab thickness.

where

\[
K_1 = \frac{\epsilon_s \sum_s (1 - e^{-\lambda_s t}) e^{-\lambda_s t_1}}{\epsilon_g \sum_g (1 - e^{-\lambda_g t}) e^{-\lambda_g t_1}}
\]  

and

- \( A_g \) cadmium-difference gold foil specific activity
- \( A_s \) sulfur pellet specific activity
- \( \epsilon_s \) sulfur pellet counting efficiency
- \( \epsilon_g \) gold foil counting efficiency
- \( t \) exposure time
- \( t_1 \) decay time
- \( \Sigma_s \) \( S^{32}(n,p) \) macroscopic cross section averaged over \( E > 0.5 \text{ eV}, 4.65 \times 10^{-4} \text{ cm}^2/\text{g} \)
- \( \Sigma_g \) \( \text{Au}^{197}(n,\gamma) \) macroscopic cross section averaged over 0 to 0.5 eV, 0.214 cm\(^2\)/g
- \( \lambda_s \) phosphorus 32 decay constant
- \( \lambda_g \) gold 198 decay constant

The gold foil activity \( A_g \) was corrected for beam thermal neutron background by correcting for activity produced by sweeping foils with no concrete in the beam path.

The mean sulfur cross section \( \Sigma_s \) corresponded to a microscopic value of 26 millibarns, which was obtained by averaging \( S^{32}(n,p) \) cross-sectional data over a diffusion-code spectral-shape calculation for the beam (fig. 3) for neutron energies greater than 0.5 eV.

In order to obtain the mean gold activation cross section over 0 to 0.5 eV, where
0.5 eV is considered the cadmium cutoff, a Maxwellian density spectrum was assumed for 0 to 0.1 eV, and a 1/E flux spectrum for 0.1 to 0.5 eV. The resulting mean cross section is 73 percent of the 2200-meter-per-second cross section value for a 1/v detector such as gold.

Measurements of $\beta_{TF}$ for borated concrete were made by replacing the first layer (6 in.) of nonborated blocks with borated blocks. These borated blocks contained approximately 0.8 weight percent of elemental boron.

**Plutonium-Beryllium Source Experiment**

The experimental arrangement is indicated in figure 4, where a 5-curie plutonium-beryllium source was placed in a boral-shielded collimator to provide a fast neutron source. The concrete slab for this measurement was not composed of the same blocks used for the reactor source measurement, but consisted of limestone-aggregate structural blocks. The same detector-sweep method of flux measurement was used as was used for the reactor-source measurement, except a lithium 6 semiconductor sandwich detector was used to count thermal neutrons.

The total number of incident fast neutrons per unit time was taken as the product of the collimator exit area and the neutron flux and was determined by the source strength and an assumed $1/r^2$ relation. Detector count rates were recorded with and without a 20-mil cadmium cover. Cadmium-difference count rates were corrected for background by placing a large boral plate over the surface of the concrete slab.

The flux albedo $\beta_{TF}$ is given in the following equation:
Figure 4. - Experiment for plutonium-beryllium measurement.

\[
\beta_{TF} = \frac{\varphi_T S_d}{\varphi_F S_C} \tag{4}
\]

where

- \( \varphi_T \) surface-averaged thermal (E = 0 to 0.5 eV) neutron flux
- \( S_d \) surface area swept out by the detector
- \( \varphi_F \) fast (E > 0.5 eV) neutron flux at collimator exit
- \( S_C \) collimator exit area

The relation of cadmium-difference detector count rate \( \dot{C}_T \) to \( \varphi_T \) is given by

\[
\dot{C}_T = K \bar{\sigma}_{Li} \varphi_T \tag{5}
\]

where \( \bar{\sigma}_{Li} = Li^6(n, \alpha) \) cross section averaged over 0 to 0.5 eV and \( K \) is the detector constant.

The flux albedo \( \beta_{TF} \) may be written then in terms of measured parameters:

\[
\beta_{TF} = \frac{4\pi d^2 S_d}{Q S_C K \bar{\sigma}_{Li}} \frac{\dot{C}_T}{\dot{C}_T} \tag{6}
\]
where \( Q \) is the source output and \( d \) is the distance from source to collimator exit.

### RESULTS AND DISCUSSION

The values obtained for albedo are summarized in table III. The value of \( \beta_{TF} \) for the plutonium-beryllium measurement was \( 0.13 \pm 0.04 \). The indicated error includes counting statistics and uncertainties in both the lithium 6 cross section and mass. The concrete used in obtaining this value was a nonborated limestone aggregate, but was not the same concrete as that used for obtaining the reactor source value. The plutonium-beryllium albedo value includes an estimated correction factor to account for detector readings being recorded over a finite surface area about the beam center and thus not recording the total emergent thermal neutron flux. This correction is described in appendix A.

Table III also includes an analytical current albedo value of \( 0.15 \pm 0.10 \), which was calculated for ordinary concrete with an age-diffusion method (see appendix B). Since the experimental and analytical albedo values are on a flux or current basis, respectively, some assumption on emergent angular distribution of neutrons must be made for a valid comparison. The measured plutonium-beryllium flux albedo implies a current albedo of \( 0.09 \pm 0.03 \) if a cosine distribution for emergent thermal neutrons is assumed, since \( \cos \theta = 2/3 \), where \( \theta \) is the angle from the outward normal to the surface. Thus the measured plutonium-beryllium albedo and the age-diffusion result are in agreement, within the ranges of error.

The use of foil data for the reactor-beam source and equations (2) and (3) produce a value for \( \beta_{TF} \) of \( 0.020 \) for nonborated concrete. The cadmium-difference gold foil

<table>
<thead>
<tr>
<th>Albedo</th>
<th>Basis</th>
<th>Source energy</th>
<th>Concrete</th>
<th>Neutron albedo value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flux</td>
<td>Measurement</td>
<td>Plutonium-beryllium</td>
<td>Nonborated B</td>
<td>0.13 \pm 0.04</td>
</tr>
<tr>
<td>Current</td>
<td>Measurement estimate (^a)</td>
<td>Plutonium-beryllium</td>
<td>Nonborated B</td>
<td>0.09 \pm 0.03</td>
</tr>
<tr>
<td>Current</td>
<td>Calculated (^b)</td>
<td>Fission Reactor</td>
<td>Nonborated</td>
<td>0.15 \pm 0.10</td>
</tr>
<tr>
<td>Flux</td>
<td>Measurement</td>
<td>Reactor</td>
<td>Borated A</td>
<td>0.020</td>
</tr>
<tr>
<td>Flux</td>
<td>Measurement</td>
<td></td>
<td></td>
<td>&lt;0.003</td>
</tr>
</tbody>
</table>

\(^a\) Based upon measured flux value assuming a cosine emergent distribution.

\(^b\) See appendix B.
activity for the borated concrete, however, was essentially the same as that for no concrete in the beam path. Since this beam thermal neutron background corresponded to a value of 0.003 for $\beta_{TF}$, the actual albedo for borated concrete would be smaller. No successful measurements of beam spectrum were possible since beam gamma intensity was too high to use lithium 6 semiconductor sandwich detectors, and beam neutron intensity was too low to use foils adequately. Consequently, a major source of uncertainty exists in the value for $\overline{\Sigma}_S$ used in equation (3).

The large difference in $\beta_{TF}$ values obtained for the reactor and plutonium-beryllium sources is probably not accountable in terms of source spectrum effects; however, a large uncertainty in the reactor source albedo value arises from the lack of beam spectrum data and the consequent error in mean sulfur cross section used for fast neutron dosimetry. Note that no flux-area correction factor (appendix A) was applied to the reactor source albedo value because of the lack of data. On the basis of the plutonium-beryllium correction, however, this effect would cause about a factor of 2 increase in the reactor source albedo value. The quoted reactor source albedo value could be in error by as much as a factor of 10 because of these combined uncertainties.

The difference between albedo values for reactor and plutonium-beryllium sources might also be in part due to compositional variation in the concretes used for each measurement. The relative elemental thermal neutron absorption contributions, based upon the nonborated concrete A composition given in table II, are presented in table IV. Approximately 93 percent of the total thermal neutron absorption is due to calcium, hydrogen, and silicon. Calcium should also be a major absorber in the nonborated concrete

<table>
<thead>
<tr>
<th>Element</th>
<th>Composition, weight percent</th>
<th>Cross section, $\sigma_{a'}$</th>
<th>Macroscopic cross section, $\overline{\Sigma}_{a'}$</th>
<th>Percent of total $\Sigma_a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hydrogen</td>
<td>0.7</td>
<td>0.332</td>
<td>140 $\times 10^6$</td>
<td>37.4</td>
</tr>
<tr>
<td>Oxygen</td>
<td>51.0</td>
<td>.0002</td>
<td>.38</td>
<td>1</td>
</tr>
<tr>
<td>Silicon</td>
<td>16.2</td>
<td>.16</td>
<td>56</td>
<td>15.0</td>
</tr>
<tr>
<td>Calcium</td>
<td>23.0</td>
<td>.44</td>
<td>150</td>
<td>40.2</td>
</tr>
<tr>
<td>Aluminum</td>
<td>.6</td>
<td>.23</td>
<td>3.1</td>
<td>.8</td>
</tr>
<tr>
<td>Iron</td>
<td>.5</td>
<td>2.53</td>
<td>14</td>
<td>3.7</td>
</tr>
<tr>
<td>Magnesium</td>
<td>1.9</td>
<td>.063</td>
<td>3.0</td>
<td>.8</td>
</tr>
<tr>
<td>Sodium</td>
<td>.1</td>
<td>.505</td>
<td>1.3</td>
<td>.3</td>
</tr>
<tr>
<td>Potassium</td>
<td>.1</td>
<td>2.07</td>
<td>3.2</td>
<td>.8</td>
</tr>
<tr>
<td>Carbon</td>
<td>5.7</td>
<td>.0034</td>
<td>1.4</td>
<td>.4</td>
</tr>
<tr>
<td>Sulfur</td>
<td>.2</td>
<td>.52</td>
<td>2.0</td>
<td>.5</td>
</tr>
</tbody>
</table>
B, used for plutonium-beryllium measurements, since this was also a limestone-aggregate concrete.

SUMMARY OF RESULTS

Measurements of thermal-from-fast neutron flux albedo $\beta_{TF}$ were made for non-borated and borated concrete by using a reactor-beam source. Flux albedo measurements were also made on a nonborated concrete with a collimated plutonium-beryllium source. The following results were obtained:

1. On the basis of reactor-beam measurements, a thickness of 12 inches of nonborated limestone concrete was found to be sufficient for essential saturation back diffusion of thermal neutrons.

2. The best value obtained for $\beta_{TF}$ is $0.13\pm0.04$ for plutonium-beryllium neutrons incident on a nonborated concrete. This value is in agreement with an age-diffusion theory calculation for ordinary concrete.

3. The reactor-beam experiment yielded a value of 0.020 for $\beta_{TF}$ from nonborated concrete. A value of $\beta_{TF}$ for borated concrete was below sensitivity limits, which corresponded to an albedo value of 0.003. The large difference between quoted plutonium-beryllium and reactor source albedo values might result from a large uncertainty in reactor-beam source spectrum, incomplete flux-area integration, and composition variation that may exist for the two different nonborated concretes.

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National Aeronautics and Space Administration,
Cleveland, Ohio, June 30, 1966,
120-27-01-03-22.
APPENDIX A

FLUX-AREA CORRECTION FACTOR FOR PLUTONIUM-BERYLLIUM SOURCE MEASUREMENT

The lithium 6 detector thermal neutron readings were recorded over the dashed-line square areas indicated in figure 5, where d is 18 inches. To account for thermal neutron flux beyond the areas of actual measurement, a simple analytical model was assumed (fig. 6), where the plutonium-beryllium neutrons incident upon the concrete were assumed to generate a unit point source of thermal neutrons at a depth equal to an average slowing-down length $\bar{L}_s$. The thermal neutron flux at the surface is

$$\varphi(r) = \frac{e^{-K\rho}}{4\pi\rho^2} \quad (A1)$$

where the coefficient $K$ is evaluated from data recorded over finite surface areas.

The relative surface flux-area integrals enclosed by radii $r_1$ and $r_2$ are first determined by a surface geometry transformation between square areas used for measurements and circular areas defined by the model. The values of $r_1$ and $r_2$ are determined by equating the area $d^2$ to the central circular area and also to that of the segment defined by $\theta$ (fig. 5). If $\alpha$ is defined as the ratio of flux-area integrals as measured for the area of squares B and A (fig. 5), then $\alpha'$, which represents the ratio of flux-area integrals for the annular region of dimension $r_2 - r_1$ to that for the region inside $r_1$, is
\[ \alpha' = \frac{2\pi}{\theta} \alpha \]  

(A2)

where \( \theta \) equals \( 2 \tan^{-1} \left[ \frac{d}{r_1 + r_2} \right] \) and is determined from the geometry transformation.

A value for \( K \) is determined by solving the following equations:

\[ \alpha' = \frac{\int_{r_1}^{r_2} \varphi(r)r \, dr}{\int_{0}^{r_1} \varphi(r)r \, dr} \]

(A3)

The quantity \( \gamma \) is experimentally evaluated and fitted to the model while \( \tau \) is evaluated with the model.

The corrected value of flux albedo \( \beta \), which includes flux-area contributions from all surface area regions, may be expressed as

\[ \beta = \beta_0 + \alpha' \beta_0 + \gamma \beta \]

(A5)

where \( \beta_0 \) is the value of flux albedo based upon measurements over the area of the center square \( A \) (fig. 5). The quantity \( \alpha' \) is experimentally evaluated and fitted to the model while \( \gamma \) is evaluated with the model.

The following is a summary of parameters including the resultant value of \( \beta \) (the value of \( L_s \) was assumed on the basis of representative values of age \( \tau \) from references 2 and 5):
\[ r_1 = 10.2 \text{ in.} \]
\[ r_2 = 29.4 \text{ in.} \]
\[ \theta = 0.855 \text{ rad} \]
\[ \bar{L}_S = 5 \pm 2 \text{ in.} \]
\[ \alpha = 0.082 \pm 0.027 \]
\[ \alpha' = 0.60 \pm 0.20 \]
\[ \gamma = 0.07 \pm 0.02 \]
\[ \beta_0 = 0.0764 \pm 0.0184 \]
\[ \beta = 0.13 \pm 0.04 \]

A disk source would have been a more reasonable source geometry to select for this model, but flux-area integral calculations for such a source cannot be evaluated simply. The validity of the point source model, however, can be investigated by displacing the point source a distance \( m \) (fig. 6) from the center of symmetry and by calculating the relative change in flux-area product over a surface area with a radius of 10 inches. This area is that included by dimension \( r_1 \) (see fig. 5) and over which the value of \( \beta_0 \) is determined. The disk source was estimated from the experiment geometry to be about 9 inches in diameter. The appropriate value of \( m \) is not the source radius, but corresponds to a radial distance averaged over the source area. Thus \( m \) was taken as two-thirds of the source radius, or 3 inches. It was estimated from the results of Rockwell (ref. 6) that less than a 5-percent change in flux-area product over the 20-inch-diameter surface area is effected by such a point-source displacement. Hence, the original point-source assumption is a reasonably valid source geometry for the surface area sizes upon which measurements were based.
ANALYTICAL ALBEDO CALCULATION

An estimate of thermal-from-fast neutron current albedo \( \beta_j \) was made by using the diffusion theory result (ref. 2):

\[
\beta_j = \frac{SL}{(L_s + L)(1 + \frac{2D}{L})}
\]  \hspace{1cm} (B1)

where

- \( S \) total source strength
- \( L \) thermal diffusion length in concrete
- \( L_s \) slowing-down length in medium appropriate to thermal neutrons
- \( D \) thermal diffusion coefficient

The total source strength \( S \) is taken as \( 1 - F(\tau_{Th}) \), where \( F(\tau_{Th}) \) represents the fraction of neutrons emerging from the concrete with age less than thermal.

An expression for \( F(\tau_{Th}) \) has been obtained (ref. 1) by using age theory with a distributed source of fast neutrons given by first collisions of incident beam neutrons. An estimate of \( F(\tau_{Th}) \) was made by utilizing a set of curves (ref. 2) plotted as functions of the parameters \( \tau_{Th}, \Sigma_R, \) and \( \ell_{Tr} \), where \( \tau_{Th} \) is the thermal neutron age in concrete, \( \Sigma_R \) is the macroscopic removal cross section for fast neutrons incident upon concrete, and \( \ell_{Tr} \) is the transport mean free path for neutrons slowing down to thermal energies in concrete.

The following is a summary of mean parameter values that corresponded to several ordinary concretes (ref. 5) (the parameter values assume a water retention of 100 percent, and parameter variation about the mean value was assigned on a confidence level of 90 percent):

\[
\begin{align*}
L &= 8.3 \pm 4.2 \text{ cm} \\
D &= 0.66 \pm 0.30 \text{ cm} \\
\tau_{Th} &= 187 \pm 98 \text{ cm}^2 \\
L_s &= 13.7 \pm 3.6 \text{ cm}
\end{align*}
\]
\[ \ell_{Tr} = 1.98 \pm 0.60 \text{ cm} \]
\[ \Sigma_R = 0.081 \pm 0.005 \text{ cm}^{-1} \]
\[ F(\tau_{Th}) = 0.55 \pm 0.10 \]

When these values are used in equation (B1), a value of $0.15 \pm 0.10$ is obtained for $\beta_J$. 
REFERENCES


