AERODYNAMIC-CENTER CONSIDERATIONS
OF WINGS AND WING-BODY COMBINATIONS

by John E. Lamar and William J. Alford, Jr.

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

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SUMMARY

Aerodynamic-center variations with Mach number are considered for wings of different planform. The normalizing parameter used is the square root of the wing area, which provides a more meaningful basis for comparing the aerodynamic-center shifts than does the mean geometric chord. The theoretical methods used are shown to be adequate for predicting typical aerodynamic-center shifts, and ways of minimizing the shifts for both fixed and variable-sweep wings are presented.

INTRODUCTION

In the design of supersonic aircraft, a detailed knowledge of the aerodynamic-center movement is important in order to minimize trim drag, maximize load-factor capability, and provide acceptable handling qualities. One of the principal contributions to the movement of the aerodynamic-center position is the well-known change in load distribution with Mach number in going from subsonic to supersonic speeds. In addition, large aerodynamic-center variations are quite often associated with variable-geometry features such as variable wing sweep.

The purpose of this paper is to review the choice of normalizing parameters and the effects of Mach number on the aerodynamic-center movement of rigid wing-body combinations at low lift. For fixed wings the effects of both conventional and composite planforms on the aerodynamic-center shift are presented, and for variable-sweep wings the characteristic movements of aerodynamic-center position with pivot location and with a variable-geometry apex are discussed.

Since systematic experimental investigations of the effects of planform on the aerodynamic-center movement with Mach number are still limited, the approach followed herein is to establish the validity of the computative processes by illustrative comparison with experiment and then to rely on theory to show the systematic variations. The two theories used in this paper are for the wing alone in unseparated flow. One is a modified Multhopp subsonic lifting-surface theory developed by the senior author (unpublished), and the other is a supersonic lifting-surface theory (ref. 1). For wings experiencing separated flow these theories are not adequate per se for predicting the aerodynamic-center movement.

### SYMBOLS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>aspect ratio</td>
</tr>
<tr>
<td>a</td>
<td>distance from apex of high-sweep wing to apex of low-sweep wing (see fig. 10)</td>
</tr>
<tr>
<td>b</td>
<td>span</td>
</tr>
<tr>
<td>$C_L$</td>
<td>lift coefficient</td>
</tr>
<tr>
<td>$C_p$</td>
<td>pressure coefficient, $\frac{p_{local} - p_{free \ stream}}{q}$</td>
</tr>
<tr>
<td>$\Delta C_p$</td>
<td>incremental pressure coefficient, $C_p,upper - C_p,lower$</td>
</tr>
<tr>
<td>c</td>
<td>local chord</td>
</tr>
<tr>
<td>$\bar{c}$</td>
<td>mean geometric chord</td>
</tr>
<tr>
<td>$c_r$</td>
<td>root chord of basic planform</td>
</tr>
<tr>
<td>$c_t$</td>
<td>tip chord of basic planform</td>
</tr>
<tr>
<td>d</td>
<td>longitudinal distance from root trailing edge to tip trailing edge</td>
</tr>
<tr>
<td>K</td>
<td>constant</td>
</tr>
<tr>
<td>$l$</td>
<td>longitudinal distance from apex to tip trailing edge</td>
</tr>
<tr>
<td>M</td>
<td>Mach number</td>
</tr>
<tr>
<td>p</td>
<td>static pressure</td>
</tr>
<tr>
<td>q</td>
<td>free-stream dynamic pressure</td>
</tr>
<tr>
<td>S</td>
<td>wing area</td>
</tr>
<tr>
<td>x</td>
<td>chordwise distance from apex of high-sweep wing to plane-of-symmetry intercept with trailing edge of free-floating apex</td>
</tr>
<tr>
<td>$\bar{x}_M$</td>
<td>chordwise distance from a reference point to aerodynamic center at any Mach number</td>
</tr>
<tr>
<td>$\bar{x}<em>{M=0}, \bar{x}</em>{M=0.2}, \bar{x}_{M=0.25}$</td>
<td>chordwise distance from a reference point to aerodynamic center at specific Mach number indicated by subscript</td>
</tr>
<tr>
<td>$\bar{x}<em>{M=2}, \bar{x}</em>{M=3}$</td>
<td></td>
</tr>
<tr>
<td>$\Delta \bar{x}$</td>
<td>incremental change in aerodynamic-center location</td>
</tr>
</tbody>
</table>
\( y \) 
spanwise distance 
\( y_b \) 
spanwise distance from plane of symmetry to leading-edge break 
\( y_p \) 
spanwise distance from plane of symmetry to pivot 
\( \alpha \) 
angle of attack 
\( \Lambda \) 
leading-edge sweep of wing 
\( \Lambda_0 \) 
leading-edge sweep of outer panel 
\( \Lambda_t \) 
leading-edge sweep of cranked wing tip 
\( \lambda \) 
taper ratio

**REQUIREMENT OF A NORMALIZING PARAMETER**

A knowledge of the actual dimensional movement of the aerodynamic center is required in order to determine the out-of-trim moments which must be balanced by the control surface. Thus, in the selection of a normalizing parameter the need for a reference length which, for a given wing area, is independent of planform is considered to be of primary importance. The reference length selected herein is the square root of the wing area \( \sqrt{S} \), which, of course, is independent of planform and therefore provides fractional aerodynamic-center movements that are proportional to the actual dimensional shifts.

The customary use of the mean geometric chord \( \bar{c} \), although adequate for normalizing the aerodynamic-center shift for a given planform, is not convenient when comparing planforms, since the magnitude of \( \bar{c} \) is dependent upon planform. As an aid in transferring aerodynamic-center shifts from one normalizing parameter to another, the relationship between \( \bar{c} \) and \( \sqrt{S} \) is given both algebraically and graphically in figure 1 for wings which fit within the geometry limitations shown. For composite planforms \( \bar{c}/\sqrt{S} \) may be determined from

\[
\frac{\bar{c}}{\sqrt{S}} = \frac{\int_0^{b/2} c^2 dy}{\sqrt{\int_0^{b/2} \left( \frac{c}{dy} \right)^{3/2}}}
\]

**DISCUSSION**

**Comparison of Theory and Experiment**

Some typical experimentally determined aerodynamic-center shifts with Mach number (ref. 2), which are useful in evaluating the theories and the previously mentioned normalizing parameters, are presented in figures 2 and 3.
The experimental shifts, together with theoretical predictions, are shown in figure 2 for a series of delta wings with aspect ratios ranging from 2 to 4. In this figure, \( \Delta x \) is the distance between the aerodynamic-center location at a Mach number of 0.25 and the aerodynamic-center location at any Mach number. The mean geometric chord \( \bar{c} \) and the square root of the wing area \( \sqrt{S} \) are used as normalizing parameters, and both \( \Delta x / \bar{c} \) and \( \Delta x / \sqrt{S} \) are plotted as functions of Mach number. When the aerodynamic-center shift is based on the respective \( \bar{c} \), the delta wing with the lowest aspect ratio has the smallest incremental change in aerodynamic-center location at the supersonic Mach numbers. However, when the aerodynamic-center shift is based on the respective \( \sqrt{S} \), all three wings exhibit essentially the same fractional change in aerodynamic-center location throughout the Mach number range. The theories predict reasonably well the aerodynamic-center shifts for these delta-wing bodies.

Figure 3 presents three wing-body combinations and illustrates the effect of wing sweep and taper ratio on the aerodynamic-center shift with Mach number. The wings are of aspect ratio 3 and have planforms ranging from a trapezoidal to a delta shape. Of the three wing-body combinations shown, the delta-wing-body configuration is seen to exhibit the smallest change in aerodynamic-center location for Mach numbers greater than 1 when \( \bar{c} \) is used as the normalizing parameter. However, when \( \sqrt{S} \) is used as the normalizing parameter, the aerodynamic-center shift for the sweptback-wing-body configuration is almost as small. Again the agreement between theory and experiment is reasonable.

An indication of the aerodynamic-center movement, when the wings of figures 2 and 3 are sized to provide comparable take-off and landing performance, can be obtained by normalizing the incremental aerodynamic-center shifts previously obtained with respect to the low-speed lift coefficient of each wing at an assumed angle of attack. Such an indication is presented in figure 4 for \( \alpha = 12^\circ \). At the supersonic Mach numbers for the delta planforms, the wing with the highest aspect ratio has the lowest aerodynamic-center shift, and for the planforms of constant aspect ratio, the sweptback wing exhibits the lowest aerodynamic-center shift.

Fixed-Wing Studies

In figures discussed subsequently, the aerodynamic-center shifts have been computed by the theoretical methods. For wings which have fixed planforms, the reference length is the \( \sqrt{S} \) of each planform.

The results of one such aerodynamic-center study for a series of conventional fixed wings with planform variation in sweep and in taper and notch ratios are presented in figure 5. For a delta wing, \( d/l = 0 \) and for arrow wings, \( d/l > 0 \). For illustrative purposes both the effect of changing the leading-edge sweep and the notch ratio when the taper ratio is zero and the effect of changing the taper and notch ratios when the leading-edge sweep angle is 60\(^\circ\) are presented.

When the taper ratio is zero, a decrease in \( \Delta x / \sqrt{S} \) of about 0.05 occurs as the notch ratio is increased from 0 to 0.5 for leading-edge sweep angles of 4
45° and 60°. For a sweep angle of 70°, $\Delta x/\sqrt{S}$ at first decreases approximately 0.01 and then increases about 0.01 above its value at $d/l = 0$. At any particular notch ratio, the wing with the lowest sweep shows the smallest aerodynamic-center shift.

When the wing leading-edge sweep angle is 60°, decreases in $\Delta x/\sqrt{S}$ of 0.05, 0.09, and 0.12 occur over the range of notch ratios considered for taper ratios of 0, 0.25, and 0.50, respectively. At any particular notch ratio, the wing with the lowest taper ratio exhibits the smallest aerodynamic-center shift. When the supersonic Mach number is other than 3, different trends in the aerodynamic-center movement may occur with increasing notch ratio.

One method of minimizing the aerodynamic-center shift of an arrow wing is to reduce the sweep of the wing tip by shearing it forward. Some calculated results illustrating this technique are presented in figure 6. The basic arrow wing has a sweep of 74°, and $\Delta x/\sqrt{S}$ is reduced to about half its original value by shearing the tip forward from 74° to 55°. The reason for this reduction is that wings with cranked tips lose some of the loading at the tip because the value of lift-curve slope on the lower swept outer portion of the wing decreases with increasing Mach number more markedly than does the value on the inner portion of the wing. This decrease in lift-curve slope on the outer portion begins at a lower Mach number and results in a change in loading which causes the aerodynamic center to move forward with increasing supersonic Mach number.

One method of reducing the aerodynamic-center shift of a delta wing is the addition of a forewing inboard. In figure 7 the effect of such an addition is presented as a function of the leading-edge-break location and apex extension. A reduction in the aerodynamic-center shift is obtained for each apex location as $\frac{y_b}{b/2}$ is increased from 0 to 0.5. At any particular value of leading-edge-break ratio within the range examined, the wing with the most forward apex or the longest root chord has the smallest aerodynamic-center shift, because the inboard sweeps are higher and therefore the inner panel has a lower aspect ratio which gives it an essentially invariant value of lift-curve slope with Mach number. However, the outer panel has a higher aspect ratio and lower sweep, and the value of lift-curve slope decreases with increasing supersonic Mach number. Thus, the inner panel carries proportionally more of the loading. The aerodynamic center is forced forward with increasing values of leading-edge-break ratio because of the area added inboard. Experimental substantiation of this low level of aerodynamic-center shift, with a model that had a wing which covered most of the body, is provided in reference 3. (See also ref. 4.)

In addition, wing-body combinations exhibit smaller aerodynamic-center shifts than does the wing alone because the body acts as a forewing with a very low value of leading-edge-break ratio.

Variable-Sweep-Wing Studies

For wings with variable sweep, a problem in aerodynamic-center variation, in addition to that caused by the Mach number effect, results from changes in the wing sweep. The shift resulting from wing-sweep changes must be minimized
in order to make variable-sweep wings competitive, from aerodynamic-center considerations, with fixed wings. To illustrate this problem, the theoretical loading distributions of a variable-sweep wing with an outboard pivot (ref. 5) at a Mach number of 0.23 and at low lift is presented in figure 8. At the top of this figure the variable-sweep wing is shown in its low-sweep and high-sweep positions, and superimposed on the low-sweep planform are its theoretical and experimental chordwise pressure loadings which are seen to be in good agreement. At the bottom of the figure the theoretical longitudinal loading distributions for both sweeps have been computed at $C_L = 0.12$ and projected onto the plane of symmetry. As the outer panel is swept back, the inner panel carries more of the loading and thus tends to balance out the additional moments created by the reduced outer-panel loadings acting through longer moment arms. In this example, because of the outboard location of the pivot, the aerodynamic center, as given by the chordwise location of the lift vector, actually shifts slightly forward.

A study was undertaken to determine the effect that the pivot location has on the aerodynamic-center shift, and the results are presented in figure 9. In this figure and in figure 10, the reference planform area is taken for the wing in its high-sweep position.

Each pivot lies on the loci of points from which the outer panel can be swept from its high-sweep position to a low-sweep position. The relative chordwise location of the pivot determines the chordwise position of the outer panel at low sweep without changing the sweep angle or the semispan.

The results of the theoretical study show that the total aerodynamic-center shift $\Delta x/\Delta S$ (see fig. 9) can be reduced from 0.2 to 0.1 by moving the pivot outboard. The dashed line is used as a reference to indicate that portion of the total shift caused by the change in Mach number from 0.2 to 2 at $A_o = 70^\circ$. The remaining shift is attributed to the change in sweep from $15^\circ$ to $70^\circ$ at $M = 0.2$. The movement of the pivot outboard changes only the part of the shift dependent on sweep. By proper positioning of the pivot, this part of the shift can be eliminated. When the sweep effect causes the aerodynamic center to move ahead of its low-speed high-sweep position, the Mach number effect is reduced.

These results are supported by experimental data for a similar wing-body combination. Figure 9 shows that a reduction in the total aerodynamic-center shift of 0.07 occurs as the spanwise location of the pivot is moved from one extreme to the other. The characteristics of this combination and how the pivot location affects maneuverability considerations are discussed in reference 6.

As noted in reference 7, if a high inboard sweep is required for supersonic flight, then at subsonic speeds and low outer-panel sweep, devices such as the double inboard pivot (ref. 8) and the free-floating apex (ref. 9) can be used to eliminate the resulting pitch-up. These devices also provide a means of controlling the aerodynamic-center movement, as illustrated in figure 10, where they are shown to have the following two features in common: (1) When the outer panel is in its low-sweep position, the forewing or apex is either pivoted inside the fuselage or allowed to free-float carrying no load; and (2) when the
outer panel is swept back, the apex is affixed to the front of the outer panel and forms a continuous leading edge.

Lifting-surface calculations have been made to illustrate the effect of the amount of the apex which is folded or free-floated. Varying amounts of the apex have been removed to represent the aerodynamic effect of both concepts. With the removal, subsonically, of an increasingly large amount of the apex (correlated with the chordwise distance x), the total aerodynamic-center shift decreases from about 0.18 to 0. Again the dashed line represents that portion of the shift due to changing the Mach number from 0 to 3 when \( \Lambda_0 = 71.5^0 \). The effect of changing the sweep \( \Lambda_0 \) from 25° to 71.5° at \( M = 0 \) makes up the remainder of the shift.

When \( x/a = 0 \), the change in wing sweep has essentially no effect; consequently, almost all the aerodynamic-center shift is due to the change in Mach number. However, when \( x/a = 1.0 \), the sweep effect is large enough to cancel all the Mach number effect.

It should be noted that the aerodynamic-center shift may also be minimized by changing the supersonic Mach number or by changing the center-of-gravity location at the different sweeps and Mach numbers.

**CONCLUSIONS**

A general conclusion of this study is that, when comparing aerodynamic-center movements of wings of different planform, a normalizing parameter independent of planform, such as the square root of the wing area, is more appropriate than the customarily used mean geometric chord, which is dependent on planform. The following specific conclusions were reached:

1. The theoretical methods have been demonstrated to be adequate for predicting the aerodynamic-center shift with Mach number for a variety of wing planforms, but are not suitable for determining the absolute aerodynamic-center location at any Mach number since body and interference effects are not included.

2. For fixed wings, the aerodynamic-center shift can be controlled by proper selection of sweep and of taper and notch ratios and by inboard and outboard area proportioning with different degrees of sweep.

3. For variable-sweep wings the aerodynamic-center shift can be controlled by pivot location and by apex devices, such as the double inboard pivot and the free-floating apex.

 Langley Research Center,  
National Aeronautics and Space Administration,  
Langley Station, Hampton, Va., May 23, 1966,  
126-13-03-22-23.
REFERENCES


Figure 1

EFFECT OF PLANFORM PARAMETERS ON $\frac{\bar{c}}{\sqrt{S}}$

\[ \lambda = \frac{c_t}{c_r} \]

\[ \frac{\bar{c}}{\sqrt{S}} = \frac{4}{3\sqrt{a}} \frac{1 + \lambda + \lambda^2}{1 + 2\lambda + \lambda^2} \]

Figure 2

EFFECT OF ASPECT RATIO AND SWEEP

$\Delta x = \bar{x}_M - \bar{x}_M = 0.25; \lambda = 0$

<table>
<thead>
<tr>
<th>EXP</th>
<th>WING-ALONE THEORY</th>
<th>A</th>
<th>$\lambda$, DEG</th>
</tr>
</thead>
<tbody>
<tr>
<td>○</td>
<td>2</td>
<td>63.5</td>
<td></td>
</tr>
<tr>
<td>□</td>
<td>3</td>
<td>53</td>
<td></td>
</tr>
<tr>
<td>◆</td>
<td>4</td>
<td>45</td>
<td></td>
</tr>
</tbody>
</table>
EFFECT OF SWEEP AND TAPER
\( \Delta x = \bar{x}_M - \bar{x}_m = 0.25 \); \( \alpha = 3 \)

**Table:**

| EXP WING-ALONE THEORY | \( \lambda \) | \( \Delta \) | DEG |
|-----------------------|---------------|-------------|
| ○                     | 20            | 0.39        |
| □                     | 45            | 0.40        |
| ▽                     | 53            | 0           |

**Figure 3**

EFFECT OF WING SIZING FOR LOW-SPEED CONDITIONS
\( \Delta \bar{x} = \bar{x}_M - \bar{x}_m = 0.25 \)

**Table:**

<table>
<thead>
<tr>
<th>EXP</th>
<th>A</th>
<th>( \lambda ) = 0</th>
<th>EXP</th>
<th>( \Delta )</th>
<th>DEG</th>
<th>A = 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>○</td>
<td>2</td>
<td></td>
<td>△</td>
<td>20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>□</td>
<td>3</td>
<td></td>
<td>△</td>
<td>45</td>
<td></td>
<td></td>
</tr>
<tr>
<td>▽</td>
<td>4</td>
<td></td>
<td>△</td>
<td>53</td>
<td></td>
<td></td>
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</tbody>
</table>

**Figure 4**
CONVENTIONAL-PLANFORM VARIATION

\[ \Delta \bar{x} = \bar{x}_{M=3} - \bar{x}_{M=0} \]

\[ \lambda = 0 \]

\[ \lambda = 0.50 \]

\[ \Delta = 60^\circ \]

\[ \Delta = 70^\circ \]

\[ 60^\circ \]

\[ 45^\circ \]

NOTCH RATIO, \( \frac{d}{t} \)

Figure 5

COMPOSITE PLANFORMS

EFFECT OF CRANKED-TIP SWEEP; \( \Delta \bar{x} = \bar{x}_{M=3} - \bar{x}_{M=0} \); \( \frac{y_{B}}{b/2} = 0.73 \)

\[ \Lambda_{1} \]

\[ 74^\circ \]

AFT

\[ \frac{\Delta \bar{x}}{\sqrt{S}} \]

\[ \Delta \bar{x} \]

\[ \Delta \bar{x} \]

0.50

0.55

0.60

0.65

0.70

0.75

\[ \Lambda_{1}, \text{ DEG} \]

Figure 6
COMPOSITE PLANFORMS
EFFECT OF LEADING-EDGE BREAK LOCATION; \( \Delta \bar{x} = \bar{x}_{M=3} - \bar{x}_{M=0} \)

Figure 7

EFFECT OF SWEEP ON LOAD DISTRIBUTION
\( M = 0.23; \ C_L = 0.12 \)

Figure 8
EFFECT OF SPANWISE LOCATION OF PIVOT

\[ \Delta \tilde{x} = (\tilde{x}_{M} = 2)_{A_0 = 70^\circ} - (\tilde{x}_{M} = 0.2)_{A_0 = 15^\circ} \]

**THEORY**

**EXPERIMENT**

Figure 9

EFFECT OF VARIABLE-GEOMETRY APEX

\[ \Delta \tilde{x} = (\tilde{x}_{M} = 3)_{A_0 = 71.5^\circ} - (\tilde{x}_{M} = 0)_{A_0 = 25^\circ} \]

Figure 10
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—National Aeronautics and Space Act of 1958

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