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A MANUAL METHOD FOR CONTROL OF THE THRUST AXIS DURING PLANAR ASCENT FROM THE LUNAR SURFACE TO A CIRCULAR ORBIT

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Page 5: In the first equation on this page, the first term on the left should be $\ddot{r}$ instead of r. The corrected equation then reads:

$$\ddot{r} - r\dot{\theta}^2 = \frac{F}{m} \sin \alpha - g_m\left(\frac{r_m}{r}\right)^2$$

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A MANUAL METHOD FOR CONTROL OF THE THRUST AXIS DURING PLANAR ASCENT FROM THE LUNAR SURFACE TO A CIRCULAR ORBIT

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SUMMARY

An analytical study has been made of the possibility of using some simple control reference as an aid to thrust-axis orientation during the ascent from the lunar surface to a circular orbit at 80 nautical miles (148 160 meters). The procedure used in the study was to compute a fuel-optimum ascent which was composed of: (1) a constant-thrust continuous-burn powered trajectory to a circular orbit at 50 000 feet (15 240 meters), (2) a short thrusting period to establish a transfer trajectory from 50 000 feet (15 240 meters) to 80 nautical miles (148 160 meters), and (3) a short thrusting period to circularize the orbit at 80 nautical miles (148 160 meters). The thrust-axis orientation relative to various references was examined for the powered portion of the trajectory. The down-range horizon was chosen as the most readily available or convenient reference.

Calculations showed that, after an initial vertical thrust time of 19 seconds, the fuel-optimum trajectory could be approximated to 50 000 feet (15 240 meters) by holding the first angle between the thrust axis and the down-range horizon at 30° for 248 seconds and a second angle at 9° for 156.9 seconds. A Hohmann transfer from 50 000 feet (15 240 meters) was established by thrusting for 5.04 seconds at 7.6° and completed at 80 nautical miles by thrusting for 4.9 seconds at 22.9°. The two-constant-angle ascent to 50 000 feet (15 240 meters) used 0.3 percent more fuel than the fuel-optimum maneuver. The fuel used for the rest of the ascent was the same for the constant-angle thrusting periods as for the fuel-optimum maneuver.

The terminal conditions were sensitive to errors in thrust direction and magnitude and were affected more by errors occurring in the powered portion of the trajectory involving the first constant down-range angle. Terminal conditions at 50 000 feet (15 240 meters) or 80 nautical miles (148 160 meters) were insensitive to thrusting-time errors except those occurring during the departure from the circular orbit at 50 000 feet (15 240 meters).

INTRODUCTION

In the future journey to the moon, it may be necessary for man to control the attitude of his spacecraft with only a limited amount of instrumentation. Thus, techniques which allow the pilot to control various phases of the lunar mission have been under consideration at the Langley Research Center for some time. (See refs. 1 to 9.)

The phase of the lunar mission to be considered here is the ascent of the lunar module (LM) from the lunar surface to rendezvous with the command and service modules (CSM) in an 80-nautical-mile (148 160-meter) circular orbit. (See refs. 10 and 11 for other studies involving the ascent from the lunar surface.) Under normal conditions, instrumentation onboard either the LM or CSM could be used for guidance for this mission phase. In an emergency, however, the LM may be limited in operational instrumentation and the CSM may be on the other side of the moon where its instrumentation would be unavailable. In such a situation, it would be desirable to have available a simplified procedure for manually guiding the LM to a satisfactory rendezvous.

The purpose of this investigation was to search for some simple ascent-control procedure that could be used to guide the LM to a circular orbit at an altitude of 50 000 feet (15 240 meters) and thence (immediately, if the phasing between the LM and CSM is as desired) to a rendezvous with the CSM at an altitude of 80 nautical miles (148 160 meters). The primary control task for the ascent to the circular orbit at 80 nautical miles (148 160 meters) is to obtain proper orientation of the thrust axis during the powered portion of the flight. The procedure used in this investigation was to examine the orientation of the thrust axis for an optimum ascent, and then to try to duplicate the characteristics of the optimum trajectory by orientation of the thrust axis relative to some available reference. This study was limited to motion in a plane.

SYMBOLS

The units for the physical quantities used in this paper are given in both U.S. Customary Units and in the International System of Units (SI). (See ref. 12.)

F thrust, 3500 pounds (15 568.775 newtons)

g_e gravitational acceleration at surface of earth, 32.2 feet/second^2 (9.8 meters/second^2)

g_m gravitational acceleration at surface of moon, 5.32 feet/second^2 (1.62 meters/second^2)

h	altitude, feet (meters)
h_a	altitude at apocynthion, feet (meters)
h_p	altitude at pericynthion, feet (meters)
m	mass, slugs (kilograms)
m_o	mass of lunar module at lunar surface, 329.192 slugs (4803 kilograms)
I_{sp}	specific impulse, 306 seconds
K	angle between thrust vector and line of sight to specified reference, degrees
r	radial distance from center of moon, feet (meters)
r_m	radius of moon, 5 702 000 feet (1738 kilometers)
t	time, sec
V	velocity, feet/second (meters/second)
ΔV	characteristic velocity, $g_e I_{sp} \ln \frac{m_o}{m_o - \dot{m}t}$, feet/second (meters/second)
γ	flight-path angle, degrees
θ	angular travel over lunar surface, degrees or radians
$r\dot{\theta}$	circumferential velocity component, feet/second (meters/second)
α	thrust attitude with respect to local horizontal, positive when thrust is directed upward, degrees (fig. 1)
θ^*	angular separation of lunar and orbiting modules measured with vertex at moon's center, $\theta^* = \theta_o^* + \theta - t\dot{\theta}_{CSM}$, degrees

$$\phi = \tan^{-1} \frac{\left(r_m + h\right)\sin \theta^*}{\left(r_m + h_{CSM}\right) - \left(r_m + h\right)\cos \theta^*}$$

Subscripts:

o initial conditions (at take-off)

m relative to moon

T circular orbit conditions at 50 000 feet (15 240 meters) or 80 nautical miles (148 160 meters)

DR down range

UR up range

TOS take-off site

CSM command and service module

v vertical portion of trajectory

1 powered portion of trajectory involving first constant down-range angle

2 powered portion of trajectory involving second constant down-range angle

3 powered portion of trajectory at beginning of transfer orbit from 50 000 feet (15 240 meters) to 80 nautical miles (148 160 meters)

c coast portion of transfer orbit from 50 000 feet (15 240 meters) to 80 nautical miles (148 160 meters)

4 powered portion of trajectory at end of transfer orbit from 50 000 feet (15 240 meters) to 80 nautical miles (148 160 meters)

Dots over symbols indicate derivatives with respect to time.

STATEMENT OF PROBLEM

The planar ascent maneuver from the lunar surface studied in the present investigation is illustrated in figure 1. The ascent is performed by the LM ascent stage and will proceed as follows:

At the appropriate time the LM ascent is initiated by thrusting vertically until the velocity is approximately 100 ft/sec (30.48 m/sec). At this point, the vehicle is rotated and the thrust direction varied so that the vehicle is injected into a circular orbit at 50 000 feet (15 240 meters). At the proper time, depending on the LM-target phasing, additional thrust is applied horizontally for about 5 seconds. This maneuver places the LM on a transfer trajectory which intercepts the orbit of the CSM at an altitude of 80 nautical miles (148 160 meters) after approximately 180° of coasting. Thrust is then applied for about 5 seconds in a horizontal direction to complete the rendezvous.

The primary control variable over the powered portion of the ascent is orientation of the vehicle thrust axis. It is desirable to vary the thrust orientation in such a manner as to accomplish the ascent as efficiently as possible with regard to fuel usage. The objective of this study was to determine a simple manual-control procedure that used easily identifiable references and resulted in an efficient ascent maneuver.

Equations of Motion

The equations of motion used in evaluating the guidance schemes of the present study were for a point mass moving in a central force field and subject to a thrust force in the plane of the motion. The equations of motion are:

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$$\ddot{r} - r\dot{\theta}^2 = \frac{F}{m}\sin\alpha - g_m\left(\frac{r_m}{r}\right)^2$$

$$r\ddot{\theta} + 2\dot{r}\dot{\theta} = \frac{F}{m}\cos\alpha$$

where

$$m = m_o + \int \dot{m}\, dt$$

and

$$\dot{m} = \frac{-F}{g_e I_{sp}}$$

The equations of motion were solved on a digital computer.

Reference Ascent Trajectory

The reference ascent trajectory selected was a fuel-optimum trajectory to a circular orbit at 80 nautical miles (148 160 meters) determined by using the PRESTO computer program. (See ref. 13.) The trajectory was obtained by combining the results obtained for a constant-thrust continuous-burn ascent from the lunar surface to a circular orbit at 50 000 feet (15 240 meters) and the results obtained for a transfer trajectory from the 50 000-foot (15 240-meter) circular orbit to a circular orbit at 80 nautical miles (148 160 meters). The transfer orbit had a pericynthion of 50 000 feet (15 240 meters) and an apocynthion of 80 nautical miles (148 160 meters) which closely approximated a Hohmann transfer trajectory.

RESULTS AND DISCUSSION

Most of the results reported herein involve the portion of the ascent trajectory from the lunar surface to the circular orbit at 50 000 feet (15 240 meters). A constant-thrust engine producing a thrust-to-initial-mass ratio of 10.63 was assumed. The thrusting time for this portion of the ascent was approximately 420 seconds as compared with a total of 10 seconds for the portion from 50 000 feet (15 240 meters) to 80 nautical miles (148 160 meters). It was clear, then, that the orientation of the thrust axis during the ascent to the circular orbit at 50,000 feet (15,240 meters) would determine the feasibility of successfully completing an efficient ascent and rendezvous with the orbiting CSM at 80 nautical miles (148 160 meters).

Fuel-Optimum Ascent to 50 000 Feet (15 240 Meters)

The fuel-optimum ascent trajectory is initiated from the surface of the moon, and power is continuously applied until the circular orbit at 50 000 feet (15 240 meters) is reached. Figure 2 is a plot of the variation of the flight-path angle γ and thrust-orientation angle α with altitude. Although the thrust-orientation angle α with respect to the local horizontal varies slowly and linearly over most of the ascent range, it does not appear that the local horizontal would be a convenient reference because it is difficult to determine accurately by simple methods. Data on this plot begin at 976 feet (297.5 meters), which is the terminal altitude (at 19 seconds) for the vertical portion of the ascent.

Orientation of Thrust Axis for Fuel-Optimum Trajectory Relative to Various References

This section is concerned with determining the orientation of the thrust axis during the fuel-optimum ascent trajectory relative to various references. The various references examined were the lunar horizons, a star, the take-off site, and the orbiting vehicle. Although the CSM is not within the range of vision of the LM for the majority of the CSM orbit, the CSM is still considered a possible reference during the limited time that it is in view.

The geometric relationship between the vehicle thrust axis and the various references is shown in figure 3. Also shown are the equations that define the angle between the thrust axis and the line of sight to the various references, and plots of these angles as functions of altitude. Figure 3(a) shows variations in K_{DR} that are similar to the variation in α shown on figure 2. This similarity is to be expected since the variation in θ_{DR} is small over the ascent to 50 000 feet (15 240 meters). This down-range reference is one of the more readily available references considered herein; and, due to the fact that this reference is located in the direction which the LM is traveling, it appears to offer some promise as a reference for thrust-axis control. Also, there is only slightly more variation in the slope of K_{DR} below 5000 feet (1524 meters) and above 45 000 feet (13 716 meters) than exists over the major portion of the trajectory, where the variation is linear with altitude. Figure 3(b) shows the variation of K_{UR} with altitude, and, in this case also, the angle K_{UR} varies slowly and linearly over most of the ascent range but varies more rapidly at altitudes below 5000 feet (1524 meters) and above 45 000 feet (13 716 meters) than for K_{DR}. Figure 3(c) is a plot of K_{star} as a function of altitude where the initial value for (K_{star}) is zero. In this case the vehicle was assumed to be pointed at a star that was directly in line with the thrust axis after the vehicle had been rotated from vertical flight. The variation shown here is similar to that for K_{UR} in figure 3(b). Figure 3(d) is a plot of the variation of K_{TOS} with altitude. In this case, the variation is highly nonlinear at altitudes below 15 000 feet (4572 meters). At the higher altitudes, between 40 000 and 50 000 feet (12 192 and 15 240 meters), the variation is quite similar to the variation for K_{UR}. Figure 3(e) is a plot that shows the variation of K_{CSM} with altitude throughout the ascent maneuver to 50 000 feet (15 240 meters). The highly nonlinear character of this curve indicates that the orbiting vehicle would be an extremely poor thrust-axis reference.

Determination of Thrust-Orientation Angle for Manual Control of Ascent

Calculations were made to determine if a single constant average angle $(K_{DR}, K_{TOS}, K_{CSM}, K_{UR}, \text{ or } K_{star})$ existed that could be used to approximate the fuel-optimum ascent trajectory. No suitable single constant average value was found for any of the references investigated. Thus, it became necessary to devise some other easily implemented scheme for manual control. In view of its simplicity and its previous successful use in other studies (e.g., refs. 4 and 8), a scheme employing two constant angles was chosen. Also, in order to limit the number of calculations required, all but one of the references were eliminated. The reference chosen for the remainder of the study was down-range horizon because, in addition to its practical attractiveness, such as visibility in direction of travel, it had less variation with altitude than did the other references. (See fig. 3.) After many trials, it was found that two different constant angles (30° and 9°, relative to the down-range horizon) held for specified times (248 and 156.9 seconds, respectively) would insert the LM in a circular orbit at 50 000 feet (15 240 meters). The ascent to this intermediate orbit would proceed as shown in figure 4 and as explained in the following section.

Down-Range Horizon as Thrust-Direction Reference

After an initial vertical flight for 19 seconds, the LM is rotated to 30° relative to the down-range horizon $\left(K_{DR} = 30^{\circ}\right)$ and is held at this angle for 248 seconds. At the end of this time period, the vehicle is rotated to reduce K_{DR} to 9°, and this heading held for 156.9 seconds. (These rotations were assumed to occur instantaneously.) The termination of thrust after 156.9 seconds (total time of 423.9 seconds) inserted the LM into a nearly circular orbit at 49 461 feet (15 075 meters) where the orbit may or may not be circularized before the ascent is continued (depending on the relative positions of the CSM and LM at $t = 423.9$ seconds). The following table shows a comparison of the characteristic velocity ΔV (a measure of the fuel consumption) and the terminal conditions at 50 000 feet (15 240 meters) obtained by maintaining two constant angles for specified times with the corresponding results for the fuel-optimum trajectory.

Condition	Terminal conditions for –	
	Fuel-optimum trajectory	Constant-angle trajectory: $K_{DR,1} = 30^{\circ}$ and $K_{DR,2} = 9^{\circ}$
$\dot{r}$	0	0.042 fps (0.012 m/sec)
$r\dot{\theta}$	5 483.69 fps (1 671.42 m/sec)	5 484.36 fps (1 671.63 m/sec)
h	50 000 ft (15 240 m)	49 461 ft (15 075.71 m)
θ	9.61°	9.964°
ΔV	5 997.54 fps (1 828.05 m/sec)	6 024.14 fps (1 836.15 m/sec)
h_p	50 000 ft (15 240 m)	49 461 ft (15 075.71 m)
h_a	50 000 ft (15 240 m)	51 181 ft (15 600 m)

These results show that the constant-angle trajectory closely approximated the terminal conditions of the fuel-optimum trajectory and that ΔV was 26.6 ft/sec (8.1 m/s) greater; about 0.3 percent more fuel was used. Some of the trajectory parameters for the fuel-optimum and constant-angle trajectories are shown in figure 5.

In order to continue the ascent to 80 nautical miles (148 160 meters) without terminating thrust at the parking orbit condition, K_{DR} must be further reduced to 7.6^{o} and thrust terminated after 5.04 seconds (total time = 428.94 seconds). The LM is allowed to follow a coast trajectory for 3 484.44 seconds or 180^{o}, terminating at the desired altitude of 80 nautical miles (148 160 meters) where the velocity is increased to orbital speed by thrusting at $K_{DR} = 22.9^{o}$ for 4.9 seconds. For the transfer orbit from 50 000 feet (15 240 meters) to 80 nautical miles (148 160 meters), the angles $K_{DR} = 7.6^{o}$ and $K_{DR} = 22.9^{o}$ result when the thrust is applied horizontally.

The following table presents a comparison of the conditions at thrust cutoff for the fuel-optimum and constant-angle trajectories when the vehicle departs from the 50 000-foot (15 240-meter) orbit and at injection into the 80-nautical-mile (148 160-meter) orbit. It is assumed the constant-angle trajectory was circularized before the departure from the 50 000-foot (15 240-meter) orbit.

Condition	Terminal conditions for –			
	Departure from 50 000 ft		Injection into 80 n. mi. orbit	
	Fuel-optimum trajectory after 5.16 sec	Constant-angle trajectory $K_{DR,3} = 7.6^{o}$ after 5.04 sec	Fuel-optimum trajectory after 4.94 sec	Constant-angle trajectory $K_{DR,4} = 22.9^{o}$ after 4.9 sec
$\dot{r}$	0.97 fps (0.29 m/sec)	0.66 fps (0.20 m/sec)	0 fps (0 m/sec)	-0.34 fps (-0.10 m/sec)
$r\dot{\theta}$	5 582.89 fps (1 701.66 m/sec)	5 583.62 fps (1 701.88 m/sec)	5 287 fps (1 611.47 m/sec)	5 286.92 fps (1 611.45 m/sec)
h	50 002 ft (15 240.6 m)	50 001 ft (15 240.3 m)	486 038 ft (148 144.38 m)	486 151 ft (148 178.82 m)
ΔV	100.6 fps (30.66 m/sec)	100.6 fps (30.66 m/sec)	97.44 fps (29.69 m/sec)	97.44 fps (29.69 m/ sec)

The data in the preceding table show that the two constant angles used, 7.6^{o} for 5.04 seconds and 22.9^{o} for 4.9 seconds, duplicate the transfer orbit from 50 000 feet (15 240 meters) to 80 nautical miles (148 160 meters). This duplication is reasonable since the thrusting times involved are too short to have any appreciable effect on the transfer orbit unless large and unusual errors are present in thrust level and direction, or errors in thrusting time exist when the LM departs from 50 000 feet (15 240 meters) or attempts to complete the rendezvous at 80 nautical miles (148 160 meters). These results indicate that the transfer trajectory from 50 000 feet (15 240 meters) to 80 nautical miles (148 160 meters) would not materially affect the economy of the ascent.

Error Analysis

The error analysis primarily consisted of determining the change in terminal conditions at 50 000 feet (15 240 meters) (after the nominal ascent time of 423.9 seconds) caused by departures from the nominal conditions for thrust direction, thrust level, and thrusting time during this portion of the ascent. Errors involving thrust direction, magnitude, and time would also affect the transfer orbit from 50 000 feet (15 240 meters) to 80 nautical miles (148 160 meters) but not as powerfully as errors in the ascent to 50 000 feet (15 240 meters). The error analysis, then, consisted of two distinct sections: the effect of errors in thrust direction, magnitude, and time on the terminal conditions when the nominal flight time of 423.9 seconds had elapsed for powered flight to the circular orbit at 50 000 feet (15 240 meters); and the effect of errors on the terminal conditions for the transfer trajectory from 50 000 feet (15 240 meters) to 80 nautical miles (148 160 meters). The individual error effects that follow were obtained by varying one initial condition at a time from the nominal value in only one of the three flight portions involved while maintaining nominal conditions for the remainder of the ascent to the intermediate orbit. Comments on combination effects in which various errors occurred in all portions are presented after the effects of individual errors are discussed.

Thrust-vector direction.- The variation of the change in terminal conditions with errors in thrust direction is shown in figure 6. As noted previously, the terminal conditions are defined as those conditions existing in the vicinity of 50 000 feet (15 240 meters) after the nominal thrust time of 423.9 seconds. Figure 6 shows that for all portions of the ascent the variation of the change in terminal conditions with change in thrust direction was generally linear with the exception of the $\Delta\dot{r}_T$ variation in the first constant-angle portion. The data in figure 6(b) deal with the first constant-angle portion only and show that the variation of $\Delta\dot{r}_T$ was more linear for positive errors in $K_{DR,1}$ than for negative errors. Thrust-direction variations occurring during this portion $\left(K_{DR,1}\right)$ of the ascent affected the terminal conditions for altitude, circumferential velocity, and range more than did the thrust-direction variations in the α_V (see fig. 6(a)) or $K_{DR,2}$ (see fig. 6(c)) portions. Despite the shorter thrusting time for the $K_{DR,2}$ portion, errors in thrust direction occurring during this portion had more effect on $\Delta\dot{r}_T$ than the thrust-direction errors occurring in either of the other two portions.

Thrust level.- The sensitivity of the change in terminal conditions to variation in thrust level is shown in figure 7. Figures 7(a), 7(b), and 7(c) present the effects of thrust error in each portion only; figures 7(d) and 7(e) show the effects of identical errors occurring in the two constant-angle and the two constant-angle plus vertical portions, respectively. The data show that the change in terminal altitude, radial and circumferential velocity, and range vary linearly with thrust level. These results show that thrust-level errors occurring in the first constant-angle portion (fig. 7(b)) affected the terminal conditions more than errors in either of the other two portions (figs. 7(a) and 7(c)). Figure 7(e) presents the results for identical and continuous thrust errors throughout the three portions of the ascent and, thus, shows the maximum terminal variation to be expected from thrust errors. Constant thrust errors of 1 percent existing throughout the entire trajectory resulted in terminal errors on the order of 5 000 feet (1 524 meters) in altitude, 18.5 ft/sec (5.64 m/s) in radial velocity, 77.7 ft/sec (23.68 m/sec) in circumferential velocity, and 12 138 feet (3 699.66 meters) in range.

Thrusting time.- The effects of thrusting-time errors (±1 second) on the terminal conditions are presented in figure 8. The variations with thrusting time are linear and show that the terminal conditions are rather insensitive to single errors occurring in each portion only. Figure 8(a) presents data that deal solely with errors occurring during the vertical portion of the ascent where the nominal time, t_V, was 19 seconds. During the course of the error analysis it was discovered that for $\Delta t_V = 1$ second ($t_V = 20$ sec) a perfect circular orbit was reached at 50 953 feet (15 530.47 meters) when the second constant-angle thrusting time, t_2, was 156.3 seconds $\left(\Delta t_2 = -0.6\right)$.

Pericynthion altitude.- The pericynthion altitude of the parking orbit established by ascent trajectories of the type considered herein is particularly sensitive to errors during the ascent. Since, due to phasing problems, the LM might have to orbit the moon one or more times prior to initiating the transfer orbit and since the lunar terrain is quite mountainous, it is desirable to maintain a pericynthion altitude of at least 30 000 feet (9 144 meters) for terrain clearance. Figures 9 and 10 show the pericynthion altitudes of the parking orbit as a function of errors in thrust direction, thrust magnitude, and time of thrust. These figures indicate that only small errors in thrust level, time, and direction can be tolerated if the procedure is followed as specified. It is advisable, therefore, that, after thrust cutoff, steps be taken to circularize the parking orbit.

Error results for transfer orbit.- Figure 11 presents the error results for the transfer orbit from 50 000 feet (15 240 meters) to 80 nautical miles (148 160 meters) and shows the effects of reasonable errors in thrust direction, thrust level, and thrusting time on the terminal conditions at 80 nautical miles (148 160 meters). Thrust-level F_3 and thrusting-time t_3 errors occurring during the departure from the 50 000-foot (15 240-meter) orbit caused appreciable errors in altitude, circumferential velocity, and range at 80 nautical miles (148 160 meters).

Combination errors.- In the previous discussions, the variations in terminal conditions were examined for single errors occurring in only one portion of the ascent to 50 000 feet (15 240 meters); that is, only one error at a time was assumed in any one portion of the ascent. Consequently, there could be some question regarding the effects of combinations of errors occurring in all portions of the ascent as the pilot attempted to follow the nominal ascent sequence. This does not present a problem, however, since it is possible to approximate errors at 50 000 feet (15 240 meters) resulting from these relatively small errors by a linear combination of the individual effects.

As a check case and for illustrative purposes, computations were made in which thrust direction, thrust level, and thrusting time were varied simultaneously in each portion of the flight and conditions at the end of each consecutive portion were used as initial conditions for the following portion. The terminal conditions obtained are compared with those predicted by using the linear addition of the individual effects. The change in terminal radial velocity, $\Delta\dot{r}_T$, can be expressed as:

$$\Delta\dot{r}_T = \frac{\partial\dot{r}_T}{\partial\alpha_V}\Delta\alpha_V + \frac{\partial\dot{r}_T}{\partial K_{DR,1}}\Delta K_{DR,1} + \frac{\partial\dot{r}_T}{\partial K_{DR,2}}\Delta K_{DR,2} + \frac{\partial\dot{r}_T}{\partial F_V}\Delta F_V + \frac{\partial\dot{r}_T}{\partial F_1}\Delta F_1 + \frac{\partial\dot{r}_T}{\partial F_2}\Delta F_2 + \frac{\partial\dot{r}_T}{\partial t_V}\Delta t_V + \frac{\partial\dot{r}_T}{\partial t_1}\Delta t_1 + \frac{\partial\dot{r}_T}{\partial t_2}\Delta t_2$$

The ratios are referred to as sensitivity coefficients and are given in this case by the slopes of the individual curves. The errors in thrust direction, thrust level, and thrusting time were as follows:

$\Delta\alpha_V = -1$	$\Delta K_{DR,1} = -1$	$\Delta K_{DR,2} = 1$
$\Delta F_V = 2$	$\Delta F_1 = 2$	$\Delta F_2 = 2$
$\Delta t_V = 1$	$\Delta t_1 = 1$	$\Delta t_2 = 0$

and from the individual error plots

$$\frac{\partial\dot{r}_T}{\partial\alpha_V} = -0.91 \text{ fps/deg} \quad (-0.277 \text{ m/sec/deg})$$

$$\frac{\partial\dot{r}_T}{\partial K_{DR,1}} = 11.91 \text{ fps/deg} \quad (3.63 \text{ m/sec/deg})$$

$$\frac{\partial\dot{r}_T}{\partial K_{DR,2}} = 42.56 \text{ fps/deg} \quad (12.97 \text{ m/sec/deg})$$

$$\frac{\partial\dot{r}_T}{\partial F_V} = 0.1887 \text{ fps}/F_V \quad (0.0575 \text{ m/sec}/F_V)$$

$$\frac{\partial\dot{r}_T}{\partial F_T} = 16.91 \text{ fps}/F_1 \quad (5.154 \text{ m/sec}/F_1)$$

$$\frac{\partial\dot{r}_T}{\partial F_2} = 4.236 \text{ fps}/F_2 \quad (1.291 \text{ m/sec}/F_2)$$

$$\frac{\partial\dot{r}_T}{\partial t_V} = 0.345 \text{ fps/sec} \quad (0.105 \text{ m/sec/sec})$$

$$\frac{\partial\dot{r}_T}{\partial t_1} = 4.167 \text{ fps/sec} \quad (1.270 \text{ m/sec/sec})$$

$$\frac{\partial\dot{r}_T}{\partial t_2} = 0.527 \text{ fps/sec} \quad (0.160 \text{ m/sec/sec})$$

Substituting these values (determined from the linear portion of the curve) into the equation for terminal radial velocity change gives a value of 78.74 ft/sec (23.99 m/sec). The computed flight trajectory indicated a value at 78.25 ft/sec (23.85 m/sec). The other terminal conditions were determined similarly and are presented in the following table for comparison with the actual values.

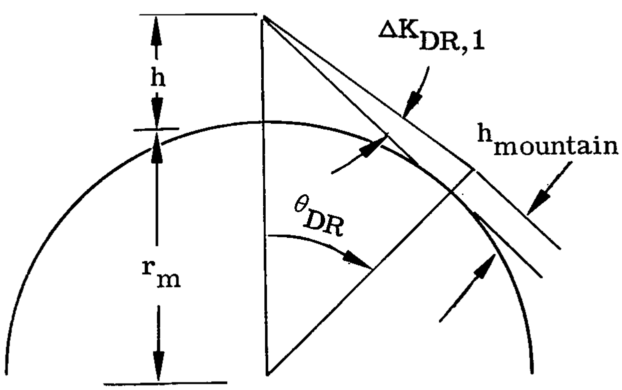
Condition	Terminal conditions for –	
	Computed value	Predicted value
Δh_T	6403 ft (1951.63 meters)	6479 ft (1974.79 meters)
$\Delta \dot{r}_T$	78.25 fps (23.85 m/sec)	78.74 fps (23.99 m/sec)
$\Delta (r\dot{\theta})_T$	207.23 fps (63.16 m/sec)	211.59 fps (64.49 m/sec)
$\Delta \theta_T$	0.394°	0.393°

This table shows close agreement between computed and predicted terminal conditions obtained for the lunar ascent to a circular orbit at 50 000 feet (15 240 meters) for this example.

Effect of horizon irregularities.- In the previous discussions no consideration was given to errors in sighting angle, $K_{DR,1}$, caused by mountainous terrain on the down-range horizon when the LM is initially rotated to $K_{DR,1} = 30^{\circ}$ at low altitude (1 000 feet (304.8 meters)). Depending upon the location previously selected for the landing site, these mountains could be close enough to the take-off site to cause appreciable errors in initial $K_{DR,1}$. For instance, a 5 000-foot (1 524-meter) mountain on the down-range horizon would, from an altitude of 1 000 feet (304.8 meters), cause an initial error in $K_{DR,1}$ of 2.68°. By the time the LM has ascended to 10 000 feet (3084 meters) the error caused by a 5 000 foot (1 524-meter) mountain on the horizon will have decreased to 0.848°. Errors of this magnitude would not be tolerable. However, if the local terrain elevations are known, the sighting angles could be corrected to account for these effects. The error in thrust direction caused by protuberances at the horizon is given by the following equation (see sketch):

$$\Delta K_{DR,1} = \tan^{-1} \frac{h_{mountain}}{\left(r_m + h\right)\sin \theta_{DR}}$$

$$= \tan^{-1} \frac{h_{mountain}}{\left(r_m + h\right)\sin\left(\cos^{-1} \frac{r_m}{r_m + h}\right)}$$



CONCLUDING REMARKS

An analytical study has been made of the possibility of using a simple control reference as an aid to thrust-axis orientation during ascent from the lunar surface to a circular orbit at 80 nautical miles (148 160 meters). The references examined were the horizons, a star, the take-off site, and an orbiting spacecraft. No significant advantage of one reference over another existed. However, the down-range horizon (in the direction of flight) was chosen for further study since it was visible from the overhead window of the lunar module for most of the launch maneuver. It was found that use of the down-range horizon as a reference permitted a near-optimum trajectory if two constant angles were used.

Terminal conditions were found to be sensitive to errors in thrust direction and magnitude.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Station, Hampton, Va., January 17, 1966.

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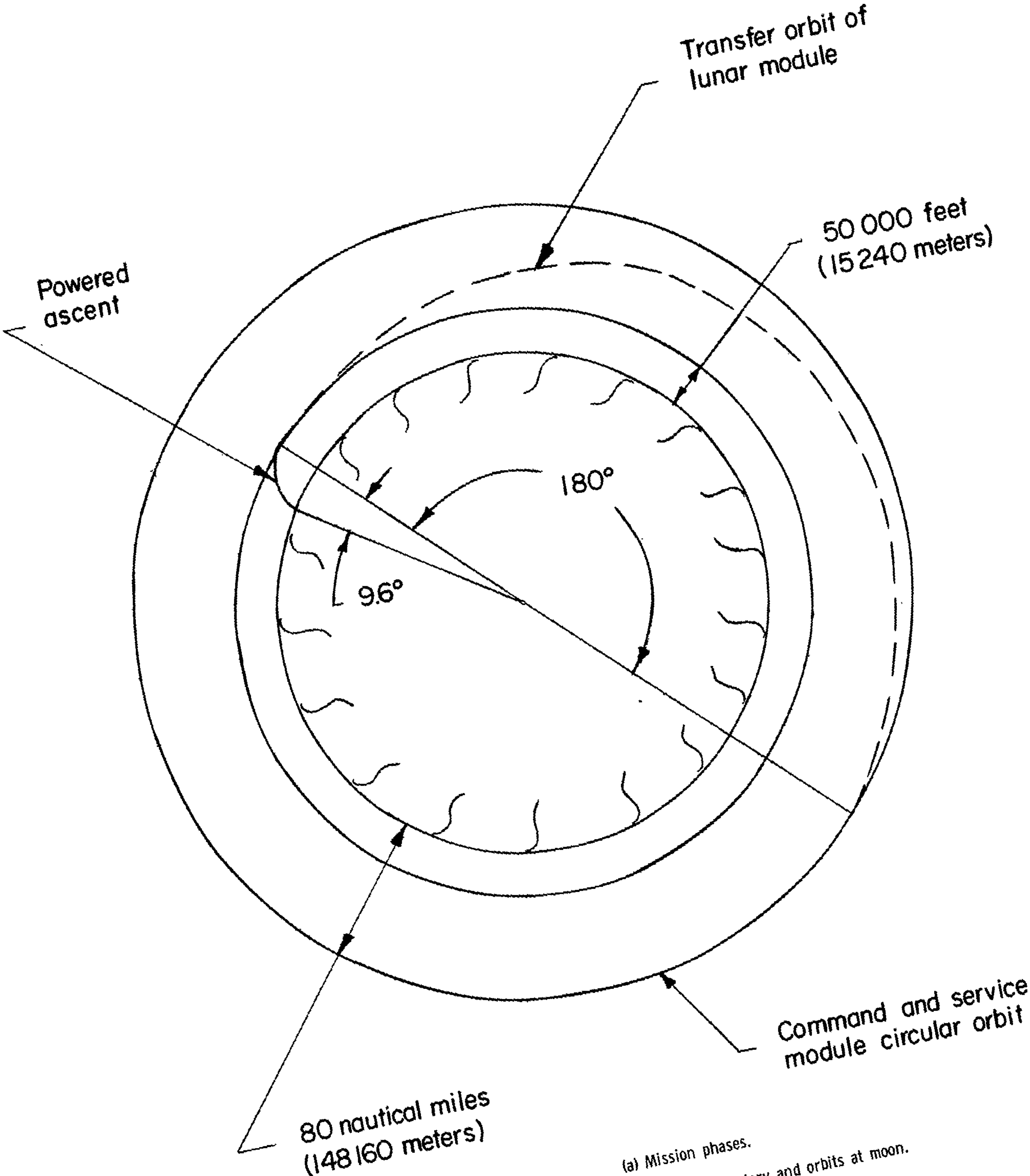
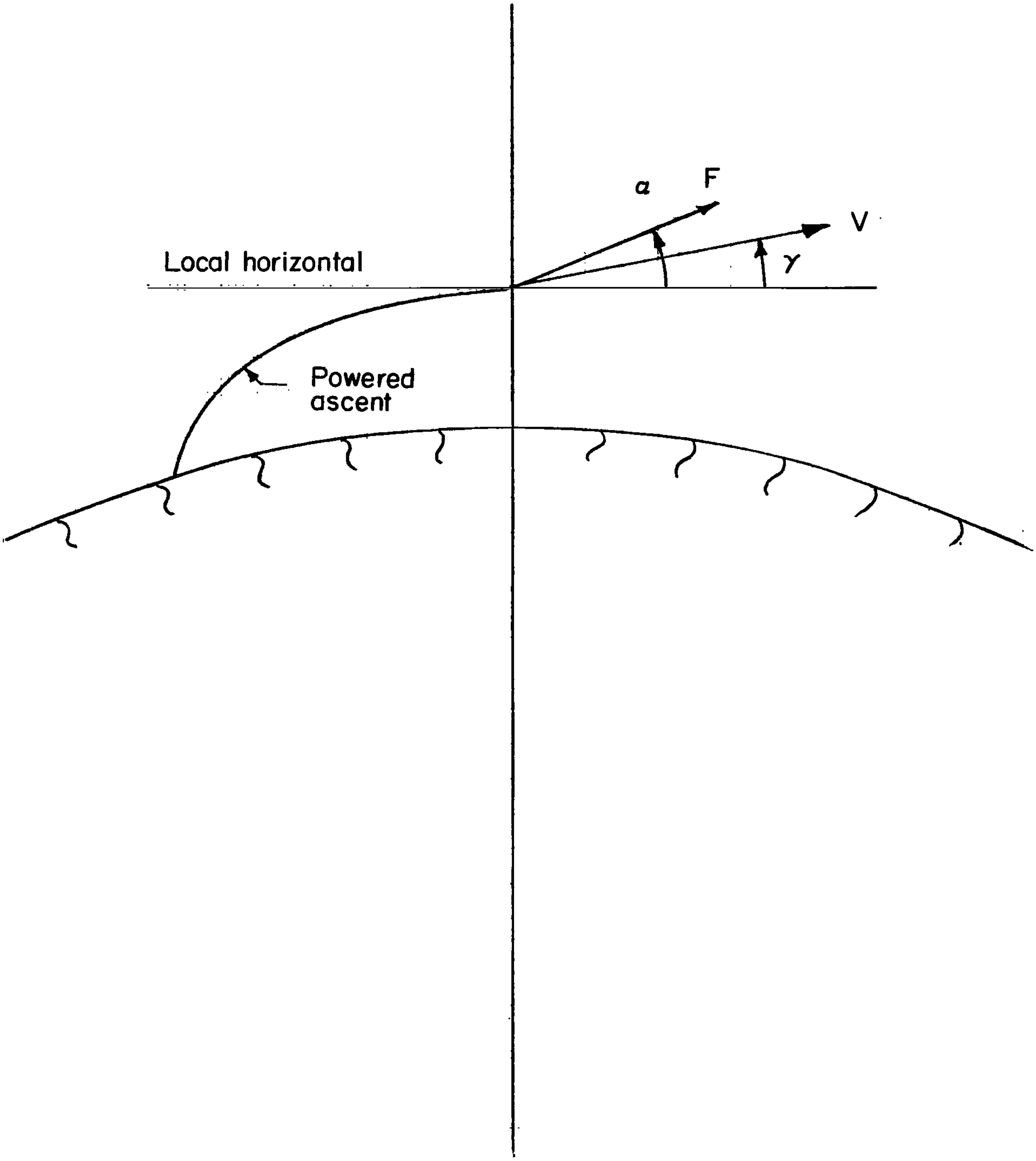


(a) Mission phases.

Figure 1.- Illustration of ascent trajectory and orbits at moon.



(b) Details of powered ascent.

Figure 1.- Concluded.

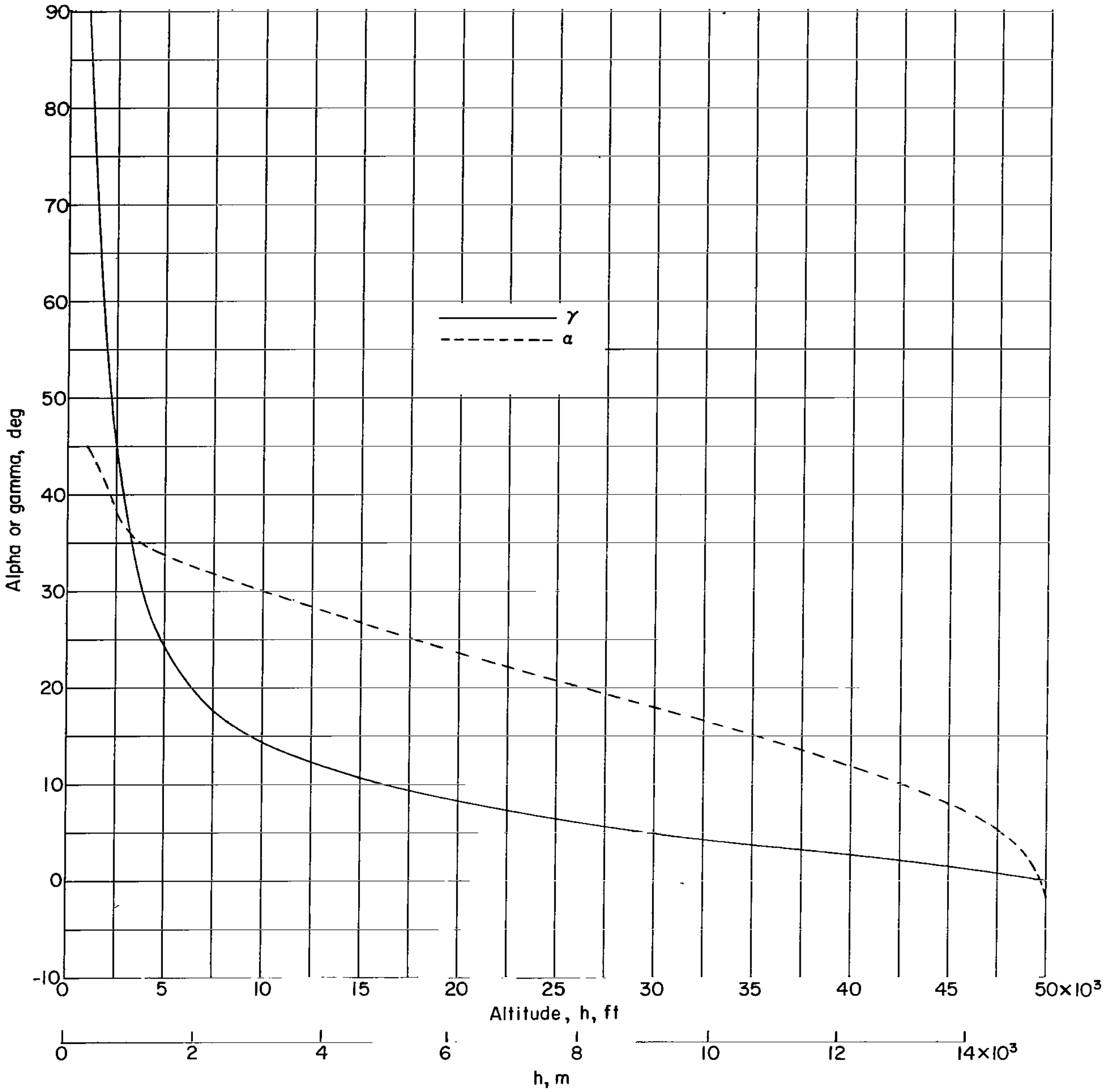
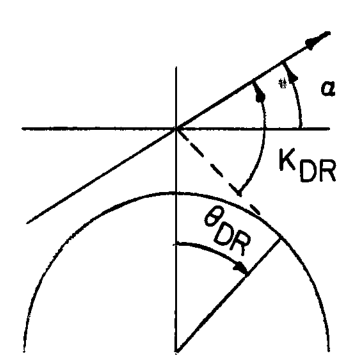


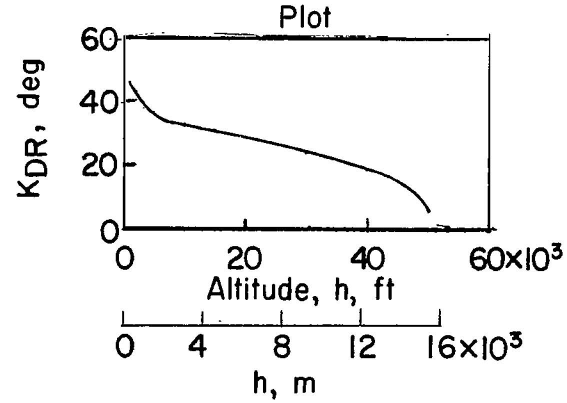
Figure 2.- Variation of flight-path angle γ and thrust-orientation angle α with altitude for fuel-optimum trajectory to 50 000-foot (15 240-meter) circular orbit.

Geometry

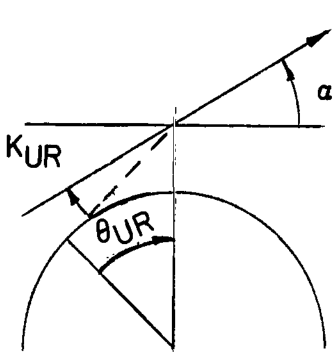


Equations

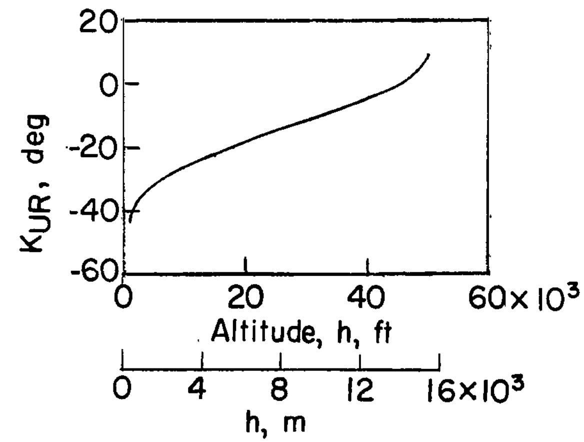
$$K_{DR} = \theta_{DR} + \alpha$$
$$\text{or } K_{DR} = \cos^{-1}\left(\frac{r_m}{r_m + h}\right) + \alpha$$



3a.- Down range reference



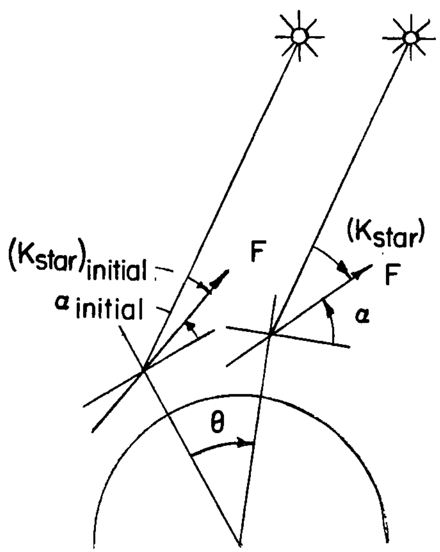
$$K_{UR} = \theta_{UR} - \alpha$$
$$\text{or } K_{UR} = \cos^{-1}\left(\frac{r_m}{r_m + h}\right) - \alpha$$



3b.- Up range reference

Figure 3.- Geometry, equations, and plots showing the orientation of the thrust axis relative to the various references and the variation of the angle defining the references with altitude. Arrows indicate positive direction of angles.

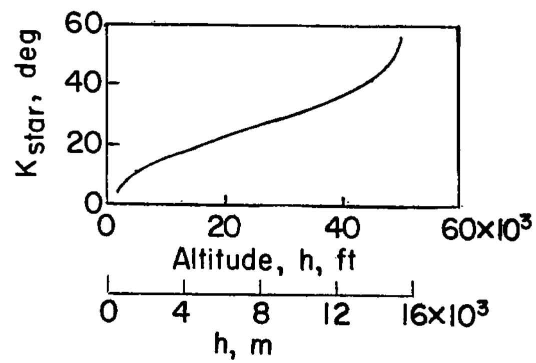
Geometry



Equations

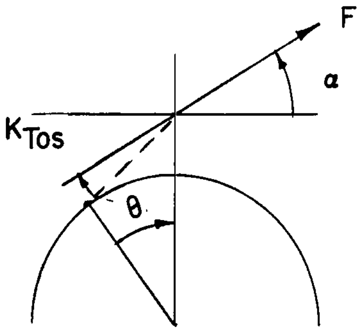
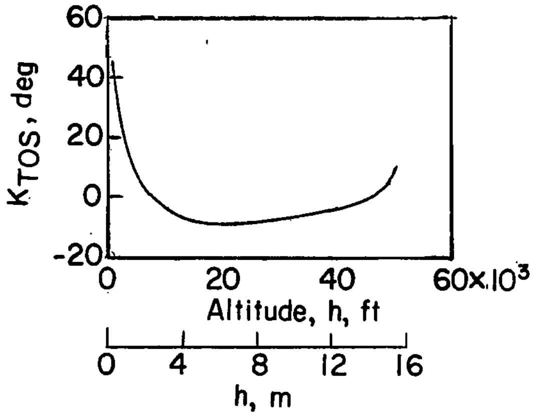
$$K_{star} = (K_{star})_{initial} + \theta + (\alpha_{initial} - \alpha)$$

Plot

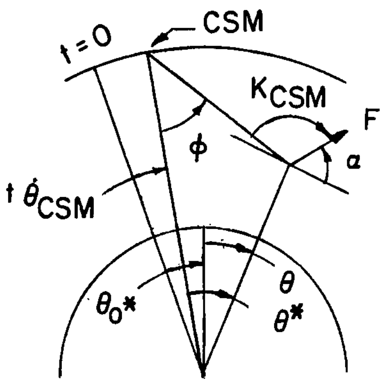
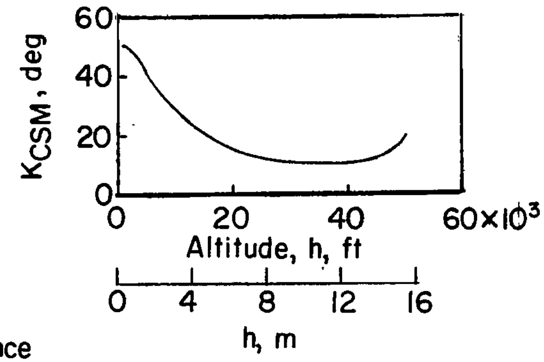


3c.– Star reference

Figure 3.- Continued.

Geometry	Equations	Plots
	$$K_{TOS} = \tan^{-1}\left(\frac{r - r_m \cos\theta}{r_m \sin\theta}\right) - \alpha$$	

3d.- Take off site reference

Geometry	Equations	Plots
	$$K_{CSM} = 90 - \alpha + \theta^* + \phi$$	

3e.- Orbiting spacecraft reference

Figure 3.- Concluded.

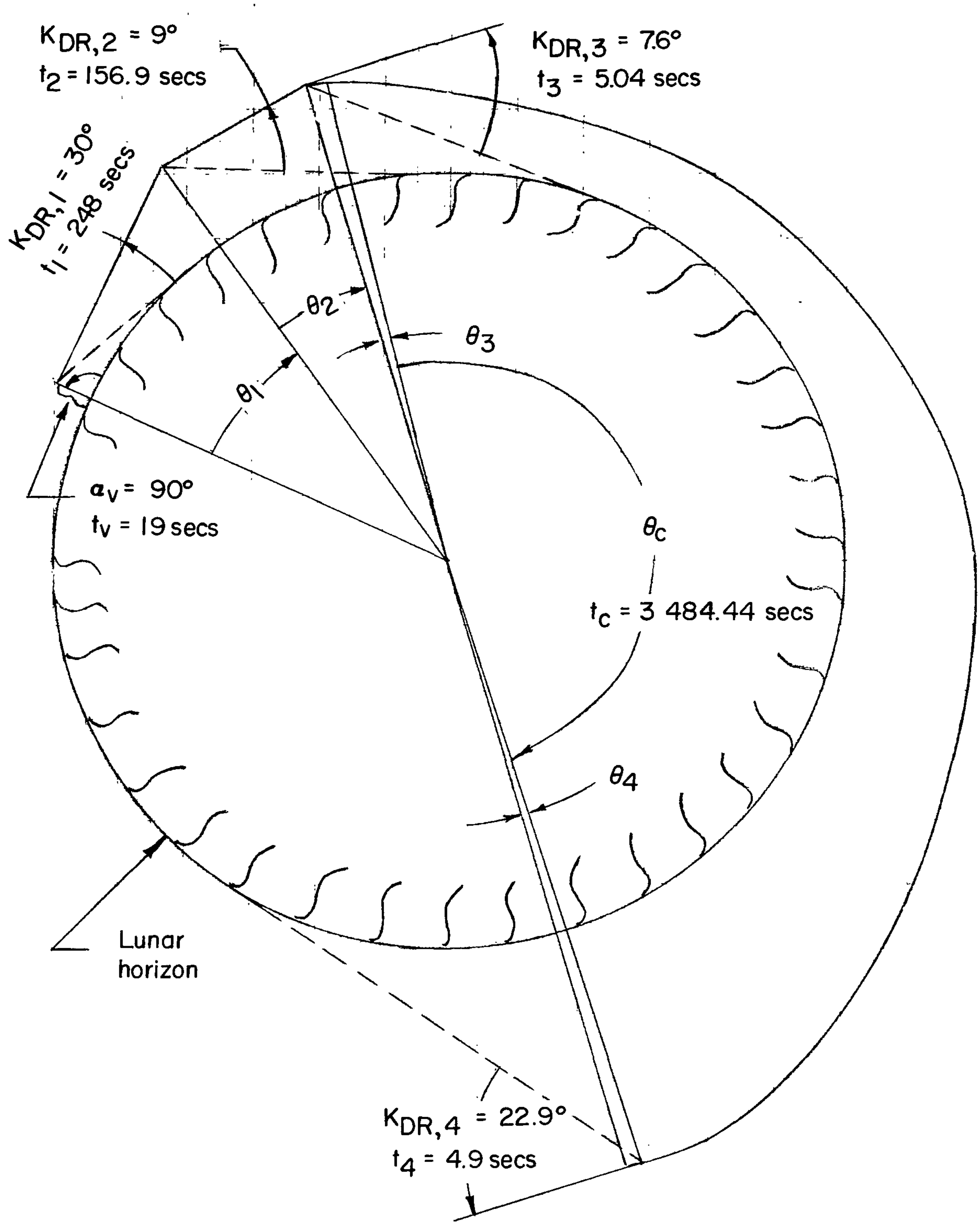
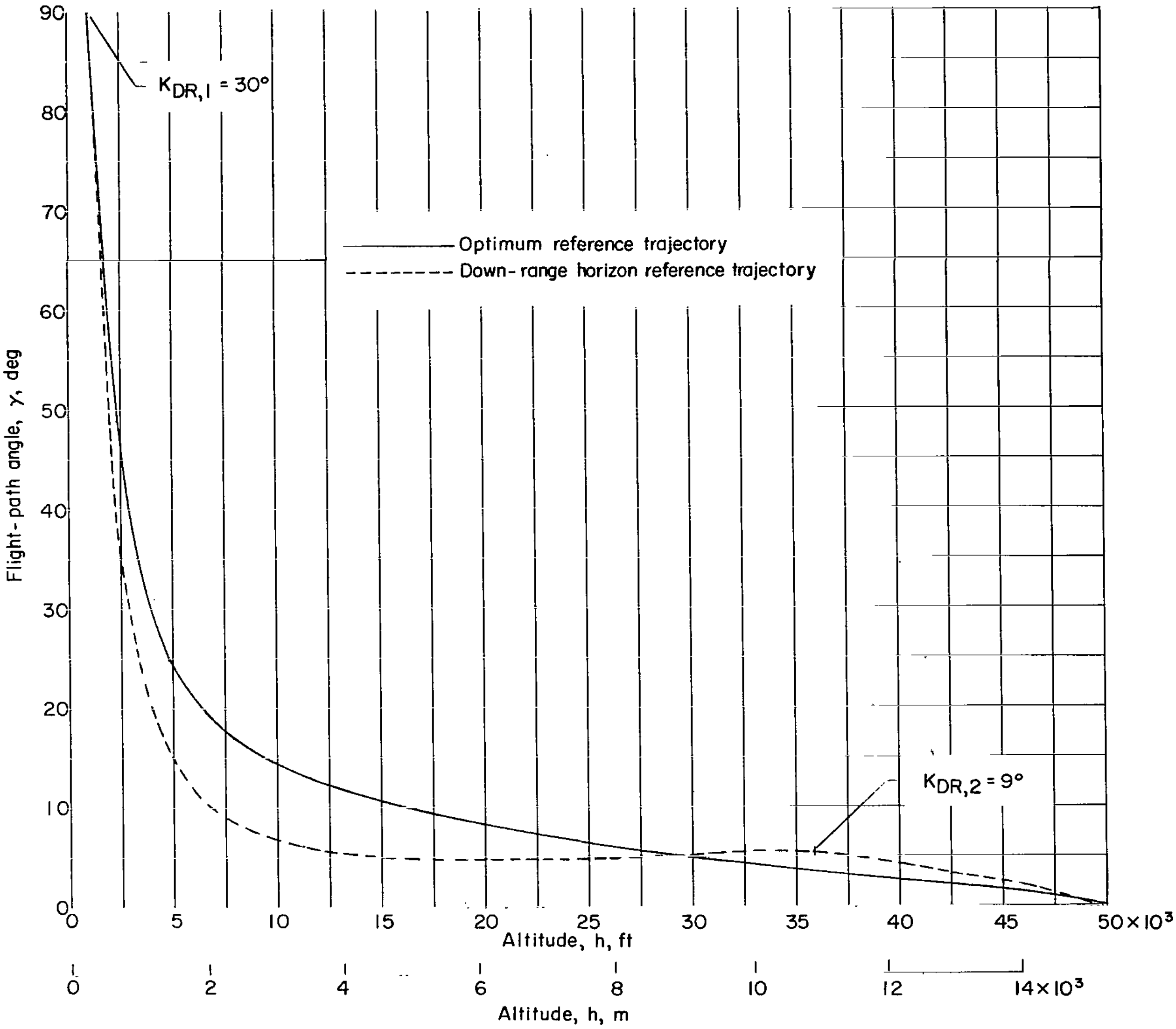
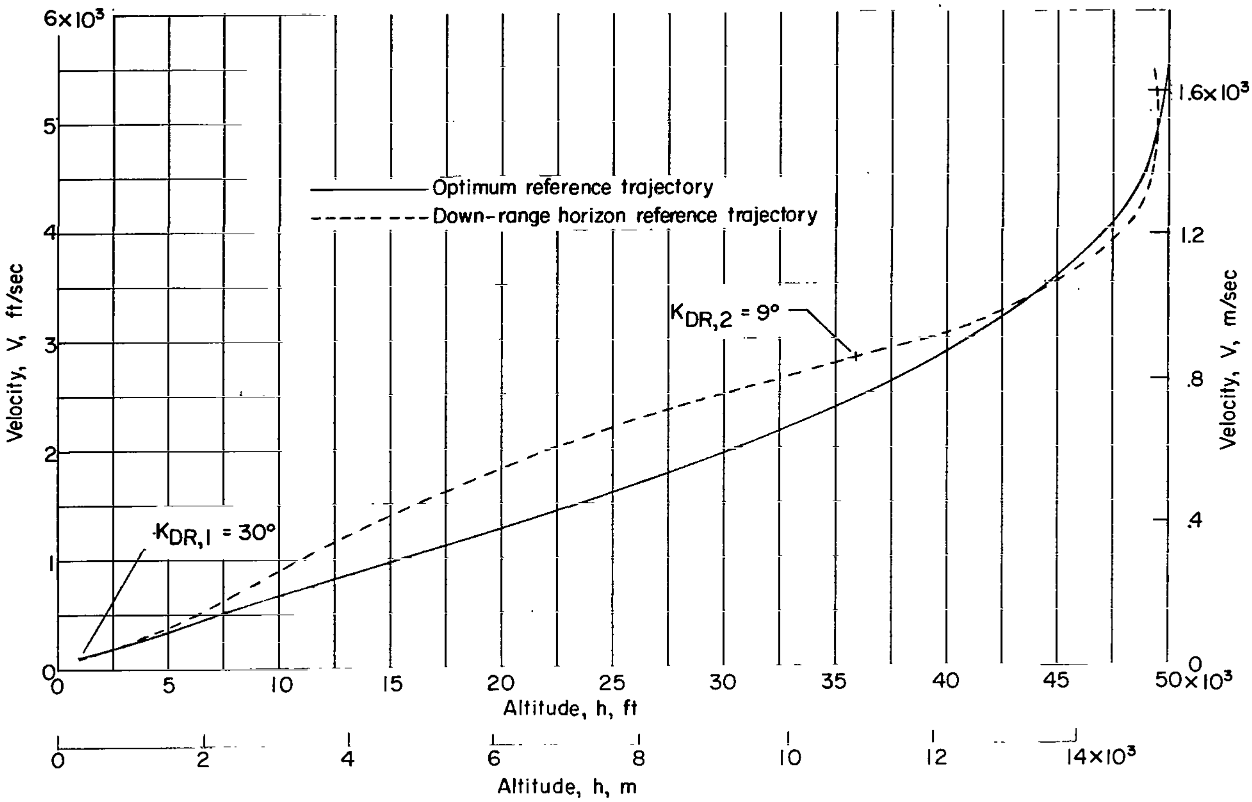


Figure 4.- Illustration of procedure for manually guided ascent to a circular orbit at 80 nautical miles (148 160 meters).



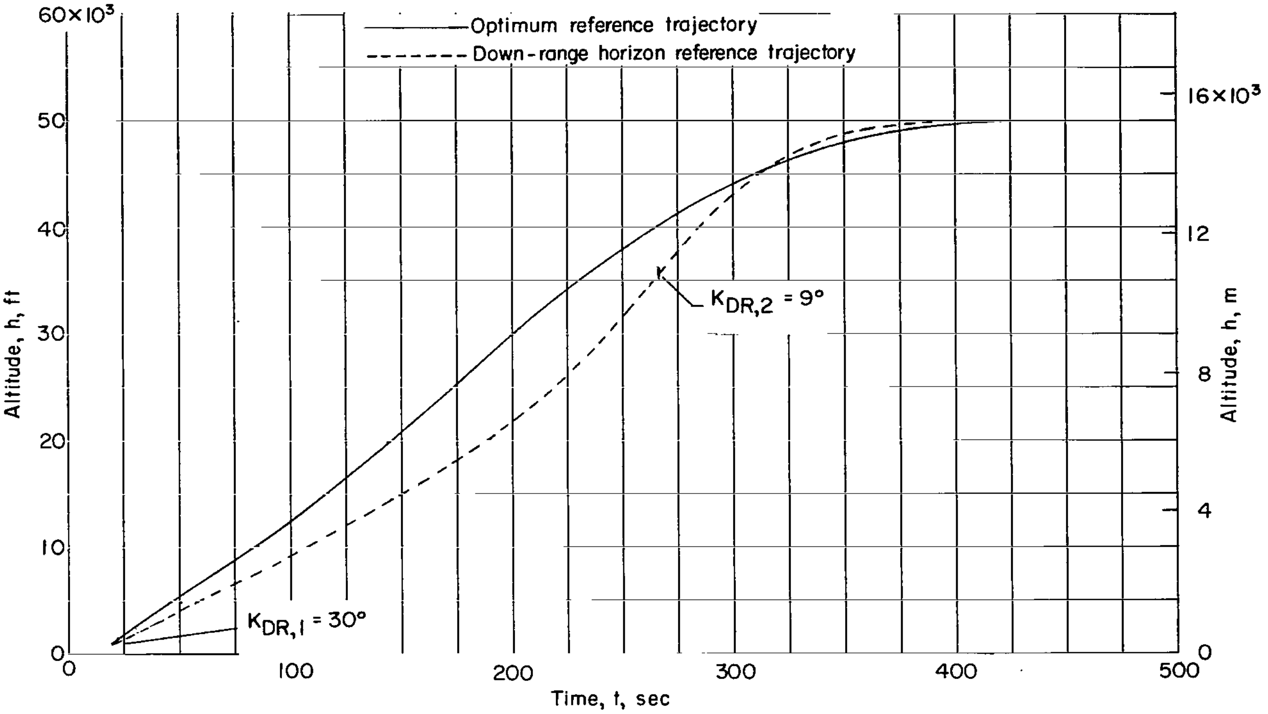
(a) Variation of flight-path angle with altitude.

Figure 5.- Comparison of characteristics of fuel-optimum ascent trajectory with those of the trajectory generated by thrusting vertically for 19 seconds, then rotating to $K_{DR,1} = 30^0$ for 248 seconds, and then to $K_{DR,2} = 9^0$ for 156.9 seconds.



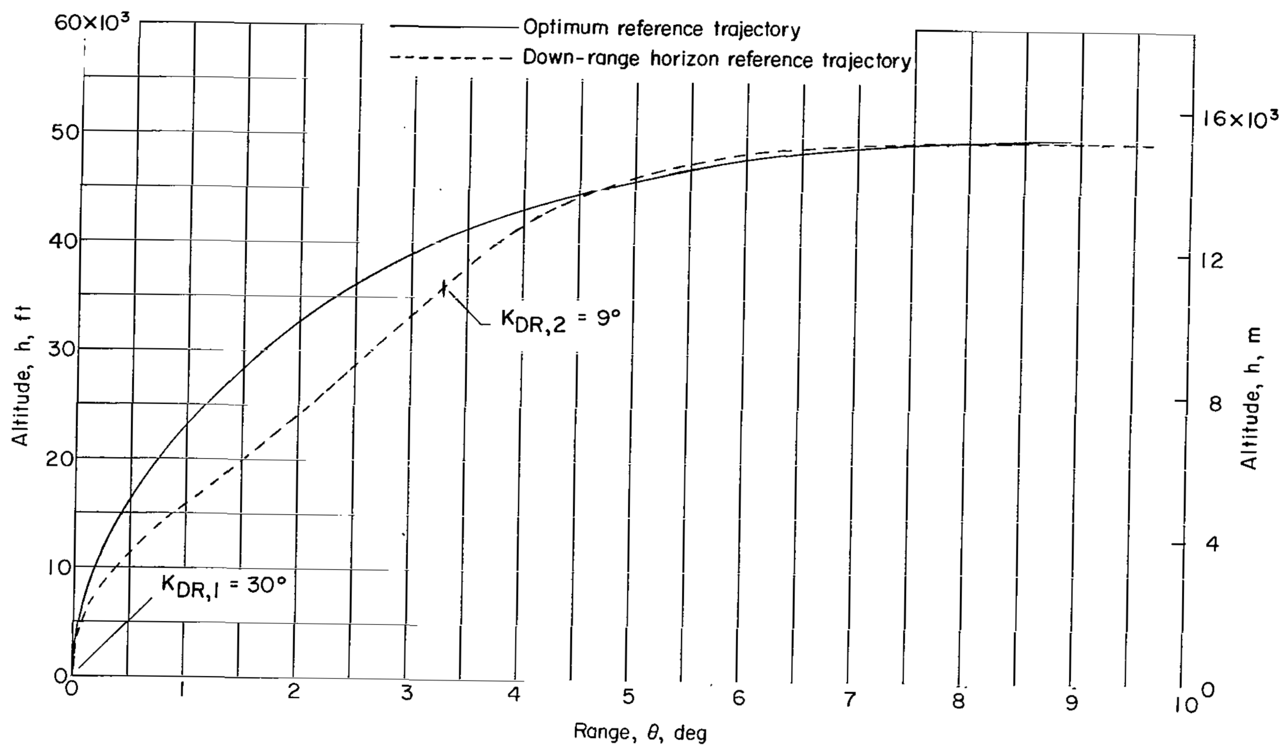
(b) Variation of velocity with altitude.

Figure 5.- Continued.



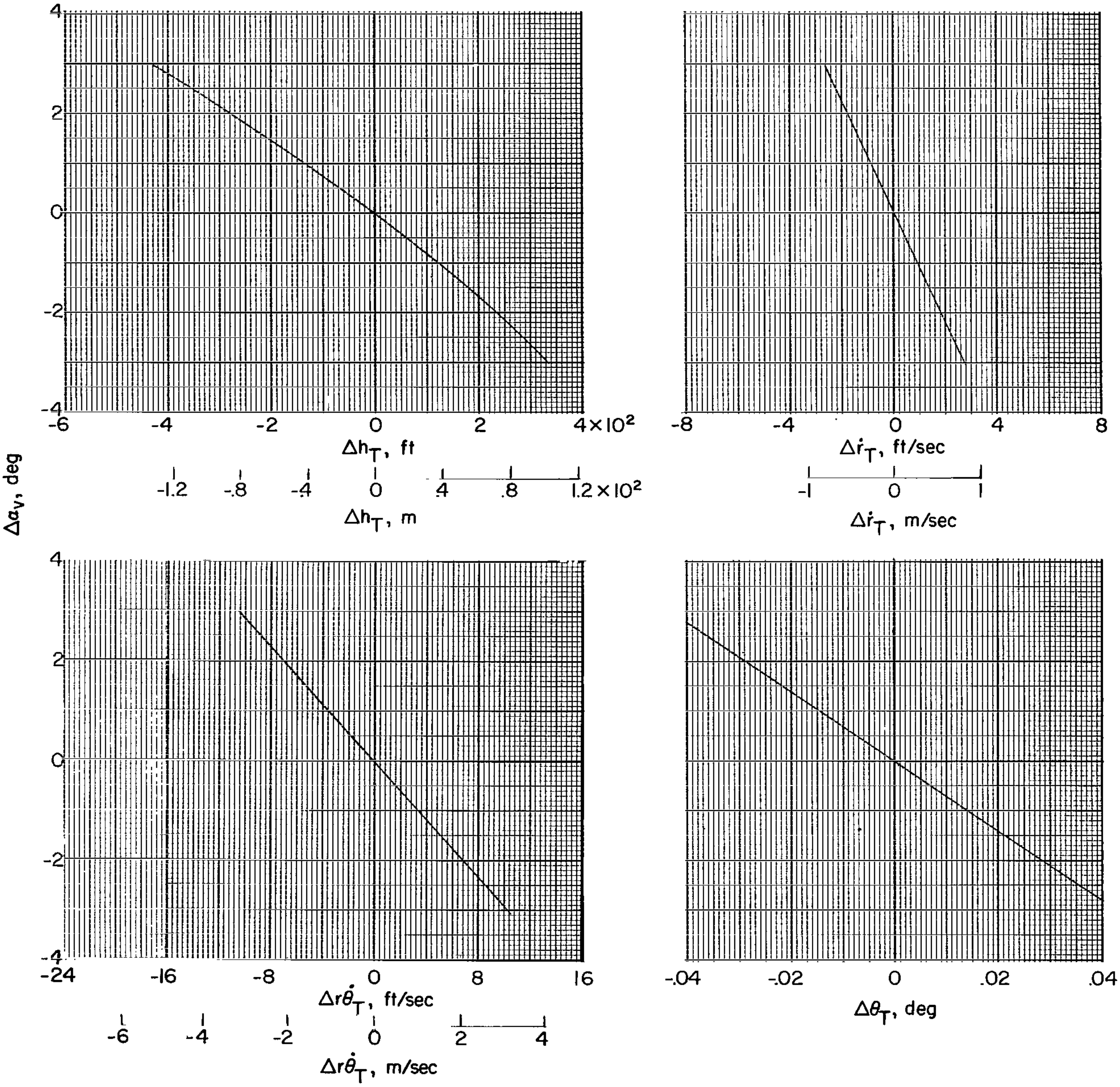
(c) Variation of altitude with time.

Figure 5.- Continued.



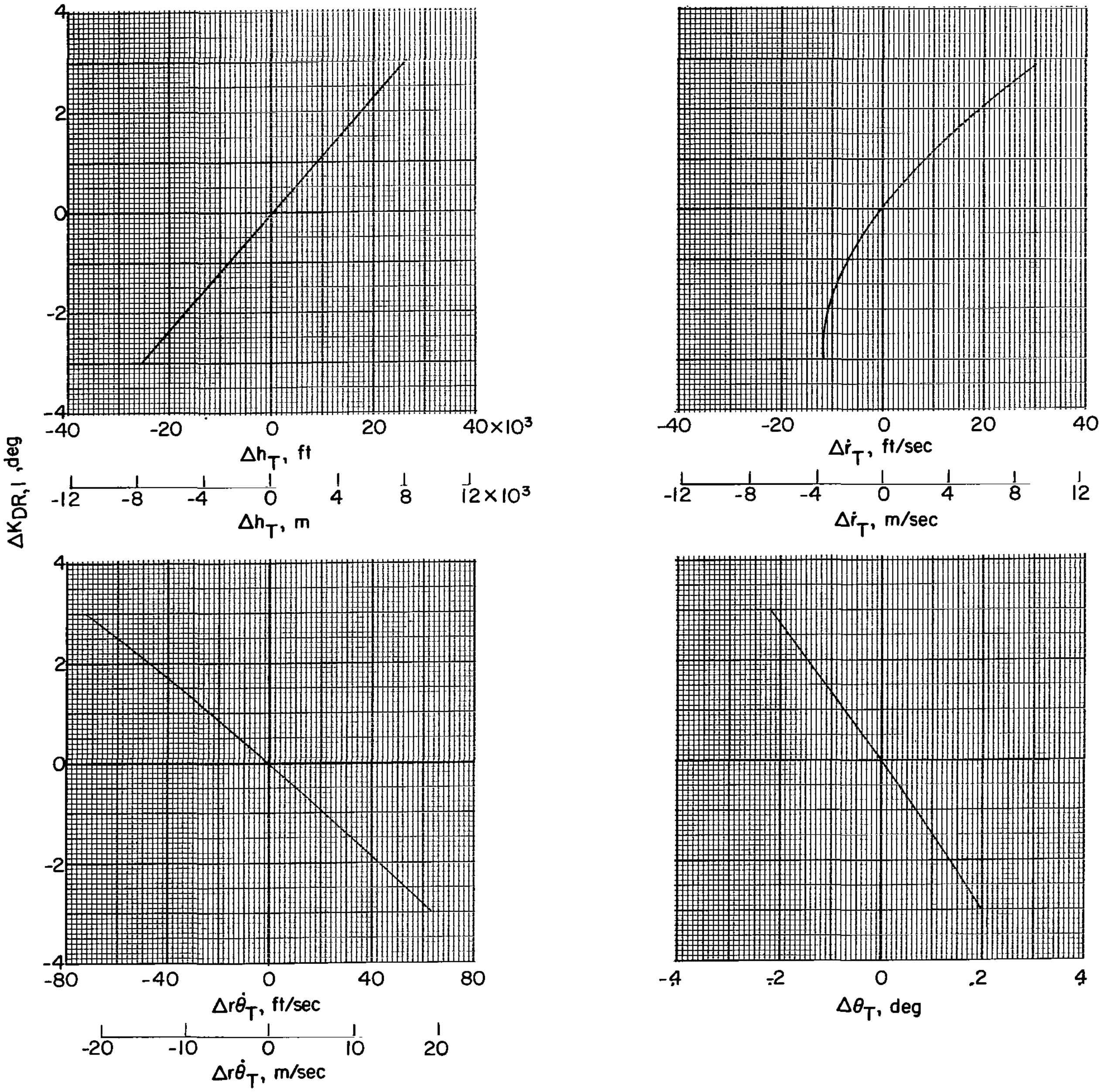
(d) Variation of altitude with range.

Figure 5.- Concluded.



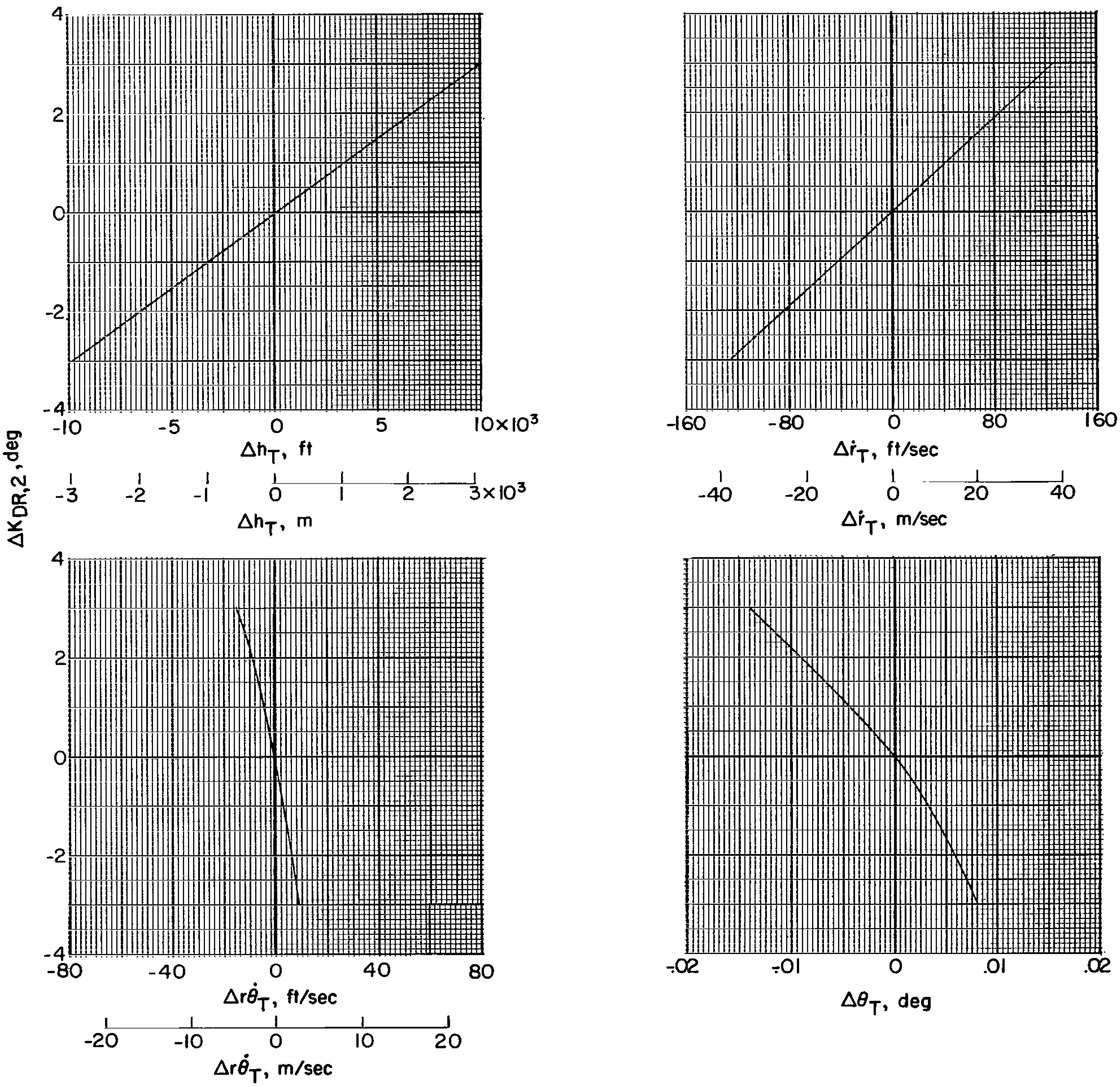
(a) Thrust-direction errors in vertical powered portion of trajectory. t_V = 19 sec.

Figure 6.- Variation of the change in terminal conditions at 50 000 feet (15 240 meters) due to thrust-direction errors.



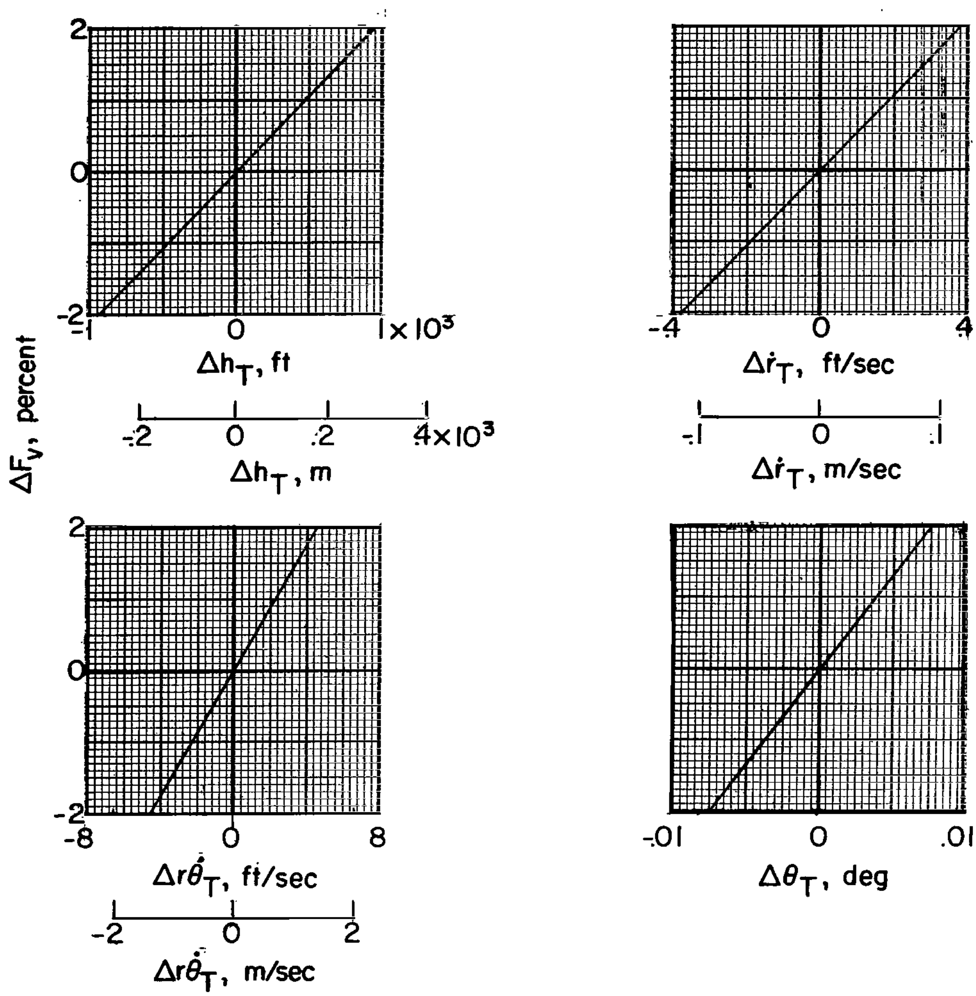
(b) Thrust-direction errors in powered portion of trajectory involving first constant down-range angle. t_1 = 248 sec.

Figure 6.- Continued.



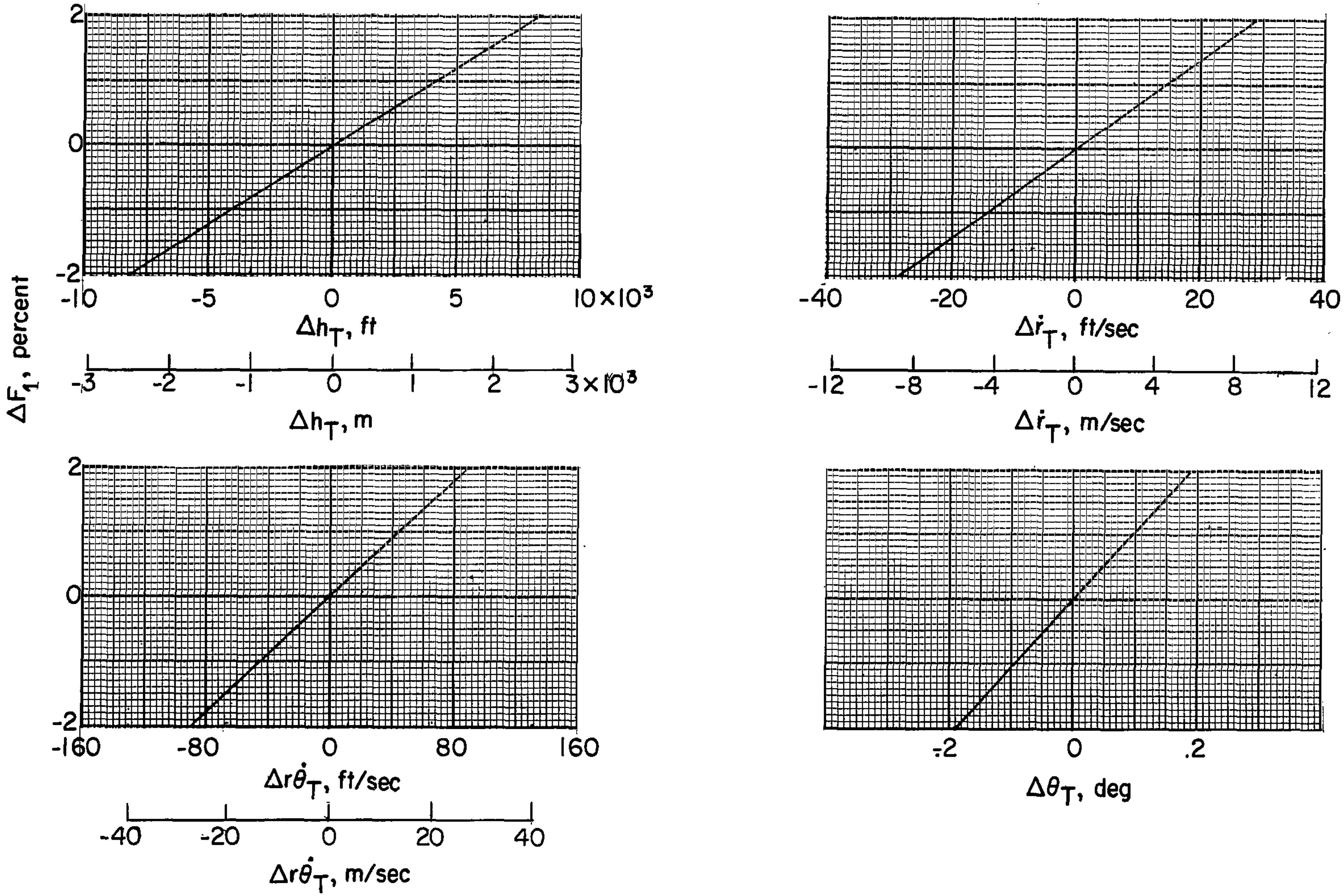
(c) Thrust-direction errors in powered portion of trajectory involving second constant down-range angle. t_2 = 156.9 sec.

Figure 6.- Concluded.



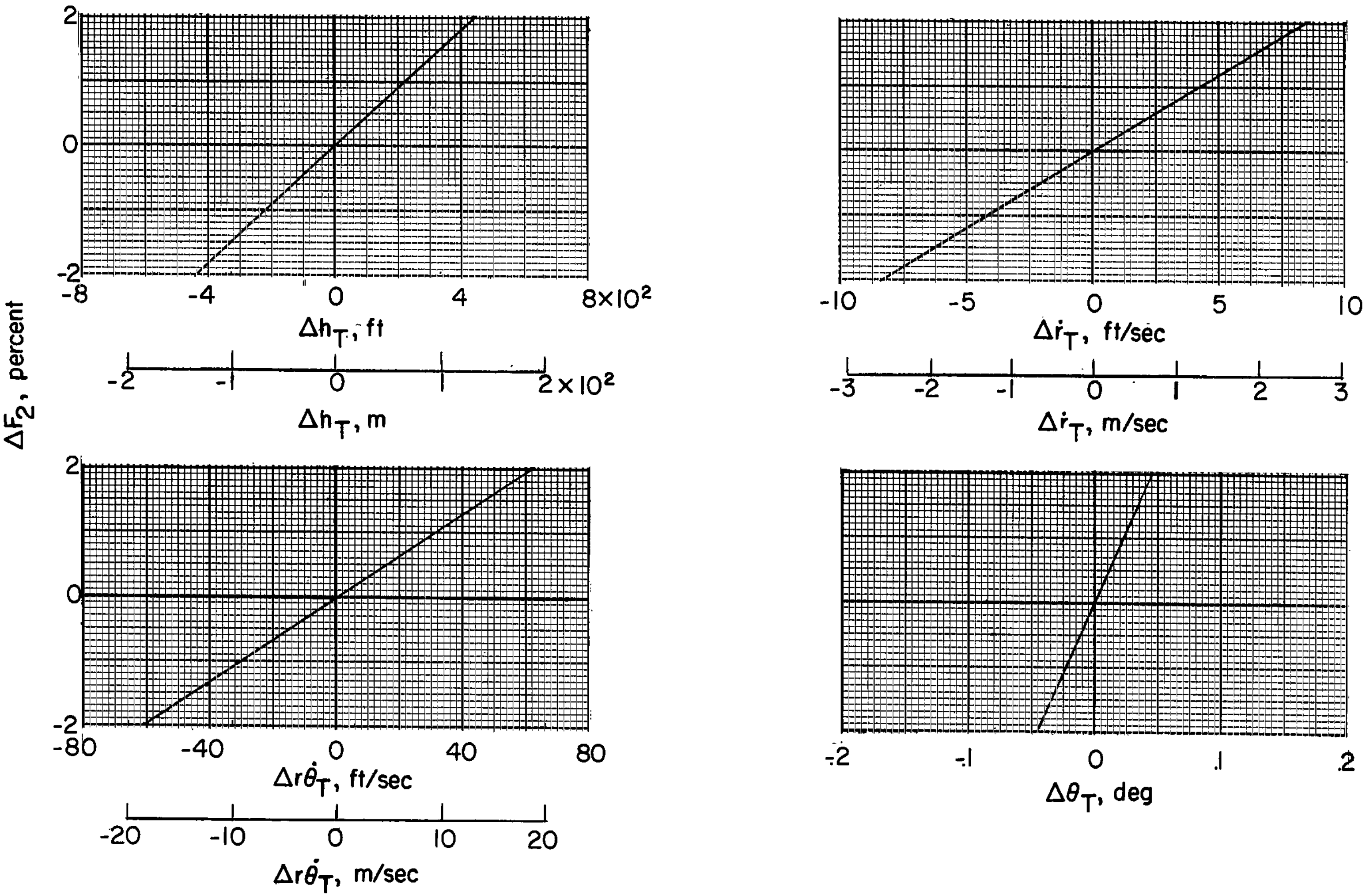
(a) Thrust magnitude errors in vertical powered portion of trajectory.

Figure 7.- Variation of the change in terminal conditions at 50 000 feet (15 240 meters) due to thrust magnitude errors.



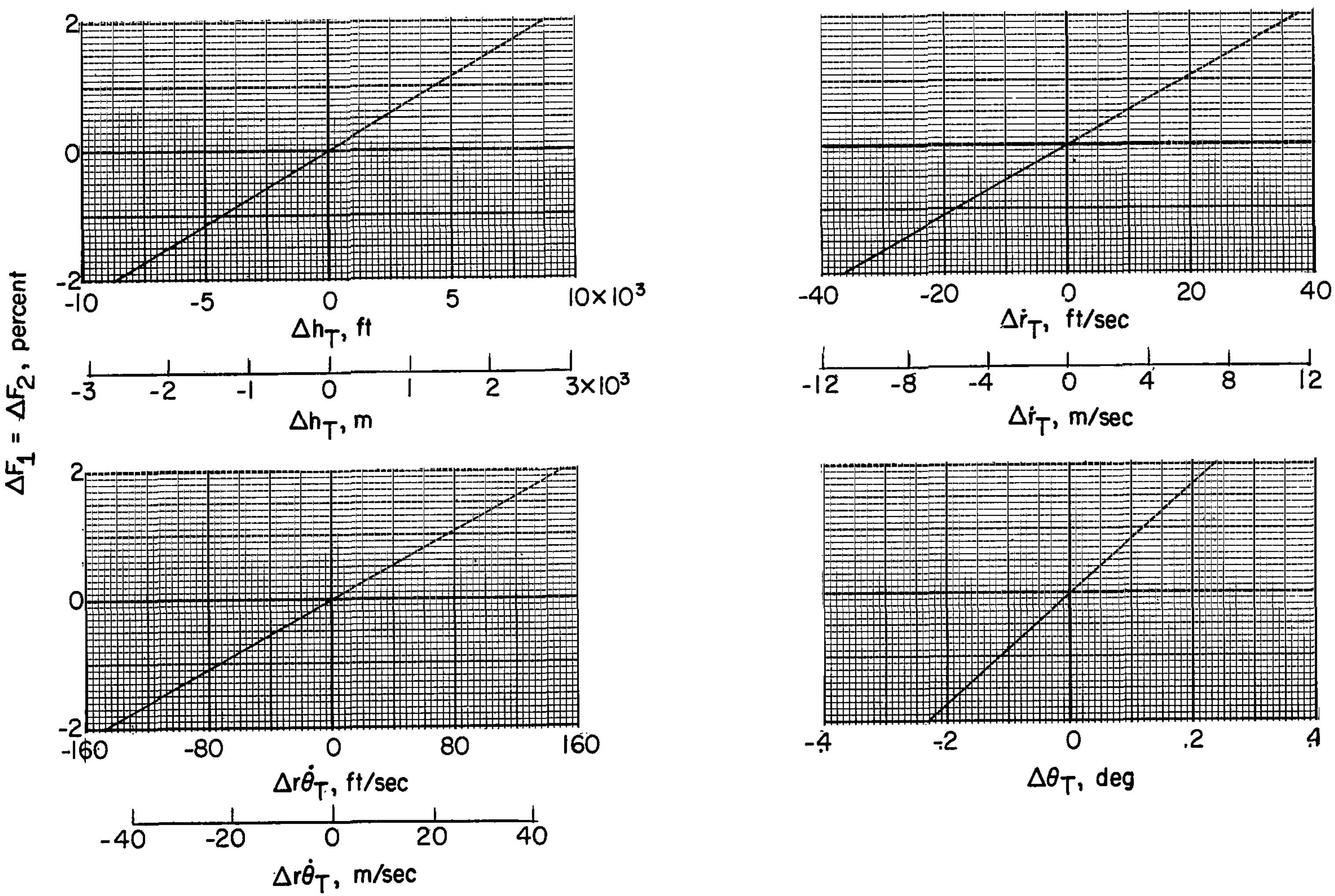
(b) Thrust magnitude errors in powered portion of trajectory involving first constant down-range angle.

Figure 7.- Continued.



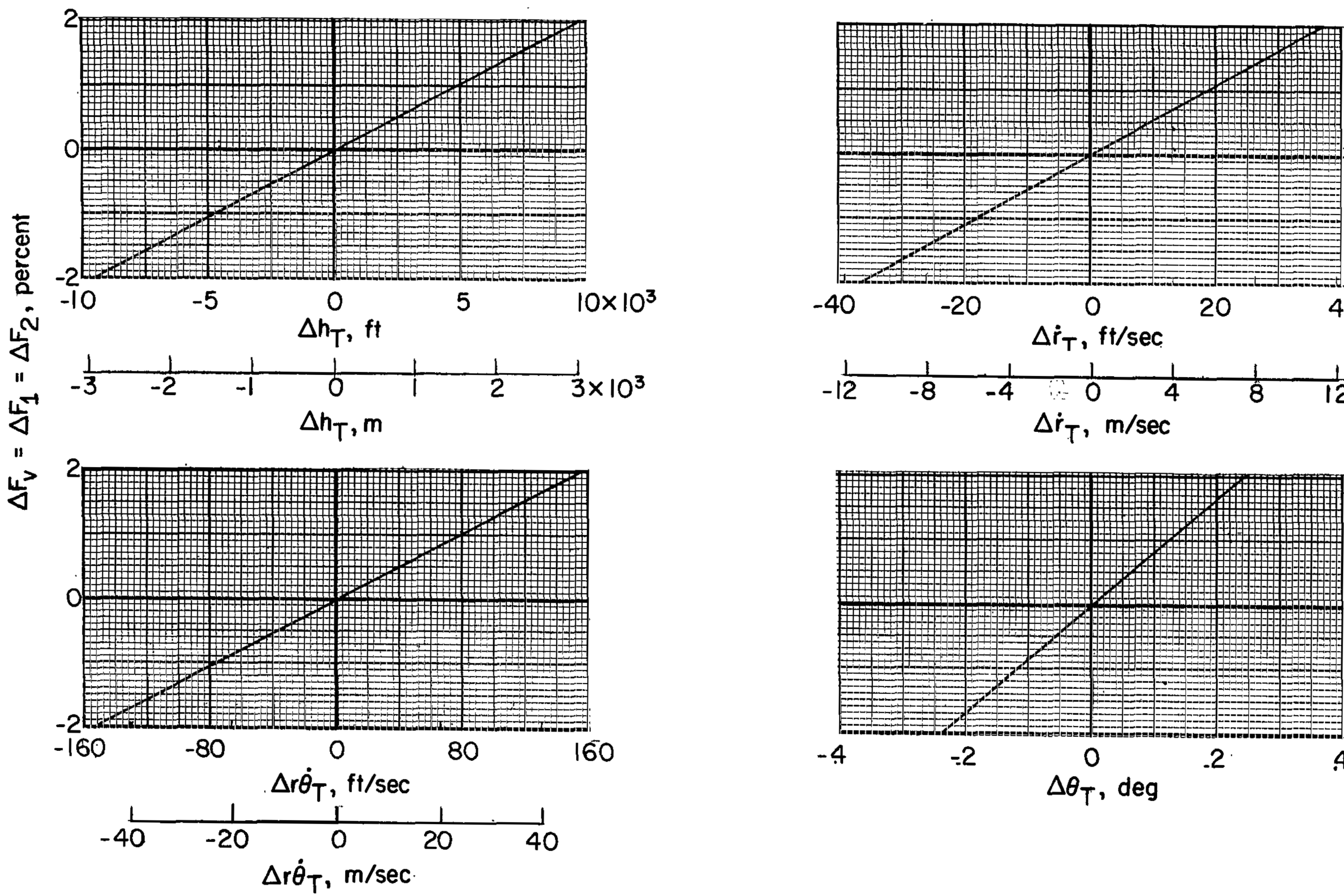
(c) Thrust magnitude errors in powered portion of trajectory involving second constant down-range angle.

Figure 7.- Continued.



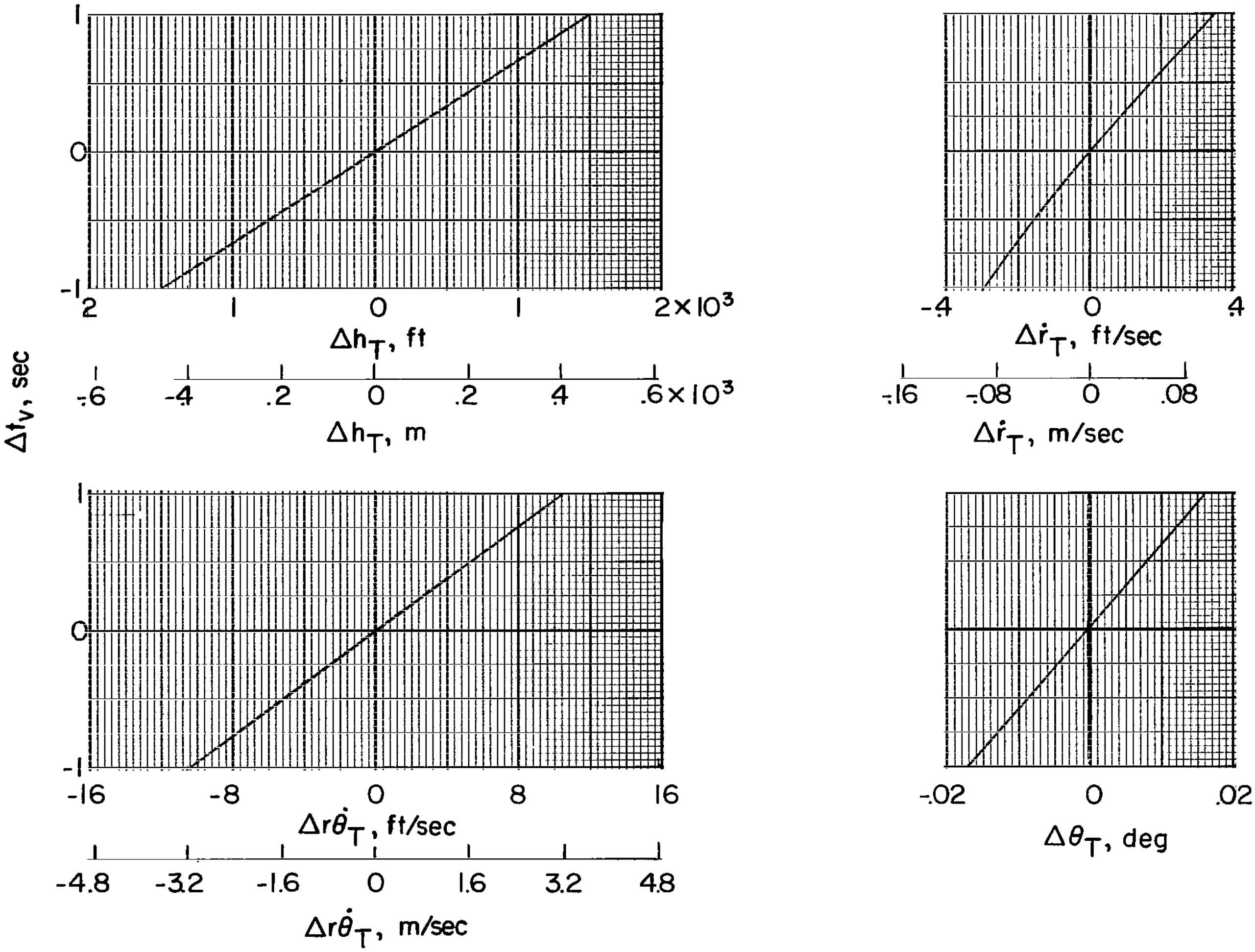
(d) Thrust magnitude errors in powered portion of trajectory involving first and second constant down-range angles.

Figure 7.- Continued.



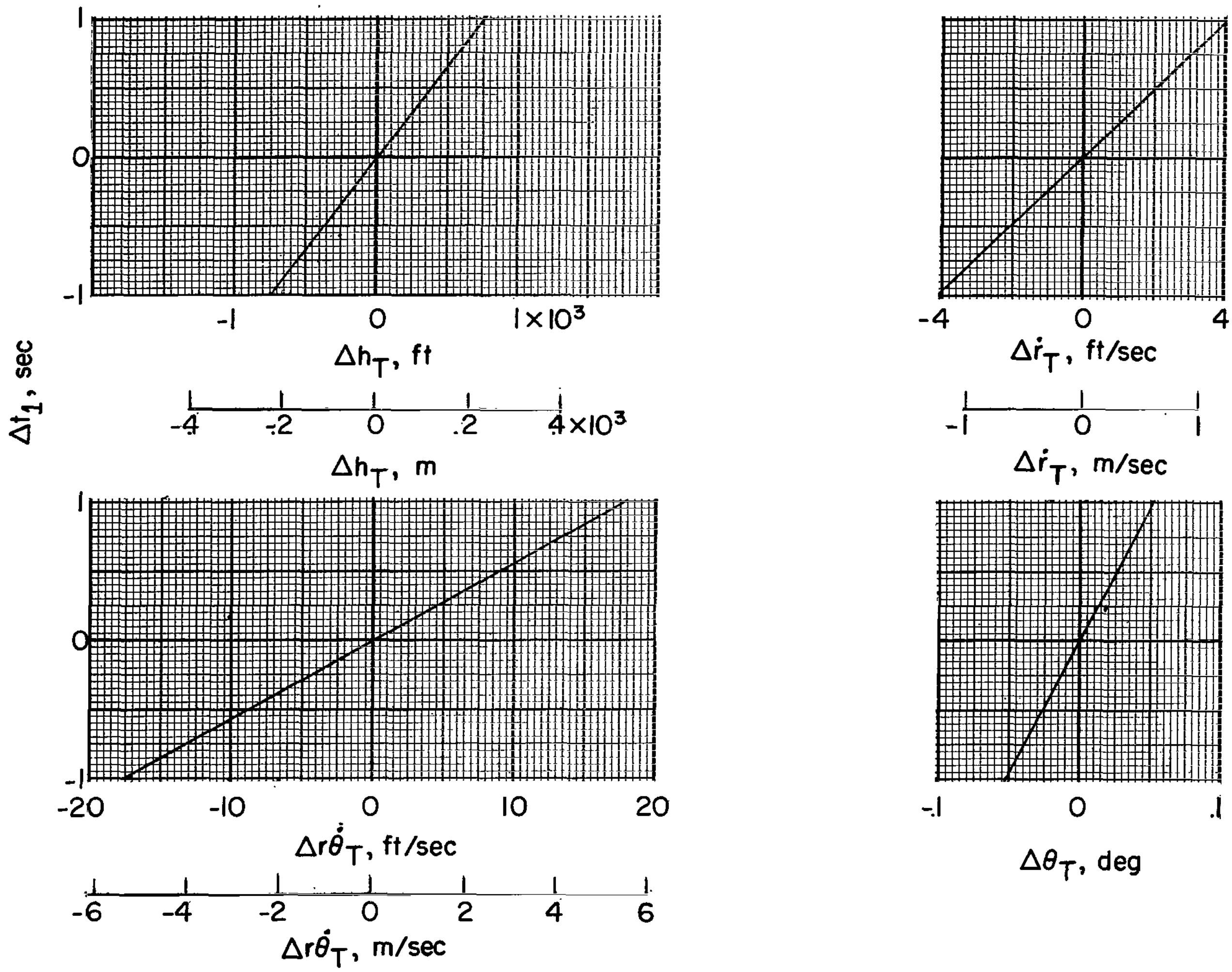
(e) Thrust magnitude errors involving vertical, first constant down-range angle, and second constant down-range angle powered portions of trajectory.

Figure 7.- Concluded.



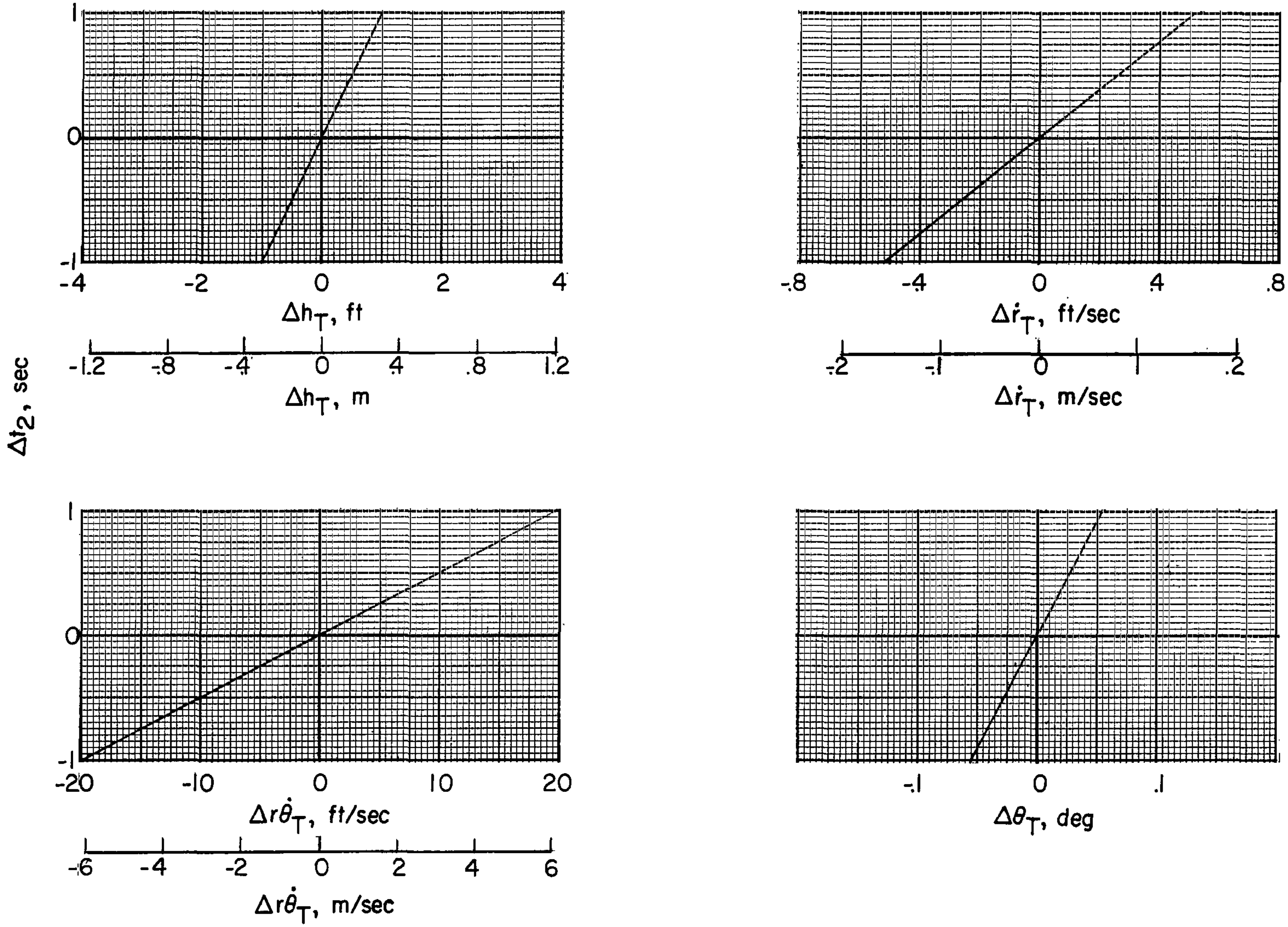
(a) Thrusting time errors in vertical powered portion of trajectory.

Figure 8.- Variation of change in terminal conditions at 50 000 feet (15 240 meters) due to thrusting time errors.



(b) Thrusting time errors in powered portion of trajectory involving first constant down-range angle.

Figure 8.- Continued.



(c) Thrusting time errors in powered portion of trajectory involving second constant down-range angle.

Figure 8.- Concluded.

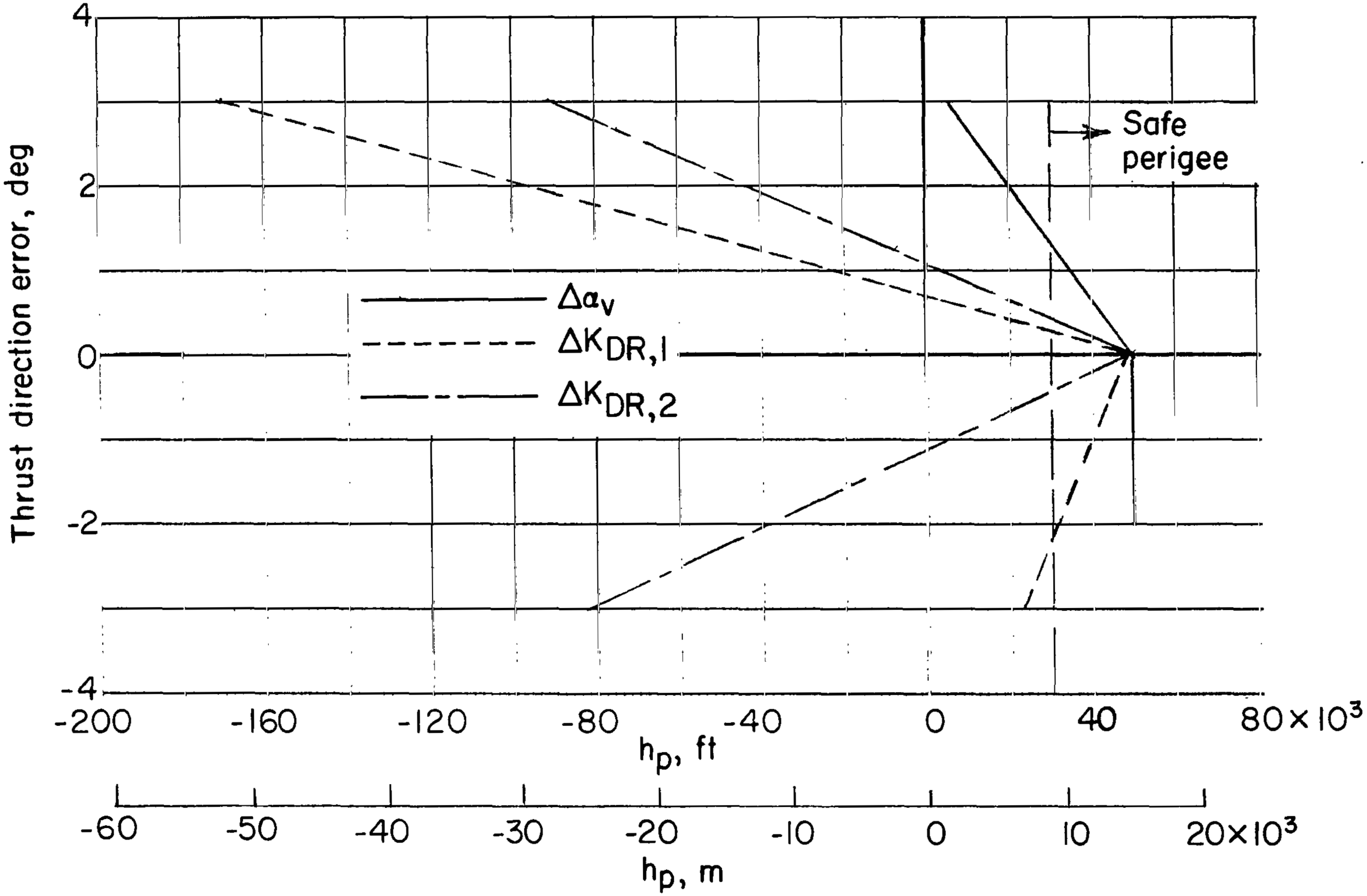


Figure 9.- Effect of variation in thrust direction on the perigee of coast trajectory resulting from thrust cutoff at t = 423.9 sec.

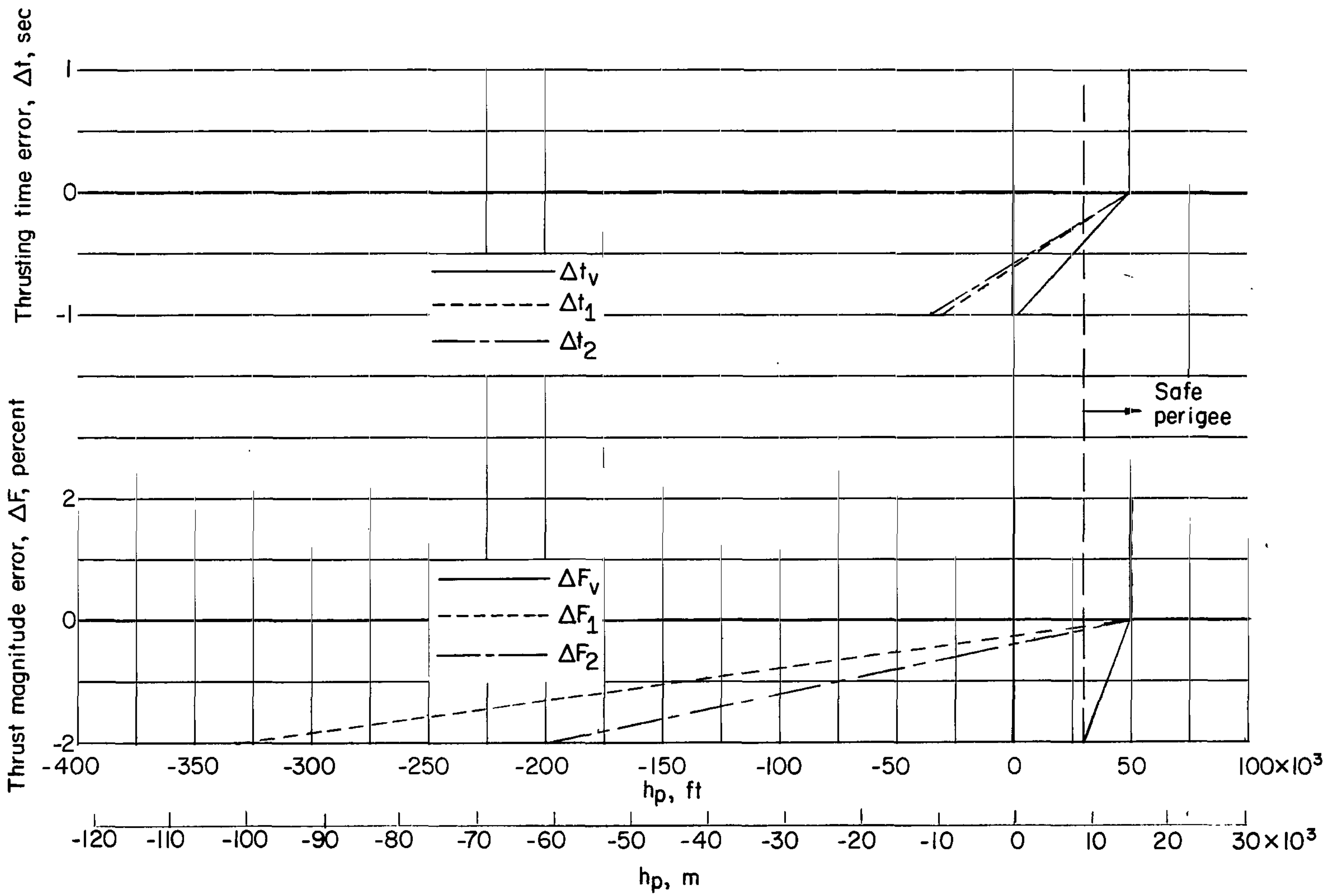
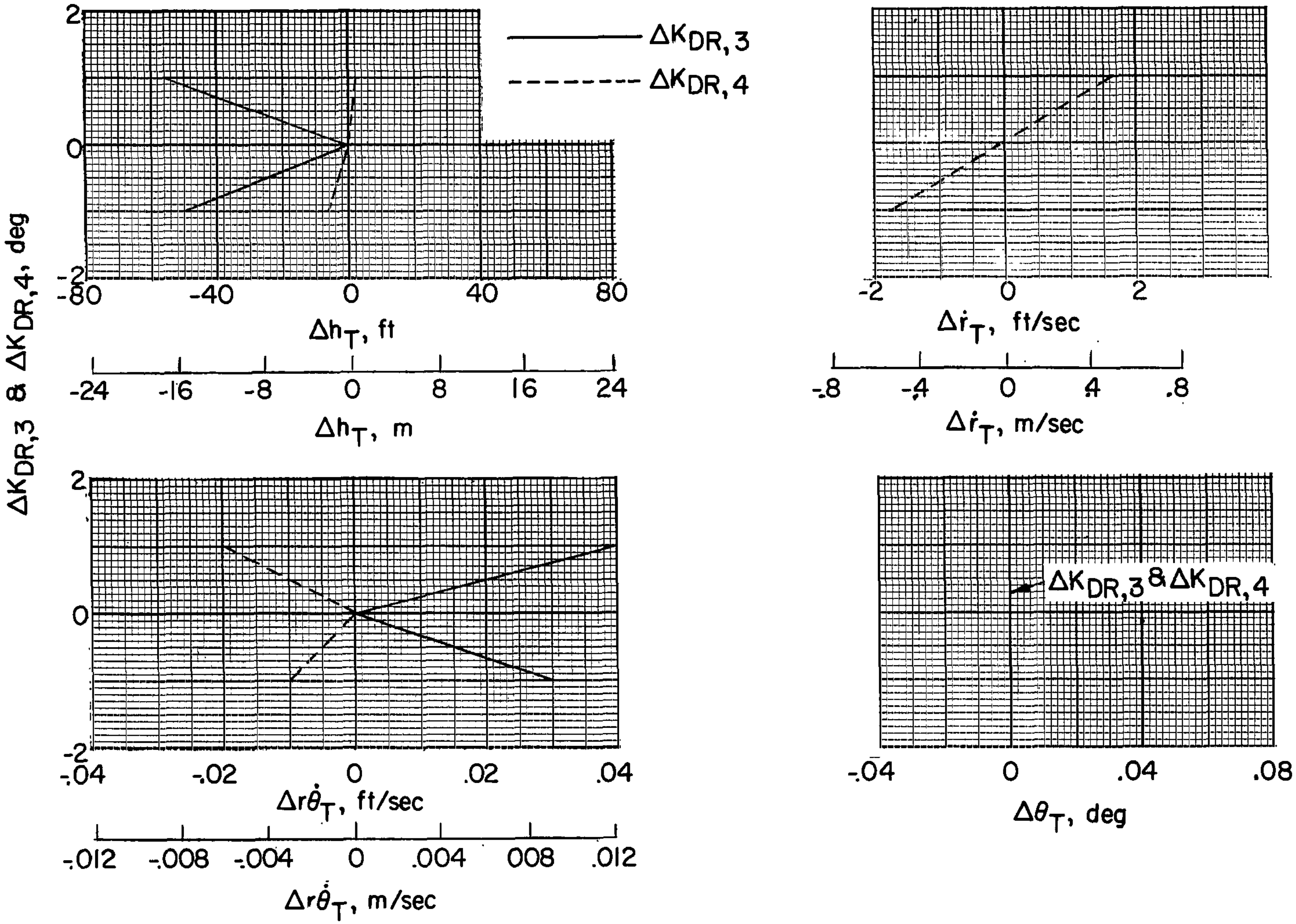
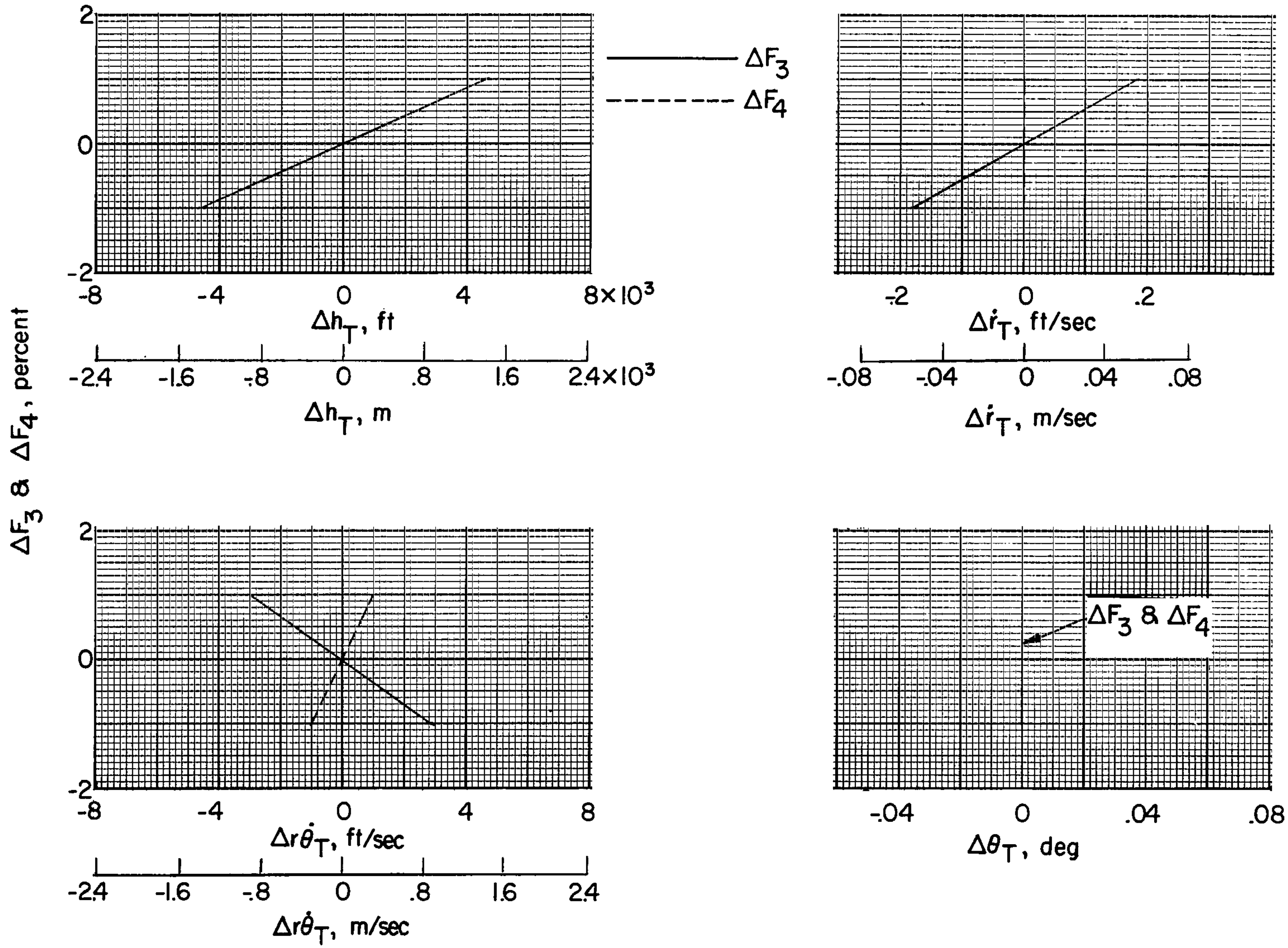


Figure 10.- Effect of variation in thrusting time and thrust magnitude on the perigee of the coast trajectories resulting from thrust cutoff at $t = 423.9$ sec.



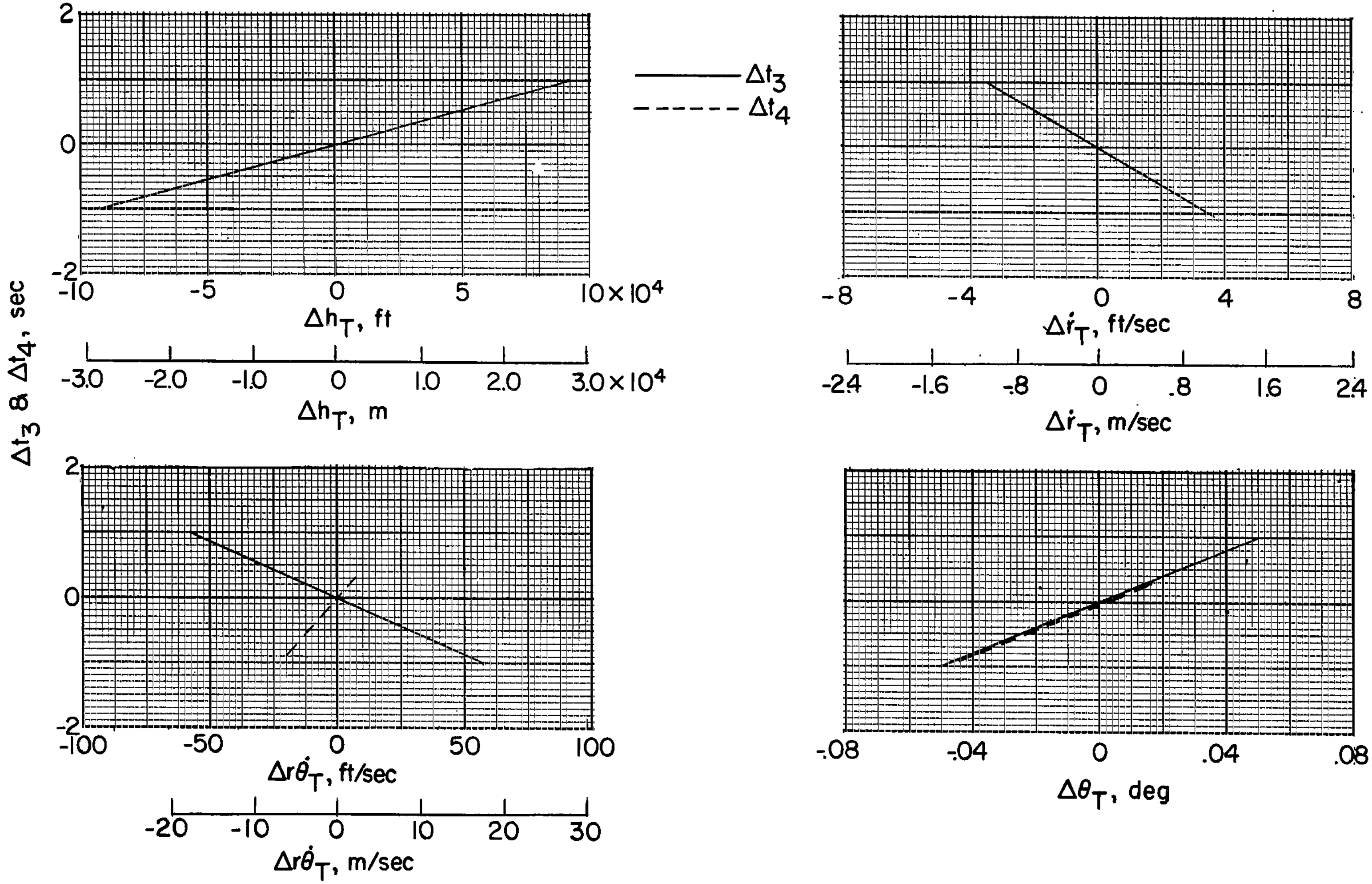
(a) Thrust-direction errors.

Figure 11.- Variation of change in terminal conditions at 80 nautical miles (148 160 meters) due to variations in thrust direction, thrust magnitude, and thrusting time for the transfer trajectory from 50 000 feet (15 240 meters) to 80 nautical miles (148 160 meters).



(b) Thrust magnitude errors.

Figure 11.- Continued.



(c) Thrusting time errors.

Figure 11.- Concluded.

NASA-Langley, 1966 L-4941

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