PHASE II REPORT
GIANT APERTURE TELESCOPE STUDY

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120-Inch Aperture Azimuth-Elevation Communications Receiver
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SECTION I

INTRODUCTION

This report presents the results of Phase II of a design study directed towards determining the optimum configuration of, and specifying engineering guidelines for, the ground receiver for a deep-space optical communications system.

During the first phase of this study two techniques for communication from space, intensity detection and coherent detection, were compared. Ground receivers were broadly examined and the communications aspects were compared.

Phase II has consisted primarily of a detailed study of a specific configuration designated by the Jet Propulsion Laboratory in a technical directive memorandum dated 8 June 1966. The designated configuration is a fully steerable telescope for coherent detection at a wavelength near 10 microns.

The JPL technical directive further specified an aperture of 80- to 120-inch diameter. We have concentrated on a 120-inch design since this instrument would present the greater design problem. All of the conclusions reached are, however, applicable to telescopes in the range between 80 to 120 inches.

We were also directed to estimate the astronomical ability of such a telescope. It is apparent that a large telescope suitable for tracking of a distant spacecraft and for coherent detection of energy from that spacecraft is also suitable for use as an astronomical telescope. In view of the high
cost of such a telescope, the probability of long periods when it will not be required for its prime function, and the great need for large aperture instruments by the astronomical community, we consider it to be of substantial importance that the telescope be suitable for astronomical work.

The telescope discussed here is very well suited to its primary role as a coherent optical deep-space communications receiver and to use as a general purpose astronomical telescope.

In this report we describe the design for a 120-inch telescope in enough detail to prove feasibility and to permit a detail design to be started. All major parts have been sized and requirements of major components have been established. There are still several areas where further development is required before a coherent deep-space communications system can become a reality; but the ground receiver, the subject of this study, presents no problems that require other than good engineering for their solution.

The JPL technical directive also authorized study of a backup design for incoherent detection to receive less emphasis than the coherent system. The directive stated that "Questions that relate to the desirability of this design as compared with a fully steerable paraboloid take precedence over details of design." During the course of the second phase of the study there has been no new information which would change the conclusions that led to the selection of the coherent system as the primary area for study. For this reason, we have not further pursued the incoherent detection system and this report is concerned only with the coherent technique.
SECTION II

GENERAL DESCRIPTION OF THE GROUND RECEIVER

A. FACILITIES

Before discussing the features of the ground receiver in detail, an overall description of its capabilities and features is presented.

The ground receiver is a 120-inch aperture telescope on an alt-azimuth mount. The telescope is capable of complete hemispheric sky coverage about its site and may be used for normal astronomical work in addition to its intended mission as a ground receiver for a deep-space laser communication system.

The entire telescope will be mounted on a tower and enclosed by a conventional dome. Figures II-1 and II-2 show the instrument and some of its details, while Figure II-3 shows a cross section of the complete installation.

All of the design features except the prime focus cage are valid for telescopes in the 80- to 120-inch aperture range.

The prime focus cage is not considered to be applicable to telescopes much smaller than 120 inch because of the large amount of light blocked by a cage large enough to hold a man.

The azimuth axis of the telescope is carried by a 96-inch diameter hydrostatic oil bearing. This was chosen for the very smooth, stiction-free,
Figure II-1. 120-Inch Aperture Azimuth-Elevation Communications Receiver
Figure II-2. Details of 120-Inch Aperture Azimuth-Elevation Communications Receiver
Figure II-3. Building and Dome - 120-inch Aperture Azimuth Elevation Communication Receiver
low friction, and accurate performance that these bearings are capable of providing, under the 70-ton load imposed by the telescope and yoke.

The elevation axis is carried on ball bearings of 28-inch bore diameter. Consideration was given to the use of hydrostatic bearings for this axis but ball bearings were chosen because of their simplicity and, in this size, the ability to carry the load of the telescope with sufficiently low friction torque.

Both axes are driven by direct coupled dc torquers to eliminate all effects of tooth noise and runout associated with gearing. The telescope is not capable of tracking precisely through zenith because of the infinite accelerations required with the alt-azimuth mount, but, on a conservative basis, it is capable of tracking a spacecraft that passes within 5 arc-minutes of the zenith. Since the probability of a spacecraft passing this close to zenith is quite remote, and loss-of-target in the worst case would last less than one minute, the "blind spot" at zenith is considered to be a minor drawback.

The telescope has provision for prime, Cassegrain, and coudé focal positions when used for astronomical work. The Cassegrain focus is used for communication. A flip secondary is proposed to change from Cassegrain to coudé focus. This mirror can be folded up when prime focus work is required. The Cassegrain focus can be used either with or without a folding mirror at the end of the elevation trunnion for astronomical work. A detachable folding mirror and mount are positioned on the fork upright at one end of the elevation axis when use of the coudé focus is required.
A digital computer control will be used to perform the computations associated with star tracking and acquisition, (See figure VII-1). The computer will have available to it position and velocity parameters of the spacecraft, the site latitude and longitude, the sidereal time, and the atmospheric pressure and temperature. The computer can then solve for the proper azimuth angle, azimuth velocity, elevation angle, elevation velocity, and point-ahead angle. It shall be assumed that a small special-purpose computer is available at the site. The availability of such a computer will eliminate the time lag inherent in serialization and transmission of data to a remote central computer facility. The presence of such a computer will also be helpful for data reduction.

Both the azimuth and elevation axes of the pedestal will have 21-bit digital shaft angle encoders, with the output of those encoders fed directly to the digital computer.

A dc tachometer will be used to sense the shaft angular velocity of each axis. The output of the tachometer will be used for servo stabilization. An analog-to-digital converter will be used to transform the tachometer output into a form suitable for digital computation.

For its communication function the telescope makes use of an argon laser to provide a cooperative beacon for tracking by the spacecraft, a CO₂ laser as a local oscillator, a servoed fine tracking mirror to take out average wave front tilt and high frequency disturbances, and associated I.R. and visible trackers. The entire communication optics group will be located in as close proximity to each other as possible, on a kinematically mounted optical bench or platform fabricated of a thermally stable material such as Invar. The close
arrangement of components on the bench will minimize boresight errors that might occur as a result of thermal variations between the trackers, beacon laser, local oscillator, and detectors.

The diameter of the upgoing beacon is limited to 10 arc-seconds by the power levels which can reasonably be anticipated on the basis of current laser technology. This limitation could change with unforeseen developments in the laser field.

The entire telescope and mount is placed atop a rigid tower of sufficient height to avoid the severe seeing degradation associated with ground-level turbulence.

The telescope is enclosed in a conventional rotating astronomical dome. The dome is supported by an enclosed structure, which surrounds the rigid tower supporting the instrument and shields it from wind loads and thermal inputs. This structure also provides ample space for auxiliary equipment and laboratories.

B. SEQUENCE OF OPERATION

Although absolute pointing accuracy is a prime objective for the proposed instrument, it is improbable that the inherent accuracy of the instrument will be sufficient to assure illumination of the spacecraft by the 10 arc-second diameter argon beacon without some external reference. The following procedure, elaborated upon in subsequent sections of this report, utilizes the positions of known stars as reference or calibration points for the instrument:
1. The telescope is commanded to point to, acquire, and closed-loop track a known navigational reference star as close as possible to the known position of the spacecraft.

2. The encoders on the azimuth and elevation axes are interrogated, and the readings employed in conjunction with the known offset angle between reference star and spacecraft to compute encoder settings for the spacecraft coordinates.

3. The telescope is offset to the computed encoder settings and driven open loop at sidereal rate.

4. The upgoing argon beacon is turned on and the spacecraft instructed via the microwave link to initiate a search routine for the argon beacon.

5. When the spacecraft acquires the earth beacon, it turns on its 10.6 micron transmitter.

6. The ground station acquires the spacecraft signal and commences closed-loop tracking thereof.

7. The spacecraft is instructed via the microwave link to begin data transmission. This transmission may continue until loss-of-signal occurs above the western horizon.
SECTION III

SELECTION OF MOUNT CONFIGURATION

A. SELECTION CRITERIA

The selection of a mount type for the Laser Communication Receiver is influenced by many variables, some of which are quite different from those important in the selection of an astronomical telescope mount. Some of the more important parameters are:

1. Sky Coverage Available

   It would, of course, be desirable to be able to look at an object anywhere in the celestial hemisphere. We can compromise somewhat in this requirement if we are willing to limit our coverage to spacecraft in the vicinity of the planets. For this case we must be able to cover any area of the sky within ±8 degrees of the plane of the ecliptic. We can further compromise this requirement when we realize that space communications will not be possible close to the horizon. The fact remains, however, that the most desirable system would give us a full hemispheric coverage around the telescope site.

2. Absolute Pointing Capability

   It would be very desirable to be able to command the telescope to the known position of the spacecraft and then be sure that the spacecraft was in the field of view of the telescope. This requires a telescope and mount that is thermally stable, extremely stiff, and possessed of transducers which are capable of reading its angular position to a high degree of accuracy.
3. **The Degree of Redesign Required for Adapting the Telescope for Use at Different Latitudes**

A complete optical space communication network will require several instruments at widely spaced points on the earth. In view of the high engineering costs associated with the design of such an instrument it would be extremely desirable to be able to use the same instrument design for any of the sites.

4. **Costs**

Because of the need for several instruments, the cost of each instrument is of paramount importance and should be an important factor in the selection of a specific type of mount.

5. **Dome Size Required**

The cost of the dome required to house and protect the telescope is a large fraction of the cost of the total site. It is therefore of importance that the mount chosen requires a minimum size dome.

6. **Astronomical Convenience**

The instrument will almost certainly be used by astronomers during periods of space communication inactivity. The mount type should be chosen to provide maximum utilization by the astronomer.

B. **MOUNT TYPES**

Astronomers have traditionally used the equatorial type of mounting. This two-axis mount has one of its axes aligned parallel to the apparent polar axis of the earth. Motion about this axis is referred to as right ascension. The other axis, the declination axis, is orthogonal to the polar axis. These mountings have two important advantages to astronomers. One, a constant speed
drive about the polar axis suffices to keep stars nominally fixed in the field of view. Two, the field of view does not rotate with respect to the telescope. There are many variations of the equatorial mount, a few of which are shown in Figure III-1. There are several disadvantages to the equatorial type of mount for the application considered here. Some of these are:

1. It is difficult to obtain complete hemispheric sky coverage with these mounts because the supporting structure invariably obstructs access to some portions of the sky.

2. It is very difficult to achieve high absolute pointing accuracy with these mounts because of unavoidable deflections in the forks, or, for some cases, cantilevered axles. It is, of course, possible to calibrate these deflections and program them out when pointing the instrument, but this presents formidable problems since the deflections of both the telescope tube and the mount vary with both right ascension and declination. Meinel has stated, "It should be realized that to achieve absolute pointing errors in the range of a few seconds of arc is currently beyond the state of the art of telescope making." Modern large telescopes may be expected to have pointing errors on the order of one to three minutes of arc. Calibration to a few seconds of arc over large angles of the sky of a telescope having errors of this magnitude would be an extremely difficult undertaking. Thermal effects within such a structure may in fact preclude such a calibration under any circumstances. Table III-1 lists some of the advantages and disadvantages of various telescope mount configurations.

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Figure III-1. Equatorial Symmetric Class Telescope Mounts
<table>
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<th>TYPE</th>
<th>CLASS</th>
<th>VARIATION</th>
<th>REPRESENTATIVE EXAMPLE</th>
<th>PRINCIPAL ADVANTAGES</th>
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<tr>
<td>EQUATORIAL</td>
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<td></td>
<td></td>
<td>Nominal sidereal tracking requires constant rate rotation about polar axis. Star field does not rotate relative to telescope.</td>
</tr>
<tr>
<td>ASYMMETRIC</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Ample space for instrumentation; improved rigidity; access to Cassegrain focus. Conveniences of observation and access to large field of view. Full sky coverage</td>
</tr>
<tr>
<td>CANTILEVER</td>
<td></td>
<td></td>
<td></td>
<td>KITT PEAK 36&quot;</td>
<td>Convenience of operation.</td>
</tr>
<tr>
<td>MODIFIED ENGLISH</td>
<td></td>
<td></td>
<td></td>
<td>McDONALD 82&quot;</td>
<td>More rigid than cantilever.</td>
</tr>
<tr>
<td>SYMMETRIC</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Improved rigidity. Reduced bearing moments.</td>
</tr>
<tr>
<td>FORK</td>
<td></td>
<td></td>
<td></td>
<td>LICK 120&quot;</td>
<td>Single pier yields compact rigidity.</td>
</tr>
<tr>
<td>ENGLISH YOKE</td>
<td></td>
<td></td>
<td></td>
<td>MT. WILSON 100&quot;</td>
<td>Small diameter bearings.</td>
</tr>
<tr>
<td>HORSeshOE YOKE</td>
<td></td>
<td></td>
<td></td>
<td>MT. PALOMAR 200&quot;</td>
<td>Provides access to polar sky.</td>
</tr>
<tr>
<td>CRADLE YOKE</td>
<td></td>
<td></td>
<td></td>
<td>KITT PEAK 150&quot;</td>
<td>More rigid and compact than Yoke.</td>
</tr>
<tr>
<td>EQUATORIAL DISK</td>
<td></td>
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<td></td>
<td>NEWTON 98&quot;</td>
<td>Combines advantages of fork and yoke rigidity.</td>
</tr>
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<td>COMMENTS</td>
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<td>Requires only a single axis. relative to ion at Cassegrain</td>
<td>Mount design must be tailored to a specific site latitude. Gravity vector rotates about two axes relative to telescope.</td>
<td>Astronomical telescopes employ mounts of this type almost without exception. Two-axis fine guiding is required.</td>
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<td></td>
<td>Severe defl. problems in large sizes. Massive counterweight required. Difficult bearing problems on declination axis. Large travel of Cassegrain focus. Asymmetric position of telescope in dome.</td>
<td>An asymmetric mount is considered inadequate for a 120&quot; telescope where absolute pointing accuracy is important.</td>
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<td></td>
<td>Lack of rigidity and pointing accuracy.</td>
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<td>Reduced sky coverage.</td>
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<tr>
<td></td>
<td>Reduced sky coverage. Cassegrain instruments require increased depth of fork or cradle or yoke.</td>
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<td>Fork deflection problems in large sizes. Limited access to Southern sky.</td>
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<tr>
<td></td>
<td>Limited access to Northern and Southern sky.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Large diameter horseshoe bearing required.</td>
<td></td>
<td>Use of a folded Cassegrain focus may be preferable to providing adequate horseshoe bearing clearance.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Horseshoe with improved length.</td>
<td>Large diameter horseshoe bearing must provide clearance for aft end of telescope.</td>
<td>Basically similar to fork.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Limited to sites of fairly high latitude.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Requires precision flat appreciably larger than primary. Limited sky coverage North and South.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MOUNTING TYPE</td>
<td>CLASS</td>
<td>VARIATION</td>
<td>REPRESENTATIVE EXAMPLE</td>
<td>PRINCIPAL ADVANTAGES</td>
<td></td>
</tr>
<tr>
<td>---------------</td>
<td>-------</td>
<td>-----------</td>
<td>------------------------</td>
<td>---------------------</td>
<td></td>
</tr>
<tr>
<td>ALT-AZIMUTH</td>
<td></td>
<td></td>
<td></td>
<td>Gravity vector is limited to a single axis relative to gravity load vector. Azimuth portion of mount provides limited angular motion about pole. Mount design is identical at both poles. Bearing design is simplified due to reduced required angular motion. Relatively low moments of inertia. Relatively low cost.</td>
<td></td>
</tr>
<tr>
<td>FORK</td>
<td></td>
<td></td>
<td></td>
<td>Compact Mount. Virtually full sky coverage. Communications package may be moved to reduce platform complexity.</td>
<td></td>
</tr>
<tr>
<td>INVERTED</td>
<td></td>
<td>VERTICAL</td>
<td></td>
<td>Fixed primary mirror. Minimum rotating mass.</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>SIDEROSTAT</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ALT-ALT</td>
<td></td>
<td></td>
<td></td>
<td>High angular rates occur where sky coverage is confined. Mount design is identical at both poles.</td>
<td></td>
</tr>
<tr>
<td>TRI-AXIAL</td>
<td></td>
<td>(Equatorial over Azimuth)</td>
<td>APRA/Michigan 60&quot;</td>
<td>May be used either as equatorial or alt-azimuth mount.</td>
<td></td>
</tr>
<tr>
<td>PRINCIPAL DISADVANTAGES</td>
<td>COMMENTS</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>----------------------------------------------------------------------------------------</td>
<td>--------------------------------------------------------------------------</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sidereal tracking requires non-linear rates about both axes.</td>
<td>Two axis tracking and coordinate conversions are not considered major obstacles.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Angular rates and accelerations are large near zenith.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coordinate conversion required to comply with astronomical convention.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Field rotates relative to telescope.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Depth of fork must be increased if Cassegrain focus is employed.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Requires precision flat appreciably larger than primary.</td>
<td>Design proposed by Bowen and Meinel.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Geometry of fork; near horizon, of little value, for any site</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gravity vector rotates about two axes relative to telescope; deflection problems are comparable to those for equatorial mounts.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sidereal tracking requires non-linear rates about both axes.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coordinate conversion required to comply with astronomical convention.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Field rotates relative to telescope.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mount design must be tailored to a specific site latitude.</td>
<td>It is doubtful that the additional flexibility of a tri-axial mount justifies the technical difficulty in a 120&quot; aperture instrument.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Third axis contributes to substantially increased complexity and cost.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Third axis creates additional source of deflection and pointing error.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Although the equatorial mount has long been favored by optical astronomers, radio astronomers have made much use of the alt-azimuth mounting. In this configuration the telescope rotates about the local vertical axis for azimuth coverage and an axis normal to it for altitude coverage. In the alt-azimuth configuration, the fork holding the telescope rotates about the vertical axis and so experiences no change in orientation of the gravity vector during tracking. The telescope rotates in elevation about an axis, which always remains normal to the vertical azimuth axis, and as a result the gravity vector always remains in one plane with respect to the telescope during use. By careful design of the telescope mounting structure, the rotation of the line of sight with respect to the elevation encoders can be held to one to two seconds of arc. Even this one to two second error can be programmed out since it is a function only of the elevation angle. This is an overriding advantage when it is desired to make a telescope with extremely high pointing accuracy.

One disadvantage of the alt-azimuth mount is that when tracking near the zenith the angular rates about azimuth become extremely high, so high in fact that it is not feasible to track an object passing through zenith.

There is one configuration of mount, which is well suited to tracking around the zenith, called the alt-alt mounting. In this two-axis mount, one axis is always horizontal and the other is normal to it. Although it conveniently permits tracking across the zenith it suffers from much the same problem that the equatorial mounts suffer in that the second axis rotates with respect to the horizontal. This means that the orientation of gravity with respect to the telescope is no longer in one plane. A further drawback is that in order for the telescope
to come down close to the horizon, large forks are required at each end of the mount. These forks are, of course, subject to varying deflections under gravity, a problem which is not present in the standard alt-azimuth mounting. These disadvantages, in our opinion, far outweigh the small advantage of being able to track across the zenith.

A problem that arises with the alt-azimuth mount that is not a factor with equatorial mounts is that of rotation about the line of sight. This presents no problem in use of the alt-azimuth mount in its communications receiver configuration since the communications function components are always operating on axis of the main telescope. When the system is used for astronomical work, however, the photographic plate at the focus of the telescope, must be rotated to prevent blurring of off-axis images. We do not feel that this is a serious drawback for astronomical use, since the correction does not require extreme precision and is easily put in on an open-loop basis. This problem is further discussed in Section VI.

Another disadvantage of the alt-azimuth telescope for astronomical use is that the instrument works in altitude and azimuth coordinates rather than in the right ascension and declination system to which astronomers are traditionally accustomed. We feel that this is not a serious detriment, as this system will always be used in conjunction with a computer that can easily and accurately convert from one coordinate system to the other. If desired, an electromechanical analog device could be used to convert from the usual astronomical coordinates to alt-azimuth coordinates continuously.
There are several significant advantages of the alt-azimuth configuration as the space communications receiver. With an equatorial mount design, the design of the system must be modified for each latitude at which it is used. For extreme changes of latitude, a drastic redesign involving a complete change of mount configuration may be required. This is not true of the alt-azimuth mount. The alt-azimuth mounting may be placed at any latitude with no change in design. Since a complete optical communications tracking network involving several sites at widely varying latitudes will eventually be required, it is extremely desirable that these be reproduced with no additional engineering work to adapt the system to each site. In addition, multiple quantity economies may be realized during fabrication of several identical systems at the same time.

Another advantage of the alt-azimuth mount is the rather small dome size required for a given aperture of telescope. Data published by the National Academy of Sciences\(^2\) indicates that the dome of a large astronomical telescope may typically be expected to constitute 20 to 30 percent of the cost of the entire installation. A mount which minimizes the size of the dome required may thus be expected to result in substantial savings on the cost of the building and dome.

Traditionally, astronomical telescopes have been driven in right ascension by large worm and wheel combinations of extreme precision. Even with the highest precision worm wheels available, astronomers have been required to do manual tracking of stars during long exposures because of errors in the drive,

\(^2\)Ground-Based Astronomy: A Ten Year Program, National Academy of Sciences, 1964.
flexures of the telescope, and changes of refraction angle during the exposure. In addition, for the tracking of planets or, in the case of the communications receiver, planetary probes, the conventionally driven telescope would require high correction rates were this type drive used. We feel that a communications receiver can best overcome these difficulties through the use of a closed-loop tracking system utilizing DC torquers for the elevation and azimuth drives. We have included provision in our design for the tracking of guide stars during astronomical use. This system will require no manual correction by the astronomer even during the longest exposures.

C. SELECTION OF ALT-AZIMUTH MOUNT

As is evident from the above paragraphs, the alt-azimuth mount is considered the logical choice for the proposed ground receiver. Although it has been historically avoided by designers of large astronomical telescopes, its shortcomings are minor in terms of modern technology. The potential gains in terms of compactness, rigidity, accuracy, cost, and applicability to multiple sites are all significant to the communications application.
SECTION IV

ACQUISITION AND TRACKING

A. ALTERNATIVE SPACECRAFT ACQUISITION TECHNIQUES

As a first step in the spacecraft to earth communications cycle we must flood the spacecraft with light from the upgoing beacon.

We prefer this approach to the alternative of requiring earth tracking and offset pointing by the spacecraft, because that technique puts an extra burden on the spacecraft. It is felt that, whenever possible, the proper trade-off is that which puts additional complexity in the ground station rather than the spacecraft.

The output power available from the argon laser is a key parameter in the design of the system. Extrapolation of current technology indicates that the maximum attainable value is on the order of 200 watts. This limitation on upgoing beacon power, in conjunction with the necessity of maintaining adequate flux density at the spacecraft, limits the angular diameter of the beacon to about 10 arc-seconds. The diameter of the beacon, in turn, defines the pointing accuracy required to assure illumination of the spacecraft by the beacon. Specifically, it is apparent that an absolute pointing error greater than 5 arc-seconds will result in failure to illuminate the spacecraft with the argon beacon. As indicated in Section IX, the telescope cannot reasonably be expected to achieve this degree of accuracy without some external reference.
The obvious source of external reference or calibration points is the known star map. The pointing accuracy of the instrument may be substantially improved by tracking a known star in the vicinity of the spacecraft and then offsetting the line of sight a precalculated amount to the spacecraft. This offset can be made to very high accuracy, since most of the sources of error have a very low rate of change with angle or time and, for angular offsets on the order of 10 degrees, are negligible.

We have considered several different alternatives for accomplishing this offset pointing, each of which is illustrated schematically in Figure IV-1.

The first makes use of a separate relatively small-aperture tracking telescope, which is mounted on the large telescope and which has a relatively large field of view. A tracking detector properly positioned in the focal plane of the sub-aperture telescope provides offset pointing from the main telescope. With this arrangement, tracking of the star can continue until the main telescope has acquired and locked onto the spacecraft. The offset could be obtained very accurately without the use of precision angle encoders by translating the detector in the tracking telescope focal plane with lead screws rather than rotating the entire tracking telescope.

The disadvantage of this system is that the tracking telescope would be located a substantial distance from the communications optics package and would be subject to errors arising from structural deflections and thermal gradients.

This approach requires a separate tracking telescope because the main optical system does not have a large enough field angle to permit the
Figure IV-1. Schematic Illustration of Alternative Methods for Offset Pointing From a Reference Star
required offset. For operation during daylight hours, a star of 5th magnitude or brighter is required. As is evident from Figure IV-2, analysis of the distribution of stars of this brightness indicates that offsets of up to 2.5 degrees from the spacecraft may be required. This field coverage is easily achieved with a small refracting telescope, but is far beyond the approximately 30-minute diameter field obtainable with the main optics.

A second possible technique for offset pointing is to use a small portion of the main telescope aperture, say a 10-inch diameter, and deflect the light coming into that portion of the aperture with Risley prisms. The light coming from that area of the aperture would be folded out at an appropriate point in the optical system and directed to the tracking detector. This arrangement is shown schematically in Figure IV-1. The use of Risley prisms in this application is particularly attractive because of their insensitivity to tilts or translations and because the introduction of precise offset angles would not require sophisticated hardware or the use of precision angle encoders.

This technique, like the auxiliary tracking telescope, would permit reference star tracking until acquisition of the spacecraft has been completed. The only disadvantage of this system is the extra complexity of the optics and servos associated with deflecting the line of sight. Note that the main optical system is always working on axis with this arrangement.

A third technique for offset pointing utilizes a fixed-star tracker which uses the primary optics and is accurately boresighted with the heterodyne detector and the beacon laser. In use, the telescope is directed to a star of known position near the spacecraft and the star tracker is permitted to lock on.
Figure IV-2. Number of Stars in Field and Signal-to-Noise Ratio vs. Magnitude for a 1/2 Hertz Bandwidth
The telescope will now track the star. For this operation the transfer mirror will be locked at its zero position.

The elevation and azimuth encoders will then be interrogated and new positions established by adding the known angular differences between the star and the spacecraft to the encoder readings. The telescope is then directed to the new position and commanded to track at earth rate and the star tracker folding mirror removed from the optical path.

This sequence of operations effectively eliminates the effects of almost all sources of error except encoder errors, differential deflections which occur as a result of the offset between the star and the spacecraft, thermal changes which occur during the time required for the acquisition cycle, and uncertainty of spacecraft position with respect to the star.

This is the technique that best suits the proposed ground receiver, and is thus the approach considered in following sections of this report. The star tracker optics are located close to the rest of the communications optics and suffer minimum loss of boresight due to thermal errors and no loss due to gravitationally induced deflections. It requires less additional mechanism than either of the other two schemes and puts minimum additional load on the computer. Open-loop tracking is required during the acquisition cycle but the computer can generate the signals to any required accuracy and the maximum error due to the encoder will remain at ± 2 bits, that is, no worse than the static case or the closed-loop tracking case.
Table IV-1 lists the parameters involved in the analysis of daytime star tracking, and Figure IV-2 plots signal-to-noise ratios obtainable with 25-cm and 3-meter apertures under day and night conditions.

Consideration has been given to the possibility of accepting reduced pointing accuracy from the telescope and utilizing a search program to find the spacecraft. This is not a reasonable approach for this system because of the transit times involved and the long cycle required for each iteration.

Consider the case of a spacecraft at one A.U. range, a 10-second diameter beacon, and a telescope with an absolute pointing capability of ± 10 seconds. We will then require four cycles to be sure of hitting the spacecraft but after each of these cycles we must wait $10^3$ seconds or 17 minutes for the light to make the round trip and, say, 13 minutes for the spacecraft to search out and lock onto the beam. We thus require up to 2 hours, a fairly large fraction of our total daily observing time, to achieve acquisition. This technique also wastes spacecraft maneuvering fuel.

This 2 hour figure is based on use of a telescope with a pointing uncertainty of only 10 seconds. The time to acquire by search increases as the square of the pointing uncertainty. Should the spacecraft be in the vicinity of Jupiter (4.2 A.U.), the time required for the previous search example would be 84 minutes per cycle or four and one-half hours to complete a four cycle search.

In view of this, it would be of substantial value to be assured of illuminating the spacecraft without a search procedure.
### TABLE IV-1

PARAMETERS FOR DAYTIME STAR TRACKING

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>SYMBOL</th>
<th>VALUE</th>
<th>COMMENT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Star Brightness</td>
<td>$I_{\lambda S}$</td>
<td>$3.7 \times 10^{-(2 + 0.4\mu)}$</td>
<td>&quot;H&quot; magnitude star at 4400°K 0.65-0.80μ</td>
</tr>
<tr>
<td>Sky Radiant Intensity</td>
<td>$H_{\lambda B}$</td>
<td>$10^{-3} W \text{ cm}^{-2}$</td>
<td>--</td>
</tr>
<tr>
<td>Atmospheric Transmission</td>
<td>$\tau_a$</td>
<td>0.65</td>
<td>--</td>
</tr>
<tr>
<td>Receiver Optical Efficiency</td>
<td>$\tau_r$</td>
<td>0.50</td>
<td>NASA CR252</td>
</tr>
<tr>
<td>Phototube Sensitivity</td>
<td>$S_{\lambda}$</td>
<td>$10^{-2} \text{ AW}^{-1}$</td>
<td>RCA S-20</td>
</tr>
<tr>
<td>Receiver Diameter</td>
<td>$D_r$</td>
<td>25 CM, 300 CM</td>
<td>Auxiliary Telescope.</td>
</tr>
<tr>
<td>Sensor Field of View</td>
<td>$\alpha_r$</td>
<td>1 Min, (2.9 x 10^{-4} rad)</td>
<td>Main Telescope</td>
</tr>
<tr>
<td>Tube Dark Current</td>
<td>$I_{DC}$</td>
<td>$2.2 \times 10^{-15} A$</td>
<td>NASA CR252</td>
</tr>
<tr>
<td>Optical Bandwidth</td>
<td>$\Delta\lambda$</td>
<td>0.15μ</td>
<td>0.65 - 0.8μ, Chosen for Day</td>
</tr>
</tbody>
</table>

$$\text{SNR} = \frac{I_{\lambda S} S_{\lambda} \tau_m \frac{\pi}{4} D_r^2 \Delta \lambda \tau_r}{\sqrt{2e^\lambda f (I_{DC} + \Delta L \tau_s \frac{\pi}{4} D_r \Delta \lambda \tau_r + H_{\lambda B} \left( \frac{\pi}{4} D_r \alpha_r \right)^2 + I_{\lambda S} \tau_s \frac{\pi}{4} D_r^2 \Delta \lambda \tau_r}}$$
B. SPACECRAFT TRACKING

The preceding paragraphs have discussed the problem of illuminating the spacecraft with the beacon laser. After the spacecraft has acquired the beacon, the downgoing laser beam from the spacecraft will enter the ground receiver. At this time, the 10.6μ tracker in the telescope will acquire the spacecraft and the fine tracking mirror will be activated. The fine tracking mirror has two functions: it reduces the tracking accuracy required of the mount and it removes the average tilt of the incoming wave front.

The tracking mirror responds to signals from the 10.6μ tracker to maintain the image of the target on that tracker and, as a result, maintains a constant phase front entering the coherent detector. The high frequency response of the tracking mirror permits it to follow the average tilt of the wave fronts entering the aperture.

As was discussed in the Phase I report, the calculated value of the effective coherence diameter of the atmosphere at 10.6μ is increased by a factor of 3.4 when the wave front tilt is tracked in this manner.

In the communication receiver mode, the position of the fine tracking mirror is sensed and the resultant signals used as inputs to the main telescope servos. The main telescope will always operate to minimize the tilt of the fine tracking mirror.
SECTION V

MECHANICAL DESIGN OF TELESCOPE AND MOUNT

A. GENERAL

The 120-inch Aperture Azimuth-Elevation Pedestal assembly and detailed sections are depicted by Figures II-1, II-2, and II-3. The main structure consists of three assemblies: the telescope; the fork; and the pedestal. Weight of the total assembly is approximately 90 tons. The preliminary design analysis of this telescope pedestal, sizing main steel structural members and selecting the particular servo drives, bearings, and data devices, is based predominantly on stiffness, compliance, and accuracy. An effort has been made to limit to a value less than 5 arc-seconds, the absolute pointing errors due to structural compliance, bearing and shaft runouts, and misalignment. The azimuth and elevation torsional locked rotor natural frequencies calculated for this design are approximately 15 and 5 cycles per second, respectively. These natural frequencies are at least 15 times greater than the servo bandwidth frequencies.

B. TELESCOPE ASSEMBLY

The telescope comprises all the assemblies on the elevation axis: namely; the secondary and its truss support; the elevation box and trunnions; the primary and its truss support; the No. 3 coude housing; fixed counterweights; and adjustable counterweights.

The secondary assembly, shown in Section CC on Figure II-2 includes a prime focus cage for astronomical purposes, and the Cassegrain and coude
secondary mirrors with the means to swing either into operation or retract both
to clear the prime focus beam. The steel vanes supporting the secondary cage
have been designed differently from those of conventional large telescopes in
that the loads of the secondary assembly are transferred directly into the two
side gravity load carrying trusses without having to go around the large outer
ring. This arrangement provides a stiffer and more efficient structure because
of shorter load paths. Also, the secondary cage assembly does not see the loads
due to the weight of the outer ring and top and bottom trusses. The top and
bottom vanes (shown in View EE) provide torsional restraint about the elevation
axis and are hinged to the outer ring so as to carry axial loads only.

The secondary and primary main tube steel truss supports are of the
Serurrier type in which the rigidity of the tubes is sized to allow the primary
cell to deflect under gravity by the same amount as the secondary cell, thus com-
penating for radial displacements at any elevation angle. Maximum lateral de-
fections of primary and secondary cells are computed to be 0.014 inch at zero
degrees elevation. The location of the elevation axis on the telescope tube is
approximately at the one-quarter balance point. Nine thousand pounds of counter-
weight at the primary cell are required to achieve this balance. A balance point
at one-third would be more desirable for minimum weight, but would result in much
longer fork uprights. This increased length cannot be tolerated, since the fork
is the critical structural component in terms of overall rigidity. To compensate
for the weight of a man in the observer's cage, and to permit fine balancing in
general, movable counterweights have been placed at the rear of the elevation box
structure shown in Figure II-1. The top and bottom secondary support trusses were
reduced in cross-sectional size as compared to the side trusses since, on an alt-
azimuth mount, they do not carry any gravity loads.
The primary and secondary mirrors are carefully supported and positioned in cells designed to maintain their figure and alignment. Section VI-A-5 discusses the details of the mirror mounting problem.

The elevation trunnion shafts are welded to the elevation steel box structure and machined as one assembly for greater precision in maintaining close fabricating tolerances.

C. TELESCOPE ELEVATION DRIVE AND DATA ASSEMBLY

The 120-inch telescope is driven in elevation by two direct-drive DC torquers, one on each end of the trunnions as shown in Section B-B of Figure II-2. The direct-drive servo eliminates all of the problems inherent in geared servos related to backlash, compliance of the gear train, and high motor inertia. Large diameter torquers have been selected in order to meet the torsional stiffness requirements for a high accuracy system. Two torquer motors are used for increased stiffness.

Precision elevation angular contact bearings, mounted as a duplex set back-to-back on each end, support the telescope assembly. These bearings are mounted in pillow blocks for ease of assembly and alignment of the elevation axis orthogonal to the azimuth axis. Both sets of bearings are preloaded to improve the accuracy of shaft position and assembly stiffness characteristics. One end of the trunnion shaft assembly is fixed to the yoke, while the other end is free to float to accommodate any differential thermal expansion and avoid internal strains.

Data pickoff is provided by means of a 21-bit encoder coupled directly to the shaft by a high accuracy flexible disc-type coupling.
A caliper-type disc brake is used as a parking brake to hold the elevation assembly when the telescope is de-energized. Stow pins will be provided for locking the telescope to zero and ninety degrees elevation positions for servicing. Hydraulic shock absorbers and interlocks will be used to de-energize the telescope at the limits of elevation travel (-5° and 95°).

D. FORK ASSEMBLY

The fork is a C-shaped hollow steel welded rectangular structure stiffened by internal partitions. The fork supports the telescope and laser communications optical assemblies. The folded Cassegrain focus comes out through one end of the fork upright and a platform is provided for the observer. Removable covers are at the top end of each fork upright for accessibility to the drive and data assemblies and installation of the telescope assembly. The inner race of the hydrostatic bearing is mounted to the base of the fork.

E. PEDESTAL ASSEMBLY

The pedestal shown in Section AA, Figure II-2, is a box ring gusseted steel weldment and main nerve center. It supports the yoke assembly and contains the fixed members of the azimuth hydrostatic bearing, the slip rings, the hydraulic rotary joint, the azimuth drive and data devices, and the leveling jack assemblies.

For minimum friction and maximum accuracy, the azimuth axis assembly, weighing approximately 70 tons, is supported on an oil-pad bearing system. A two-row stepped configuration with 6 pads per row has been selected. This arrangement provides axial, radial, and torsional restraint and is self-positioning. The bearing has been designed for an average operating oil pressure of 300 pounds per square inch and flow requirements of up to 10 gallons per minute. Oil film
The thickness between pads will range from 0.0015 to 0.003 inch. The oil used for this application will have a high viscosity index with a relatively small change of viscosity over the normal range of operating temperature. The hydraulic system will contain orifices in the bearing for metering oil flow; pressure switches to protect against failure of oil supply; a system of filters to assure freedom from suspended particles; a constant displacement pump; and a reservoir. To seal the azimuth bearing system from dust and foreign matter, an oil seal has been placed between the turntable and base. The oil seal exhibits no stiction and is almost frictionless compared to a conventional lip seal of this diameter.

The azimuth servo drive assembly shown in Section AA, Figure II-2, utilizes the same DC torquer motor and data encoder as used in the elevation drive assemblies.

The inefficiency of the argon laser results in substantial heat generation on the order of 20,000 watts or more for the required output. Convective cooling of this heat load cannot be used because the large temperature differential would result in thermal distortion of the mount and the stream of heated air would degrade the seeing. Provision has been included in the design to bring cooling water onto the mount for cooling the argon laser and any other substantial heat sources such as power supplies and the CO₂ laser.

Power will be brought onto the mount through slip rings surrounding the coudé focus optical path through the center of the azimuth platform. It is advisable to bring only primary power through the slip rings and convert this to the required DC and AC levels with on-board power supplies to minimize the number of slip rings required.
Consideration was also given to bringing power onto the mount via cables, which were transferred from one drum to another. This would be feasible because continuous rotation is not required but we feel that, since slip rings do not present severe problems, they are preferable to cables.

Leveling provisions have been included for possible foundation settlement. The complete telescope assembly is positioned for leveling by three mechanical screw jacks triangularly located around the pedestal base. Leveling is carried out by utilizing precision level dials on the azimuth rotating assembly to determine when the axis is plumb. Nine hydraulic jacks are positioned around the pedestal between the mechanical jacks. These hydraulic jacks are hydraulically interconnected and the hydraulic pressure is maintained so as to equally share the total weight on all twelve jacks. This arrangement minimizes deflection of the pedestal structure between the mechanical screw jacks.
SECTION VI

OPTICAL SYSTEM

A. BASIC TELESCOPE

1. General

Selection of an appropriate optical system for the basic telescope is governed by the primary requirement for optimum performance of the communication function and by the secondary but very desirable objective of maximum utility for general-purpose astronomy. Fortunately, these requirements do not conflict to any substantial degree, and thus permit the design of an instrument that, in addition to fulfilling communications requirements, will be of major interest to the astronomical community.

Restricting consideration for the moment to the communications function, a few requirements which influence the selection of the telescope optical system may be defined:

(a) The telescope must form a good, diffraction-limited image at the 10.6 micron wavelength in order to permit efficient optical heterodyning.

(b) The image should be available at a fixed location with respect to the gravity vector in order that critically aligned communications equipment and sensitive cryogenically cooled detectors are not subjected to varying gravity loads.
(c) The system must be as consistent as possible with the necessity for extremely good absolute pointing capability.

(d) The use of a 10.6 micron received wavelength and a 4880 angstrom transmitted wavelength by common optics strongly suggests a necessity for an all-reflective telescope. The desire for maximum flexibility of the instrument for astronomical work adds further impetus for an all-reflective instrument.

Consideration of the above requirements leads to the conclusion that the basic telescope should be a conventional two-mirror configuration with the optical path folded to bring the focal plane to a fixed position with respect to the gravity vector. The need to bring the focal plane to a fixed location rules out the possibility of using a single mirror at prime focus and, in combination with alignment considerations and the need for broad spectrum capabilities, outweighs the possible advantages of 3-mirror or catadioptric designs.

2. Traditional Parabolic - Hyperbolic Design

The traditional two-mirror telescope configuration is the Cassegrain or, in the case where the focal plane is brought to a fixed location, the coude. In either case, the primary and secondary mirrors are parabolic and hyperbolic, respectively. Such an arrangement produces a good image with the parabolic primary alone as well as with the two mirrors in combination. This characteristic permits use of the telescope without correctors at prime or Newtonian
focus, as well as at the Cassegrain and coude foci. This combination of foci
is, in fact, typical of that available on large astronomical telescopes.

An additional advantage of the traditional Cassegrain/coude con-
figuration is that the two mirrors can be figured and tested using conventional
methods. Each mirror, when tested at the proper conjugates, forms a theoretic-
ally perfect diffraction image. This advantage, however, decreases in impor-
tance as modern techniques increase the practicality of producing more difficult
surfaces.

The major shortcoming of the traditional parabolic-hyperbolic
system is the relatively narrow uncorrected field of view. The predominant
aberration is coma. For either prime or Cassegrain/coude foci, the coma is
directly proportional to the angular distance off-axis and inversely pro-
portional to the square of the f-number at the focal plane. Astigmatism and
field curvature are also present.

A variety of wide-field correctors have been developed to sub-
stantially improve the off-axis performance at the prime or Cassegrain foci.
These correctors are of great value for astronomical applications but, since
they involve refractive elements, would present severe problems in the com-
munications transceiver with simultaneous use of 10.6 micron and 4880 angstrom
wavelengths.

3. Ritchey-Chretien Design

An increasingly popular variation in large telescope design is the
Ritchey-Chretien configuration, in which both primary and secondary mirrors are
hyperbolic. The major attraction of this design is the complete absence of
coma at the Cassegrain and coude foci. Good performance is thus obtained with no transmitting elements, although a single correcting element may be employed near the focus to correct residual astigmatism and field curvature.

Since the primary mirror does not form a good image when used alone, correcting elements are a necessity for prime focus use.

4. Selection of Communications Telescope

It would at first appear that the Ritchey-Cretien design, with its absence of coma, would be the optimum choice for the communications telescope. However, as will be shown in a later section, the auxiliary optics associated with the communications equipment exhibit coma characteristics equal and opposite to those of the traditional parabolic-hyperbolic telescope. Thus, while selection of a coma-free Ritchey-Chretien telescope would leave unaffected the coma of the auxiliary optics, selection of a parabolic-hyperbolic telescope results in exact cancellation of the telescope's coma by that of the auxiliary optics. On this basis, the parabolic-hyperbolic design has been selected as most advantageous for the communications telescope.

The use of an elevation-over-azimuth mount, for reasons discussed elsewhere in this report, suggests two possible focal positions for the communications equipment. Either would comply with the requirement for a fixed position relative to the gravity vector. The first alternative would be a true coude focus, with the optical path folded out through the elevation axis and then down through the azimuth axis to a fixed position below the pedestal. The second possibility is to mount the communications equipment on the azimuth turntable and to employ what is best described as a folded Cassegrain focus.
brought out through the elevation axis. This alternative is permissible because rotation about the azimuth axis does not result in variations of gravity loading.

The true coude focus would avoid the inconveniences of transferring electrical and/or plumbing lines across the azimuth bearing. However, such a focal position requires a number of additional reflections and, of critical importance, adds to the absolute pointing error of the system those errors associated with transferring the optical path across an additional bearing axis.

Because best possible absolute pointing capability is a prime objective of the proposed instrument, and because the physical size and nature of the communications equipment is compatible with on-pedestal mounting, this alternative has been selected. The advantages in terms of absolute pointing capability are considered to outweigh the inconvenient design problems associated with electrical and/or plumbing connections to the azimuth turntable. It is also felt that the motion of a multi-ton instrument driven by DC torquers on a hydrostatic bearing will be so smooth that no problems will be encountered with vibration or microphonics of the sensitive detectors.

Having made the above selections, the optical parameters of the basic telescope may be determined.

The speed of the primary mirror is to some extent an arbitrary compromise between optical performance, ease of optical figuring, and insensitivity to collimation errors, which favor high f-numbers; and tube flexure,
tube length, dome size, and cost factors which favor low f-numbers. The current trend in astronomical telescopes is towards primary f-numbers in the 2.5 to 3.0 range. Since the considerations influencing the astronomers are, in this instance, substantially the same as those pertinent to the communications telescope, an f-number of 2.8 has been selected for the primary mirror of the proposed instrument. This value matches that for the 150-inch Kitt Peak telescope, which presumably represents the current state-of-the-art.

A nominal primary-to-secondary diameter ratio of 4:1, based on obscuration considerations, leads to a secondary mirror diameter of 30 inches for a primary diameter of 120 inches. (The actual secondary diameter must be somewhat larger than 30 inches in order to accommodate a finite field diameter, but the 30-inch value will be utilized for preliminary sizing purposes.) This, in turn, defines the primary-secondary spacing to be 202 inches for an f/2.8 primary. A rotatable, two-sided secondary mirror is proposed, as described elsewhere. One surface would provide a coude focus for astronomical use, and the second surface, when rotated into position, would provide the folded Cassegrain focus for either communications or astronomical use. It is apparent that, with this approach, the diameter and axial position of the secondary mirror are essentially the same for the coude and Cassegrain foci and that the final f-numbers in each case are determined solely by the required positions of the focal planes.

For the folded Cassegrain, the desired position of the focal plane outboard of one end of the elevation axis results in an f/11.5 system. Similarly, an optical path folded out through the elevation axis and then down through the azimuth axis to a workable location beneath the pedestal results in an f/24 system.
coudé focus. In both cases the resulting f-numbers are reasonably in line
with those for modern astronomical telescopes of large aperture.

5. Mirrors and Mounting

(a) General

A fundamental parameter in the design of the primary and secondary
mirrors (as well as of the rest of the optics) is the degree of surface pre-
cision required. The basic requirement of the system is efficient optical
heterodyning of the received signal, and since heterodyning efficiency is
directly dependent on the wavefront accuracy, this leads inevitably to the
necessity for diffraction-limited performance of the optical system.

A realistic specification for mirror surfaces in diffraction-limited
optical system is a maximum root-mean-square surface error of 1/50 of the
operating wavelength. (This corresponds to a transfer function of approximately
0.95; see pp. 84-85 of the Phase I Report on this program.)

Alternatively, it is possible to consider directly the effects of
surface errors on heterodyne efficiency. The heterodyne signal may be defined
as

\[ k \cos \left[ (\Delta \omega) t - (\phi_o - \phi_s) \right] \]

where \( k \) is a constant, \( \Delta \omega \) is the beat frequency between the signal and the
local oscillator, and \( \phi_o \) and \( \phi_s \) are the phases of the local oscillator and
the signal, respectively. The average signal is then proportional to the
average value of \( \cos (\phi_o - \phi_s) \) which, in turn, depends on the variation
of \( (\phi_o - \phi_s) \) across the aperture. It is reasonable to assume
that this variation is Gaussian, in which case the average signal is proportional to
\[ e^{-1/2 (\phi_o - \phi_s)^2} \]
But \((\phi_o - \phi_s)^2\) is simply the square of the rms phase error. The phase error is related to the figure error by
\[ \phi_o - \phi_s = 2 (2\pi\delta) \]
where \(\delta\) is the figure error in waves. The heterodyne efficiency thus becomes
\[ e^{-1/2(16 \pi^2 \delta^2)} = 1 - 8\pi^2 \delta^2. \]
For an efficiency of 95 percent, the above expression yields a value for \(\delta\) of 1/40 wave rms.

The above analysis considers only the effects of a single surface, whereas the actual system involves many surfaces. It is thus apparent that 1/50 wave rms remains a realistic specification of surface figure requirements.

An rms error of 1/50\(\lambda\) at \(\lambda = 10.6\) microns corresponds to about 1/3\(\lambda\) in the visible region. Attaining this accuracy in any of the optics other than the primary mirror presents no problem of consequence. Even in the case of the 120-inch primary mirror, 1/3\(\lambda\) rms is not a major figuring problem, since such a value is typical of modern astronomical mirrors of large diameter.
It is apparent, however, that maintenance of the desired surface accuracy requires very careful attention to the manner in which the secondary and especially the primary mirrors are supported and located. The importance of this aspect of the design is further emphasized by the requirement for the ultimate in absolute pointing accuracy. By way of illustrating the nature of the problem, consider a tilt of the primary mirror of 0.001 inch across its 120-inch diameter. In angular terms, this is a tilt of nearly 1.7 seconds of arc. Because of the 2:1 magnification inherent in mirrors, the resulting pointing error is 3.4 seconds of arc.

Figures VI-1, VI-2, VI-3, and VI-4 illustrate the geometric derivation of the pointing error resulting from tilts or lateral translations of the primary and secondary mirrors. These derivations assume that the focal plane (detector) remains fixed with respect to the elevation axis, and that deflections of the folding mirror and other optics are negligible. In each instance, the primary mirror forms an image, which, in turn, acts as the object for the secondary mirror. A straight line passing from this point through the vertex of the secondary mirror then defines the location of the final image in the focal plane. Note that tilts of the secondary do not introduce pointing errors and that lateral displacements of the two mirrors in the same directions introduce pointing errors which are of opposite sign and hence at least partially cancel.

The considerations enumerated in the above paragraphs make it clear that, if an instrument capable of diffraction-limited performance at 10.6 microns and absolute pointing accuracy within seconds of arc is to be achieved, the utmost care must be exercised in the design of the mirrors and their mountings.
\[ d = D_e N_p \]
\[ f = D_e N_o \]
\[ \text{EFL} = D_p N_2 \]
\[ N = \frac{f}{n_0}. \]

**Defining Positive Displacements Downwards, the Image Displacement Due to a Lateral Secondary Displacement \( \delta_s \) is**

\[ e = -\delta_s \left( \frac{N_2}{N_1} - 1 \right) \]

**In Angular Terms,**

\[ \alpha = -\delta_s \left( \frac{N_2}{N_1} - 1 \right) = -\frac{\delta_s}{D_p} \left( \frac{1}{N_p} - \frac{1}{N_1} \right) \]

---

**Example:**

\[ D_e = 120 \text{ in}, \quad N_p = 2.8, \]

\[ \begin{array}{c|c|c}
   N_2 & 11.5 & 2.8 \\
   \delta_s & -4.5 \delta_s \text{ sec} & -53.7 \delta_s \text{ sec}
\end{array} \]

(\( \delta_s \) in inches)
LATERAL DISPLACEMENT OF PRIMARY MIRROR

\[ e = \delta_p \left( \frac{N_3}{N_r} \right) \]

\[ \frac{e}{EFL} = \frac{\delta_p}{D_p N_r} \]

For \( D_p = 120 \) in, \( N_r = 2.5 \),

\[ \frac{e}{EFL} = 614 \frac{\delta_p}{EFL} \]

\( \delta_p \) in inches.
(b) Primary Mirror

Consider first the primary mirror, since it is certainly the most critical. Choice of the material to be employed is strongly influenced by the fact that the mirror must be figured to excellent astronomical quality, i.e., $1/3\lambda$ rms in the visible, and must maintain that figure at all elevation angles and in daytime as well as nighttime operation. The requirement for day-night operation inevitably means thermal perturbations considerably more severe than those encountered by the typical astronomical instrument. On this basis, the only sensible choice among the traditional mirror blank materials is fused silica, with its low coefficient of thermal expansion. However, it may well be that by the time fabrication of the optical communications system is actually undertaken, sufficient experience with and confidence in some of the new "zero coefficient" ceramic glasses will have been accumulated to make them competitive with fused silica for blanks of the size herein contemplated.

For purposes of this study, a solid 120-inch diameter mirror with an average thickness of 18 inches and a total weight of 16,200 pounds has been selected. The 6.7:1 aspect ratio is conventional in terms of comparison with existing large astronomical mirrors. It is entirely possible that detailed analysis accompanying an actual design might suggest an alternative to a solid disc, e.g., a ribbed structure to improve thermal response time characteristics. This possibility has not been pursued in detail as part of this study because it is not a significant factor in the overall system concept or in the basic feasibility of the instrument.
The mount for the primary mirror, as previously indicated, must perform two distinct functions:

(1) It must provide means for reacting the gravity loads on the mirror in a manner which reduces the resulting distortion to a level consistent with the required figure tolerance.

(2) It must provide means for precisely defining the position of the mirror with respect to the rest of the system.

The attempt to implement these fundamentally different functions with common hardware is perhaps the most common shortcoming of support systems for large mirrors. It is strongly recommended that, particularly for the case herein considered where the positioning requirements are so critical, the two functions of load reaction and position definition be handled separately and independently to the greatest degree practical.

Figure VI-5 illustrates a possible primary cell configuration which conforms with the above recommendations. The component of weight acting normal to the mirror's axis is reacted in the plane of the mirror's center of gravity by a circumferential flexible tube partially filled with mercury. The level of the mercury is adjusted with the mirror on edge until the resultant of the pressure forces applied to the mirror exactly balances its weight. Once so adjusted, the magnitude of the supporting forces are automatically self adjusting for any elevation angle of the system. This approach was originated by Boller and Chivens and has proven highly successful in a
Pressurized Chamber for Axial Support

Mercury-Filled Support Tube

Spherical Bearing

Locating Link with Load Limit Springs

Bracket

Insulating Support Ring

$\text{SiO}_2$ Primary Mirror

Aft Cover

Padded Support Block

Primary Cell - Structural Ring

Nominal Cell Cross Section 21-1/2" x 12" x 1" Wall

$I = 1475 \text{ IN}^4 \quad A = 61 \text{ IN}^2$

Mean Dia. = 136 In.

1/4 Scale

Figure VI-5. Primary Mirror Cell
number of applications. In addition to its simplicity, the concept has the advantage of a relatively favorable distribution of edge forces. As shown, an insulating material between the mercury-filled tube and the cell wall is recommended to minimize the possibly deleterious effect of a good conduction path between the two.

The gravity component parallel to the axis of the mirror is reacted by air pressure on the rear surface. The magnitude of the pressure is controlled as a function of elevation angle to exactly balance the axial component at all times. This approach is felt to have significant advantages over the conventional use of multiple mechanical counterweights. Counterweight systems, while simple in concept, are notoriously complex, difficult to adjust, and sensitive to bearing stiction problems in actual application. Pneumatic support suffers from none of these drawbacks and provides a precisely uniform distribution of support loads over the back of the mirror.

The latter characteristic has one implication which influences the design of the mirror blank: uniform support pressure is strictly correct only in the case of uniform mass distribution across the mirror. Obviously, if the mirror is concave on the front and flat on the back, the mass distribution is not uniform. An approximate computation of the magnitude of the resulting deflection for the f/2.8 mirror herein considered yields a result of 1/3 wavelength in the visible. Although the nature of this deflection is such that it could be largely compensated by a slight focus change, the problem could be eliminated by making the rear surface of the blank with a convex radius such that an optimum balance between mass distribution and air pressure is obtained.
The seal necessary to close the air chamber required to pressure-support the mirror could be accomplished either by utilizing a flexible membrane across the back of the mirror or by allowing the mercury-filled edge-support tube to act also as a peripheral air seal. The maximum pressure differential required for an 18-inch thick fused silica mirror is 1.44 psi.

The mercury-filled tube and the air pressure system balance the weight of the mirror but do not define its position. Locating elements must thus be added to the system. These elements must control the axial and lateral positions and the tilts of the mirror, and should do so in a non-redundant manner to prevent the introduction of unwanted constraining forces. They should also permit relative thermal expansions of the cell and the mirror while maintaining concentricity of the two.

The locating elements shown in Figure VI-5 consist of three links parallel to the axis of the mirror, equally spaced around its periphery, and connecting attachment points on the mirror to brackets on the structural ring of the cell. Each link connects to its mirror attachment point through a spherical bearing and to the cell bracket through a widely spaced pair of bearings, permitting rotation only about a tangential axis (normal to the plane of Figure VI-5). The result is that, at each mirror attachment point, translation in a radial direction is permitted, introduction of bending moments is prevented, and axial or tangential displacements are prevented. This combination of constraints satisfies all of the requirements for properly defining the position of the mirror.
With proper adjustment of the mercury level in the peripheral tube and the air pressure behind the mirror, the positioning links will not carry any loads and hence will not undergo any deflections. In fact, an interesting possibility is the instrumentation of the positioning links to measure transmitted forces, e.g., with strain gages, and the nulling of those forces by controlling the mercury level and/or the air pressure with a simple servo loop.

This system circumvents the near-impossibility of defining the position of the mirror within a small fraction of one thousandth of an inch by means of elements which must also carry a share (even a small percentage) of the gravity load. The latter approach frequently succeeds in less critical applications only because structural symmetry leads to deflections of the axial locating points which are approximately the same, so that, while appreciable translation occurs, tilts are minimized.

A major benefit of the approach herein described is the insensitivity of mirror alignment to deflections of the structure closing the aft end of the telescope tube. As a result, this structure can consist of a simple, relatively flexible plate or bulkhead rather than the massive, rigid assembly otherwise required.

(c) Secondary Mirror

Considerations affecting the design of the secondary mirror and its support system are much the same as those for the primary. Although the problems must be treated with equal care, the 4:1 reduction in diameter appreciably reduces the mechanical difficulty of effecting appropriate solutions. For this reason, the following discussion of the secondary mirror is in terms of concepts rather than mechanical detail.
As with the primary mirror, material selection is influenced by the thermal environments inherent in day-night operation of the instrument. The most promising candidates, once again, are fused silica and the new "zero expansion" ceramic glasses. The latter alternative is an especially attractive one in this instance because good results are being obtained with some of these materials in mirrors comparable in size to the proposed secondary.

The secondary mirror could be either a homogeneous solid disc or a relatively lightweight sandwich structure. The sandwich configuration would offer a weight reduction in that portion of the instrument where weight is of most concern, and would probably reduce somewhat the static bending deflections of the mirror. These benefits would be countered by increased costs and the added complications associated with fabrication of the desired double-sided mirror with Cassegrain and coudé surfaces on the same blank.

For either a solid or a sandwich secondary, however, the deflections on a simple edge support would be excessive. Consequently, a support system similar to that described for the primary is recommended, with, in this case, a partial vacuum applied to the back of the mirror to balance the axial component of gravity. To permit rotation of the mirror from the Cassegrain to the coudé position, the vacuum chamber could include a hinged door.

Since central location of the secondary is not practical, locating elements consisting of links permitting only radial motion at three edge points are again recommended.
B. COMMUNICATIONS SYSTEM OPTICS

1. General Description

The complete optical system for the proposed instrument is shown schematically in Figure VI-6 and as a tentative mechanical configuration in Figure VI-7.

The basic telescope is the parabolic-hyperbolic Cassegrain design described in the preceding section. A flat mirror, rigidly mounted in the telescope tube, folds the downcoming f/11.5 beam of 10.6 micron light out through the elevation axis to a second smaller flat which folds the beam into a plane parallel to the exterior surface of the fork upright. This second folding flat and the rest of the communications optics are mounted on a rigid, dimensionally stable plate which, in turn, is kinematically attached to the fork upright.

The f/11.5 beam converges to a focal point and continues as a diverging f/11.5 beam to a collimating paraboloid whose focal point coincides with that of the Cassegrain telescope. The reflected collimated beam returns towards the original focal plane, where it is intercepted by a third folding flat. This flat has a central hole to permit passage of the f/11.5 beam, and is mounted on a servo-controlled two-axis gimbal, controlled in such a manner that, upon reflection from the flat, the collimated beam is propagated in a fixed direction. This gimballed mirror constitutes the fine guidance element of the system.
NO. 3 FOLDING FLAT

2 SIDED CASSEGRAIN/COUDE SECONDARY MIRROR

FOLDING FLAT INDEXES OR SWINGS AWAY

OPTIONAL f/11.5 CASSEGRAIN FOCUS

NO. 4 COUDE FLAT

-120" DIA f/2.8 PRIMARY MIRROR

FINE TRACKING MIRROR ON SERVO-CONTROLLED 2 AXIS GIMBAL

COLLIMATING PARABOLOID

NO. 5 COUDE FLAT

NO. 6 COUDE FLAT

f/24 COUDE FOCUS

SEGMENTED REFLECTIVE CHOPPERS

PHOTOMULTIPLIER TUBES

VISIBILE
Figure VI-6. 120-Inch Aperture Communications Receiver Optical Schematic
Figure VI-7. Tentative Mechanical Configuration of Communications Equipment
The next component in the system is another flat mirror, this one capable of rotating into or out of the beam in a precise manner. When this mirror is in the collimated beam, it redirects the beam through a filter and converging optics to a visible-spectrum star tracker. This tracker enables boresighting of the equipment and calibration of the encoders on stars of known position.

With the rotatable mirror withdrawn from the collimated beam, the next element in the path is a beamsplitter. The design of the beamsplitter is such that 10 percent of the 10.6 micron energy is diverted through converging optics to an infrared tracker. The remainder of the energy passes through the beamsplitter, which is also designed for maximum transmission of the 4880 angstrom upgoing beacon.

The 10.6 micron beam transmitted by the first beamsplitter then encounters and is transmitted by a second specially coated dichroic beamsplitter/mirror designed for maximum transmission at 10.6 microns and maximum reflectivity at 4880Å. At this point, the beam is combined with a properly polarized local oscillator beam from a CO₂ laser, reflected from the second beamsplitter as shown. (The efficiency of this reflection is low, but available local oscillator power is more than ample.) The combined beam is then focused by converging optics onto the heterodyne detector.

The dichroic beamsplitter also serves to introduce the upgoing 4880Å beam, as illustrated. The argon laser beam passes through a beam expander, through a pair of Risley prisms which permit introduction of the required point-ahead angle, and thence off the beamsplitter and back along the same optical path employed for the downcoming 10.6 micron beam.
2. **Collimating Paraboloid**

Conversion to a collimated beam for passage through the communications optics was selected as a desirable alternative to traversing these optics with a converging beam direct from the telescope. Factors influencing this selection included the following:

(a) Use of a collimated beam avoids the introduction of defocusing effects by rotations of the fine tracking mirror.

(b) Use of a collimated beam avoids potential problems with varying angles of incidence on the specially designed beamsplitters, as well as with spherical aberrations introduced by plane-parallel transmitting elements.

(c) Field angle, detector size, and baffling considerations would require additional optics at each tracker and at the heterodyne detector in any event.

(d) Utilization of a collimating paraboloid permits cancellation of coma, the predominant aberration of the Cassegrain telescope.

(e) The collimated beam provides maximum flexibility for system modification.
The size of the collimating paraboloid, and hence diameter of the collimated beam, were determined in the following manner.

(a) The obscuration ratio (i.e., the ratio of obscured diameter to aperture diameter) for the Cassegrain telescope is 30:120 or 1:4.

(b) Consideration of operating techniques and characteristics of the instrument leads to a desire for a Cassegrain field of view 2 arc-minutes in diameter. At f/11.5 and with a 120-inch aperture, this corresponds to a linear diameter of 0.8 inch in the focal plane.

(c) It is thus apparent that the required diameter of the hole through the fine tracking mirror, and hence of the minimum obscuration in the collimated beam, is on the order of 1.0 inch.

(d) In order to match the obscuration ratio of the collimated beam to that of the telescope (1:4), the diameter of the collimated beam must be 4.0 inches.

Since the paraboloid must be f/11.5 to match the telescope, its focal length is thus specified as 4.0 x 11.5 = 46.0 inches.
It is now appropriate to consider the effects of coma on the off-axis imagery of the proposed system. An important characteristic of the parabolic-hyperbolic telescope in this regard is that the angular length of the comatic image for such an instrument is identical to that for a paraboloid with the same final f-number. Multiplication of the angular image size by focal length yields the linear size of the astigmatic image, the formula for which may be written as

$$\Delta l_c = \frac{K \beta F}{N^2}$$

where $K$ is a constant, $\beta = \text{angular distance off axis}$, $F = \text{focal length}$, and $N = \text{f-number}$. It is furthermore apparent that, with a 30:1 ratio of telescope aperture diameter to paraboloid diameter,

$$F_{\text{Telescope}} = 30 F_{\text{Paraboloid}}$$

and

$$\beta_{\text{Telescope}} = \frac{1}{30} \beta_{\text{Paraboloid}}.$$  

This shows that $BF$ and hence $\Delta l_c$ are constant for the two cases. In other words, a plane wavefront incident 30 arc-minute off-axis on the 4.0-inch paraboloid would form exactly the same comatic image the same linear distance off-axis as that formed by a plane wavefront incident 1.0 arc-minute off-axis on the 120-inch aperture telescope. Applying the principle of reversibility to the foregoing, it is evident that a plane wavefront incident 1.0 arc-minute off-axis on the telescope will form a comatic image, that this image will become the object for the 4.0-inch paraboloid, and that a coma-free plane wavefront will emerge 30 arc-minute off-axis from the paraboloid. As previously indicated,
this cancellation of coma is the principal reason for selecting a parabolic-hyperbolic telescope in preference to a Ritchey-Chretien design.

3. **Fine Tracking Mirror**

The fine tracking mirror is a folding flat with an elliptical clear aperture and an inclined 1.0-inch diameter central hole, as indicated above. The mirror is mounted on a two-axis gimbal system controlled by error signals from either the visible or the infrared tracker.

In order to accommodate the 2.0 arc-minute diameter field of view at the Cassegrain focus, the gimballed fine tracking mirror must have sufficient excursion capability to correct ±30 arc-minute of deviation in the collimated beam. The values of these excursions depend on the selected orientations of the gimbal axes and upon the angle of incidence of the collimated beam. For the tentative configuration shown in Figure VI-7, the angle of incidence is near enough to normal so that, if both gimbal axes are in the plane of the mirror, the required mirror tilts will be roughly half of the beam deviations or ±15 minutes of arc.

Similarly, it is evident that tracking to 1/5 arc-second, for example, requires positioning of the fine tracking mirror (by the servo loop, and not with respect to any absolute reference) to 3 arc-seconds. This is not an unreasonable ratio in terms of practical hardware.

4. **Beamsplitters**

The first beamsplitter encountered by the downcoming 10.6 micron beam is that required to divert 10 percent of the energy to the infrared tracker. The requirements for this element are, plainly, for 10 percent reflectivity at 10.6 microns, as close as possible to 90 percent transmission at 10.6 microns,
and maximum transmission at 4880Å for the upgoing beam. This combination of properties does not present any unusual problems, and the fabrication of such a beamsplitter is considered to be a straightforward application of existing techniques. On this basis, detailed design of such a beamsplitter, leading to optimized multilayer coatings on an appropriate substrate (perhaps barium fluoride), has not been undertaken as part of this study.

In contrast, the second beamsplitter is less straightforward. Its requirements are for maximum transmission at 10.6 microns and maximum reflection at 4880Å. Since it was not immediately obvious that this combination of properties was amenable to practical solution, a tentative multilayer coating was designed and applied to small samples of Irtran II and barium fluoride. The barium fluoride sample proved slightly the better of the two. The reflectivity at 4880Å was virtually 100 percent, and the transmission at 10.6 microns, after correction for a 4 percent loss at the uncoated surface, was 86 percent. These results are considered ample demonstration of the feasibility of such a beamsplitter. It is also possible that further development could increase the transmission at 10.6 microns over the 86 percent obtained in the initial trial.

The high transmission at 10.6 microns would, at first glance, seem incompatible with the requirement for reflection of the 10.6 micron beam from the local oscillator. This apparent paradox is circumvented by the fact that, in terms of the power required for heterodyning with the received signal, the power available from the local oscillator is virtually unlimited. Consequently, a few percent reflectance at 10.6 microns is ample for introduction of the local oscillator beam.
5. \textbf{CO}_2 Local Oscillator

The local oscillator required for optical heterodyning with the received signal is provided by a relatively low power, frequency controlled, \textit{CO}_2\textit{ laser.}

The plane polarized output beam from this laser passes through a polarization control device (to be discussed in a subsequent section) and thence through beam expanding optics to the second beamsplitter.

Frequency control and stability of the local oscillator is, of course, a major problem and one which is the subject of concerted study by a number of investigators. The details of these studies are beyond the scope of this report, but omission of further discussion of the problem herein should not be interpreted as de-emphasis of its importance.

6. \textbf{Argon Beacon}

The energy for the upgoing beacon is provided by an argon laser operating at 4880Å. As with the local oscillator, the output beam from the laser first passes through beam expanding optics to match the 4.0-inch diameter of the collimating paraboloid.

The upgoing beacon must, in general, be offset with respect to the downcoming beacon to account for the velocity of the spacecraft relative to the ground station. Capability of accomplishing this offset has been provided by the inclusion of adjustable Risley prisms in the upgoing beam, as shown in Figure VI-6. These prisms are simply plano-plano transmitting elements with small but identical wedge angles. The resulting deviation of the transmitted
beam thus varies from zero when the thick side of one prism is in line with the 
thin side of the other, to a maximum value when the two thick sides are in line. 
Relative rotation of the two prisms thus controls the magnitude of the offset 
angle, while rotation of both prisms as a unit controls the angular orientation 
of the offset around the line of sight. The maximum anticipated value of the 
offset angle, as indicated elsewhere, is on the order of 36 minutes of arc.

Both the magnitude and the orientation of the offset angle are, of 
course, time varying functions, and the Risley prisms must therefore be con-
tinuously readjusted. Fortunately, however, the required rotational positioning 
accuracy for the prisms is on the order of degrees rather than seconds, so that 
the mechanical aspects of this adjustment are not demanding.

Because of the angular beam spread required for the upgoing beacon 
tentatively 10 seconds of arc), the argon laser beam must be deliberately 
defocused. The most convenient method for accomplishing this defocusing is 
alx motion of one element in the beam expanding optics used to increase the 
diameter of the argon laser beam.

The argon laser itself represents a significant development problem, 
with the principal objectives being attainment of the required output power and 
effective handling of the cooling requirements. As indicated elsewhere in this 
report, pulsed operation of the argon laser offers potential advantages. Again 
however, detailed study of laser development possibilities is beyond the im-
mediate scope of this report.
7. **Trackers and Heterodyne Detector**

The communications system includes a visible spectrum tracker for acquisition, calibration, and boresight purposes, and a 10.6 micron tracker for active tracking of the spacecraft transmitter.

The basic concept of these trackers will be the same. Light from the target is focused on the apex of a four-sided image divider and split into four bundles. The intensities of these four bundles are measured and compared to each other. The pointing error is considered to be zero when all four bundles contain the same amount of energy.

Figure VI-6 includes schematic diagrams of the two trackers. For clarity, only two of the four beams are shown in each case. The light reflected from the four sides of the image divider impinges on four elliptical mirrors and is refocused at the detectors. The four beams are intersected by a rotating reticle which contains an odd number of reflecting sectors and an odd number of clear sectors. Each of the four cones is either transmitted to detector No. 1 or reflected to detector No. 2. A pickup on the chopping reticle is used to generate a demodulator reference signal.

Figure VI-8 shows an electrical block diagram of the system. Figure VI-9 shows typical waveforms from the detectors and the modes of processing in the case of a chopper with 60-degree segments. Note that a complete failure of one detector would not cause a failure of the system, but only a signal-to-noise degradation of 3 db. By similar reasoning, sensitivity differences in the two detectors would not result in a systematic error.
Figure VI-8. Electrical Block Diagram of Trackers
Figure VI-9. Typical Waveforms from Detectors
As indicated schematically, the detectors in the visible tracker are photomultiplier tubes and those in the 10.6 micron tracker are cryogenic solid state sensors, probably mercury-doped germanium. In the latter case, the dewar employed to cool the detector would also enclose appropriate baffles, filters, and re-imaging optics designed to minimize background inputs originating in the optical system itself. Appendix B considers some of the characteristics of appropriate infrared detectors.

Proposed field-of-view diameters for the visible and 10.6 micron trackers are 1.0 minute of arc and 10 seconds of arc, respectively. The one minute field for the visible tracker was selected to provide ample latitude for rapid acquisition of reference stars. The 10-second infrared field represents a tradeoff between background noise and the absolute pointing error accumulated in the time interval between encoder calibration with a reference star and acquisition of the spacecraft beacon.

The heterodyne detector would be similar to those employed in the 10.6 micron tracker, i.e., a cryogenically cooled solid-state device enclosed with re-imaging optics and filters in an appropriately baffled dewar. The field of view in this instance has been tentatively specified as one second of arc, based primarily on background noise considerations. The field of view is determined by the size of the detector and by the f-number of the beam incident on the detector. A probable value for detector size is 0.3mm, for which a one arc-second field of view requires a final f-number of approximately f/20.
8. **Polarization Considerations**

The CO$_2$ laser aboard the spacecraft emits linearly polarized light. If nothing is done in the transmitter to alter the state of polarization, the signal arriving at the ground receiver will also consist essentially of linearly polarized light. The plane of polarization will, in general, rotate with time about the optical axis of the ground receiver. The position of the plane of polarization will be calculable at any time from the known attitude of the spacecraft with respect to the ground station.

Upon transmission through the ground station telescope and the communications optical system, the signal will, in general, arrive at the heterodyne detector in an elliptically polarized state. The change from linear to elliptical polarization results from differential phase shifts upon reflection from the various surfaces. A portion of these shifts will occur at the curved surfaces, where the angle of incidence varies somewhat with radius, but the major effect will be introduced by the various folding flats and at the beam-splitters, where the angles of incidence are relatively large. The degree and orientation of the ellipticity will be functions of azimuth and elevation angles, i.e., of time. The following paragraphs consider quantitatively the degree of ellipticity resulting from reflection by the six aluminized mirrors in the proposed system.

If light is reflected from a metal at other than normal incidence, the p and s components of any electric vector will be reflected with a phase difference. In the case of incident light which is linearly polarized, the reflected light will generally be elliptically polarized.
If the electric vector of linearly polarized light vibrates in the plane of incidence or perpendicular to it, the reflected light will be linearly polarized.

The amplitude reflectances $r_p$ and $r_s$ may be found from the following equations:

$$r_s = \left[ R_s \right]^{1/2} = \left[ \frac{(\cos \theta - n)^2 + k^2}{(\cos \theta + n)^2 + k^2} \right]^{1/2}$$

$$r_p = \left[ R_p \right]^{1/2} = \left[ R_s \tan^2 \psi \right]^{1/2}$$

where

$$\tan^2 \psi = \frac{\left[ n \cos \theta - \sin^2 \theta + k^2 \cos^2 \theta \right]^2 + k^2 \cos^2 \theta}{\left[ n \cos \theta + \sin^2 \theta + k^2 \cos^2 \theta \right]^2 + k^2 \cos^2 \theta}$$

and $R_s$ is the intensity reflection coefficient for the $s$ component, $R_p$ the intensity reflection coefficient for the $p$ component, $\theta$ the angle of incidence, $n$ the index of refraction of the metal, $k$ the absorption coefficient of the metal, and $\psi$ the angle the direction of vibration in the reflected ray makes with respect to the normal of the plane of incidence.

The phase difference $\Delta$ between the $p$ and $s$ components is given by the following relation.

$$\tan \Delta = \frac{-2k \sin \theta \tan \theta}{\left[ n^2 + k^2 - \tan^2 \theta \right]}$$

The complex index of refraction of a metal is given by $N = n - ik$ where $k$ is the absorption coefficient. The American Institute of Physics Handbook\(^3\) gives

the values of \( n \) and \( k \) for evaporated aluminum at 10 microns as \( n = 26 \) and \( k = 67.3 \).

Figure VI-10 shows the system under consideration. The maximum angles of incidence on \( M_1, M_2, \) and \( M_5 \) are 5, 6.9, and 1.3 degrees, respectively. The angle of incidence on the two plane mirrors varies between 42.5 and 47.5 degrees. The angle of incidence on \( M_6 \) is 45 degrees.

Substituting these angles of incidence and material properties into equations (1) through (4) yields the corresponding reflectances and phase shifts. Table VI-1 shows the values obtained for aluminized mirrors at 10 microns for angles of incidence of 0, 6.9, 42.5, and 47.5 degrees.

<table>
<thead>
<tr>
<th>( \theta ) (degrees)</th>
<th>( \tan^2 \psi )</th>
<th>( R_s )</th>
<th>( R_p )</th>
<th>( \Delta ) (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>0.980</td>
<td>0.980</td>
<td>0.0</td>
</tr>
<tr>
<td>6.9</td>
<td>1</td>
<td>0.980</td>
<td>0.980</td>
<td>0.02</td>
</tr>
<tr>
<td>42.5</td>
<td>0.988</td>
<td>0.985</td>
<td>0.973</td>
<td>0.92</td>
</tr>
<tr>
<td>47.5</td>
<td>0.984</td>
<td>0.986</td>
<td>0.970</td>
<td>1.19</td>
</tr>
</tbody>
</table>

From the results in Table VI-1 it is seen that the total phase difference introduced into linearly polarized light after reflection from the six mirrors will be about 3 degrees. The corresponding degree of ellipticity is small, and for all practical purposes the reflected beam may be considered to be linearly polarized.
Figure VI-10. Reflective Portion of Optical System
If the differential phase lag introduced by the beamsplitters is similarly small, the signal energy incident on the detector will be essentially linearly polarized, but with the orientation of the plane of polarization varying with time, as previously noted. Since efficient heterodyning requires alignment of the plane of polarization of the local oscillator beam with that of the received signal, some provision for rotating the plane of polarization of the local oscillator is required. Mechanical rotation of the laser itself is inconsistent with alignment tolerances, so that the logical solution is a rotatable half-wave plate in the output beam. Rotation of such a plate through an angle \( \theta \) rotates the plane of polarization through an angle \( 2\theta \). Rotation of such a device would be controlled by the digital computer on the basis of system geometry computations.

If, on the other hand, the effect of the beamsplitters is such that a significant degree of ellipticity results, efficient heterodyning requires that the local oscillator be elliptically polarized to match the received signal. This requirement can be met by addition of a rotatable quarter-wave plate to the system. The combination of a half-wave plate and a quarter-wave plate permits matching any degree and orientation of right- or left-handed elliptical polarization.

The polarizing effect of the beamsplitters is highly dependent on the exact nature of the coatings employed. Determination of the need for the quarter-wave plate must thus await final design of those coatings.
Implementation of half-wave or quarter-wave plates at 10.6 microns poses some problems. Fortunately, however, the power available from the local oscillator presents no difficulty, so that the transmission of the retarding plates can be quite low. On this basis, several candidate birefringent materials are available. Alternatively, the use of Fresnel rhombs in a material such as germanium appears feasible.

An interesting approach which could lead to system simplification is the transmission of circularly polarized rather than linearly polarized light by the spacecraft transmitter. This would eliminate any time variation of the state of polarization with respect to the telescope. However, conversion from linear to circular polarization on the spacecraft again requires a 10.6 micron quarter-wave plate or the equivalent, and this could lead to an untenable energy loss. It is possible that the modulator, a key development item for the spacecraft, will incidentally convert to circular polarization. In any event, it is felt that the decision between linear and circular polarization is a system decision rather than a ground station decision.

9. Mechanical Configuration

Figure VI-7 illustrates a plausible physical arrangement of the various communications system components described in the preceding sections. The configuration shown attempts reasonable estimates of sizes of key components such as the argon laser, and represents an emphasis on compactness and compatibility with the rest of the system. The system shown should not be interpreted as a detailed or optimized design, since the development status in a number of areas previously indicated makes extensive mechanical design effort rather pointless.
As shown, the various components of the communications system are attached to a common mounting plate. This plate, in turn, is attached kinematically, i.e., in a nonredundant manner, to the upright of the mount fork. The design of the mounting plate would emphasize rigidity and maximum dimensional stability to assure continued optical alignment. Use of a low-expansion material such as Invar should be considered to minimize sensitivity to thermal perturbations.

Collimation of the optical beam through the communications package and the use of a common mounting plate results in a system with a high degree of adaptability. Addition or substitution of individual components is not a major undertaking. If, for example, continued advances in laser technology should result in a desire for operation at a wavelength other than that for which the system was originally built, conversion would consist only of replacement of a few components such as the beamsplitters. The basic system concept and configuration would remain unchanged.

10. Optical Efficiency

The percentage of the collected energy transmitted to the heterodyne detector by the proposed system has been estimated as shown in the following table.
### TABLE VI-2

<table>
<thead>
<tr>
<th>Element</th>
<th>Percent of Transmission or Reflection at 10.6(\mu)</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Clear Aperture</td>
<td>91</td>
<td>36-inch dia. obscuration</td>
</tr>
<tr>
<td>Primary Mirror</td>
<td>98</td>
<td></td>
</tr>
<tr>
<td>Secondary Mirror</td>
<td>98</td>
<td></td>
</tr>
<tr>
<td>1st Folding Flat</td>
<td>97.8</td>
<td>Aluminized mirrors; refer to Section VI-B-8</td>
</tr>
<tr>
<td>2nd Folding Flat</td>
<td>97.8</td>
<td></td>
</tr>
<tr>
<td>Collimating Paraboloid</td>
<td>98</td>
<td></td>
</tr>
<tr>
<td>Fine Tracking Mirror</td>
<td>97.8</td>
<td></td>
</tr>
<tr>
<td>1st Beamsplitter</td>
<td>90</td>
<td>Refer to Section VI-B-4</td>
</tr>
<tr>
<td>2nd Beamsplitter</td>
<td>86</td>
<td></td>
</tr>
<tr>
<td>Imaging Lens</td>
<td>96</td>
<td>VLR Coated Germanium</td>
</tr>
<tr>
<td>Relay Lens</td>
<td>96</td>
<td></td>
</tr>
<tr>
<td><strong>System</strong></td>
<td><strong>57.</strong></td>
<td></td>
</tr>
</tbody>
</table>

The 57 percent efficiency provides some margin over the 50 percent value assumed in Section VIII. In each case, the specified efficiency excludes effects of the predetection filter.

11. **Boresighting Procedure**

It is essential that the various components of the communications optical system be properly boresighted. The following steps constitute a procedure whereby the initial boresighting can be accomplished to a high degree of precision. The procedure requires a bright star with emission at 10.6 microns as well as in the visible, a high precision retroreflector, and a mount for the swing-away mirror which allows that mirror to be accurately
translated in its plane. Reference is made to Figure VI-6, the optical schematic of the proposed system.

(a) With the swing-away mirror positioned to divert the beam to the visible tracker, acquire and initiate closed-loop tracking of the bright star.

(b) Rotate the swing-away mirror partially out of the beam, maintaining the visible tracking but allowing part of the energy to continue towards the first beamsplitter.

(c) Adjust the position of the 10.6 micron tracker to produce a null signal.

(d) Similarly, adjust the position of the optical heterodyne detector to give a peak signal.

(e) At this point, tracking of the star may be discontinued. The two trackers and the heterodyne detector are boresighted.

(f) Turn on the CO$_2$ local oscillator laser and adjust its position until a peak signal from the heterodyne detector is obtained.

(g) With the swing-away mirror still partially in the beam, position the precision retroreflector so that
a portion of the beam from the argon laser will bypass the swing-away mirror, enter the retro-reflector, and be redirected back to the swing-away mirror and thence to the visible tracker. Rotate the Risley prisms to their zero setting.

(h) Turn on the argon laser at a low power level and with appropriate filtration, and adjust its position to obtain a null signal from the visible tracker.

C. ASTRONOMICAL CONFIGURATIONS

1. Focal Positions

As indicated by Figure VI-6, the proposed system includes three focal positions appropriate for astronomical use. These focal positions are an f/2.8 prime focus, an f/11.5 folded Cassegrain focus, and an f/24 coude focus. This combination of foci permits a wide variety of astronomical applications, and results in an instrument which, in addition to serving as a communications station, would be an immensely powerful and versatile astronomical research tool.
As described elsewhere in this report, the f/2.8 prime focus would be implemented by swinging the Cassegrain/coudé secondary mirror out of the optical path and allowing the beam to reach the prime focal plane in the observer's cage. Correcting elements would be employed for wide field coverage.

Actually, it is felt that incorporation of prime focus capability should be carefully evaluated, and its inclusion in the preliminary design outlined by this report is not intended as a positive recommendation. The uncertainty about the inclusion of prime focus capability stems from the feeling among some segments of the astronomical community that, with an f/2.8 primary mirror and with the speed of modern photographic films, prime focus is of limited usefulness. The importance of careful evaluation of this point receives added impetus from the fact that inclusion of prime focus does, to some degree, compromise the basic communications function. Aspects of this compromise include the added weight and complexity of the secondary package, the resulting increase in system structural requirements and/or deflections, some increase in obscuration, and increased dome size requirements.

The selection of a folded Cassegrain focus rather than the typical hole-in-the primary configuration is based on requirements of the communications system. The focal position at the end of the elevation axis permits mounting of the communications optical system in a fixed position with respect to gravity. Additional benefits include the short and rigid fork uprights compared to those which would be required if a focal point behind the primary were to be utilized. The configuration also permits permanent, fixed mounting of the large folding flat in front of the primary.
Astronomical use of the folded Cassegrain system must thus accept the slight loss in efficiency introduced by the additional reflection off the folding mirror. Counteracting this minor shortcoming, however, is the considerable convenience of a stable, non-tilting work platform. Instruments employed at the Cassegrain focus need not resist the effects of a rotating gravity vector.

Access to the Cassegrain focus may be obtained either by swinging out of the path the small folding mirror which directs the beam to the communications optics, or, at the expense of an additional reflection, rotating that mirror about the elevation axis. In the first case, the optical axis at the focal plane would coincide with the elevation axis, while in the latter instance it could be rotated to any convenient position normal to the elevation axis. Figure VI-6 indicates both possibilities. In either case, a general purpose mounting surface on the fork upright would facilitate installation of instrumentation.

The coudé focus, in which an axial image point remains fixed with respect to ground, requires folding the beam out through the elevation axis and thence down through the azimuth axis to an observing position beneath the pedestal. Utilization of this configuration requires installation or indexing into the beam of a folding flat at a position somewhat outboard of the Cassegrain focal position, as shown in phantom on Figure VI-6. Because of the short length of the fork uprights and the limited fork clearance behind the aft end of the telescope, two additional folding mirrors are required to direct the beam down the azimuth axis.

2. Tracking

The classical guiding or tracking technique applied to astronomical telescopes consists of driving the instrument at a nominal sidereal rate around
a polar axis, and introducing manual corrections to keep a reference star centered on cross hairs in the presence of drive errors, atmospheric refraction variations, etc. The same technique could be employed with the instrument herein proposed, with the computer generating the required sidereal rate commands. However, in view of the capabilities provided for performance of the communications function, it would be almost unthinkable not to employ automatic star tracking for astronomical uses.

A visible star tracker similar in principle to that described for the communications system could be employed at any of the astronomical focal positions. Tracking accuracy would be substantially improved over manual guiding, particularly for relatively short period perturbations such as wavefront tilts.

The operational sequence would be similar to that for the communication operation. A bright star outside of field would be chosen as a guide star. The star tracker detector would be placed such that, when the guide star is centered on the detector, the object of interest is centered in the photographic field of view. The digital computer controller will compute the proper velocity and position of the azimuth and elevation gimbals such that the guide star is in the acquisition field of the star tracker. Once this track has been achieved, the system will automatically be switched to guide star operation.

Figure VI-11 plots signal-to-noise ratios for nighttime tracking of 10th and 12th magnitude stars with a 3-meter telescope.

Typical values applicable to the Cassegrain focus might be a 30 arc-minute photographic field surrounded by a 60 arc-minute diameter guide star field. If the guide star image is somewhat comatic or astigmatic, a constant systematic error results which has no effect on blur of the photographic image.
Figure VI-11. Signal-to-Noise Ratios for Nighttime Tracking with a 3-Meter Telescope
There is, of course, an important additional aspect of the astronomical tracking problem: with the alt-azimuth mount, the star field rotates with time about the optical axis. If uncorrected, this effect would destroy the telescope's utility in photographic work and requires some provision be made to rotate the photographic plate and the star tracker detector to compensate for the star field rotation. Fortunately the accuracy requirements of field rotation are much less severe than for pointing and tracking. If the total imaging field of view is \( \pm \theta_f \) and the field rotation error is \( \theta_e \), then a star at the edge of the field would move an amount equivalent to \( \theta_e \theta_f \).

For a \( \pm 15 \) minute field of view, for instance, a 2-minute field rotation would result in a translation at the edge of the field of 1 second of arc. For this reason, the rotation of the field may be controlled "open loop" by calculating the proper field rotation on a continuous basis with a digital computer and rotating the plate correspondingly.

At each focus, then, a mechanism would be required for positioning the photographic plate, positioning the star tracker sensor an adjustable distance from the optical axis, rotating the two as a unit to align the sensor with the desired guide star, and rotating the same assembly in response to computer commands for field rotation correction. Note that with the star tracker sensor rotating in conjunction with the photographic plate, the error signal will not be about the same axis at all times. A simple resolver-type coordinate transformation may be performed to generate the proper error signals. Since the system is nominally at null, a precise coordinate transformation is not required.

In the event of failure of the star tracker, the digital computer still has the capability of positioning the pedestal to the shaft
encoder accuracy.

Similarly, the initial acquisition could be performed with a sighting telescope and a "joystick" controller, but such an operation would be more time consuming.
SECTION VII

COMMUNICATIONS SYSTEM TRACKING SERVO

A. INTRODUCTION

The communication tracking system must be capable of tracking the satellite as it moves across the sky and, at the same time, tracking out the random wavefront tilts induced by the atmosphere.

These two requirements suggest the use of a dual piggy-back servo, consisting of a "fast" servo for tracking the low amplitude "high" frequency wavefront tilts, and a slower servo for tracking the spacecraft across the sky.

The following design realizes such a system, while also allowing for simple modification for astronomical use.

The velocities of the spacecraft with respect to inertial space are such that any system capable of tracking the spacecraft can similarly track any celestial object with only slight modification.

The following sections consider one practical design approach and estimate the servo system's performance in a communication mode.

Figure VII-1 shows the overall system and indicates how the system can be modified to become a versatile astronomical instrument.
Figure VII-1. Servo System Block Diagram
B. SERVO ANALYSIS

The difference between the desired position of the image at the coherent detector and the true position of the image is sensed by the photoelectric detector. The photoelectric detector in turn generates a command signal to move the fine tracking mirror so that the image remains at the desired position. The average position of the fine tracking mirror, in turn, is sensed and used to position the mount such that the fine tracking mirror remains nominally at the center of its range of travel. The basic block diagram is shown in Figure VII-1. Note that there are actually two ways in which the image error may be corrected:

1. By moving the fine tracking mirror, or
2. By moving the pedestal.

The system can correct for the higher frequency errors with the fine tracking mirror, and for the lower frequency long-term errors with the pedestal.

By making the response of the fine tracking mirror an order of magnitude faster than the pedestal gimbal, the two loops will be effectively decoupled.

The design parameter of the fine tracking mirror servo shall be chosen on the assumption that the pedestal remains fixed while the mirror is operative; while the design of the pedestal shall be based on the assumption that the fine tracking mirror position is an exact reproduction of the error. This is reasonable if the fine mirror response is an order of magnitude higher than the pedestal response.
1. Pedestal Gimbal Loop

The pedestal gimbal loop will be a torque motor servo loop with tachometer feedback. Torque motors were chosen for the gimbal drive because of their extreme smoothness over a wide range of speeds. The torque motors tentatively selected are commercially available 3000 pound-feet units (Inland Motor Corporation, Type T-36001).

The gimbal transfer function may be written as:

\[
\frac{\theta_p}{V_{in}} = \frac{A_G}{A_G K_T s + J s^2} = K_T s \left[ \frac{1}{1 + \frac{s}{J A_G K_T}} \right]
\]

where

- \( A_G \) is the amplifier-motor transfer gain in ft-lb/volt
- \( J \) is the moment of inertia in ft-lb-sec\(^2\)
- \( K_T \) is the tachometer scale factor volts/rad/sec

There are two adjustable parameters in this loop: \( K_T \) and \( J/A_G \). \( K_T \) is chosen as 1/1.5 and \( J/A_G K_T \) is chosen as 6.3. In this way, choosing \( V_{in} = \theta_{EM} \) where \( \theta_{EM} \) is the difference between the mirror position and the desired mirror position \( \theta_m \), the following result is obtained:

\[
\frac{\theta_p}{\theta_m} = \frac{1}{\left(1 + \frac{s}{3.84}\right)\left(1 + \frac{s}{2.46}\right)}
\]

This is equivalent to a slightly overdamped 1/2 Hz response.
2. The Fine Tracking Mirror Servo

The positional image error is sensed by the photoelectric detector and causes the mirror to move until that error is zero. The position of the mirror is sensed by a capacitor type pickoff, whose output controls repositioning of the pedestal.

Output of the mirror position sensor is also differentiated and used to provide (in a minor loop fashion) a rate damping signal for the fine tracking mirror loop.

A block diagram of this loop is shown in Figure VII-2.

The transfer function of this minor loop (capacitor pickoff loop) is given by:

\[
\frac{\theta_m}{V_d} = \frac{1}{s K_0} \frac{1 + \frac{s}{\omega_2}}{1 + \frac{J_m}{A K_0} s + \frac{J_m}{A K_0 \omega_2} s^2}
\]

where

- \( A \) is the servo amplifier and motor gain, ft-lb/volt
- \( J_m \) is the mirror inertia, slug-ft\(^2\)
- \( K_0 \) is the capacitor pickoff sensitivity, volts/radian
- \( \omega_2 \) is the reciprocal of the differentiator time constant, radian/second
- \( s \) is the Laplace operator
- \( V_d \) is an input driving voltage
Figure VII-2. Block Diagram
It is desirable to make $\omega_2$ as large as possible. It has been found that 750 is fairly readily attained.

The values of $A$ and $K_0$ are chosen to make the loop critically damped, which requires that

$$\frac{AK_0}{J} = \frac{\omega_2}{4}$$

Substituting this into the transfer function gives:

$$\frac{\theta_m}{V_d} = \frac{1}{sK_0} \frac{1 + \frac{s}{\omega_2}}{\left[1 + \frac{s}{\omega_2}\right]^2}$$

but

$$V_d = \left(\frac{K_oK_1}{s} \left[1 + \frac{s}{\omega_1}\right] \theta_e\right)$$

where

$K_oK_1$ is the integrator and transducer gain, volts/sec

$\omega_1$ is the ratio of integrator gain to proportional gain, sec$^{-1}$

$$\frac{\theta_m}{\theta_e} = K_1 \frac{\left[1 + \frac{s}{\omega_1}\right]}{s^2} \frac{1 + \frac{s}{\omega_2}}{\left[1 + \frac{s}{\omega_2}\right]^2}$$

It is desired that the loop response exceed 20 Hz and that the overall loop have a damping ratio of 0.7 of critical damping. In order that the lagging phase shift due to the integrator be small at 20 Hz, $\omega_1$ is chosen as 6.3
radians/second (1 Hz). The value of $K_o K_1/\omega_1$ is then chosen as 90 to give a damping ratio of 0.707 at 25 Hz. The response of the transfer lens servo loop (assuming the pedestal remains fixed and $\omega_2 = 750$ rad/sec).

\[
\frac{\theta_m}{\theta_{LOS}} = \left(1 + \frac{s}{530}\right) \left(1 + \frac{s}{110 + j110}\right) \left(1 + \frac{s}{110 - j110}\right)
\]

where $\theta_{LOS}$ is the line of sight angle.

3. **Combined Loops**

An expression for the angular error as a function of line of sight perturbation frequency may be obtained by drawing the entire loop and performing block diagram reduction.

\[
\frac{\theta_e}{\theta_{LOS}} = \frac{s^3 (1 + \frac{s}{375})^2}{9.35^3 (1+\frac{s}{4.6})(1+\frac{s}{2.3}) (1 + \frac{s}{110+j98}) (1 + \frac{s}{110-j98}) (1 + \frac{s}{506})}
\]

A Bode plot of the function is shown in Figure VII-3.

4. **Performance Estimates**

The dual servo loop described in the above paragraphs is subject to pointing errors arising from three sources: the line-of-sight rates incurred by the alt-azimuth mount, random wavefront tilts induced by atmospheric inhomogeneity, and system noise.
(a) Line-of-Sight Tracking

The servo design yields a type three system. As such, it has no steady-state positional, velocity, or acceleration error.

The line-of-sight input, however, is not a constant velocity or acceleration type. Figure VII-4 shows a typical required azimuth and elevation acceleration for a particular site location.

The nature of the acceleration is such that it may be approximated by an infinite series of sine waves in azimuth and cosine waves in elevation, since the inputs are respectively odd and even angular functions of time, where the time reference is the meridian crossover.

The nature of a type three servo is such that the error is proportional to the third derivative of the input (jerk) if the jerk does not contain frequency components above the first time constant of the servo loop.

For a given track, the worst inputs occur about the azimuth axis. Appendix C considers the angular rate problem in some detail. For a given site location, the worst condition occurs when the target declination is equal to the site latitude. Under this condition, both the azimuth and elevation input accelerations become infinite.

For a given slight difference between the target declination and the site latitude, the maximum acceleration is a function of site latitude and the worst case is a site located at the equator, i.e., zero latitude. Figure VII-5 shows how the maximum azimuth angle acceleration varies with declination.
Figure VII.4: Typical Required Gimbal Accelerations vs Time

- Elevation Acceleration (Divide Scale by 500)
- Azimuth Acceleration

Site Location: Equator
Target Declination: 6 Minutes of Arc

Time to Zenith Crossover - seconds
Figure VII-5. Maximum Azimuth Acceleration Versus (Latitude minus Declination) in Degrees.
for a site located at the equator. As the declination approaches the latitude, not only does the maximum acceleration increase but the equivalent frequency of the acceleration (the reciprocal of twice the time between the maxima of acceleration) also increases, as shown in Figure VII-6.

With a 3000 ft-lb azimuth torquer and an inertia about the azimuth axis of $2.2 \times 10^6$ in-lb-sec$^2$ at a 90 degree elevation angle (see Appendix A), the maximum available azimuth acceleration is $16.4 \times 10^{-3}$ radians/sec$^2$. Figure VII-5 shows that this level of acceleration corresponds to a target track passing within $\frac{1}{40}$ degree of zenith. For purposes of subsequent analysis, a factor of four has been applied to this number, i.e., the nominal "worst case" has been taken as a zenith miss of $\frac{1}{10}$ degree of 6 arc-minutes. This, in effect, represents a substantial derating of the azimuth torquer.

Figure VII-7 plots the third time derivative of azimuth angle for the case of a target passing within 6 arc-minutes of zenith. Calling the function $J(t)$, and since $J(t) = J(-t)$, the frequency content of this wave may be found from $F(\omega) = \int_{-\infty}^{\infty} J(t) \cos(\omega t) dt$. This integral was numerically evaluated and the results are shown in Figure VII-8. From the plot, it can be seen that the jerk has no components above 1 radian/second and the error will have the same form as the jerk, but reduced by $(9.35)^3$. On this basis, the peak servo error will be

$$\frac{144 \times 10^{-6}}{(9.35)^3} \text{ radian/second}^3 = 0.18 \times 10^{-6} \text{ radian}$$

$$= 0.036 \text{ arc-second}$$

The azimuth error due to tracking may, therefore, be neglected.
Figure VII-6. Equivalent Fundamental Frequency of Azimuth Acceleration versus Declination

Latitude = 0°
Latitude $= 0$
Declination $= 0.1^\circ$

Figure VII-7. Azimuth Jerk versus Time
Figure VII-8. Fourier Component of Azimuth Jerk Function Versus Angular Frequency
If the same basic controller is used in the elevation axis, the tracking error would be even less, since the elevation maximum inputs are smaller than the maximum azimuth inputs by two orders of magnitude.

The presence of bearing friction means that the servo loop will have to generate constant output force to overcome this friction. The error due to constant friction is the same as the error due to constant acceleration, which will be zero.

The elevation axis, however, will see a change in friction as the elevation velocity reverses at zenith crossover. However, at this time, the elevation acceleration is $3 \times 10^{-6}$ radians/second$^2$. Assuming that the elevation gimbal sticks for $1/2$ second at this time, the angular error, $\theta_e$, would be $\theta_e = \frac{1}{2} \alpha t^2$, where $\alpha$ is the required acceleration and $t$ is the time the gimbal stuck. The maximum error would then be $0.75 \times 10^{-6}$ radian or 0.15 second of arc. The actual error would be considerably less than this, since the transfer mirror time constant is much smaller than $1/2$ second, and would track the line-of-sight during this time.

(b) The Tracking of the Wavefront Tilt

The variations in the angle of arrival of the wave at the telescope are caused by the turbulent atmosphere between the receiver and the transmitter. The major contribution to this variation is made by the turbulent region extending from the receiving telescope to a few hundred meters above the receiving telescope.

The angle-of-arrival variation can be considered to be caused by "clumps" of atmosphere (called turbulons) moving normal to the line-of-sight.
with a velocity \( V_n \). Chase \(^4\) has shown that if the size of the turbulon is \( l_0 \), the line-of-sight will appear to oscillate at a rate equal to \( V_n/l_0 \) Hertz. If the turbulon is smaller than the receiving aperture, wavefront corrugations, rather than angle of arrival variation, will occur. It is only turbulons equal to or greater in size than the receiving aperture which will cause angle of arrival variations. The highest frequency component of the angle of arrival variation is then \( V_n/D_r \), where \( D_r \) is the receiving aperture diameter. For a wind velocity of 10 meters per second and a 3-meter aperture, good tracking capability is needed at 3.3 Hertz. The servo is capable of attenuating angle of arrival variation at 3.3 Hz by 16 db and will consequently be able to realize the increase in coherence diameter associated with perfect tracking.

(c) Noise Induced Error

The equivalent noise bandwidth of the system is 25.5 rad/sec. Then rms error will be

\[
T_e = \frac{L.R \sqrt{ENB}}{SNR_p}
\]

where

- \( T_e \) is the rms error
- \( L.R \) is the radius of the linear range of the servo detector
- \( ENB \) is the equivalent noise bandwidth
- \( SNR_p \) is the signal-to-noise ratio in a 1 Hz bandwidth

If the linear range is set at \( \pm 3 \sec \) and \( SNR_p = 95; T_e = 0.15 \sec \) rms,

This is smaller than the diffraction-limit (0.88 \( \sec \)) of the telescope at a wavelength of 10.6 microns.

SECTION VIII

COMMUNICATION SYSTEM PERFORMANCE ANALYSES

The basic optical configuration will be as shown in Figure VI-6.

The bundle of rays coming toward the detector is assumed to be controlled by a fine tracking mirror which directs the bundle of light to a 10-percent reflecting, 90-percent transmitting beam divider. The reflected light will be directed to an error-detecting pyramidal beamsplitter, while the remaining light will be directed toward the coherent detector.

The error detector will reposition the fine tracking mirror to maintain constant phase front for the rays transmitted toward the coherent detector.

The system parameters were chosen on the basis of tracking the laser with a 3-meter telescope and a 10 second of arc field of view. The power transmitted by the spacecraft is assumed to be 200 watts.

The following tables show the system parameters as well as the rationale for their selection for the cases of vehicles in the vicinity of Mars and Venus.

A calculation for the spacecraft's acquisition of an earth-based beacon is also included.

The calculated values for the power required for the earth-based beacon, such that the spacecraft may adequately acquire it (more than 90 watts)
are presently beyond the state of the art. The upper limit on argon power is approximately 20 watts. There are, however, other techniques which may be used to optimize the spacecraft system.

One method is to pulse the ground-based laser at some base frequency, $f_b$, with a duty cycle of $1/D$. If the same average power, $P_{sa}$, is maintained, the peak signal is $DP_{sa}$. The bandwidth necessary to recover this pulse is $Df_b$. If the spectral power density of the noise is $P_n$ watt (Hz)$^{-1/2}$, the peak-signal-to-rms noise would be

$$\frac{P_D}{P_n \sqrt{Df_b}} = \frac{P_{sa}}{P_n} \left( \frac{D}{f_b} \right)^{1/2}$$

If $D$ is larger than $f_b$, there will be a peak-signal-to-rms-noise enhancement. This may be increased still further by an exact knowledge of the pulse wave shape.

A delay line integrator$^5$ could be used, for instance, to increase the peak-to-rms ratio by a factor of 10 and effectively lowering the processor bandwidth to below $Df_b$.

A more detailed analysis of this technique is an area for further investigation.

### TABLE VIII-1
**SYSTEM PARAMETERS - TRACKING OF LASER VEHICLE NEAR MARS**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sky Background Radiant Intensity</td>
<td>$N_{\lambda B}$</td>
<td>$3.4 \text{ W m}^{-2} \text{ sr}^{-1} \mu^{-1}$</td>
<td>60-degree viewing. See &quot;Spectral Irradiance of Sky and Terrain,&quot; JOSA, 50, 12</td>
</tr>
<tr>
<td>Range</td>
<td>$R$</td>
<td>$1.5 \times 10^8 \text{ SM}$</td>
<td>Mars</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$= 2.3 \times 10^{11} \text{ M}$</td>
<td></td>
</tr>
<tr>
<td>Wavelength</td>
<td>$\lambda_T$</td>
<td>10.6 microns</td>
<td></td>
</tr>
<tr>
<td>Predetector Filter Bandwidth</td>
<td>$\Delta \lambda$</td>
<td>0.03 micron</td>
<td>Chosen for Maximum $\tau_f/\Delta \lambda$</td>
</tr>
<tr>
<td>Predetector Filter Transmission</td>
<td>$\tau_f$</td>
<td>0.6</td>
<td></td>
</tr>
<tr>
<td>Atmospheric Transmission</td>
<td>$\tau_a$</td>
<td>0.36</td>
<td>at 30° Elevation Angle</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>See Phase I Report (8393)</td>
</tr>
<tr>
<td>Optical Telescope Efficiency</td>
<td>$\tau_r$</td>
<td>0.50</td>
<td></td>
</tr>
<tr>
<td>Transmitter Efficiency</td>
<td>$\tau_t$</td>
<td>0.5</td>
<td>See NASA CR252</td>
</tr>
<tr>
<td>Beam Distribution Factor</td>
<td>$\tau_d$</td>
<td>0.5</td>
<td>Far field loss</td>
</tr>
<tr>
<td>Scintillation Factor</td>
<td>$\tau_s$</td>
<td>1.0</td>
<td>Chosen on the basis of aperture averaging</td>
</tr>
<tr>
<td>Beamsplitter Reflectance (to pointing detector)</td>
<td>$\tau_{mp}$</td>
<td>0.1</td>
<td>Chosen for minimum interference with communications</td>
</tr>
<tr>
<td>Transmitting Aperture</td>
<td>$D_T$</td>
<td>1.5 meters</td>
<td>See Report 8393</td>
</tr>
<tr>
<td>Receiving Aperture</td>
<td>$D_R$</td>
<td>3.0 meters</td>
<td>Chosen on the basis of coherence diameter</td>
</tr>
<tr>
<td>Detector Operating Temperature</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>--</td>
<td>30°K</td>
<td></td>
</tr>
</tbody>
</table>
TABLE VIII-1
SYSTEM PARAMETERS - TRACKING OF LASER
VEHICLE NEAR MARS (Cont'd)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transmission Divergence</td>
<td>( \alpha_T )</td>
<td>1.7 arc-second, 0.87 ( \times 10^{-5} ) rad.</td>
<td>Diffraction limit</td>
</tr>
<tr>
<td>Receiver Field of View, Diameter</td>
<td>( \alpha_R )</td>
<td>10 arc-seconds (48.5 ( \mu ) rad)</td>
<td>Chosen on the basis of acquisition considerations</td>
</tr>
<tr>
<td>Transmitter Power</td>
<td>( P_T )</td>
<td>200 watts</td>
<td></td>
</tr>
<tr>
<td>Mars Spectral Irradiance</td>
<td>( H_{\lambda M} )</td>
<td>( 3.9 \times 10^{-9} ) ( \text{w m}^{-2} ) ( \mu )</td>
<td>NASA CR252</td>
</tr>
</tbody>
</table>
### TABLE VIII-2
CALCULATED VALUES - TRACKING OF LASER VEHICLE NEAR MARS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Calculated Value</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Signal Power at Photodetector</td>
<td>$P_S$</td>
<td>0.31 nanowatts</td>
<td></td>
</tr>
<tr>
<td>Background Power</td>
<td>$P_B$</td>
<td>0.18 nanowatts</td>
<td>Day Sky Background</td>
</tr>
<tr>
<td>Photon Flux</td>
<td>$N$</td>
<td>$10^{14} \frac{\text{Photon}}{\text{sec-cm}^2}$</td>
<td></td>
</tr>
<tr>
<td>Detectivity</td>
<td>$D$</td>
<td>$3 \times 10^{13} \frac{\text{cm(cps)}^{1/2}}{\text{Watt}}$</td>
<td></td>
</tr>
<tr>
<td>Noise Power</td>
<td>NEP</td>
<td>$3.3 \times 10^{-15}$ Watts</td>
<td></td>
</tr>
<tr>
<td>Pointing Signal-to-Noise Ratio in One Hertz Bandwidth</td>
<td>$\text{SNR}_P$</td>
<td>94</td>
<td></td>
</tr>
<tr>
<td>Signal-to-Noise Ratio in 25 Hertz Bandwidth</td>
<td>$\text{SNR}$</td>
<td>19</td>
<td></td>
</tr>
</tbody>
</table>
TABLE VIII-3
SYSTEM PARAMETERS -
LASER TRACKING OF VEHICLE NEAR VENUS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range</td>
<td>R</td>
<td>$8.8 \times 10^{10}$ M</td>
<td>H. Seifert (editor), &quot;Space Technology,&quot; John Wiley</td>
</tr>
<tr>
<td>Sky Background Radiant Intensity</td>
<td>$N_{\lambda B}$</td>
<td>$3.4 \text{wm}^{-2} \text{sr}^{-1} \mu^{-1}$</td>
<td>60-degrees from zenith</td>
</tr>
<tr>
<td>Wavelength</td>
<td>$\lambda_{T}$</td>
<td>10.6 microns</td>
<td>CO$_2$ laser</td>
</tr>
<tr>
<td>Predetector Optical Bandwidth</td>
<td>$\Delta\lambda$</td>
<td>0.03 micron</td>
<td>Chosen for maximum transmission with minimum bandwidth</td>
</tr>
<tr>
<td>Predetector Optical Efficiency</td>
<td>$\tau_{f}$</td>
<td>0.6</td>
<td>Chosen for maximum transmission with minimum bandwidth</td>
</tr>
<tr>
<td>Atmospheric Transmission</td>
<td>$\tau_{a}$</td>
<td>0.36</td>
<td>Report 8393</td>
</tr>
<tr>
<td>Receiving Telescope Efficiency</td>
<td>$\tau_{r}$</td>
<td>0.5</td>
<td>NASA CR252</td>
</tr>
<tr>
<td>Beam Distribution Factor</td>
<td>$\tau_{d}$</td>
<td>0.5</td>
<td>Far field diffraction loss</td>
</tr>
<tr>
<td>Scintillation Factor</td>
<td>$\tau_{s}$</td>
<td>1.0</td>
<td>Aperture averaging</td>
</tr>
<tr>
<td>Mirror Transmission</td>
<td>$\tau_{mp}$</td>
<td>0.1</td>
<td>Chosen for minimum interference with communications</td>
</tr>
<tr>
<td>Transmitter Diameter</td>
<td>$D_{T}$</td>
<td>1.5 M</td>
<td>Report 8393</td>
</tr>
<tr>
<td>Receiver Diameter</td>
<td>$D_{R}$</td>
<td>3.0 M</td>
<td>Coherence diameter</td>
</tr>
<tr>
<td>Transmitter Beam Divergence</td>
<td>$\alpha_{T}$</td>
<td>0.6 second</td>
<td>Diffraction limit</td>
</tr>
<tr>
<td>Receiver Field of View</td>
<td>$\alpha_{R}$</td>
<td>10 arc-seconds, 48.5 $\mu$rad.</td>
<td>Chosen on basis of acquisition</td>
</tr>
<tr>
<td>Transmitter Power</td>
<td>$P_{T}$</td>
<td>To be Determined</td>
<td>Report 8393 (30°K)</td>
</tr>
<tr>
<td>Equivalent Dark Current Power</td>
<td>$P_{DC}$</td>
<td>$6 \times 10^{-10}$ W</td>
<td></td>
</tr>
</tbody>
</table>

110
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Calculated Value</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Signal Power</td>
<td>$P_S$</td>
<td>$4.3 \times 10^{-14}$ watts</td>
<td>One watt ${7.5$ watts}</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$3.3 \times 10^{-13}$ watts</td>
<td></td>
</tr>
<tr>
<td>Background Power</td>
<td>$P_B$</td>
<td>$0.18 \times 10^{-9}$ watts</td>
<td></td>
</tr>
<tr>
<td>Noise Equivalent Power</td>
<td>NEP</td>
<td>$3.3 \times 10^{-15}$ watts</td>
<td></td>
</tr>
<tr>
<td>Signal-to-Noise Ratio in One Hertz Bandwidth</td>
<td>SNR$_P$</td>
<td>13.5</td>
<td>One watt ${7.5$ watts}</td>
</tr>
<tr>
<td></td>
<td></td>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>

$$P_S = P_T \left[ \frac{D_D D_T}{1.22 \lambda R} \right]^2 \tau \tau_d \tau_s \tau_t \tau_m \tau_p$$

$$P_B = N \lambda_B \left[ \frac{\pi}{4} D_D r \alpha \right]^2 \tau \tau_f \tau_m \tau_p \lambda$$

$$\text{SNR}_P = \frac{P_S}{\text{NEP}}$$
TABLE VIII-5
CALCULATIONS FOR ACQUISITION OF AN EARTH BEACON BY SPACECRAFT NEAR MARS

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range</td>
<td>R</td>
<td>$2.3 \times 10^{11}$ meters</td>
<td></td>
</tr>
<tr>
<td>Transmitted Optical Power</td>
<td>$P_T$</td>
<td>Variable</td>
<td></td>
</tr>
<tr>
<td>Operating Wavelength</td>
<td>$\lambda_T$</td>
<td>0.488 micron</td>
<td>Argon</td>
</tr>
<tr>
<td>Dark Current</td>
<td>$P_{DC}$</td>
<td>$10^{-15}$ watts cooled 3 x $10^{-14}$ watts at room temperature</td>
<td>NASA CR252</td>
</tr>
<tr>
<td>Equivalent Power (S20 Surface)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Receiver Diameter</td>
<td>$D_R$</td>
<td>1.5 meters</td>
<td>Report 8393</td>
</tr>
<tr>
<td>Transmitter Diameter</td>
<td>$D_T$</td>
<td>3 meters</td>
<td>Coherent reception at 10.6 microns</td>
</tr>
<tr>
<td>Receiver Field of View</td>
<td>$R$</td>
<td>10 arc-seconds 4.85 $\mu$ radians</td>
<td>Based on one sigma or 3-arc-seconds</td>
</tr>
<tr>
<td>Transmitter Beam Divergence</td>
<td>$\alpha_T$</td>
<td>10 seconds 3 seconds 1 second</td>
<td></td>
</tr>
<tr>
<td>Radiant Intensity of Earth (Entire Earth in Field of View)</td>
<td>$H_{\lambda E}$</td>
<td>$5.6 \times 10^{-7}$ w/m$^2$$\cdot$$\mu$</td>
<td>Extrapolation from NASA CR252</td>
</tr>
<tr>
<td>Atmospheric Transmission</td>
<td>$\tau_a$</td>
<td>0.5</td>
<td>60-degree zenith angle (AD 261-585)</td>
</tr>
<tr>
<td>Transmitter Efficiency</td>
<td>$\tau_T$</td>
<td>0.5</td>
<td>NASA CR252</td>
</tr>
<tr>
<td>Beam Distribution Factor</td>
<td>$\tau_d$</td>
<td>0.5</td>
<td>Diffraction</td>
</tr>
<tr>
<td>Receiver Optical Efficiency</td>
<td>$\tau_r$</td>
<td>0.5</td>
<td>NASA CR252</td>
</tr>
<tr>
<td>Spike Filter Bandwidth Transmission</td>
<td>$\tau_f$</td>
<td>0.65</td>
<td>Fabry-Perot and Lyot</td>
</tr>
<tr>
<td>Spike Filter Bandwidth</td>
<td>$\Delta\lambda$</td>
<td>$10^{-5}$ microns</td>
<td>Fabry-Perot</td>
</tr>
<tr>
<td>Detector Quantum Efficiency</td>
<td>$\eta$</td>
<td>0.12 micron</td>
<td>S-20 Surface</td>
</tr>
</tbody>
</table>
TABLE VIII-6
CALCULATED SIGNAL-TO-NOISE RATIOS FOR ONE WATT OF TRANSMITTED OPTICAL POWER

<table>
<thead>
<tr>
<th>Conditions</th>
<th>( P_S )</th>
<th>( P_B )</th>
<th>( P_{DC} )</th>
<th>( SNR_p ) (1 Hz Bandwidth)</th>
<th>Power Required for SNR of 6 in a 1.0 Hz Bandwidth</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transmitted Beam Divergence 10 arc-sec, Receiver FOV 10 arc-sec</td>
<td>( 2.9 \times 10^{-16} ) watts</td>
<td>( 2.55 \times 10^{-12} ) watts</td>
<td>( 3 \times 10^{-14} ) watts</td>
<td>0.068</td>
<td>90 watts</td>
</tr>
<tr>
<td>Transmitter Beam Divergence 3 arc-sec, Receiver FOV 10 arc-sec</td>
<td>( 3.2 \times 10^{-15} ) watts</td>
<td>( 2.55 \times 10^{-12} ) watts</td>
<td>( 3 \times 10^{-14} ) watts</td>
<td>0.80</td>
<td>7.5 watts</td>
</tr>
<tr>
<td>Transmitter Beam Divergence 3 arc-sec, Receiver FOV 3 arc-sec</td>
<td>( 3.2 \times 10^{-15} ) watts</td>
<td>( 7.2 \times 10^{-13} ) watts</td>
<td>( 3 \times 10^{-14} ) watts</td>
<td>1.53</td>
<td>3.9 watts</td>
</tr>
</tbody>
</table>
SECTION IX

ERROR ANALYSIS

The need for accurate pointing of the communications receiver telescope was discussed in Section IV where it was shown that the upgoing argon beacon is limited by available power to a diameter of 10 arc-seconds.

As a first step in the communication process, the ground station must illuminate the spacecraft with light from the beacon so that acquisition can be accomplished. This obviously requires that the error in pointing the upgoing beam be less than 5 arc-seconds.

The following section outlines a nonrigorous preliminary compilation of the various sources contributing to the net system error. The conclusion reached is that an absolute pointing capability of 5 arc-seconds, while not out of the question, cannot realistically be depended upon without detailed design analysis and system tests. This conclusion forms the basis for the approach of striving for best possible pointing accuracy but including provision for offset pointing from reference stars.

A. UNCERTAINTY OF SPACECRAFT POSITION

The celestial coordinates of the spacecraft can be determined to within a 1000 KM diameter sphere on a real-time basis.\(^6\) This is approximately equivalent to 1 arc-second at a range of 1.5 A.U.

---

We can assume that these coordinates can be transferred to the ground receiver site without loss of accuracy. This assumes that the geographic location of the site is known to sufficient accuracy.

We will therefore take 1 arc-second as the error uncertainty of the spacecraft position.

B. UNCERTAINTY OF REFRACTION CORRECTION

The telescope position must be corrected for bending of the line of sight by atmospheric refraction. The refraction correction varies from zero at zenith to over one minute of arc at low elevation angles. The refraction correction can be computed from Comstock's formula

\[ r = \frac{983b \tan \phi}{460t} \]

Where

- \( b \) = the barometer reading in inches of mercury
- \( t \) = the temperature in °F
- \( \phi \) = the zenith distance
- \( r \) = the refraction angle in seconds

Russell\textsuperscript{7} states that the error in this formula is less than 1 second for zenith distances less than 75° except in extreme conditions of temperature and pressure.

During operation of the telescope the computer will continually correct for refraction with changing elevation angle. The local temperature

\textsuperscript{7}Russell, Duggan, Stewart, Astronomy, Ginn and Co. 1945.
and barometer inputs will be updated manually. More elaborate correction tech-
niques exist but these involve use of balloons to obtain temperature and pressure
profiles of the atmosphere above the site. We do not consider that the small
additional accuracy obtainable justifies the additional effort required and will
use one arc-second as the error inherent in refraction correction.

C. MOUNT AND TELESCOPE ERRORS

The mount and telescope will contribute to the absolute pointing error
in several areas including deflection as a result of changing orientation with
respect to gravity, temperature variations from one part of the mount to another,
manufacturing errors such as imperfect axis orthogonality and bearing runout,
imperfect leveling of the azimuth bearing whether due to set up errors or to
bending of the concrete tower, and encoder and servo errors.

1. Gravity Induced Elevation Errors

A major advantage of the Alt-Azimuth mount for precision pointing is
that the only part of the system that experiences a change in the direction of
the gravity vector is the telescope, and even this is limited to changes which
remain in one plane.

The correction of these errors is a relatively simple matter.

The telescope structure consists of a massive box structure to which
the elevation trunnions and two sets of trusses, which position the primary and
secondary mirrors, are attached. These trusses are designed such that, under the
influence of gravity, the two trusses deflect an equal amount and the ends remain
parallel to each other. This arrangement, called a Serurrier truss, is of great
importance because, as the telescope changes its orientation, the primary and
secondary mirror move equal amounts parallel to each other and, as a result, no
collimation errors are introduced. We are interested in two things here; one,
that the optics remain in proper collimation and two, that the line of sight does
not rotate with respect to the elevation encoder. Figures VI-1 through VI-4
illustrate the rotation of the line of sight as a function of four types of
mirror deflections: tilt of primary and secondary, and lateral displacement
of primary and secondary.

(a) Lateral Displacement of Secondary Mirror

Table IX-1 shows the lateral displacement of the secondary mirror and
the resultant rotation of the line of sight due to structural deflections of the
secondary assembly. The tabulated values are derived from the preliminary analyses
summarized in Appendix A.

(b) Lateral Displacement of Primary Mirror

There will also be a displacement of the primary in the same direc-
tion as a result of elevation changes.

Measurements of primary mirror deflection by the Boller and Chivens
Division of Perkin-Elmer on the 60-inch astrometric telescope indicate a lateral
shift of about 0.001 inch over the 90-degree range of elevation angles. This
mirror is radially and axially supported by the mercury filled tube and hydro-
static support proposed in Section VI. This deflection would result in a
rotation of the line of sight by 0.61 second. Note that although the displace-
ments of the two mirrors are in the same direction, the rotations of the line of
sight are in opposite directions and very nearly cancel each other.
<table>
<thead>
<tr>
<th>ELEVATION ANGLE</th>
<th>LATERAL DISPLACEMENT (Inches)</th>
<th>ROTATION (Seconds)</th>
<th>MAXIMUM ROTATION (30°-90°) SECONDS</th>
</tr>
</thead>
<tbody>
<tr>
<td>ITEM</td>
<td>0°</td>
<td>30°</td>
<td>60°</td>
</tr>
<tr>
<td>Cage Support</td>
<td>0.00015</td>
<td>0.00013</td>
<td>0.00008</td>
</tr>
<tr>
<td>Flip Cell</td>
<td>0.0001</td>
<td>0.00009</td>
<td>0.00005</td>
</tr>
<tr>
<td>Cell Support</td>
<td>0.0001</td>
<td>0.00009</td>
<td>0.00005</td>
</tr>
<tr>
<td>Linear Screw</td>
<td>0.0001</td>
<td>0.00009</td>
<td>0.00005</td>
</tr>
<tr>
<td>Support Plates</td>
<td>0.00015</td>
<td>0.00013</td>
<td>0.00008</td>
</tr>
<tr>
<td>Secondary Truss Vanes</td>
<td>0.001</td>
<td>0.00086</td>
<td>0.0005</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
(c) Serurrier Truss Errors

The Serurrier truss is calculated to equalize the deflections of both the primary and secondary mirrors but, as a result of manufacturing errors, analysis assumptions, and other as yet unknown causes, they will probably be unequal.

The calculated total deflection of the trusses is 0.014 inch. If we assume residual errors and hysteresis result in a 7 percent or 0.001 inch variation from calculated we get another 0.6 second error.

(d) Deflection of Box Structure

The calculated value of the deflection of the secondary due to bending of the box structure is 0.0002 inch or 0.12 second.

(e) Torsional Windup

Another source of elevation error is torsional windup of the entire structure below the elevation axis caused by reaction of torque due to bearings, seals, brush friction, and inertia. This windup rotates the outer case of the elevation encoder to produce a reading error of up to 0.23 second.

(f) Bending of Primary Cell

The primary cell will bend out of round under the load of the primary when pointing close to the horizon. The calculated deflection is 0.0006 inch or about 0.4 second.

(g) Tilt of Primary Mirror

Because of the symmetry of both the loading and of the structure there is no calculated tilt of the primary as a result of elevation changes.
We will, however, allow for a tilt of the primary of 0.0005 inch across its diameter. This tilt causes a rotation of the line of sight of 1.6 second. As shown in Figure VI-3 rotation of the secondary has no effect.

(h) Summary of Gravity Induced Elevation Errors

All of the errors listed in paragraphs (a) through (g) result from elevation changes and should be summed algebraically.

<table>
<thead>
<tr>
<th>Paragraph</th>
<th>Elevation Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>+ 0.93 sec</td>
</tr>
<tr>
<td>(b)</td>
<td>- 0.61</td>
</tr>
<tr>
<td>(c)</td>
<td>+ 0.60</td>
</tr>
<tr>
<td>(d)</td>
<td>+ 0.12</td>
</tr>
<tr>
<td>(e)</td>
<td>+ 0.23</td>
</tr>
<tr>
<td>(f)</td>
<td>- 0.40</td>
</tr>
<tr>
<td>(g)</td>
<td>+ 1.60</td>
</tr>
</tbody>
</table>

+ 2.47 sec

These errors can be measured by reading the elevation angles of stars with known positions and a calibration curve of elevation versus error can be prepared. If we assume that this calibration will permit error corrections with an accuracy of ± 20 percent, we are left with a gravity induced residual error of ± 0.5 seconds.

2. Other Systematic Elevation Errors

(a) Elevation Bearing Radial Runout

Assuming an eccentricity of 0.0002 inch, a 15-inch radius and a 90-degree range of rotation leads to a worst-case elevation error of 1.34 arc-seconds. This error will, however, be indistinguishable from gravity deflection errors and subject to the same calibration procedure. Applying the same 20 percent factor used above yields a residual error of 0.27 arc-second.
(b) Mount Leveling Errors

It is assumed that the mount can be leveled to 1.0 arc-second. Although, in principle, elevation errors arising from this source are also subject to calibration, it is here assumed that such is not the case. In other words, it is assumed that elevation angle will not be calibrated as a function of azimuth angle, and that the 1.0 arc-second leveling error can appear directly as an elevation error.

3. Random Elevation Errors

(a) Thermal Errors

Assume that, with the telescope pointed to the horizon, the temperature of the upper part of the truss is 1°F different from the lower part of the truss. This temperature difference will result in a rotation of the line of sight of approximately 0.2 second.

The air-conditioning system will circulate air in the dome to minimize this effect, but a one degree difference is probably a reasonable residual value to be expected.

(b) Encoder Errors

Assume that the 21-bit digital encoders are subject to a random error of 1 bit or 0.6 arc-second.

(c) Tracking Errors

As indicated in Section VII, the tracking servo will have an error of approximately 0.2 second, due primarily to noise.
4. **Total Elevation Errors**

Arithmetic addition of the error values quoted in the above paragraphs yields a total elevation angle error of 4.8 arc-seconds. This is admittedly a pessimistic method of adding errors, but the importance of illuminating the spacecraft with the upgoing beacon warrants a conservative approach.

5. **Azimuth Angle Errors**

(a) **Azimuth Bearing Runout**

A hydrostatic bearing tends to be self-centering even in the presence of a slight out-of-round condition of the journal. We will, however, allow for an eccentricity of 0.00025 inch, which is equivalent to an azimuth reading error of 1 second.

(b) **Mount Leveling and Axis Orthogonality Errors**

Assume that the mount can be leveled to 1.0 arc-second and that the elevation axis can be made orthogonal to the azimuth axis to 1.0 arc-second. These errors result in azimuth angle errors ranging from negligible values at low elevation angles to appreciable values close to zenith. Bearing in mind that spacecraft acquisition will normally occur at low elevation angles, arbitrarily estimate a 1.0 arc-second azimuth error from each of these sources.

(c) **Encoder Errors**

Again assume a 1 bit or 0.6 arc-second error from the 21-bit azimuth encoder.

(d) **Tracking Errors**

Again in conformance with Section VII, estimate a dynamic tracking error of 0.2 arc-second.
(e) Thermal Errors

A 1°F temperature differential across the telescope in a direction parallel to the elevation axis will produce a cross-elevation line-of-sight rotation of 0.2 arc-second due to differential expansions of the Serrurier truss, and an additional orthogonality error of 1.0 arc-second due to differential expansion of the fork uprights. By reasoning similar to the above, arbitrarily assign an azimuth angle error of 1.0 arc-second to this source.

6. Total Azimuth Angle Error

Again employing arithmetic addition of individual error components, a maximum azimuth error of 6.8 arc-seconds is obtained.

D. TOTAL POINTING ERROR

At low elevation angles, where acquisition would normally take place, elevation and azimuth errors of 4.8 and 6.8 arc-seconds, respectively, result in a pointing angle error of about 8.5 arc-seconds. An error of this magnitude would plainly result in failure to illuminate the spacecraft with a 10 arc-second diameter beacon. The need for offset pointing from a reference star therefore results.

It should be emphasized that while estimates of individual error components are felt to be realistic, the above error summary is pessimistic in terms of the way in which those error components were combined. Combination on an rms or statistical basis would result in considerably lower net errors, well within the required ± 5 arc-seconds. This might well be a realistic approach, but experience with large precision equipment and appreciation of the magnitude of an arc-second suggests caution in this sort of evaluation. Inclusion of offset pointing capabilities is straightforward, and although it might well prove to be unnecessary, assures successful performance of the system.
SECTION X

SITE LOCATION CONSIDERATIONS

The basic requirement for operation of the system at any time, day or night, results in site selection criteria which are quite different from those appropriate for the usual astronomical instrument. Astronomical telescopes are located on mountain tops to optimize night observing conditions at the tradeoff of cloudiness and convenience of access.

Under night conditions the ground temperature of the earth in a clear environment is significantly lower than the free air temperature. Thus radiation-chilled air tends to collect in low areas and to drain downslope. A mountain top is, therefore, the best place to be since the free (ambient) air tends to lower over the summit. This condition means that the thermal optical inhomogeneities are minimized. The astronomer must then place his telescope a sufficient height above the mountain top terrain to avoid the dynamic mixing of the surface-chilled air with the free air. Recent studies made at KPNO by Dr. C. R. Lynds indicate that this mixing has become small at a height of 100 feet.

Under day conditions the ground temperature of the earth in a clear environment is significantly higher than the free air temperature, exactly the opposite to night conditions. The surface-heated air tends to rise and a mountain upslope leads to the early onset of strong vertical convection. A site that is a good night observing station actually is likely to be a very poor daytime observing station. The combined usage of a mountain site for stellar and solar observing is usually a complicated logistics and economics compromise, and in
the case of this system, these factors may be such as to further de-emphasize
a mountain site.

The excellence of night seeing at a mountain site and the gains
attendant with a small seeing disc for a star is a factor of dominant importance
for astronomers, even though the percentage of clear sky is apt to be a minimum
for the local climate. In the case of the communications receiver, the need for
maximum percentage of clear sky changes the weighting factor again away from a
mountain site. A mountain site generally is cloudier than nearby less mountainous
terrain for two reasons: 1. the orographic effect of the mountain area tends to
produce saturation of the air and localized cloudiness, and 2. the daytime con-
vection tends to form cumulus over mountain peaks.

A site that optimizes day and night seeing would appear to be one
that is above local fog or low stratus conditions (as along the Pacific Coast
in the N30-40° and S30-40° latitudes) but on sufficiently gentle terrain to
avoid local steep slopes on sunlit sides.

A desert floor site, such as at Goldstone, would appear to have
many factors in its favor as a compromise site of near optimum day-night criteria.
For example, there are elevated slopes available to provide avoidance of chilled
night air and convection day air. The percentage of clear sky is significantly
larger than available on the nearby coastal mountain barrier. Dr. Meinel has
pointed out that during the 200-inch site survey in the 1930's Dr. J. A. Anderson
made a few tests of the seeing in the Barstow area and reported surprisingly good
results. It would, therefore, appear very desirable to do some day-night seeing
studies in the Goldstone area to ascertain the performance of this site, since the
very important questions of logistics and peripheral facilities make this a very attractive location.

In regard to the performance of a seeing test of a Goldstone area site it is most important to make the test a valid one. A test conducted from the surface, say within the lower 10-20 feet, would not be valid due to ground effects. The execution of an optical evaluation at operating heights significant for the final site, say 50-100 feet, are expensive since the structure to support the test telescope must be sufficiently stable to withstand to make measurements in the fractional second of arc range. One scientifically acceptable alternate to the optical evaluation test is a microthermal evaluation as extensively used in KPNO studies. In this case, a set of very short response time temperature sensors are mounted on a light scaffold tower. It is obvious that if the microthermal sensor detects blobs of air at different temperatures (~0.05°C in ~0.1 sec) there will also be optical index of refraction variations that will cause wavefront deformations. The advantage of the microthermal method of site investigation, even though it is an indirect measure of local optical disturbances, is that it can be done at lower cost and with relatively unsophisticated unmanned instrumentation.

In regard to high altitude seeing disturbances, these will vary only slowly from nearby geographical areas, and one needs only to take recourse to the grosser meteorological criteria as climate and large-scale orographic effects, such as the turbulence in the wake of the "Sierra wave."
If the above type of study does not show that a compromise day-night site can be found, then the possibility of requiring separate "day" and "night" installations would need to be considered.
SECTION XI

BUILDING AND DOME CONSIDERATIONS

The proposed communications telescope must, of course, be shielded from external environments by some sort of building or enclosure. The most appropriate form of enclosure is considered to be a relatively conventional rotatable astronomical dome with an openable slot. Figure II-3 shows the instrument enclosed in such a dome. The inside diameter of the dome as shown is 64 feet. This is a large and comfortable dome providing adequate room for the prime focus cage and for any conceivable astronomical use. If the prime focus cage were omitted and a "minimum" dome were used, the inside diameter of the dome could be reduced to 48 feet. This diameter dome for a 120-inch, fast primary, alt-azimuth telescope may be compared with the 100-foot dome required for the 120-inch, slow primary, equatorial telescope at Lick Observatory.

A large portion of the cost of the system will go for the building and dome. Meinel\textsuperscript{8} has stated that the cost of a dome goes nearly with the 2.7 power of the diameter. The choice of dome diameter should be made with due considerations of intended uses, user comfort, auxiliary equipment, and cost.

The telescope will have to be mounted on a tower to raise it above severe seeing disturbances at ground level. Figure II-3 shows a 100-foot tower, and all of the tower bending calculations in Appendix D are based on a 100-foot height. While Appendix D demonstrates that the 100-foot tower is feasible from a bending point of view, it may not be required. The height of the tower for any

\textsuperscript{8} A.B. Meinel, Introduction to the Design of Astronomical Telescopes. University of Arizona, 1965, p. 70
particular site should be based on 10.6μ coherence diameter measurements and, if astronomers are to use the telescope, visible seeing measurements, at different heights above ground, for that site. As indicated in Section X, site selection requirements for this instrument are unique and require careful evaluation.

The structure supporting the dome should be entirely separate from and independent of the tower supporting the telescope. Wind loads, thermally induced deflections, and mechanically induced vibrations in the dome or its supporting structure will thus be isolated from the telescope.

Figure II-3 indicates that the dome support structure provides ample space for equipment rooms, laboratories, and the variety of requirements associated with an installation of this magnitude.

The requirements for air-conditioning and thermal control of the dome are essentially the same as those for a purely astronomical facility, but the magnitude of the task is enhanced by the necessity of operation during daytime as well as nighttime hours.

The entire dome must be air-conditioned by a system that can hold the temperature inside the dome at the expected outside temperature when the dome is opened. The requirement for day or night operation means that the air-conditioning system must be capable of maintaining the telescope at any constant temperature over a fairly wide range of temperatures. The air-conditioning system will also have to maintain a constant temperature throughout the dome to minimize temperature gradients in the telescope.
In addition to active conditioning, the building should be provided with large capacity air exhaust fans to purge the dome with ambient air drawn in through the dome slot just before (and possibly during) the most critical performance period. Special care should also be taken to avoid unusual heat sources in the building or dome that can locally upset the optical homogeneity of the air in the optical path of the telescope.

A potentially troublesome source of thermal perturbation, and one that telescopes limited to night operation do not encounter, is infrared radiation from the daytime sky. Preliminary analysis indicates, however, that at least in terms of temperature changes of the primary mirror, sky radiation is not appreciable in comparison to typical diurnal variations in ambient air temperature.
SECTION XII

ASTRONOMICAL UTILITY

Although the telescope was designed as a ground receiver for a deep-
space communications system, it is very well suited for normal astronomical work
and ideally suited for special applications which can make full use of its pre-
cision pointing capability and its ability to track targets on a closed-loop
basis.

For routine observational astronomy the telescope, as designed for
this study, provides prime, Cassegrain, and coudé focal positions. Recent de-
velopments in photographic plate technology have somewhat reduced the importance
of the low f-number prime focus position and some astronomers now consider the
prime focus to be superfluous. The prime focus cage is thus optional and if
deleted will result in a reduced dome size.

The proposed design has its Cassegrain focus brought out through the
elevation trunnion, an advantage to astronomers who wish to use large, heavy,
equipment at that focus or to astronomers who are using equipment such as liquid
cooled detectors that are sensitive to gravity vector changes.

The absolute pointing accuracy is advantageous to infrared astronomers
making measurements on stars which are too cool to be visible.

The telescope has many advantages for experiments that require a laser
transceiver capable of tracking artificial satellites. This telescope is ideally
suited as the ground station for the lunar laser ranging experiment proposed by C.O. Alley, et al.  

Astronomers may find that the altitude and azimuth coordinate system used by the telescope is more difficult to use than the more familiar right ascension and declination, but the availability of the computer for coordinate transformation should minimize this problem.

A more serious problem is that of rotation of the field of view in the telescope. This rotation requires that the photographic plate be rotated during the exposure. We do not consider this to be a serious drawback since the plate rotation can be easily accomplished on an open-loop basis.

Radio astronomers have been using alt-azimuth mount configurations for many years.

Meinel has discussed the features of alt-azimuth mounts and pointed out the advantages that they offer, especially for very large telescopes (apertures greater than 200 inches).

---


SECTION XIII

EVALUATION OF A MULTIPLE APERTURE RECEIVER

A. SUMMARY

The following section considers the possibility of a ground receiver employing several relatively small telescopes rather than a single, large aperture telescope for coherent detection at 10.6 μ. The results indicate that, in principle, a gain in signal-to-noise ratio can be obtained with the use of multiple apertures. However, the magnitude of this gain is not considered sufficient to justify the concomitant increase in system complexity.

B. COMMUNICATION CONSIDERATIONS FOR A MULTIPLE APERTURE ARRAY

It has been assumed that the total surface area would be the same as one 3-meter telescope (for instance three 1.73-meter telescopes, or five 1.34-meter telescopes).

The signal developed by coherent detection is proportional to the square of the diameter, while the noise is proportional to the diameter. Consider one large telescope whose diameter is $D_L$. The signal-to-noise ratio is

$$\frac{K_1 D_L^2}{K_2 D_L} = K_3 D_L$$

where $K_1$, $K_2$, and $K_3$ are gain constants.
If this single aperture is replaced by "n" apertures, each will have a diameter \( D = \frac{D_L}{\sqrt{n}} \) to give the same total area. The output voltages of these receivers are then summed coherently to give the total signal.

The signal voltage from each receiver will be

\[
\sqrt{p} K_1 D^2 = \frac{K_1 D_L^2}{n} \sqrt{p}
\]

where \( p \) is the signal power improvement factor due to higher heterodyne efficiency of a smaller aperture. (Refer to paragraph XIII-D.)

The noise voltage will be:

\[
K_2 D = \frac{K_2 D_L}{\sqrt{n}}
\]

If the signal outputs of all the receivers are coherent, the result of coherent addition would be:

\[
\sqrt{p} n K_1 D^2 = \sqrt{p} K_1 D_L^2
\]

The noise voltages generated at the individual receivers are not coherent, and they are added on a root-sum square basis. The noise is then given by

\[
N = \left( \sum_{L=1}^{n} K_2 D_L^2 \right)^{1/2} = K_2 D_L;
\]

and the signal-to-noise ratio is

\[
\sqrt{p} K_3 D_L.
\]

The signal-to-noise ratio is, therefore, improved by a factor of \( \sqrt{p} \) for a multiple aperture array compared to a single, large aperture. This assumes

1. The signals can be added coherently.
(2) That the detector noise is very small compared to the signal noise.

Coherent addition of signals requires additional equipment.

C. MULTI-APERTURE RECEPTION

The laser wave transmitted by the satellite is essentially a plane wave when it reaches the receiving telescope. The wave field intensity (for a simple amplitude-modulated case) may be written as:

\[ E_i(t, x, y, z) = E_s(1 + M e^{jv_m t}) e^{j(2\pi v_s t - \beta z + \phi(x, y, t))} \]

where the wave is assumed to be traveling in the z direction.

\[ M \] is the modulation index
\[ v_m \] is the modulation frequency
\[ v_s \] is the carrier frequency
\[ \beta \] is the wave number
\[ \phi(x, y, t) \] is phase retardation which is assumed to vary in a stochastic manner in plane of constant z.

At some plane, say the \( z = 0 \) plane for convenience, the wave has another wave (the local oscillator wave)

\[ E_o(x, y, z, t) = E_o e^{j(2\pi \nu_0 t + \phi_0)} \]

added to it, where \( \nu_0 \) is the frequency of the local oscillator signal, and \( \phi_0 \) is the local oscillator phase.

The wave impinging upon the photodetector will then be:

\[ E_{TOT} = E_o(x, y, z, t) + E_i(x, y, z, t) \]
Since the photodetector responds to the power in the wave, the current in the photodetector, \( J_p \), per square meter will then be:

\[
J_p = (E_{TOT}) (E_{TOT}^*) \left( \frac{en}{hv \mu_o} \right)
\]

where

- \( J_p \) is the current density, \( \text{amp/cm}^2 \)
- \( n \) is the detector quantum efficiency
- \( h \) is Planck's constant
- \( v \) is the wave frequency (Hz)
- \( \varepsilon_0 \) is the permittivity of free space
- \( \mu_0 \) is the permeability of free space
- \( e \) is the electronic charge (coulombs)

After the signal has been processed and demodulated, it will have the general form

\[
S(t) = S_o \cos \left( 2\pi v_m t + \phi (x, y, t) - \phi_0 \right)
\]

where \( S(t) \) is the processed output. Note that the phase fluctuations in the carrier appear as phase fluctuations in the signal. If two receivers are located in the x-y plane such that \( \phi (x, y, t) \) is essentially a constant, then their outputs will have phase coherence. Outside of their coherence region, the signals will not have phase coherence and some auxiliary method must be found to restore the phase.

One possible method of achieving this coherence is to have the satellite broadcast an auxiliary constant frequency tone. The difference in phase of that tone at each receiver can be sensed and used to vary the local oscillator phase \( \phi_0 \) at each receiver. Such a local oscillator phase variation may be achieved by changing the index of refraction of a crystal for
instance, but this requires that the same local oscillator laser be used for each receiver.

If this is not possible, a double heterodyne system may be used to provide the necessary phase-matching, with the phase error used to drive a voltage-controlled microwave oscillator.

D. DETECTION EFFICIENCY IN THE PRESENCE OF TURBULENCE

Fried has shown\(^\text{11}\) that in the presence of atmospheric turbulence the signal power (across a 1Ω resistor), \(S(D)\), as a function of diameter is given by:

\[
S(D) = K \pi \left( \frac{2n}{q} \frac{E E_0}{hv} \right)^2 \int_0^{D} 1/2 rdr \left\{ D^2 \cos^{-1} \frac{r}{D} - r \sqrt{D^2 - r^2} \right\} e^{-3.44 \left( \frac{r}{r_0} \right)^{5/3}}
\]

where \(K\) is a constant, \(r_0\) is the coherence diameter, and \(D\) is the aperture diameter.

In the absence of turbulence

\[
S(D) = (1/2)K \left( \frac{q}{\pi} \frac{\pi/2}{D^2} \frac{E_0^2}{E_s^2} \right)
\]

Defining the detection efficiency as \(n_p\), a function of \((D/r_0)\), and setting \(v = D/r_0\), we find

\[
n_p(v) = \frac{32}{\pi v^4} \int_0^v x dx 1/2 \left( v^2 \cos^{-1} \left( \frac{x}{v} \right) - x \sqrt{v^2 - x^2} \right) e^{-3.44 x^{5/3}}
\]

where \(x\) is the variable of integration.

This function is plotted in Figure XIII-1. At $D = r_o$, the efficiency is 44 percent. Assuming that the coherence diameter is 4 meters, a 3-meter telescope would have a detection efficiency of 56 percent. Defining $p(n) = n \left( \frac{3}{\sqrt{n/r_o}} \right) / n \left( \frac{3}{r_o} \right)$, it can be seen that the maximum value of $p$ is 1.80, which assumes no error in phase matching at the local oscillator.

Figure XIII-2 shows $p$ as a function of $n$, for $r_o = 4$ meters and for $r_o = 3$ meters.

E. CONCLUSIONS

The use of several smaller telescopes, rather than one telescope can be made to yield a higher signal-to-noise ratio. Under good seeing condition ($r_o$ with tracking = 4 meters) the improvement can approach 6 decibels with the use of more than five telescopes. The local oscillator phase must be matched at all of these telescopes to realize this slight increase. The complexity of using more than one telescope does not seem to justify this increase in signal-to-noise ratio.
Figure XIII-1. Detection Efficiency Versus Diameter Ratio
Figure XIII-2. Detection Improvement Versus Number of Receiving Telescopes for Two Coherence Diameters.
APPENDIX A

TELESCOPE AND MOUNT
PRELIMINARY DESIGN ANALYSIS

The following is a preliminary mechanical design analysis of the telescope and of the alt-azimuth mount. Included are size determinations of key components, weight and moment of inertia estimates, deflection and resonant frequency computations, and sketches of key mechanical details.

The intent is not to arrive at detailed final design, but rather to demonstrate feasibility and to arrive at reasonable estimates of those parameters which influence system performance.
WEIGHTS OF SECONDARY CASE ASSY

1. WT. of COUDE-CASSEGRAIN MIRROR

\[
0.08 \left( \frac{\pi \times 3^2}{4} \right) 6 = 0.08 \left( \frac{\pi \times 16 \times 20}{4} \right) 6 = 385 \# \quad \text{USE 400\#}
\]

2. FLIP CELL

\[
2P = 400 \\
P = 200 \\
t = 16
\]

ASSUME COSINE LOADING

VERTICAL DEFLECTION \( W = 0.033 \frac{Pl^3}{Et} \)

\[
I = \frac{2 \times 10 \times 14^3 \times 0.33}{30 \times 10^6 \times 10^{-6}} = 9 \text{ in}^4
\]

for a cross section

\[
I = (7 \times 3^3 - 6.5 \times 2.5^3) \frac{1}{12} = 8.4
\]

\[
A = 21 - 16.2 = 4.8 \text{ in}^2
\]

\[
wt = 4.8 \times 2 \pi \times 17.5 \times 3 = 156 \# \quad \text{USE 200}\#
\]

3. CELL SUPPORT

\[
4P = 600 \\
P = 150
\]
\[ U = \frac{0.04bph^3}{EI} \]

Let \( U = 0.0001 \)

\[ b = 21 \]

\[ I = \frac{0.046 \times 150 \times 21^3}{30 \times 10^6 \times 10^{-4}} = \frac{6.9 \times 9300}{3000} = 21.4 \]

For a cross section

\[ I = \left( 7 \times 4^3 - 6.5 \times 3.5^3 \right) \frac{1}{12} \]

\[ = (450 - 278) \frac{1}{12} = 172 \frac{1}{12} = 14.4 \]

\[ A = 28 - 22.7 = 5.3 \text{ in}^2 \]

\[ Wt = 5.3 \times 2\pi \times 21 \times 3 = 210 \# \quad \text{USE} 250 \# \]

4. Linear Screw

For focus adjustment

\[ 1'' \text{ TRAVEL} \]

\[ \frac{s}{48EI} \]

\[ \text{LET} \ s = 0.001 \]

\[ I = \frac{250 \times 9^3}{30 \times 10^6 \times 10^{-4}} \]

\[ I = 1.26 \text{ in}^4 \]

\[ \text{USE} \ d = 2'' \]

\[ w_t = \pi \times 2^4 \times 10 \times 4 \times 3 = 37.7 \# \quad \text{USE} 50 \# \]

5. Allow 100\# for drives

6. Total \( Wt = 400 + 200 + 250 + 50 + 100 = 1000 \# \)
7. CAGE STRUCTURE WT.

\[ t = \frac{3}{8}'' \]

\[ I = \pi \times 24.5^3 \times \frac{3}{8} = 1760 \]

\[ J = \frac{P \ell^3}{3EI} = \frac{1500 \times 24^3}{90 \times 10^6 \times 1760} = 0.0015 \]

\[ W_t = 2\pi \times 24.5 \times \frac{3}{8} \times 6.5 \times 7 = 1120\# \]

\[ W_e = 2\pi \times 24.5 \times \frac{3}{8} \times 52 \times 3 = 300\# \]

ASSUME A RADIAL STIFFENER FLANGED

\[ \frac{1}{4}'' \text{ WIDE} \times 3'' \text{ THK} \]

\[ A = \frac{\pi}{4} (48^2 - 42^2) = \frac{\pi}{4} (1100) = 880\,\text{in}^2 \]

\[ W_t = \frac{1}{4} \times 880 \times 4 \times 3 = 264.0\# \]

**Total Cage Structure wt = 1700\#**

8. FOR FLIP ASSY DRIVES & HARDWARE, ASSUME 200\#

9. VANE RING SUPPORT

\[ W_t = 2\pi \times 25 \times \frac{3}{8} \times 2 \times 8 \times 3 = 280\# \]

10. **Total approx. wt of secondary package** 3700\#
SECONDARY TRUSS VANE WEIGHTS

(1) VANES FOR LATERAL LOADING
8 VANES, 6" x 1" CROSS SECTION x 48" LONG
wt = 8 x 6 x 48 x 3 = 690 #

(2) TORSIONAL VANE
\[ wt = \frac{1}{2} (6 + 40) \times \frac{1}{4} \times 3 = 66 # \]

Total Vane wt = 756 #

\[ \max \ 800 # \]

OUTER RING (VANE & TRUSS INTERFACE SUPPORT)

Assume same truss size as in original analysis
10" O.D TUBE x 1/2" wall, LENGTH = 220"

\[ wt = 15 \pi \Delta x 220 x 0.3 x 4 = 4000 # \]

\[ \delta_h = 0.15 \frac{PR^3}{EI} \]
\[ P = 3500 # \]
\[ R = 68 \]
\[ I = \frac{1}{12} (12 \times 12^3 - 11.5 \times 11.5^3) \]
\[ = \frac{1}{12} (20700 - 17500) = \frac{3200}{12} = 265 \]

\[ \delta_h = \frac{525 (312 \times 10^3)}{795 \times 10^5} = 2.07 \times 10^{-3} \]

K = \[ \frac{3500}{0.0207} = 170000 \text{#} / \text{in} \]
Horizontal displacement due to AZ. accel.

\[ F = ma = \frac{W}{g} = \frac{3500}{386} \times 250 \times 0.011 \]

\[ F = 25 \text{ lb} \]

\[ \delta = \frac{2.5}{0.17 \times 10^6} = 14.7 \times 10^{-6} \text{ in negligible} \]

If ring deflection is combined with truss deflection \( \delta_{\text{total}} = 0.040 \)

\[ f_n = \frac{1}{2\pi} \sqrt{\frac{386}{4 \times 10^{-2}}} = \frac{1}{2\pi} \sqrt{9700} = \frac{1}{2\pi} \times 90 = 15 \text{ cycles/sec.} \]

For a cross section

\[ I = \frac{1}{12} (12 \times 14^3 - 11.5 \times 13.5^3) \]

\[ = \frac{1}{12} (33700 - 28200) = \frac{1}{12} (5500) = 458 \]

\[ S_h = 0.0207 \left( \frac{265}{400} \right) = 0.014'' \quad \text{Truss deflection: } 0.014'' \]

\( \delta = 19 \times 12 - 11.5 \times 13.5 = 168 - 155 = 13 '' \)

\( W = 13 \times 2\pi \times 70 \times 3 = 1700 \text{ lb} \)

\[ f_n = \frac{1}{2\pi} \sqrt{\frac{386}{3.5 \times 10^{-2}}} = \frac{1}{2\pi} \sqrt{107000} = \frac{1}{2\pi} (103) = 167 \text{ cycles/sec.} \]
LOAD DISTRIBUTION
ON PRIME FOCUS CAGE

(Without)

\[ \bar{x} (3450) = 1000(8) + 100(70) + 700(65) + 150(90) + 1700 \times 65 \]

\[ 3450 \bar{x} = 8000 + 7000 + 12000 + 13500 + 11500 = 105500 \]

\[ \bar{x} = 30.5'' \]

(with)

\[ \bar{x} (3650) = 900 \times 8 + 600 \times 35 + 1700 \times 45 + 100 \times 70 + 200 \times 82 + 150 \times 90 \]

\[ 3650 \bar{x} = 142100 \]

\[ \bar{x} = 39.6'' \]

With man, unbalance moment ≈ 4000 in-lb
BENDING DEFORMATION OF TWO PLATES HAVING SECONDARY MIRROR CELL SUPPORT (axial)

\[
\sigma = \frac{2\mathcal{E} w a^2}{E t^3} \quad a = \frac{2L}{b} \quad \frac{a}{b} = 1.19 \quad L = 0.21
\]

\[
t^3 = \frac{21 \times 5 \times 0 \times 0.25}{30 \times 10^6 \times 1 \times 10^{-3}}
\]

\[
t^3 = \frac{105 \times 0.25}{3 \times 10^{-6}} = 2.18
\]

\[t = 1.28\text{"}
\]

\[
wt = \frac{\pi (50^2 - 40^2) \times 2.56 \times 0.3}{4} = \frac{\pi (2.56)(0.3)}{4} \approx 540 \text{#}
\]

radially \[w = 0.046 \frac{PL^3}{EI} \]

\[4P = 850 \quad P = 212 \]

\[
I = \frac{256(5^3)}{12} = \frac{125}{6} = 20.5
\]

\[u = 0.046 \times 212 \times 22^3 \div 30 \times 10^6 \times 26.5 = 0.006 \times 10^{-6} = 1.3 \times 10^{-6}
\]

\[u = 0.0013\text{"} \quad = 0.00015\text{"
[Page A-8]
WEIGHT OF SECONDARY TRUSS ASSY

SIDE TRUSSES: $W = 55\#/ft$

$W = \frac{200\text{"}}{12} \times 55\#/ft \times 9 = 3670\#$

TOP & BOTTOM TRUSSES: $W = 34.25\#/ft$

$W = \frac{200}{12} \times 34.25 \times 9 = 2280\#$

**total wt** = 5950#

ASSUME $\frac{1}{4}$ LOAD $= \frac{6000}{4} = 1500\#$ AT END OF EACH SIDE TRUSS FOR TRUSS ANALYSIS

PLUS $\frac{1}{2}$ SECONDARY CAGE LOAD $= \frac{6000}{2} = 3000\#$

USE 4500#
Support for Secondary Assembly

Determination of deflection of pt. "O" in Y-direction due to W at any tilt angle $\phi$

First find forces acting on members OA, OB, OC, OD due to unit load at pt. O in Y-direction.

Since it is symmetrical, we know

$U_A = U_D = +U_C = +U_B = U$

$\Sigma F_y = 0, \quad 1 = \sin\alpha (U_A + U_B + U_C + U_D)$

$= 4U \sin\alpha$

$\therefore \quad U = \frac{1}{4 \sin\alpha}$

$U_A = U_D = +\frac{1}{4 \sin\alpha}$, Tension; $U_B = U_C = -\frac{1}{4 \sin\alpha}$, Compression
Then find forces acting on these members due to $W_y$:

\[ \overrightarrow{OA} = \overrightarrow{BO} \]
\[ 2 \overrightarrow{OA} \sin \alpha = \frac{W_y}{2} \]
\[ \overrightarrow{OA} = \overrightarrow{BO} = \frac{W_y}{4 \sin \alpha} \]

\[ \overrightarrow{OA} = \overrightarrow{OD} = + \frac{W_y}{4 \sin \alpha} \text{, Tension} \]
\[ \overrightarrow{BO} = \overrightarrow{CO} = - \frac{W_y}{4 \sin \alpha} \text{, Compression} \]

Due to $W_x$:

\[ \overrightarrow{AO} = \overrightarrow{OD} \]
\[ 2 \overrightarrow{AO} \sin \beta = \frac{W_x}{2} \]
\[ \overrightarrow{AO} = \frac{W_x}{4 \sin \beta} \]
\[ \overrightarrow{AO} = \overrightarrow{BO} = - \frac{W_x}{4 \sin \beta} \text{, Compression} \]
\[ \overrightarrow{OC} = \overrightarrow{OD} = \frac{W_x}{4 \sin \beta} \text{, Tension} \]
### Summary of Forces on Members

<table>
<thead>
<tr>
<th>Member</th>
<th>( S )</th>
<th>( U )</th>
<th>( S_U )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>( \frac{W_y}{4 \sin \alpha} - \frac{W_x}{4 \sin \beta} )</td>
<td>( \frac{1}{4 \sin \alpha} )</td>
<td>( \frac{1}{16 \sin \alpha} \left( \frac{\sin \alpha}{\sin \beta} \right) \frac{W_y}{W_x} )</td>
</tr>
<tr>
<td>B</td>
<td>( \frac{-W_y}{4 \sin \alpha} - \frac{W_x}{4 \sin \beta} )</td>
<td>( \frac{1}{4 \sin \alpha} )</td>
<td>( \frac{1}{16 \sin \alpha} \left( \frac{\sin \alpha}{\sin \beta} \right) \frac{W_y + W_x}{W_x} )</td>
</tr>
<tr>
<td>C</td>
<td>( \frac{-W_y}{4 \sin \alpha} + \frac{W_x}{4 \sin \beta} )</td>
<td>( \frac{1}{4 \sin \alpha} )</td>
<td>( \frac{1}{16 \sin \alpha} \left( \frac{\sin \alpha}{\sin \beta} \right) \frac{W_y - W_x}{W_x} )</td>
</tr>
<tr>
<td>D</td>
<td>( \frac{W_y}{4 \sin \alpha} + \frac{W_x}{4 \sin \beta} )</td>
<td>( \frac{1}{4 \sin \alpha} )</td>
<td>( \frac{1}{16 \sin \alpha} \left( \frac{\sin \alpha}{\sin \beta} \right) \frac{W_y + W_x}{W_x} )</td>
</tr>
</tbody>
</table>

\[ \sum S_U = \frac{W_y}{4 \sin^2 \alpha} \]

\[ \delta_{oy} = \sum \frac{S_U l}{AE} = \frac{l}{AE} \sum S_U = \frac{l}{AE} \frac{W_y}{4 \sin \alpha} \]

But \( W_y = W \cos \phi \)

\[ \therefore \delta_{oy} = \frac{l}{4AE} \frac{W}{\sin \alpha} \cos \phi \]

**Numerical Results:** For \( \alpha = 20^\circ \), \( A = 6 \text{ in}^2 \), \( E = 30 \times 10^6 \text{ psi} \),
\( l = 48" \), \( W = \frac{3500}{2} = 1750 \text{ lb} \)

\[ \delta_{oy} = 0.000997 \cos \phi \text{ in.} \]

<table>
<thead>
<tr>
<th>( \phi )</th>
<th>( \delta_{oy} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0°</td>
<td>0.0010&quot;</td>
</tr>
<tr>
<td>30°</td>
<td>0.00086&quot;</td>
</tr>
<tr>
<td>60°</td>
<td>0.0005&quot;</td>
</tr>
<tr>
<td>85°</td>
<td>0.00009&quot;</td>
</tr>
<tr>
<td>90°</td>
<td>0.00000&quot;</td>
</tr>
</tbody>
</table>
Support for Secondary Assembly

Zero Tilt Angle Condition:

Load on one section of truss:

\[ \sum F_V = 0 \]

\[ AB \sin 20° + AC \sin 20° - \frac{W}{4} = 0 \]

\[ AB = AC \]

\[ AB = AC = \frac{W}{8 \sin 20°} = 0.3654W \]

\[ U = \frac{AB}{W} \times 1 = \frac{4AB}{W} = \frac{1}{25 \sin 20°} = 1.462 \# \]

Deflection at Pt. A in plane ABC:

\[ \delta_A = \sum \frac{SUL}{AE} = \sum \frac{(AB)(4AB)l}{AE} = \sum \frac{WLl}{16AE} \frac{1}{\sin^2 20°} \]

\[ = 1.0686 \frac{WL}{AE} \]

\[ \text{I-13} \]
For \( E = 30 \times 10^6 \text{ psi}, \ A = 6 \text{ in}^2, \ l = 48'' \)
\[
\delta_a = 0.2667 \times 10^{-6} \text{ in},
\]

If \( W = 3500 \) lb

Then \( \delta_a = 0.00093 '' \)

\( AB = 1,279 \) lb

Check for buckling for both ends hinged:

\[
P' = \frac{\pi^2 EI}{l^2} = \frac{\pi^2 (30 \times 10^6) (\frac{l}{2})}{48^2} = 64,255 \text{ lb}
\]

Factor = \( \frac{P'}{AB} = 50 \), O.K.
TORSIONAL STIFFNESS OF SECONDARY TRUSS SUPPORT

\[ M = 6P \]
\[ P = \frac{M}{6} \]

Let \( M = 6 \times 10^6 \text{ in}-\text{lb} \).

Then \( P = 10^6 \) #

\[ 2AB \sin 20^\circ = P \]
\[ AB = \frac{P}{2 \sin 20^\circ} = 1.462 \times 10^6 \] #

\[ U = \frac{AB}{P} \times 1 = 1.462 \] #

\[ \delta_A = \sum \frac{SUL}{AE} = \frac{2 \left[ 1.462 \times 10^6 \times 1.462 \times 48 \right]}{6 \times 30 \times 10^6} = 1.14'' \]

\[ \theta = \frac{\delta_A}{3} = .38 \]

\[ K_t = \frac{M}{\theta} = \frac{6 \times 10^6}{.38} = 15.8 \times 10^6 \text{ in}-\text{lb}/\text{rad.} \]

Too low
\[ \overrightarrow{OA} = \overrightarrow{BO} \]

\[ 2 \overrightarrow{OA} \sin 20^\circ = \overrightarrow{F} \]

\[ \overrightarrow{OA} = \frac{\overrightarrow{F}}{2 \sin 20^\circ} = 1.462 \overrightarrow{F}, \text{ Tension} \]

\[ \overrightarrow{BO} = -1.462 \overrightarrow{F}, \text{ Compression} \]

\[ U_A = \frac{\overrightarrow{OA} \cdot \overrightarrow{l}}{F} = 1.462 \# , \text{ Tension} \]

\[ U_B = \frac{\overrightarrow{BO} \cdot \overrightarrow{l}}{F} = -1.462 \# , \text{ Compression} \]

\[ \Delta = \frac{\sum SU \cdot l}{AE} = \frac{\overrightarrow{l}}{AE} \sum SU = \frac{\overrightarrow{l}}{AE} \left[ 1.462 \overrightarrow{F} + (-1.462) \overrightarrow{F} \right] \]

\[ = 4.274 \frac{lF}{AE} = 3.8 \times 10^{-6} \text{ in,} \]

\[ \Theta = \frac{\Delta}{62.5} = 0.0608 \times 10^{-6} \]

\[ K_e = \frac{M}{\theta} = 2.056 \times 10^6 \text{ in-lb/} \text{rad} \]
SECONDARY TRUSS DEFORMATIONS

EL. ANGLE

EL. ANGLE = 0°

\[ W = 4500 \text{ #} \]
\[ A = 200 \text{ in} \]
\[ A = 16.015 \text{ in}^2 \]
\[ E = 30 \times 10^6 \]

\[ \delta = \frac{ESUL}{AC} = \frac{l}{AE} \]
\[ \varepsilon SU = \frac{200}{10.1 \times 10^6} \]
\[ \delta = 0.0138'' \]

EL. ANGLE

EL. ANGLE = 30°

\[ P_2 \cos 45° - P_1 \sin 75° - W = 0 \]
\[ P_1 \sin 75° - P_2 \sin 45° = 0 \]
\[ 0.71P_2 - 0.26P_1 - W = 0 \]
\[ 0.97P_1 - 0.71P_2 = 0 \]

\[ P_2 = 73P_1 \]
\[ 0.71P_2 - 0.19P_1 = 4500 \]
\[ 0.52P_2 = 4500 \]
\[ P_1 = 8700 \]

\[ P_1 = 63\pi \]
EL. ANGLE = $60^\circ$

\[ f = 4.14 \times 10^{-6} \times 1.9 \left( 8700 \right) \]
\[ = 0.0069'' \]

EL. ANGLE = $85^\circ$

\[ f = 4.14 \times 10^{-6} \times 1.9 \left( 3100 - 1550 \right) \]
\[ = 0.0012 \]

\[ P_2 \sin 80^\circ + P_1 \sin 70^\circ - W = 0 \]
\[ P_2 \sin 80^\circ - P_1 \sin 70^\circ = 0 \]
\[ P_2 = 0.17P_1 \]
\[ P_1 = 2P_2 \]
\[ 0.98P_2 + 0.94P_1 = 4500 \]
\[ P_1 = 4500 \]
\[ P_2 = 3100 \]
Primary Truss Deflections

\[ \begin{align*}
W &= 20,000 \text{#} \\
AB &= 90^\circ + AC \sin 40^\circ - W = 0 \\
0.64AB + 0.64AC &= 20,000 \\
AB &= AC = 15,625 \text{#}
\end{align*} \]

Let \( \delta_A @ 0^\circ \) el be same as secondary = 0.014

\[ \delta = \frac{lw}{AE} \quad \text{and} \quad l = 66'' \] 
\[ A = ? \] 
\[ E = 30 \times 10^6 \]

\[ A = \frac{66 \times 0.78 \times 31200}{30 \times 10^6 \times 1.4 \times 10^{-2}} \]
\[ A = 3.85 \text{in}^2 \]

Let \( D_0 = 5'' \)
\[ 3.85 = \frac{\pi}{4} (25 - D_i^2) \]
\[ 4 \times (3.85) = \pi (25 - D_i^2) \]
\[ D_i^2 = 25 - 4.9 = 20 \]
\[ D_i = 4.5 \]
\[ t = \frac{D_i}{2} = 2.25'' \]

4-19
Buckling of Primary Tubes

\[ A = 3.85 \text{ in}^2 \]
\[ l = 66 \text{ in} \]
\[ I = \frac{\pi d^4}{64} = \frac{\pi}{64} (625 - 441) = \frac{\pi}{64} (184) = 10.6 \text{ in}^4 \]
\[ t = \sqrt{\frac{I}{A}} = \sqrt{\frac{10.6}{3.85}} = 1.65 \]
\[ \frac{l}{t} = 40 \]
\[ S_{allow} = 16220 \#/\text{in}^2 \text{ (AISC)} \]
\[ S_{actual} = \frac{15600}{3.85} = 4060 \#/\text{in}^2 \text{ okay} \]

for a 6" O.D. tube

\[ A = \frac{\pi}{4} (L^2 - D_i^2) = 3.85 \]
\[ 36 - D_i^2 = 4.9 \]
\[ D_i^2 = 36 - 4.9 = 31.1 \]
\[ D_i = 5.58 \approx 5.6'' \]
\[ t = 0.2'' \]
\[ I = \frac{\pi}{64} (1350 - 985) = \frac{\pi}{64} (365) = 15.4 \]
\[ t = \sqrt{\frac{15.4}{3.85}} = \sqrt{4} = 2 \]
\[ \frac{l}{t} = 33 \]
\[ S_{allow} = 16470 \#/\text{in}^2 \text{ (AISC)} \]
\[ \text{very little change} \]
TORSIONAL STIFFNESS
of PRIMARY TRUSS
SUPPORT

\[ M = 194P \]
\[ P = \frac{M}{194} \]

Let \( M = 1,000,000 \) in-lb
\[ P = 7,000\# \]

\[ AB = 90^\circ + AC \cos 90^\circ - 7,000 = 0 \]

\[ 0.77AB + 0.77AC = 7,000 \]

\[ AB = AC \]
\[ 1.58AB = 7,000 \]

\[ AB = 4,450 \]

\[ 1.54U_{AC} = 1 \]
\[ U_{AB} = U_{AC} = 0.65 \]
\[ s_A = \frac{\Sigma SUL}{AE} \]

<table>
<thead>
<tr>
<th>Member</th>
<th>S</th>
<th>U</th>
<th>L</th>
<th>SUL</th>
<th>SUL/\AE</th>
<th>A</th>
</tr>
</thead>
<tbody>
<tr>
<td>AB</td>
<td>445</td>
<td>65</td>
<td>66</td>
<td>190x10^6</td>
<td>.0021</td>
<td>3.85in^-2</td>
</tr>
<tr>
<td>AC</td>
<td>445</td>
<td>65</td>
<td>66</td>
<td>190x10^6</td>
<td>.0021</td>
<td>3.85in^-2</td>
</tr>
</tbody>
</table>

\[ \delta = .0032'' \]

\[ \theta = \frac{2V_{1}L_{02}}{144} = \frac{42x10^{-4}}{72} = .45x10^{-4} \]

\[ K_t = \frac{M_t}{\theta} = \frac{1x10^6}{.45x10^{-4}} = 2.20 \times 10^{10} = \frac{22000 \times 10^6}{\text{rad}} \]

\[ \text{Japprox} = 250,000 \]

\[ f_m = \frac{1}{2\pi} \sqrt{\frac{22000 \times 10^6}{250,000}} \cdot \frac{1}{2\pi} \sqrt{8.8 \times 10^6} = 47 \text{cycles/see} \]
## WEIGHT & INERTIA BREAKDOWN OF SECONDARY CAGE ASSEMBLY

$I_{1-1}$ is about mass center

<table>
<thead>
<tr>
<th>Item</th>
<th>Weight</th>
<th>$I_o$</th>
<th>$md^2$</th>
<th>$I_{1-1} = I_o + md^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coudé-Cassegrain Mirror</td>
<td>400#</td>
<td>$\frac{W \times R^2}{g \times 4} = \frac{400 \times 16^2}{386 \times 4}$</td>
<td>$\frac{400 \times 24^2}{386}$</td>
<td>661 in-#-sec$^2$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>= 66 in-#-sec$^2$</td>
<td>= 595 in-#-sec$^2$</td>
<td></td>
</tr>
<tr>
<td>Flip Cell</td>
<td>200#</td>
<td>$\frac{W \times R^2}{g \times 2} = \frac{200 \times 18^2}{386 \times 2}$</td>
<td>$\frac{200 \times 24^2}{386}$</td>
<td>381</td>
</tr>
<tr>
<td></td>
<td></td>
<td>= 84</td>
<td>= 297</td>
<td></td>
</tr>
<tr>
<td>Cell Support</td>
<td>250#</td>
<td>$\frac{250 \times 22^2}{386 \times 2}$</td>
<td>$\frac{250 \times 24^2}{386}$</td>
<td>527</td>
</tr>
<tr>
<td></td>
<td></td>
<td>= 156</td>
<td>= 371</td>
<td></td>
</tr>
<tr>
<td>Linear Screw</td>
<td>50#</td>
<td>$\frac{50}{386 \times 12} \times 15^2$</td>
<td>$\frac{50 \times 24^2}{386}$</td>
<td>77</td>
</tr>
<tr>
<td></td>
<td></td>
<td>= 3</td>
<td>= 74</td>
<td></td>
</tr>
<tr>
<td>Flip Cell Drives</td>
<td>100#</td>
<td>$\frac{100}{386 \times 24^2}$</td>
<td>$\frac{100 \times 24^2}{386}$</td>
<td>148</td>
</tr>
<tr>
<td></td>
<td></td>
<td>= 148</td>
<td>= 148</td>
<td></td>
</tr>
<tr>
<td>Cage Tube</td>
<td>1700#</td>
<td>$\frac{W \left( \frac{L^2}{12} + \frac{R^2}{2} \right)}{g}$</td>
<td>$\frac{300 \times 57^2}{386}$</td>
<td>5340</td>
</tr>
<tr>
<td></td>
<td></td>
<td>= $\frac{1400 \times 65^2}{386 \times 12} + \frac{25^2}{2}$</td>
<td>= 2520</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>+ $\frac{300 \times 52^2}{386 \times 12} + \frac{25^2}{2}$</td>
<td>= 2820</td>
<td></td>
</tr>
<tr>
<td>Controls and TV Console</td>
<td>150#</td>
<td>$\frac{150}{386 \times 59^2}$</td>
<td>$\frac{150 \times 59^2}{386}$</td>
<td>1350</td>
</tr>
<tr>
<td></td>
<td></td>
<td>= 1350</td>
<td>= 1350</td>
<td></td>
</tr>
<tr>
<td>Item</td>
<td>Weight</td>
<td>$I_0$</td>
<td>$md^2$</td>
<td>$I_{1-1} = I_0 + md^2$</td>
</tr>
<tr>
<td>----------------------------------</td>
<td>--------</td>
<td>---------------</td>
<td>------------</td>
<td>------------------------</td>
</tr>
<tr>
<td>Pedestal and Seat</td>
<td>100#</td>
<td>$\frac{100}{386} \times 39^2$</td>
<td>$393$</td>
<td></td>
</tr>
<tr>
<td>Cell Support Plates</td>
<td>500#</td>
<td>$\frac{500}{386} \times \frac{23^2}{2}$</td>
<td>$340$</td>
<td>$\frac{500}{386} \times 24^2 = 750$</td>
</tr>
<tr>
<td>Vanes</td>
<td>800#</td>
<td>$\frac{800}{386} \times 10^2$</td>
<td>$207$</td>
<td></td>
</tr>
<tr>
<td>Secondary Truss Support Ring</td>
<td>1700#</td>
<td>$\frac{1700}{386} \times \frac{70^2}{2}$</td>
<td>$10800$</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>5950#</td>
<td></td>
<td></td>
<td>$20,974$ in-#-sec$^2$</td>
</tr>
</tbody>
</table>
TORSIONAL STIFFNESS
OF SECONDARY TRUSS SUPPORT

\[ M = 144P \]
\[ P = \frac{M}{144} \]
\[ \text{let } M = 1 \times 10^6 \text{ in} - \text{lb} \]
\[ P = 7000 \text{ lb} \]

\[ \begin{align*}
AB & \approx 13.2^\circ \\
AC & \approx 13.2^\circ \\
A & \approx 7000 \text{ lb}
\end{align*} \]

\[ AB \cos 13.2^\circ + AC \cos 13.2^\circ - 7000 = 0 \]
\[ 0.973 AB + 0.973 AC = 7000 \]
\[ 1.946 AB = 7000 \]
\[ AB = AC = 3600 \text{ lb} \]

\[ 1.946 U_{AB} = 1 \]
\[ U_{AB} = 0.516 \text{ lb} = U_{AC} \]

\[ \delta_A = \frac{\varepsilon S L}{AE} = \frac{2 \times (3600)(0.516)(2.20)}{10.07 \times 30 \times 10^6} = 2700 \times 10^{-6} \]

\[ \delta = \frac{2700 \times 10^{-6}}{72} = 37.6 \times 10^{-6} \]

\[ K_T = \frac{110 \times 10^6}{37.6 \times 10^{-6}} = 2.65 \times 10^{10} \text{ in} - \text{lb/rad} \]

\[ J_{support} = 47,000 \]

\[ f_n = \frac{1}{2\pi} \sqrt{\frac{2.65 \times 10^{10}}{0.047 \times 10^6}} = \frac{1}{2\pi} \sqrt{58.5 \times 10^4} = \frac{750}{2\pi} = 120 \text{ cycles/} \text{sec} \]
ELEVATION DRIVE ANALYSIS

\[ J = 3.5 \times 10^6 \text{ in}-\# \cdot \text{sec}^2 = 29 \times 10^4 \text{ ft}-\# \cdot \text{sec}^2 \]

for 900 ft-lb torque

\[ \text{max. accel.} = \frac{900 \text{ ft-lb}}{29 \times 10^4 \text{ ft}-\# \cdot \text{sec}^2} = \frac{900}{29 \times 10^4} \text{ ft/sec}^2 = 0.0031 \text{ rad/sec}^2 \]

\[ T_f = \frac{\mu}{2 \times \text{bearing load}} \]

assume \( \mu = 0.0015 \)

for a ball bearing \( \mu = 0.0015 \)

\[ T_f = 0.0015 \times 8 \times 80000 = 960 \text{ in}-\# = 8 \text{ ft}-\# \]

If \( L = 10^{-1} \text{ milliradian/sec}^2 = 0.0001 \text{ rad/sec}^2 \),

\[ T = 3.5 \times 10^6 \times 1 \times 10^{-9} = 350 \text{ in}-\# \]

\[ \theta = \frac{M_1 L}{J_\phi} \]

\[ I = \frac{\pi}{2} \times 10^3 \times 2 = 6282 \]

\[ \theta = \frac{35 \times 10^6}{6282 \times 10^3} = 0.56 \times 10^{-3} \text{ rad} = 0.56 \times 10^{-3} \text{ mile} = 112 \times 10^{-6} \text{ sec} \]

\[ \theta = 0.0112 \text{ sec} \]

\[ K_t = \frac{T_\phi}{L} = \frac{6282 \times 12 \times 10^6}{12} = 6282 \times 10^6 \]

\[ f_m = \frac{1}{2\pi} \sqrt{\frac{6282 \times 10^6}{3.5 \times 10^6}} = \frac{1}{2\pi} \sqrt{1800} = 6.8 \text{ cycle/sec} \]

A too low
NEXT SIZE TORQUER 3000 ft-lb

\[ I_p = \pi \times 13.5^3 \times 2 = \pi \times 1820 \times 2 = 11400 \]

\[ K t = \frac{11400 \times 12 \times 10^6}{10} = 13700 \times 10^6 \]

\[ f_m = \frac{1}{2\pi} \sqrt{\frac{13700 \times 10^6}{3.5 \times 10^6}} = \frac{1}{2\pi} \sqrt{3950} \approx 10 \text{ cfs} \text{sec} \]

for two torquers, \( K t = 2 \times 13700 \times 10^6 = 27400 \times 10^6 \text{ in.} \text{lb} \text{rad}^{-1} \)

SPRING CONSTANTS

for ELEVATION DRIVE BOX

\[ \theta = \frac{Ml}{12EI} \]

\[ I \approx \frac{1}{12} (16 \times 48^3 - 14 \times 46^3) \]

\[ I \approx \frac{4 \times 10^3}{12} \approx 33330 \text{ in.}^2 \]

\[ K t = \frac{M}{\theta} = \frac{24 \times 30 \times 10^6 \times 33330}{54} = 440000 \text{ in.} \text{lb} \text{rad}^{-1} \]
wt. of ELEVATION BOX

\[ wt = \left[ \left( \frac{150^2 - \pi \times 125^2}{4} \right) \times 1 + 98 \times (5.281) \right] \times 3 + 2\pi \times 2.5 \times 1 \times 48 \times 3 \]

\[ wt = \left[ (20900) + 25000 \right] \times 3 + 5900 = 19520 \]
## Weights and Inertias About Elevation Axis

**Telescope Assembly**

<table>
<thead>
<tr>
<th>Item</th>
<th>Weight</th>
<th>$I_o$</th>
<th>$md^2$</th>
<th>$I_o + md^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Secondary Cage Assy.</td>
<td>6000#</td>
<td>$\frac{W}{g} \left( \frac{L^2}{12} + \frac{R^2}{4} \right)$</td>
<td>$\frac{6000}{386} \left( \frac{117^2}{12} + \frac{25^2}{4} \right)$</td>
<td>$6000 (230)^2$</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>15.5 (53,000)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$= 20,000 \text{ in}-#-\text{sec}^2$</td>
<td></td>
</tr>
<tr>
<td>Secondary Truss</td>
<td>6000#</td>
<td>$\frac{6000}{386} \left( \frac{200^2}{12} \right)$</td>
<td>$\frac{6000}{386} (125)^2$</td>
<td>$= 294,000 \text{ in}-#-\text{sec}^2$</td>
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<td></td>
<td></td>
<td></td>
<td>52,000 \text{ in}-#-\text{sec}^2</td>
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<tr>
<td>Elevation Box Structure</td>
<td>20,000#</td>
<td>$\frac{20,000}{386} \left( \frac{48^2 + 150^2}{12} \right)$</td>
<td>$\frac{20,000}{386} (100)^2$</td>
<td>$= 108,000 \text{ in}-#-\text{sec}^2$</td>
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<tr>
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<td>108,000</td>
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<tr>
<td>Primary Truss</td>
<td>610#</td>
<td>$\frac{610}{386} \left( \frac{66^2}{12} \right)$</td>
<td>$\frac{610}{386} (50)^2$</td>
<td>$= 4490$</td>
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</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>570</td>
<td></td>
</tr>
<tr>
<td>Primary Mirror</td>
<td>17,200#</td>
<td>$\frac{17200}{386} \left( \frac{20^2 + 120^2}{12} \right)$</td>
<td>$\frac{17200}{386} (57)^2$</td>
<td>$= 322,000$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>170,000</td>
<td></td>
</tr>
<tr>
<td>Primary Mirror Cell</td>
<td>6,000#</td>
<td>$\frac{6000}{386} \left( \frac{25^2 + 65^2}{4} \right)$</td>
<td>$\frac{6000}{386} (60)^2$</td>
<td>$= 73,000$</td>
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<td>17,000</td>
<td></td>
</tr>
<tr>
<td>Axial Support &amp; Cwt.</td>
<td>9,000#</td>
<td>$\frac{9000}{386} \left( \frac{20^2 + 65^2}{12} \right)$</td>
<td>$\frac{9000}{386} (85)^2$</td>
<td>$= 193,400$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>25,400</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>64,810#</td>
<td>392,970</td>
<td>1,441,920</td>
<td></td>
</tr>
<tr>
<td>Add 15% For Misc.</td>
<td>74,500#</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Upright Spring Constant

Scale 1" = 50"

Take \( I_{AA} \) as average moment of inertia.
Assume walls ½" thick.

\[
I_{AA} = \frac{42(83)^3 - 41(82)^3}{12} = \frac{42(575 \times 10^3) - 41(550 \times 10^3)}{12}
\]

\[
I_{AA} = \frac{(240000 - 225000)}{12} = 126000 \text{ in}^4
\]

\[
\Theta_A = \frac{ML^2}{2EI}
\]

\[
K = \frac{M}{\Theta_A} = \frac{2EI}{L^2} = \frac{60 \times 10^6 \times 126000}{125^2} = 490 \times 10^6 \text{ in-lb}
\]

\( \text{Too low} \)
Assume wall 1" thick

\[ I_{A-A} = \frac{50(74)^3 - 48(72)^3}{12} = \frac{(20300 - 18000)10^3}{12} \]

\[ = \frac{2300 \times 10^3}{12} = 192,000 \text{ in}^4 \text{ (still too small)} \]

Increase depth of cross section to 85" and 1" thick wall

\[ I_{A-A} = \frac{42(85)^3 - 40(83)^3}{12} = \frac{(26700 - 22800)10^3}{12} \]

\[ I_{A-A} = \frac{3900 \times 10^3}{12} = 325,000 \text{ in}^4 \]

\[ \kappa = \frac{326 \times 490}{126} = 1280 \text{ in}^3/\text{sec} \]

If both sides of square were effective

\[ \kappa_e = 2560 \times 10^6 \]

\[ f_n = 1.4 \times 3.0 = 4.2 \text{ cycles/see} \]

Rotation from \( p \) to \( q \)

\[ \Theta = \frac{Ml}{4A^2G} \phi \frac{ds}{t} \]

\[ \phi \frac{ds}{t} = \frac{2 \times 48 + 2 \times 95}{1} = 286 \]

\[ A = 47 \times 94 = 4400 \]

\[ \kappa = \frac{4A^2G}{l \phi \frac{ds}{t}} = \frac{4 \times 19.4 \times 10^6 \times 12 \times 10^6}{286 \times 4.0} \]

\[ \kappa = 32600 \times 10^6 \text{ in}^3/\text{sec} \]

\[ \Theta = \frac{3.26 \times 10^6}{\text{rad}} \]
INCREASE DEPTH OF CROSS SECTION TO 83" AND 1" THICK WALL. REDUCE LENGTH FROM 125" TO 112"

\[ I_{A-A} = \frac{35(4.5)^3 - 30(4.3)^3}{12} = \frac{(21500 - 17200)10^3}{12} \]

\[ I_{A-A} = \frac{4300 \times 10^3}{12} = 360,000 \text{ in}^4 \]

\[ K_t = \frac{60 \times 360,000 \times 10^6}{12600} = 1700 \text{ in} - \text{in}-\text{#}/\text{rad} \]

WITH BOTH SIDES OF YOKE UPRIGHTS EFFECTIVE

\[ K_t = 3400 \times 10^6 \text{ in} - \text{in}-\text{#}/\text{rad} \]

Rotation from

\[ \theta = \frac{Ml}{4A^2G} \phi \frac{di}{E} \]

\[ \phi \frac{di}{E} = \frac{2 \times 39 + 2 \times 100}{1} = 276 \]

\[ A = 37 \times 99 = 3660 \]

\[ K = \frac{4A^2G}{l \phi \frac{di}{E}} = \frac{4 \times 13.4 \times 10^6 \times 12 \times 10^6}{276 \times 30} \]

\[ K = 77500 \text{ in} - \text{in}-\text{#}/\text{rad} \]
WEIGHT OF UPRIGHTS

\[ A = \frac{1}{4} (36 + 60) 160x2 + \frac{1}{4} (95 + 50) 140x2 \]

\[ A = 15400 + 20500 = 35900 \text{ in}^2 \]

\[ \text{wt} = 35900 \times 1.3 = 10800 \# \]

\[ 2 \times 10800 = 21600 \# \text{ for two uprights} \]

WEIGHT OF TURNTABLE

\[ \text{wt} = \left( (150 \times 55) ^2 + (95 \times 55) ^2 + (95 \times 150) ^2 \right) \times 1.3 \]

\[ \text{wt} = \left[ 16500 + 10500 + 14200 \right] \times 1.3 = 12900 \# \]

wt. of 3000 ft.-# motor = 1200 #

wt. of ELEV. BRG'S =

\[ 27 \times 30.5 \times 2.5 \times 2 \times 1.2 \times 1.3 = 304 \# \]

wt. of ELEV. DC TACH = 51 #

wt. of DECLINATION

axis, coude, focus

MIRRORS

\[ \text{wt} = \frac{\pi}{4} (27^2) \times 0.08 + \frac{\pi}{4} (22^2) \times 0.08 \]

\[ \text{wt} = 184 + 61 = 245 \# \]

ASSUME 1000 # including support structure

wt. of LASER = ASSUME 3000 #

\[ 21600 + 1 \frac{9}{4} 000 + 1200 + 3 \times 51 = 43000 \# \]

add 10% for unknowns

\[ = 43000 \times 1.1 = 39500 \# \]

Total wt on g. & y. = 43000 + 39500 = 122500 #
AZIMUTH HYDROSTATIC BEARING

3 ROW CONFIGURATION

18 PADS EQUALLY SPACED

12 PADS EQUALLY SPACED

TYPICAL PAD

WIDTH OF RECESS $y = 0.75''$
LENGTH OF $x = 12''$
WIDTH OF PAD $X = 14''$
LENGTH OF PAD $Y = 3$
MEAN WIDTH $b_1 = 1.87''$
MEAN LENGTH $b_2 = 13''$
$L_1 = 1''$
$L_2 = 1.12''$
for total vertical load $W$, use 125,000#

$$W = P_L(x_1y + r_L(3y-xy))$$

$$W = P_L(12x.75) + P_L(3x14 - 12x.75)$$

$$W = 9P_L + \frac{P_L}{2}(42-9) = 9P_L + 16.5P_L = 25.5P_L$$

for 1 pad

$$W = 1(25.5)P_L$$

$$W = \frac{125,000}{18} = 7000\#/pad$$

$$P_L = \frac{7000}{25.5} = 273\#/in^2$$

$$Q = \frac{P_Lh^3}{12\mu} \left( \frac{2b_2}{L_2} + \frac{2b_1}{L_1} \right)$$

$$h = 0.002$$

$$W = 9 \times 10^{-4} Nya's\left(\frac{1}{in^2}\right)$$

for SAE 20 oil @ 100°C

$$Q = \frac{500 \times 8 \times 10^{-9}}{12 \times 9 \times 10^{-6}} \left( \frac{2 \times 13}{1.12} + 2 \times 1.57 \right) = 22.3 \times 10^{-3} (26.99) = 1\ in^3/sec$$

$$Q = 18\ in^3/sec$$ for bottom row for 18 pads

1 ft $P_L = 500\# / in^2$ for bottom row
for equilibrium, top row $P_L = 200\# / in^2$

$$Q = 18 \times \frac{2}{5} = 7.2\ in^3/sec$$ for top row

for side row $P_L = 200\# / in^2$

$$Q = \frac{12}{18} \times 7.2 = 4.8\ in^3/sec$$

$$\varepsilon Q = \frac{30\ in^3/sec \times 30 \times 60}{231} = 7.8\ gal/min$$
2 ROW
STEPPED CONFIGURATION

LOWER PAD DIMENSIONS:
- Width of recess $y = 1''$
- Length $x = 16''$
- Length of pad $x' = 20''$
- Width $y' = 5''$
- Mean width $b_1 = 3''$
- Mean length $b_2 = 17''$
- $L_1 = 3''$
- $L_2 = 3''$

$X - x = Y - y$

$\frac{x}{y} = 2.75$

$\frac{y}{y'} = 0.14$

UPPER PAD DIMENSIONS:
- Width of recess $y = 1''$
- Length of recess $x = 16''$
- Length of pad $x' = 20''$
- Width $y' = 5''$
- Mean width $b_1 = 3''$
- Mean length $b_2 = 18''$
- $L_1 = 2''$
- $L_2 = 2''$

$X - x = Y - y$

$\frac{x}{y} = 4$

$\frac{y}{x} = 0.25$
ASSUME UPPER PAD PRESSURE = 300#/in$^2$ = $p_L$

$$W = p_L \left( xL \right) + \frac{p_L}{2} \left( xL - xH \right)$$

$$W = 300 \left( 16 \right) + 150 \left( 140 - 16 \right)$$

$$W = 4800 + 12600 = 17400 \# / \text{pad}$$

VERTICAL LOAD / PAD = 17400 \times 0.707 = 12,200 \# DUE TO UPPER PAD PRESSURE

VERTICAL LOAD / PAD = \frac{125,000 \#}{6} = 21,000 \# DUE TO DEAD WT

TOTAL VERT LOAD / PAD = 33,200 \#

For lower pad: $W = \frac{p_L}{2} \left( xL - xH \right)$

$$33200 = \frac{p_L}{2} \left( 16L - 14 \right)$$

$$33200 = 14 \frac{p_L}{2} + 13 \frac{p_L}{2} = 77 \frac{p_L}{2}$$

$$p_L = 430 \# / \text{in}^2$$

STIFFNESS / PAD:

BULK MODULUS of FLUID $N = \Delta p$

for lower pad: $\Delta p_{avg} = 200 \# / \text{in}^2$

Assume 75% of area is effective

$$A = 140 \text{in}^2 \times 0.75 = 100 \text{in}^2$$

$$Y = 100 \times 0.003 = .03 \text{in}^3$$

$$F = NA$$

$$\delta = \frac{Y}{A}$$

$$K = \frac{F}{\delta} = \frac{NA}{\frac{Y}{A}} = \frac{NA^2}{Y}$$

A-37
\[ K_1 = \frac{200 \times 10^2}{30 \times 10^{-3}} - \frac{200 \times 1.70 \times 10^4}{30 \times 10^{-3}} = \frac{66 \times 10^6}{\text{lb/lin}} \]

For upper pad:\ \[ \Delta F_{w,g} = 150 \text{ lb/lin}^2 \]

\[ A = 100 \text{ in}^2 \times 75 \text{ in} = 75 \text{ in}^3 \]

\[ Y = 75 \times 0.003 = 0.221 \text{ in}^3 \]

\[ K = \frac{100 \times 75^2}{22 \times 10^{-3}} = 26 \times 10^6 \text{ lb/lin} \]

\[ K_2 = 0.707 \times 26 \times 10^6 = 18.4 \times 10^6 \text{ lb/lin} \]

\[ F_2 = 39 \]

\[ F_1 = -39 \]

\[ 39F_1 + 39F_2 = M \]

\[ F_1 = F_2 = F \]

\[ 72F = M \]

\[ F = \frac{M}{78} \]

Let \( M = 100 \times 10^6 \text{ in-lb} = \# \)

\[ F = \frac{100 \times 10^6}{78} = 1.3 \times 10^6 \text{ #} \]

\[ \delta_1 = \frac{F}{K_1} \]

\[ K_1 = \frac{1300 \times 10^3}{66 \times 10^6} = 20 \times 10^{-3} \]
\[ \delta_2 = \frac{1300 \times 10^3}{18.4 \times 10^6} = 7.1 \times 10^{-3} \]

\[ \Theta = \frac{(70 + 20) \times 10^{-3}}{78} = \frac{900 \times 10^{-4}}{78} = 11.5 \times 10^{-4} \text{ rad} \]

\[ K_{bearing} = \frac{M}{\Theta} = \frac{100 \times 10^6}{11.5 \times 10^{-4}} = 8.6 \times 10^{10} \text{ in.-lb} \]

\[ J_0 \text{ (approx.)} = 3.5 \times 10^6 + \frac{80,000 \times 140^2}{386} + \frac{21,600 \times 95^2}{386} \]

\[ = 3.5 \times 10^6 + 7.4 \times 10^6 + 0.5 \times 10^6 = 11.4 \times 10^6 \text{ in.-lb} \]

\[ \text{Bearing} \quad f_m = \frac{1}{2\pi} \sqrt{\frac{8.6 \times 10^{10}}{11.4 \times 10^6}} = \frac{1}{2\pi} \sqrt{750000} = 87 \approx 14 \text{ cycles/sec} \]

\text{TYPICAL LOWER PAD (} \frac{3}{8}'' = 1''\text{)}
for Lower Pad:
\[
Q = \frac{P_h h^3}{12 \mu} \left( \frac{2b_2}{L_2} + \frac{2b_1}{L_1} \right) \quad \text{Use } P_h = 500 \text{#/#}
\]
\[
Q = \frac{500 \times 6 \times 10^{-9}}{12 \times 9 \times 10^{-6}} \left( \frac{2 \times 17}{3} + \frac{2 \times 4}{3} \right)
\]
\[
Q = \frac{400 \times 10^{-3}}{108 \times 3} (34 + 8) = 12.4 \times 10^{-3} (42) = 0.52
\]
\[
Q = 1.52 \text{in}^3/\text{sec}
\]

for Upper Pad:
\[
Q = \frac{300 \times 8 \times 10^{-9}}{12 \times 4 \times 10^{-6}} \left( \frac{2 \times 18}{2} + \frac{2 \times 3}{2} \right)
\]
\[
Q = \frac{2400 \times 10^{-3}}{108} (21) = 465 \times 10^{-3} = 0.465 \text{in}^3/\text{sec}
\]

\[\leq Q \leq 1 \text{in}^3/\text{sec} \times 6 = 6 \text{in}^3/\text{sec} \times \frac{60}{231} = 1.6 \text{gal/min}\]

If h is tolerated 0.002 ± 0.005

\[h_{max} = 0.0025\]

\[Q_{max} = \frac{2.5^3}{2^3} (1.6) = 3.1 \text{gal/min}\]

for a 3½ ft. s; use a 10 gal/min pump

Note: 1. Stepped bearing is most efficient
2. On DWG's mean dia., increased from 7' to 8'
**Azimuth Axis**

**Spring Constants**

**Elevation Trunnion Assy:**

![Diagram of trunnion assembly](image)

\[ P = \frac{M}{160} \]

\[ M = 160P \]

\[ I_{\text{trunn at } b} = \pi \times 13.5^3 \times 1 = 7750 \]

\[ \delta = \frac{M \times 8^3}{160 \times 90 \times 10^6 \times 7750} = \frac{512 M}{14400 \times 7750 \times 10^6} = 4.6 \times 10^{-12} M \]

\[ \theta = \frac{2 \delta}{160} = \frac{\delta}{80} = 5.8 \times 10^{-14} M \]

\[ K_t = \frac{M}{\theta} = \frac{1}{5.8 \times 10^{-14}} = 173 \times 10^{12} \text{ in-mm/mrad} \]

(negligible)

**Uprights:**

![Diagram of uprights](image)

\[ \delta = \frac{M \times 125^3}{180 \times 90 \times 10^6 \times 326,000} = \frac{M \times 1.96 \times 10^6}{16,200 \times 326,000 \times 10^6} \]

\[ \delta = 3.7 \times 10^{-10} M \]

\[ \theta = \frac{2 \delta}{780} = \frac{\delta}{390} = 9.7 \times 10^{-10} M \]

\[ K_t = 2.4 \times 10^{19} \text{ mm}^{-2}/\text{mrad} \]
PEDESTAL BASE:

\[ \theta = \frac{m_1 \ell}{I_p b} \]

\[ I_p = 2 \times 680 \times 10^3 = 1360 \times 10^3 \]

\[ K_t = \frac{I_p c}{L} = \frac{1360 \times 10^3 \times 12 \times 10^6}{84} = \frac{194540 \times 10^6}{100} \]

\[ \frac{1}{K_c} = \frac{1}{240,000} + \frac{1}{194,000} \]

\[ \frac{200,000 \times 10^6}{K_c} = .83 + 1.03 = 1.86 \]

\[ K_c = 107,000 \times 10^6 \]

\[ f_m = \frac{1}{2\pi} \sqrt{\frac{107,000}{5}} = \frac{1}{2\pi} \sqrt{21300} \]

\[ f_m = \frac{146}{2\pi} = 23 \text{ cycles/sec} \]

**If 20% is allowed for shear deflection**

\[ f_m = 23 \sqrt{\frac{1}{12}} \approx 21 \text{ cycles/sec} \]
**FRICION TORQUES:**

**ELEV AXIS BAGS:**

\[ T = \frac{w d_m N}{2} \]

\[ w = 0.015 \]
\[ d_m = 32 \]
\[ N = 80,000 \text{ lb} \]

\[ T = 0.0015 \times 16 \times 80,000 \]
\[ T = 1920 \text{ in-lb} = 160 \text{ ft-lb} \]

**HYDROSTATIC BEARING:**

\[ F = w A_s \frac{v}{h} \]

\[ T = F x t = w A_s \frac{v^2}{h} \]

\[ w = 0.03 \text{ rad/sec} \]
\[ t = 42 \]
\[ h = 0.003 \]
\[ A_s = 12 \times (10^2) = 1200 \text{ in}^2 \]

\[ T = \frac{9 \times 10^{-6} \times 1200 \times 1760 \times 0.3}{3 \times 10^{-3}} = 190 \text{ in-lb} = 15.8 \text{ ft-lb} \]

**DC TORQUER:** 18 foot-pounds

**SEALS:** 10 foot-pounds

**TOTAL ELEV. FRICTION:** 180 ft-pounds

**TOTAL AZ. FRICTION:** 34 ft-pounds
### PERKIN-ELMER

#### WEIGHTS AND INERTIA'S (TELESCOPE @ 0° ELEV.)

**ABOUT AZIMUTH AXIS**

<table>
<thead>
<tr>
<th>Item</th>
<th>Weight</th>
<th>( I_0 )</th>
<th>( I_0 + m d^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Telescope Assembly</td>
<td>74,500#</td>
<td>( \frac{4000}{386}(95)^2 = 93500 )</td>
<td>( 2.1 \times 10^6 ) in-#-sec^2</td>
</tr>
<tr>
<td>Elev. Brg's Motor &amp; Tachometer</td>
<td>4000#</td>
<td>( \frac{21600}{386}(95)^2 )</td>
<td>0.0935 x 10^6</td>
</tr>
<tr>
<td>Uprights</td>
<td>21,600#</td>
<td>( \frac{21600}{386}(95)^2 )</td>
<td>0.5054 x 10^6</td>
</tr>
<tr>
<td>Coude' &amp; Cassegrain No.4 Mirror &amp; Support</td>
<td>2000#</td>
<td>( \frac{2000}{386}(125)^2 = 80,000 )</td>
<td>0.08 x 10^6</td>
</tr>
<tr>
<td>Laser and Support</td>
<td>3000#</td>
<td>( \frac{3000}{386}(125)^2 =120,000 )</td>
<td>0.12 x 10^6</td>
</tr>
<tr>
<td>Movable Cwt. Assy</td>
<td>5000#</td>
<td>( \frac{5000}{386}(70)^2 = 63500 )</td>
<td>0.0635 x 10^6</td>
</tr>
<tr>
<td>Turntable &amp; Lower Yoke</td>
<td>15000#</td>
<td>( \frac{15000}{386}(120^2 + 120^2) )</td>
<td>0.77 x 10^6</td>
</tr>
<tr>
<td>Hydrostatic Brg. Inner Race</td>
<td>2500#</td>
<td>( \frac{2500}{386}(48^2 + 42^2) )</td>
<td>0.026 x 10^6</td>
</tr>
<tr>
<td>Motor, Tach, Encoder Slip Ring, Hydraulic</td>
<td>2500#</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Rotary Joint Cables</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>130,100#</td>
<td></td>
<td>( 3.76 \times 10^6 ) in-#-sec^2</td>
</tr>
<tr>
<td>Add 15% For Miscellaneous</td>
<td>138,400#</td>
<td></td>
<td>( \frac{14}{10^6} ) in-#-sec^2</td>
</tr>
<tr>
<td>Item</td>
<td>Inertia</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-------------------------------------------</td>
<td>----------------------------------------------</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Secondary Cage Assembly</td>
<td>$\frac{4250}{386} \left( \frac{625}{2} \right) = 3400 \text{ in}-\text{#-sec}^2$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Secondary Truss Support Ring</td>
<td>$\frac{1700}{386} (70^2) = 21500$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Secondary Vanes</td>
<td>$\frac{800}{386} (2500) = 5200$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Elevation Box Structure</td>
<td>$\frac{20,000}{386} (65)^2 = 220,000$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Primary Truss</td>
<td>$\frac{610}{386} (65)^2 = 6700$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Secondary Truss</td>
<td>$\frac{6000}{386} (65)^2 = 6600$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Primary Mirror and Cell</td>
<td>$\frac{23200}{386} \left( \frac{65}{2} \right)^2 = 126,000$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cwt. Primary</td>
<td>$\frac{9000}{386} (50)^2 = 58,000$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Remaining Azimuth Items From Previous</td>
<td>$1.65 \times 10^6 \text{ in}-\text{#-sec}^2$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sheet</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>$2.16 \text{ in}-\text{#-sec}^2$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Add 10% to All Items Except Last</td>
<td>$2.2 \times 10^6 \text{ in}-\text{#-sec}^2$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
wt of BASE PEDESTAL ASSY

**OUTER BRG RACE:** \( W = 2\pi \times 51 \times 5.5 \times 5.5 \times 3 = 2900 \) #

**BRG PAD:** \( W = \pi (150^2 - 43^2) \times 3 \times 3 = 1840 \) #

**TOP FLANGE:** \( W = \pi (58^2 - 36^2) \times 3 \times 3 = 11900 \) #

**INNER RING:** \( W = 2\pi \times 42 \times 2 \times 16 \times 3 = 2550 \) #

**OUTER RING:** \( W = 2\pi \times 56 \times 2 \times 16 \times 3 = 3400 \) #

**BOTTOM FLANGE:** \( W = \pi (46^2 - 40^2) \times 2 \times 3 = 5200 \) #

**GUSSETS:** \( W = 5221\times 3(3/4)(2^9).3 = 2850 \) #

**JACKS:** \( W = \frac{\pi}{4} (6^2) 12 \times 3 \times 12 = 1230 \) #

**TOTAL:** \( = 31870 \) #

ADD 10% #

**TOTAL wt = 139,400 + 35,000 = 173,400 #**

TELESCOPE PED,
ASSY

**PEDESTAL BASE**

**SPRING CONSTANT (TO RISION)**

\( K_t = \frac{I_p \cdot G}{L} \)

\( I_p = 2\pi R^3 t = 2\pi \times 56^3 \times 2 = 2200 \times 10^3 \)

\( L = 20 \) "

\( K_t = \frac{2200 \times 10^3 \times 12 \times 10^6}{20} = 1,320,000 \times 10^6 \) in-lb #
LEVELING JACK & BASE ANALYSIS

Total wt of ped. approx. = 90 tons
Assume 12 support points

\[ \frac{90}{12} = 7.5 \text{ tons/Jack} \]

\[ \delta = \frac{SWL^3}{384EI} = \frac{5(150,000)16^3}{384 \times 30 \times 10^6 \times 14,243} \]

\[ \delta = \frac{750,000(4100)}{11500(14243 \times 10^6)} = 1.9 \times 10^{-6} \text{ in} \]

Okay

Axial stiffness of jack

\[ \delta = \frac{PL}{AE} \]

\[ M = 120P \]

\[ K_t = \frac{M}{\theta} \]

\[ K_t = \frac{120P}{\delta/60} \]

\[ K_t = \frac{7200P}{\delta} \]
\[
\delta = \frac{AE}{l}
\]

\[
K_t = \frac{7200 \ AE}{l}
\]

Let \(K_t = 100,000 \times 10^6 \text{ in}-\text{lb/rev}\)

\(l = 4''\)

\(E = 30 \times 10^6\)

\[
A = \frac{K_t \ l}{7200 \ E} = \frac{100,000 \times 10^6 \times 4}{7200 \times 30 \times 10^6} = \frac{4000}{2160}
\]

\[A = 1.85 \text{ in}^2\]

**Torsional Stiffness (Azimuth)**

**Leveling Jack Locations**

\[
M = 12P(60) = 720P
\]

\[K_t = \frac{M}{\delta} = \frac{720P}{\delta/60} = \frac{43200P}{\delta}
\]

\[l = 3''\]

\[
\delta = \frac{PL^3}{3EI}
\]

\[
I = \frac{\pi}{64} (625 - 39) = \frac{\pi}{64} (586) = 29
\]

\[
P = \frac{3EI}{L^3} = \frac{90 \times 10^6 \times 29}{27} = 90 \times 10^6
\]

\[A-48 \quad K_t = 43200 \times 90 \times 10^6 = 3888000 \times 10^6 \text{ in}-\text{lb/rev only}\]
MECHANICAL LEVELING

JACK ASSEMBLY
LOCATION OF LEVELING JACKS

MECHANICAL SCREW JACKS (3) FOR LEVELING

HYDRAULIC JACKS (9)

SCALE: 1" = 20"
Torsion of Lower Base

Angular Rotation due to Radial Load on Other Race of Hydrostatic BRG

Assumed Effective Section (.1" = 1"

\[ A = 52 + 72 + 50 + 33 = 207 \text{ in}^2 \]

\[ \overline{g} = \frac{52 + 72(11) + 50(21) + 33(22.5)}{207} \]

\[ \overline{g} = \frac{2632}{207} = 127'' \]

\[ F = .707(300)(100) = 21400\# \]

\[ \frac{F}{l} = \frac{21400}{48\pi \frac{2}{3}} = \frac{21400}{16\pi} = 425\#/\text{in} \]

\[ M_{c} = 425 \times 15 = 6400 \text{ in} - \text{lb/in (due to oil pressure)} \]

\[ P = \text{total load on BRG} = 125,000\# \]
\[ \theta = \frac{\Delta M_e R^2}{EI} \]
\[ \theta = \frac{550(51)^2}{30 \times 10^6(14.293)} = \frac{550 \times 2600}{30 \times 10^6 \times 14.293} \]
\[ \theta = 3.4 \times 10^{-6} \text{ rad} \]

Displacement of \( P \) = 18\( \theta \) = 18 \times 3.4 \times 10^{-6} = 0.000061" okay

**Radial Displacement of Base Due to Internal Pressure on Outer Ring**

\[ \sigma = \frac{a b}{r} \left( \frac{a^2 + b^2}{b^2 - a^2} + \mu \right) \]
\[ a = 24 \]
\[ b = 5.8 \]
\[ \mu = 0.3 \]
\[ \sigma = \frac{24(212)}{30 \times 10^6} \left( \frac{3936}{2084} \right) \]
\[ \sigma = 170 \times 10^{-6}(142) = 240 \times 10^{-6} = 0.00024" \]

Plate free to deform as a thin cylinder.
COUNTERWEIGHT
FOR BALANCING
WT. OF MAN IN
OBSERVER'S CAGE

LOCATION OF MAN FROM EL AXIS = 300"
WT. OF MAN = 225#

MOMENT = 300 x 225 = 67500 in.-#

WT. OF STL CWT EACH
= 48 x 12 x 12 x 3 = 2160#

\[
\frac{67500}{4200} = 16" 
\]

LOCATION OF
C'WTS

EL. AXIS

LINEAR ACTUATOR

SERVO MOTOR
DRIVE PACKAGE

C'W'T FULLY
EXTENDED

SCALE: 1" = 50"
TORSIONAL DEFLECTION OF A THIN PLATE

STRAIN ENERGY OF AN ELEMENT IN A THIN SHEET

\[ dU = \frac{V^2 dx}{2AG} \]

\[ f = \frac{dV}{dV} = \int \frac{2Vdvdx}{2AG} = \frac{1}{G} \int \frac{vdvdx}{A} \]

ASSUME THE WALL OF THE SHEET TO BE BROKEN UP INTO 360 SPOKES, DEFLECTION OF EACH SPOKE

\[ \delta = \frac{1}{360G} \int \frac{V}{A} \text{ d}x \]

WHERE \( V = \frac{\text{Torque}}{360R} \) AND R IS TAKEN AT 2° INCREMENTS

LENGTH OF EACH INCREMENT \( l = \frac{\text{Length}}{\text{Increment}} \)

\[ A = \tan 1° \]

\[ 1 + t = 3° \]

\[ A = 0.522R \]

\[ \delta = \frac{2T}{360G} \frac{1}{AR} \]

\[ \delta = \frac{T}{180G} \frac{1}{0.0522R^2} \]

\[ \delta = \frac{T}{4.96} \frac{1}{R^2} \]

\[ \delta = \frac{T}{128G} \frac{1}{A^2} \]

\[ \delta = \frac{T}{128G} \frac{1}{A^2} \]

FOR BASE:

<table>
<thead>
<tr>
<th>R</th>
<th>R²</th>
<th>1/R²</th>
<th>R</th>
<th>R²</th>
<th>1/R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>21</td>
<td>440</td>
<td>2.21x10⁻³</td>
<td>39</td>
<td>1520</td>
<td>0.625x10⁻³</td>
</tr>
<tr>
<td>23</td>
<td>530</td>
<td>1.89x10⁻³</td>
<td>41</td>
<td>1680</td>
<td>0.610x10⁻³</td>
</tr>
<tr>
<td>25</td>
<td>625</td>
<td>1.60x10⁻³</td>
<td>43</td>
<td>1840</td>
<td>0.540x10⁻³</td>
</tr>
<tr>
<td>27</td>
<td>730</td>
<td>1.37x10⁻³</td>
<td>45</td>
<td>2020</td>
<td>0.445x10⁻³</td>
</tr>
<tr>
<td>29</td>
<td>840</td>
<td>1.19x10⁻³</td>
<td>47</td>
<td>2200</td>
<td>0.450x10⁻³</td>
</tr>
<tr>
<td>31</td>
<td>960</td>
<td>1.04x10⁻³</td>
<td>49</td>
<td>2400</td>
<td>0.42x10⁻³</td>
</tr>
<tr>
<td>33</td>
<td>1090</td>
<td>0.92x10⁻³</td>
<td>51</td>
<td>2600</td>
<td>0.39x10⁻³</td>
</tr>
<tr>
<td>35</td>
<td>1220</td>
<td>0.82x10⁻³</td>
<td>53</td>
<td>2800</td>
<td>0.34x10⁻³</td>
</tr>
<tr>
<td>37</td>
<td>1370</td>
<td>0.73x10⁻³</td>
<td>55</td>
<td>3000</td>
<td>0.33x10⁻³</td>
</tr>
<tr>
<td>39</td>
<td>1540</td>
<td>0.68x10⁻³</td>
<td>57</td>
<td>3200</td>
<td>0.32x10⁻³</td>
</tr>
</tbody>
</table>

\[ \Sigma = 11.82x10⁻³ \]

\[ \Sigma = 4.21x10⁻³ \]
\[ \delta_{21-37} = \frac{T}{E} \left( \frac{1}{9.4 \times 11.82 \times 10^{-3}} \right) = \frac{T}{12 \times 10^6} \times 1.25 \times 10^{-3} \]

\[ \delta_{21-37} = 1.05 \times 10^{-10} T \]

\[ \Theta_{21-37} = \frac{1.05 \times 10^{-10} T}{37} = 0.285 \times 10^{-10} T \]

\[ \delta_{39-55} = \frac{\Theta_{39-55}}{12 \times 10^6} \left( \frac{3.36 \times 10^{-3}}{0.01} \right) = \frac{1.28 \times 10^{-10} T}{55} = 0.005 \times 10^{-10} T \]

\[ \Theta_{\text{total}} = \left[ 0.285 + 0.005 \right] \times 10^{-10} T \]

\[ \Theta_{\text{total}} = 0.0336 \times 10^{-10} T \]

\[ K_e = \frac{1}{0.0336 \times 10^{-10}} = 30 \times 10^{10} = \frac{30,000,000}{10^{6}} \text{ in.-#/rad} \]

**Equivalence K_e for Pedestal Base**

\[ \frac{1}{K_e} = \frac{1}{1.32 \times 10^{12}} + \frac{1}{0.3 \times 10^{12}} \]

\[ \frac{10^{12}}{K_e} = .76 + 3.3 = 4.06 \]

\[ K_e = 0.245 \times 10^{12} = \frac{245,000}{10^{6}} \text{ in.-#/rad} \]

**For Turntable:**

**Using Same Analysis**

\[ K_e = 250,000 \text{ x} 10^{6} \text{ in.-#/rad} \]
FOR DRIVE PLATE ON ELEVATION BOX

t = 1"
A = .0179R

δ = \frac{I}{180G} \leq \frac{1}{.0179R^2}

δ = \frac{I}{3.14G} \leq \frac{1}{R^2}

from pt 16 to pt 24
2" INCREMENTS

<table>
<thead>
<tr>
<th>R</th>
<th>R^2</th>
<th>1/R^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>256</td>
<td>3.9 \times 10^{-3}</td>
</tr>
<tr>
<td>18</td>
<td>324</td>
<td>3.1 \times 10^{-3}</td>
</tr>
<tr>
<td>20</td>
<td>400</td>
<td>2.5 \times 10^{-3}</td>
</tr>
<tr>
<td>22</td>
<td>485</td>
<td>2.1 \times 10^{-3}</td>
</tr>
<tr>
<td>24</td>
<td>576</td>
<td>1.74 \times 10^{-3}</td>
</tr>
</tbody>
</table>

\delta = \frac{I}{3.14 \times 12 \times 10^6} (13.34 \times 10^{-3})

\delta = 0.355 \times 10^{-9} \tau

\theta = \frac{\delta}{24} = \frac{35.5 \times 10^{-11}}{24} = 1.48 \times 10^{-11} \tau

K_t = \frac{1}{1.48 \times 10^{-9}} = 0.675 \times 10^{-11}

K_t = 675 \times 10^6 \text{ in} \cdot \text{lb/rd}

Both sides of Box effect, K_t = 2(67500 \times 10^6) = 135000 \text{ in} \cdot \text{lb/rd}
**NO. 3 COUDE MIRROR SUPPORT**

Mirror wt. \( \approx 0.08 \left( \frac{\pi}{4} \right) (20^2)(.3) = 138 \) #

With Cell, Assume Total \( \approx 250 \) #

Wt of Top & Bottom Vanes = \( \frac{1}{4} \times 20 \times 2 \times 65 \times .3 = 195 \) #

Total Wt \( \approx 450 \) #

Main Optical Axis in Plane of Paper

**INERTIA'S ABOUT A'Z. AXIS**

\[ I_{\text{mirror}} = \frac{250 \times (25^2 + 26^2)}{12 \times 386} = 70 \text{ in}^2 \text{-in} \text{-sec}^2 \]

\[ I_{\text{vanes}} = \frac{200 \times \left( \frac{1^2}{386} + 20^2 \right)}{400 \times 4850} = 17 \text{ in}^2 \text{-in} \text{-sec}^2 \]

Total \( \approx 90 \text{ in}^2 \text{-in} \text{-sec}^2 \)

**K of Rods (Approx.)**

\[ M = 2P(44) = 88P \]

\[ \delta = \frac{PL}{AE} \]

\[ \theta = \frac{\delta}{L} = \frac{PL}{11AE} \]

A-57
\[ k_t = \frac{M}{\Theta} = \frac{88P}{11AE} = 970 \frac{AE}{I} \]

\[ l = 66'' \]
\[ A = 9.41\text{in}^2 \]
\[ I = 0.155\text{in}^4 \]

\[ k_t = 970 \frac{(4.41)30 \times 10^6}{66} = 195 \times 10^6 \text{in}^{-4}/\text{rad} \]

\[ f_m = \frac{1}{2\pi} \sqrt{\frac{195 \times 10^6}{90}} = \frac{1}{2\pi} \sqrt{2.17 \times 10^6} = \frac{1}{2\pi} (1270) \]

\[ f_m = 2.35 \text{cycles/sec} \quad \text{okay} \]

**Vibration of Rod as a Uniformly Loaded Fixed-Fixed Beam**

\[ f_m = K \sqrt{\frac{2EI}{W^2L^2}} \]

\[ W = A (L)(3) = 0.841(3)l = 0.132l \]
\[ W = \frac{W}{l} = 0.132 \text{#/in} \]

\[ K = 3.56 \]

\[ f_m = 3.56 \sqrt{\frac{286 \times 30 \times 10^6 \times 15.5 \times 10^{-6}}{132 \times 10^{-3} \times 190 \times 10^4}} \]

\[ f_m = 3.56 \sqrt{715} = 3.56 (8.45) = 30 \text{cycles/sec} \quad \text{okay} \]

**Max. Axial Deflection of Coudé Cell due to Gravity**

\[ I = \frac{1}{12} \times \frac{2.4^3}{l} = \frac{f_{rod}}{48} = 160 \]

\[ f = \frac{250 \times 55^3}{190 \times 11 \times 160} = 250 \times 166 \times 10^3 \times 570 \times 160 \times 10^4 = 0.045 \times 10^{-3} \text{in} \quad \text{okay} \]
ELEVATION SPRING CONSTANTS

(MAJOR CONTRIBUTORS TO OVERALL STIFFNESS)

<table>
<thead>
<tr>
<th>ITEM</th>
<th>K (in-lb/ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ELEVATION DRIVE</td>
<td>27,400 x 10^6</td>
</tr>
<tr>
<td>ELEVATION DRIVE UPRIGHTS</td>
<td>3400 x 10^6</td>
</tr>
<tr>
<td>HYDROMATIC BEARING</td>
<td>86000 x 10^6</td>
</tr>
<tr>
<td>PEDESTAL BASE</td>
<td>97500 x 10^6</td>
</tr>
<tr>
<td>ELEV. BOX</td>
<td>100,104 x 10^6</td>
</tr>
</tbody>
</table>

\[
\frac{10''}{K_e} = \frac{1}{27,400} + \frac{1}{3400} + \frac{1}{86,000} + \frac{1}{97,500} + \frac{1}{100,104}
\]

\[
\frac{10''}{K_e} = 3.65 + 2.95 + 1.16 + 1.02 = 8.88
\]

\[
K_e = 2800 \times 10^6 \text{ in-lb/ft}
\]

\[
J_{lew} = 2.1 \times 10^6 \text{ in-lb-sec}^2
\]

\[
\frac{1}{f^2} = \frac{1}{472} + \frac{1}{518^2} \quad \text{SECONDARY CONTRIBUTION IS SMALL}
\]

\[
\frac{1}{f^2} = \frac{1}{2200} + \frac{1}{38.6} = \frac{2800}{f^2} = 1 + 66 = 67
\]

\[
f^2 = 33
\]

\[
f = 5.7 \text{ cycles/sec}
\]

A-39
# Azimuth Spring Constants

(Major contributors to overall stiffness)

<table>
<thead>
<tr>
<th>Item</th>
<th>$K$ (in.*$^{-1}$/rad)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Uprights</td>
<td>$370,000 \times 10^6$</td>
</tr>
<tr>
<td>Pedestal base</td>
<td>$245,000 \times 10^6$</td>
</tr>
<tr>
<td>Azimuth drive</td>
<td>$164,000 \times 10^6$</td>
</tr>
<tr>
<td>Turntable</td>
<td>$250,000 \times 10^6$</td>
</tr>
</tbody>
</table>

\[
\frac{10^{12}}{K_e} = \frac{1}{370,000} + \frac{1}{245,000} + \frac{1}{164,000} + \frac{1}{250,000}
\]

\[
\frac{10^{12}}{K_e} = 2.7 + 4.1 + 6.1 + 4.0
\]

\[
K_e = \frac{1,000,000 \times 10^6}{16.1} = 60,000 \times 10^6 \text{ in.-ft./rad}
\]

\[
J = 4 \times 10^6 \text{ in.-ft.}-\text{sec}^2
\]

\[
f_m = \frac{1}{2\pi} \sqrt{\frac{60,000 \times 10^6}{4 \times 10^6}} = \frac{1}{2\pi} \sqrt{15,000}
\]

\[
f_m = 19.6 \text{ cycle/sec}
\]

Combination of primary & secondary masses with main mass, using Dunkelly's eq.

\[
\frac{1}{f^2} = \frac{1}{47^2} + \frac{1}{19.6} + \frac{1}{120^2}
\]

\[
\frac{1}{f^2} = \frac{1}{2200} + \frac{1}{385} + \frac{1}{14400}
\]

\[
f^2 = \frac{14,400}{45.1} = 317
\]

\[f = 17.8 \text{ cycle/sec}\]
APPENDIX B

INFRARED DETECTORS FOR TRACKING AND COMMUNICATION AT 10.6 MICRONS

The noise power of an extrinsic photodetector is twice that for a photo-emissive detector due to the random combination effect.\(^1\)

The noise current is given by:

\[ \overline{I}_n^2 = 4e \overline{I} \Delta f \]

where

- \( \overline{I}_n^2 \) is the average noise power (amps\(^2\))
- \( e \) is the electronic charge (coulombs)
- \( \overline{I} \) is the average current (amps)
- \( \Delta f \) is the measurement bandwidth (Hertz)

\( \overline{I} \) is related to the average power, \( \overline{P} \), incident upon the detector by:

\[ \overline{I} = \frac{e \eta_q \overline{P}}{hv} \]

where

- \( \eta_q \) is the quantum efficiency (electrons/photon)
- \( h \) is Planck's constant (joule-second)
- \( v \) is the wave frequency (Hertz)

Therefore:

\[ \overline{I}_n^2 = 4e^2 \Delta f \frac{\overline{P}}{hv} \eta_q \]

Calling $\bar{W}$ the power density, watts per unit area, and $A$ the detector area:

$$I_n^2 = 4e^2 \Delta f A \bar{W}/hv$$

but $\bar{W}/hv$ is $\bar{N}$ the average photon flux density (photon - cm$^{-2}$ - sec$^{-1}$); therefore,

$$I_n^2 = 4e^2 \Delta f A \bar{N} \eta_q$$

If $P_s$ is the signal power, which must impinge upon the detector such that the signal current power is equal to the noise power, the current due to this power is

$$I_s^2 = \left[ \frac{\eta_q e P_s}{hv} \right]^2 = I_n^2 = 4e^2 \eta_q A \bar{N} \Delta f$$

$$P_s = \sqrt{\frac{4e^2 \eta_q A \bar{N} \Delta f}{hv/\eta_q e}}$$

By the above definition, $P_s$ is the noise equivalent power. The detectivity is defined by:

$$D^* = \sqrt{\frac{A \Delta f}{P_s}}$$

Therefore:

$$D^* = \frac{\eta_q e}{hv \sqrt{4e^2 \eta_q N}} = \frac{1}{2hv} \sqrt{\frac{\eta_q}{N}}$$

So, for an ideal extrinsic photoconductor working at 10.6 microns with unity quantum efficiency:

$$D^* = \frac{2.5 \times 10^{19}}{\sqrt{N}}$$

(1)
The $D^*$ given by equation (1) is the ideal limit. The measured value of $D^*$ is always less than the ideal value due to the presence of amplifier noise, lattice noise, and contact noise. Until recently, the actual $D^*$ was one order of magnitude lower than the ideal at low flux levels. A new improvement in mercury-doped germanium detectors\(^2\) has brought the $D^*$ closer to the ideal. $D^*$'s of $2.5 \times 10^{13} \text{ cm(Hz)}^{1/2} \text{ watt}^{-1}$ have been observed at $5 \times 10^{11} \text{ photon/second/cm}^2$. Under these conditions, the ideal $D^*$ is $4.0 \times 10^{13}$. However, the higher values of $D^*$ are accompanied by larger time constants. Figures B-1 and B-2 show data taken for several photodetectors at Santa Barbara Research Center. The part number of the detector is shown next to the plotted points.

The present state of the art\(^3\) is about 10 pf of lead and detector capacitance. On the basis of semiconductor physics, the resistance of a square of photoconductive material can be given by $R = K/N$, where $K$ is a constant dependent upon carrier lifetime, transit time, and electronic mobility. From empirical data, the resistance can be found from $R = 10^{22}/N$ ohms per square at flux levels of $> 10^{10}$. At a flux level of $10^{14}$ photons/cm\(^2\)/second, the detector resistance will be 100 megohms and the time constant will be 100 microseconds, giving an equivalent frequency of 150 Hz. This upper frequency is still suitable for transfer mirror tracking.

The situation is better in the case of coherent detection, since the number of photons/cm\(^2\)/second impinging upon the detector is vastly increased.

Assuming 20 microwatts at 10.6 microns impinging upon a 0.3mm x 0.3mm detector\(^4\), the detector impedance would be 10 K ohms. This would give a 1.5 MHz bandwidth.

\(^3\)Private communication with Dr. Bode of Santa Barbara Research Center.
\(^4\)Perkin-Elmer Report No. 8393
Figure B-1. Dependence of Ge-Hg Detectivity as a Function of Background Level

Figure B-2. Dependence of Ge:Hg Detector Resistance as a Function of Background Level
Using the detector to drive a 50 Ω load would raise the response to 300 MHz, but would place a larger burden on the IF amplifier following the detector, since the signal would be reduced by a factor of 200.

It is interesting to note that it has been theoretically shown that a semiconductor driven by a microwave electric field can be used as a broadband photoconductive detector with high current gain. In an insulator photoconductor, the high frequency response is difficult to attain, and the gain bandwidth product is limited by dielectric relaxation time. Utilizing an rf bias, the contacts to the material need not be ohmic and the above restriction is alleviated. Therefore, the gain bandwidth product is not limited by material parameters, but by drive frequency.

Drive frequencies may be extremely high (in the GHz region) in order to obtain bandwidths in the MHz region. This condition can be ameliorated if an impedance transformation can also be performed which will provide additional gain. Ross indicates that, theoretically at least, the photoconductive detector with rf bias can operate like a photomultiplier in the quantum limited condition. This may yield a system with a bandwidth higher than 300 MHz, but remains an area for continued development.

---

5 Sommers, H.S., Jr., Teutsch, W.B., Proc. IEEE, 52, 144, (1964)
APPENDIX C

ANGULAR RATES AND ACCELERATIONS OF AN ALT-AZ MOUNT TRACKING A CELESTIAL TARGET

The possibility of utilizing an Alt-Az mounted telescope to track an object in deep space leads to an interest in the time variation of the angular rates and accelerations around the azimuth and elevation axes. The following discussion considers the geometry involved and derives expressions for azimuth angle, elevation angle, and the first two derivatives of each in terms of observer latitude, object declination, and time. Also included are expressions for angular position and angular rate about the line of sight, since these parameters are also important in some applications.

A computer program has been prepared which tabulates these quantities as functions of time for any desired combination of latitude and declination.

The basic geometry is shown in Figure C-1, which shows an observer at point 0 at latitude \( L \) observing a deep-space object at point \( P \). The polar axis intersects the celestial sphere at an angle \( L \) above the northern horizon, and is normal to the plane of the celestial equator, which intersects the horizontal along the east-west line.

If it is assumed that the motion of the object in question is essentially equivalent to that of a distant star, the observer will see the object follow a circular path across the sky parallel to the celestial equator and separated from it by the declination angle \( \delta \).

The position of the object with respect to the observer may be defined by an azimuth angle \( \alpha \), measured from north as shown, and by an elevation angle \( \eta \), measured up from the horizon. The rotation of the target around the line-of-sight may be defined by an angle \( \phi \), as shown.
Figure C-1. Basic Geometry
For a given combination of L and $\delta$, the values of $\alpha$, $\epsilon$, and $\phi$ depend only on $\theta$, the angular position of the object along its path. The angle $\theta$ is really a time variable, since it is plain that $\theta = \omega t$ where $\omega = \text{sidereal rate} = 2\pi$ radians/day. For purposes of subsequent analysis, we define $\theta$ such that an object rises in the east at a negative value of $\theta$ (negative time), crosses the north-south meridian when $\theta = 0$ ($t = 0$), and sets in the west at positive $\theta$ (positive time).

From Figure C-1, the following dimensions may be evaluated, assigning a value of unity to the radius of the sphere:

\[
\begin{align*}
(PR) &= \cos \delta \cos \theta \\
(PK) &= \cos \delta \cos \theta \cos L + \sin \delta \sin L \\
(KM) &= \cos \delta \cos \theta \sin L - \sin \delta \cos L \\
(MO) &= -\cos \delta \sin \theta \\
(KO) &= \left[ (KM)^2 + (MO)^2 \right]^{1/2} \\
(TP) &= \frac{PK}{KO} \\
(TU) &= \frac{KM}{KO} \frac{MO}{MO} \\
(UV) &= \frac{\sin \delta \cos L}{KO}/(KO)/(MO)
\end{align*}
\]

It is then apparent that

\[
\begin{align*}
\alpha &= 90^\circ + \tan^{-1} \left[ \frac{(KM)}{(MO)} \right] \\
\epsilon &= \tan^{-1} \left[ \frac{(PK)}{(KO)} \right] \tag{3} \\
\phi &= \tan^{-1} \left[ \frac{(TV) + (UV)}{(TP)} \right] \tag{4}
\end{align*}
\]

C-3
Equations (2), (3), and (4) could be differentiated to obtain angular rates, but it is somewhat simpler to consider Figure C-2, which shows the vector representing the earth's rotation about the polar axis resolved into components along the azimuth, elevation, and line-of-sight axis. As is evident from the figure,

\[
\dot{\alpha} = \omega \left[ \sin L + \cos L \sin (\alpha - 90^\circ) \tan \epsilon \right] \\
= \omega \left[ \sin L + \cos L \left( \frac{(KM)(PK)}{(KO)^2} \right) \right] \\
\dot{\epsilon} = \omega \cos L \cos (\alpha - 90^\circ) \\
= \omega \cos L \left( \frac{(MO)}{(KO)} \right) \\
\dot{\phi} = \omega \cos L \sin (\alpha - 90^\circ) \sec \epsilon \\
= \omega \cos L \left( \frac{(KM)}{(KO)^2} \right)
\]

Equations (5) and (6) may now be differentiated to obtain angular accelerations about the azimuth and elevation axes. The results of this differentiation are:

\[
\ddot{\alpha} = \omega^2 \left( \frac{(MO)}{(KO)(\cos L)} \right) \left\{ \frac{(KO)^2 \left[ (KM) \cos L + (PK) \sin L \right] + 2(KM)(PK)[(PR)-(KM)\sin L]}{(KO)^4} \right\} \\
\ddot{\epsilon} = \omega^2 \left( \frac{\cos L}{KO} \right) \left\{ \frac{-(KO)^2(PR) - (MO)^2(KM)\sin L - (PR)K0^2}{(KO)^3} \right\}
\]

Substitution of equation (1) and the relation \( \theta = \omega t \) into equations (2) through (9) yields the desired expressions in terms of latitude, declination, and time.

A computer program has been prepared which tabulates equations (2) through...
Figure C-2. Angular Velocity Components
(9) as functions of time for any desired combination of latitude and declination. The program also computes the maximum value of $\dot{\chi}$ and the time at which it occurs. The maximum value of $\ddot{\epsilon}$ occurs at time zero and may be read from the tabulation.

The attached figures show representative results obtained from the above equations with the aid of the computer program. (See Figures C-3 through C-9.)
Azimuth Angle vs. Time
at 20° North Latitude For Various Declination Angles

Figure C-3. Azimuth Angle Versus Time
Figure C-4. Maximum Azimuth Rate versus Declination Angle at various latitudes.

Declination Angle - Degrees

$\frac{\alpha_{\text{max}}}{\alpha_{\text{m}}}$

Maximum Azimuth Rate

Sidereal Rate
Figure C-5. Angular Rates vs. Time
Sidereal Rate

For $L = 20^\circ$
$\delta = 15^\circ$
Figure C-6. Angular Accelerations versus Time

For $L = 20^\circ$, $\delta = 15^\circ$
Figure C-7
Maximum Azimuth Rate
vs.
(Latitude - Declination)

\[ \dot{\chi}_{\text{Max}} \text{ - Millirad/Sec} \]

\( (L - \delta) \text{ in Degrees} \)

- \( L = 0^\circ \)
- \( L = 40^\circ \)
Figure C-8
Maximum Azimuth Acceleration
vs.
(Latitude - Declination)
For various latitudes

\( \dot{\alpha} \) - Milliradians per sec\(^2\)

- \( L = 0^\circ \)
- \( L = 20^\circ \)
- \( L = 40^\circ \)

\((L - \delta)\) in degrees
Figure C-9. Maximum Elevation Angle Acceleration vs. Latitude-Declination

(L - 6 in degrees)
APPENDIX D

TOWER ANALYSIS

The following analysis is a preliminary study of a tower suitable for the support of the telescope and its mount. The principal intention of the analysis is to show that a support tower of practical proportion is technically consistent with the performance objectives for the system.

A tower height of 100 feet has been assumed, since the consensus of available information is that this height is sufficient to avoid the bulk of the seeing degradation associated with ground-level turbulence. Specification of the tower height to be employed at a specific site would, of course, be based on careful seeing studies at that site.

PEDESTAL C.G.

Elev. Assy. wt ≈ 80,000 lbs.

Yoke & Base Assy. wt ≈ 100,000 lbs.

180,000 \( \bar{y} \) = 80,000(15) + 100,000(6)

\[ \bar{y} = \frac{180 \times 10^4}{18 \times 10^4} = 10' \]

Concrete Properties

\( w = 150 \text{ lb/ft}^3 \)

\( E = 3.0 \times 10^6 \times 144 = 4.3 \times 10^8 \text{ lb/ft}^2 \)

\( G = 3.3 \times 10^8 \text{ lb/ft}^2 \)

Soil Properties

\( E = 25,000 \text{ lb/in}^2 = 3.6 \times 10^6 \text{ lb/ft}^2 \)

Bearing pressure = 2000 lb/ft² (Sandy Clay)
Figure D-1. Tower
ROTATION ABOUT EL. AXIS

\[ I_{\text{Tower}} = \frac{16 \times 16^3 - 12 \times 12^3}{12} = \frac{(6.6 - 2.07)10^4}{12} = \frac{4.5 \times 10^4}{12} = 3.75 \times 10^3 \text{ft}^4 \]

Torsional stiffness of tower about elevation axis:

\[ \theta = \frac{M^2}{2EI} \quad \text{let } M = 100 \text{ ft-lb.} \]

\[ \theta = \frac{(100)(100)^2}{2(4.3 \times 10^8)(3.75 \times 10^3)} = \frac{1 \times 10^6}{32.3 \times 10^{11}} \]

\[ \theta = 0.031 \times 10^{-5} \text{rad/100 ft-lb.} \]

Max. elev. torque = 217 ft-lb.

\[ \theta = 0.031 \times 10^{-5} \times (217) = 0.068 \times 10^{-5} \text{rad} = 0.068 \times 10^{-2} \text{mils} \]

\[ \theta = 0.136 \text{ secs.} \]

Foundation stiffness:

\[ \alpha = \frac{3M(\mu^2 - 1)}{4\mu^2 E_s R^3 \chi} \]

\[ \mu = 3 = \frac{1}{\text{Poisson's ratio}} \]

\[ R = 10', \quad E_s = 3.6 \times 10^6 \text{lb/ft}^2 \]

\[ \chi = 1.7 \]

\[ M = 100 \text{ ft-lb} \]

\[ \alpha = \frac{3(100)(8)}{36 \times 3.6 \times 10^6 \times 10^3 \times 1.7} \]

\[ \alpha = \frac{2400}{220 \times 10^{12}} = 11 \times 10^{-9} \text{rad} = 11 \times 10^{-6} \text{mils} = 0.002 \text{ seconds} \]

ROTATION ABOUT AZ. AXIS

\[ K_{\text{tower}} = \frac{\pi^3 G}{12} = \frac{2(14)^3 3.3 \times 10^8}{100} \]

\[ K = \frac{2(2740)3.3 \times 10^8}{100} = 165 \times 10^8 \text{ ft-lb/ rad} \]

Max. Az. Torque \(\approx 150 \text{ ft-lb}\)

\[ \theta = \frac{150}{165 \times 10^8} = 0.9 \times 10^{-8} \text{ rad} = 0.9 \times 10^{-5} \text{ mils} \]

\[ \theta = 180 \times 10^{-5} \text{ secs} = 0.002 \text{ seconds} \]
$K_{soil} = \frac{16}{3} \left( \frac{w}{2W} \right) G_s R^3$  

$K = \frac{16}{3} \left( \frac{3}{2} \right) \cdot 1.36 \times 10^6 \times 10^3$

$K = 10.88 \times 10^9 \text{ ft-lb/rad}$

$\theta = \frac{150}{109 \times 10^8} = 1.37 \times 10^{-5} \text{ mils}$

$\theta \approx 0.003 \text{ seconds}$

**Torsional Natural Frequency About Azimuth Axis**

\[ f_n^2 = \frac{1}{4\pi^2} \left[ \frac{K_1 + K_2}{2J_1} + \frac{K_2}{2J_2} \pm \sqrt{\left( \frac{K_1 + K_2}{2J_1} + \frac{K_2}{2J_2} \right)^2 - \frac{K_1 K_2}{J_1 J_2}} \right] \]

$J_2 = J_p + \frac{1}{2} J_T$

$J_1 = J_F + \frac{1}{2} J_T$

$J_p = 5 \times 10^6 \text{ in-lb-sec}^2 = 42 \times 10^4 \text{ ft-lb-sec}^2$

$J_T = \frac{MS^2}{6} = \frac{WS^2}{6g}$

$W = (16 \times 16 - 12 \times 12)(100)(150) = 112(100)(150)$

$W = 1.68 \times 10^6 \text{ lbs.}$

$J_T = \frac{1.68 \times 10^6}{6 \times 32.2} (112) = 97.5 \times 10^4 \text{ ft-lb-sec}^2$

$J_F = \frac{W}{6g} S^2$

$W = 400 \times 5 \times 150 = 300,000 \text{ lbs.}$
\[ J_F = \frac{300,000}{6 \times 32.2} \times 400 = 620,000 \text{ ft-lb-sec}^2 \]

\[ K_1 = 10.9 \times 10^9 \text{ ft-lb/rad} \]

\[ K_2 = 16.5 \times 10^9 \text{ ft-lb/rad} \]

\[ J_2 = 42 \times 10^4 + \frac{1}{2} \times 97.5 \times 10^4 = 91 \times 10^4 \text{ ft-lb-sec}^2 \]

\[ J_1 = 62 \times 10^4 + 49 \times 10^4 = 111 \times 10^4 \text{ ft-lb-sec}^2 \]

\[
\begin{align*}
\frac{f_n^2}{4\pi^2} &= \frac{1}{4\pi^2} \left[ \frac{27.4 \times 10^9}{222 \times 10^4} + \frac{16.5 \times 10^9}{182 \times 10^4} \pm \sqrt{\left( \frac{27.4 \times 10^5}{222} + \frac{16.5 \times 10^5}{182} \right)^2 - \frac{10.9 \times 16.9 \times 10^{18}}{111 \times 91 \times 10^8}} \right] \\
\frac{f_n^2}{4\pi^2} &= \frac{1}{4\pi^2} \left[ 1.23 \times 10^4 + 0.9 \times 10^4 \pm \sqrt{4.6 \times 10^8 - 1.8 \times 10^8} \right] \\
\frac{f_n^2}{4\pi^2} &= \frac{1}{4\pi^2} \left[ 2.13 \times 10^4 \pm \sqrt{2.8 \times 10^8} \right] = \frac{1}{4\pi^2} \left[ 2.13 \times 10^4 \pm 1.7 \times 10^4 \right] \\
\frac{f_n^2}{4\pi^2} &= \frac{1}{4\pi^2} (0.43 \times 10^4) = 107
\end{align*}
\]

\[ f_n = 10.4 \text{ cycles/second} \quad \text{1st Mode} \]

**Lateral Natural Frequency**

\[
\delta = \frac{M \ell^2}{2EI} + \frac{P l^3}{3EI} + \frac{W l^3}{8EI}
\]

\[
\delta = \frac{\ell^2}{2EI} \left[ M + \frac{2}{3} P l + \frac{W l}{4} \right]
\]

\[ M = 1.8 \times 10^6 \text{ ft-lb} \]

\[ P = 180,000 \text{ lb} \]

\[ W = 1.7 \times 10^6 \text{ lb} \]

D-5
\[
\delta = \frac{98^2}{860 \times 10^6 \times 3.75 \times 10^3} \left[ 1.8 \times 10^6 + \frac{2}{3} \times 180,000 \times 98 + \frac{1.7 \times 10^6 \times 98}{4} \right]
\]

\[
\delta = \frac{9600}{3240 \times 10^9} \left[ 1.8 \times 10^6 + 11.8 \times 10^6 + 41.5 \times 10^6 \right]
\]

\[
\delta = \frac{9600 \times 55.1 \times 10^6}{3240 \times 10^9} = 164 \times 10^{-3} = 0.164 \text{ ft.}
\]

**SHEAR TRANSLATION**

\[
\delta_S = F \left( \frac{PA}{AG} \right) + \frac{F}{2} \left( \frac{WA}{AG} \right)
\]

\[
F = 2
\]

\[
\delta_S = \frac{2PA}{AG} + \frac{WA}{AG}
\]

\[
\delta_S = \frac{A}{AG} \left[ 2P + W \right]
\]

\[
A = 112 \text{ in}^2
\]

\[
\delta_S = \frac{98}{12 \times 3.3 \times 10^8} \left[ 360,000 + 1.68 \times 10^6 \right]
\]

\[
\delta_S = \frac{98 \times 2.04 \times 10^6}{39.6 \times 10^8} = 5 \times 10^{-2} = 0.05 \text{ ft.}
\]

**FOUNDATION TILT**

Loc. of C.G. of PED., Tower, and Fdn.

\[
\bar{y} = \frac{(2,160,000) = 180,000 \times 120 + 1,680,000 \times 55 + 300,000 \times 2.5}{2.16\bar{y}} = 21.6 + 92 + 0.75 = 114.4
\]

\[
\bar{y} = 53 \text{ ft.}
\]

\[
\alpha = \frac{11 \times 10^{-9} \text{ rad}}{100 \text{ ft}-\text{lb}} \times 114.4 \times 10^6 \text{ ft-lb} = 12.6 \times 10^{-3} \text{ rad.}
\]

D-6
\[
\delta_c = 12.6 \times 10^{-3} \times 53 = 670 \times 10^{-3} = 0.67 \text{ ft.}
\]

Total Deflection = 0.164 + 0.05 + 0.67 = 0.884 ft.

\[
f = \frac{1}{2\pi} \sqrt{\frac{32.2}{0.884}} = \frac{1}{2\pi} \sqrt{36} = \frac{6}{2\pi} = 0.95 \text{ cycles/second}
\]