THE EVOLUTION OF THE SUN

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The sun, being the star nearest to us, is the only one whose disc can be resolved and whose outer structure can be studied with great precision. Yet despite its proximity, only a small amount of information regarding the past history and the future course of evolution of the sun comes from direct observation. Most knowledge about the evolution of the sun comes from indirect reasoning from planetary observations and from comparing the sun with other stars which are so distant that they appear as points of light even in the largest telescope.

To the naked eye, the sun appears to be a strongly illuminated disc with an angular diameter of thirty minutes of arc. Although during solar eclipse the sun is seen to possess an extended outer envelope known as the corona, which extends to several solar radii away from the surface of the sun, the mass of the corona is only a minute fraction of the entire mass.

Although the corpuscular radiation from the sun, in the form of solar cosmic rays and the solar wind, can seriously affect the condition of the earth's ionosphere and magnetosphere, the energy of these particles is insignificant compared to the total energy radiated by the sun. At the surface of the earth, each square centimeter of area receives two calories of energy per minute, enough to raise the temperature of one
gram of water by two degrees per minute. The sun is therefore the main energy source to the earth. At present, the energy output rate is surprisingly constant and so far no variations in the luminosity of the sun have been detected.

Fossil evidence from the pre-Cambrian era have indicated that life has existed on earth for more than two, and possibly three billion years. [1]. During this period of time, if the energy output of the sun varies by more than a factor of ten, for example, the development of elementary forms of life into higher ones will be hampered and may even be terminated. Thus, fossil evidence strongly suggests that the energy output of the sun has remained fairly constant in the past two billion years. From the present solar luminosity \( L_\odot = 4 \times 10^{33} \) ergs per second, the total energy output in the past two billion years is estimated to be \( 2.4 \times 10^{50} \) ergs. Averaged over the mass of the sun (solar mass \( \odot = 2 \times 10^{33} \) grams), this amounts to over \( 1.2 \times 10^{17} \) ergs per gram. This is over 100 times the gravitational energy of the present sun. Since the beginning of this century, the energy source has been a major problem of astrophysics. Since the development of nuclear physics, it has been accepted that the nuclear energy of hydrogen is the energy source of the sun and the stars.

Fossil evidence only gives the lower limit for the age of the sun. By using radioactive material in the rock as a clock,
the oldest rock on earth has been estimated to have an age of
over 3.2 billion years. Still, if the history of the earth is
not known, this number can hardly be used as the true age of
the sun. The oldest matter that men have been able to acquire
is — not too surprisingly — the meteorites. Recent deter-
mination of the age of iron meteorites gives an age of \(4.5 \times 10^9\)
years. [2] From discussions that will be given below, this
is the closest to the age of the sun.

Figure 1 shows the site where the oldest rock in North
American was found. [3]

**Formation of the Sun and the Planets**

There is good evidence from radio astronomy that inter-
stellar space is almost uniformly filled with hydrogen gas
throughout the Galaxy, at a density of one hydrogen atom per
cubic centimeter. For a gas cloud of uniform density \(\rho\),
dimension \(R\), mass \(M\) (\(=\frac{4\pi}{3}\rho R^3\)), the gravitational energy
\(E_G\) is given by:

\[
E_G = -\frac{GM^2}{R} = -\left(\frac{4\pi}{3}\right)^2 G \rho^2 R^5
\]  

(1)

where \(G\), the gravitational constant, is \(6.670 \times 10^{-8}\) in c. g. s.
units. The kinetic energy of the gas cloud \(E_T\), is:
\[ E_T = M R g T \]  

where \( R_g \) is the gas constant \((R_g = 8.27 \times 10^7 \text{ in c.g.s. units})\) and \( T \) is the kinetic temperature of the interstellar gas. The total energy \( E \) is the sum of \( E_T \) and \( E_g \):

\[ E = E_T + E_g = M R_g T \left( 1 - \frac{4\pi \rho R^2}{3 R_g T} \right) \]

\[ = M R_g T \left[ 1 - \left( \frac{R}{R_o} \right)^2 \right] \]

where

\[ R_o = \left( \frac{3 R_g T}{4\pi \rho} \right)^{\frac{1}{2}} \]

If \( R > R_o \), then \( E < 0 \), and the gas cloud is gravitationally bound. Equation 3 thus states that, if the dimension of the gas cloud is sufficiently large, then it can be bound by its own gravitational field. A gravitationally bound gas cloud can continue condensing to form stars and planets.

\( R_o \) generally decreases with decreasing \( T \) and increasing \( \rho \).

If we substitute into Equation 4 the observed value of the interstellar gas density (one hydrogen atom per cubic centimeter) and the observed interstellar temperature of \( 4^\circ \text{K} \), then \( R_o \) is around \( 10^{21} \) cm, or a little over 1000 light years.
The mass contained in this gas cloud is around 1000 solar masses. This is too large to become the solar system. On the other hand, there exists cosmic dust in interstellar space. This cosmic dust will contribute to $E_g$ but not to $E_T$. Thus, the dimension critical to condensation can become much less than that of $R_o$. It is generally believed that cosmic dust plays an important role in the formation of the solar system.

Equation 3 expresses what is known as the "necessary condition" for the formation of the sun and the planets. However, in a cloud, a magnetic field and rotation adds complications to the problem. So far, details about the formation of stars and the sun and planets are not well known.

As the cloud condenses further, the speed of rotation will increase and the shape of the cloud becomes flattened. Its density also increases, decreasing the value of $R_o$. Consequently, condensation of smaller bodies can take place. It is therefore believed that planets and the sun are formed at approximately the same time. In this manner the planets are formed at various distances from the sun. The present theory is able to explain the Titius-Bode law, which relates the location of planets relative to the sun. One of the proto-planets happened to have too little mass to form a complete planet, and the material never condensed. It became the astroid
belt. Hence, it is believed that the asteroids are the primordial matter left over from the formation of the solar system. The orbit of some of the asteroids intercept that of the earth's, and occasionally one or two asteroids plunge into earth's atmosphere. Many of them are completely destroyed during re-entry, but sometimes they are large enough to survive the re-entry heating. The remaining fragment is the meteorite. Thus, meteorites are of extremely high value to us in adding knowledge to the past history of the solar system.

As we have said, most meteorites have an age of $4.55 \times 10^9$ years, and none of them have an age greater than this value. This age is therefore adopted as the age of the solar system.

Hertzsprung-Russell Diagram (H-R diagram).

Two important parameters which characterize the structure of a star are the surface temperature $T_e$ and the luminosity $L$. The radius of a star can be obtained from $L$ and $T_e$ from the blackbody radiation law:

$$L = 4\pi R^2 \sigma T_e^4$$

where $\sigma$, the Steffan-Boltzmann radiation constant, is $5.67 \times 10^{-5}$ in c.g.s. system of units.
A plot of $L$ against $T_e$ for stars is known as the Hertzsprung-Russell diagram (H-R diagram), named after the two astronomers who first used them. Figure 2 shows the H-R diagram for some nearby stars. Figure 3 shows the nomenclature of stars in the H-R diagram.

Around ninety percent of all stars are found in a narrow strip cutting across the middle of the H-R diagram. This narrow strip is called the main sequence, and to it the sun belongs. About ten percent of the stars belong to a group of very high surface temperature and low luminosity ($L \propto 10^{-3} L_\odot$). From Equation 5 it is thus concluded that the radius of these stars is ten thousand kilometers (a few times the earth's radius). The mass of these stars is a third less than that of the sun. Consequently the density of these stars is very great, about $10^6$ grams (one ton) per cubic centimeter. These stars are known as white dwarfs. It is believed that this white dwarf population represents the graveyard of stars.

A small fraction of stars is found to have high luminosity and low surface temperature (hence a large radius). They are known as red giants. An even smaller fraction of stars is in the population known as the planetary nebula. These are stars of extremely high temperature (over one hundred thousand degrees) with most of their radiation in the ultraviolet. These stars appear to be in a state of ejecting mass, the ejected mass
forming a beautiful nebula surrounding the star, illuminated by the ultraviolet light from the central star. This nebula is called "planetary nebula", but they have nothing to do with planets.

There is also a sizeable population of stars in the "horizontal branch". These stars include the RR Lyrae type variables, which are stars whose luminosity varies regularly within the period of a few hours.

In the following we shall use the H-R diagram to discuss the history and the future course of evolution of the sun.

**Condensation of the Solar Nebula to form the Proto-Sun.**

One way to express the equilibrium condition for a star is the virial theorem, which states that the thermo-kinetic energy $E_T$ is roughly one half the gravitational energy $E_G$. For a given star, using Equations (1) and (2) for $E_T$ and $E_G$, we find the following relation between the interior temperature of a star and its radius:

\[
\frac{1}{2} \frac{GM}{R} = R_g T \tag{6}
\]

Thus, as a gas cloud loses energy through radiation, it contracts and its radius thus decreases. Paradoxically, however, its
temperature increases. Half the gravitational energy released is radiated by the star, but the other half is used to increase the temperature of the star. Thus, the temperature of the star increases as condensation proceeds. In this way the temperature of the solar nebula increases from \(4 \, ^\circ K\) to over ten million degrees.

The actual condensation process is not so simple, but we shall not be concerned with the detailed theory. Instead, we shall discuss the results. Ezer and Cameron have studied the early evolution of the sun, and have found that the sun follows a path nearly coinciding with that of the red giants. [4] This path is known as the Hayashi track, after the astrophysicist who first predicted this phenomena. However, the time during which the sun stays in the upper part of the Hayashi track is very short, being around a few thousand years or less. It is therefore believed that very few of the red giants are actually proto-stars contracting towards the main sequence. Figure 4 shows Ezer and Cameron's results for the evolution of the proto-sun. From this graph it can be seen that the temperature at the surface of the earth must have been at one time very hot.

The Sun as a Nuclear Furnace: The Beginning.

Substituting into Equation (4) the mass and the present
radius of the sun, we find that the interior temperature of the sun is presently around $10^7 \, ^\circ K$. As we have said, the radiation rate of the sun has remained practically constant in the past $2 \times 10^9$ years. This means that nuclear energy must be released to replenish the energy radiated at the surface of the sun.

In 1938 Bethe and Critchfield suggested the following reaction sequence for the generation of nuclear energy from hydrogen burning processes: [5]

\[
p + p \rightarrow D^2 + e^+ + \nu_e \\
D^2 + p \rightarrow \text{He}^3 \\
\text{He}^3 + \text{He}^3 \rightarrow \text{He}^4 + 2p
\]

(7)

In short, hydrogen is converted into helium. This is accompanied by an energy release amounting to $6 \times 10^{18}$ ergs per gram of hydrogen. If the structure of the sun were not affected by nuclear fuel depletion, the total nuclear energy content from hydrogen burning could keep the sun shining for over one hundred billion years! This is however, an over-optimistic calculation, for as we shall see, after around ten percent of the hydrogen of the sun is used up, the structure of the sun will change drastically. In any case, there is plenty of nuclear energy
available from hydrogen reactions.

In more massive stars, the proton-proton reaction sequence (5) is replaced by the more rapid carbon-nitrogen cycle, which was first suggested by Bethe in 1939: [6]

\[
\begin{align*}
  p + ^{12}\text{C} &\rightarrow ^{13}\text{N} \\
  ^{13}\text{N} &\rightarrow ^{13}\text{C} + e^+ + v_e \\
  p + ^{13}\text{C} &\rightarrow ^{14}\text{N} \\
  p + ^{14}\text{N} &\rightarrow ^{15}\text{O} \\
  ^{15}\text{O} &\rightarrow ^{15}\text{N} + e^+ + v_e \\
  p + ^{15}\text{N} &\rightarrow ^{12}\text{C} + \text{He}_4
\end{align*}
\]

In this reaction cycle, carbon catalyzes four protons to become a helium nucleus. In the sun, around ten percent of nuclear energy is released through the carbon-nitrogen cycle.

The Sun as a Nuclear Furnace: Hydrogen Exhaustion and Red Giants.

As hydrogen exhaustion proceeds in the core, the energy producing layer moves outwards, resulting in a gradual increase in the overall luminosity as well as in the radius. When ten percent of the hydrogen in the core is used up, the increase in both luminosity and radius suddenly speeds up. In a little
over ten million years the luminosity will increase to around one hundred to one thousand times its present value, and the radius will increase to almost one to three hundred times its present value. In the H-R diagram the sun will move along the red giant branch. When the sun becomes a red giant, its radius will have extended to beyond the orbit of Venus. The surface temperature of the earth will be a little over 800 K (500°C). All life forms on the earth will then become terminated, and all oceans will boil away. The estimated lifetime of the sun from the onset of hydrogen burning to its ultimate evolution to a red giant is around ten billion years. From the presently accepted age for the sun (4.5 billion years), we conclude that terrestrial life forms will disappear at the end of another 5.5 billion years.

Helium Reaction and Ultimate Fate of the Sun.

At the tip of the red giant branch the interior temperature of the sun will be around $10^8$ K and this is high enough to burn helium through the following reaction:

$$3 \text{He}^4 \rightarrow \text{C}^{12}$$

(9)

As helium burning proceeds, the sun moves along the red giant branch to the horizontal branch of the H-R diagram. Across the
gap the sun will become an RR Lyrae type variable star, its luminosity varying regularly within a period of a few hours. The sun will spend a few million years at this stage, as if it were sending out the last distress signals to the rest of the universe that it is dying.

**Planetary Nebulae and White Dwarfs.**

From this stage on, for a star like the sun, the death is rather uneventful. It is believed that the sun will become a planetary nebula, continuing to eject its mass for a period of around twenty thousand years. At this time, the sun emits a great number of neutrinos. Eventually, when the last bit of mass the sun can get rid of is gone, the sun quietly becomes a white dwarf. From then on the sun is nothing but a corpse and it will shine as long as its dead body is still warm. Figure 5 shows the evolution track of the sun in the H-R diagram and indicates the time scale at various stages.

When the sun becomes a white dwarf, it will still shine for another one billion years, but with greatly reduced intensity. It is not likely that after the scorching heat from the red giant stage, there will be any inhabitants on the earth. But if there were any, the sun will appear to them to be no larger than the planet Venus, and the brightness of the sun will be around one
thousand times that of a full moon. (The brightness of the present sun is one million times that of the moon.) Although reading by sunlight would still be possible, the chance of sustaining life is all but gone. The surface temperature of the earth would be only a few degrees above absolute zero. The sun, after having shone for ten billion years and sustained life on the fourth planet, for eons of time now will rest until eternity.
REFERENCES


FIGURE CAPTIONS

Figure 1. Morton Gneiss from the Minesota River Valley, southwestern Minnesota from which zircon, dated by the uranium-lead isotopic method, gives an age 3.6 billion years. (Courtesy of Professor Goldich [1]).

Figure 2. Hertzsprung-Russell diagram for a number of stars and clusters.

Figure 3. Nomenclature of stars in the H-R diagram for low mass stars.

Figure 4. Pre-main sequence contraction of the sun as computed by Ezer and Cameron [47]. The evolution times are indicated at several positions.

Figure 5. The evolution of the sun after leaving the main sequence. Compare with Figure 3. The arrows show the direction of evolution and the numbers refer to time at each stage.