ON THE CALCULATION OF AN INERTIAL NAVIGATION SYSTEM

by

N. Ya. Vovchenko

Moscow, Izdatel' stvo "Mashinostroeniye," 76-85 (1965)

Translated from the Russian

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A geometrical inertial navigation system with a gyrostabilized platform oriented along the axes of an inertial frame of reference and with an accelerometer platform oriented in a horizontal coordinate system finds use in the navigation near the earth. The problems of the theory of such systems are set forth by Fridlender¹, Karakashev², and Seleznev³. In the present article the equations of dynamics cited in one of the works⁴ are defined more precisely, and an evaluation of the systematic errors due to the translational and Coriolis accelerations is given.

A simplified diagram of a geometrical inertial navigation system is shown in Figure 1. The main elements of the system are the gyro-stabilized platform 1 which preserves its orientation of the axes relative to inertial space with the help of three quadratic integrating float gyroscopes 2, and the platform 3 with two accelerometers $A_x$ and $A_y$ which defines the normal of the system and which simulates the geographical coordinate system in an object. The platform with the accelerometers is an analog of Schuler pendulum and is set into the local normal with the help of a servodrive.

We will examine the operation of the system during the motion of an object in a great circle relative to the earth at the velocity $W$.

We will introduce the frames of reference. The inertial frame of reference $\xi\eta\zeta$ has the following orientation of the axes (Figure 2). The axis $\zeta$ is oriented upward, parallel to the earth's axis of rotation. At the initial instant of time, the axis $\eta$ is oriented toward the east. The axis $\xi$ forms a right-handed coordinate system with the axes $\zeta$ and $\epsilon$ and is oriented toward the north at the initial instant.

We will take the coordinate system $\xi_0\eta_0\zeta_0$ as the horizontal coordinate system. The axis $\zeta_0$ is oriented upward along the geocentric normal. The axis $\xi_0$ lies in a horizontal plane and is oriented toward the east. The axis $\eta_0$ lies in a horizontal plane, is oriented toward the north, and forms a right-handed coordinate system with the axes $\zeta_0$ and $\epsilon_0$.

⁴ Karakashev, loc. cit.
1 - The Gyroscopic Platform; 2 - The Float Integrating Gyroscopes; 3 - The Platform with the Accelerometers.

Figure 1. A Simplified Kinematic Diagram of a Geometric Navigation System
Figure 2. The Coordinate Systems Chosen for the Calculation
We will couple with the gyroplatform the system of the coordinates \( X_g, Y_g, \) and \( Z_g \) the axes of which at a zero value of the angles \( \psi_1, \psi_2, \) and \( \psi_3 \) are arranged in the following manner. The axis \( Z_g \) is oriented upward along the rotation axis of the gyroplatform relative to the inner ring of the gimbal. The axis \( Y_g \) coincides with the rotation axis of the inner in the outer ring of the gimbal. The axis \( X_g \) coincides with the rotation axis of the outer ring, relative to the framework of the device, and forms a right-handed coordinate system with the axes \( Y_g \) and \( Z_g. \)

We will couple the coordinate system \( xyz \) with the accelerometer platform. The axis \( Z \) is oriented upward, perpendicularly to the platform. The axis \( X \) lies in the plane of the platform and is oriented toward the east along the rotation axis of the platform. The axis \( Y \) lies in the plane of the platform and forms a right-handed coordinate system with the axes \( X \) and \( Z. \) The axis \( Y \) is oriented toward the north.

As shown in one of the works the precession equations of motion of the gyroplatform are written in a simplified form in the following manner:

\[
\begin{align*}
\psi_1 &= \frac{1}{H} \int_0^t M_{V_3}(t) \, dt, \\
\psi_2 &= \frac{1}{H} \int_0^t M_{V_1}(t) \, dt, \\
\psi_3 &= \frac{1}{H} \int_0^t M_{V_2}(t) \, dt,
\end{align*}
\]

where \( \psi_1, \psi_2, \) and \( \psi_3 \) are the angles of deflection of the system of the coordinates \( X_g, Y_g, \) and \( Z_g \) from the coordinate system \( \xi \); \( H \) is the kinetic moment of the gyroscope rotor and \( M_{V_1}, M_{V_2}, \) and \( M_{V_3} \) are the moments around the output axes of the gyroscopes, which cause a departure of the gyroplatform in the course of time from the inertial frame of reference.

The platform with the accelerometers has two degrees of freedom relative to the gyroplatform. It can rotate about the axes \( Ox \) and \( Oz. \)

In Figure 3 is shown a block diagram of the measuring system with accelerometers for two channels. The signals of the accelerations

\[\text{Karakaşhev, loc. cit.}\]
Figure 3. The Block Diagram of the Measuring System with the Accelerometers for Two Channels.

\[ W_a(p) = \text{The transfer function of the accelerometer.} \]
\[ W_i(p) = \text{The transfer function of the integrator.} \]
\[ W_c(p) = \text{The transfer function of the correction device.} \]
\[ W_{ch}(p) = \text{The transfer function of the clockwork.} \]
\[ W_s(p) = \text{The transfer function of the adding device.} \]
\[ W_u(p) = \text{The transfer function of the amplifier.} \]
\[ W_{sp}(p) = \text{The transfer function of the servodrive.} \]
with which the accelerometer platform moves in the directions \( x \) and \( y \) are sent to the input of the channels and the absolute angles \( \alpha_{abs} \) and \( \beta_{abs} \) of the turn of the platform about the respective axes \( z_g \) and \( x \), i.e., the turns of the platform relative to the inertial frame of reference are received at the output. A feedback circuit with the transfer function 
\[
\frac{1}{R_p^2}
\]
registers the turn of the horizontal coordinate system \( \xi_0 \) relative to the inertial \( \xi_0^\xi \). The difference between the angles \( \alpha_{abs} - \alpha_{trans} = \alpha \) and \( \beta_{abs} - \beta_{trans} = \beta \) constitutes the angle of inclination of the accelerometer platform relative to the horizontal plane. Because of this inclination, the accelerometers measure the components \( g\alpha \) and \( g\beta \) of the vertical acceleration \( g \).

Denoting by \( W_a(p) \), \( W_i(p) \), \( W_u(p) \), \( W_sp(p) \), \( W_c(p) \), \( W_{ch}(p) \), and \( W_s(p) \) the transfer functions of the elements: accelerometer, integrator, amplifier, servodrive, correction, clock, and adding mechanisms respectively; denoting by \( a_x \) and \( a_y \) the signals received at the input of the accelerometers \( A_x \) and \( A_y \) and denoting by \( \omega_3 \) the angular velocity of the earth's rotation we will obtain, in accordance with the block diagram shown in Figure 3, the following equations of the accelerometer platform for each one of the channels:

\[
\begin{align*}
\alpha_{abs} &= \left[ (a_x - ga)W_a(p)W_i^2(p)W_{cx}(p) + \omega_3 W_{ch}(p) \right] \times W_s(p)W_u(p)W_{sp}(p); \\
\beta_{abs} &= (a_y - g\beta)W_a(p)W_i^2(p)W_{cy}(p)W_u(p)W_{sp}(p) .
\end{align*}
\]

The angle \( \beta_{abs} \) is read counterclockwise if viewed from the side of the positive direction of the axis \( x \). The minus sign in front of the \( \beta_{abs} \) is explained by the fact that integration of a positive value of the acceleration \( a_x \) leads to a decrease of the angle \( \beta_{abs} \).

We will take into account that

\[
\begin{align*}
\alpha_{abs} &= \frac{\omega_zg}{p} \text{ and } \beta_{abs} = \frac{\omega_x}{p},
\end{align*}
\]

where \( \alpha_{abs} \) and \( \beta_{abs} \) are the angles of deflection of the axes \( x \) and \( y \) of the accelerometer platform relative to the axes \( \eta \) and \( \varepsilon \) of the inertial frame of reference, and where \( \omega_x \) and \( \omega_zg \) represent the angular

\*i = integrator, ch = clockwork, c = correction device, s = adding device, u = amplifier, sp = servodrive
velocity of the rotation of the accelerometer platform about the axes $x$ and $zg$ relative to the vertical coordinate system.

The transfer functions of the separate elements of the system are as follows:

For the accelerometers

$$W_a(p) = \frac{k_a}{T_{1a}p^2 + T_{2a}p + 1},$$

where $k_a$ is the transfer ratio of the accelerometer and $T_{1a}$ and $T_{2a}$ are the time constants of the accelerometer.

For the integrators

$$W_i(p) = \frac{k_i}{(T_i p + 1) p},$$

where $k_i$ is the transfer ratio of the integrator and $T_i$ is the time constant of the integrating drive.

For the correction devices

$$W_{cx}(p) = \frac{k_c}{\cos \varphi} \quad \text{and} \quad W_{cv}(p) = k_c,$$

where $k_c$ is the transfer ratio of the correction device.

For the amplifiers

$$W_u(p) = \frac{k_u}{T_up + 1},$$

where $k_u$ is the amplification factor of the amplifier and $T_u$ is the time constant of the amplifier.

For the clockwork

$$W_{ch}(p) = \frac{k_{ch}}{p},$$

where $k_{ch}$ is the transfer ratio of the clockwork.
For the adding device

\[ W_s(p) = k_s, \]

(8)

where \( k_s \) is the transfer ratio of the adding device.

For the servodrives

\[ W_{sp}(p) = \frac{k_{sp}}{T_{sp}p + 1}, \]

(9)

where \( T_{sp} \) is the time constant of the servodrive and \( k_{sp} \) is the transfer ratio of the servodrive.

We will write the equations (2) in the following form:

\[
\frac{\omega_{zg}}{p} = \left( a_x - g \alpha \right) \frac{k_a k_i^2 k_c}{p^2(T_{1a} p^2 + T_{2a} p + 1)} \left( T_i p + 1 \right)^2 \cos \varphi + \omega_3 \frac{k_{ch}}{p} + \frac{k_{sp} k_u k_{sp}}{T_u p + 1} \frac{k_s k_u k_{sp}}{T_{sp} p + 1},
\]

(10)

Usually the system is designed in such a manner that the lag of the separate elements does not affect materially the result of the measurement. To achieve this, the parameters \( T_{1a}, T_{2a}, T_i, T_u, \) and \( T_{sp} \) of the system's elements must be sufficiently small. Without taking the lag of the system's elements into account the equations (10) assume the form:

\[
\frac{\omega_{zg}}{p} = \left[ (a_x - g \alpha) \frac{k_a k_i^2 k_c}{p^2 \cos \varphi} + \omega_3 \frac{k_{ch}}{p} \right] \frac{k_s k_u k_{sp}}{T_u p + 1} \frac{k_s k_u k_{sp}}{T_{sp} p + 1};
\]

(11)

\[- \frac{\omega_x}{p} = (a_y - g \beta) \frac{k_a k_i^2 k_c k_u k_{sp}}{p^2}. \]

We will pass on to an examination of the dependence of the output signals of the accelerometers on the parameters of the object's motion. Signals measured by the accelerometers are determined in the following
manner. The platform with the accelerometers participates in a complex motion. The absolute acceleration of the platform is determined by the relative, translational, and Coriolis accelerations.

During the motion in a great circle, the relative acceleration is dependent on the motion of the object relative to the earth. During the motion of the object without a variation in the altitude of the flight, the relative acceleration consists of the normal or centripetal acceleration directed along the radius of the earth toward its center, and tangential acceleration directed along the travelling velocity of the object.

Translational acceleration appears due to the earth’s rotation. It also consists of the normal and tangential acceleration. The vector of normal acceleration lies in the meridian plane and is directed perpendicularly to the earth’s axis of rotation. The vector of tangential acceleration is directed perpendicularly to the vectors of centripetal acceleration and angular velocity of the earth’s rotation. Usually this acceleration may be neglected since the earth rotates almost uniformly.

Coriolis acceleration develops due to the earth’s rotation and the motion of the object in a great circle. The vector of this acceleration is perpendicular to the plane in which the vectors of the earth’s rotation lie and to the relative velocity of the motion of the object, and is set in that direction where the point of the relative-velocity vector tends under the effect of angular velocity.

In the projections on the axes $\xi_0$ and $\eta_0$ of the horizontal coordinate system, the absolute acceleration acting on the accelerometer platform has the following form according to Fridlender:\(^6\)

\[
\begin{align*}
\alpha_{\xi_0} &= \dot{W}_E - 2\omega_3 \sin \varphi \, W_N - \frac{WE}{R} \sec \varphi ; \\
\alpha_{\eta_0} &= \dot{W}_N + \frac{1}{2} R \omega_3^2 \sin 2 \varphi + 2 \omega_3 \sin \varphi \, W_E + \frac{W_E^2}{R \cos \varphi} .
\end{align*}
\]

We will introduce the following notations:

\[
\Delta \alpha_{\xi_0} = - 2\omega_3 \sin \varphi \, W_N - \frac{WE}{R} \sec \varphi ;
\]

\(^6\)Fridlender, loc. cit.
\[ \Delta a_{\eta_0} = \frac{1}{2} R \omega_3^2 \sin 2 \varphi + 2 \omega_3 \sin \varphi W_E + \frac{W_E^2}{R \cos \varphi}. \]  

(13)

In that case we have

\[ a_{\epsilon_0} = \dot{W}_E + \Delta a_{\epsilon_0}; \]

\[ a_{\eta_0} = \dot{W}_N + \Delta a_{\eta_0}. \]  

(14)

If it is taken into consideration that under the actual conditions the angles \( \alpha \), \( \beta \), and \( \delta \) and the quantities \( \dot{\alpha} \), \( \dot{\beta} \), and \( \dot{\delta} \) are small, then the sines of the angles may be replaced with the angles themselves and the cosines may be assumed to be equal to unity. In addition to this, we will consider as being infinitely small those items in which \( \alpha \), \( \beta \), \( \delta \), \( \dot{\alpha} \), \( \dot{\beta} \), and \( \dot{\delta} \) or the products of their multiplication by the quantities of the order of \( \omega_3 \) and \( W/R \) are the co-factors. With this taken into account in the projections on the axes \( x \), \( y \), and \( z \), the formulas (13) are written in a simplified form in the following manner:

\[ a_x = \dot{W}_E + \Delta a_{\epsilon_0}; \]  

(15)

\[ a_y = \dot{W}_N + \Delta a_{\eta_0}. \]

With account taken of the formula

\[ p\epsilon = \frac{pa}{\cos \varphi} \]

(16)

where \( \epsilon \) is the angle of turn of the accelerometer platform about the axis \( z_g \), the angular velocities of the rotation of the accelerometer platform in the case of an ideal stabilization of the axes of the gyro-platform have the form:

\[ \omega_x = -p\beta - \frac{W_N}{R}; \]

(17)

\[ \omega_{zp} = \frac{pa}{\cos \varphi} + \omega_3 + \frac{W_E}{R \cos \varphi}. \]

After substituting the expressions (15) and (17) into formula (10) in conformity with Schuler's conditions according to which in the case under consideration
we will obtain differential equations of motion of the accelerometer platform in the following form:

\[
\ddot{a} + \frac{g}{R} \dot{a} = \frac{\Delta a \varepsilon_0}{R},
\]

\[
\ddot{\beta} + \frac{g}{R} \dot{\beta} = \frac{\Delta a \eta_0}{R}.
\]

(19)

Setting a particular solution of the system (19) in the form of

\[ a_g = a_g(t) \quad \text{and} \quad \beta_g = \beta_g(t), \]

the errors of the inertial system may be written as follows when determining:

the normal of the location

\[ a_g = \frac{\Delta a \varepsilon_0}{g} \left( 1 - \cos \sqrt{\frac{g}{R} t} \right), \]

\[ \beta_g = \frac{\Delta a \eta_0}{g} \left( 1 - \cos \sqrt{\frac{g}{R} t} \right); \]

the flight velocity

\[ \Delta W_N = \Delta a \eta_0 \sqrt{\frac{R}{g}} \sin \sqrt{\frac{g}{R} t}, \]

\[ \Delta W_E = \Delta a \varepsilon_0 \sqrt{\frac{R}{g}} \sin \sqrt{\frac{g}{R} t}; \]

the position coordinates

\[ \Delta \Phi = -\Delta \eta_0 \frac{1}{g} \cos \sqrt{\frac{g}{R} t}, \]

\[ \Delta \lambda = -\Delta \varepsilon_0 \frac{1}{g} \cos \sqrt{\frac{g}{R} t}. \]

(21)

(22)
As a numerical calculation indicates, the systematic errors of a geometric inertial navigation system due to the effect of the translational and Coriolis accelerations are considerable and must be compensated in an exact measurement of the parameters by the system.

The method used in the work for writing the differential equations of the system on the basis of its block diagram makes the solution of the problem easier and also makes it possible to evaluate the effect of the dynamics of the separate elements in a more complete examination of the processes using the methods of the control theory.

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Translated from the Russian

N. Ya. Vovchenko

12 December 1966

RSIC-621

None

Same as No. 1

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