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EFFECT OF SUBCOOLING AND RADIATION ON FILM-BOILING HEAT TRANSFER FROM A FLAT PLATE

by Thomas D. Hamill and Kenneth J. Baumeister

Lewis Research Center
Cleveland, Ohio

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SUMMARY

A theoretical analysis of film boiling from a horizontal plate with subcooling and radiation is presented. The analysis is based on the postulate that the rate of entropy production is maximized. The general solution enables the overall heat-transfer coefficient h_{tot} to be calculated when the following heat-transfer coefficients and subcooling group are specified: (1) saturated film-boiling coefficient, h_{fb}; (2) radiation coefficient, h_{rad}; (3) turbulent free-convection coefficient for subcooled liquid, h_{tcl}; (4) subcooling parameter, $\theta = (T_s - T_b)/(T_w - T_s)$, where T_s, T_b, and T_w are the saturation temperature of the liquid, the temperature of the bulk liquid, and the wall temperature, respectively. The general solution consists of a unique relation between $(h_{tot} - h_{rad})/h_{fb}$ and $(h_{rad} - h_{tcl}\theta)/h_{fb}$.

A simple formula, which is accurate when $|(h_{rad} - h_{tcl}\theta)/h_{fb}| < 1/2$, was derived and is given by

$$h_{tot} = h_{fb} + 0.88h_{rad} + 0.12h_{tcl}\theta$$

where the total heat flux q_{tot} into the system is given by

$$q_{tot} = h_{tot}(T_w - T_s)$$

Theory predicts that film boiling is impossible for values of the group $(h_{rad} - h_{tcl}\theta)/h_{fb} < -1.27$, that is, for strong subcooling.

INTRODUCTION

Radiation and subcooling are fundamental variables in film-boiling heat transfer that

are particularly important in the quenching of metals. In quenching, radiation is a significant heat-transfer mechanism because the metal is initially at a high temperature. The temperature of the quenching bath is also important, because the properties of a metal resulting from an ice water quench can be markedly different from those obtained with a saturated quench. Subcooling, therefore, is of practical significance in quenching techniques. The purpose of this report is to extend the analysis of reference 1, in which film boiling of saturated liquids was considered with radiation neglected. The present analysis, which employs some of the concepts and results of reference 1, takes into account both radiation and bulk subcooling. It is hoped that the results will be of interest from a fundamental and a practical viewpoint.

A concrete example is useful to clarify the general problem. A box with insulated sides, as shown in figure 1, is considered. The bottom of the box is made of steel plate which has been heated to a red glow. Cold water is then poured into the box. The plate is continually heated, keeping the temperature very high. The liquid that hits the plate is vaporized quickly, thus establishing an insulating layer of steam between the plate and the liquid. This condition is referred to as film boiling. The liquid that is evaporated is assumed to be simultaneously condensed by a reflux condenser inserted into the vapor space above the liquid. This condenser cools the condensate below the saturation temperature before the condensate falls back into the liquid pool.

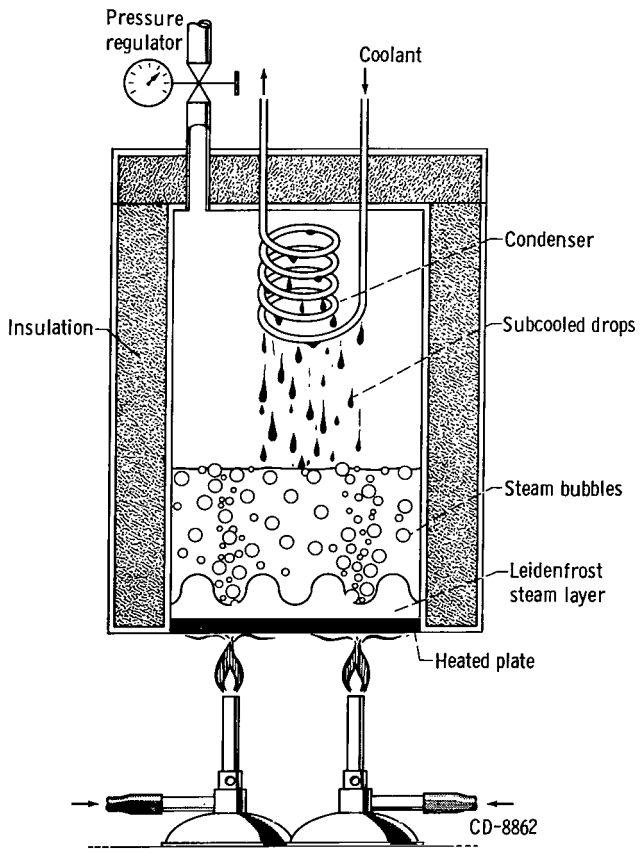


Figure 1. - Film boiling of water with bulk subcooling.

The coolant temperature and flow rate in the condenser determine the various degrees of subcooling with respect to the saturation temperature that can be achieved. As a particular example, the wall may be at 1000° F, the interface at 212° F, and the bulk of the liquid at 50° F. The important point is that there are three temperatures that characterize the system:

(1) Wall temperature, T_w

(2) Saturation temperature, T_s

(3) Bulk temperature of the liquid, T_b

Heat is transferred across the vapor

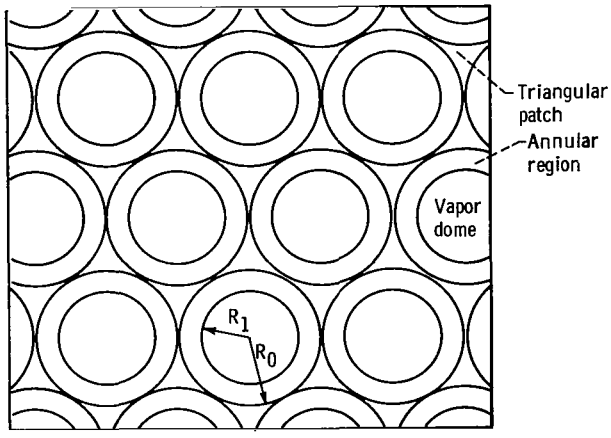
layer to the interface by conduction, convection, and radiation. Some of the heat that reaches the interface evaporates the liquid. The rest escapes into the bulk liquid by virtue of the temperature difference between the interface and the bulk liquid. Because of the agitation induced by the steam bubbles, heat transfer from the liquid-vapor interface is by turbulent free convection.

In this report are considered two common problems that arise in discussions of film-boiling heat transfer:

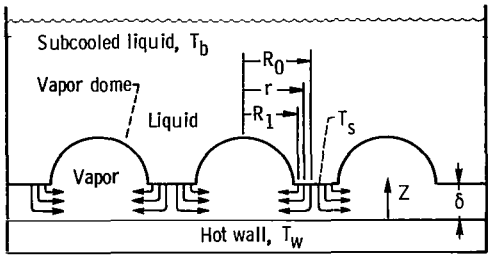
(1) The effects of subcooling on the results

(2) When radiation is important

From a design viewpoint, a method or formula is needed for computing the overall heat-transfer coefficient in those cases where both subcooling and radiation are present. This analysis indicates how the three heat-transfer coefficients (the saturated film-boiling coefficient, the radiation coefficient, and the liquid free-convection coefficient) are to be combined to yield the overall heat-transfer coefficient. The analysis is based on the postulate that the liquid-vapor interface attains an average configuration that maximizes the rate of entropy production of the system and surroundings.



(a) Top view. Symmetrical distribution of cells on surface.



(b) Side view. Direction of vapor flow into domes; axial coordinate, z; radial coordinate, r; cell radius, R_0; dome radius, R_1; gap thickness, δ.

Figure 2. - Idealized model of film boiling on horizontal surface.

BASIC MODEL ASSUMPTIONS AND GOVERNING EQUATIONS

An idealized model of film boiling is depicted in figure 2. A vapor film covers the entire plate with vapor domes spaced symmetrically. When a liquid is supported by a layer of vapor, as in film boiling, the liquid-vapor interface is inherently unstable in a gravitational field. At certain locations on the liquid-vapor interface, vapor will break through and escape, under the influence of gravity, into the bulk liquid. Photographs of film boiling show that these escape points are dome-shaped cavities arranged in a cell-type pattern (see fig. 2).

The major portion of the heat transfer to the liquid (see fig. 2) occurs across the thin portions of the vapor film. The vapor

domes are so thick that essentially no heat is conducted into them. Physically, their function is to act as hydrodynamic sinks in which the generated vapor is collected. The assumed symmetrical distribution of these sinks implies that the velocity field in the thin film is radially symmetric. The proposed model is based on the assumption that there is some time-averaged or ensemble-averaged configuration of the system where all velocity, pressure, and temperature fields are at steady state, and that this statistically idealized configuration represents the average behavior of the actual system.

The outer boundary of each unit cell in figure 2 is a circle of radius R_0; in reality, however, no plane surface can be mapped completely by nonoverlapping circles. A hexagonal boundary would be more accurate but this pattern leads to intractable mathematics. Mapping by circles does not account for a large number of semitriangular patches. The heat transfer to these curved triangular patches is assessed in an a posteriori manner as used in reference 1. The correction to the heat-transfer coefficient which accounts for these patches is of the order of 10 percent.

The heat-transfer coefficient can be obtained by solving the momentum and thermal energy equations for flow and heat transport in the thin annular vapor film of a single cell. The liquid-vapor interface is at the saturation temperature T_s, and the bulk liquid is at a colder temperature T_b. The bulk liquid is, therefore, subcooled with respect to the saturation temperature. The wall temperature T_w is held constant. Radiation is assumed to occur across the vapor gap with negligible absorption within the vapor, and with no net radiative heat transfer within the liquid phase. All radiation is either absorbed or reflected at the liquid-vapor surface. The radiation rate depends on a "view factor" determined by the geometric shape of the liquid-vapor interface. An emitted photon can hit either the flat portion of the interface or the dome region. Reflection from these two surfaces would be different. A rigorous treatment of this problem would obviously be complex, and for simplicity, the view factor is assumed to be 1. This assumption comprehends the first-order effects of radiation.

Computation of the rate of heat transfer into the system for fixed wall, bulk, and saturation temperatures requires a solution of the momentum, continuity, and energy equations for all the boundary conditions. These equations are simplified by the following assumptions:

(1) The inertia terms in the momentum equations are negligible, as justified in references 1 to 3.

(2) The vapor is incompressible.

(3) Convective effects in the vapor boundary layer are negligible.

In addition, viscous dissipation is neglected, the flow field is assumed to be radially symmetric, and the physical properties are regarded as constant, even though they are evaluated at the film temperature $T_f = (T_w + T_s)/2$. With the previous assumptions, the governing differential equations in cylindrical coordinates are as follows:

Momentum:

$$0 = -g_c \frac{\partial P}{\partial r} + \mu\left\{\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}(ru)\right] + \frac{\partial^2 u}{\partial z^2}\right\} \tag{1}$$

$$0 = -g_c \frac{\partial P}{\partial z} + \mu\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{\partial^2 w}{\partial z^2}\right) \tag{2}$$

Continuity:

$$\frac{1}{r}\frac{\partial}{\partial r}(ru) + \frac{\partial w}{\partial z} = 0 \tag{3}$$

Energy:

$$\frac{d^2 T}{dz^2} = 0 \tag{4}$$

Entropy production:

$$\left(\frac{dS}{dt}\right)_{\text{universe}} = \text{maximum} \tag{5}$$

Equation (5) is the maximization postulate to be discussed more fully in the DISCUSSION OF MAXIMUM ENTROPY RATE POSTULATE section. The subscript "universe" is used in a formal sense. In this report, the word universe means system and surroundings. The system is composed of those regions where temperature gradients exist, as well as any bubbles rising through the liquid. Thus, both the vapor layer and the liquid thermal boundary layer are part of the system. The surroundings consist of a heat source maintained at T_w and a heat sink (the bulk of the liquid as well as a condenser) maintained at T_b. The energy of the universe (system and surroundings) is constant, and thus, it can be considered an isolated system. In classical thermodynamics, the entropy principle applies only to isolated systems; that is, the entropy of an isolated system attains a maximum value in the final equilibrium state.

In the mean statistical steady state, the total entropy of the vapor layer and the liquid

thermal boundary layer is constant. That is, all the entropy generated in the vapor and liquid boundary layers per unit time is convected into the heat sink. Thus, in the steady state, only the reservoirs (source and sink) experience a net change in entropy with time. The entropy production of the universe is then given by

$$\left(\frac{dS}{dt}\right)_{universe} = Q_{tot}\left(\frac{1}{T_b} - \frac{1}{T_w}\right) \tag{6}$$

and since T_b and T_w are fixed temperatures, the entropy production is maximized if the rate of heat transfer Q_{tot} is maximized. Therefore, the entropy production postulate implies that the rate of heat transfer will be a maximum subject to the following boundary conditions (see fig. 2, p. 3):

$$z = 0 \quad u = 0 \quad w = 0 \quad T = T_w \tag{7}$$

$$z = \delta \quad u = 0 \quad w = w_\delta \quad T = T_s \tag{8}$$

$$z = \infty \quad T = T_b \tag{9}$$

$$r = R_0 \quad u = 0 \tag{10}$$

where w_δ is the evaporation velocity at the interface.

The momentum and energy equations are coupled at the liquid interface by a heat balance over the annular area:

$$-\rho\lambda w_\delta = -k\left(\frac{dT}{dz}\right)_\delta - h_{tcl}(T_s - T_b) + h_{rad}(T_w - T_s) \tag{11}$$

The term on the left side is the rate of release of latent heat (w_δ is a negative quantity). The first term on the right is the rate of heat transport by conduction through the vapor film to the interface. The second term on the right is the heat escaping from the interface into the bulk liquid because of subcooling. The symbol h_{tcl} is the turbulent free-convection coefficient for heat transport within the liquid. The third term on the right represents the heat reaching the interface by radiation. The symbol h_{rad} is the radiation coefficient, a function of the plate and liquid emissivities and the wall and saturation temperatures. As stated previously, h_{rad} is assumed to be independent of the shape of the liquid-vapor interface.

Additional boundary conditions apply to the pressure field. A static-force balance on the annular liquid region requires that the average pressure on the flat portion of the interface be equal to the saturation pressure P_s:

$$2\pi \int_{R_1}^{R_0} P(r, \delta) r \, dr = \pi\left(R_0^2 - R_1^2\right)P_s \tag{12}$$

This requirement must be met in order for the vapor to support the liquid. One further condition on the pressure field is that

$$P(R_1, \delta) = P_s - \frac{R_1(\rho_l - \rho)g}{g_c} + \frac{2\sigma}{R_1} \tag{13}$$

This equation, which is derived in reference 4, relates the pressure at the entrance to the dome $P(R_1, \delta)$ to the saturation pressure and to the difference in head within the dome corrected for surface tension effects caused by curvature. The pressure at the radial entrance must always be less than the system pressure in order for the dome to function as a hydrodynamic sink. A minimum dome size can be computed from equation (13) by letting $P(R_1, \delta) = P_s$. This requires that

$$R_1 > \sqrt{\frac{2\sigma g_c}{(\rho_l - \rho)g}} \tag{14}$$

If R_1 was less than this value, the pressure inside the dome would be greater than the pressure within the annular film and flow into the dome could not occur.

GENERAL METHOD OF SOLUTION

Momentum Equations

The momentum equations that apply in this analysis are solved in reference 1. The essential result is that in order to support the liquid phase at a height δ above the plate, the evaporation velocity w_δ required is given by

$$w_\delta = -\frac{1}{2}\beta^2\delta^3 \tag{15}$$

where β^2, a function of geometry, gravity, and physical properties, is

$$\beta^2 = -\frac{g_c}{3\mu R_0^4}\left[\frac{R_0^3(\rho_l - \rho)\frac{g}{g_c}\frac{f(1-f^2)}{2} - \frac{\sigma R_0(1-f^2)}{f}}{\frac{1-f^4}{8} + \frac{(1-f^2)^2}{4} + \frac{\ln f}{2}}\right] \tag{16}$$

and f, the ratio of the dome radius to the cell radius, is

$$f = \frac{R_1}{R_0} \tag{17}$$

Equation (15) is a functional relation between the evaporation velocity and the thickness of the gap that follows solely from momentum considerations. No matter how the vapor is generated, equation (15) must always apply. For example, a relation identical in form to equation (15) is derived by Whitney (ref. 5) and verified experimentally by Pearson and Bradfield (ref. 6) in their study of the gas-levitated disk. In the gas-levitated disk experiment, a stream of air is blown through a hole in a small solid disk at a rate high enough so that the disk floats on a cushion of air. The relation between the velocity of air flow and the distance the disk equilibrates above the plate is given by an equation identical in form to equation (15). While in the levitated disk problem the vapor flow is generated mechanically from a source of compressed air, in film boiling the source of vapor is evaporation at the interface due to the heat transfer across the gap.

Energy Equations

To keep the mathematics simple, the effect of heat capacity C_p (or equivalently of convective effects in the vapor layer) has been ignored for the present. This simplifies the solution of the energy equation to a linear drop of temperature across the gap. This defect is ultimately corrected by attributing the effect of C_p to the saturated film-boiling coefficient derived in reference 1. Thus, the temperature profile in the annular vapor layer is given by

$$T(z) = T_w - (T_w - T_s)\frac{z}{\delta} \tag{18}$$

Substituting equations (18) and (15) into the interface energy balance (eq. (11)) yields

$$\frac{1}{2}\rho\lambda\beta^2\delta^3 = \frac{k(T_w - T_s)}{\delta} - h_{tcl}(T_s - T_b) + h_{rad}(T_w - T_s) \tag{19}$$

Equation (19) is a quartic equation for the gap thickness in terms of the system parameters.

It is emphasized that equation (19) is a heat balance over the annular area only. The total rate of heat transfer from the plate per unit cell is given by

$$Q_{tot} = \pi R_0^2\left[\frac{k}{\delta}(T_w - T_s)(1 - f^2) + h_{rad}(T_w - T_s)\right] + \pi R_1^2 q_{dome} \tag{20}$$

In equation (20), the heat flux at the wall is being considered. In appendix B, the heat flux by conduction under the dome q_{dome} is shown to be negligible in comparison to conduction under the annulus if the gap thickness is much smaller than the dome radius. Thus, if conduction under the dome is negligible, the total heat-transfer coefficient per unit cell is

$$h_{tot} = \frac{Q_{tot}}{\pi R_0^2(T_w - T_s)} = \frac{k}{\delta}(1 - f^2) + h_{rad} \tag{21}$$

Optimization Procedure

The maximization postulate requires that the amount of heat transferred compatible with the boundary conditions be a maximum; that is, that R_1 and R_0 adjust themselves in such a manner that h_{tot} is maximized:

$$\left(\frac{\partial h_{tot}}{\partial R_0}\right)_{R_1} = 0 \tag{22}$$

$$\left(\frac{\partial h_{tot}}{\partial R_1}\right)_{R_0} = 0 \tag{23}$$

From a mathematical viewpoint, the problem is now completely formulated. The dependent variables (unknowns) at this point are β^2, δ, h_{tot}, R_0, and R_1, and there are five independent equations (eqs. (16), (19), and (21) to (23)). The maximization postulate, in essence, specifies what values of R_0 and R_1 will exist physically. Without the postulate, empirical equations (or some other theory, such as hydrodynamic stability considerations) would have to be used for R_0 and R_1. From this point on, the solution of the equations is merely a matter of algebraic manipulation and numerical techniques. Details of the algebraic manipulations and rationale of the numerical procedure are shown in appendix C.

Dimensionless Groups and Parametric Solution

The parameters of the problem are combined into dimensionless groups. A dimensionless gap thickness η is defined as

$$\eta = \frac{\delta}{l} \tag{24}$$

where

$$l = \sqrt{\frac{\sigma g_c}{(\rho_l - \rho)g}} \tag{25}$$

The characteristic length l emerges naturally from the analysis presented in appendix C. The optimal dome and cell radii are multiples of l.

Radiation and turbulent free-convection Nusselt numbers are defined, respectively, by

$$N_{rad} = \frac{h_{rad} l}{k} \tag{26}$$

$$N_{tcl} = \frac{h_{tcl} l}{k} \tag{27}$$

where k is the thermal conductivity of the vapor in all Nusselt numbers in this report. The film-boiling Nusselt number N_{fb} in the absence of subcooling and radiation was derived from the application of the maximization postulate in reference 1 and is given by

$$N_{fb} = \frac{h_{fb}\ell}{k} = 0.41\left(Ra\,\frac{\lambda^*}{C_p(T_w - T_s)}\right)^{1/4} \tag{28}$$

where Ra is the Rayleigh number and λ^* a modified latent heat; both are defined further in appendix A. In addition, it is convenient to define the subcooling parameter as

$$\theta = \frac{T_s - T_b}{T_w - T_s} \tag{29}$$

The symbol θ is the ratio of liquid subcooling to vapor superheat. For a particular problem involving subcooling and radiation, these three Nusselt numbers and the subcooling group can be computed a priori. From a design viewpoint, a method for relating these four groups to an overall Nusselt number for the system is required. The overall Nusselt number N_{tot} is defined by

$$N_{tot} = \frac{h_{tot}\ell}{k} \tag{30}$$

where

$$q_{tot} = h_{tot}(T_w - T_s) \tag{31}$$

As shown in appendix C, the general solution was reduced to two parametric equations (see eqs. (C35) and (C38)) of the form

$$\frac{N_{tot} - N_{rad}}{N_{fb}} = \varphi(f) \tag{32}$$

$$\frac{N_{rad} - N_{tcl}\theta}{N_{fb}} = \psi(f) \tag{33}$$

where φ and ψ are complicated functions of f. If values of f are substituted into the right sides of equations (32) and (33), a numerical correspondence between the two left sides of the equations is generated. In other words, the group $(N_{tot} - N_{rad})/N_{fb}$ is a unique function of the group $(N_{rad} - N_{tcl}\theta)/N_{fb}$ which can be calculated from equations existing in the literature. Thus an overall Nusselt number N_{tot} can be obtained.

RESULTS

As stated in the preceding section, the general solution consists of a unique relation between the group $(N_{tot} - N_{rad})/N_{fb}$ and the group $(N_{rad} - N_{tcl}\theta)/N_{fb}$. The general trends obtained after numerically evaluating equations (32) and (33) by substituting values of f are given in table I.

TABLE I. - HEAT-TRANSFER REGIME

$\frac{N_{rad} - N_{tcl}\theta}{N_{fb}}$	Characteristic	$\frac{N_{tot} - N_{rad}}{N_{fb}}$
Positive	Radiation	<1
Zero	No subcooling or radiation (exactly balanced)	1
Negative	Subcooling	>1
<-1.27	No film boiling possible	Undefined

A graph of $(N_{tot} - N_{rad})/N_{fb}$ plotted against $(N_{rad} - N_{tcl}\theta)/N_{fb}$ is given in figure 3, and this graphical function is the general solution to the film-boiling problem with radiation and subcooling. Table III (in appendix C, p. 31) contains selected tabular values, including values of f, corresponding to the two groups.

A peculiar anomaly, inherent in the parametric equations, arose in computing the general solution. When values of f less than about 0.4 were substituted into the parametric equations, a new branch solution was generated (indicated by the dotted line in fig. 3). This branch solution obtained for values of f higher than 0.4. Since this branch solution leads to a smaller heat transport, it was first thought to be physically meaningless; however, this branch turned out to be a pure free-convection line. That is, except for slight numerical errors introduced by the correction for the triangular paths, the equation for the dotted line is

$$\frac{N_{tot} - N_{rad}}{N_{fb}} = -\frac{N_{rad} - N_{tcl}\theta}{N_{fb}} \tag{34}$$

or

$$N_{tot} = N_{tcl}\theta \tag{35}$$

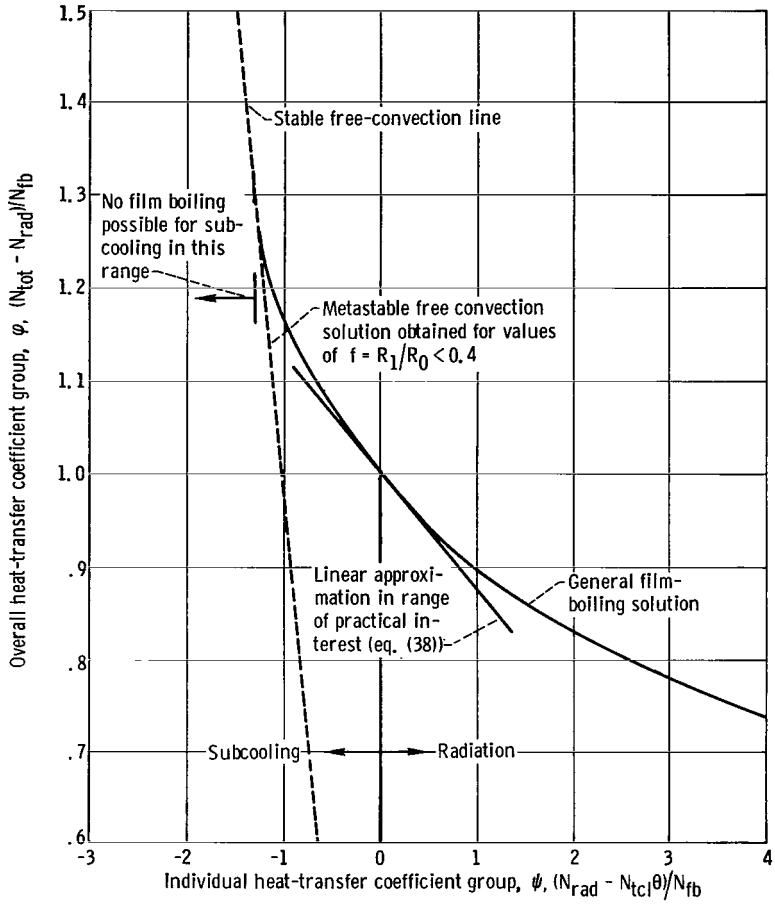


Figure 3. - General graphical solution of effect of subcooling and radiation on film boiling.

and the total heat flux is then

$$q_{tot} = h_{tot}(T_w - T_s) = h_{tcl}(T_w - T_s)\frac{T_s - T_b}{T_w - T_s} \tag{36}$$

or simply

$$q_{tot} = h_{tcl}(T_s - T_b) \tag{37}$$

What this means, then, is that the optimization procedure allows two solutions: one stable (film boiling), and one metastable (pure free convection).

However, the analysis makes an even more remarkable prediction. The intersection of the film-boiling line with the free-convection line places a theoretical bound on the

possibility of film boiling in the presence of subcooling. That is, theory predicts that film boiling is impossible whenever the value of the group $(N_{rad} - N_{tcl}\theta)/N_{fb}$ is less than -1.27 (i.e., strong subcooling). For values of this group below -1.27, something must happen to film boiling; either it reverts to pure convection or to nucleate boiling. No physically meaningful value of f can be found for values of the group below -1.27. However, it is apparent from examining figure 3 that the film-boiling line becomes tangent to the free-convection solution at -1.27. Therefore, for values of $(h_{rad} - h_{tcl}\theta)/h_{fb}$ below this critical value, the free-convection line is the correct and only solution.

At extremely high positive values of $(h_{rad} - h_{tcl}\theta)/h_{fb}$ corresponding to an f value of 0.793, the gap thickness becomes zero and the model no longer applies. As $(h_{rad} - h_{tcl}\theta)/h_{fb}$ approaches infinity (in the vicinity of f = 0.793), the group $(h_{tot} - h_{rad})/h_{fb}$ approaches zero. This implies that $h_{tot} = h_{rad}$ in the limit of infinite radiation, which is physically correct. Why the limit of infinite radiation should correspond to an f value of 0.793 is not known.

The optimum dome size R_1^*, which is given by equation (C6) in appendix C, is independent of subcooling and radiation:

$$R_1^* = \sqrt{\frac{6\sigma g_c}{(\rho_l - \rho)g}} \tag{38}$$

Theory predicts, therefore, an upper limit to the spacing of the vapor domes when f = 0.4 which is given by

$$D_{0,\,max} = \frac{2}{0.4}\sqrt{\frac{6\sigma g_c}{(\rho_l - \rho)g}} = 12.24l \tag{39}$$

For purposes of comparison, the most dangerous wavelength computed from hydrodynamic stability theory is given by $10.85l$ (ref. 4). The two numbers are of the same order of magnitude. The vapor domes become more widely spaced with subcooling and more closely spaced with increased radiation. The average size of the vapor domes, however, should remain constant under all conditions.

A simple formula for the overall heat-transfer coefficient was obtained by drawing a tangent to the graphical function at $(N_{rad} - N_{tcl}\theta)/N_{fb}$ equal to zero. The formula is

$$h_{tot} = h_{fb} + 0.88h_{rad} + 0.12h_{tcl}\frac{(T_s - T_b)}{(T_w - T_s)} \tag{40}$$

This linear approximation ought to be valid for many practical conditions encountered. It is quite accurate in the range $\left|(N_{rad} - N_{tcl}\theta)/N_{fb}\right| < 0.5$; that is, if the film-boiling coefficient is twice the value of $\left|h_{rad} - h_{tcl}\theta\right|$, equation (40) is accurate. A numerical example of the film boiling of nitrogen is presented in appendix D to illustrate the theory.

The simplified formula (eq. (40)) is similar to one derived by Bromley (ref. 7) for forced-convection film boiling from the outside of pipes. For fully developed turbulent (liquid) flow with no subcooling but with radiation, Bromley recommends the equation

$$h_{tot} = h_{fb} + \frac{7}{8} h_{rad}$$

where h_{fb} is the saturated film-boiling coefficient appropriate to the pipe geometry and flow conditions in the absence of radiation.

The coefficients 0.88 and 7/8 are so close as to lead to the speculation that equation (40) may be a general relation for many geometries and flow conditions, if the correct individual coefficients are used.

DISCUSSION OF MAXIMUM ENTROPY RATE POSTULATE

The present analysis and its predecessor (ref. 1) rely heavily on the postulate that the system will attain a state in which the rate of entropy production of the universe is a maximum. The universe (system and surroundings) is always an isolated system, and it is to isolated systems that the postulate is limited in analogy with classical thermodynamics. Since the word "entropy" has been used, it is natural to look to equilibrium thermodynamics for clarification.

The central problem of classical thermodynamics is to predict the equilibrium state of a system resulting from the removal of a barrier. For example, a rigid insulated cylinder separated into two parts by a barrier is considered. On one side is a gas at high pressure, on the other a gas at low pressure. The problem is to determine what final state will result for the overall system when the barrier is removed.

Callen (ref. 8, p. 24), in discussing this problem, states:

> . . . the tentative postulation of the simplest formal solution of a problem is a conventional and frequently successful mode of procedure in theoretical physics. What then is the simplest criterion that can reasonably be imagined for the determination of the final equilibrium state? From our experience with many physical theories we might expect that the most economical form of the equilibrium criterion is in terms of an extremum principle.

The extremum principle of thermodynamics is that the entropy of a fixed-energy system is a maximum in the final equilibrium state.

In other words, the simplest formal solution to problems involving uncertainty or randomness is to maximize or minimize some quantity; the idea is that the extremum principle generates the required number of equations to make the problem determinate. Although there is no obvious connection between equilibrium and nonequilibrium processes, both would appear to be amenable to the same type of solution. Something is known about the problem; for example, in the first problem, perhaps the energy is constant, or in the film-boiling problem, perhaps the boundary temperatures are fixed. But other boundary conditions cannot be known in sufficient detail to make the problem determinate. It is logical to hope that a mathematical procedure that worked for the first problem will also work for the second.

This kind of thinking led Malkus (refs. 9 and 10) to a possible breakthrough in the study of turbulence. First, Malkus considered turbulent natural convection from a horizontal surface (ref. 9). He postulated that the turbulent fluid motions would attain an extreme state in which the maximum amount of heat compatible with the boundary conditions would be transferred. By using this postulate, he deduced the relation of the Nusselt number against the Rayleigh number to the one-third power, an expression for the temperature profile, and an estimate of the mean-square velocity distribution - a significant achievement considering the a priori approach. Zuber (ref. 11) subsequently adapted the results of Malkus to the problem of nucleate boiling. The present analysis and reference 1 apparently complete a series of papers on heat-transfer processes from horizontal surfaces in which the heat-transfer rate is a maximum (see table II).

Malkus (ref. 10) then used the same postulate (maximum rate of entropy production) to determine the turbulent-velocity profile in a flat channel. He deduced profiles similar to the defect laws of von Karman. Recently, Nihoul (ref. 12) applied the Malkus theory to magnetohydrodynamic turbulent channel flow.

One of the differences between the film-boiling analyses and the Malkus papers is the degree of complicacy of the mathematical techniques. Malkus considers turbulent flow. Associated with this condition are stability arguments and complicated mathematics. In film boiling the vapor flow is laminar, so the mathematics are simple. In fact, the turbulence in film boiling exists only at the liquid-vapor interface and not in the vapor flow itself. Yet, the Malkus principle still seems to be correct: maximize the rate of entropy production to make the problem determinate.

Just as Carnots' reasoning on steam engines was the beginning of a much wider principle (Second Law), it may well be that the Malkus postulate has wide applicability in engineering problems characterized by macroscopic uncertainty. Turbulent flows are time dependent, yet average values emerge. Many other engineering problems, such as two-phase flows, are time dependent, and for these problems the maximization postulate

TABLE II. - PROBLEMS SOLVED BY MAXIMUM ENTROPY RATE POSTULATE[a]

Process	Author	Results	Source
Shear turbulence	Malkus	Universal velocity profile	Ref. 10
Magnetohydrodynamic (MDH) turbulence	Nihoul	Effect of magnetohydrodynamic variables	Ref. 12
Turbulent free convection	Malkus	Heat-transfer coefficient Temperature profile Eddy velocities	Ref. 9
Nucleate boiling	Zuber	Heat-transfer correlation using Malkus theory	Ref. 11
Film boiling:			
Flate plate Saturated	Hamill and Baumeister	Heat-transfer coefficient Optimum dome size	Ref. 1
Subcooling and radiation		Optimum spacing of domes Effect of subcooling and radiation	Present report
Horizontal tube		Effect of geometry	Ref. 16

[a]Physical problems involving rate processes that are governed by unknown or chaotic boundary conditions are resolved by maximizing the rate of entropy production of the universe.

may work, whether or not the flow itself is turbulent. Time dependence or uncertainty seems to be the key condition. In any event, the maximum entropy rate postulate is a technique worth trying whenever an analysis is hindered by a lack of information.

CONCLUDING REMARKS

The problem of film boiling with subcooling and radiation has been treated by means of the maximum entropy rate postulate. The overall heat-transfer coefficient for the process can be computed by

$$h_{tot} = h_{rad} + \varphi\, h_{fb}$$

where φ is a unique function of the individual heat-transfer coefficients (radiation, h_{rad}; turbulent liquid free convection, h_{tcl}; saturated film boiling, h_{fb}) and a subcooling parameter $\theta = (T_s - T_b)/(T_w - T_s)$. That is, φ is given functionally by

$$\varphi = \varphi\left(\frac{h_{rad} - h_{tcl}\theta}{h_{fb}}\right)$$

which is given graphically in figure 3 (p. 13). A simple formula that approximates the numerical results is given by

$$h_{tot} = h_{fb} + 0.88h_{rad} + 0.12h_{tcl}\frac{T_s - T_b}{T_w - T_s}$$

which is valid for

$$\left|\frac{h_{rad} - h_{tcl}\theta}{h_{fb}}\right| < 0.5$$

The analysis predicts that film boiling is physically impossible for strong subcooling - in particular, when

$$\frac{h_{rad} - h_{tcl}\theta}{h_{fb}} < -1.27$$

This prediction and the heat-transfer relations should be verified experimentally.

The results obtained could possibly be valid for systems other than pool film boiling on a flat plate. For example, they may be valid for film boiling in other geometries if the appropriate individual heat-transfer coefficients are employed. The same thing may be true for forced convection with film boiling.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, December 15, 1966,
129-01-09-04-22.

APPENDIX A

SYMBOLS

A	$1/Ra\Lambda$
B	$(N_{rad} - N_{tcl}\theta)/Ra\Lambda$
C_p	specific heat of vapor, Btu/lb mass
$C_{p,l}$	specific heat of liquid, Btu/lb mass
$D_{0,max}$	maximum predicted diameter of cell (eq. (39)), ft
$F(f)$	function of f (eq. (C8))
f	ratio of vapor-dome radius to cell radius, R_1/R_0
g	local value of gravity, ft/sec^2
g_c	Newton's law conversion factor, 32.174 (lb mass)(ft)/(lb force)(sec^2)
h_{fb}	saturated film-boiling coefficient, $Btu/(sec)(ft^2)(^oR)$, $0.41\left[\frac{k^3\lambda^*\rho g(\rho_l - \rho)}{\mu(T_w - T_s)l}\right]^{1/4}$
h_{rad}	radiation coefficient, $Btu/(sec)(ft^2)(^oR)$
h_{tcl}	turbulent-free-convection coefficient for subcooled liquid, $Btu/(sec)(ft^2)(^oR)$
h_{tot}	overall heat-transfer coefficient (eq. (31)), $Btu/(sec)(ft^2)(^oR)$
k	thermal conductivity of vapor, $Btu/(sec)(ft)(^oR)$
k_l	thermal conductivity of liquid, $Btu/(sec)(ft)(^oR)$
l	characteristic length, $\sqrt{\frac{\sigma g_c}{g(\rho_l - \rho)}}$, ft
N_{fb}	saturated film-boiling Nusselt number, $h_{fb}l/k$
N_{rad}	radiation Nusselt number, $h_{rad}l/k$
N_{tcl}	turbulent free-convection Nusselt number for subcooled liquid, $h_{tcl}l/k$
N_{tot}	overall Nusselt number for system, $h_{tot}l/k$
N^*	$N_{tot} - N_{rad}$, see eq. (C29)
P	pressure, lb force/ft^2
Pr_l	Prandtl number for liquid, $C_{p,l}\mu_l/k_l$
P_s	saturation pressure, lb force/ft^2
Q_{dome}	rate of heat transfer under dome, Btu/sec
Q_{tot}	total rate of heat transfer into system, Btu/sec

q_{dome} heat flux under dome, $Btu/(hr)(ft^2)(^oR)$

q_{tot} total heat-flux into system, $Btu/(hr)(ft^2)(^oR)$

Ra Rayleigh number for film boiling, $l^3 g\rho C_p(\rho_l - \rho)/\mu k$

R_0 radius of cell, ft

R_1 radius of vapor dome, ft

R_0^* optimal cell radius, R_1^*/f, ft

R_1^* optimal dome radius, $\sqrt{\frac{6\sigma g_c}{g(\rho_l - \rho)}}$, ft

r radial coordinate (see fig. 2), ft

S entropy of universe, $Btu/^oR$

T temperature, ^oR

T_b temperature of bulk liquid, ^oR

T_f film temperature, $T_f = (T_w + T_s)/2$, ^oR

T_s saturation temperature of liquid, ^oR

T_w wall temperature, ^oR

t time, sec

u radial velocity of vapor, ft/sec

w axial velocity of vapor, ft/sec

w_δ evaporation velocity at the interface, ft/sec

y dummy variable used in appendix B

z distance from plate to arbitrary point in vapor, ft

β_l thermal expansion coefficient for liquid, $(^oR)^{-1}$

β^2 parameter given by eq. (C7), $1/(ft^2)(sec)$

δ vapor-layer thickness, ft

$\zeta(f)$ correction factor, eq. (C7),

η dimensionless gap thickness, δ/l

θ subcooling parameter, $(T_s - T_b)/(T_w - T_s)$

Λ enthalpy ratio, $\lambda^*/C_p(T_w - T_s)$

λ latent heat of vaporization, Btu/lb mass

λ^* modified latent heat, $\lambda\left(1 + \frac{19}{20} C_p \frac{(T_w - T_s)}{\lambda}\right)$, Btu/lb mass

μ viscosity of vapor, lb mass/(ft)(sec)

μ_l viscosity of liquid, lb mass/(ft)(sec)

ξ perpendicular distance from wall to point on dome surface, ft

ρ density of vapor, lb $mass/ft^3$

ρ_l density of liquid, lb $mass/ft^3$

σ surface tension, lb force/ft

φ $\varphi(f) = (N_{tot} - N_{rad})/N_{fb}$ (see also eq. (C38))

ψ $\psi(f) = (N_{rad} - N_{tcl}\theta)/N_{fb}$ (see also eq. (C35))

APPENDIX B

JUSTIFICATION FOR NEGLECTING CONDUCTION UNDER VAPOR DOME

Conduction under the dome can be estimated by assuming a linear drop of temperature from the plate to any arbitrary position on the curved interface. Thus, the temperature profile at any radial position under the dome will be

$$T(z) = T_w - (T_w - T_s)\frac{z}{\xi} \tag{B1}$$

where ξ is the perpendicular distance from the wall to any point on the surface of the dome (see fig. 4). The symbol ξ, which is a function of radial position, is given by

$$\xi = \delta + \sqrt{R_1^2 - r^2} \tag{B2}$$

The total heat flow under the dome is obtained by integrating the heat flux over the dome area:

$$Q_{dome} = 2\pi k(T_w - T_s) \int_0^{R_1} \left(\frac{1}{\delta + \sqrt{R_1^2 - r^2}} \right) r\, dr \tag{B3}$$

This integral can be evaluated exactly by letting

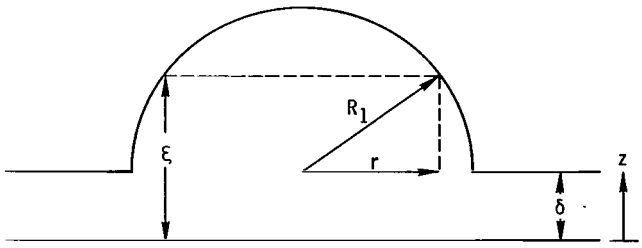


Figure 4. - Typical vapor dome with coordinate system for estimating conduction under dome.

$$y = \frac{1}{\delta + \sqrt{R_1^2 - r^2}}$$

or

$$\sqrt{R_1^2 - r^2} = \frac{1}{y} - \delta$$

and noting that

$$dr^2 = 2r\,dr$$

$$dy = \frac{1}{2}\left(\frac{1}{\delta + \sqrt{R_1^2 - r^2}}\right)^2 \left(\frac{1}{\sqrt{R_1^2 - r^2}}\right) dr^2 = \frac{1}{2}\frac{y^2}{\frac{1}{y} - \delta}\,dr^2$$

or

$$dr^2 = 2\left(\frac{1}{y} - \delta\right) y^{-2}\,dy$$

Therefore,

$$Q_{dome} = 2\pi k(T_w - T_s) \int_{y = \frac{1}{\delta + R_1}}^{y = \frac{1}{\delta}} \left(\frac{1}{y} - \delta\right) \frac{1}{y}\,dy$$

$$= 2\pi k(T_w - T_s) \left| -\frac{1}{y} - \delta \ln y \right|_{1/(\delta + R_1)}^{1/\delta}$$

$$= 2\pi k(T_w - T_s) \left[\delta + R_1 - \delta - \delta \ln \frac{\frac{1}{\delta}}{\frac{1}{\delta + R_1}}\right]$$

$$= 2\pi k(T_w - T_s) \left(R_1 + \delta \ln \frac{\delta}{\delta + R_1}\right) \tag{B4}$$

Substituting this value of Q_{dome} for $\pi R_1^2 q_{dome}$ in equation (20) yields the total rate of heat transfer at the wall as

$$Q_{tot} = \pi R_0^2 (T_w - T_s)\left[\frac{k}{\delta}(1 - f^2) + h_{rad}\right] + 2\pi k(T_w - T_s)\left[R_1 + \delta \ln\left(\frac{\delta}{\delta + R_1}\right)\right] \tag{B5}$$

The overall heat-transfer coefficient h_{tot} is then

$$h_{tot} = \frac{Q_{tot}}{\pi R_0^2 (T_w - T_s)} = \frac{k}{\delta}(1 - f^2) + h_{rad} + \frac{2kR_1}{R_0^2} + \frac{2k\delta}{R_0^2} \ln\left(\frac{\delta}{\delta + R_1}\right) \tag{B6}$$

or

$$h_{tot} = \frac{k}{\delta}(1 - f^2) + h_{rad} + \frac{2kf}{R_0} + \frac{2k\delta}{R_0^2} \ln\left(\frac{\delta}{\delta + R_1}\right) \tag{B7}$$

It is seen that if R_0 and R_1 are much greater than δ, then

$$h_{tot} = \frac{k}{\delta}(1 - f^2) + h_{rad} \tag{B8}$$

which is equation (21) in the main text. A question might arise as to the order of magnitude of the third term in equation (B7) with respect to the first. Dividing one by the other yields

$$\frac{\frac{2kf}{R_0}}{\frac{k}{\delta}(1 - f^2)} \sim \frac{\delta}{R_0}$$

Calculations indicate that δ/R_0 is of the order of 10^{-2}; therefore, the third term can be safely dropped.

APPENDIX C

METHOD OF SOLUTION

Algebraic Manipulations

A simultaneous solution of equations (19), (21), (22), and (23) in conjunction with the expression for β^2 (eq. (16)) is required. The maximization with respect to R_0 is considered first. Equation (22) can be rewritten in terms of new variables as

$$\frac{\partial}{\partial R_0}\left[h_{tot}(R_1, R_0)\right]_{R_1} = \frac{\partial}{\partial R_0}\left[h_{tot}(f, R_0)\right]_f = 0 \tag{C1}$$

(see appendix E). From equation (21), the following equation can be deduced:

$$\left(\frac{\partial h_{tot}}{\partial R_0}\right)_f = \left(\frac{\partial \delta}{\partial R_0}\right)_f = 0 \tag{C2}$$

This condition requires that the gap thickness be minimized with respect to R_0 in order to maximize the heat transfer. Equation (19) gives an implicit equation for δ in terms of R_0. In equation (19), β^2 is a function of R_0 as shown in equation (16). Taking the derivative on both sides of the interface energy balance (eq. (19)) with respect to R_0 and solving for $\left(\partial\delta/\partial R_0\right)_f$ give

$$\left(\frac{\partial \delta}{\partial R_0}\right)_f = \frac{\left(\frac{1}{2}\rho\lambda\delta^4\frac{\partial\beta^2}{\partial R_0}\right)_f}{h_{rad}(T_w - T_s) - h_{tcl}(T_w - T_s) - 2\rho\lambda\beta^2\delta^3} \tag{C3}$$

The only way for this equation to be satisfied for all arbitrary values of the parameters is for

$$\left(\frac{\partial \beta^2}{\partial R_0}\right)_f = 0 \tag{C4}$$

Taking the derivative of β^2 (eq. (16)) with respect to R_0, setting this equal to zero, and solving for R_0 yield

$$R_0^* = \frac{1}{f}\sqrt{\frac{6\sigma g_c}{(\rho_l - \rho)g}} \tag{C5}$$

or

$$R_1^* = \sqrt{\frac{6\sigma g_c}{(\rho_l - \rho)g}} \tag{C6}$$

The optimum vapor-dome radius R_1^* given by equation (C6) is identical to the result obtained in reference 1. The determination of the optimum f is not as straightforward - a numerical approach is necessary. Substituting R_0^* into the expression for β^2 (eq. (16)) yields

$$\beta^2 = \frac{2g(\rho_l - \rho)}{\mu l\, F(f)} \tag{C7}$$

where l is a characteristic length given by equation (25). The motivation for its introduction is based on R_1^*. A function only of f is given by

$$F(f) = (-3)6^{3/2}\left[\frac{\frac{\left(1 - f^2\right)^2}{4} + \frac{1 - f^4}{8} + \frac{\ln f}{2}}{f^2(1 - f^2)}\right] \tag{C8}$$

A characteristic length given by $R_1^*/\sqrt{6}$ emerged quite naturally from the first maximization. This characteristic length is used to make the equations dimensionless. Multiplying equation (21) by l/k gives the dimensionless equation

$$N_{tot} = \frac{1 - f^2}{\eta} + N_{rad} \tag{C9}$$

The interface energy balance (eq. (19)) becomes

$$\frac{\eta^4}{F(f)} = \left[\frac{k(T_w - T_s)\mu}{\rho\lambda g(\rho_l - \rho)l^3}\right] + \eta\left[\frac{h_{rad}(T_w - T_s) - h_{tcl}(T_s - T_b)}{\frac{\rho\lambda(\rho_l - \rho)gl^2}{\mu}}\right] \tag{C10}$$

The characteristic Rayleigh number for film boiling is given by

$$Ra = \frac{l^3 g\rho C_p(\rho_l - \rho)}{\mu k} \tag{C11}$$

Employing this dimensionless group yields

$$\eta^4 = F(f)\left[\frac{1}{Ra\Lambda} + \eta\left(\frac{N_{rad} - N_{tcl}\theta}{Ra\Lambda}\right)\right] \tag{C12}$$

where

$$\theta = \frac{T_s - T_b}{T_w - T_s}$$

is a subcooling parameter and replacing λ by λ^* gives

$$\Lambda = \frac{\lambda^*}{C_p(T_w - T_s)} \tag{C13}$$

The maximization with respect to f will now be undertaken. At this point, there are three unknowns (η, f, and N_{tot}) and three equations (eqs. (C9) and (C12) and the maximization with respect to f) to be considered. Thus, for maximum heat transport,

$$\frac{dN_{tot}}{df} = \frac{-(1 - f^2)}{\eta^2}\frac{d\eta}{df} - \frac{2f}{\eta} = 0 \tag{C14}$$

see equation (E3). Solving for $d\eta/df$ yields

$$\frac{d\eta}{df} = -\frac{2f}{1 - f^2}\eta \tag{C15}$$

Before taking the derivative of equation (C12) with respect to f, it is convenient to let

$$A = \frac{1}{Ra\Lambda} \tag{C16}$$

and

$$B = \frac{N_{rad} - N_{tcl}\theta}{Ra\Lambda} \tag{C17}$$

Thus, the energy interface balance becomes

$$\eta^4 = (A + B\eta)F(f) \tag{C18}$$

From this point on, F = F(f). Taking the derivative of η with respect to f yields

$$4\eta^3 \frac{d\eta}{df} = (A + B\eta)\frac{dF}{df} + BF\frac{d\eta}{df} \tag{C19}$$

Solving for $d\eta/df$ gives

$$\frac{d\eta}{df} = \frac{(A + B\eta)\frac{dF}{df}}{4\eta^3 - BF} = \frac{\eta^4 \frac{dF}{df}}{(4\eta^3 - BF)F} \tag{C20}$$

Equating (C15) to (C20) gives

$$-\frac{2f}{1 - f^2}\eta = \frac{\eta^4 \frac{dF}{df}}{(4\eta^3 - BF)F} \tag{C21}$$

The goal of these manipulations is a set of parametric equations relating A and B to f. Solving for B from equation (C21) gives

$$B = \eta^3 \left(\frac{4}{F} + \frac{1 - f^2}{2f} \frac{1}{F^2} \frac{dF}{df} \right) \tag{C22}$$

but from equation (C18)

$$A = \frac{\eta^4}{F} - \eta B \tag{C23}$$

Substituting equation (C22) into equation (C23) gives

$$A = \eta^4 \left(- \frac{3}{F} - \frac{1 - f^2}{2f} \frac{1}{F^2} \frac{dF}{df} \right) \tag{C24}$$

Thus,

$$\eta = \frac{A^{1/4}}{\left(- \frac{3}{F} - \frac{1 - f^2}{2f} \frac{1}{F^2} \frac{dF}{df} \right)^{1/4}} \tag{C25}$$

Cubing equation (C25) and substituting for η^3 in equation (C22) give

$$\frac{B}{A^{3/4}} = \frac{\frac{4}{F} + \frac{1 - f^2}{2f} \frac{1}{F^2} \frac{dF}{df}}{\left(- \frac{3}{F} - \frac{1 - f^2}{2f} \frac{1}{F^2} \frac{dF}{df} \right)^{3/4}} \tag{C26}$$

Substituting equation (C25) into the expression for the total Nusselt number (eq. (C9)) gives

$$N_{tot} = \frac{(1 - f^2) \left(- \frac{3}{F} - \frac{1 - f^2}{2f} \frac{1}{F^2} \frac{dF}{df} \right)^{1/4}}{A^{1/4}} + N_{rad} \tag{C27}$$

Equations (C26) and (C27) constitute a pair of parametric equations for N_{tot}, the individual Nusselt numbers, and the value of f. That is, if an arbitrary value of f is substituted into equations (C26) and (C27), unique values of $B/A^{3/4}$ and $A^{1/4}(N_{tot} - N_{rad})$ result. Numerically computing these values for the entire range of admissible f values results in a complete correspondence of N_{tot} to the individual Nusselt numbers. This constitutes the general solution. It is convenient to define

$$N^* = \frac{1 - f^2}{A^{1/4}} \left(- \frac{3}{F} - \frac{1 - f^2}{2f} \frac{1}{F^2} \frac{dF}{df} \right)^{1/4} \tag{C28}$$

Therefore,

$$N_{tot} = N^* + N_{rad} \tag{C29}$$

The solution for the case where there is no radiation or subcooling is given in reference 1 as

$$N_{fb} = 0.41(Ra\Lambda)^{1/4} \tag{C30}$$

When equation (C16) is used, $A^{1/4}$ is

$$A^{1/4} = \frac{1}{(Ra\Lambda)^{1/4}} = \frac{0.41}{N_{fb}} \tag{C31}$$

Thus,

$$\frac{N^*}{N_{fb}} = \frac{1 - f^2}{0.41} \left(- \frac{3}{F} - \frac{1 - f^2}{2f} \frac{1}{F^2} \frac{dF}{df} \right)^{1/4} \tag{C32}$$

Changing the form of equation (C26) by expressing $B/A^{3/4}$ in terms of Nusselt numbers through the use of equations (C16) and (C17) yields

$$\frac{B}{A^{3/4}} = \frac{\dfrac{N_{rad} - N_{tcl}^{\theta}}{Ra\Lambda}}{(Ra\Lambda)^{-3/4}} = \frac{N_{rad} - N_{tcl}^{\theta}}{(Ra\Lambda)^{1/4}} \tag{C33}$$

which from equation (C31) reduces to

$$\frac{B}{A^{3/4}} = 0.41\left(\frac{N_{rad} - N_{tcl}^{\theta}}{N_{fb}}\right) \tag{C34}$$

Thus, substituting equation (C34) into equation (C26) gives

$$\frac{N_{rad} - N_{tcl}^{\theta}}{N_{fb}} = \frac{1}{0.41} \frac{\dfrac{4}{F} + \dfrac{1 - f^2}{2f} \dfrac{1}{F^2} \dfrac{dF}{df}}{\left(-\dfrac{3}{F} - \dfrac{1 - f^2}{2f} \dfrac{1}{F^2} \dfrac{dF}{df}\right)^{3/4}} \tag{C35}$$

A small correction factor must be applied to the conduction term in the total Nusselt number. This correction accounts for the triangular patches shown in figure 1 (p. 2). The correction factor, which is derived in reference 1, is given by

$$\zeta(f) = \frac{\pi(1 - f^2) + 2\sqrt{3} - \pi}{2\sqrt{3}} \frac{1}{1 - f^2} \tag{C36}$$

Numerical Procedure

The important relations of the preceding section are

$$N_{tot} = \varphi N_{fb} + N_{rad} \tag{C37}$$

where

$$\varphi = \frac{N^*}{N_{fb}} = \frac{N_{tot} - N_{rad}}{N_{fb}}$$

and is given parametrically in terms of f by

$$\varphi(f) = \varphi = \frac{\zeta(f)(1 - f^2)}{0.41}\left(-\frac{3}{F} - \frac{1 - f^2}{2f}\frac{1}{F^2}\frac{dF}{df}\right)^{1/4} \tag{C38}$$

and where θ is related parametrically to radiation and subcooling by

$$\frac{N_{rad} - N_{tcl}\theta}{N_{fb}} = \psi(f) = \frac{1}{0.41}\frac{\frac{4}{F} + \frac{1 - f^2}{2f}\frac{1}{F^2}\frac{dF}{df}}{\left(-\frac{3}{F} - \frac{1 - f^2}{2f}\frac{1}{F^2}\frac{dF}{df}\right)^{3/4}} \tag{C35}$$

The function F is given by equation (C8), and ζ by equation (C36). These equations constitute the general solution of the problem, since a method for computing φ when N_{rad}, N_{fb}, N_{tcl}, and θ are given is apparent. Simply stated, φ is a unique function of the grouping $(N_{rad} - N_{tcl}\theta)/N_{fb}$. The numerical procedure is to substitute values of f in the range 0 to 1 into the right sides of equations (C38) and (C35). In this way, the complete functional dependence of φ on radiation and subcooling can be determined. The rather involved algebraic manipulations of the preceding section have thereby eliminated any need for numerical iteration. The calculations are presented graphically in figure 3

TABLE III. - TABULATED NUMERICAL RESULTS SHOWING EFFECT OF SUBCOOLING AND RADIATION ON DOME SPACING AND OVERALL HEAT-TRANSFER COEFFICIENT

Ratio of vapor-dome radius to cell radius, $f = R_1/R_0$	Individual heat-transfer coefficient group, $\psi = (h_{rad} - h_{tcl}\theta)/h_{fb}$	Overall heat-transfer coefficient group, $\varphi = (h_{tot} - h_{rad})/h_{fb}$
0.40	-1.266	1.262
.45	-1.235	1.252
.50	-1.165	1.228
.55	-1.032	1.188
.60	-.7926	1.130
.65	-.3431	1.049
.673+	.0000	1.000
.700	.6098	.9355
.750	3.471	.7598
.790	41.56	.3877

(p. 13) and selected values are tabulated in table III.

When values of $f = R_1/R_0$ less than 0.4 are substituted into the parametric equations, a new branch solution is obtained (shown by the dotted line in fig. 3). This branch is discussed in the RESULTS section; it is the degenerate case of pure free convection. Also, for a value of $f = 0.793$ the gap thickness becomes zero and the value of $(h_{rad} - h_{tcl}\theta)/h_{fb}$ becomes infinite. At the same time, the ordinate $(h_{tot} - h_{rad})/h_{fb}$ becomes zero. Thus, for infinite radiation, $h_{tot} = h_{rad}$, which is physically correct. However, why infinite radiation should correspond to $f = 0.793$ is unresolved.

APPENDIX D

SAMPLE CALCULATION

To illustrate the heat-transfer relations obtained, a numerical example of the film boiling of liquid nitrogen is considered. The various temperatures are shown in figure 5. These temperatures were chosen because the saturation temperature of nitrogen at atmospheric pressure is very nearly 140° R and the triple point is 114° R. A wall temperature of 2740° R was selected arbitrarily to assess the effect of radiation. To evaluate the heat-transfer coefficients the vapor properties are evaluated at the film temperature:

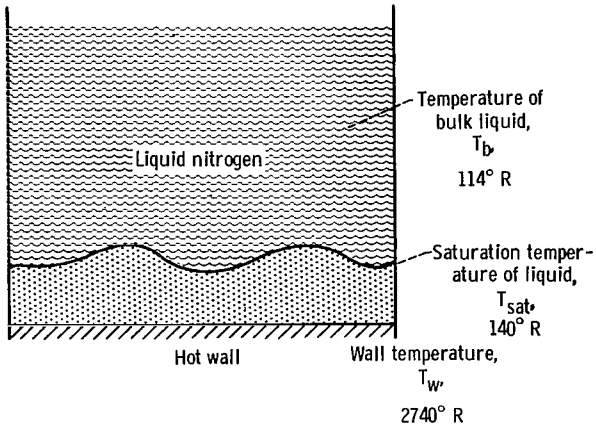


Figure 5. - Film boiling of liquid nitrogen.

$$T_f = \frac{T_w + T_s}{2} = \frac{2740 + 140}{2} = 1440^{\circ}\ R = 980^{\circ}\ F$$

Physical Properties

The following vapor properties were taken from Eckert and Drake (ref. 13, p. 506) and converted to consistent units:

$$\rho = 0.0267 \text{ lb mass/ft}^3$$

$$C_p = 0.2681 \text{ Btu/lb mass}$$

$$\mu = 23.41 \times 10^{-6} \text{ lb mass/(sec)(ft)}$$

$$k = 0.09 \times 10^{-4} \text{ Btu/(sec)(ft)}(^{\circ}R)$$

The liquid properties used were taken from reference 14 and evaluated at T_s. The number of the section in which it appears is given after each property.

$$\rho_l = 50.4 \text{ lb mass/ft}^3 \qquad \text{(Sec. 1.004)}$$

$$\mu_l = 1.055\times10^{-4} \text{ lb mass/(sec)(ft)} \qquad \text{(Sec. 10.004)}$$

$$k_l = 2.24\times10^{-5} \text{ Btu/(sec)(ft)}(^{\circ}\text{R}) \qquad \text{(Sec. 3.004)}$$

$$C_{p,l} = 0.49 \text{ Btu/(lb mass)}(^{\circ}\text{R}) \qquad \text{(Sec. 4.004)}$$

$$\lambda = 85.81 \text{ Btu/lb mass} \qquad \text{(Sec. 5.004)}$$

$$\sigma = 6.03\times10^{-4} \text{ lb force/ft} \qquad \text{(Sec. 9.004)}$$

$$\beta_l = 3.23\times10^{-3} (^{\circ}\text{R})^{-1} \text{ (calculated)} \qquad \text{(Sec. 1.004)}$$

Evaluation of Saturated Film-Boiling Coefficient

The saturated film-boiling coefficient is evaluated as

$$h_{fb} = 0.41\left[\frac{k^3\lambda^*\rho g(\rho_l - \rho)}{\mu(T_w - T_s)l}\right]^{1/4}$$

where

$$l = \left(\frac{g_c\sigma}{g(\rho_l - \rho)}\right)^{1/2} = \left(\frac{6.03\times10^{-4}}{50.4 - 0.0267}\right)^{1/2} = 3.46\times10^{-3} \text{ ft}$$

and

$$\lambda^* = \lambda\left(1 + \frac{19}{20}\frac{C_p(T_w - T_s)}{\lambda}\right) = 85.81\left(1 + \frac{19}{20}\frac{0.2681\times2600}{85.81}\right) = 748.01 \text{ Btu/lb}$$

Therefore,

$$h_{fb} = 0.41\left[\frac{(0.09\times10^{-4})^3 \times 748.01 \times 0.0267 \times 32.2 \times (50.4 - 0.0267)}{23.41\times10^{-6} \times 2600 \times 3.46\times10^{-3}}\right]^{1/4} \times 3600$$

or

$$h_{fb} = 27.00 \text{ Btu/(hr)(ft}^2\text{)(}^\circ\text{R)}$$

Evaluation of Radiation Coefficient

The plate and liquid emissivities are assumed to be 1 in the following evaluation of h_{rad}:

$$h_{rad} = 0.1713\times10^{-8} \frac{T_w^4 - T_s^4}{T_w - T_s} = 0.1713 \frac{27.4^4 - 1.4^4}{2600} = 37.50 \text{ Btu/(hr)(ft}^2\text{)(}^\circ\text{R)}$$

Subcooling Parameter

For the film boiling of liquid nitrogen, the subcooling parameter is evaluated as

$$\theta = \frac{T_s - T_b}{T_w - T_s} = \frac{26}{2600} = 0.01$$

Liquid Free-Convection Coefficient

One of the major questions left unanswered by the present analysis is what to use for the turbulent liquid free-convection coefficient, since the vapor bubbles induce strong turbulence in the liquid. Experimental data in this area would be most helpful. For the purposes of this calculation, a coefficient based on ordinary turbulent free convection from a horizontal plate given by McAdams (ref. 15, p. 180) is used. The liquid properties were evaluated at T_s, though McAdams recommends using the liquid film temperature:

$$h_{tcl} = 0.14 \left[\frac{k_l^3 \rho_l^2 g \beta_l (T_s - T_b)}{\mu_l^2} Pr_l \right]^{1/3}$$

The Prandtl number is given by

$$Pr_l = \frac{C_{p,l}\mu_l}{k_l} = \frac{0.49 \times 1.055\times10^{-4}}{2.24\times10^{-5}} = 2.31$$

Therefore,

$$h_{tcl} = 0.14\left[\frac{\left(2.24\times10^{-5}\right)^3 \times 50.4^2 \times 32.2 \times 3.23\times10^{-3} \times 26 \times 2.31}{\left(1.055\times10^{-4}\right)^2}\right]^{1/3} \times 3600$$

$$= 127.15 \text{ Btu/(hr)(ft}^2\text{)(}^\circ\text{R)}$$

Evaluation of Overall Heat-Transfer Coefficient

To evaluate the overall heat-transfer coefficient

$$\frac{h_{rad} - h_{tcl}\theta}{h_{fb}} = \frac{37.50 - 127.15 \times 0.01}{27.00} = 1.342$$

is first computed. From figure 3 (p. 13), the value of φ is 0.871. Thus,

$$h_{tot} = \varphi h_{fb} + h_{rad} = 0.871 \times 27.00 + 37.50 = 61.02 \text{ Btu/(hr)(ft}^2\text{)(}^\circ\text{R)}$$

and from equation (31)

$$q_{tot} = 61.02 \times 2600 = 1.59\times10^5 \text{ Btu/(hr)(ft}^2\text{)}$$

The simple formula given by equation (38) predicts

$$h_{tot} = h_{fb} + 0.88h_{rad} + 0.12h_{tcl}\theta = 27.00 + 0.88 \times 37.5 + 0.12 \times 127.15 \times 0.01$$

$$= 60.16 \text{ Btu/(hr)(ft}^2\text{)(}^\circ\text{R)}$$

This value is very close to the graphical solution, indicating the utility of the simple formula.

APPENDIX E

TRANSFORMATION OF VARIABLES

The optimization constraints, equations (22) and (23), are rewritten in terms of the parameter f for convenience in the mathematical operations, as given by equation (C1).

The total heat-transfer coefficient can be rewritten in terms of the parameter f as

$$h_{tot}(R_0, R_1) = h_{tot}(R_0, f) \tag{E1}$$

The functional forms of h_{tot} on the left and right side of this equation are, of course, different.

The optimal constraint, equation (23), can now be rewritten as

$$\frac{\partial}{\partial R_1}\left[h_{tot}(R_1, R_0)\right]_{R_0} = \frac{1}{R_0}\frac{\partial}{\partial f}\left[h_{tot}(f, R_0)\right]_{R_0} = 0 \tag{E2}$$

or

$$\frac{\partial}{\partial f}\left[h_{tot}(f, R_0)\right]_{R_0} = 0 \tag{E3}$$

which is used in arriving at equation (C14).

Now, by the chain rule of differentiation, the optimal constraint, equation (22), can be rewritten as

$$\frac{\partial}{\partial R_0}\left[h_{tot}(R_1, R_0)\right]_{R_1} = \frac{\partial}{\partial R_0}\left[h_{tot}(f, R_0)\right]_f + \frac{\partial}{\partial f}\left[h_{tot}(f, R_0)\right]_{R_0}\frac{\partial f}{\partial R_0} = 0 \tag{E4}$$

However, because of equation (E3), equation (E4) simplifies to

$$\frac{\partial}{\partial R_0}\left[h_{tot}(R_1, R_0)\right]_{R_1} = \frac{\partial}{\partial R_0}\left[h_{tot}(f, R_0)\right]_f = 0 \tag{E5}$$

which is equation (C1).

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