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by

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ABSTRACT

Due to the low electron density and the large mean free path in the magnetosphere it is necessary to modify the equation of the heat flux parallel to the magnetic field. It is shown that the heat conductivity is effectively decreased by this modification, which can explain the high temperatures in the upper magnetosphere observed by Serbu and Maier. Since the effective perpendicular heat conductivity may be enormously enhanced due to plasma turbulence, it is then conceivable that both conductivities may be of comparable magnitude. This suggests a heat flux from the interplanetary space through the magnetosphere into the thermosphere, which could also effect the magnetospheric electron temperature distribution.

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INTRODUCTION

Measurements of the low energy electron spectrum by Serbu and Maier (1966) on IMP II have shown that in the magnetosphere these electrons have almost a Maxwellian velocity distribution.

In the equatorial plane the electron temperature T_e increases with distance r from the earth's center as

$$T_e \propto r^2 \tag{1}$$

above $r = 2a$ (a = earth's radius), reaching values of the order $T_e \sim 20\,000\ ^\circ K$ in heights between $5a$ and $15a$. Otherwise relatively low temperatures of the order of $3000\ ^\circ K$ have been observed at the base of the protonosphere in heights of about 1000 km above the ground at low and high latitudes (Brace, Reddy and Mayr, 1967).

According to the classical formulae of heat conduction within an anisotropic plasma the predominant heat flow parallel to the geomagnetic fieldlines within the protonosphere is due to electrons (see e.g. Kaufman, 1966). The coefficient of heat conductivity is

$$\kappa_{\parallel} \sim \kappa_{\parallel}^{(e)} = A_1 T_e^{5/2} \quad \text{erg/cm sec deg} \tag{2}$$

with

$$A_1 \sim 1 \times 10^{-6}.$$

Perpendicular to the magnetic field the ionic component is predominant and the coefficient of heat conductivity is

$$\kappa_{\perp} \sim \kappa_{\perp}^{(i)} = \kappa_{\parallel}^{(e)} \frac{\nu_i^2}{\omega_i^2} \sqrt{\frac{m_e}{m_i}} = A_2 \frac{N_i^2}{B^2 T_i^{1/2}} \text{ erg/cm sec deg} \tag{3}$$

with

$$A_2 \sim 3 \times 10^{-16}.$$

Here T_i, N_i and ν_i are temperature, density and collision number of the protons, B and ω_i are the earth's magnetic field and the ion gyrofrequency, and m_i and m_e are ion and electron masses.

Taking mean numerical values of $T_e \sim T_i \sim 10^4$ °K, $N_i \sim 10^2$ cm^{-3} and $B \sim 10^{-3}$ Gauss leads to a ratio between the coefficients of heat conductivity of

$$\frac{\kappa_{\perp}}{\kappa_{\parallel}} = 3 \times 10^{-12}. \tag{4}$$

If the classical formulae of heat conductivity equations (2) and (3) are valid within the protonosphere one would expect an extremely large anisotropy of temperature: namely nearly isothermy along the fieldlines and an almost complete heat isolation orthogonal to the fieldlines.

In order to derive a first approximation of the temperature profile within the protonosphere we consider a heat isolated fieldline, neglect the geometrics of the dipole field and assume no energy source within the protonosphere. The static energy equation has the form

$$\mathrm{div}\,\vec{j} = -\,\mathrm{div}\,(\kappa\ \mathrm{grad}\ T) = Q. \tag{5}$$

$\vec{j}$ is the heat flux vector and Q is the external energy input per unit volume. Introducing the coefficient $\kappa_{\shortparallel}$ of equation (2) into equation (5) and taking $Q = 0$ leads to the solution

$$T = \left[T_I^{7/2} + \frac{2}{7}\,\frac{\kappa_I T_I'}{A_1}\,(s - s_I) \right]^{2/7} \sim C\, s^{2/7} \tag{6}$$

where T_I and $T_I' = \left(\frac{\partial T}{\partial s}\right)_I$ are the lower boundary values at the height s_I. From equation (6) follows the expected small temperature gradient along the field lines proportional to $s^{2/7}$ in greater heights ($s \gg s_I$). To maintain even this small gradient and to reach a temperature $T_I \approx 20\,000°$ at the top of the field line a heat flux of the order of

$$j_I = \kappa_I T_I' = 1\ \mathrm{erg/cm^2\ sec}$$

had to flow into the thermosphere in medium and higher latitudes. This would imply a temperature gradient of the order

$$T_I' = 100 \ °/\text{km}$$

in 1000 km height. Tamao (1966) taking into account the geometry of the field lines came to the same result. He assumed that this heat flow must be supplied from the interplanetary space via turbulent diffusion across the field lines in greater heights. But the observations made by Brace, Reddy and Mayr (1967) give evidence that a realistic upper limit of the electron temperature gradient in 1000 km height is at least smaller by two orders of magnitude:

$$T_I' \lesssim 1 \ °/\text{km} .$$

If we introduce this value into equation (6) together with $T_I = 3000 \ °K$ we obtain a temperature at the top of the fieldlines of $T_{II} \lesssim 7000 \ °K$ in $r = 8a$ distance which is considerably smaller than the measurements of Serbu and Maier (1966) show.

This obvious and striking discrepancy between theory and observations is the subject of the following investigation. We shall show that a more rigorous treatment of heat conduction leads to an effective decrease of the heat flow in regions of low electron density and to a general increase of the temperature at greater heights.

Parallel Heat Conduction

An examination of the mean free path approximation of heat conduction reveals that it contains the assumption of a linear temperature distribution within the range of the mean free path. In the magnetosphere where the mean free path

$$\lambda = K \cdot \frac{T_e^2}{N_e} \; ; \; K = \text{const} . \qquad (7)$$

is large (in the order of 10^9 cm), due to high electron temperatures T_e and low electron densities N_e, it seems questionable whether this assumption remains valid. Mayr (1965) generalized the mean free path method by considering the actual free path distribution and by allowing for any temperature distribution. With this generalization the heat flux is represented as an integral that covers the entire temperature range, implying that rigorously the heat flux depends on the entire temperature distribution. For an investigation on the magnetospheric temperature which requires the integration of the energy equation, this integral representation is inconvenient because of the necessarily involved analysis. For this reason we shall approach the problem by merely generalizing the mean free path derivation for a non linear temperature distribution.

Expanding the temperature into a power series in s around $s_o = 0$ gives to

$$T = T_o + \left(\frac{\partial T}{\partial s}\right)_o s + \frac{1}{2}\left(\frac{\partial^2 T}{\partial s^2}\right)_o s^2 + \frac{1}{6}\left(\frac{\partial^3 T}{\partial s^3}\right)_o s^3 .$$

when higher order terms are assumed to be neglegible small. If ζ is the angle of the velocity direction toward the s axis, each electron carries the energy

$$\frac{3}{2} k T_o - \lambda \cos \zeta \left[\frac{\partial}{\partial s}\left(\frac{3}{2} k T\right)\right]_o + \lambda^2 \cos^2 \zeta \left[\frac{\partial^2}{\partial s^2}\left(\frac{3}{2} k T\right)\right]_o - \frac{\lambda^3}{6} \cos^3 \zeta \left[\frac{\partial^3}{\partial s^3}\left(\frac{3}{2} k T\right)\right]_o \tag{8}$$

k is Boltzmann's constant

through the plane at $s = 0$. Correspondingly, electrons going in exactly the opposite direction transport the energy

$$\frac{3}{2} k T_o + \lambda \cos \zeta \left[\frac{\partial}{\partial s}\left(\frac{3}{2} k T\right)\right]_o + \frac{\lambda^2}{2} \cos^2 \zeta \left[\frac{\partial^2}{\partial s^2}\left(\frac{3}{2} k T\right)\right]_o + \frac{\lambda^3}{6} \cos^3 \zeta \left[\frac{\partial^3}{\partial s^3}\left(\frac{3}{2} k T\right)\right]_o . \tag{9}$$

The difference of these two energies is

$$- 2\lambda \cos \zeta \left[\frac{\partial}{\partial s}\left(\frac{3}{2} k T\right)\right]_o - \frac{\lambda^3}{3} \cos^3 \zeta \left[\frac{\partial^3}{\partial s^3}\left(\frac{3}{2} k T\right)\right]_o . \tag{10}$$

During unit time

$$c \cos \zeta \, N_e \, f \, d\zeta$$

electrons traverse the unit area at $s = 0$, if f is the distribution function for the velocities c, given by

$$f = 2\pi \left(\frac{1}{2\pi m_e k T}\right)^{3/2} \exp\left(\frac{m_e c^2}{2kT}\right) m_e^3 c^2 \sin \zeta \, dc . \tag{11}$$

The net energy transported by electrons with the velocity c and the angle ζ is therefore

$$\begin{aligned} &- 2\lambda \cos^2 \zeta \, c \frac{\partial}{\partial s}\left(\frac{3}{2} k T\right) N_e f \, d\zeta \; - \\ &- \frac{\lambda^3}{3} \cos^4 \zeta \, c \frac{\partial^3}{\partial s^3}\left(\frac{3}{2} k T\right) N_e f \, d\zeta . \end{aligned} \tag{12}$$

Integration over $o \leq \zeta \leq \frac{\pi}{2}$ and $o \leq c \leq \infty$ then leads in a straight forward manner to the heat conduction flux in the form

$$j = - \frac{\bar{c}\lambda}{3} N_e \frac{\partial}{\partial s}\left(\frac{3}{2} k T\right) - \bar{c} \frac{\lambda^3}{15} N_e \frac{\partial^3}{\partial s^3}\left(\frac{3}{2} k T\right) \tag{13}$$

where $\bar{c}$ is the mean thermal velocity.

Introducing in equation (13) the dependance of the mean free path on temperature and density (equation (7)) yields

$$j = - \kappa_{\shortparallel} \frac{\partial T}{\partial s} - \kappa_{\shortparallel} \frac{K^2}{N_e^{\,2}} T^4 \frac{\partial^3 T}{\partial s^3} . \tag{14}$$

The heat flux j in equation (14) contains now an additional term which is proportional to the third temperature derivative. As can be seen, this term becomes of increasing importance for high temperatures and low densities. Thus it may have an important effect on the temperature distribution in the magnetosphere such as to decrease the conductivity of the plasma in a way that could allow the very high temperatures measured by Serbu and Maier.

We have employed the new heat flux formula equation (14) in the energy equation (5) which leads to

$$\kappa_{\shortparallel} \frac{\partial T}{\partial s} + \kappa_{\shortparallel} \frac{K^2}{N^2} T^4 \frac{\partial^3 T}{\partial s^3} = \kappa_{\shortparallel} \left(\frac{\partial T}{\partial s}\right)_{II} + \kappa_{\shortparallel} \frac{K^2}{N_e^{\,2}} T_{II}^{\,4} \left(\frac{\partial^3 T}{\partial s^3}\right)_{II} + \int_{s_2}^{s} Q \, ds . \tag{15}$$

Equation (15) was solved with the following boundary conditions.

At $s = s_{II}$, where s_{II} is the field line distance to the equator we assume $T_{II} = 20\,000\ °K$ according to Serbu and Maier's measurements, and we assume

$$\left(\frac{\partial T}{\partial s}\right)_{II} = \left(\frac{\partial^3 T}{\partial s^3}\right)_{II} = 0$$ which implies that at the equator the heat flux is zero and that our solution is symmetrical with respect to the equatorial plane. The second derivative was choosen such that for $s = s_I$, in 1000 km height $T_I = 3.000\ °K$. Q, the energy input along the field line was assumed to be $2 \times 10^{-12}\ erg/cm^3$. This is an upper limit of the heating rate expected from fast electrons that escape the ionosphere (Geisler and Bowhill, 1965) and could quite well be produced in part by the solar wind which is another possible energy source. The electron density choosen to be $10^2/cm^3$ according to the proton density trough at high latitudes observed by Taylor et al. (1967), was assumed to be constant along the field line.

With these inputs the computation of equation (15) leads to the temperature profile shown in Figure 1 as full line. It reveals a magnetospheric temperature structure consistant with Serbu and Maier's measurements and a temperature gradient of the order of 1°/km in 1000 km height. The resulting heat flux is $j_I = -6 \times 10^{-3} erg/cm^2$ sec in this height and is comparable with the energy flux expected to flow downwards into the thermosphere (see section 3). For comparison we show also the temperature distribution (dashed line) derived with the classical heat flux formula, which illustrated the strong effect of the additional flux term introduced here. Thus it appears that our approach offers an appropriate mean of explaining high temperatures in the magnetosphere.

We have shown that the heat flux equation (14) is actually density dependent. With decreasing concentration the third derivative of the temperature distribution becomes more effective in a way to decrease the thermal conductivity. The result was an increase of the temperature in greater hights. This density dependant temperature behavior could also be responsible for the measured increase of the equatorial temperature (equation (1)) because at the equator the electron density decreases like

$$N_e \propto r^{-3} \text{ to } r^{-4}$$

(Carpenter and Smith, 1964; Serbu and Maier, 1966). An alternative explanation of the temperature variation with height according to equation (1) will be given in the next section.

Perpendicular Heat Conduction

Up to now our investigation was based on the assumption of an extremely low heat conductivity perpendicular to the field lines and therefore a complete heat isolation of the different field lines within the magnetosphere. But experiments as well as theory in plasma physics reveal that plasma turbulence enhances transport processes perpendicular to the magnetic field, such that the effective transport coefficients are by several orders of magnitudes larger than the classical coefficients. (Bohm, 1949; Kadomtsev, 1965; Tsuda, 1967).

In the previous section we have shown that the effective parallel heat conductivity can be significantly decreased due to the low electron concentration in the magnetosphere. Thus it is conceivable that the effective conductivities parallel ($\kappa_{\parallel eff}$) and perpendicular ($\kappa_{\perp eff}$) to the magnetic field are of the same order of magnitude and therefore the anisotropy of the temperature behaviour may diminish in the magnetosphere. The consequence would be that the perpendicular heat transport could also effect the temperature distribution.

In order to estimate the equatorial temperature profile of such "quasi isotropic" magnetosphere we employ a transverse heat coefficient ($\kappa_{\perp eff}$) which has the same temperature, density and magnetic field dependance as the classical term in equation (3), an assumption which is rather arbitrary in view of the very different energy transport mechanism involved. If we take $B \sim r^{-3}$ and a density distribution $N_i \propto r^{-4}$ in agreement with Carpenter and Smith (1964) and in the order of that measured by Serbu and Maier, the spherical energy equation (5) with $Q = 0$ leads to

$$T = \left\{ T_I + \text{Const} \times T_I' \; (r - r_I) \right\}^2 \sim C r^2 \tag{16}$$

which gives the measured r dependance of T [Eq. (1)].

From the interpretation of satellite drag measurements a global heat flux from the interplanetary space into the thermosphere in the order of

$$j = -10^{-2} \text{ erg/cm}^2 \text{ sec}$$

is suggested, which could possibly explain the geomagnetic activity effect, the semiannual effect and the second heat source of the Harris and Priester model (Priester, Roemer and Volland, 1967). Such a heat flux would require a perpenticular heat conductivity

$$\kappa_{\perp eff} \simeq 10^{10} \times \kappa_{\perp} . \tag{17}$$

From Tsuda's (1967) calculations a factor of $\kappa_{\perp eff}/\kappa_{\perp} > 10^7$ can be derived which gives evidence that the value of equation 17 is not completely inconceivable.

The present investigation served primarily to indicate that the classical energy transport formulae for a plasma are not necessarily valid within the magnetosphere. Our calculations are only crude estimates, which appear to be justified by the qualitative agreement between our results and measurements.

LITERATURE

D. Bohm in "The characteristics of electric discharges in magnetic fields", edited by A. Guthrie and R. K. Wakerling, Chapter 2, Section 5, McGrawhill, New York, 1949

L. H. Brace, B. M. Reddy and H. Mayr, "Global behavior of the ionosphere at 1000 km altitude", Journ. Geophys. Res. 72, 1967, 265-183

D. L. Carpenter and R. L. Smith, "Whistler measurements of electron density in the magnetosphere", Rev. Geophys. 2, 1964, 415-441

J. E. Geisler and S. A. Bowhill, "Exchange of energy between the ionosphere and the protonosphere", Journ. Atm. Terr. Phys. 27, 1965, 1119-1146

P. Kadomtsez, "Plasma turbulence", Academic Press, London-New York, 1965

A. N. Kaufman, "Dissipative effects" in "Plasma physics in theory and application" (W. B. Kunkel, ed.) McGrawhill, New York, 1966, p. 92

H. Mayr, "Ein Ansatz zur Behandlung von Transportprozessen in Gasen", Acta physica Austria, 1965

W. Priester, M. Roemer and H. Volland, "The physical behavior of the upper atmosphere deduced from satellite drag data", Space Sci. Rev. 6, 1967

LITERATURE (Continued)

G. P. Serbu and E. J. R. Maier, "Low energy electrons measured on IMP 2", Journ. Geophys. Res. 71, 1966, 3755-3766

T. Tamao, "Temperature distribution in the magnetosphere" Report Ionosph. and Space Res. in Japan, 20, 1966, 312-321

T. Tsuda, "Effective viscosity of a streaming collision free plasma in a weekly turbulent magnetic field", AGU-Meeting, Washington, D. C., April 1967

H. A. Taylor, Jr., A. C. Brinton and L. R. Muenz, "First results from OGO-C Ion Composition Experiment" Presented at the 47th Annual Meeting of AGU, March 1966.

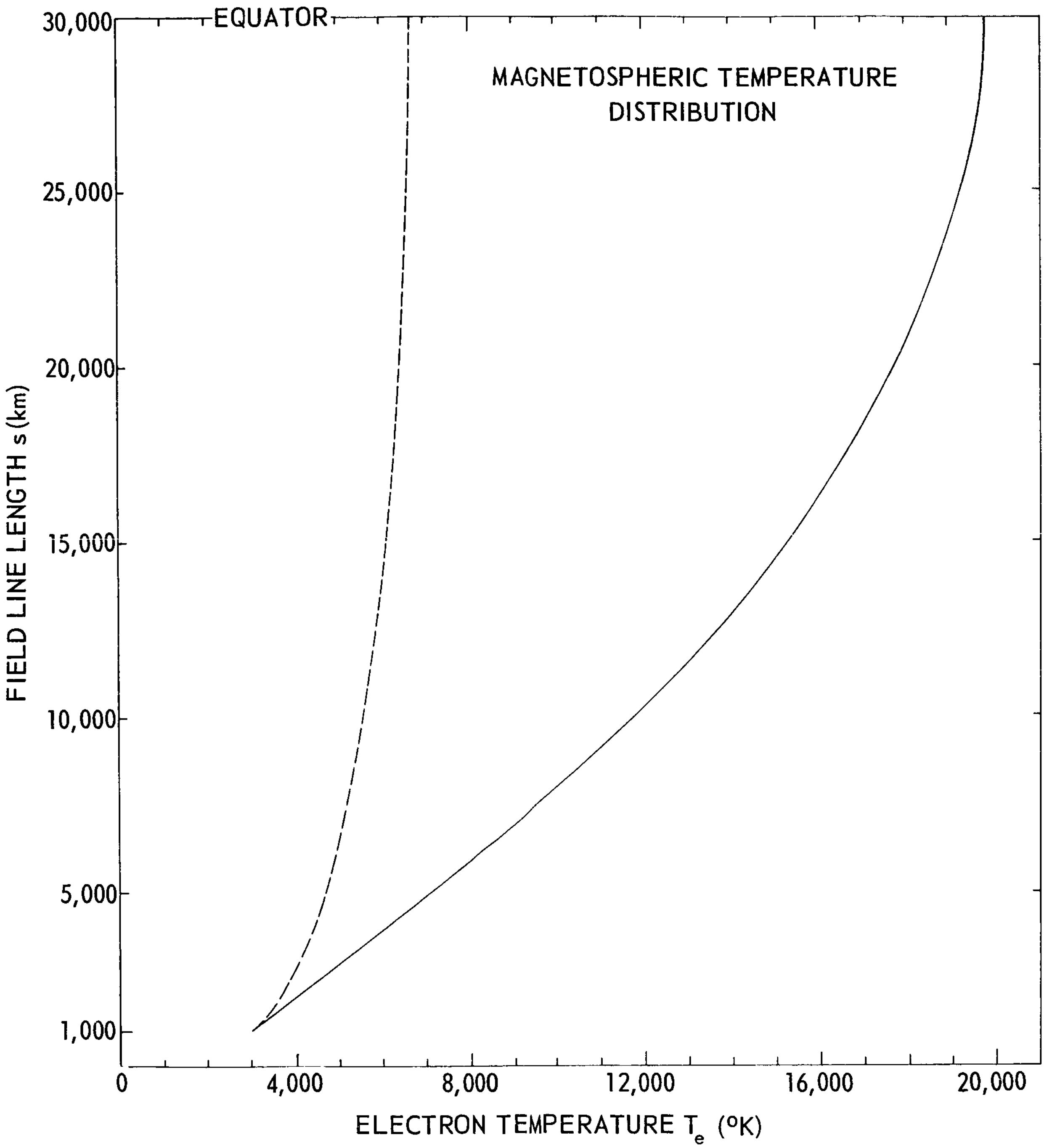

Figure 1. Calculated electron temperatures along a field line. The dashed line represents the result obtained from the classical formula of the heat flux, the solid line was derived employing a modified formula.