SCALE FACTORS FOR PARACHUTE OPENING

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ABSTRACT

A method is derived for relating model to full-scale parachute-opening characteristics by comparing like forces in the equation of motion. Experimental results from a drop test of two scaled, ringsail parachutes are presented using the scaling method. Model laws are determined for earth simulations of planetary deployments under the restriction of matching the Mach number.
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SUMMARY

A method has been derived to relate model to full-scale parachute-opening characteristics by applying the principle of dynamic similarity to the parachute equation of motion. Scale factors for each parameter in the equation of motion have been shown to be a function of three independent ratios: diameter, gravity, and density.

Earth simulation of planetary deployments has been considered under the restriction that at deployment the earth Mach number must equal the planetary Mach number. The diameter ratio was thus forced to be a discrete function of the ratio of earth speed of sound to planetary speed of sound. Therefore, the three independent ratios became (1) the speed of sound, (2) the gravity, and (3) the density.

An experimental test has been conducted using the derived scale factors. The test subjects were two scaled, ringsail parachutes. Initially the parachutes were held in a cylindrical shape by rigid internal hoops. The test procedure consisted of dropping the parachutes from a crane in this forced-reefed condition. The internal hoops allowed the parachute to open freely. The measured opening characteristics of the model motion (riser tension, acceleration, velocity, and distance), as a function of time, were then scaled to predict the full-scale measured opening characteristics. The comparison of the measured full-scale motion with the predicted full-scale motion showed close similarity.

INTRODUCTION

The successful design of parachute systems to date has required extensive experimental testing, especially in the deployment area. Since it is much simpler and less expensive to work with small parachutes, scaled models of the full-scale parachute are normally used in the early design stages. Unfortunately, in the past, the model opening characteristics were not directly applicable to the full-scale opening characteristics because of the lack of an acceptable scaling technique. The purpose of this paper is to present a method of predicting the opening characteristics of a full-scale parachute from the results of model deployments. Also, model laws for earth simulations of planetary deployments will be derived under the restriction that the test Mach numbers must match.
SYMBOLS

\( C_D \) drag coefficient

\( D \) drag force, lb

\( d \) diameter, ft

\( F \) force, lb

\( g \) gravity, \( \text{ft/sec}^2 \)

\( k \) spring constant, lb/ft

\( M \) Mach number

\( m \) mass, slugs

\( \dot{m} \) mass flow, slugs/sec

\( N \) force scale factor

\( q \) dynamic pressure, \( \text{lb/ft}^2 \)

\( R \) ratio

\( S \) projected area, \( \text{ft}^2 \)

\( T \) riser tension, lb

\( t \) time, sec

\( V \) volume, \( \text{ft}^3 \)

\( v \) velocity, \( \text{ft/sec} \)

\( \dot{v} \) acceleration, \( \text{ft/sec}^2 \)

\( W \) weight, lb

\( x \) distance, ft

\( \rho \) atmospheric density, slugs/\( \text{ft}^3 \)

\( \rho_S \) structural mass density, slugs/\( \text{ft}^3 \)
Subscripts:

\( \text{e} \) earth
\( \text{f} \) full scale
\( \text{m} \) model
\( \text{o} \) refers to the velocity of the speed of sound
\( \text{p} \) planetary
\( \text{S} \) structural

All subscripts used with the symbol \( \text{R} \) (ratio) are defined in the basic symbols list.

SCALING METHOD THEORY

To scale the opening characteristics of parachutes, the governing equation of motion must be considered. Applying Newton's second law to the parachute motion yields

\[
\sum F = \frac{d}{dt} (mv) = m\ddot{v} + \dot{m}v = W - D
\]

as shown in the accompanying sketch. The symbols used in equation (1) are defined as follows:

- \( m \) = the total mass of the system, including the apparent and the included mass
- \( \ddot{v} \) = the acceleration of the system
- \( v \) = the velocity
- \( \dot{m} \) = the mass flow per unit time
- \( W \) = the total structural weight of the payload-parachute system
\[ D = \text{the system drag force} = \frac{1}{2} \rho v^2 S C_D, \text{ where } \rho \text{ is the atmospheric density,} \]
\[ S \text{ is the projected area, and } C_D \text{ is the drag coefficient.} \]

The laws of dynamic similarity (ref. 1) require that, for a scaled test, all forces in the model equation of motion be scaled similarly to the same force in the full-scale equation of motion.

\[
\frac{(m \dot{v})_m}{(m \dot{v})_f} = \frac{(\dot{m} v)_m}{(\dot{m} v)_f} = \frac{(W)_m}{(W)_f} = \frac{(D)_m}{(D)_f} = N
\]

(2)

where the subscripts \( m \) and \( f \) refer to the model and full-scale tests, and the symbol \( N \) represents the force scale factor equal to each of the force ratios. In a more convenient notation, equation (2) becomes

\[
R_m R_v = R_m R_v = R_W = R_D = N
\]

(3)

where each of the scale factors refers to the model-to-full-scale ratio.

\[ \text{RESULTS} \]

To determine the force scale factor \( N \) in its most basic form, the weight scale factor is expanded to yield

\[
R_W = R_m R_g = R_\rho R_v R_g = R_\rho R_d^3 R_g = N
\]

(4)

where use is made of the relations \( W = mg = \rho_S V g, \ V \propto d^3 \), and \( R_\rho = R_\rho \). It will be noted that the structural density ratio is set as being equal to the air density ratio. This is not required for a solution, but otherwise, the ensuing scale factors become somewhat more complicated.
After determining \( N \), the acceleration scale factor is determined by examining the mass-acceleration force ratio

\[
R_m \frac{R_c}{R_v} = \frac{\rho}{\rho_v} V^2 R_c = N
\]

\[
R_v = \frac{N}{R_v^2} = \frac{\rho}{\rho_v} \frac{R^3_c}{R_d^3} = R_g
\]

In a similar fashion, the velocity scale factor is determined from the drag-force scale factor

\[
R_D = \frac{R}{\rho_v} V^2 R_S R_C D = N
\]

\[
R_v^2 = \frac{R^3}{\rho_v^2 R_d^2} = R_g R_d
\]

Here, it is assumed that the ratio of drag coefficients between the two tests is unity. This requires that the drag coefficient be insensitive to Mach number and Reynolds number over the range being tested. Also, the relation \( R_s = R_{d^2} \) is employed from the relation \( S \propto d^2 \). The time scale factor is

\[
R_t = \frac{R_v}{R_v^2} = \frac{R_g^{1/2} R_{d^{1/2}}}{R_g} = R_g^{-1/2} R_d^{1/2}
\]

Similarly, the distance scale factor becomes

\[
R_x = R_v R_t = R_g^{1/2} R_{d^{1/2}} R_g^{-1/2} R_d^{1/2} = R_d
\]
Finally, considering the mass-accumulation force, the following is obtained:

\[
\begin{align*}
R_m R_v &= N = \frac{R \rho g R_d^3}{R_g^{1/2} R_d^{1/2} R_d^{5/2}} \\
R_m &= \frac{R \rho g R_d^3}{R_g^{1/2} R_d^{1/2}} = \frac{R \rho g R_d^{1/2} R_d^{5/2}}{R_g^{1/2} R_d^{1/2}} \tag{9}
\end{align*}
\]

Identical results can be obtained in considering the following relations:

\[
\begin{align*}
\dot{m} &= \rho S v \\
R_m &= \frac{R \rho S R_v}{\rho g R_d^{2/3} R_g^{1/3} R_d^{1/3}} = \frac{R \rho g^{1/2} R_d^{5/2}}{R_g^{1/2} R_d^{1/2}} \tag{10}
\end{align*}
\]

Therefore, each of the parameters in the governing equation of motion of the model is scaled to the same parameter in the full-scale equation of motion. Table I lists the scale factors for easy reference.

All the scale factors shown in the table are functions of three independent ratios: diameter, gravity, and density. These ratios are arbitrary and will be determined by the scaled test under consideration.

EARTH-TO-PLANET SCALE FACTORS

The scale factors presented in table I would apply to an earth simulation of a planetary deployment if the ratios are considered as earth-to-planet scale factors. Unfortunately, the initial conditions on a planet at parachute deployment may cause the earth simulation to have a wide variance in the Mach number. Since the drag coefficient ratio must be unity, consider the Mach number as a scaling basis.

On earth, the Mach number is defined as

\[
M_e = \frac{v_e}{v_{o,e}} \tag{11}
\]

where \( v_o \) is the speed of sound.
TABLE I. - SCALE FACTORS

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Ratio</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter</td>
<td>$R_d$</td>
<td>$R_d$</td>
</tr>
<tr>
<td>Area</td>
<td>$R_S$</td>
<td>$R_d^2$</td>
</tr>
<tr>
<td>Volume</td>
<td>$R_V$</td>
<td>$R_d^3$</td>
</tr>
<tr>
<td>Gravity</td>
<td>$R_g$</td>
<td>$R_g$</td>
</tr>
<tr>
<td>Density</td>
<td>$R_\rho$</td>
<td>$R_\rho$</td>
</tr>
<tr>
<td>Distance</td>
<td>$R_x$</td>
<td>$R_d$</td>
</tr>
<tr>
<td>Velocity</td>
<td>$R_v$</td>
<td>$R_g^{1/2} R_d^{1/2}$</td>
</tr>
<tr>
<td>Acceleration</td>
<td>$R_\dot{v}$</td>
<td>$R_g$</td>
</tr>
<tr>
<td>Time</td>
<td>$R_t$</td>
<td>$R_g^{-1/2} R_d^{1/2}$</td>
</tr>
<tr>
<td>Mass</td>
<td>$R_m$</td>
<td>$R_\rho^{1/2} R_d^{1/2}$</td>
</tr>
<tr>
<td>Mass flow</td>
<td>$R_{\dot{m}}$</td>
<td>$R_\rho R_g^{1/2} R_d^{5/2}$</td>
</tr>
<tr>
<td>Weight</td>
<td>$R_W$</td>
<td>$R_\rho R_g R_d^{3}$</td>
</tr>
<tr>
<td>Drag coefficient</td>
<td>$R_{C_D}$</td>
<td>1.0</td>
</tr>
<tr>
<td>Mach number</td>
<td>$R_M$</td>
<td>$R_g^{1/2} R_d^{1/2}$</td>
</tr>
<tr>
<td>Speed of sound</td>
<td>$R_{v_o}$</td>
<td>1.0</td>
</tr>
</tbody>
</table>
On a planet, the Mach number is

\[ M_p = \frac{v_p}{v_o, p} \]  

\( v_p \) 

Hence, the earth-to-planet Mach number ratio is

\[ R_M = \frac{v_e}{v_o, e} = \frac{R_v}{R_{v_o}} \]  

\[ R_v \] 

Since \( R_{C_D} \) must equal unity, the earth test Mach number should be restricted to match the planet test Mach number

\[ R_M = 1 = \frac{R_v}{R_{v_o}} \]  

This restriction assumes that the drag coefficient is primarily dependent on the Mach number while being insensitive to the Reynolds number. If this assumption is met, then the restriction of matching the test Mach number yields for the velocity ratio

\[ R_v = R_{v_o} \]  

\( R_v \) 

but, from table I,

\[ R_v = R_g^{1/2} R_d^{1/2} \]  

\( R_g \) 

\( R_d \)
Hence,

\[ R_g \frac{1}{2} R_d \frac{1}{2} = R_{v_0} \]  \hspace{1cm} (17)

Then,

\[ R_d \frac{1}{2} = \frac{R_{v_0}}{R_g} \]  \hspace{1cm} (18)

or

\[ R_d = \frac{R_{v_0}^2}{R_g} \]  \hspace{1cm} (19)

Therefore, the earth-to-planet diameter ratio is fixed so that the Mach number ratio is unity. Table II presents the results determined by substituting equation (19) into table I. Now, the three independent scale factors have become the speed of sound, the gravity, and the density. While the speed of sound and the gravity ratio are fixed for the specific deployment condition to be tested, the density ratio is completely arbitrary. Therefore, the density at deployment for the earth test may be at any desired altitude. No restriction, such as one in which the earth-to-planet density ratio should be 1, is observed.

EXPERIMENTAL RESULTS

Technique

To check the validity of the scaling method, an experimental test of the opening characteristics of parachutes was considered necessary. It was required that the initial conditions of the test be scaled and repeatable.

The method of testing selected for the scaled deployment study was a crane drop (fig. 1). This method consisted of dropping the parachute in a forced-reefed condition at an initial velocity of zero. Measurements were made of the riser-line tension as the parachute opened. For tests where high opening loads would be expected, consideration should be given to scaling the riser-line spring constant \( k \), since the
TABLE II. - PLANETARY SCALE FACTORS

<table>
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<tr>
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<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter</td>
<td>$R_d$</td>
<td>$R_g^{-1}R_{v_0}^2$</td>
</tr>
<tr>
<td>Area</td>
<td>$R_S$</td>
<td>$R_g^{-2}R_{v_0}^4$</td>
</tr>
<tr>
<td>Volume</td>
<td>$R_V$</td>
<td>$R_g^{-3}R_{v_0}^6$</td>
</tr>
<tr>
<td>Gravity</td>
<td>$R_g$</td>
<td></td>
</tr>
<tr>
<td>Density</td>
<td>$R_\rho$</td>
<td>$R_\rho$</td>
</tr>
<tr>
<td>Distance</td>
<td>$R_x$</td>
<td>$R_g^{-1}R_{v_0}^2$</td>
</tr>
<tr>
<td>Velocity</td>
<td>$R_v$</td>
<td>$R_{v_0}$</td>
</tr>
<tr>
<td>Acceleration</td>
<td>$R_\dot{v}$</td>
<td>$R_g$</td>
</tr>
<tr>
<td>Time</td>
<td>$R_t$</td>
<td>$R_g^{-1}R_{v_0}$</td>
</tr>
<tr>
<td>Mass</td>
<td>$R_m$</td>
<td>$R_\rho R_g^{-3}R_{v_0}^6$</td>
</tr>
<tr>
<td>Mass flow</td>
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<td>$R_\rho R_g^{-2}R_{v_0}^5$</td>
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<td>Drag coefficient</td>
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</tr>
<tr>
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<td>$1.0$</td>
</tr>
<tr>
<td>Speed of sound</td>
<td>$R_{v_0}$</td>
<td>$R_{v_0}$</td>
</tr>
</tbody>
</table>
riser lines might absorb some of the opening shock force

\[
\frac{(kx)_m}{(kx)_t} = N
\]

(20)

\[
R_k = \frac{R_\rho R_\rho R_\rho}{R_d} = R_\rho R_\rho R_\rho 2
\]

But for tests where very moderate loads would be expected, the spring constant would not be considered as a scaling basis.

Data Reduction

The measured tension \( T \) versus the time \( t \) for each of the drops allowed the acceleration-time history to be determined by considering the payload equation of motion in vertical descent

\[
F = \ddot{m}v + \dot{m}v = W - T - D
\]

Since the payload itself has a constant mass, then \( \ddot{m} = 0 \). Also, the payload aerodynamic drag force is considered as negligible — hence, the acceleration becomes

\[
\ddot{v} = \frac{W - T}{m} = g - \frac{T}{m}
\]

(22)

Integrating, with respect to time, yields the velocity

\[
v = \int \ddot{v} \, dt = gt - \frac{1}{m} \int T \, dt
\]

(23)

The distance fallen is

\[
x = \int v \, dt = \frac{1}{2} gt^2 - \frac{1}{m} \int \int T \, dt \, dt
\]

(24)
In this manner the tension, the acceleration, the velocity, and the distance time histories can be determined for each deployment.

Initial Conditions

Because of its symmetry, its ease of reefing, and its availability, a ringsail parachute was chosen as the test parachute (fig. 2). The model ringsail had an 18.00-foot nominal diameter, while the full-scale ringsail had a 29.33-foot nominal diameter. The reefing, based on the nominal diameter, was 17.3 percent.

The model weight was 25 pounds, thereby forcing the full-scale weight to be

\[ W_f = \frac{W_m}{\frac{3}{R_d}} = 108.0 \text{ lb} \]  

(25)

Because the tests were on earth at sea level, the gravity and density ratios were unity. The initial conditions of time, of velocity, and of acceleration were all zero for both model and full-scale tests, thus scaling correctly as a matter of course.

Discussion of Experimental Results

The measured tension versus time for eight model drops is shown in figure 3. Initially, the tension was 25 pounds, which was the payload weight. At release, the tension quickly fell to zero before the drag force became measurable. As the velocity increased, the drag force increased causing the tension to increase. After about 2 seconds, the drag force reached a maximum and thereafter decreased toward the payload weight at terminal conditions. Although all of the eight drops were similar, a significant scatter was evident. This was attributed to a slight wind and to nonsymmetrical initial reefing conditions.

The measured tension versus the time for the full-scale drop is compared in figure 4. Similar results to those given in figure 3 are noted, except that the payload weight used in figure 4 was 108 pounds.

To check the scaling technique, the tension-time history of the eight model drops was averaged. Then, the scaling laws were applied, and the full-scale tension-time history was predicted. Figure 5 presents this prediction so that it can be compared to
the measured (average) full-scale results. The solid line is the average of the full-scale drops, and the dotted line is the prediction derived from the average of the model drops. The two results are quite close, that is, well within the data repeatability.

The acceleration versus time was calculated for each drop and averaged for the model and the full scale. The model results were then used to predict the full-scale results shown in figure 6. Initially, the acceleration was zero. It quickly reached 32.2 ft/sec² during the initial part of the flight. As the drag force increased, the acceleration decreased to zero and to a negative maximum at maximum tension. Thereafter, the acceleration decreased, tending toward zero at the steady state. The full-scale results were quite accurately predicted from the model test.

Computing the velocity-versus-time history, averaging and predicting as before, the results shown in figure 7 were obtained. The velocity started from zero, increased almost linearly, leveled out at the maximum, and finally decreased to the steady-state value. Again, the model drops accurately predicted the full-scale results.

Figure 8 presents the results of the distance-versus-time comparison. Initially, the distance increased quadratically with time. As the velocity became maximum and leveled off, the distance increased linearly with time, and the slope decreased slowly as the velocity decreased to the steady state. The comparison of experimental full-scale results with model predictions lends support to the scaling method.
29.33-foot ringsail parachute
108-pound payload

Figure 4. - Measured tension versus time for the six full-scale drops.
Figure 5. - Prediction of the full-scale tension versus time for the model drops.

Figure 6. - Prediction of the full-scale acceleration versus time for the model drops.
Figure 7. - Prediction of the full-scale velocity versus time for the model drops.

Figure 8. - Prediction of the full-scale distance versus time for the model drops.
CONCLUDING REMARKS

A method has been derived to relate model to full-scale parachute-opening characteristics by comparing like forces in the equation of motion. Model laws were derived for earth simulations of planetary deployments. Some results obtained by experimentation have shown the practicality of the method.

Manned Spacecraft Center
National Aeronautics and Space Administration
Houston, Texas, June 9, 1967
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REFERENCE

"The aeronautical and space activities of the United States shall be conducted so as to contribute . . . to the expansion of human knowledge of phenomena in the atmosphere and space. The Administration shall provide for the widest practicable and appropriate dissemination of information concerning its activities and the results thereof."

—National Aeronautics and Space Act of 1958

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