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MINIMUM FUEL ASCENT FROM THE LUNAR SURFACE

by Albert E. Brown

Langley Research Center
Langley Station, Hampton, Va.

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SUMMARY

The steepest-descent method is used to compute minimum fuel lunar launch trajectories consisting of two thrust phases separated by a coast phase terminating in an 80-nautical-mile circular orbit around the moon. Propulsion is provided by a constant-thrust and mass-flow-rate engine with a shutdown and restart capability. Three-dimensional point-mass equations of motion subjected to a spherical inverse-square gravitational field are used in simulating the lunar launch trajectories. Provision is made to optimize initial azimuth, first thrust duration, ballistic coast duration between the first and second thrusting phases, second thrust duration, and thrust orientation. The launch sites considered range from a launch site in the plane of the target orbit up to and including one displaced 30° (offset) from the target orbit. It was found that as the initial launch-site offset is increased, the trajectories steepen, the required angular travel to orbit injection is reduced, and for the larger offset values the altitude during coast exceeds the altitude of the target orbit. The amount of fuel required for orbit injection was found to increase almost linearly with initial launch-site offset.

INTRODUCTION

In the past several years there has been much interest shown in the optimization of lunar launch trajectories. The studies most often seen used either impulsive thrust with coasts (for example, ref. 1) or direct ascent with continuous finite thrust which was generally in-plane (ref. 2). Little work has apparently been done in the area of optimal (minimum fuel) launch trajectories using finite thrust, a coast period, and a launch site out of the plane of a target orbit. The present study was conducted to fill this void.

This study presents results on the characteristics of launch trajectories typical of those which might occur during the course of the manned lunar mission as currently envisioned. Accordingly, the target orbit altitude and the lunar module mass and engine characteristics for the basic vehicle in this study were chosen to conform to current design values. In addition to results presented for the basic vehicle, results are presented for two additional vehicles with the same engine characteristics as the basic engine except for thrust level. One of these vehicles has a very high thrust level engine (called pseudo-impulsive) which is used to simulate impulsive thrust studies. The other vehicle's engine (simply called low thrust) has approximately one-half the thrust level of the basic engine.

SYMBOLS

The units used for the physical quantities defined in this paper are given both in U.S. Customary Units and in the International System of Units (SI). Where distances are expressed in nautical miles or in feet, the international nautical mile and the international foot, respectively, are intended. Factors relating these two systems of units are given in reference 3.

F	thrust, pounds (newtons)
F/m_o	initial thrust to mass ratio, applied take-off acceleration, feet/second^2 (meters/second^2)
g_e	weight-mass conversion factor, 32.17405 feet/second^2 (9.80665 meters/second^2)
h	altitude above lunar surface, feet or nautical miles (meters)
Δh	overshoot altitude, maximum increment of coast altitude above injection altitude, feet (meters)
I_{sp}	specific impulse, 305 seconds
i	inclination of ballistic coast with respect to equator, degrees
m	mass, slugs (kilograms)
m_o	initial mass of launch vehicle at lunar surface, slugs (kilograms)
m_f	mass of fuel consumed for orbit injection, slugs (kilograms)
r_m	radius of moon, 5 702 100 feet (1.738×10^6 meters)
t	time, seconds

t_B thrust duration, boost phase, seconds

t_f total trajectory time, $t_B + t_\beta + t_T$, seconds

t_T thrust duration, terminal phase, seconds

t_β ballistic coast duration, seconds

V total velocity, feet/second (meters/second)

W weight, mg_e, pounds (newtons)

W_B weight of fuel consumed, boost phase, pounds (newtons)

W_f total weight of fuel consumed, $W_B + W_T$, pounds (newtons)

W_T weight of fuel consumed, terminal phase, pounds (newtons)

β central angle of coast, degrees

γ vehicle flight-path angle, total velocity vector relative to local horizontal, degrees

ζ angle between projection of thrust vector onto local horizontal plane and due east, degrees

η thrust-vector pitch angle with respect to velocity vector, degrees

θ thrust-vector inclination angle relative to local horizontal, degrees

λ latitude, degrees

Λ initial offset, perpendicular great-circle distance from vehicle at lift-off to plane of target orbit, degrees

μ lunar gravitational constant, 1.7282387×10^{14} feet^3/second^2 (4.893827×10^{12} meters^3/second^2)

τ longitude, degrees

$\Delta\tau_B$ longitude travel, boost phase, degrees

$\Delta\tau_T$ longitude travel, terminal phase, degrees

χ thrust-vector yaw angle with respect to velocity vector, degrees

χ_{av} average value of thrust-vector yaw angle during terminal phase, degrees

ψ azimuth, angle measured clockwise from north to projection of velocity vector onto local horizontal plane, degrees

ψ_o launch azimuth defined in section "Problem Description," degrees

Ω rendezvous lead angle, separation between orbiter and meridian through launch site of vehicle at lift-off, positive when launch vehicle leads orbiter, degrees

PROBLEM DESCRIPTION

The problem, herein, is to determine the characteristics of lunar launch trajectories with finite times of thrust duration and launch sites which, in general, are not in the plane of a given circular target orbit. These trajectories are to be minimum-fuel trajectories and are considered to be composed of three arcs, consisting of a boost phase initiated with a vertical lift-off, a ballistic coast phase, and a powered terminal phase.

A digital computer program, which is based on the method of steepest descent (ref. 4), has been used to compute these minimum-fuel trajectories. This program, which uses three-dimensional point-mass equations of motion, simultaneously determines the optimal initial launch azimuth and time duration of each of the three separate trajectory arcs, in addition to the thrust-vector orientation. In this paper launch azimuth is defined as the azimuth of the velocity vector immediately after the termination of vertical lift-off. A brief description of the program is as follows. Starting with given initial conditions and an estimate of a thrust-attitude time history, the program computes a nominal trajectory to specified terminal stopping condition. The program then automatically proceeds to shape this nominal trajectory by successive corrections until a trajectory is determined which satisfies within acceptable limits the terminal constraints of the problem. Once this guidance problem is solved, the program attempts to reshape the trajectory such that terminal mass is increased in addition to still satisfying the desired terminal constraints. When the attempted mass increase becomes less than a specified value, the program computes a final guidance trajectory which satisfies to very close tolerances all the desired terminal conditions. A typical solution was obtained with about 12 iterations and required about 3 minutes of IBM 7094 computer time.

For the purposes of this study, the moon was assumed to be a nonrotating sphere with a radius, $r_m = 5.7021 \times 10^6$ ft (1.738×10^6 m), and a gravitational constant, $\mu = 1.7282387 \times 10^{14}$ ft^3/sec^2 (4.893827×10^{12} m^3/sec^2). From the assumption of a nonrotating sphere, it could be further assumed, without loss of generality, that the plane of the target orbit was in the equatorial plane of the moon. This assumption permitted the use of lunar latitude for launch-site initial offset, since the latter by definition is the great-circle arc distance from the launch site to the plane of the target orbit.

The target orbit chosen was an 80-nautical-mile (148 160 m) circular orbit which corresponds to current plans for the lunar parking orbit. The desired terminal values of injection conditions are given in the following table:

Cutoff velocity, ft/sec (m/sec)	5284.7 (1610.8)
Altitude, ft (m)	486 089 (148 160)
Flight-path angle, deg	0.0
Latitude, deg	0.0
Azimuth, deg	90.0

Three launch vehicles were investigated in this study: a basic vehicle corresponding to the proposed lunar launch vehicle, a low-thrust-level vehicle, and a pseudo-impulsive thrust level vehicle. Each vehicle had the same launch weight and engine characteristics with the exception of thrust level. The values used for launch weight W and specific impulse I_{sp} were 9185.0 earth pounds (40 857 N) and 305.0 seconds, respectively. The values of thrust level used and applied take-off accelerations are given in the following table:

Launch vehicle	F		F/m_o	
	lb	N	ft/sec^2	m/sec^2
Basic	3 500	15 568.8	12.260	3.737
Low thrust	1 800	8 006.8	6.305	1.922
Pseudo-impulsive	130 000	578 268.8	455.375	138.797

For each of the launch vehicles studied, a set of minimum-fuel trajectories were determined for each launch site by systematically varying launch-site latitude (initial offset) from 0° to a maximum value of 30° while maintaining launch-site longitude at 0°. At the start of each trajectory, the computer program simulated a vertical take-off until a velocity of approximately 150 ft/sec (46 m/sec) was reached. At that point it is

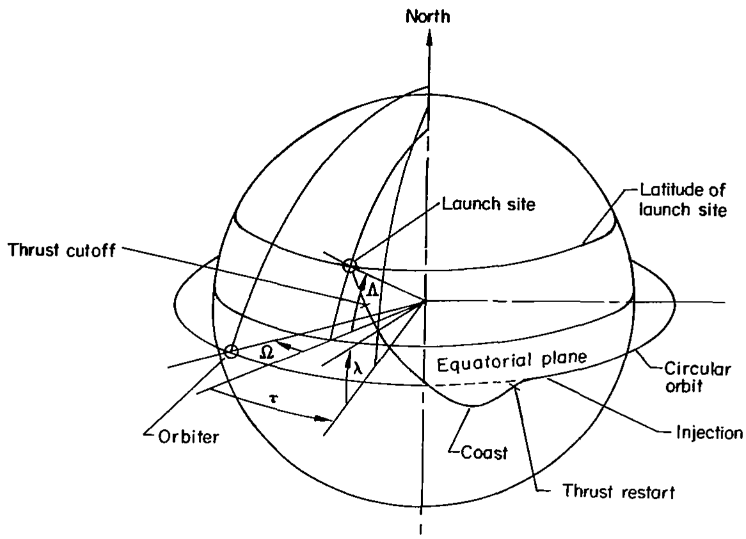

Figure 1.- Trajectory illustration.

assumed that the vehicle thrust attitude was instantaneously rotated downward in a vertical plane defined by the launch azimuth. The computer program then optimally shaped the trajectory by using the method of steepest descent as formulated and described in reference 4. The typical situation is depicted in figure 1. Shown in this figure are the lunar sphere, the launch-site latitude at Λ degrees initial offset, the typical trajectory consisting of the boost phase, the coast phase, and the terminal phase, the equatorial plane containing the target orbit, and an orbiting vehicle in the target orbit Ω degrees downrange from the launch-site longitude. The angle Ω, defined as the rendezvous lead angle, fixes the position of the target vehicle at the time of lunar launch vehicle lift-off for subsequent rendezvous at the end of the trajectory terminal phase. The rendezvous lead angle has been computed for each trajectory.

PRESENTATION OF RESULTS

A compilation of results for all the trajectories computed during this study is presented in tables I and II. The variables which describe the trajectories at various points of interest, boost-phase termination, coast phase, and terminal-phase initiation and termination are presented. In addition to tables I and II, figures 2 to 14 have been prepared to show the shapes of the trajectories and the changes in trajectory characteristics as initial offset increases. The format generally followed in discussing these data is to discuss first the overall trajectory shape, and then separately discuss each of the three trajectory phases – boost, coast, and terminal – for the basic vehicle. After the discussion on the basic vehicle, similar data are presented for the other two vehicles and comparison shown.

TABLE I.- TRAJECTORY CHARACTERISTICS OF BASIC VEHICLE

Parameter	Values of trajectory parameters for initial offsets, Λ, degree, of –									
	0	1	2	3	4	5	8	12	20	30
Boost-phase termination										
Thrust duration, t_B, sec	366.79	367.45	366.89	366.05	365.06	364.32	362.73	361.12	358.60	356.83
Fuel consumed, W_B:										
lb	4 209	4 217	4 210	4 201	4 189	4 181	4 162	4 144	4 115	4 095
newton	18 722	18 758	18 727	18 687	18 633	18 598	18 513	18 433	18 304	18 215
Altitude, h:										
ft	11 110	12 192	28 619	41 514	51 670	56 094	66 969	78 748	101 280	122 383
m	3 386	3 716	8 723	12 653	15 749	17 097	20 412	24 002	30 870	37 302
Velocity, V:										
ft/sec	5 608.5	5 590.2	5 539.2	5 491.9	5 446.8	5 416.4	5 353.4	5 288.2	5 180.4	5 094.5
m/sec	1 709.5	1 703.9	1 688.3	1 673.9	1 660.2	1 650.9	1 631.7	1 611.8	1 579.0	1 552.8
Flight-path angle, γ, deg	0.22	1.91	3.36	4.29	5.04	5.54	6.48	7.50	9.40	11.22
Azimuth, ψ, deg	90.00	89.44	89.70	90.18	90.75	91.20	92.51	93.99	96.76	100.20
Latitude, λ, deg	0.00	0.71	1.70	2.62	3.53	4.46	7.29	11.11	18.83	28.52
Longitude, τ, deg	8.67	8.71	8.61	8.51	8.42	8.36	8.25	8.18	8.19	8.55
Coast phase										
Coast duration, t_β, sec	3 281.58	2 508.24	1 983.68	1 724.25	1 580.32	1 528.52	1 470.90	1 483.19	1 544.20	1 614.60
Central angle, β, deg	170.16	128.35	99.70	85.95	77.93	74.49	70.48	69.33	69.33	69.33
Inclination, i, deg	0.00	0.90	1.73	2.63	3.61	4.62	7.71	11.80	19.97	30.14
Overshoot altitude, Δh:										
ft					53	1 260	9 120	31 640	98 000	178 800
m					16	384	2 780	9 644	29 870	54 500
Terminal-phase initiation										
Altitude, h:										
ft	486 013	486 022	485 993	486 002	486 142	486 423	487 650	490 702	500 961	516 963
m	148 137	148 140	148 131	148 133	148 176	148 262	148 636	149 566	152 693	157 570
Velocity, V:										
ft/sec	5 178.1	5 159.3	5 121.0	5 083.1	5 044.8	5 016.3	4 958.5	4 897.7	4 795.7	4 710.5
m/sec	1 578.3	1 572.6	1 560.9	1 549.3	1 537.7	1 529.0	1 511.4	1 492.8	1 461.7	1 435.8
Flight-path angle, γ, deg	0.17	0.24	0.22	0.14	-0.06	-0.34	-1.06	-2.18	-4.41	-6.51
Azimuth, ψ, deg	90.00	90.90	91.72	92.63	93.61	94.62	96.70	101.80	109.96	120.12
Latitude, λ, deg	0.00	0.00	0.01	0.01	0.03	0.04	0.10	0.23	0.60	1.23
Longitude, τ, deg	178.83	136.80	108.39	94.23	86.07	82.80	78.47	77.19	76.39	75.79
Terminal-phase termination										
Thrust duration, t_T, sec	4.73	6.64	9.93	13.58	17.58	21.47	32.93	47.93	76.17	107.45
Fuel consumed, W_T:										
lb	54	76	114	155	202	246	378	550	874	1 233
newton	240	338	507	689	899	1 094	1 682	2 447	3 888	5 485
Altitude, h:										
ft	486 048	486 094	486 090	486 086	486 084	486 084	486 087	486 076	486 088	486 089
m	148 147	148 161	148 160	148 159	148 158	148 158	148 159	147 242	148 160	148 160
Velocity, V:										
ft/sec	5 284.7	5 284.7	5 284.7	5 284.7	5 284.7	5 284.7	5 284.7	5 284.7	5 284.7	5 284.7
m/sec	1 610.8	1 610.8	1 610.8	1 610.8	1 610.8	1 610.8	1 610.8	1 610.8	1 610.8	1 610.8
Flight-path angle, γ, deg	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Azimuth, ψ, deg.	90.00	90.00	90.00	90.00	90.00	90.00	90.00	90.00	90.00	90.00
Latitude, λ, deg	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Longitude, τ, deg	179.06	137.12	108.87	94.88	86.91	83.82	80.02	79.43	79.83	80.41
Overall characteristics										
Total time, t_f, sec	3 653.10	2 882.33	2 360.50	2 103.88	1 962.96	1 914.31	1 866.56	1 892.24	1 978.97	2 078.88
Total fuel consumed, W_f:										
lb.	4 263	4 293	4 324	4 356	4 391	4 427	4 540	4 694	4 989	5 328
newton	18 962	19 096	19 234	19 376	19 532	19 692	20 195	20 880	22 192	23 700
Rendezvous lead angle, Ω, deg	-0.31	3.91	6.63	8.06	9.14	9.85	11.31	13.16	17.00	12.31

TABLE II.- TRAJECTORY CHARACTERISTICS OF LOW THRUST AND PSEUDO-IMPULSIVE VEHICLES

Parameter	Low thrust					Pseudo-impulsive					
	Values of trajectory parameters for initial offsets, Λ, degree, of –					Values of trajectory parameters for initial offsets, Λ, degree, of –					
	0	3	5	12	20	0	1	5	12	20	30
Boost-phase termination											
Thrust duration, t_B, sec	796.70	800.25	800.41	800.34	798.80	9.58	9.45	9.15	9.02	8.95	8.88
Fuel consumed, W_B:											
lb	4 702	4 723	4 724	4 723	4 714	4 083	4 029	3 900	3 846	3 816	3 787
newton	20 915	21 009	21 013	21 009	20 969	18 162	17 921	17 348	17 108	16 974	16 845
Altitude, h:											
ft	10 453	28 689	38 482	53 265	64 759	735	2 371	3 795	4 523	5 151	5 951
m	3 186	8 744	11 729	16 235	19 739	224	723	1 157	1 379	1 570	1 814
Velocity, V:											
ft/sec	5 609.0	5 547.4	5 502.6	5 414.4	5 338.4	5 615.6	5 518.3	5 284.3	5 189.4	5 137.3	5 089.1
m/sec	1 709.6	1 690.8	1 677.2	1 650.3	1 627.1	1 711.6	1 682.0	1 610.7	1 581.7	1 565.8	1 551.2
Flight-path angle, γ, deg	-0.18	3.10	4.11	5.62	6.79	0.36	4.53	8.80	11.08	13.16	15.73
Azimuth, ψ, deg	90.00	89.46	90.17	93.12	96.32	90.00	90.02	92.67	95.12	96.88	98.80
Latitude, λ, deg	0.00	2.19	3.89	9.95	17.34	0.00	1.00	4.99	11.98	19.97	29.97
Longitude, τ, deg	16.07	16.14	16.09	16.11	16.44	0.24	0.23	0.21	0.21	0.21	0.22
Coast phase											
Coast duration, t_β, sec	3 538.78	2 051.43	1 740.28	1 456.44	1 421.86	3 311.10	1 788.56	1 326.36	1 510.10	1 682.86	1 853.97
Central angle, β, deg	183.93	103.71	87.10	71.05	67.61	171.30	88.82	61.88	67.04	71.63	75.06
Inclination, i, deg	0.00	2.26	3.89	10.42	18.42	0.00	1.00	5.66	13.01	21.08	31.12
Overshoot altitude, Δh:											
ft				100	11 960			11 320	83 640	183 200	318 500
m				30	3 645			3 450	25 490	55 840	97 080
Terminal-phase initiation											
Altitude, h:											
ft	486 112	485 878	485 722	486 304	491 939	486 087	486 089	486 146	486 417	486 846	487 586
m	148 167	148 096	148 048	148 225	149 943	148 159	148 160	148 177	148 260	148 391	148 616
Velocity, V:											
ft/sec	5 177.8	5 130.0	5 091.9	5 011.3	4 936.1	5 175.0	5 071.1	4 816.8	4 713.2	4 656.0	4 602.9
m/sec	1 578.2	1 563.6	1 552.0	1 527.4	1 504.5	1 577.3	1 545.7	1 468.2	1 436.6	1 419.1	1 403.0
Flight-path angle, γ, deg	0.02	0.25	0.28	-0.03	-1.01	-0.03	0.02	-1.54	-4.87	-7.79	-11.07
Azimuth, ψ, deg	90.00	92.26	93.90	100.42	108.41	90.00	91.00	95.65	103.01	111.08	121.12
Latitude, λ, deg	0.00	0.02	0.05	0.31	0.89	0.00	0.00	0.00	0.01	0.02	0.04
Longitude, τ, deg	200.00	119.61	102.90	86.94	83.36	171.54	89.30	62.03	66.82	70.72	72.93
Terminal-phase termination											
Thrust duration, t_T, sec	8.22	19.60	30.39	71.51	119.11	0.13	0.29	0.86	1.59	2.36	3.21
Fuel consumed, W_T:											
lb	48	115	179	422	703	57	120	363	679	1003	1370
newton	214	511	796	1 877	3 127	253	534	1 615	3 020	4 462	6 094
Altitude, h:											
ft	486 122	486 094	486 086	486 036	486 087	486 087	486 089	486 089	486 089	486 074	486 087
m	148 170	148 161	148 159	148 144	148 159	148 159	148 160	148 160	148 160	148 155	148 159
Velocity, V:											
ft/sec	5 284.7	5 284.7	5 284.7	5 284.7	5 284.7	5 284.7	5 284.7	5 284.7	5 284.7	5 284.7	5 284.7
m/sec	1 610.8	1 610.8	1 610.8	1 610.8	1 610.8	1 610.8	1 610.8	1 610.8	1 610.8	1 610.8	1 610.8
Flight-path angle, γ, deg	0.01	0.00	0.00	-0.01	0.00	0.00	0.01	0.00	0.00	0.00	0.00
Azimuth, ψ, deg	90.00	90.00	90.00	90.00	90.00	90.00	90.00	90.00	90.00	90.00	90.00
Latitude, λ, deg	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Longitude, τ, deg	200.40	120.55	104.36	90.33	88.87	171.55	89.31	62.07	66.90	70.82	73.07
Overall characteristics											
Total time, t_f, sec	4 343.70	2 871.28	2 571.08	2 328.29	2 339.77	3 320.81	1 798.30	1 336.37	1 520.71	1 694.17	1 866.06
Total fuel consumed, W_f:											
lb	4 750	4 838	4 903	5 145	5 417	4 140	4 149	4 263	4 525	4 819	5 157
newton	21 129	21 520	21 809	22 886	24 096	18 415	18 455	18 963	20 128	21 436	22 939
Rendezvous lead angle, Ω, deg	12.14	19.94	21.44	23.59	25.61	-9.06	-1.32	3.32	7.51	12.08	18.24

DISCUSSION OF RESULTS

Basic Vehicle

General.- A family of trajectories for the basic vehicle is presented in figure 2 for a launch-site latitude (initial offset) range of 0° to 30°. In the upper half of figure 2, variations of altitude above the lunar surface with longitude are shown and in the lower half the corresponding variations of latitude with longitude are shown. As might be

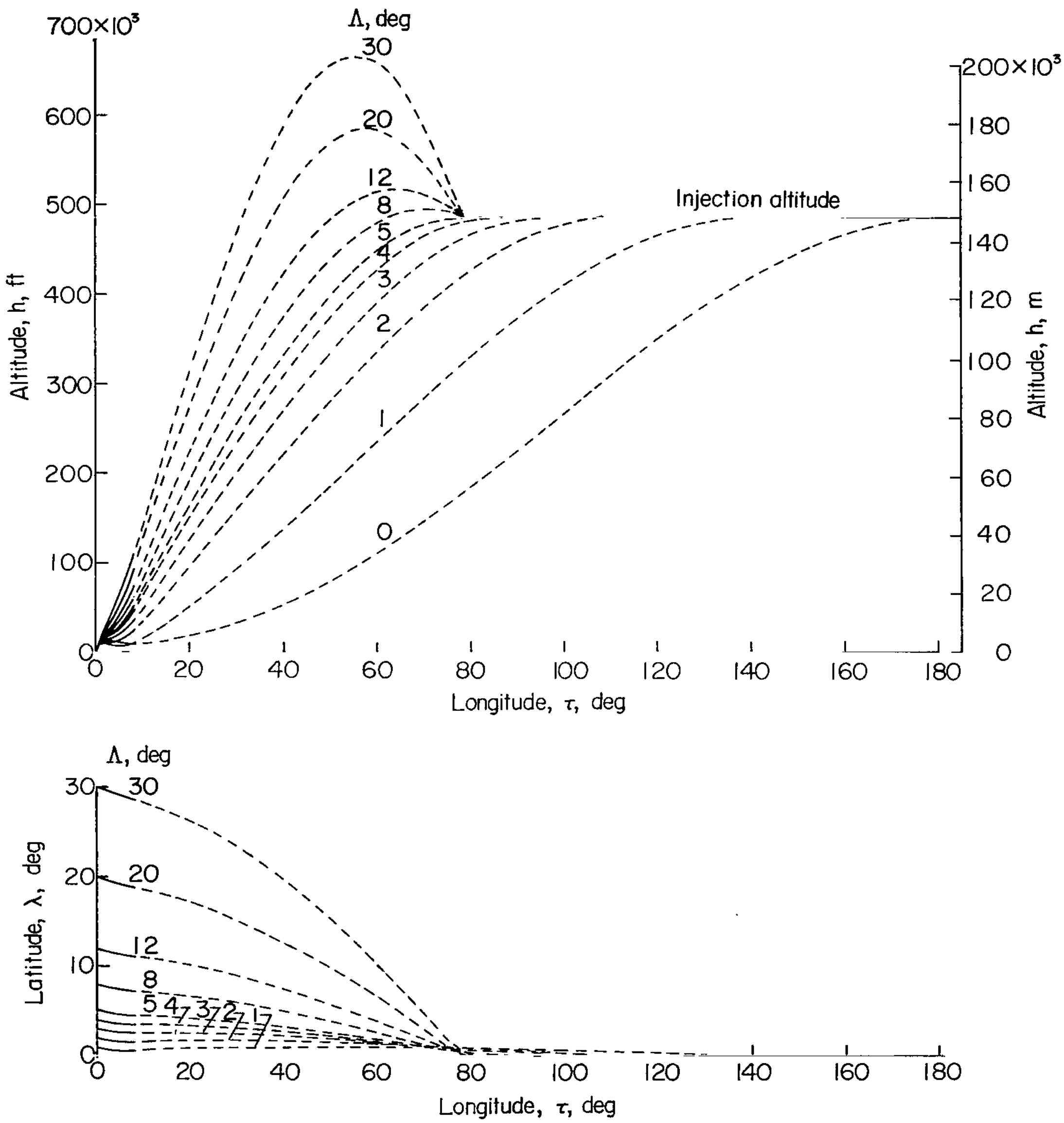

Figure 2.- Variations of altitude and latitude with longitude of trajectories for various initial offsets. Basic vehicle.

expected, the in-plane trajectory ($\Lambda = 0^{o}$) tends to simulate a Hohmann transfer maneuver. The boost phase is terminated at a low altitude with a near horizontal velocity vector (see table I for values of h and γ), after which the vehicle coasts for nearly 180^{o} to the desired injection altitude and then a short-time-duration terminal phase makes the orbit circular. However, as the launch site moves out of the plane of the target orbit, the trajectories shorten up significantly until at an initial offset of approximately 4^{o} and above, the required longitude travel to injection is less than 90^{o}. Values of initial offset larger than about 8^{o} require nearly the same longitude travel, approximately 77^{o}. These results agree qualitatively with the two-impulse trajectory results in reference 1. As the trajectories steepen and reduce the longitude travel to orbit injection, the maximum altitude during the coast increases until eventually altitude overshoot occurs; that is, the vehicle ascends to an altitude during the coast phase that exceeds the altitude of the target orbit. An interesting related aspect of these trajectories may be noted in figure 3. Figure 3 presents the total trajectory time from take-off to injection and a quantity which is the sum of three nonplanar angles, the coast central angle, and the amounts of longitude travel during the boost and terminal phases. The shapes of these two curves are similar, both dropping sharply as initial offset increases from 0^{o} until a minimum is reached and then increasing gradually as initial offset further

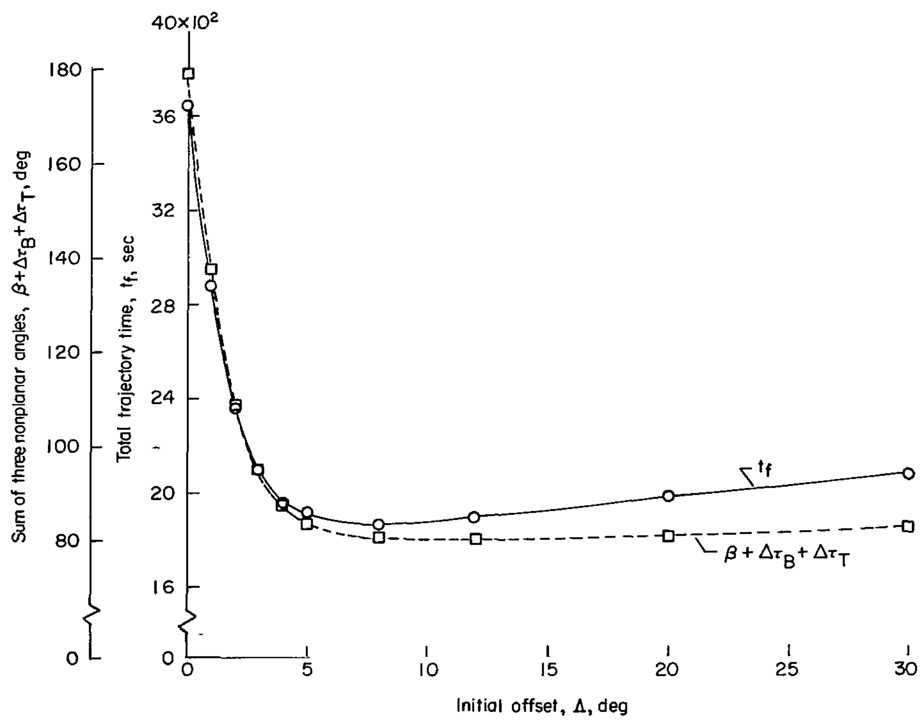

Figure 3.- Variations of total trajectory time and sum of three nonplanar angles $\beta + \Delta\tau_B + \Delta\tau_T$ with initial offset. Basic vehicle.

increases; however, an increasing spread between the two curves is shown. This divergence is indicative of increasing altitude overshoot with increasing initial offset since the angle summation is almost constant from about 4^{o} offset out to 30^{o} offset.

Boost phase.- The variables available to the computer program in optimally shaping the trajectory during the boost phase are launch azimuth, thrust-time duration, and thrust orientation. The nomenclature used in relating the thrust and velocity vectors to each other and to certain references, namely, the horizontal plane and the north direction, are shown in figure 4. As shown in figure 4, γ and θ are, respectively, the inclinations of the velocity and thrust vectors with respect to the local horizontal plane, and the azimuth ψ is the direction of the component of the velocity vector projected onto the horizontal plane with respect to north. The angles χ and η fix the position of the thrust vector with respect to the velocity vector and are the yaw and pitch angles, respectively. Positive directions of angular displacements are shown.

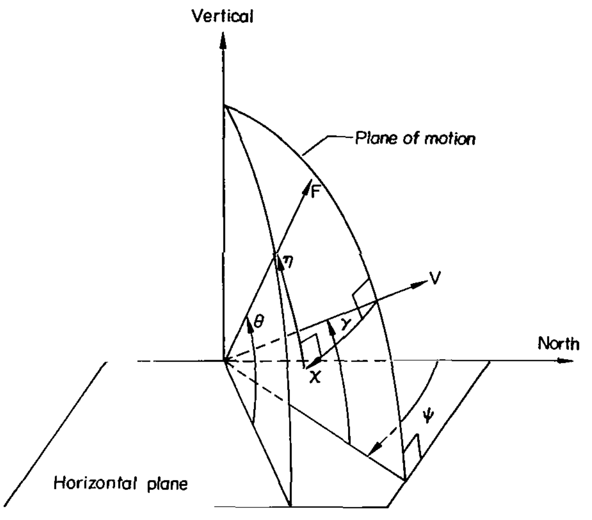

Figure 4.- Orientation for three-dimensional powered trajectory. Positive directions of angular displacements are shown.

Typical time histories of the thrust attitude with respect to the local horizontal θ and the yaw angle χ are shown in figure 5. The thrust axis remains vertical, $\theta = 90^{o}$, for the first 21 seconds of flight. The vehicle is then assumed to rotate instantaneously to the attitude shown in the upper half of figure 5 at $t = 21$ seconds. From that time on, a characteristic pitch-up maneuver occurs after which the vehicle pitches down. Note that the initial pitch attitude is relatively low ($23^{o} \pm 2^{o}$) for all four cases shown. Each of the offset cases other than 0^{o} shows a tendency to pitch up near the end of the boost phase. The yaw angle χ shown in the lower half of the figure for each of the cases varies almost linearly with time. The data on this plot start at a value of t greater than 40 seconds because of a programing peculiarity which does not allow integration of the $\dot{\psi}$ equation of motion until the quantity $V \cos \gamma$ exceeds 250 feet per second. It should be pointed out that the control-variable time histories should be taken as indicating trends rather than as absolute values because of the technique used in obtaining the optimal trajectories. This technique is a direct method that does not guarantee optimal control time histories as opposed to indirect techniques in which certain necessary conditions for optimality are satisfied.

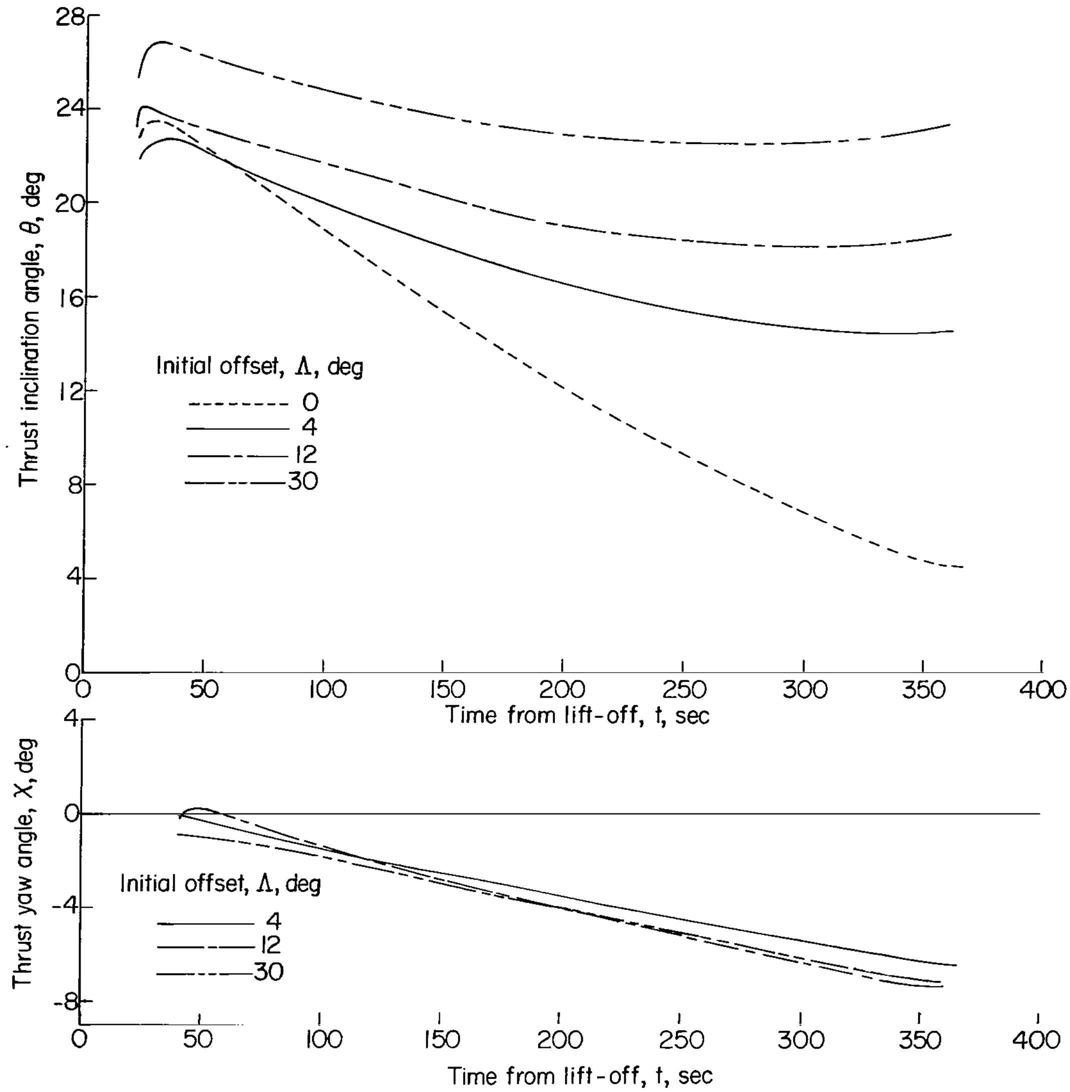

Figure 5.- Typical time histories of boost-phase thrust attitude. Basic vehicle.

Another important control variable for the out-of-plane launch trajectory is the launch azimuth. Launch aximuth is plotted in the upper half of figure 6 for each of the offsets considered. As expected, the in-plane case launches due east, $\psi_o = 90^o$, and as offset increases, the launch azimuth changes to a more southerly direction. Note that, while the launch azimuth is in a southerly direction, the sign of χ in figure 5 is such as to turn the vehicle in a northerly direction. This turning apparently represents a flight mechanics compromise of minimizing the latitude λ at the end of the boost phase and having an azimuth ψ of 90^o at this time. ($\psi = 90^o$ at the end of the boost phase minimizes the angle between the plane of the coast trajectory and the equatorial plane.) The variation of the time duration of the boost phase with initial offset is shown in the lower half of figure 6. Starting with 1° offset, the time duration of the boost phase decreased about 11 seconds, from approximately 368 seconds down to approximately 357 seconds at 30° offset. This decrease represents about a 3-percent variation in thrust duration. The 0° offset case required only 1.5 seconds less thrust duration than the 1° offset case.

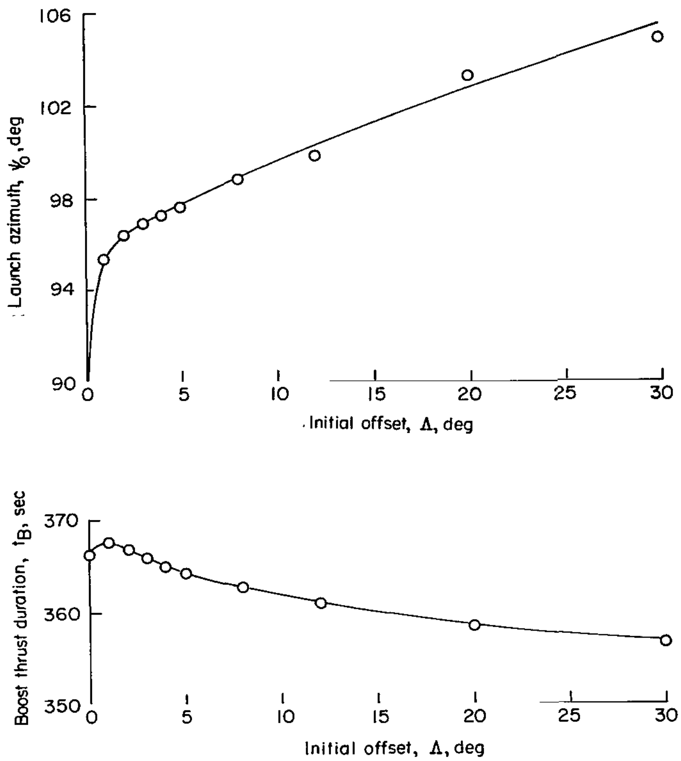

Figure 6.- Variations of launch azimuth and boost-phase thrust duration with initial offset. Basic vehicle.

Coast phase.- Time, central angle, and altitude overshoot of the coast orbit are shown in figure 7 plotted against initial offset angle. The same trend previously noted on the plot of total trajectory time and angle summation with initial offset (fig. 3) may be noted in this figure also. The central angle remains fairly constant from about 8° out to 30° offset while the coast time increases some 144 seconds over the same offset range. The plot of altitude overshoot against offset shows that altitude overshoot occurs at about 4° offset and rises exponentially out to 10° offset and almost linearly thereafter as offset increases.

Terminal phase.- Terminal phase duration and an average value of the yaw angle χ during this phase are shown in figure 8 plotted against initial offset. For the in-plane launch only a short thrust duration, about 5 seconds, is required. The required thrust duration increases nearly linearly to a value of about 108 seconds at 30° offset. The average yaw angle varies smoothly with offset, rapidly increasing for small offsets and gradually tapering off to a low slope at the higher offsets. Variations of θ and χ with time during the terminal phase are shown in figure 9 for typical initial offset cases. Notice that both sets of curves are linear. Another interesting plot during the terminal phase is thrust direction with respect to the target orbit plane, which in this case is due east, as a function of time for typical initial offsets as shown in figure 10. These curves are also linear with surprisingly high average values.

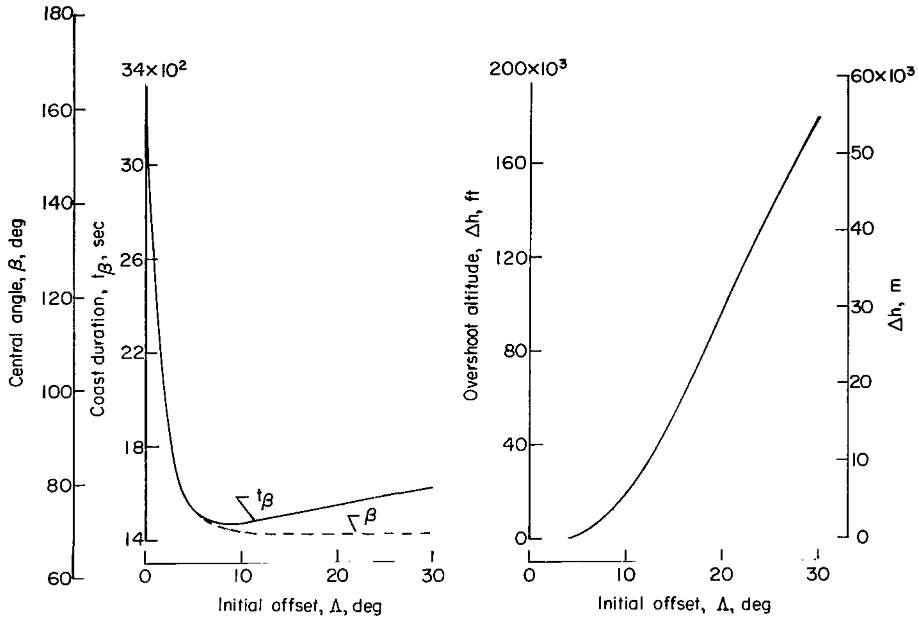

Figure 7.- Variations of coast duration, central angle, and overshoot altitude with initial offset. Basic vehicle.

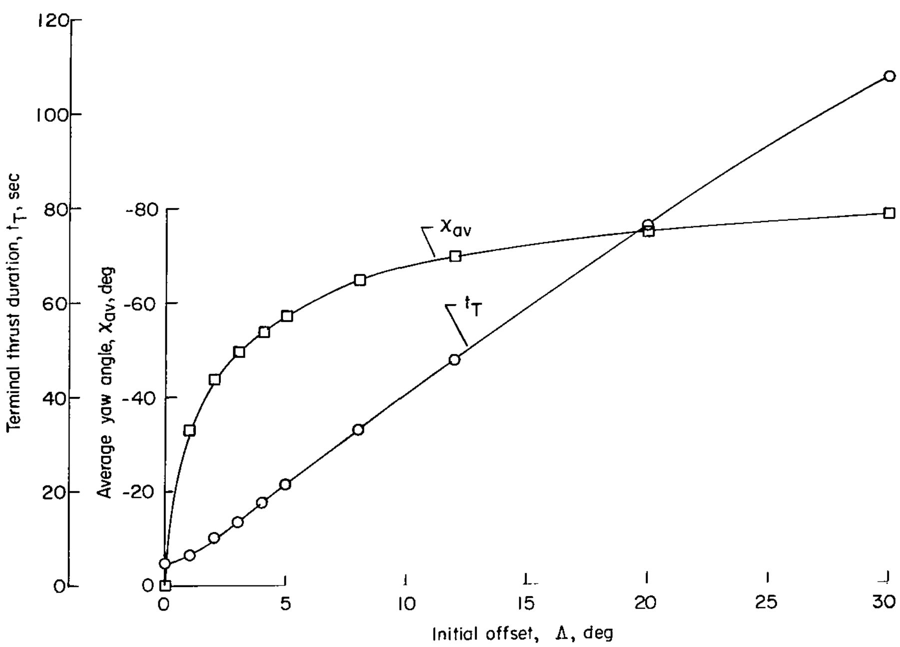

Figure 8.- Variations of terminal-phase thrust duration and average yaw angle with initial offset. Basic vehicle.

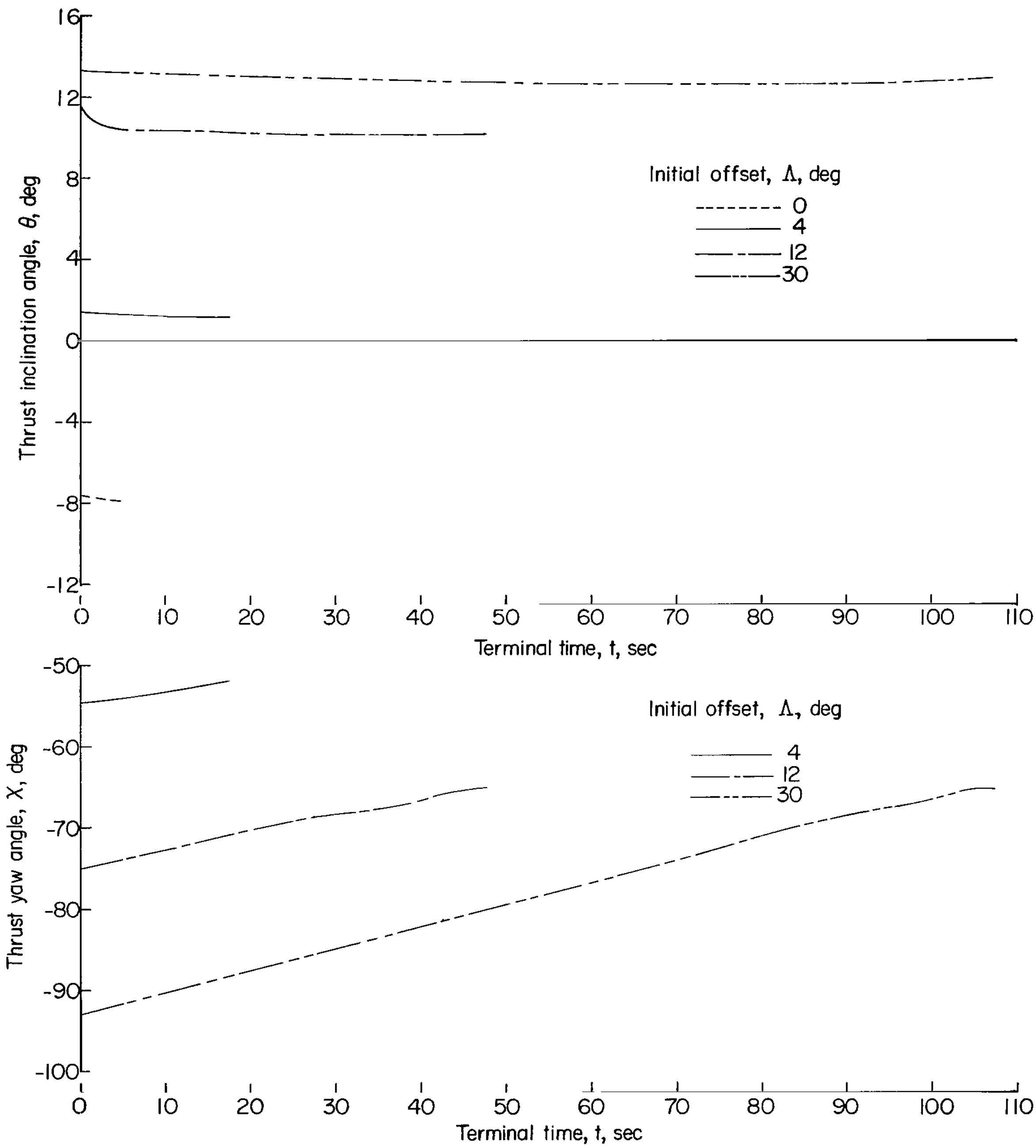

Figure 9.- Typical time histories of terminal-phase thrust attitude. Basic vehicle.

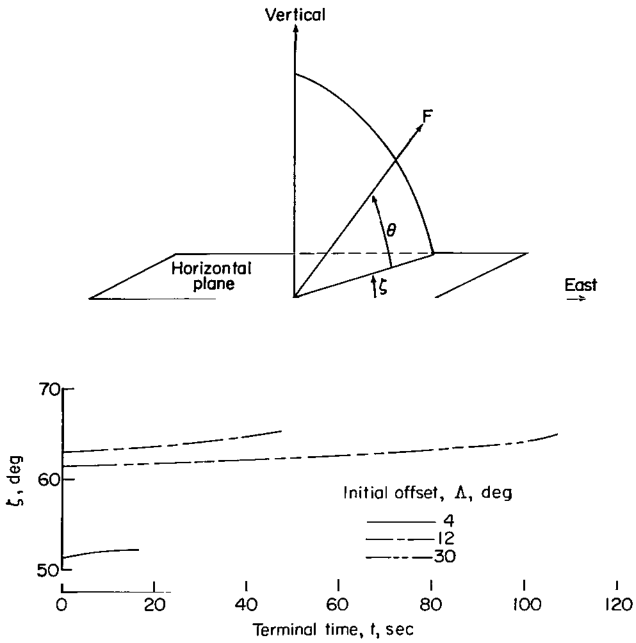

Figure 10.- Typical time histories of terminal-phase thrust direction with respect to due east. Basic vehicle.

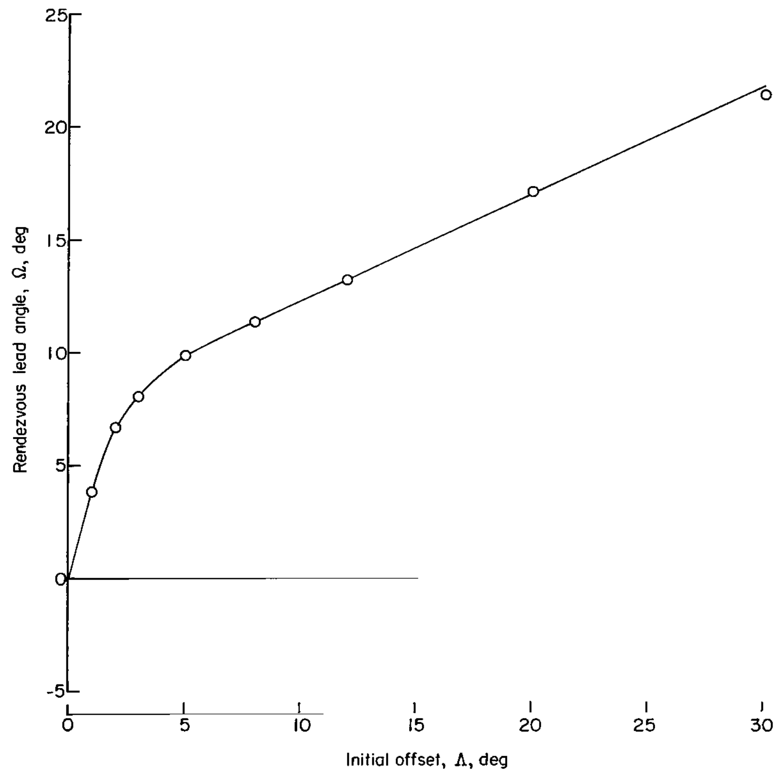

Figure 11.- Variation of rendezvous lead angle with initial offset. Basic vehicle.

Direct ascent to rendezvous.- The angular velocity of the target vehicle in an 80-nautical-mile circular orbit is about 3° per minute. By using this fact in conjunction with the total time required to achieve orbit injection conditions, it is possible to compute the rendezvous lead angle Ω required in order to accomplish rendezvous with the target vehicle. Although the requirement of rendezvous did not enter into the calculation of these fuel optimal launch trajectories, it is of interest to know where the target vehicle was at launch relative to the launch site. A plot of this information showing lead angle Ω against initial offset is presented in figure 11. It can be seen from this figure that the target vehicle is nearly overhead for the in-plane launch. As the launch site is moved out of the target orbit plane, the lead angle Ω increases to about 21° for $\Lambda = 30^{\circ}$. The plotted values of Ω correspond to optimal positioning of the target vehicle in order to accomplish a minimum fuel launch to rendezvous trajectory.

High and Low Thrust Vehicles

Families of trajectories in terms of altitude and latitude plotted against longitude, for the low and the high thrust vehicles, are presented in figures 12 and 13, respectively. Although trajectories have not been computed for every initial offset considered with the basic vehicle, it is felt that sufficient data have been computed to identify trends. Superposed on figure 13, by using solid lines, are trajectories for both the basic and low thrust vehicles for an offset of 20°. It is very apparent that the higher the thrust level, the steeper the trajectory and the larger the altitude overshoot. There was one problem with the low thrust which had to be overcome in the computation of the trajectories. There was a tendency for the low-acceleration vehicle to dive and impact the lunar surface. In order to retain realism, the trajectory was constrained to prevent the vehicle from going below an altitude of 10 000 feet during the dive. This constraint resulted in the trajectories having a roller coaster appearance during the early portion of the boost phase. A similar but less pronounced tendency was noted in the cases involving the basic vehicle and small offset angles. No trajectory constraints were deemed to be necessary for the basic-vehicle cases.

Other trends shown for the basic vehicle were also exhibited for both the low and high thrust vehicles and may be seen by a comparison of the detailed information presented in tables I and II.

Fuel Requirements

The fuel requirements for each of the vehicles studied and some results from an early impulsive study are presented in figure 14. For reference purposes, the ratio of the radius of the 80-nautical-mile target orbit to the radius of the moon has been computed to be 1.086. By using this value, the results of a paper by Carstens and Edelbaum

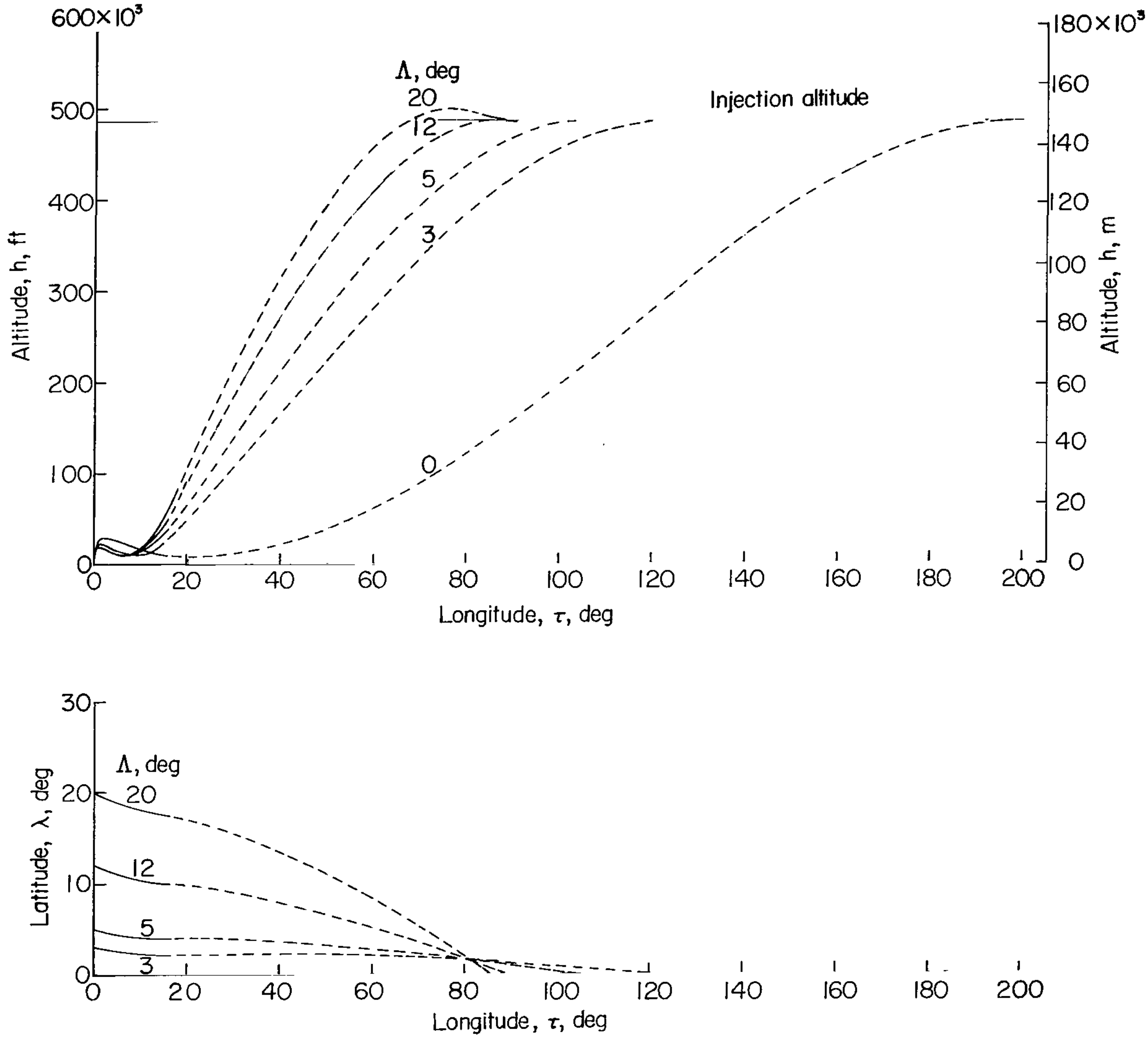

Figure 12.- Variations of altitude and latitude with longitude of trajectories for various initial offsets. Low-thrust vehicle.

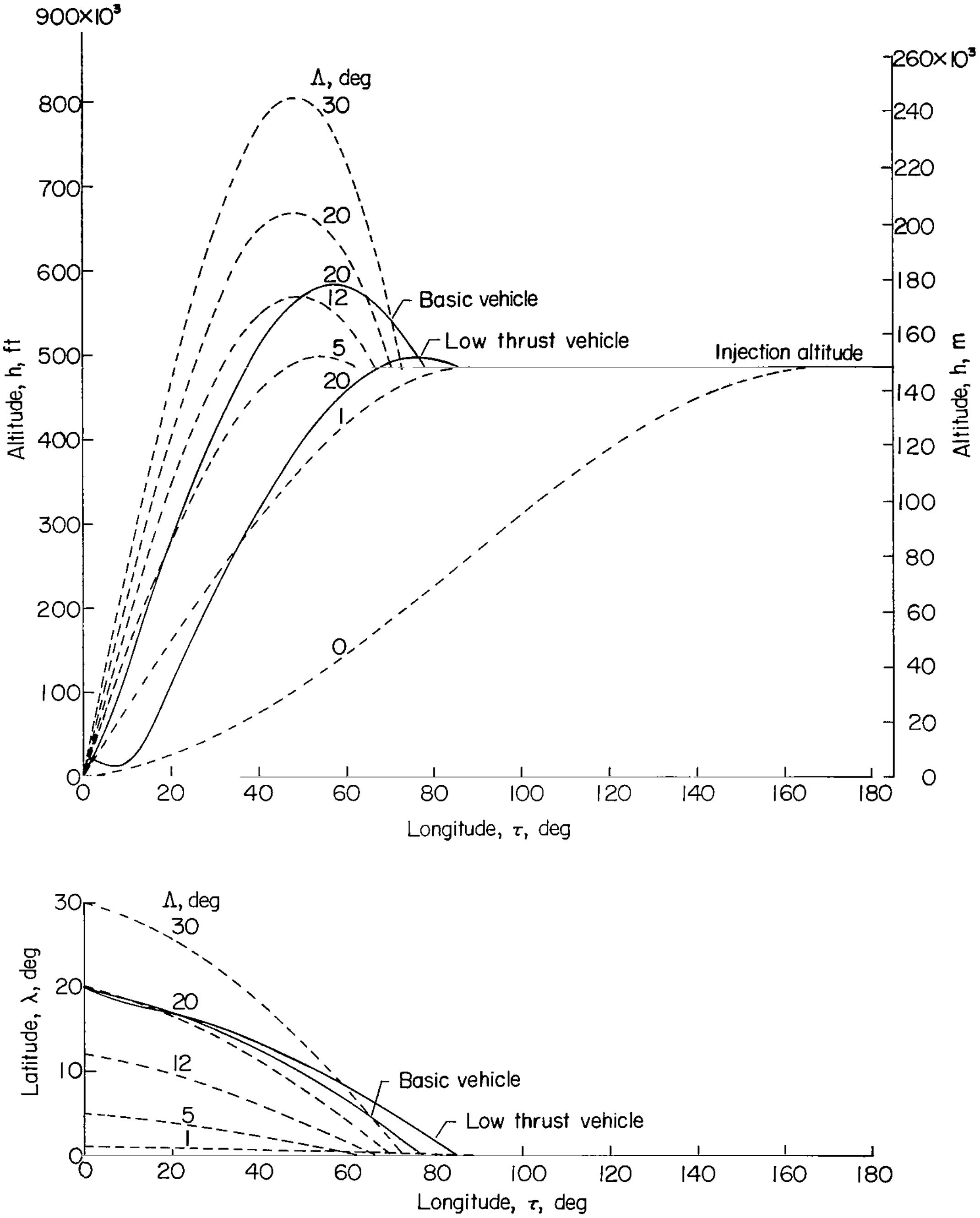

Figure 13.- Variations of altitude and latitude with longitude of trajectories for various initial offsets. Pseudo-impulsive vehicle.

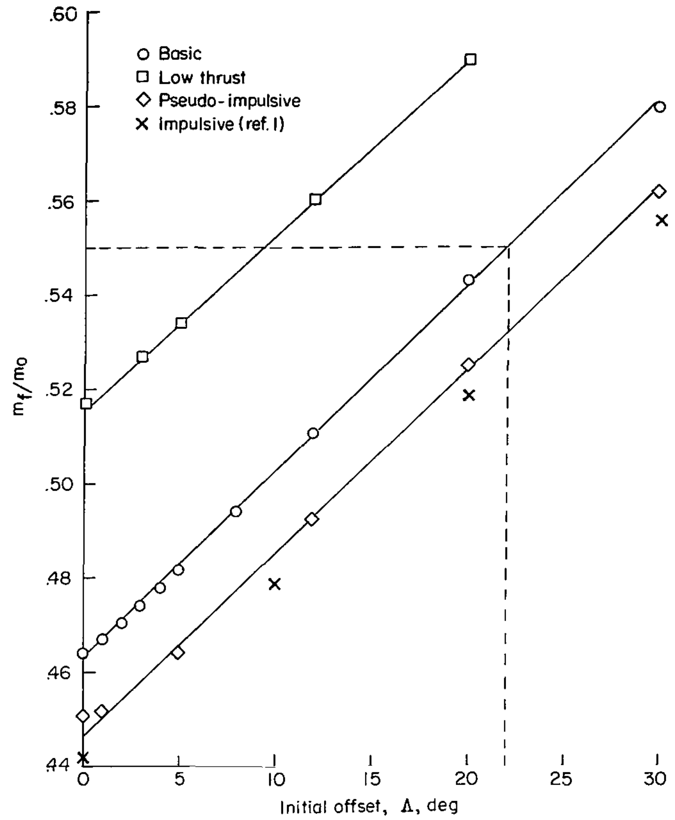

Figure 14.- Mass ratio of fuel required for orbit injection as a function of initial offset.

(ref. 1) are shown in figure 14 (crossmarks), where propellant requirements as a ratio of propellant required to total vehicle mass are plotted against initial offset angle.

The trends of the curves shown in figure 14 are very similar. The curves are each very nearly linear and their slopes differ only slightly. The pseudo-impulsive thrust case computed by using the method of steepest descent agrees well with the analytical results of reference 1, differing by about 0.7 percent in fuel required. The basic vehicle requires about 2.5 percent more fuel than the impulsive case but, of course, is characterized by realistic values of thrust acceleration (only about $3/4g_e$ at injection). The low-thrust-level vehicle requires about 7.5 percent more fuel than the impulsive vehicle.

Current plans for the fuel loading of the lunar module are on the order of 55 percent of the lunar take-off mass. From the results the maximum initial offset angle for the basic vehicle with 55 percent available fuel is less than 22°.

CONCLUDING REMARKS

The steepest-descent method has been used to compute minimum fuel ascent trajectories from the lunar surface which require lateral steering and terminate in an 80-nautical-mile circular target orbit around the moon. The lunar launch sites range from in-plane to 30° initial offset. The trajectories each involve a powered boost phase, a ballistic coast, and a terminal powered phase.

The results show similar trajectory characteristics for the three different vehicles investigated. Each of the vehicles tend to simulate a Hohmann transfer maneuver for the in-plane trajectory. However, as the launch site is moved out of the target orbit plane, the trajectories steepen and rapidly reduce the required angular travel to orbit injection. As the trajectories steepen, the maximum altitude during coast increases until eventually altitude overshoot occurs; that is, the vehicle ascends to an altitude during the coast phase which exceeds the altitude of the target orbit. It is shown that the higher the vehicle thrust level, the steeper the trajectory and the larger the altitude overshoot. In addition, it is shown for each of the three vehicles that the amount of fuel required for orbit injection increases very nearly linearly with initial offset. The basic vehicle requires about 2.5 percent more fuel than the impulsive vehicle. The pseudo-impulsive and low thrust vehicles require about 0.7 and 7.5 percent more fuel, respectively, than the impulsive vehicle.

Langley Research Center,
National Aeronautics and Space Administration,
Langley Station, Hampton, Va., May 18, 1967,
125-17-05-01-23.

REFERENCES

1. Carstens, J. P.; and Edelbaum, T. N.: Optimum Maneuvers for Launching Satellites Into Circular Orbits of Arbitrary Radius and Inclination. ARS J., vol. 31, no. 7, July 1961, pp. 943-949.

2. Burns, Rowland E.; and Singleton, Larry G.: Ascent From the Lunar Surface. NASA TN D-1644 (Corrected copy), 1965.

3. Mechtly, E. A.: The International System of Units – Physical Constants and Conversion Factors. NASA SP-7012, 1964.

4. Willwerth, Robert E., Jr.; Rosenbaum, Richard C.; and Chuck, Wong: PRESTO – Program for Rapid Earth-to-Space Trajectory Optimization. NASA CR-158, 1965.

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