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THE DESIGN OF A LARGE ANGLE THREE-AXIS ATTITUDE SERVOMECHANISM FOR SPACECRAFT

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1. Introduction

In the present paper an attitude servo is defined as a control system which adjusts the attitude and angular velocity of the spacecraft so as to follow the reference which may be varying in time. Such systems are multi-dimensional multivariable, nonlinear, and nonautonomous with infinite sets of forcing functions and initial conditions. A procedure for designing such systems for spacecraft which are controlled by means of an arbitrary angular momentum exchange and storage device is presented in the paper. The discussion is restricted neither to small angle motion nor to single axis motion of the spacecraft. Indeed, the nonlinear effects arising from kinematics and gyroscopic action, the presence of bounds on angular momentum exchange rate and storage capacity of the controlling device, and variations in system parameters are taken into account. The proposed control scheme forces the spacecraft into the desired attitude by generating torque about all three axes simultaneously.

2. A Model of Attitude Servos

Consider three right-handed orthonormal triplets of vectors $\hat{s}$, $\hat{a}$, and $\hat{d}$ whose common origin is the fixed point of rotation of the spacecraft. Let $\hat{s}$ and $\hat{a}$ be fixed in inertial space and spacecraft, respectively, and let $\hat{d}$ represent the reference. The servo input, output, and error will be defined by direction cosine matrices: the input A_{ds} defines $\hat{d}$ relative to $\hat{s}$; the output A_{as} defines $\hat{a}$ relative to $\hat{s}$; the error

$R = A_{as}A_{ds}^{t}$ (superscript t denotes transpose) defines $\hat{a}$ relative to $\hat{d}$. Servo null occurs when $R = I$, the identity matrix.

The model shown in figure 1 is proposed for the description of an attitude servo in which the controller is an arbitrary angular momentum exchange and storage device. The body coordinates of the angular momentum exchange rate are assumed to be the controlling variables of the servo. External torques are assumed to be negligible.

$$\text{STATE } \alpha = (A_{ds}, R, w_a, h_s)$$

$$\Theta = \left\{ \alpha : A_{ds}A_{ds}^{t} = I = RR^{t}, \| w_a \| \le w_{max}, \| h_s \| \le h_{smax} \right\}$$

$$\dot{A}_{ds} = S(w_d)\, A_{ds}$$

$$\dot{R} = S(w_a - Rw_d)\, R$$

$$\dot{w}_a = J_a^{-1}\, z(\alpha) + n(\alpha, \lambda)$$

$$\dot{h}_s = 0$$

$$\underline{W} = \left\{ \underline{w}_d : \| w_d(t) \| \le w_{dmax}, t \ge 0 \right\}$$

$$\underline{\Lambda} = \left\{ \underline{\lambda} : \| \lambda(t) \| \le 1, t \ge 0 \right\}$$

$$Z = \left\{ (\underline{z}, \underline{n}) : \| z(\alpha) \| \le z_{max} \text{ ON } \Theta \text{ AND } \| w_a \|' \le 0 \text{ FOR } \| w_a \| = w_{max} \right\}$$

Fig. 1 A model of attitude servos.

The states of the servo are imbedded in a 24-dimensional space whose points α are represented mnemonically by the quadruplet whose elements are the input matrix, error matrix, body coordinates of angular velocity of spacecraft, and inertial coordinates of total angular momentum of the system. The subset θ is the region of operation of the servo on which orthogonality and saturation constraints are satisfied. The latter arise because any practical controlling device has limited angular momentum storage capacity which will be assumed to be spherically bounded by h_{max}. $h_{smax} + j_{max}w_{max} = h_{max}$, where j_{max} is the maximum principal moment of inertia of the main body.

The state equations are defined on θ. The first two are the kinematic equations of input and error, respectively (for any column x, $S(x)$ is defined by the skew symmetric matrix whose entries in the upper triangle are x_3, $-x_2$, and x_1). Kinematic equations automatically preserve the orthogonality of A_{ds} and R. The second pair of state equations are the dynamic equations. The controlling variable is $z_a = -(d/dt)(A_{as}h_s - J_a w_a)$ where the matrix J_a represents the moment of inertia of the main body in body coordinates. The angular acceleration depends on the control law $\underline{z}$ which is assumed in the form of a state feedback $z(\alpha)$, and on the perturbation function $\underline{n}$ which is assumed to depend on α and the perturbation variable λ. The last dynamic equation is a consequence of the assumption that external torques are negligible, and automatically preserves the norm of h_s.

There are two forcing functions. One is the angular velocity of the reference whose d-coordinates are denoted by the matrix w_d. The other is the perturbation variable λ. They are restricted to the sets of vector functions of time which are spherically bounded by w_{dmax} and 1, respectively.

The set Z of admissible control laws and perturbation functions is defined by two conditions. One requiring the control to be spherically bounded everywhere on θ accounts for the fact that a practical controlling device has limited angular momentum exchange rate. The other requiring the time rate of change of the magnitude of angular velocity to be not greater than zero on the velocity boundary of θ forces the servo to remain in θ.

3. Global Description of Attitude Servos

Suppose that a definite admissible control law and perturbation function are given. The question whether the servo is fast enough for the mission requirements of the spacecraft may be resolved as follows.

It is a consequence of Euler's theorem on rotations that an angle (error angle φ) and an axis (error axis c) determine R. Thus, $R = \exp[\varphi S(c)]$. Conversely, $\varphi = \arccos[0.5(tr(R) - 1)]$, and for

$\varphi \in (0, \pi)$, $c = 0.5 \csc \varphi (r_{23} - r_{32}, r_{31} - r_{13}, r_{12} - r_{21})^t$. The error angle φ is a mathematically convenient and intuitively appealing scalar representation of attitude servo error, and forms a basis of the notion of a response envelope introduced next.

For fixed admissible control law and perturbation function the state equations determine the history $\underline{\varphi} = \varphi(t, \alpha, \underline{w}_d, \underline{\lambda})$, $t \geq 0$ of the error angle for each admissible initial state α, reference velocity $\underline{w}_d$, and perturbation variable $\underline{\lambda}$. The response envelope $\underline{\varphi}^{**}$ is defined as the envelope of all such histories. That is, $\underline{\varphi}^{**} = \varphi^{**}(t)$, $t \geq 0$, where

$$\varphi^{**}(t) = \max_{\alpha \in \theta} \left[\max_{\underline{w}_d \in \underline{W},\ \underline{\lambda} \in \underline{\Lambda}} \varphi(t, \alpha, \underline{w}_d, \underline{\lambda}) \right] \quad \text{fixed } t$$

An approximation, which is in the spirit of the Liapunov theory, of the response envelope may be obtained as follows. Consider the surface $V(\alpha) = v(t)$. The function $V(\alpha)$ is assumed to be given explicitly, to have a gradient with respect to α everywhere on θ, and to include θ for some finite $v(0)$. Let the expansion rate of the surface be defined by $\dot{v} = W(v)$, where

$$W(v) = \max_{\{\alpha \in \theta : V(\alpha) = v,\ \|w_d\| \leq w_{d\max},\ \|\lambda\| \leq 1\}} (\nabla_\alpha V(\alpha)\dot{\alpha})$$

An upper estimate $\underline{\varphi}_u$ of the response envelope $\underline{\varphi}^{**}$ such that $\varphi_u(t) \geq \varphi^{**}(t)$ for every $t \geq 0$, may be obtained by maximizing at each t the error angle φ on this moving surface thus,

$$\varphi_u(t) = \max_{\{\alpha \in \theta : V(\alpha) = v(t)\}} (\varphi)$$

4. A Set of Control Laws

Euler's theorem gives rise to the concepts of error angle and error axis. The first was used above to describe an attitude servo. The second is used next for control. The set of control laws having the form indicated by the following equation (1) is proposed

$$z(\alpha) = -J_a[g_1(\varphi)g_2(\|w_a\|^2)c + g_3(\varphi)w_a] \qquad (1)$$

Qualitatively, a control of this form may be thought as a generalization of the familiar position plus rate feedback used in one-dimensional servos. The first term in the brackets in equation (1) acts along the error axis; the second term acts along the velocity vector. The scalar functions g_1, g_2, and g_3 are restricted only by the two conditions defining Z.

An upper estimate of the response envelope corresponding to any control law of this form may be computed by means of the V-function given by the following equation.

$$V(\alpha) = \int_0^{\varphi} \left[g_1(x) + \mu g_3(x) \sin \frac{x}{2} \right] dx + \frac{1}{2} \int_0^{\|w_a\|^2} \frac{dx}{g_2(x)} + \mu \sin \frac{\varphi}{2} c^t w_a$$

μ is an adjustable parameter. The actual computation of an upper estimate may be performed in a three-dimensional space whose points are related to states in θ by the function

$$q(\alpha) = \left(\varphi, \|w_a\|^2, \sin \frac{\varphi}{2} c^t w_a \right)$$

5. Example

Consider the Orbiting Astronomical Observatory (OAO). It is controlled by means of three identical orthogonally placed reaction wheels. The ith component of z_a is the negative of the ith motor shaft torque. J_a is the moment of inertia of the spacecraft with locked wheels minus the moment of inertia of the wheels about their spin axes. In what follows, h_{max} and z_{max} will denote the bounds on angular momentum storage capacity of each wheel and torque capacity of each motor, respectively.

Let the spacecraft be controlled by (2), below, which is a special case of (1).

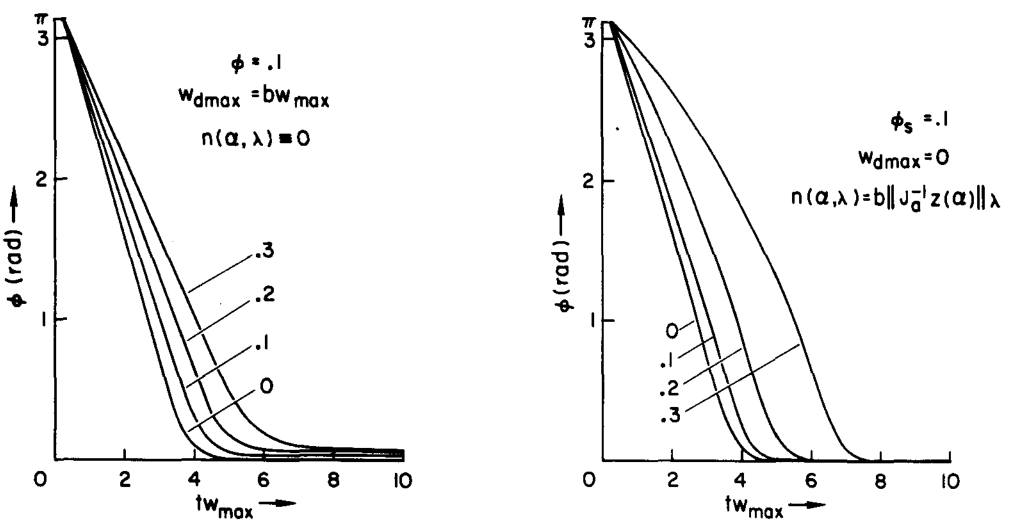
$$z(\alpha) = -J_a \frac{z_{max}}{2 J_{max}} \left[\operatorname{sat}(\varphi, \varphi_s) c + \frac{1}{w_{max}} w_a \right] \qquad (2)$$

sat(φ, φ_s) equals φ/φ_s for $\varphi \leq \varphi_s$ and 1 for $\varphi \geq \varphi_s$. The dimensionless constant $\varphi_s = (2h^2_{max})/(j_{max}z_{max})$. For OAO, $\varphi_s = 0.1$.

Upper estimates of several response envelopes were computed on a digital computer using the following V-function.

$$V(\alpha) = \int_0^{\varphi} \text{sat}(x, \varphi_s)dx + \frac{2}{\varphi_s}\left(1 - \cos\frac{\varphi}{2}\right) + \frac{1}{2}\varphi_s\left(\frac{w_a}{w_{max}}\right)^2 + \sin\frac{\varphi}{2}\, c^t\left(\frac{w_a}{w_{max}}\right)$$

The following plots show the results in dimensionless form. Consequently, they apply to any spacecraft, with $\varphi_s = 0.1$, which is controlled by (2).



(a) Nominal servo. (b) Spherical error in acceleration.

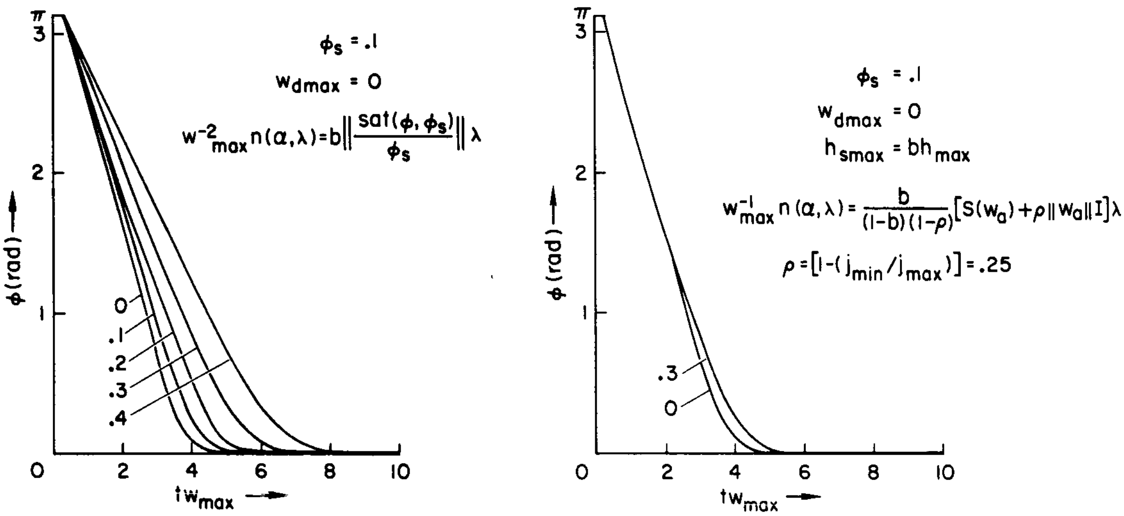
Fig. 2 Spacecraft response.

Figure 2a shows upper estimates of response envelopes for the nominal servo for various bounds w_{dmax} on angular velocity of the input. The case with $b = 0$ corresponds to the state regulator. The plots indicate that regardless of in which admissible state the servo is initially, the attitude error will be less than 6° after about 5 units of time (28 minutes for the OAO) even when the input varies arbitrarily with one quarter of maximum angular velocity allowed for the spacecraft.

Figure 2b is relevant to the design of angular momentum exchange and storage devices. For example, suppose that the spacecraft is to be controlled not by reaction wheels but by a set of control moment gyros in some configuration. If the angular momentum of each gyro is approximated by its spin momentum, the angular momentum stored in the complete device may be expressed as a function $h(x)$ of the gimbal angles x of the gyros. The control $z_a = -(\partial h/\partial x)\dot{x}$. Let the exchange law of the device be $\dot{x} = F(x)z(\alpha)$. If $z(\alpha)$ is given by (2), $\varphi_s = 0.1$, and the following inequality holds for all possible x, then the response envelope of the spacecraft is bounded from above by the curve $b = 0.3$.

$$\frac{j_{max}}{j_{min}} \left\| I - \left(\frac{\partial h}{\partial x}\right) F(x) \right\| = b \leq 0.3$$

Two reasons why b may be greater than zero are desired simplicity of $F(x)$ and failure of some gyros in the device.



(a) Spherical error in error axis. (b) Gyroscopic effects.

Fig. 3 Spacecraft response.

Part (a) of figure 3 is relevant to the design of attitude sensors. For example, let the attitude of the spacecraft be measured with a set of star trackers. The gimbal angles x may be expressed as a function $g(A_{as}, x_o)$ of the attitude of the spacecraft A_{as} and inertial coordinates x_o of the guide stars. Let the output y of the gimbal processor be some function $f(x, A_{ds}, x_o)$. Suppose that y is used in place of $\text{sat}(\varphi, \varphi_s)c$ in the control law (2). Then for $\varphi_s = 0.1$ the response envelope of the spacecraft is bounded from above by the curve $b = 0.4$ if the following inequality holds for all orthogonal R and all expected inputs and guide stars.

$$\left\| \frac{f(g(RA_{ds}, x_o), A_{ds}, x_o) - sat(\varphi, \varphi_s)c}{sat(\varphi, \varphi_s)} \right\| = b \leq 0.4$$

Two reasons why b may be greater than zero are the desired simplicity of the processor f and failure of some trackers.

Part (b) of figure 3 shows the influence of gyroscopic coupling arising from nonzero total angular momentum. The perturbation function $n(\alpha, \lambda)$ defined in the figure generates a set of vectors which always includes the gyroscopic terms. It is seen that in the case of the OAO gyroscopic coupling is not very significant even when the system is loaded with as much as 0.3 of its angular momentum storage capacity.